Experiment 1a

Zhe Sha

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1 Introduction

In this report we present results of Experiment 1a that focuses on updating a uni-variate process with single source of observations. The main purpose is to develop a complete implementation of the Bayesian hierarchical model framework on the sphere that is simple enough for debugging and tuning (model or algorithm related) parameters.

In Experiment 1a, we test on the GIA process with synthetic GPS data. Over the period of 2005 to 2015, we assume the GIA process is a time invariant spatial process that represent a yearly trend in mm. The GPS data are processed to reflect only the vertical movement of the earth, also in mm/yr. Denote the GIA process by X_{GIA} and the GPS observation by Y_{GPS}, then the Bayesian hierarchical model can be written as

$$\begin{cases} \boldsymbol{Y}_{GPS}|\boldsymbol{X}_{GIA}, \boldsymbol{e} = \boldsymbol{A}\boldsymbol{X}_{GIA} + \boldsymbol{e} \\ \boldsymbol{X}_{GIA}|\boldsymbol{\mu}_{GIA}, \boldsymbol{Z}_{GIA} = \boldsymbol{\mu}_{GIA} + \boldsymbol{Z}_{GIA} \\ \boldsymbol{Z}_{GIA}|\rho, \sigma^2 \sim \mathcal{GP}(0, K_\nu(\cdot, \cdot\,|\rho, \sigma^2)),\ \boldsymbol{e} \sim \mathcal{N}(0, \sigma_e^2 \boldsymbol{I}) \end{cases} \tag{1.1}$$

where A is the linear mapping operator from the process to the observations, $\boldsymbol{e}$ is a vector of the measurement errors of the GPS data, $\boldsymbol{\mu}_{GIA}$ is the prior GIA mean field which can usually be derived from some physical models, $\boldsymbol{Z}_{GIA}$ is a stationary zero mean Gaussian spatial process with Matérn covariance function, and $\rho, \sigma^2, \sigma_e^2$ are the hyper-parameters to be estimated.

2 Priors for the hyper parameters

2. Transformation of the hyper parameter and choosing the prior

3 Test error size

3. Test the "Principle of stable inference" theorem

3.1 Change error size of a single point

Compare the effect of error size using a single isolated point.

