Applications of Discrete Mathematics in Computing: Forbidden Positions, Shortest Paths, and Apportionment

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ABSTRACT

The purpose of this study is to demonstrate select applications of discrete mathematics in computer science. In particular, we consider computing arrangements given forbidden positions, finding the shortest path between two points in a network, and the apportionment problem. While these problems are not novel, they are foundational to many tangible, real-world problems such as scheduling, the traveling salesman problem, and gerrymandering.

FORBIDDEN POSITIONS

The forbidden positions problem is concerned with counting arrangements of objects where there are restrictions in object positioning. For example, we may wish to make a schedule for a workplace where some employees cannot work certain times; or, we may wish to program a chess-playing robot that beats all but the world's best players. To solve this problem we use Rook polynomials, which generate the number of ways to place non-attacking rook chess pieces on an mxn-sized board.

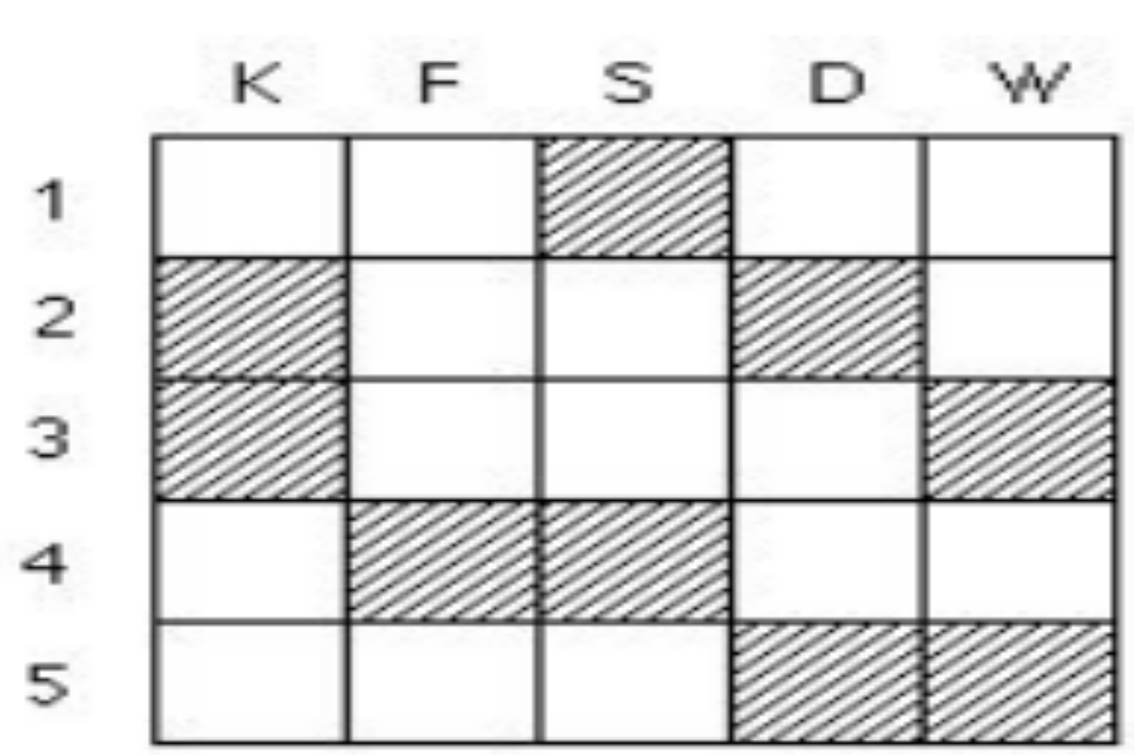

Figure A. This is an example of a board with forbidden positions that could be filled using Rook polynomials. The shaded pieces are the forbidden spaces. In this example, K cannot be placed either in row 2 or 3. Given this information, should K be placed in row 1, 4, or 5; assuming K,F,S,D, and W share no row?

SHORTEST PATHS

In the shortest path problem, we are interested in finding the shortest path between points in a network. A modern example of this would be how to transmit a message between two points along a network of thousands of computers. This is extendable to other problems as well where we want to minimize a given quantity. Dijkstra's algorithm was designed to find the shortest path in a weighted graph between two points, β and δ. This is done by initially labeling each vertex other than β with ∞, labeling β with 0, and then modifying the labels as shortest paths within the graph. These labels are temporary; they become permanent when it becomes apparent that a label could never become smaller.

