15.1 Quickselect

Quickselect is an algorithm that selects the k^{th} smallest element in a list. Ex: Running quickselect on the list (15, 73, 5, 88, 9) with k = 0, returns the smallest element in the list, or 5.

For a list with N elements, quickselect uses quicksort's partition function to partition the list into a low partition containing the X smallest elements and a high partition containing the N-X largest elements. The k^{th} smallest element is in the low partition if k is ≤ the last index in the low partition, and in the high partition otherwise. Quickselect is recursively called on the partition that contains the k^{th} element. When a partition of size 1 is encountered, quickselect has found the k^{th} smallest element.

Quickselect partially sorts the list when selecting the k^{th} smallest element.

The best case and average runtime complexity of quickselect are both O(N). In the worst case, quickselect may sort the entire list, resulting in a runtime of O(N^2).

Figure 15.1.1: Quickselect algorithm.

```
// Selects kth smallest element, where k is 0-based
Quickselect(numbers, first, last, k) {
   if (first >= last)
      return numbers[first]

   lowLastIndex = Partition(numbers, first, last)

   if (k <= lowLastIndex)
      return Quickselect(numbers, first, lowLastIndex, k)
   return Quickselect(numbers, lowLastIndex + 1, last, k)
}
```

PARTICIPATION ACTIVITY 15.1.1: Quickselect.

1) Calling quickselect with argument k equal to 1 returns the smallest element in the list.
   - ○ True
   - ○ False

2) The following function produces the same result as quickselect, albeit with

a different runtime complexity.

```
Quickselect(numbers, first, last,
k) {
   Quicksort(numbers, first, last)
   return numbers[k]
}
```

- True
- False

3) Given k = 4, if the quickselect call `Partition(numbers, 0, 10)` returns 4, then the element being selected is in the low partition.

- True
- False

CHALLENGE ACTIVITY 15.1.1: Quickselect.

361856.2187296.qx3zqy7

Start

What is returned when running quickselect on (74, 43, 28, 52, 91) with k = 3?

Ex: 25

1 2 3

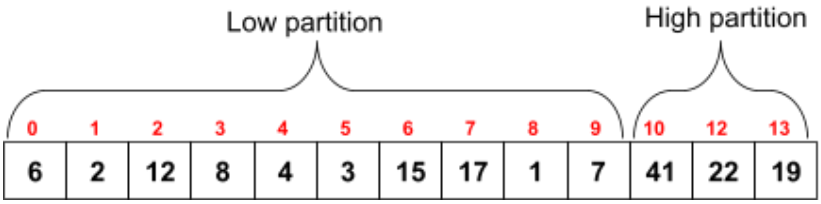
Check Next

15.2 Java: Quickselect

Java: Quickselect

The Quickselect algorithm finds the k^{th} smallest item in an array. The quickselect() method uses the same partition() method as quicksort(). partition() selects a pivot and creates a low partition ≤ the pivot and a high partition ≥ the pivot. quickselect() and quicksort() are also both recursive, but unlike quicksort(), quickselect() is recursively applied only to one of the partitions.

Figure 15.2.1: An array after being partitioned into low and high partitions.



The example above shows an array after the partition() method is called. To find the 7th smallest value in the array (that is, $k = 7$), the value must appear in the low partition, because the low partition holds the 10 smallest values. Similarly, the 11th smallest value has to be one of the three values in the high partition. The quickselect() method partitions the array and then recursively calls quickselect() on whichever partition the k^{th} element is in. The recursion ends when the current partition has only one element, and that element has to be the k^{th} element that is being searched for.

Figure 15.2.2: quickselect() method.

```
int quickselect(int[] numbers, int startIndex, int endIndex, int k) {
   if (startIndex >= endIndex) {
      return numbers[startIndex];
   }

   int lowLastIndex = partition(numbers, startIndex, endIndex);
   if (k <= lowLastIndex) {
      return quickselect(numbers, startIndex, lowLastIndex, k);
   }
   return quickselect(numbers, lowLastIndex + 1, endIndex, k);
}
```

PARTICIPATION ACTIVITY 15.2.1: quickselect() method.

