Spatio-temporal modelling for global sea level change

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1. Introduction and framework

Motivation
Future sea level rise (SLR) is one of the most serious consequences of climate change. Traditionally, the Earth system components, including oceans, land ice, terrestrial water storage, and solid Earth effects, that contribute to SLR were treated separately and often led to inconsistencies between discipline-specific estimates of each part of the sea level budget.

Our project aims at producing a physically-based, data-driven solution for the complete coupled land-ocean-solid Earth system that is consistent with the full suite of observations, prior knowledge and fundamental geophysical constraints.

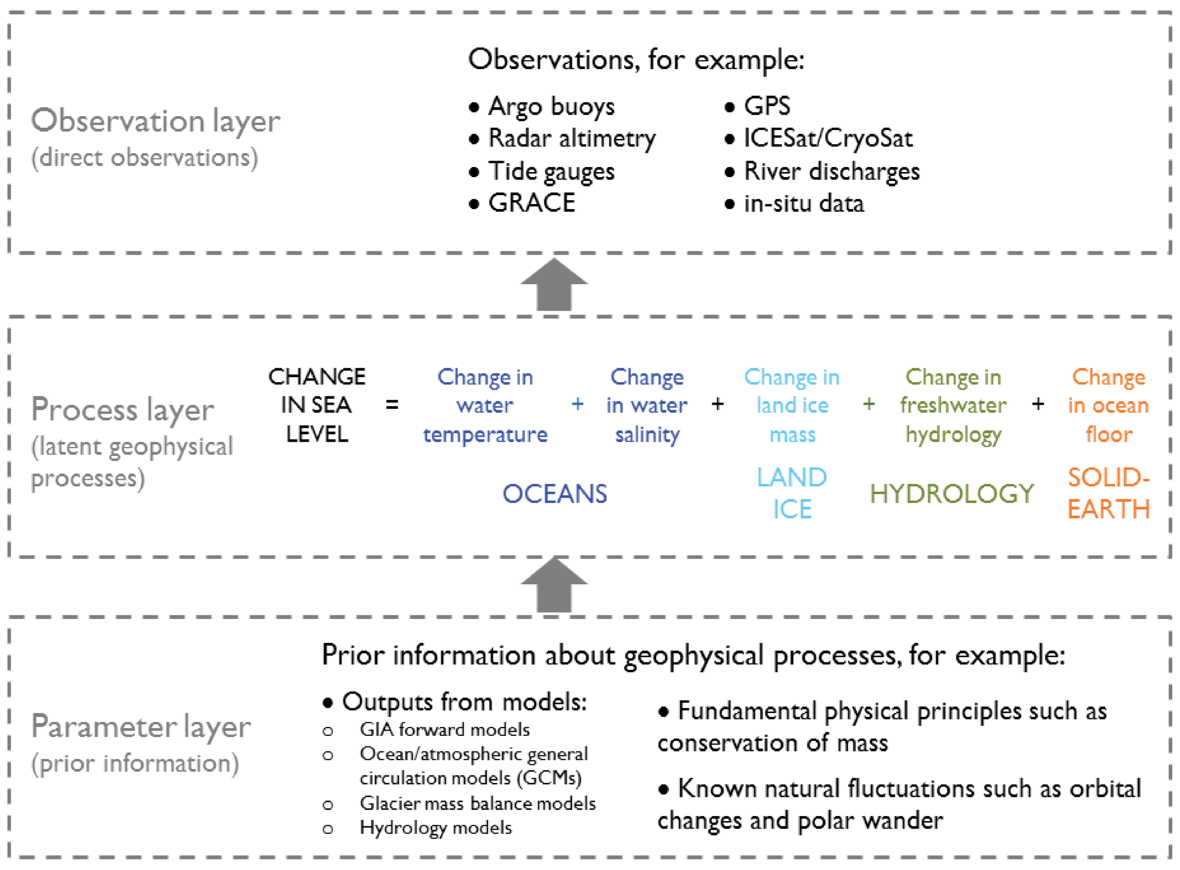

Figure 1: Global Mass concept and framework.