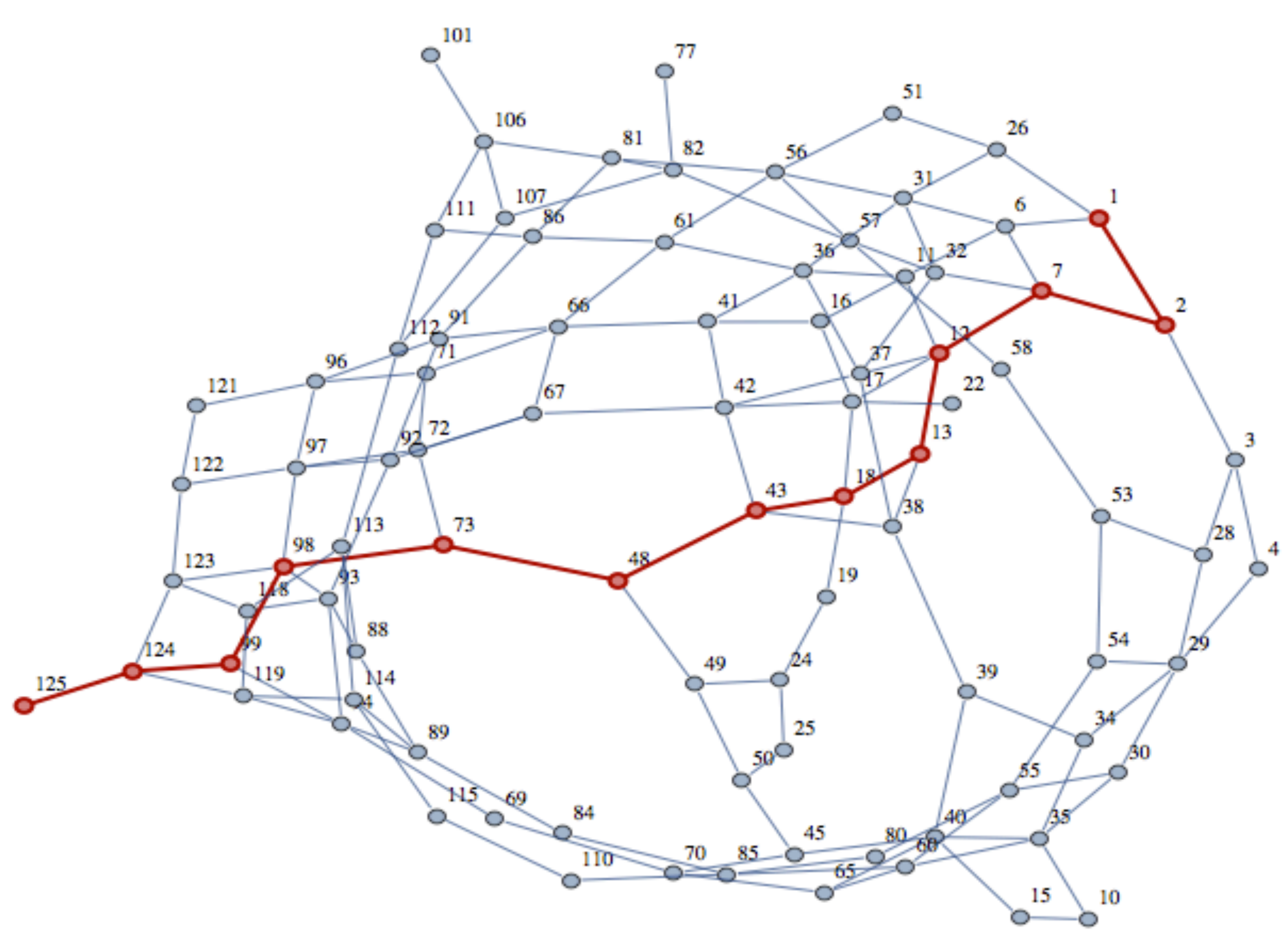

Figure B. This is a weighted graph showing the shortest (least expensive) path from the largest to the smallest weighted vertex.

Efficient algorithms for finding the shortest path from β to δ need only keep track of the vertex from which the shortest path entered. One example that does this is Hedetniemi's Algorithm.

APPORTIONMENT PROBLEM

The apportionment problem deals with how to "fairly" divide discrete objects among competing demands. One famous example of this is how to divide the number of seats in the U.S. House of Representatives to states. The crux of this problem is that although proportional representation usually requires that states have a fractional number of representatives, an integral number must be assigned. Since 1941, the Huntington Sequential Method has been used to apportion the House. This algorithm works by starting with no seats assigned to each state, and then sequentially assigning each seat to the "most deserving" state.

STATE	CURRENT	PROJECTED	NET
AL	7	9	+2
AK	1	2	+1
AZ	9	12	+3
AR	4	6	+2
CA	53	SECEDED	-53
CO	7	10	+3
CT	5	7	+2
DE	1	2	+1
FL	27	37	+10
GA	14	19	+5
HI	2	3	+1
ID	2	3	+1
IL	18	24	+6
IN	9	13	+4
IA	4	6	+2
KS	4	6	+2
KY	6	8	+2
LA	6	9	+3
ME	2	3	+1
MD	8	11	+3
MA	9	13	+4
MI	14	19	+5
MN	4	6	+2
MS	8	11	+3
MO	1	2	+1

STATE	CURRENT	PROJECTED	NET
MT	1	2	+1
NE	3	4	+1
NV	4	6	+2
NH	2	3	+1
NJ	12	17	+5
NM	3	4	+1
NY	27	39	+12
NC	13	19	+6
ND	1	2	+1
OH	16	22	+6
OK	5	7	+2
OR	5	8	+3
PA	18	24	+6
RI	2	3	+1
SC	7	9	+2
SD	1	2	+1
TN	9	12	+3
TX	36	52	+16
UT	4	6	+2
VT	1	1	+0
VA	11	16	+5
WA	10	14	+4
WV	3	4	+1
WI	8	11	+3
WY	1	1	+0

Figure C. This shows how each of the state's number of House Representatives would change if California seceded based on July 2016 population projections.

REFERENCES

[1] J. Michaels and K. Rosen, *Applications of Discrete Mathematics*, 1st ed. New York [etc.]: McGraw-Hill, 1992, pp. 2-18, 158-173, 187-202.