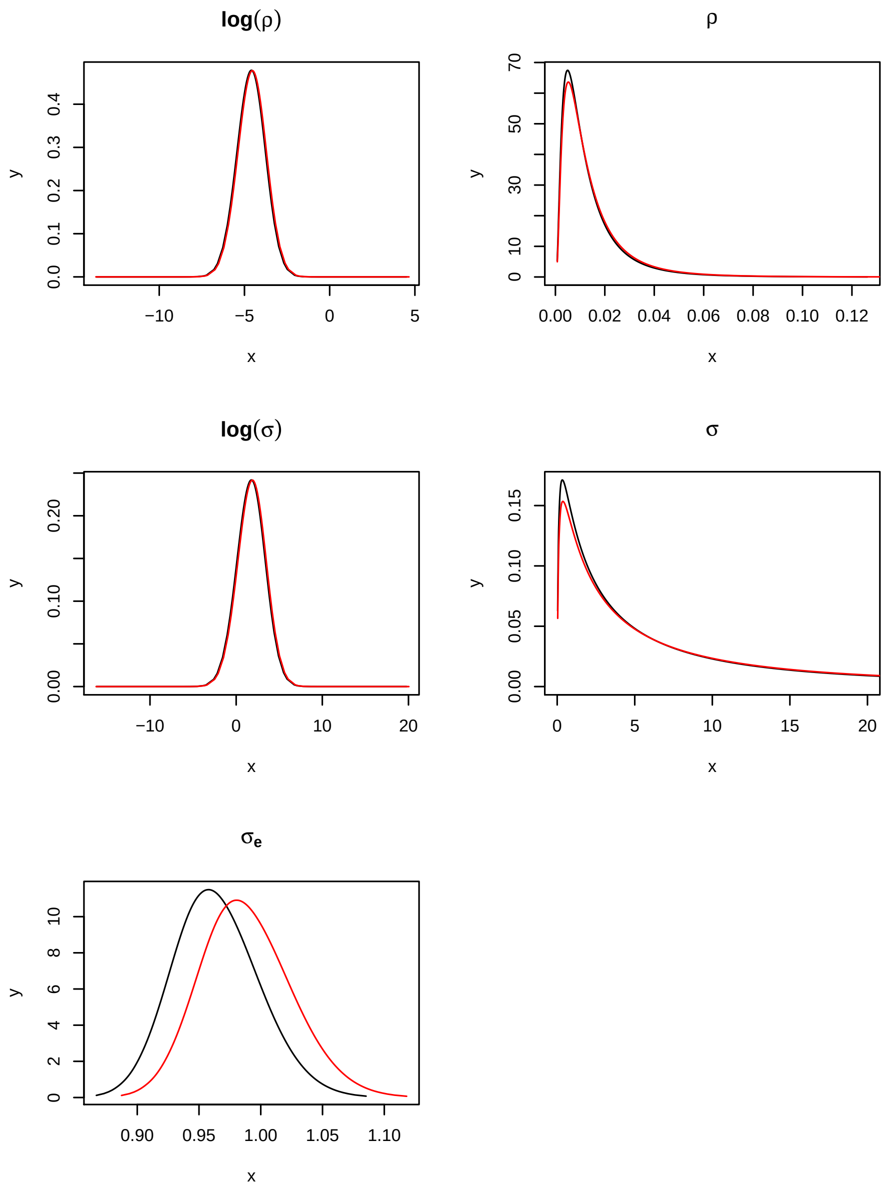


Figure 1: Posteriors of the hyper parameters using different error size for a selected point. Black: small error, Red: large error.