Bayesian Hierarchical Model (BHM) [2]

Define $\boldsymbol{y}$ as the observations, $\boldsymbol{x}$ the latent process at the spatial unit of $\boldsymbol{y}$, $\boldsymbol{X}$ the latent process, and $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ the parameters to be estimated.

$$
\begin{aligned}
\boldsymbol{y}|\boldsymbol{\beta}, \boldsymbol{x}, \boldsymbol{\theta} &\sim \mathcal{N}(\boldsymbol{P}(\boldsymbol{\beta})\boldsymbol{x}, \boldsymbol{\Sigma_{obs}}),\\
\boldsymbol{x}|\mathcal{A}, \boldsymbol{X}, \boldsymbol{\theta} &\sim \mathcal{N}(\mathcal{A}\boldsymbol{X}, \boldsymbol{Q}^{-1}(\boldsymbol{\theta})),\\
\boldsymbol{X}(\boldsymbol{s})|\boldsymbol{\theta}, &\sim \mathcal{GP}(\boldsymbol{\mu}(\boldsymbol{s}), k(\boldsymbol{s}, \boldsymbol{r}; \boldsymbol{\theta})),\\
\boldsymbol{\beta} &\sim \mathcal{N}(\boldsymbol{\mu_\beta}, \boldsymbol{\Sigma_\beta}),\\
f(\boldsymbol{\theta}) &\sim \mathcal{N}(\boldsymbol{\mu_\theta}, \boldsymbol{\Sigma_\theta}).
\end{aligned}
$$

where $\boldsymbol{P}$ maps the latent process to the observations, $\boldsymbol{\Sigma_{obs}}$ is a given measurement error matrix, $\mathcal{A}$ maps $\boldsymbol{X}$ into the spatial unit of $\boldsymbol{y}$, $\boldsymbol{Q}$ is a sparse precision matrix given by the stochastic partial differential equation (SPDE) approach [1].

2. Big data challenges

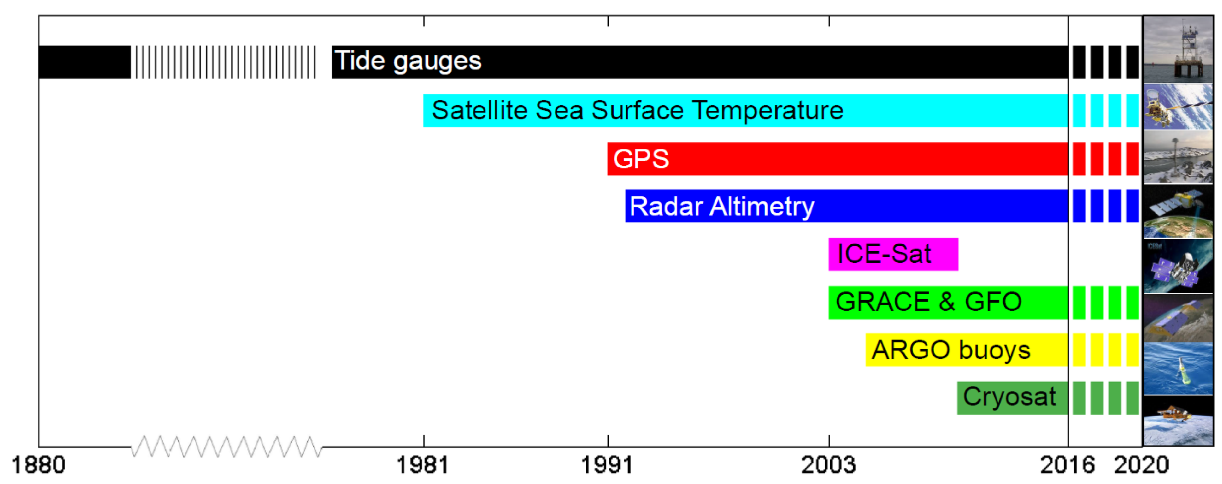

Figure 2: Temporal coverage of the observations.

Challenges

- Massive volume of data sets with uncertainty in error estimates.
- Inconsistent temporal coverages and frequencies between the data sets.
- In-situ and satellite measurements exhibit various spatial footprints.
- Different spatially-varying mesh grids in high resolutions for SPDE approximation.

Example: Glacio-Isostatic Adjustment (GIA)
GIA describes the ongoing movement of land once burdened by ice-age glaciers and makes a crucial contribution to sea level rise evaluation.

