16-720A Computer Vision:

Homework 4 - 3D Reconstruction

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1.1 In the given figure:

$$\tilde{x}_1 = \tilde{x}_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

We know that:

$$\tilde{x}_2^T F \tilde{x}_1 = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow F_{33} = 0$$

Hence Proved.

1.2

$$t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = tR$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} t_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

Let $\tilde{x}_1^T = \begin{bmatrix} u_1 & u_2 & 1 \end{bmatrix}$

Let $\tilde{x}_2^T = \begin{bmatrix} v_1 & v_2 & 1 \end{bmatrix}$

Then, $l_1^T = \tilde{x}_2^T E$

$$= \begin{bmatrix} v_1 & v_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & t_1 & -v_2 t_1 \end{bmatrix}$$

Epipolar line in Camera 1 $\rightarrow t_1 y_1 - v_2 t_1 = 0$ — (1)

$$l_2^T = \tilde{x}_1^T E$$

$$= \begin{bmatrix} u_1 & u_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_1 \\ 0 & -t_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -t_1 & u_2 t_1 \end{bmatrix}$$

Epipolar line in Camera 2 $\rightarrow -t_1 y_2 + u_2 t_1 = 0$ — (2)

Since eqns (1) & (2) don't have any 'x' component, thus they are parallel.

1.3 If $[x \; y \; z]^T$ is the coordinate of the object in the 3D world & $[u_i \; v_i]^T$ is the position at time 'i',

At time t_1:
$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = K\left(R_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} + t_1\right)$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_1^{-1}\left(K^{-1}\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} - t_1\right)$$

$$= R_1^T K^{-1}\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} - R_1^T t_1 \quad \text{— (1)}$$

At time t_2:
$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = K\left(R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} + t_2\right)$$

(Replacing from (1)) $$= K\left(R_2\left(R_1^T K^{-1}\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} - R_1^T t_1\right) + t_2\right)$$

$$= KR_2R_1^TK^{-1}\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} - KR_2R_1^Tt_1 + Kt_2$$

Now thus,

$R_{rel} = KR_2R_1^TK^{-1}$

$t_{rel} = -KR_2R_1^Tt_1 + Kt_2$

$$E = t_{rel} \times R_{rel}$$

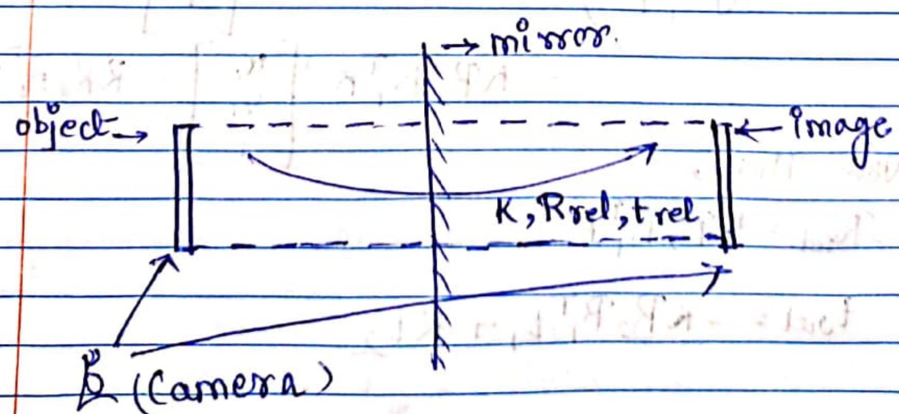
$$F = (K^{-1})^T E K^{-1}$$

$$\Rightarrow F = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

1.4 Assuming all pts on an object are equal distance to the mirror, when a camera views the object & its reflection in the plane mirror, the transformation between the object & its reflection is pure translation.

$$R_{rel} = I$$

$$t_{rel} = [t_x, t_y, t_z]$$



$$F = (K^{-1})^T E K^{-1} = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

$$\Rightarrow F = (K^{-1})^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1}$$

$$F^T = (K^{-1})^T \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} K^{-1}$$

