Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

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GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

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Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \underbrace{\Delta\text{barystatic}(t)}_{\text{mass}} + \underbrace{\Delta\text{steric}(t)}_{\text{density}} + \underbrace{\text{GIA}}_{\text{ocean basins}}$$

- GIA: glacial isostatic adjustment
- inconsistencies between the dicipline-specific etimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierachical model

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS (Change in water temperature, Change in water salinity) | LAND ICE (Change in land ice mass) | HYDROLOGY (Change in freshwater hydrology) | SOLID-EARTH (Change in ocean floor)

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

First step – modelling GIA

What is GIA?

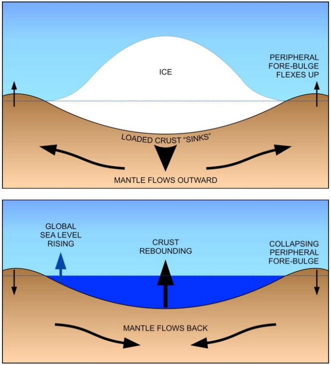


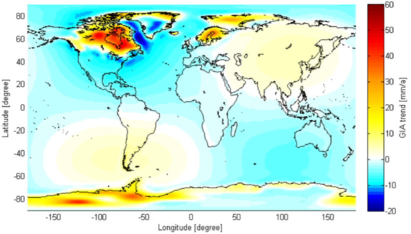
Figure courtesy of the Canadian Geodetic Survey, Natural Resources Canada.

GIA - glacial isostatic adjustment

- vertical displacement of the Earth's crust once burdened by ice-age glaciers (∼20,000 yrs ago)
- constant over the time scale of this study (1981-2020)
- Regional descrepencies in estimates from physical models, e.g. ICE-6G (Peltier et al., 2015)

The GIA process

Denote the true GIA process by $\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$, and the prior mean trend by $\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$.



GIA estimates from a ICE-6G model.

After detrend, the residuals become stationary and assume

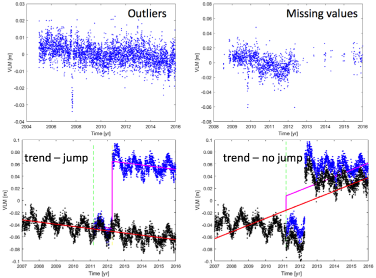
$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\mathbf{0}, \kappa(\rho, \sigma^2)) \tag{1}$$

$\kappa(\rho, \sigma^2)$ - Matérn covariance function with length-scale and variance parameters ρ and σ^2.

The GPS observations

The GPS observation at the i^{th} selected station Z_i can be decomposed as yearly trend of vertical movment in the Earth's surface and measurement error $\varepsilon_i \sim \mathcal{N}(0, e_i^2)$.

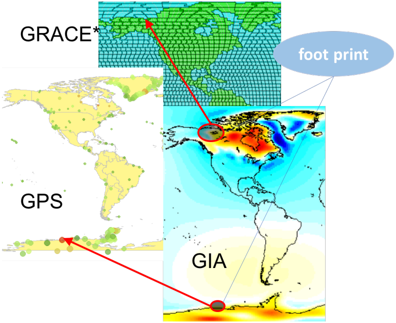
A GPS station in Antarctic.



e_i^2 is the estimated from raw GPS time series and fixed.

Forward modelling

From process to observations



*GRACE: the Gravity Recovery And Climate Experiment satellites

Define $\mathcal{A}_i$ as the map from the latent process over the appropriate spatial ***foot print*** to the i^{th} observations,

$$Z_i = \mathcal{A}_i \boldsymbol{Y} + \varepsilon_i, \qquad (2)$$

$\mathcal{A}_i$ is usually a linear operator

- point $\rightarrow$ point
- area $\rightarrow$ point
- area $\rightarrow$ area

The BHM for GIA

Denote by $\boldsymbol{\mathcal{A}}^T = \left[\boldsymbol{\mathcal{A}}_1^T, \cdots, \boldsymbol{\mathcal{A}}_N^T\right]$ Then we can have the vector form for equation 2

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}}\boldsymbol{Y} + \boldsymbol{\varepsilon} \tag{3}$$

Bayesian hierarchical model for GIA

Combine the observation equation 3 and the process equation 1

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{X} + \varepsilon, \ \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\mathbf{0}, \kappa(\rho, \sigma^2)) \\ \rho \sim \boldsymbol{\pi}(\rho), \quad \sigma^2 \sim \boldsymbol{\pi}(\sigma^2) \end{cases} \tag{4}$$