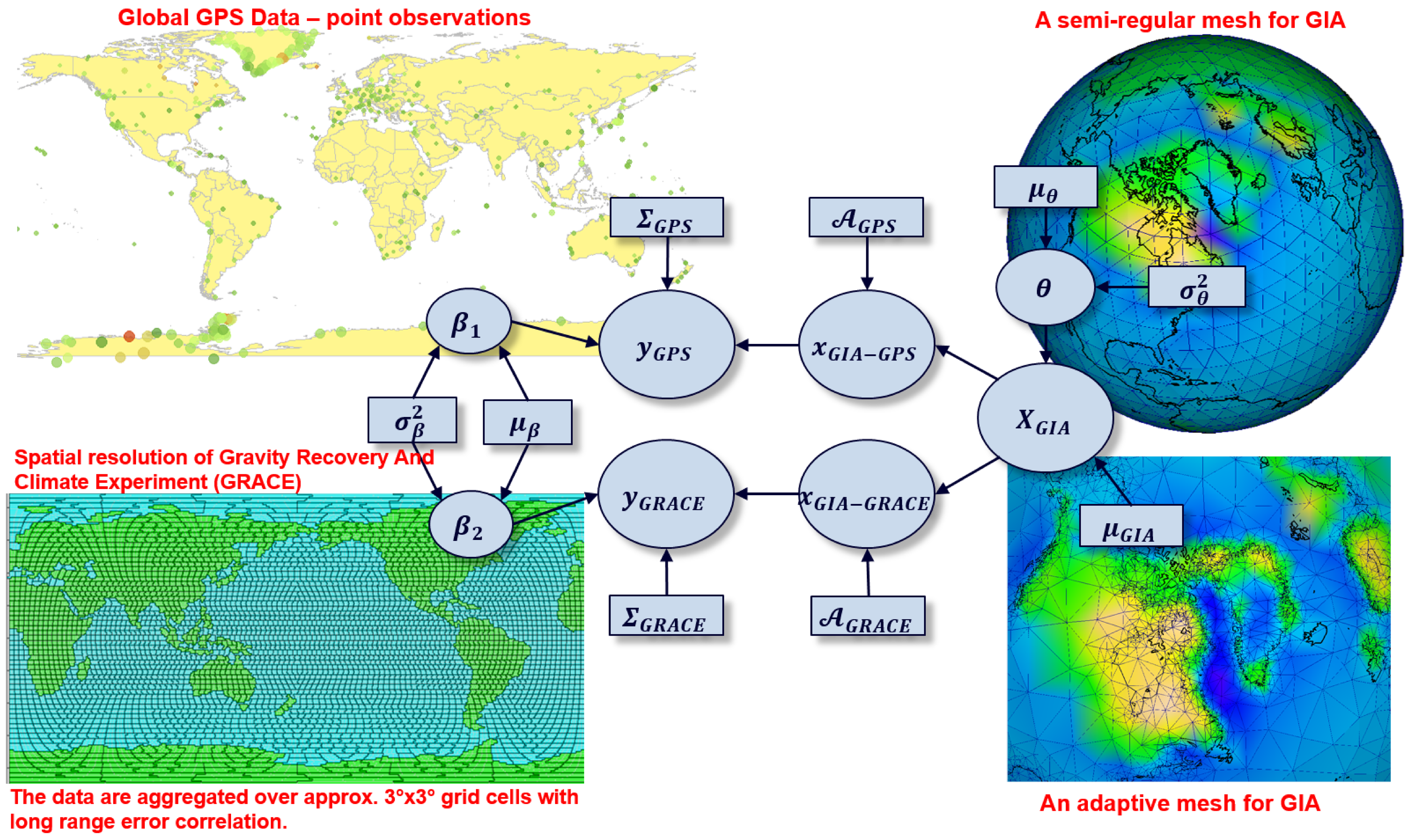

Figure 3: The data sets, meshes and graphical model for GIA.

3. Initial solution for the GIA process

Initial GIA solution combines global GPS data and prior means from a physical forward model, using a semi-regular mesh on a sphere at approximately 10 degree resolution.
Right shows the predicted GIA mean field. The predicted errors at the GPS locations and a set of regular grid points are shown as blue disks with disk radius proportional to the error size.
The data present strong signals at the GPS locations with much smaller predicted errors than elsewhere.