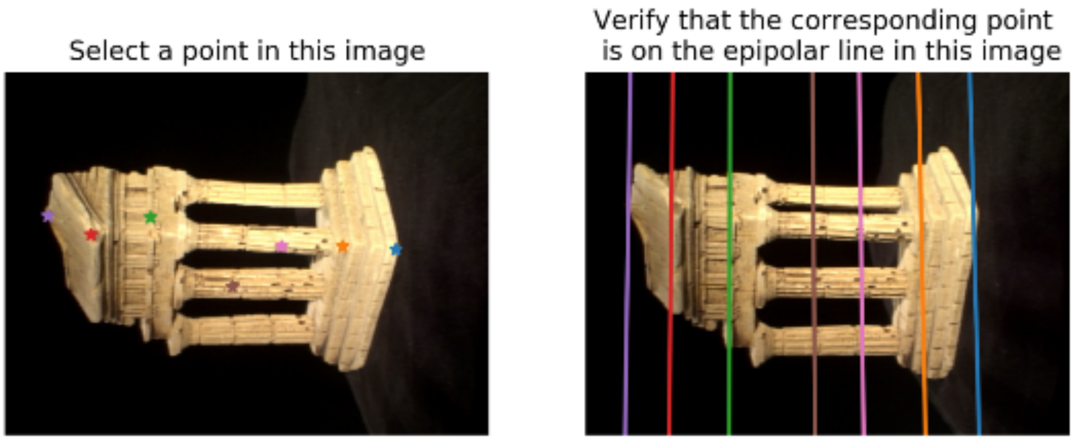
$\Rightarrow \boxed{F^T = -F}$ $\longrightarrow$ Thus the fundamental matrix is <u>skew symmetric</u>

Q).2.1. The recovered **F** Matrix is :

$$F = \begin{bmatrix} 9.80213863e-10 & -1.32271663e-07 & 1.12586847e-03 \\ -5.72416248e-08 & 2.97011941e-09 & -1.17899320e-05 \\ -1.08270296e-03 & 3.05098538e-05 & -4.46974798e-03 \end{bmatrix}$$

[[9.80213863e-10 -1.32271663e-07 1.12586847e-03]
[-5.72416248e-08 2.97011941e-09 -1.17899320e-05]
[-1.08270296e-03 3.05098538e-05 -4.46974798e-03]]

The image below is the visualization :



Q)3.1. The **Essential Matrix** computed from F is :

E = [[2.26587821e-03 -3.06867395e-01 1.66257398e+00]
[-1.32799331e-01 6.91553934e-03 -4.32775554e-02]
[-1.66717617e+00 -1.33444257e-02 -6.72047195e-04]]

Q) 3.2

3.2 We are given two Camera Matrices

For Camera C1:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} — C_{11}^T — \\ — C_{12}^T — \\ — C_{13}^T — \end{bmatrix} X = 0$$

$$\Rightarrow \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} C_{11}^T X \\ C_{12}^T X \\ C_{13}^T X \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} y_1 C_{13}^T X - C_{12}^T X \\ C_{11}^T X - x_1 C_{13}^T X \\ x_1 C_{12}^T X - y_1 C_{11}^T X \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

→ Since 3rd eqn is linear combn of first two, we avoid it!

$$\therefore \begin{bmatrix} y_1 C_{13}^T X - C_{12}^T X \\ C_{11}^T X - x_1 C_{13}^T X \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2)$$

Similarly for camera 2:

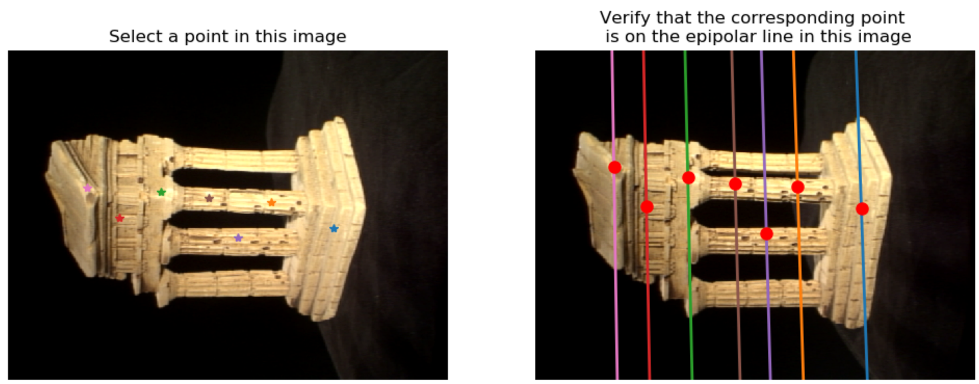
$$\begin{bmatrix} y_2 C_{23}^T X - C_{22}^T X \\ C_{21}^T X - x_2 C_{23}^T X \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3)$$

We concatenate eq$^{\text{ns}}$ (2) & (3) :

