Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

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GlobalMass

A 5-year project for global sea level rise re-evaluation

GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

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Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \Delta\text{barystatic}(t) + \Delta\text{steric}(t) + \text{GIA}$$

mass | density | ocean basins

- GIA: glacial isostatic adjustment
- inconsistencies between the discipline-specific estimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierarchical model

Framework concept (Zammit-Mangion et al., 2015)

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS | LAND ICE | HYDROLOGY | SOLID-EARTH

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

First step – modelling GIA

What is GIA?

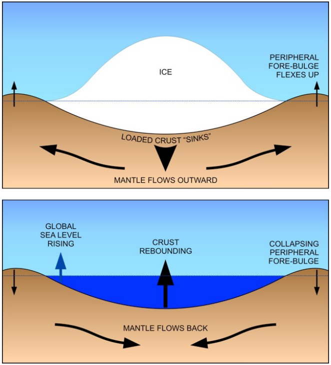


Figure courtesy of the Canadian Geodetic Survey, Natural Resources Canada.

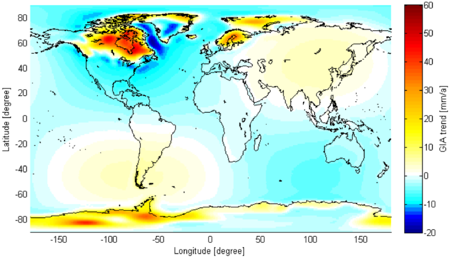
GIA - glacial isostatic adjustment

- vertical displacement of the Earth's crust once burdened by ice-age glaciers (∼20,000 yrs ago)
- constant over the time scale of this study (1981-2020)
- Regional discrepancies in estimates from physical models, e.g. ICE-6G (Peltier et al., 2015)

The GIA process

true GIA process
$\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$

prior mean trend
$\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$



GIA estimates from a ICE-6G model.

After de-trend, the residuals become stationary and assume

$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \tag{1}$$

$\kappa(\boldsymbol{\theta})$ – Matérn covariance function with parameter $\boldsymbol{\theta} = (\rho, \sigma^2)$ for the length-scale and variance.

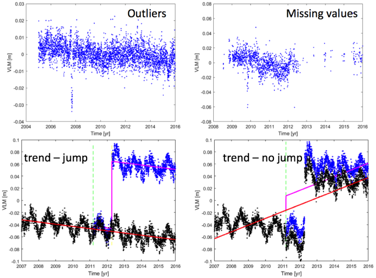
The GPS observations

Vertical movement in the Earth’s surface

GPS times series signals at Station i,

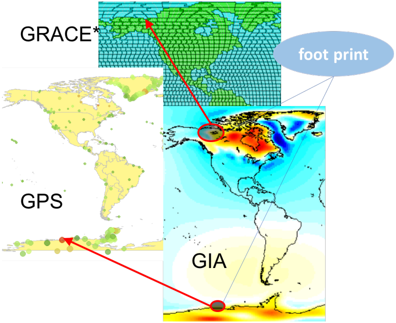
- an uplift rate Z_i (mm/yr)
- a measurement error $\varepsilon_i \sim \mathcal{N}(0, e_i^2)$
- $e_i = \text{s.d.}(Z_i)$ estimated from the time series.

A GPS station in Antarctic.



Forward modelling

From process to observations



*GRACE: the Gravity Recovery And Climate Experiment satellites

$\mathcal{A}_i$ maps the latent process over the appropriate spatial ***foot print*** to the i^{th} observation,

$$Z_i = \mathcal{A}_i \boldsymbol{Y} + \varepsilon_i, \qquad (2)$$

$\mathcal{A}_i$ is usually a linear operator

- point $\rightarrow$ point
- area $\rightarrow$ point
- area $\rightarrow$ area

The BHM for GIA

Denote by $\boldsymbol{\mathcal{A}}^T = \left[\boldsymbol{\mathcal{A}}_1^T, \cdots, \boldsymbol{\mathcal{A}}_N^T\right]$, then we can have the vector form for equation (2)

