Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

Zhe Sha[1], Maike Schumancher[1],
Jonathan Rougier[2] and Jonathan Bamber[1]

[1]School of Geographical Sciences,[2]School of Mathematics
University of Bristol

26 July 2017

GlobalMass

University of
BRISTOL

GLOBAL
MASS

GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

www.globalmass.eu
@GlobalMassTeam

Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta \text{sea level}(t) = \Delta \text{barystatic}(t) + \Delta \text{steric}(t) + \text{GIA}$$

mass   density   ocean basins

- GIA: glacial isostatic adjustment
- inconsistencies between the dicipline-specific etimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierachical model

University of BRISTOL

GLOBAL MASS

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE

- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS   LAND ICE   HYDROLOGY   SOLID-EARTH

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models

- Fundamental physical principles such as conservation of mass

- Known natural fluctuations such as orbital changes and polar wander

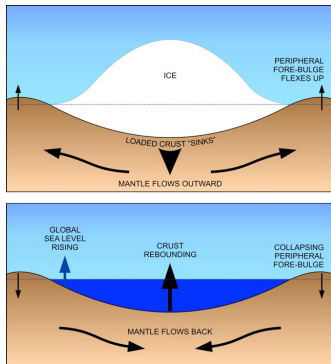# First step – modelling GIA

What is GIA?

Figure courtesy of the Canadian Geodetic Survey, Natural Resources Canada.

GIA - **glacial isostatic adjustment**

- vertical displacement of the Earth's crust once burdened by ice-age glaciers (∼20,000 yrs ago)

- constant over the time scale of this study (1981-2020)

- Regional descrepencies in estimates from physical models, e.g. ICE-6G (Peltier et al., 2015)

The GIA process

Denote the true GIA process by $\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$, and the prior mean trend by $\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$.
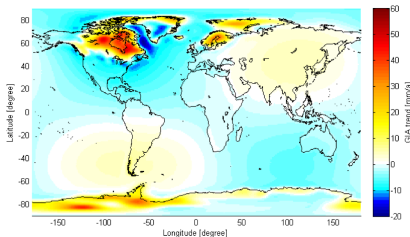
GIA estimates from a ICE-6G model.

After detrend, the residuals become stationary and assume

$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\boldsymbol{0}, \kappa(\rho, \sigma^2)) \tag{1}$$
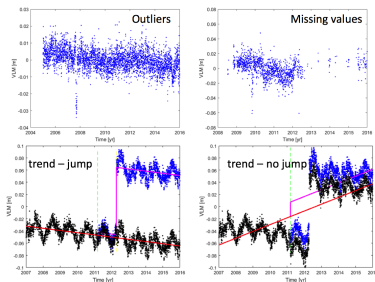
$\kappa(\rho, \sigma^2)$ - Matérn covariance function with length-scale and variance parameters ρ and σ^2.

The GPS observations

The GPS observation at the ith selected station Z_i can be decomposed as yearly trend of vertical movment in the Earth's surface and measurement error $\varepsilon_i \sim \mathcal{N}(0, e_i^2)$.
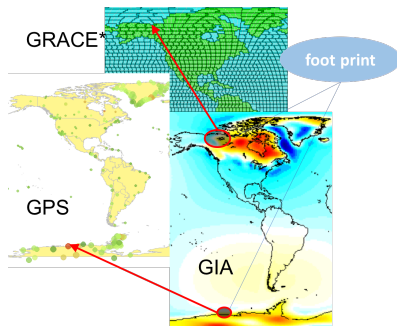

A GPS station in Antarctic.

e_i^2s are the estimated from raw GPS time series and fixed.

Forward modelling
From process to observations

GRACE*

foot print

GPS

GIA

*GRACE: the Gravity Recovery And Climate Experiment satellites

Define $\mathcal{A}_i$ as the map from the latent process over the appropriate spatial **_foot print_** to the ith observations,

$$Z_i = \mathcal{A}_i \boldsymbol{Y} + \varepsilon_i, \qquad (2)$$

$\mathcal{A}_i$ is usually a linear operator

- point $\rightarrow$ point
- area $\rightarrow$ point
- area $\rightarrow$ area

The BHM for GIA

Denote by $\boldsymbol{\mathcal{A}}^T = \left[\boldsymbol{\mathcal{A}}_1^T, \cdots, \boldsymbol{\mathcal{A}}_N^T\right]$, then we can have the vector form for equation 2

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}}\boldsymbol{Y} + \varepsilon \qquad (3)$$

Bayesian hierarchical model for GIA

Combine the observation equation 3 and the process equation 1

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{X} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\boldsymbol{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\boldsymbol{0}, \kappa(\rho, \sigma^2)) \\ \rho \sim \boldsymbol{\pi}(\rho), \quad \sigma^2 \sim \boldsymbol{\pi}(\sigma^2) \end{cases} \qquad (4)$$