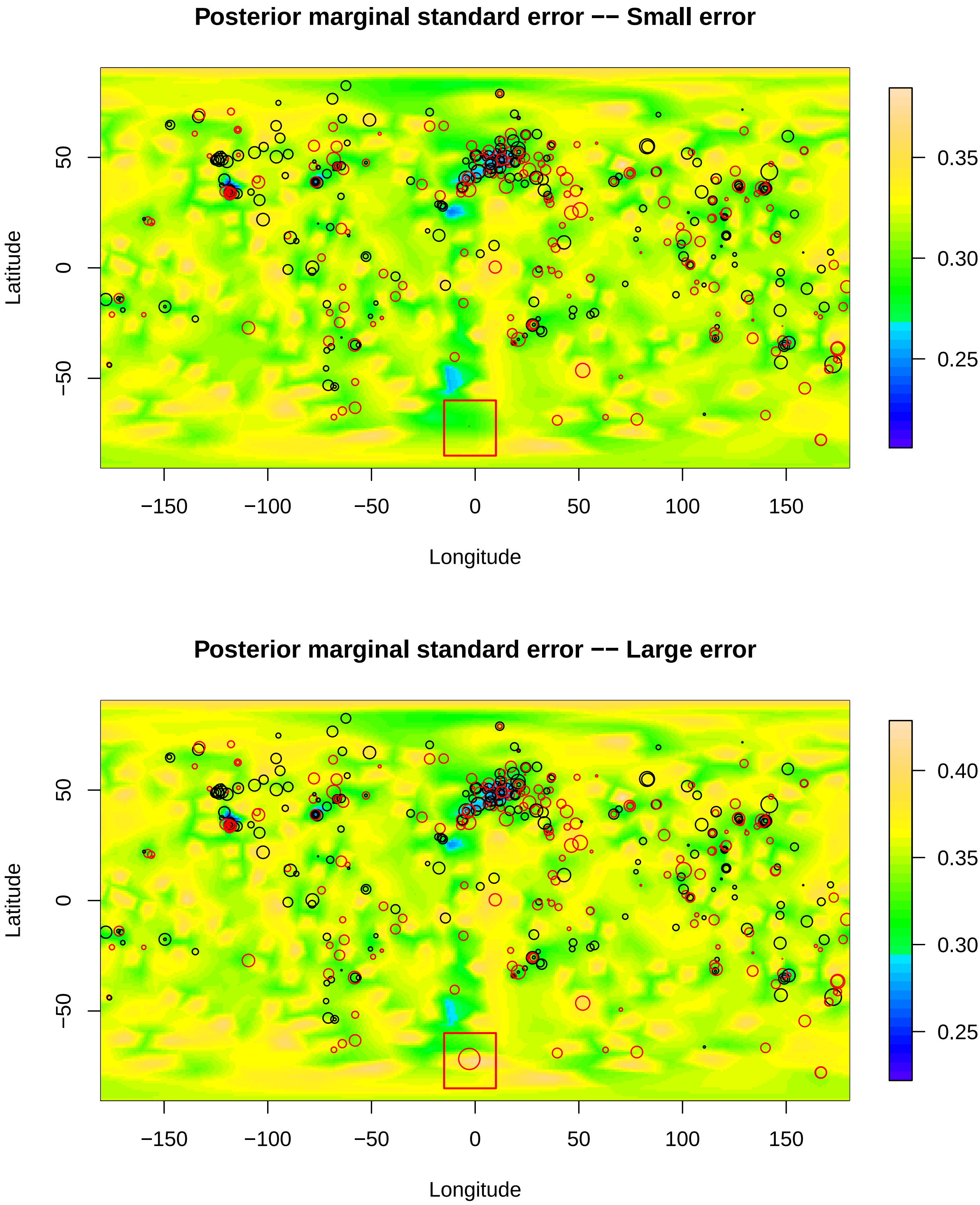


Figure 2: Posteriors marginal standard errors of the latent field using GPS data with different error on a single selected point. The selected point is marked as the circle within the red rectangle. Points are the GPS locations. Red points are positive errors and black negative. The circle size in the bottom plot represents the error size.

3.2 Change global error size

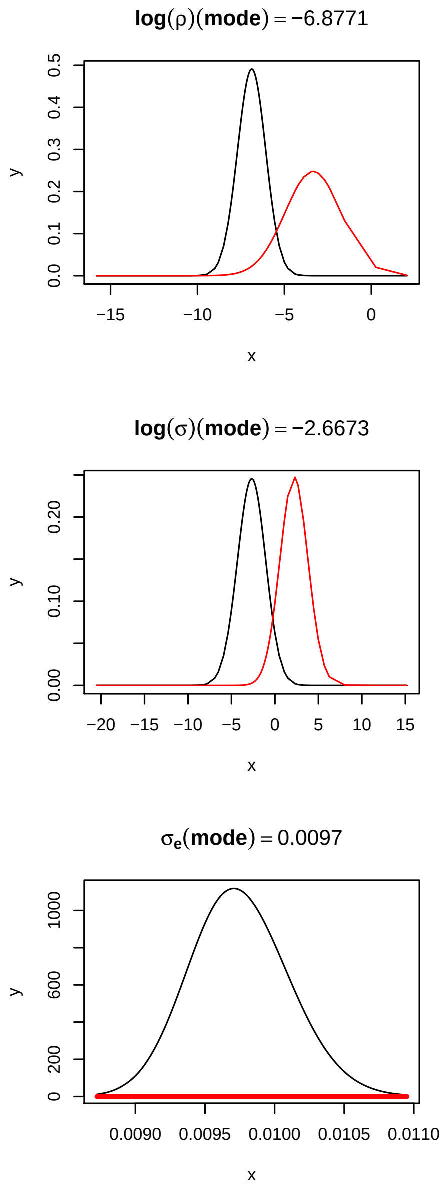


Figure 3: Posteriors of hyper parameters using GPS data with small measurement errors. Black: posterior, Red: prior.

