

## M.Sc. Qualification exam - 2010

Date: 16.04.10

Time: 3 hours

Answer **4 out of 6 questions**

**Important:** In the following table, circle the numbers of the four questions you chose to answer

Question	Grade
<b>1</b>	/25
<b>2</b>	/25
<b>3</b>	/25
<b>4</b>	/25
<b>5</b>	/25
<b>6</b>	/25

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### Question 1: Algorithms

1. For each of the two following questions, copy to the notebook the correct statement and justify your answer in 3-10 lines. In each question, 2 points will be given for selecting the correct statement and 4 points for the justification.

Remark: Only one statement is true in each question.

**1.1**  $G$  is a **directed acyclic graph**. Given are two vertices  $u, v$ . When  $\text{DFS}(v)$  is performed, the resulting spanning forest consists of two trees. When  $\text{DFS}(u)$  is performed, the resulting spanning forest consists of a single tree.

- There might be a topological sort in which  $v$  appears before  $u$ .
- In every topological sort,  $u$  appears before  $v$ .
- $G$  is strongly connected (for every two vertices  $a, b$ , there is a directed path from  $a$  to  $b$ ).

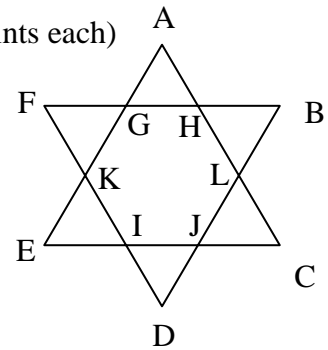
**1.2** Given is a **network flow**,  $G=(V,E)$ . It is known that the max-flow has value  $F > 0$ .

Let  $p$  be a directed path from the source,  $s$ , to the target,  $t$ . Let  $G'$  be the network flow produced from  $G$  by increasing by one all edge capacities along the path  $p$ . The max-flow in  $G'$

- Is exactly  $F+1$
- Might be  $F+2$
- Might remain  $F$ .

1. Answer the following questions. **Justify your answers.** (6 points each)

2.1. G is a Star-of-David graph with labeled vertices (see figure). Each of the 6 internal edges has weight 1. Each of the 12 outer edges has weight 2.



- What is the weight of a **minimum spanning tree** in G?
- How many different MSTs does G have?

2.2.  $T=(V,E)$  is a tree with  $|V|=100$  and  $|E|=99$ . It is known that T has an **Euler path**. Let  $v$  be a leaf in T. Assume that **BFS(v)** is performed.

Is it possible that two vertices  $a,b$  in T will be labeled such that  $label(a)=label(b)=2$  ?

## Question 2: Operating System

The processes P1, P2, P3 should run in the following order:

P1 starts and runs section work\_p1. Once P1 finished its work section, P2 and P3 run their sections (work\_p2 & work\_p3 respectively) and only after they both finish their work, P1 can start again running work\_p1 and the whole cycle starts over. This scenario repeats forever. Three different solutions were proposed. Express your opinion on each one of them (according to the following questions).

### Solution A

Semaphores S1\_2 and S1\_3 are initialized to 0, Semaphores S2, S3 are initialized to 1

#### Process P1

```
While(true){
  Down(S2)
  Down(S3)
  Work_p1
  Up(S1_2)
  Up(S1_3)
}
```

#### Process P2

```
While(true){
  Down(S1_2)

  Work_p2

  Up(S2)
}
```

#### Process P3

```
While(true){
  Down(S1_3)

  Work_p3

  Up(S3)
}
```

1. Does the solution solve the problem? Explain.
2. If not, can the solution lead to a deadlock?

### Solution B

Semaphores S1,S2,S3 are initialized to 0

#### Process P1

```
While(true){
  Work_p1
  Up(S2)
  Up(S3)
  Down(S1)
  Down(S1)
}
```

#### Process P2

```
While(true){
  Down(S2)

  Work_p2

  Up(S1)
}
```

#### Process P3

```
While(true){
  Down(S3)

  Work_p3

  Up(S1)
}
```

1. Does the solution solve the problem? Explain.
2. If not, can the solution lead to a deadlock?

### Solution C

Semaphores S1,S2,S3 are initialized to 0

```
Process P1  
While(true){  
  Work_p1  
  Up(S1)  
  Up(S1)  
  Down(S2)  
  Down(S3)  
}
```

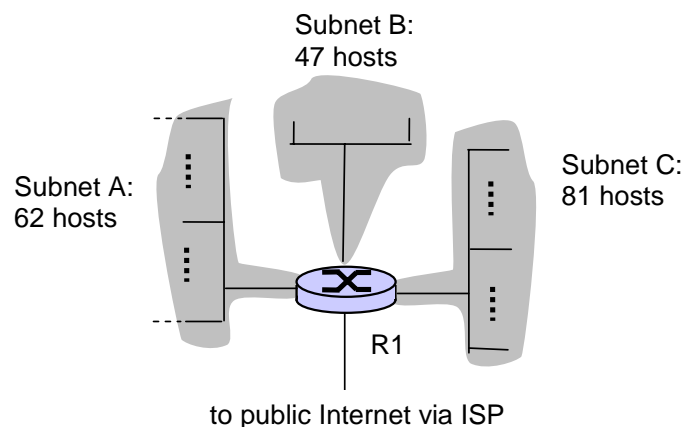
```
Process P2  
While(true){  
  Down(S1)  
  
  Work_p2  
  
  Up(S2)  
}
```

```
Process P3  
While(true){  
  Down(S1)  
  
  Work_p3  
  
  Up(S3)  
}
```

1. Does the solution solve the problem? Explain.
2. If not, can the solution lead to a deadlock?

### Question 3: Computer Networks

Consider the network shown below, consisting of a single router, R1, with three subnets A, B and C, with 62, 47, and 81 hosts respectively.