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}}\boldsymbol{Y} + \boldsymbol{\varepsilon} \tag{3}$$

Bayesian hierarchical model for GIA

Combine the observation equation (3) and the process equation (1)

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{X} + \boldsymbol{\varepsilon}, \ \boldsymbol{\varepsilon} \sim \mathcal{N}(\boldsymbol{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\boldsymbol{0}, \kappa(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \tag{4}$$

Predicting GIA

The predictive distribution of the latent process

Predictive Distribution of GIA

The prediction is based on the posterior marginal distribution of the latent process

$$\pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \int_{\boldsymbol{\Theta}} \pi(\boldsymbol{X}, \theta|\tilde{\boldsymbol{Z}})\, \mathrm{d}\theta \qquad (5)$$

– Integrating the uncertainty of the parameters.

Other choices include the "plug-in" predictive distribution $\pi_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \pi_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}, \hat{\theta})$, where $\hat{\theta}$ are the estimated values of the parameters, e.g. posterior mean or mode.

Predicting GIA

Point-wise prediction of the latent process

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Point-wise predicted mean and uncertainty

For point-wise update at X_i on a fine grid, we use

$$\text{predicted mean:} X_i^* = \mathbb{E}(X_i|\tilde{\boldsymbol{Z}})$$
$$\text{predicted uncertainty:} u_i^* = \text{s.d.}(X_i|\tilde{\boldsymbol{Z}})$$

where $\pi(X_i|\tilde{\boldsymbol{Z}}) = \int_{\mathcal{X}_{-i}} \pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}})\,\mathrm{d}\boldsymbol{X}_{-\boldsymbol{i}}$.

When using the "plug-in" predictive distribution, $\pi_p(X_i|\tilde{\boldsymbol{Z}})$ is Gaussian and the Bayesian update can be written in closed form.

The GMRF approximation

- Bayesian update of the GP on a grid of m points scales as $\mathcal{O}(m^3)$, $m \sim 10^5$ for a 1° global grid.
- Gaussian Markov random field (GMRF) with sparse precision matrix has a much better scaling property.

The SPDE approach (Lindgren et al., 2011)

Denote by $\mathbf{x}$ the GMRF approximation of $\boldsymbol{X}$ on a triangulation of m vertices with basis functions $\{\phi_i\}_{i\in\mathbb{N}}$, then $s \in \mathbb{S}^2$

$$\boldsymbol{X}(s) \approx \phi_i(s)^T \mathbf{x} \tag{6}$$

For a grid $\boldsymbol{S} \subset \mathbb{S}$ of locations, we find the weight matrix $\boldsymbol{C}$ and write $\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}\mathbf{x}$.

The GMRF approximation

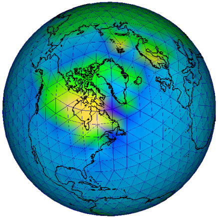
Now the BHM in equation (4) becomes

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{C}\mathbf{x} + \varepsilon, \;\; \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \mathbf{x} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}^{-1}(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \tag{7}$$

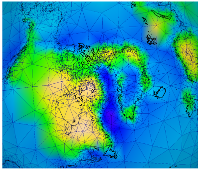
$\mathbf{x}$ is GMRF approximation defined by the precision matrix $\boldsymbol{Q}$.