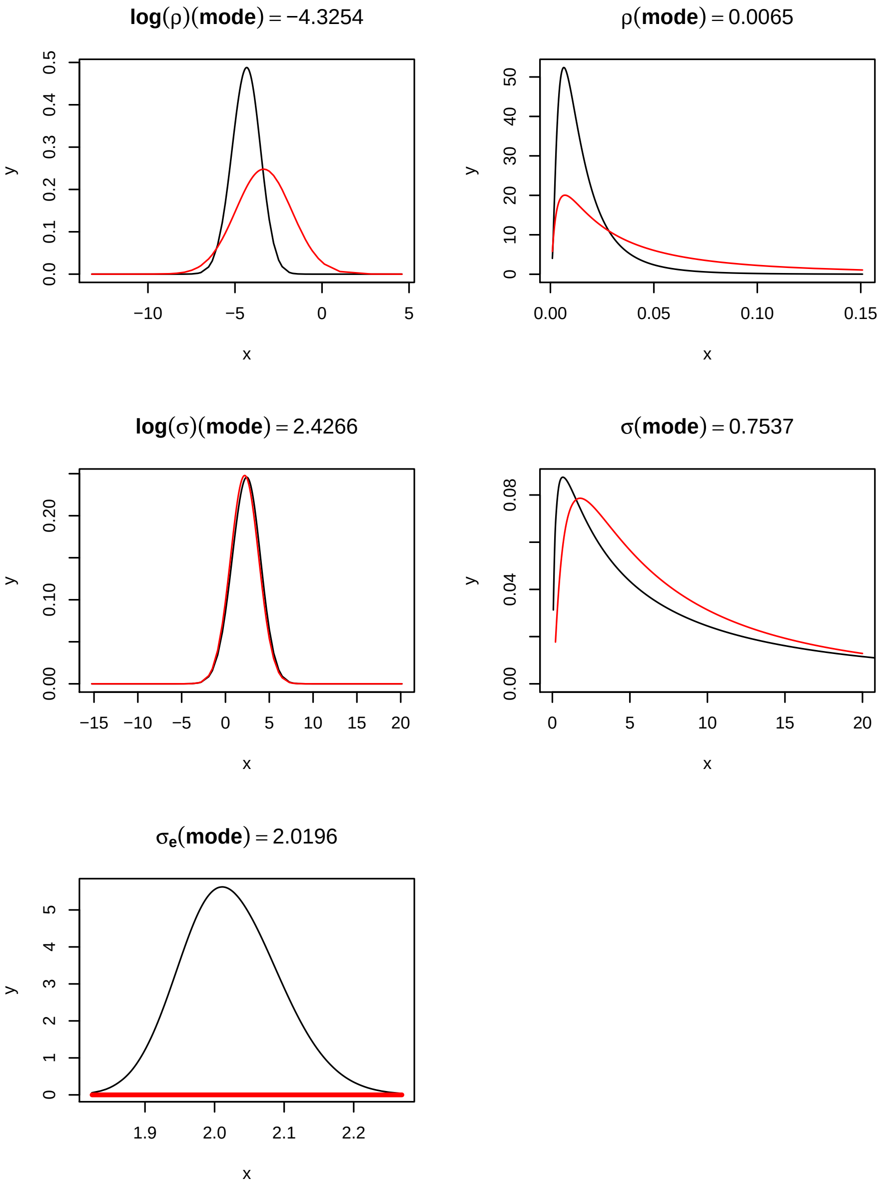
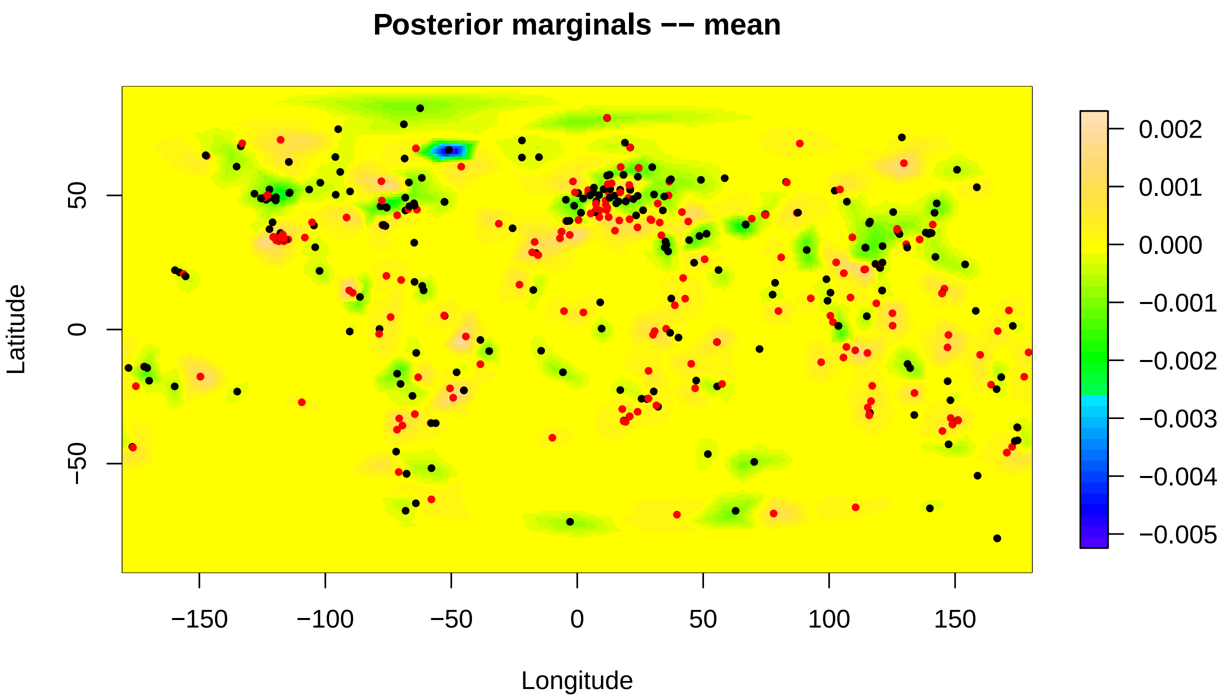


