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Global Mass Framework Documentation

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Summary: Correctly separating the sources of sea level rise (SLR) is crucial for improving future SLR predictions. Traditionally, changes in each component of the integrated signal have been tackled separately, which has often lead to inconsistencies between the sum of these components and the integral as measured by satellite altimetry. In this paper, we produce the first physically-based and data-driven solution for the complete coupled land-ocean-solid Earth system that is consistent with the full suite of observations, prior knowledge and fundamental geophysical constraints.

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Keywords: Bayesian hierarchical models; Gaussian Markov random fields; sea-level rise; spatial-temporal statistics; stochastic partial differential equations.

1. INTRODUCTION

2. BAYESIAN HIERARCHICAL MODELLING

We follow the framework in Zammit-Mangion A. et al. (2014) and define the Bayesian hierarchical model with (i) an observation layer describing the interaction between the process and instruments, (ii) a process layer containing information on the geophysical and

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spatial-temporal nature of the processes and (iii) a parameter layer specifying prior beliefs on unknowns.

2.1. The process layer

All the processes are measured as yearly hight change in water equivalent (mm/yr) and denoted by $\boldsymbol{X}(\boldsymbol{s})$. The global sea level change depends on the volume change of water in the world's oceans and the shape of the ocean basins. The glacio-isostatic adjustment (GIA), denoted by $\boldsymbol{X}_{GIA}$, can be used to account for the change of ocean basins (cite Wu et al 2010). The volume change of ocean is mainly due to the mass change (barystatic) and the density change (steric) of water. Denote the barystatic process by $\boldsymbol{X}_B$ and the steric process by $\boldsymbol{X}_S$. The steric process consists of two parts: the thermosteric process $\boldsymbol{X}_{ST}$ induced by temperature and the halosteric process $\boldsymbol{X}_{SH}$ due to the salinity change.

When investigating locally, the sea level change can be also strongly affected by ocean circulation $\boldsymbol{X}_C$ and gravity's fingerprint $\boldsymbol{X}_F$ on a fine scale (cf Bamber and Riva 2010). Hence, the total sea-level change $\boldsymbol{X}_{tot}(\boldsymbol{s})$ at a given location $\boldsymbol{s}$ can be decomposed as

$$\boldsymbol{X}_{tot}(\boldsymbol{s}) = \boldsymbol{X}_{SH}(\boldsymbol{s}) + \boldsymbol{X}_{ST}(\boldsymbol{s}) + \boldsymbol{X}_B(\boldsymbol{s}) + \boldsymbol{X}_{GIA}(\boldsymbol{s}) + \boldsymbol{X}_C(\boldsymbol{s}) + \boldsymbol{X}_F(\boldsymbol{s}) \tag{1}$$

In the following, we specify the spatial-temporal nature for each process.

2.1.1. The steric process

The steric process consists of halosteric process $\boldsymbol{X}_{SH}(\boldsymbol{s})$ and the thermosteric process $\boldsymbol{X}_{ST}$. How to model these two processes?

1. time invariant?
2. independent?
3. stationary?

2.1.2. The barystatic process

1. stationary spatial process?
2. time invariant?

2.1.3. The GIA process

The GIA process is modelled as spatial process that does not change over the time. In this first study, we will assume it as known.

1. We need to decide what error inflation to use, 20%?
2. Maike provide the fixed GIA value for the first study and prior information for later use?

2.1.4. The fine scale processes

For regional sea level change, the ocean circulation and gravity's finger print can both modelled as fine scale variations.

$$\boldsymbol{X}_C(\boldsymbol{s}) = \sum_{i=1}^{n_C} \phi_i(\boldsymbol{s})\eta_i + \delta_C(\boldsymbol{s}) \tag{2}$$

$$\boldsymbol{X}_F(\boldsymbol{s}) = \sum_{i=1}^{n_F} \psi_i(\boldsymbol{s})\beta_i + \delta_F(\boldsymbol{s}) \tag{3}$$

where $\{\phi_i\}$ and $\{\psi_i\}$ are some basis functions, η_i and β_i are the corresponding coefficients, and $delta_C$ and δ_F are Gaussian errors.

In this first study, we will ignore $\boldsymbol{X}_C(\boldsymbol{s})$ and $\boldsymbol{X}_F(\boldsymbol{s})$ since we do not have a satisfactory model to relate them with data yet. Including these two latent process will make the model under-determined. One possible solution is to smooth these two processes out by using historic data. *My memory of using a emulator for the finger print now faded away... What data can we use to estimate β_i and what basis functions? Green's function??*

2.2. The data layer

We are going to use the following datasets:

1. Altimery – Rory – not ready
2. Argo buoys – who – not ready
3. GRACE – Maike – ready

2.2.1. Data processing

Describe the data source and how we process the data.

2.2.2. Data modelling

The data layer is modelled by the following linear system

$$\boldsymbol{Z}_{ALT}(\boldsymbol{s}) = \boldsymbol{X}_{tot}(\boldsymbol{s}) + \boldsymbol{\varepsilon}_{ALT}(\boldsymbol{s}) \tag{4}$$

$$\boldsymbol{Z}_{ARGO}(\boldsymbol{s}) = \boldsymbol{X}_{SH}(\boldsymbol{s}) + \boldsymbol{\varepsilon}_{ARGO}(\boldsymbol{s}) \tag{5}$$

$$\boldsymbol{Z}_{GRACE}(D_i) = \int_{D_i} \rho_0(\boldsymbol{s})\boldsymbol{X}_B \, \mathbf{d}\boldsymbol{s} + \int_{D_i} \rho_E(\boldsymbol{s})\boldsymbol{X}_{GIA} \, \mathbf{d}\boldsymbol{s} + \int_{D_i} \rho_0(\boldsymbol{s})\boldsymbol{X}_C \, \mathbf{d}\boldsymbol{s} + \boldsymbol{\varepsilon}_{GRACE}(D_i) \tag{6}$$

where $\boldsymbol{\varepsilon}$ are the measurement errors for each instrument (provided by the person who processed the data), rho are fixed density (or density map).

1. I'm not entirely sure about the last equation, especially the density notation.

2.3. The priors layer

3. PARAMETER ESTIMATION

4. RESULTS

5. DISCUSSION

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APPENDIX

(This appendix was not part of the original paper by AV Raveendran and is included here just for illustrative purposes. The references are not relevant to the text of the appendix, they are references from the bibliography used to illustrate text before and after citations.)