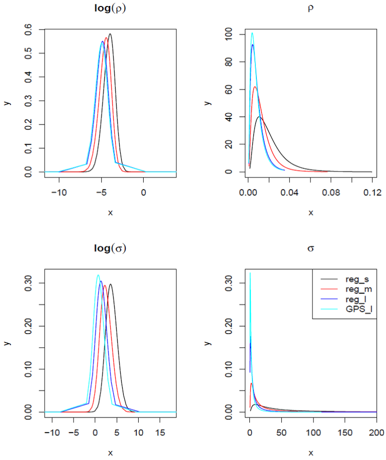
Choosing the mesh on a sphere

semi-regular mesh



irregular mesh

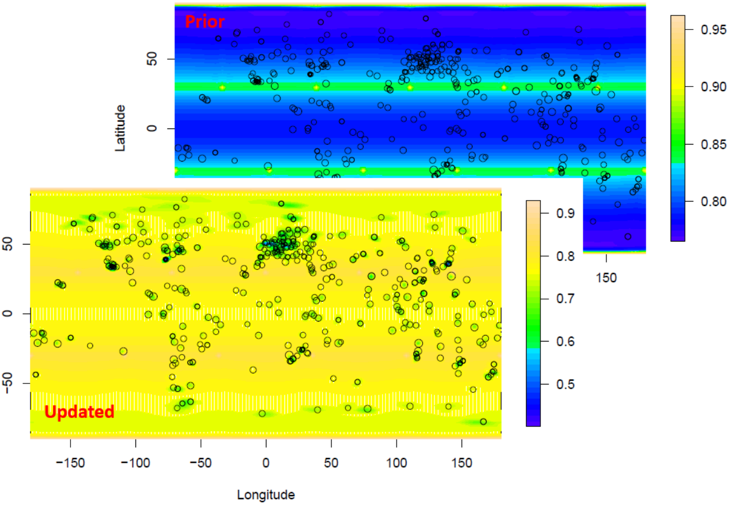




Parameter estimation using different meshes.

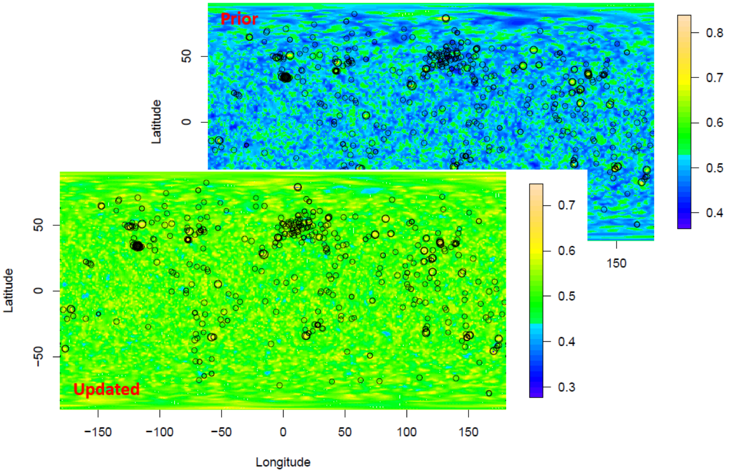
Mesh effect

The predicted uncertainties using regular mesh

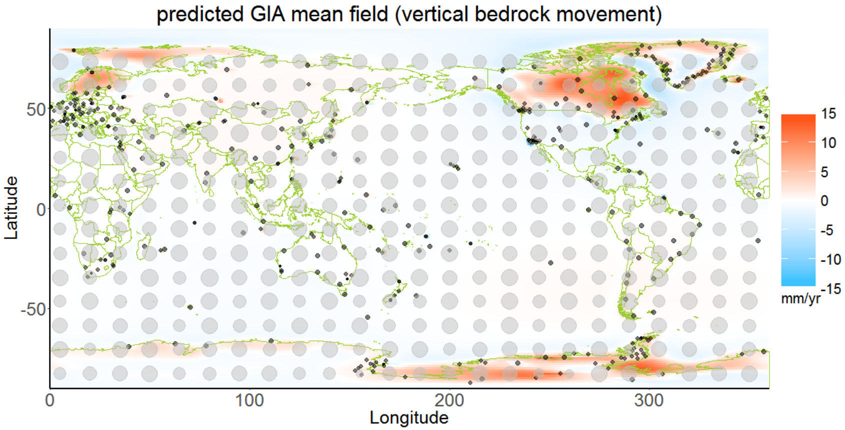


Mesh effect

The predicted uncertainties using irregular mesh



First solution for GIA



GIA disc size (grey) proportional to predicted uncertainty

Conclusions and outlook

- Testing the statistical framework for the GIA.
- Requiring improved mesh.
  - Adaptive – fast computation
  - Regular – stable approximation
- Improving GPS data quality.
- Extending to the full system.

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Questions

Thank you!