Figure 4: Posteriors of hyper parameters using GPS data with large measurement errors. Black: posterior, Red: prior.



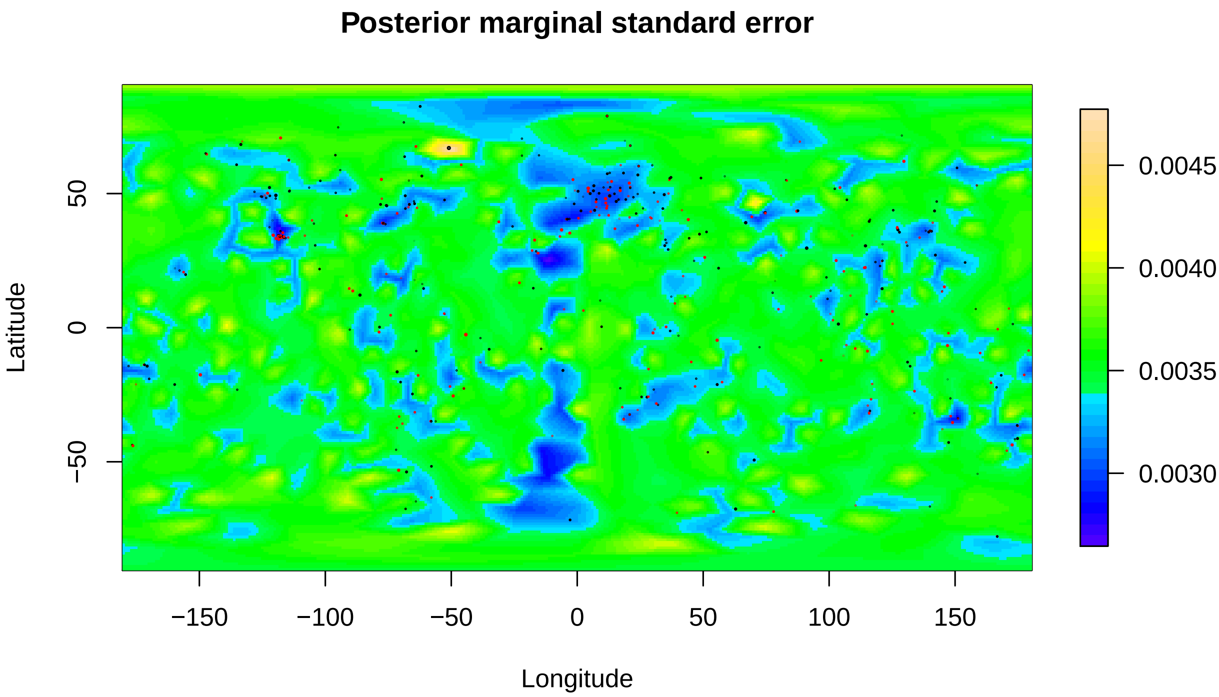


Figure 5: Posteriors marginal means and standard errors of the latent field using GPS data with small measurement errors. Points are the GPS locations. Red points are positive errors and black negative. The circle size in the bottom plot represents the error size.