Which action will the quickselect() method take for each condition?

1) numbers: { 6, 2, 12, 8, 4, 3, 19, 17, 22, 41, 7, 1, 15 }
startIndex: 0
endIndex: 12
k: 5
The first action is:

- partition(numbers, 0, 12);
- partition(numbers, 5, 12);
- quickselect(numbers, 0, 5, 5);
- quickselect(numbers, 5, 12, 5);

2) numbers: { 1, 2, 3, 4, 8, 12, 7, 6, 17, 15, 41, 22, 19 }
startIndex: 4
endIndex: 7
k: 5
lowLastIndex: 6
After partition(numbers, 4, 7) returns:

- quickselect(numbers, 4, 7, 5);
- quickselect(numbers, 4, 6, 5);
- quickselect(numbers, 6, 7, 5);

3) numbers: { 6, 2, 12, 8, 4, 3, 15, 17, 1, 7, 41, 22, 19 }
startIndex: 0
endIndex: 9
k: 5
lowLastIndex: 3
After partition(numbers, 0, 9) returns:

- quickselect(numbers, 0, 3, 5);
- quickselect(numbers, 0, 3, 3);
- quickselect(numbers, 4, 9, 5);

zyDE 15.2.1: Working with quickselect().

The following program uses quickselect() to find the k^{th} smallest item. The value of k is from user input.

Run the program with some different values for k. Observe the following:

- The quickselect() method always returns the k^{th} smallest item. The sorted array shown at the end so the value of the k^{th} smallest item can be confirmed.
- The array is partially sorted after calling quickselect(). Which partitions become so depends on the value of k. Try values k=3, k=7, and k=11.

QuickselectDemo.java Load default templ

```
1  import java.util.Arrays;
2  import java.util.Scanner;
3
4  public class QuickselectDemo {
5     private static int partition(int[] numbers, int startIndex, int endInde
6        // Select the middle value as the pivot.
7        int midpoint = startIndex + (endIndex - startIndex) / 2;
8        int pivot = numbers[midpoint];
9
10       // "low" and "high" start at the ends of the array segment
11       // and move towards each other.
12       int low = startIndex;
13       int high = endIndex;
14
15       boolean done = false;
16       while (!done) {
17          // Increment low while numbers[low] < pivot
```

5

Run

15.3 Bucket sort

Bucket sort is a numerical sorting algorithm that distributes numbers into buckets, sorts each bucket with an additional sorting algorithm, and then concatenates buckets together to build the sorted result. A ***bucket*** is a container for numerical values in a specific range. Ex: All numbers in the range 0 to 49 may be stored in a bucket representing this range. Bucket sort is designed for arrays with non-negative numbers.

Bucket sort first creates a list of buckets, each representing a range of numerical values. Collectively, the buckets represent the range from 0 to the maximum value in the array. For N buckets and a maximum value of M, each bucket represents $\frac{M+1}{N}$ values. Ex: For 10 buckets and a maximum value of 49, each bucket represents a range of $\frac{49+1}{10}$ = 5 values; the first bucket will hold values ranging from 0 to 4, the second bucket 5 to 9, and so on. Each array element is placed in the appropriate bucket. The bucket index is calculated as $\left\lfloor number * \frac{N}{M+1} \right\rfloor$. Then, each bucket is sorted with an additional sorting algorithm. Lastly, all buckets are concatenated together in order, and copied to the original array.