Predicting GIA

Predicted mean and uncertainty of GIA

Pointwise Bayesian update of the GIA process on a fine grid. For grid point X_i, the predictive distribution is

$$\pi(X_i|\tilde{\boldsymbol{Z}}) = \int_{\mathcal{X}_{-i}} \pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}})\,\mathrm{d}\boldsymbol{X}_{-i} \tag{5}$$

predicted mean $X_i^* = \mathbb{E}(X_i|\tilde{\boldsymbol{Z}})$

predicted uncertainty $u_i^* = \mathrm{sd}(X_i|\tilde{\boldsymbol{Z}})$

The GMRF approximation

- Bayesian update of the GP on a grid of m points scales as $\mathcal{O}(m^3)$, $m \sim 10^5$ for a 1° global grid.
- Gaussian Markov random filed (GMRF) with sparse precision matrix has much better scaling property.

The SPDE approach (Lindgren et al., 2011)

Denote by $\tilde{\boldsymbol{X}}$ the GMRF approximation of $\boldsymbol{X}$ on a given triangulation with basis functions $\{\phi_i\}_{i \in \mathbb{N}}$, then $s \in \mathbb{S}^2$

$$\boldsymbol{X}(s) \approx \phi_i(s)^T \tilde{\boldsymbol{X}} \tag{6}$$

For a grid $\boldsymbol{S} \subset \mathbb{S}$ of locations, we find the weight matrix $\boldsymbol{C}$ and write $\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}\tilde{\boldsymbol{X}}$.

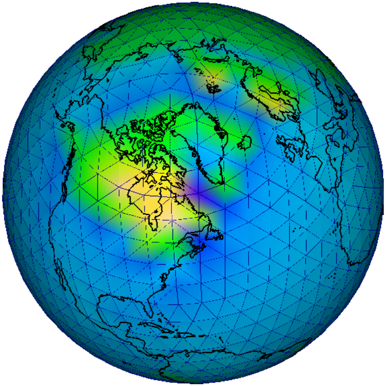
The GMRF approximation

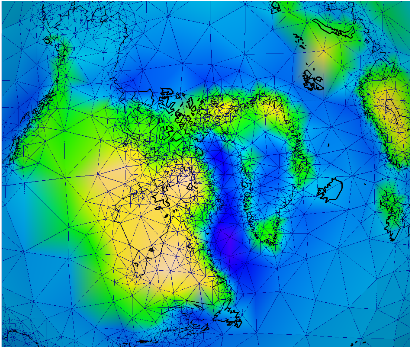
Now the BHM 4 becomes

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{C}\tilde{\boldsymbol{X}} + \varepsilon, \ \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \tilde{\boldsymbol{X}} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}^{-1}(\rho, \sigma^2)) \\ \rho \sim \boldsymbol{\pi}(\rho), \quad \sigma^2 \sim \boldsymbol{\pi}(\sigma^2) \end{cases} \tag{7}$$

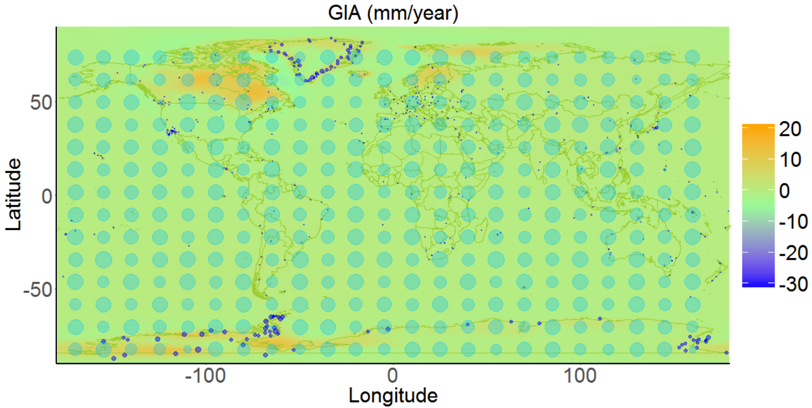
where $\tilde{\boldsymbol{X}}$ is defined by the precision matrix $\boldsymbol{Q}$

Choosing the mesh on sphere





First solution for GIA



Conclusion

- Test the our statistical framework on the GIA.
- Find a better mesh.
  - Adaptive – fast computation
  - regularity – stable approximation
- Need better quality GPS data.
- Extend to the full system.

References

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Questions

Thank you!