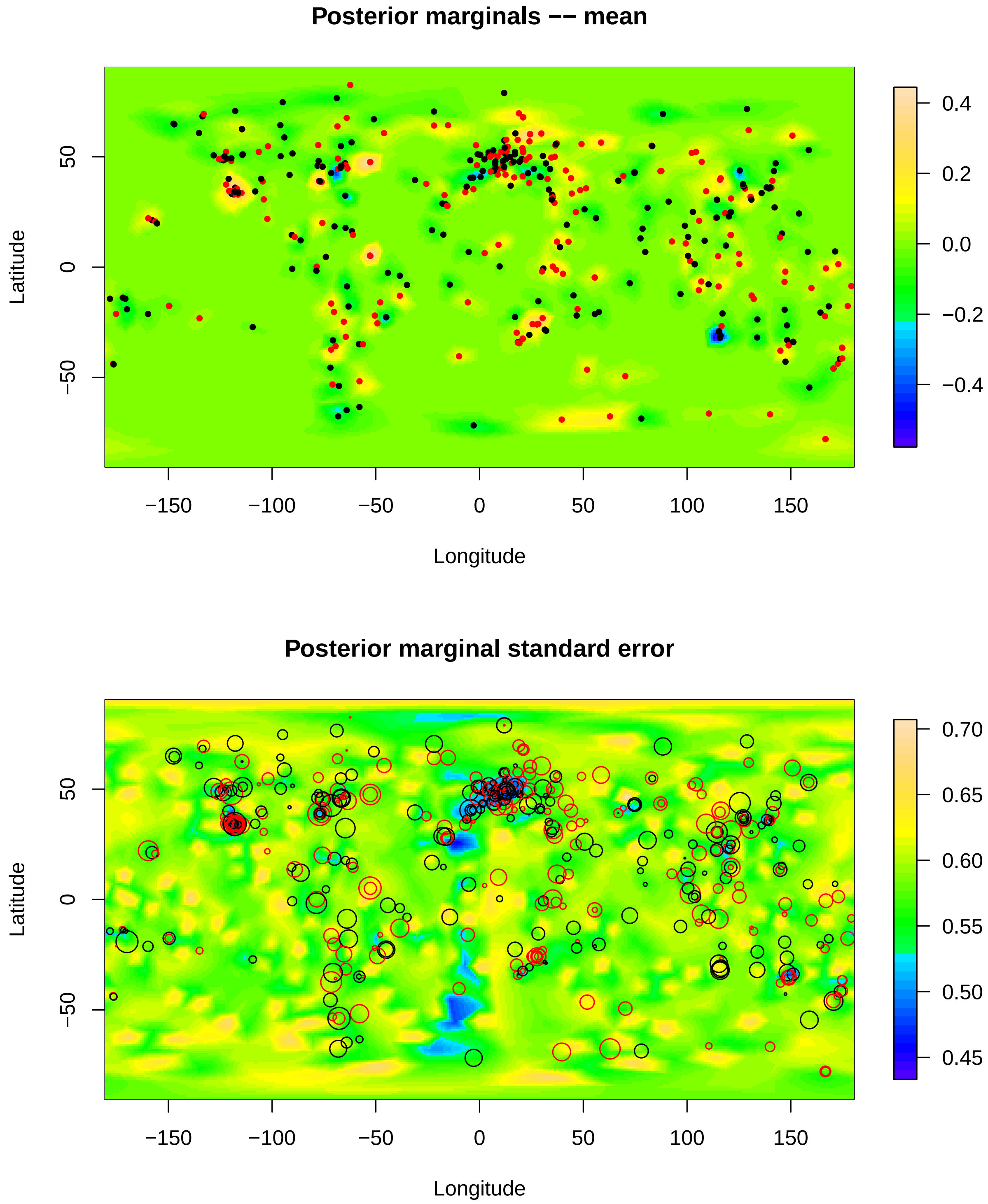


Figure 6: Posteriors marginal means and standard errors of the latent field using GPS data with large measurement errors. Points are the GPS locations. Red points are positive errors and black negative. The circle size in the bottom plot represents the error size.