Starting from the address 413.37.241.0, assign an address range to the hosts in subnets A, B, and C such that only a single aggregated address need be advertised by R1 to the public Internet, and that the size of the aggregated address ranges that is advertised is minimized. The address 413.37.241.0 must be included in one of the subnets. Fill in the following table, and justify your answer.

Subnet	address -- a.b.c.d/x
A	
B	
C	
A+B+C	

## Question 4: Programming Languages

We wish to create a class called `CensoredStringTokenizer`. This class can be used to break a given text into tokens while eliminating tokens that are considered censored. Here is an example of a typical use of this class:

```
public class CensoredStringTokenizerDemo {
    public static void main(String[] args) {
        String text = "eat the apple or the banana but not the sushi";
        String[] censoredWords = {"the", "and" , "or", "not"};
        CensoredStringTokenizer cst =
            new CensoredStringTokenizer(text, censoredWords);
        while (cst.hasMoreTokens()) {
            System.out.println(cst.nextToken());
        }
    }
}
```

Running this program will yield the following result:

```
eat
apple
banana
but
sushi
```

1. (15 points) Design and implement in a Java-like language the `CensoredStringTokenizerDemo` class. You need to write a constructor, two public methods (you can tell which are needed from the example above), and – if you want – some private variables and methods – as per your judgement. The API of the public methods is something that you can figure out from the usage example above. As usual, we seek an elegant, minimal, and correct implementation. Your implementation can make use of the services of the `StringTokenizer` class, whose API is given below.
2. (5 points) How can we make sure that the `CensoredStringTokenizerDemo` class will perform efficiently even when the censored list includes thousands of words? There is no need to write code, just explain precisely what has to be done.
3. (5 points) Pick one object-oriented principle that comes to play in your implementation, and explain briefly its benefits.

## StringTokenizer API

```
public class StringTokenizer
    extends Object
    implements Enumeration<Object>
```

The string tokenizer class allows an application to break a string into tokens.

The set of delimiters (the characters that separate tokens) may be specified either at creation time or on a per-token basis.

## Constructor Summary

<code>StringTokenizer</code> ( <code>String</code> str)	Constructs a string tokenizer for the specified string.
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## Method Summary

int	<code>countTokens</code> () Calculates the number of times that this tokenizer's <code>nextToken</code> method can be called before it generates an exception.
boolean	<code>hasMoreElements</code> () Returns the same value as the <code>hasMoreTokens</code> method.
boolean	<code>hasMoreTokens</code> () Tests if there are more tokens available from this tokenizer's string.
Object	<code>nextElement</code> () Returns the same value as the <code>nextToken</code> method, except that its declared return value is <code>Object</code> rather than <code>String</code> .
String	<code>nextToken</code> () Returns the next token from this string tokenizer.
String	<code>nextToken</code> (String delim) Returns the next token in this string tokenizer's string.

### Question 5: Data structures

- Given an array of  $n$  elements, for each of the following cases suggest an efficient algorithm for computing in sorted order the  $k$  minimal values:
  - $k=5$
  - $k=n/2$
  - $k=\sqrt{n}$

For each case, give a short description of the algorithm and the analysis of the worst case complexity. Explain your answer.

- Let  $n=m^2$  be distinct elements. Let  $S_1, \dots, S_m$  be sets of elements, each consisting of  $m$  elements. Assume that  $\forall a \in S_i$  and  $\forall b \in S_j$   $a < b$  iff  $i < j$ . What is the best lower bound on the number of comparisons that a comparison-based algorithm needs to perform in order to sort the  $n$  elements given  $S_1, \dots, S_m$ ? Explain your answer.

### Question 6: Computability and Complexity

Prove all your answers

- Suppose that  $A \in \text{PSPACE}$ ,  $B \in \text{NP}$  and  $C \in \text{P}$ . Which of the following statements is necessarily correct (can be more than one)?

- a.  $A \in P$
- b.  $C \leq_p B$
- c.  $C \in PSPACE$

2. Suppose that  $A, B \in NP$ . Prove that  $A \cup B \in NP$ .

3. Consider the following languages:

$Ham = \{ \langle G \rangle \mid G \text{ is a graph with } k \text{ vertices that contains a simple cycle of length } k \}$

$Ham3Cycle = \{ \langle G \rangle \mid G \text{ is a graph with } k \text{ vertices such that } k=4n \text{ for some } n>2, G \text{ contains a simple cycle of length } k/2 \text{ and two simple cycles of length } k/4 \}$

- a. Is  $Ham \in NP$ ?
- b. Is  $Ham3Cycle \in PSPACE$ ?
- c. True or false: If  $Ham$  is NP-hard then  $Ham3Cycle$  is NP-complete.

**GOOD LUCK**

**For questions call:**

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