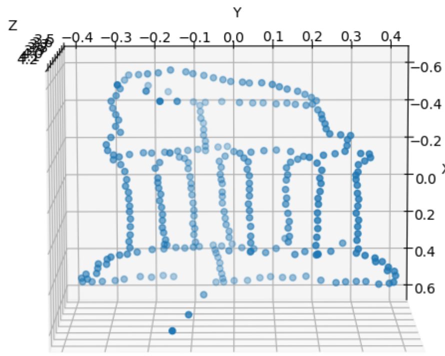
$$\begin{bmatrix} y_1 C_3^T - C_2^T \\ C_1^T - x_1 C_3^T \\ y_2 C_3^T - C_2^T \\ C_1^T - x_2 C_3^T \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

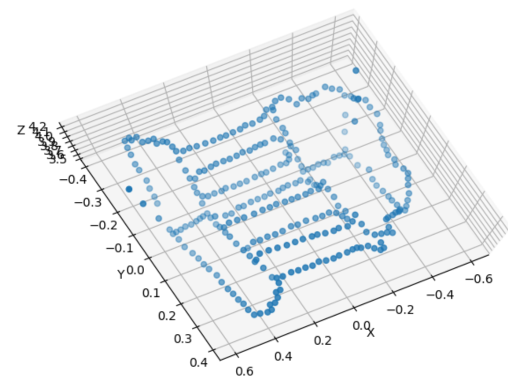
$$\therefore \quad A = \begin{bmatrix} y_1 C_{13}^T - C_{12}^T \\ C_{11}^T - x_1 C_{13}^T \\ y_2 C_{23}^T - C_{22}^T \\ C_{21}^T - x_2 C_{23}^T \end{bmatrix}$$

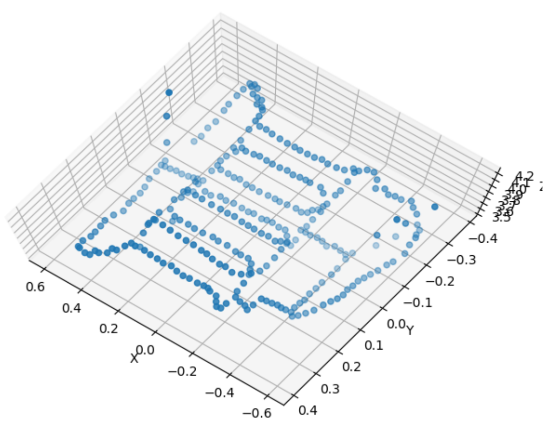
Q) 4.1. The following is a screenshot of epipolarMatchGUI with some detected correspondences :

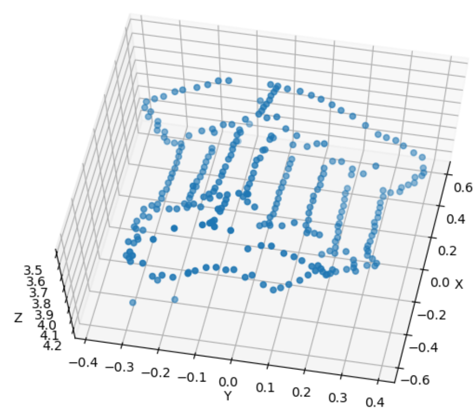


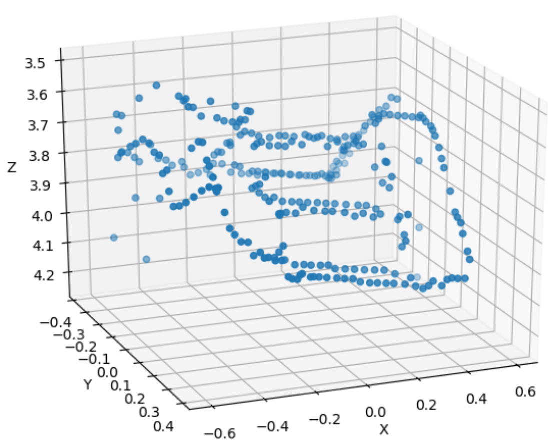
Q) 4.2. Following are 3D Visualizations of the reconstructed temple:











Q) 5.1. While finding the Fundamental Matrix with RANSAC with the eight point algorithm, when tolerance value is increased the no of inliers increases. Also the Fundamental Matrix obtained has the same values except that the values at every entry are negated. After 1000 iterations at tolerance value=0.42, with RANSAC, 108 inliers are obtained.

Q) 5.3. The image of the original 3D points and the optimized points with the initial M2 and P are shown below. The reprojection error initially is 59.67and the reprojection error after optimization is 6.597