Figure 15.3.1: Bucket sort algorithm.

```
BucketSort(numbers, numbersSize, bucketCount) {
   if (numbersSize < 1)
      return

   buckets = Create list of bucketCount buckets

   // Find the maximum value
   maxValue = numbers[0]
   for (i = 1; i < numbersSize; i++) {
      if (numbers[i] > maxValue)
         maxValue = numbers[i]
   }

   // Put each number in a bucket
   for each (number in numbers) {
      index = floor(number * bucketCount / (maxValue + 1))
      Append number to buckets[index]
   }

   // Sort each bucket
   for each (bucket in buckets)
      Sort(bucket)

   // Combine all buckets back into numbers list
   result = Concatenate all buckets together
   Copy result to numbers
}
```


PARTICIPATION ACTIVITY 15.3.1: Bucket sort.

Suppose BucketSort is called to sort the list (71, 22, 99, 7, 14), using 5 buckets.

1) 71 and 99 will be placed into the same bucket.
   - True
   - False

2) No bucket will have more than 1 number.
   - True
   - False

3) If 10 buckets were used instead of 5, no bucket would have more than 1 number.
   - True
   - False

Bucket sort terminology

The term "bucket sort" is sometimes used to refer to a category of sorting algorithms, instead of a specific sorting algorithm. When used as a categorical term, bucket sort refers to a sorting algorithm that places numbers into buckets based on some common attribute, and then combines bucket contents to produce a sorted array.

15.4 List data structure

A common approach for implementing a linked list is using two data structures:

1. List data structure: A ***list data structure*** is a data structure containing the list's head and tail, and may also include additional information, such as the list's size.
2. List node data structure: The list node data structure maintains the data for each list element, including the element's data and pointers to the other list element.

A list data structure is not required to implement a linked list, but offers a convenient way to store the list's head and tail. When using a list data structure, functions that operate on a list can use a single parameter for the list's data structure to manage the list.

A linked list can also be implemented without using a list data structure, which minimally requires using separate list node pointer variables to keep track of the list's head.

PARTICIPATION ACTIVITY 15.4.1: Linked lists can be stored with or without a list data structure.

Animation content:

undefined

Animation captions:

1. A linked list can be maintained without a list data structure, but a pointer to the head and tail of the list must be stored elsewhere, often as local variables.
2. A list data structure stores both the head and tail pointers in one object.

PARTICIPATION ACTIVITY 15.4.2: Linked list data structure.

1) A linked list must have a list data structure.
   - ○ True
   - ○ False

2) A list data structure can have additional information besides the head and tail pointers.
   - ○ True
   - ○ False

3) A linked list has O(n) space complexity, whether a list data structure is used or not.
   - ○ True
   - ○ False

15.5 Circular lists

A ***circular linked list*** is a linked list where the tail node's next pointer points to the head of the list, instead of null. A circular linked list can be used to represent repeating processes. Ex: Ocean water evaporates, forms clouds, rains down on land, and flows through rivers back into the ocean. The head of a circular linked list is often referred to as the *start* node.

A traversal through a circular linked list is similar to traversal through a standard linked list, but must terminate after reaching the head node a second time, as opposed to terminating when reaching null.

PARTICIPATION ACTIVITY 15.5.1: Circular list structure and traversal.

Animation content:

undefined

Animation captions:

1. In a circular linked list, the tail node's next pointer points to the head node.
2. In a circular doubly-linked list, the head node's previous pointer points to the tail node.
3. Instead of stopping when the "current" pointer is null, traversal through a circular list stops when current comes back to the head node.

PARTICIPATION ACTIVITY 15.5.2: Circular list concepts.

1) Only a doubly-linked list can be circular.
   - True
   - False

2) In a circular doubly-linked list with at least 2 nodes, where does the head node's previous pointer point to?
   - List head
   - List tail
   - null

3) In a circular linked list with at least 2 nodes, where does the tail node's next pointer point to?

- List head
- List tail
- null

4) In a circular linked list with 1 node, the tail node's next pointer points to the tail.

- True
- False

5) The following code can be used to traverse a circular, doubly-linked list in reverse order.

```
CircularListTraverseReverse(tail)
{
   if (tail is not null) {
      current = tail
      do {
         visit current
         current =
current⇢previous
      } while (current != tail)
   }
}
```

- True
- False