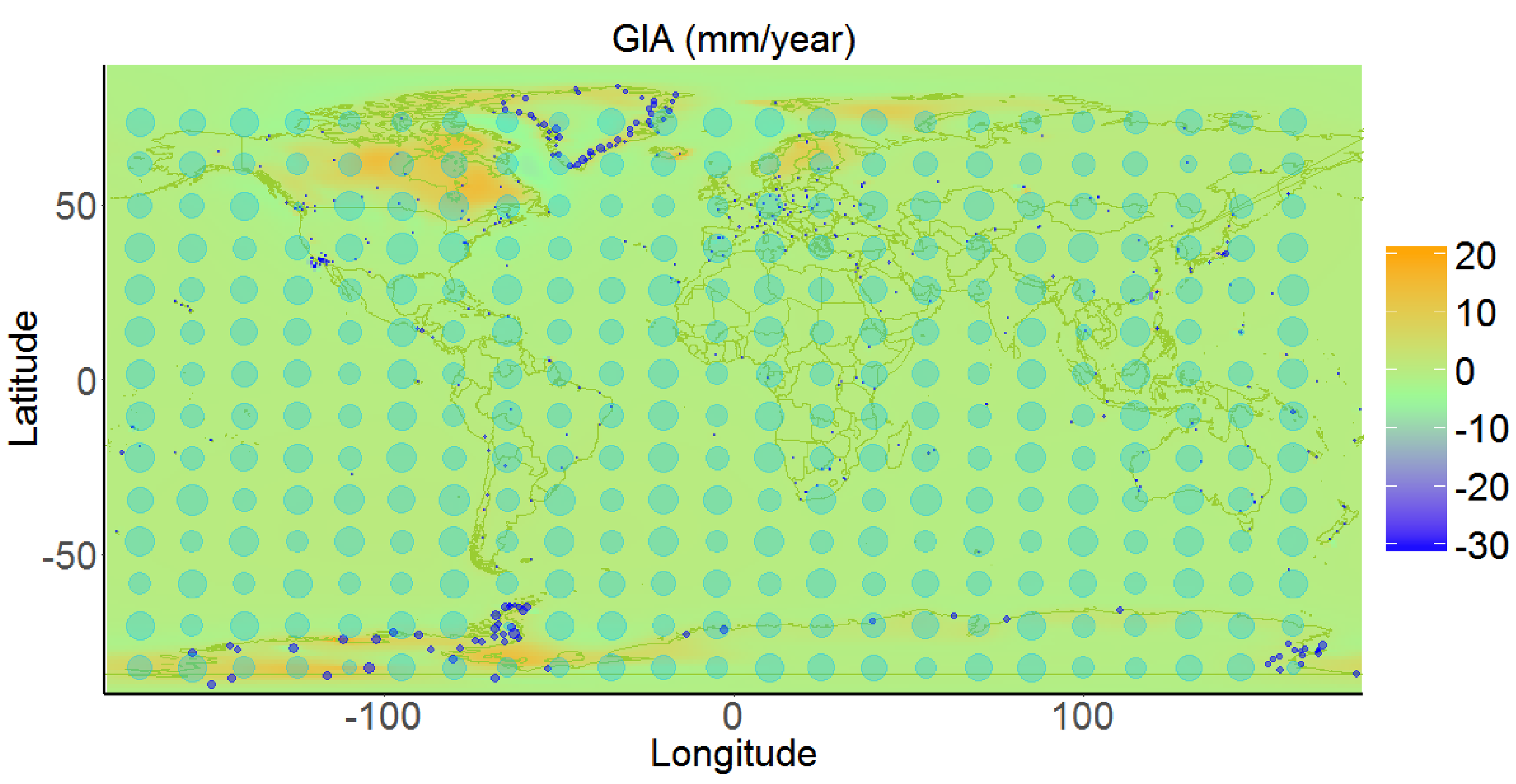

Figure 4: Predicted GIA mean field.

4. Future Work

- Integrating flexible temporal structure into the multivariate processes.
- Improving mesh quality and resolution.
- Estimating a data-driven GIA solution based on the full framework.
- Closing the sea level budget for the last four decades by adding physical constraints and more datasets.

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Keep in touch

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References

[1] F. Lindgren, H. Rue, and J. Lindström. An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 73(4):423–498, 2011.

[2] A. Zammit-Mangion, J. Rougier, N. Schön, F. Lindgren, and J. Bamber. Multivariate spatio-temporal modelling for assessing Antarctica's present-day contribution to sea-level rise. *Environmetrics*, 26(3):159–177, 2015.