Predicting GIA

Predicted mean and uncertainty of GIA

Pointwise Bayesian update of the GIA process on a fine grid. For grid point X_i, the predictive distribution is

$$\pi(X_i|\tilde{\boldsymbol{Z}}) = \int_{\boldsymbol{\mathcal{X}}_{-i}} \pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) \, d\boldsymbol{X}_{-i} \tag{5}$$

predicted mean $X_i^* = \mathbb{E}(X_i|\tilde{\boldsymbol{Z}})$

predicted uncertainty $u_i^* = \text{s.d.}(X_i|\tilde{\boldsymbol{Z}})$

The GMRF approximation

- Bayesian update of the GP on a grid of m points scales as $\mathcal{O}(m^3)$, $m \sim 10^5$ for a $1°$ global grid.
- Gaussian Markov random filed (GMRF) with sparse precision matrix has much better scaling property.

The SPDE approach (Lindgren et al., 2011)

Denote by $\tilde{X}$ the GMRF approximation of X on a given triangulation with basis functions $\{\phi_i\}_{i \in \mathbb{N}}$, then $s \in \mathbb{S}^2$

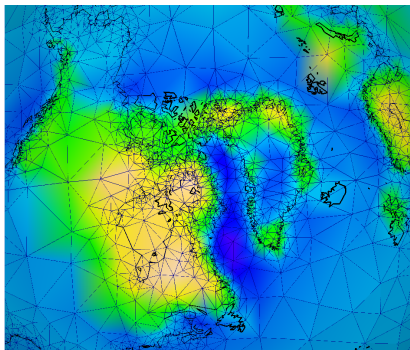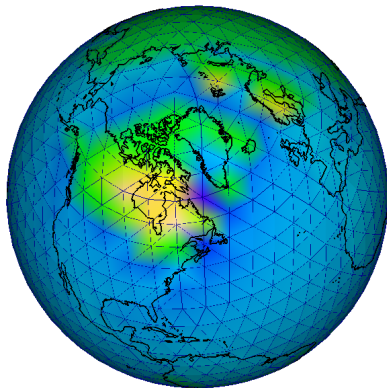$$\boldsymbol{X}(s) \approx \phi_i(s)^T \tilde{\boldsymbol{X}} \tag{6}$$

For a grid $\boldsymbol{S} \subset \mathbb{S}$ of locations, we find the weight matrix $\boldsymbol{C}$ and write $\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}\tilde{\boldsymbol{X}}$.

The GMRF approximation

Now the BHM 4 becomes

$$\begin{cases} \tilde{Z} = \mathcal{A}C\tilde{X} + \varepsilon, \ \varepsilon \sim \mathcal{N}(0, \text{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \tilde{X} \sim \mathcal{N}(0, Q^{-1}(\rho, \sigma^2)) \\ \rho \sim \pi(\rho), \quad \sigma^2 \sim \pi(\sigma^2) \end{cases} \quad (7)$$

where $\tilde{X}$ is defined by the precision matrix Q

Choosing the mesh on sphere

First solution for GIA

GIA (mm/year)

Conclusion
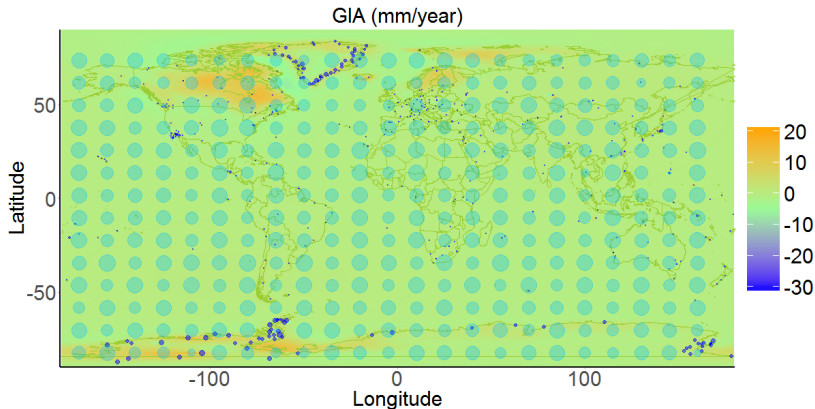
- Test the statistical framework on the GIA.
- Find a better mesh.
  - Adaptive – fast computation
  - regularity – stable approximation
- Need better quality GPS data.
- Extend to the full system.

Lindgren, F., Rue, H., and Lindström, J. (2011). An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 73(4):423–498.

Peltier, W. R., Argus, D. F., and Drummond, R. (2015). Space geodesy constrains ice age terminal deglaciation: The global ICE-6G_C (VM5a) model. *Journal of Geophysical Research: Solid Earth*, 120(1):450–487.

www.globalmass.eu
@GlobalMassTeam

Thank you!

www.globalmass.eu
@GlobalMassTeam