4 Effect of Mesh size

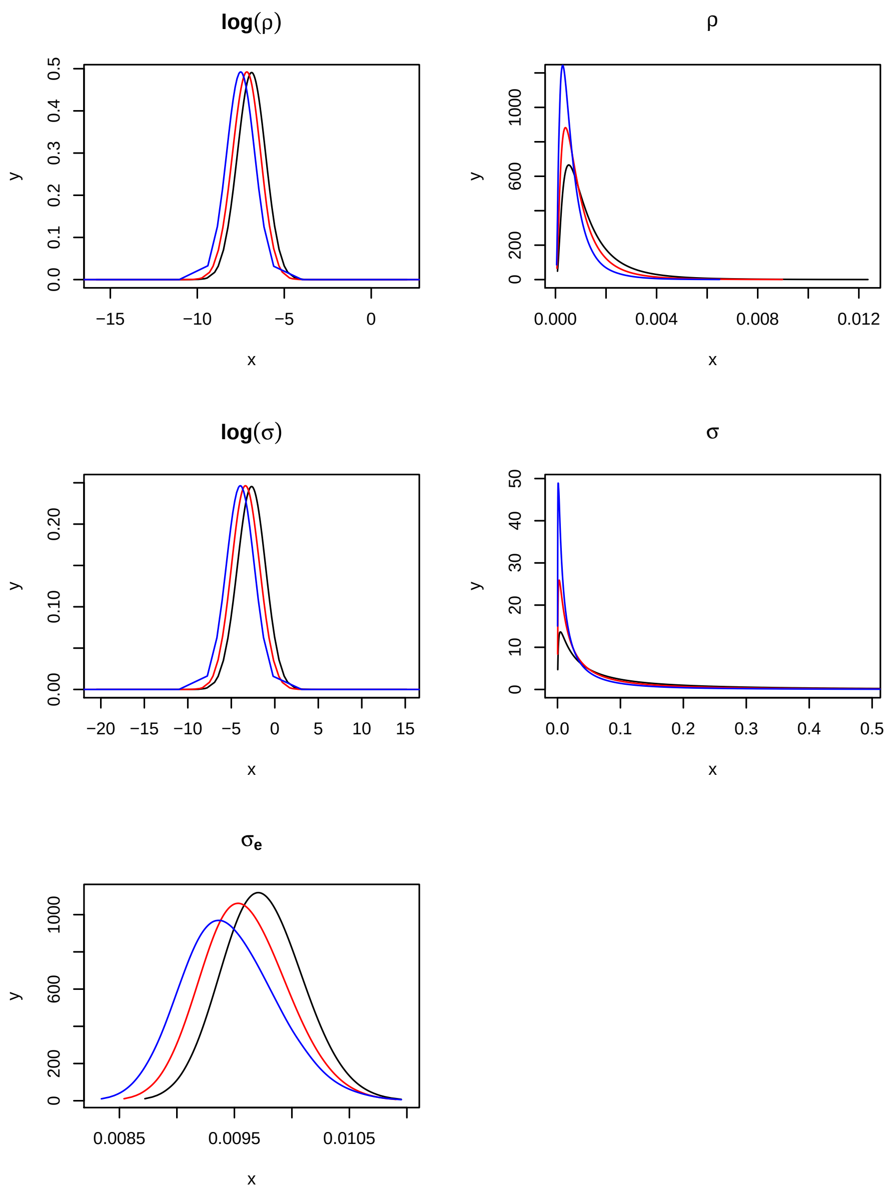


Figure 7: Posteriors of hyper parameters estimated using different mesh sizes for the latnet process. Black: small, Red: medium, blue: large.

