Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

Zhe Sha[1], Maike Schumancher[1],
Jonathan Rougier[2] and Jonathan Bamber[1]

[1]School of Geographical Sciences,[2]School of Mathematics
University of Bristol

26 July 2017

GlobalMass

A 5-year project for global sea level rise re-evaluation

GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

Funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 69418.

Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \Delta\text{barystatic}(t) + \Delta\text{steric}(t) + \text{GIA}$$

mass density ocean basins

- GIA: glacio-isostatic adjustment
- inconsistencies between the dicipline-specific etimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierachical model

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS | LAND ICE | HYDROLOGY | SOLID-EARTH

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

What is GIA?

The GIA process

We assume that the true GIA process is a real-valued spatial process continues on the sphere and denote it by $\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$. We use one of the GIA solution, say from one of the *ice6g* models, as the prior mean of the true process and denote it by $\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$. Then the residuals between the true process and forward model solutions can be modelled as a stationary Gaussian process on the sphere

$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \tag{1}$$

where $\kappa(\boldsymbol{\theta})$ defines the covariance function with hyper parameters $\boldsymbol{\theta}$.

The GPS data

The GPS data are the yearly trends of vertical movements in millimetre at the observed locations.
In practice, e_i^2 can usually be estimated from raw GPS data and therefore we set them to be fixed values from prior information.

From process to observations

These observations can be regarded as a linear map of the GIA process with measurement errors

$$Z_i = \boldsymbol{\mathcal{A}}_i \boldsymbol{Y} + \varepsilon_i, \ i = 1, \dots N. \tag{2}$$

where $\boldsymbol{\mathcal{A}}_i$ is the linear operator that maps the GIA process to the i^{th} GPS observation and ε_i are assumed to be independent Gaussian errors $\mathcal{N}(0, e_i^2)$.
Denote the linear operator for the GPS observation vector $\boldsymbol{Z}$ by

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} \boldsymbol{\mathcal{A}}_1 \\ \vdots \\ \boldsymbol{\mathcal{A}}_N \end{bmatrix}$$

Then we can write equation 2 into the vector form

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}} \boldsymbol{Y} + \boldsymbol{\varepsilon} \tag{3}$$

Bayesian update of the GIA process

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{X} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{p}(\boldsymbol{\theta}) \end{cases} \tag{4}$$

The GMRF approximation

The Gaussian process model can be computationally expensive for large scale inference since the Bayesian update scales as $\mathcal{O}(m^3)$ mainly due to the inverse of a dense covariance matrix.

The Gaussian process with Matérn covariance function can be treated as solutions to a class of SPDEs (**?**) which can then be approximated by GMRF using finite element methods.

Denote by $\tilde{\boldsymbol{X}}$ the GMRF approximation of $\boldsymbol{X}$ on a given triangulation of the sphere with piecewise linear basis functions $\{\phi_i\}_{i\in\mathbb{N}}$, then given any location $s \in \mathbb{S}^2$

$$\boldsymbol{X}(s) \approx \phi_i(s)^T \tilde{\boldsymbol{X}} \tag{5}$$

and for a given set $\boldsymbol{S}$ of locations, we have

$$\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}(\boldsymbol{S})\tilde{\boldsymbol{X}} \tag{6}$$

where the matrix $\boldsymbol{C}$ contains basis functions for all locations.

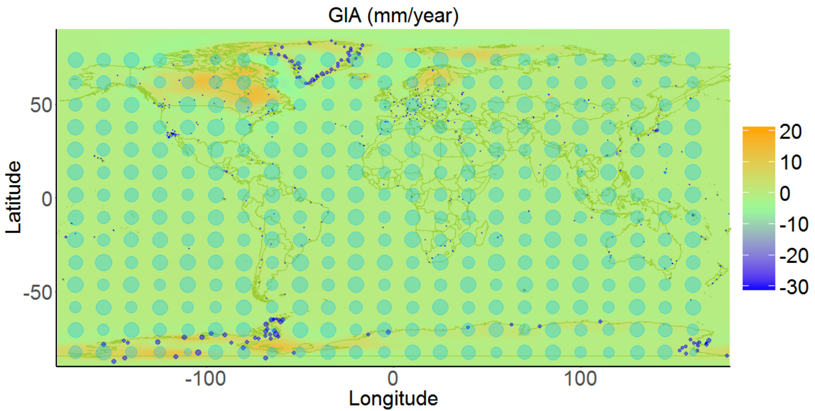
The GMRF approximation

$$
\begin{cases}
\tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{C}\tilde{\boldsymbol{X}} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\
\tilde{\boldsymbol{X}} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}^{-1}(\boldsymbol{\theta})) \\
\boldsymbol{\theta} \sim \boldsymbol{p}(\boldsymbol{\theta})
\end{cases}
\tag{7}
$$

where $\boldsymbol{Q}$ is the precision matrix of the GMRF approximation.

Choosing the mesh

First solution for GIA



Conclusion and future work

Thank you! Questions?

References