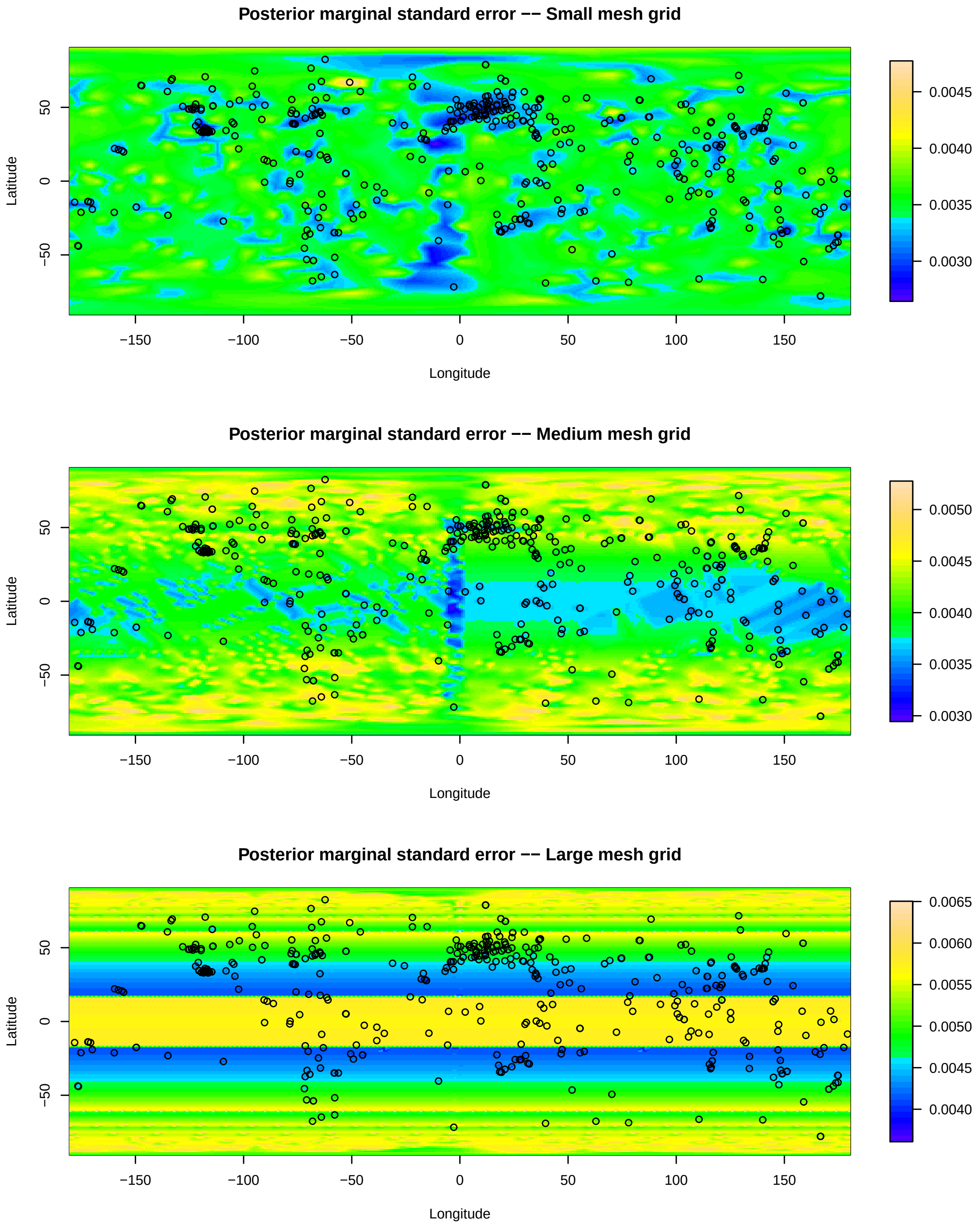


Figure 8: Posteriors marginal standard errors of the latent field with different mesh sizes. Points are the GPS locations.