Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

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GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

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Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \underbrace{\Delta\text{barystatic}(t)}_{\text{mass}} + \underbrace{\Delta\text{steric}(t)}_{\text{density}} + \underbrace{\text{GIA}}_{\text{ocean basins}}$$

- GIA: glacial isostatic adjustment
- inconsistencies between the discipline-specific estimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierarchical model

Framework concept (Zammit-Mangion et al., 2015)

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS (Change in water temperature, Change in water salinity) | LAND ICE (Change in land ice mass) | HYDROLOGY (Change in freshwater hydrology) | SOLID-EARTH (Change in ocean floor)

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

First step – modelling GIA

What is GIA?

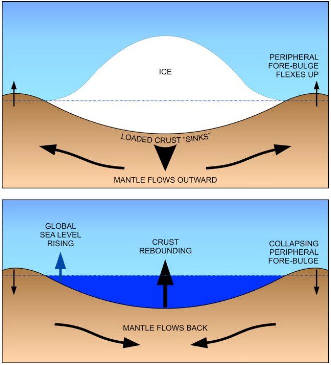


Figure courtesy of the Canadian Geodetic Survey, Natural Resources Canada.

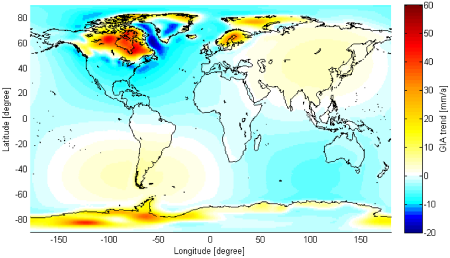
GIA - glacial isostatic adjustment

- vertical displacement of the Earth's crust once burdened by ice-age glaciers (~20,000 yrs ago)
- constant over the time scale of this study (1981-2020)
- Regional discrepancies in estimates from physical models, e.g. ICE-6G (Peltier et al., 2015)

The GIA process

true GIA process
$\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$

prior mean trend
$\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$



GIA estimates from a ICE-6G model.

After de-trend, the residuals become stationary and assume

$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \tag{1}$$

$\kappa(\boldsymbol{\theta})$ – Matérn covariance function with parameter $\boldsymbol{\theta} = (\rho, \sigma^2)$ for the length-scale and variance.

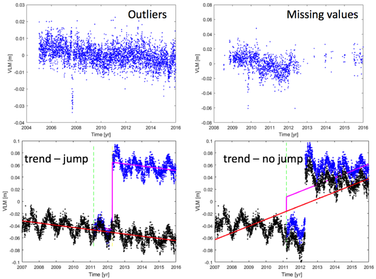
The GPS observations

Vertical movement in the Earth's surface

GPS times series signals at Station i,

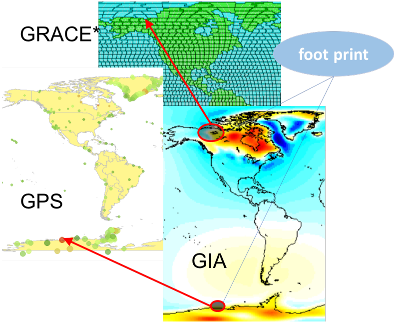
- an uplift rate Z_i (mm/yr)
- a measurement error $\varepsilon_i \sim \mathcal{N}(0, e_i^2)$
- $e_i = \text{s.d.}(Z_i)$ estimated from the time series.

A GPS station in Antarctic.



Forward modelling

From process to observations



*GRACE: the Gravity Recovery And Climate Experiment satellites

$\mathcal{A}_i$ maps the latent process over the appropriate spatial ***foot print*** to the i^{th} observation,

$$Z_i = \mathcal{A}_i \boldsymbol{Y} + \varepsilon_i, \qquad (2)$$

$\mathcal{A}_i$ is usually a linear operator

- point $\rightarrow$ point
- area $\rightarrow$ point
- area $\rightarrow$ area

The BHM for GIA

Denote by $\boldsymbol{\mathcal{A}}^T = \left[\boldsymbol{\mathcal{A}}_1^T, \cdots, \boldsymbol{\mathcal{A}}_N^T\right]$, then we can have the vector form for equation (2)

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}}\boldsymbol{Y} + \varepsilon \tag{3}$$

Bayesian hierarchical model for GIA

Combine the observation equation (3) and the process equation (1)

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{X} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \tag{4}$$

Predicting GIA

The predictive distribution of the latent process

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Predictive Distribution of GIA

The prediction is based on the posterior marginal distribution of the latent process

$$\pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \int_{\boldsymbol{\Theta}} \pi(\boldsymbol{X}, \boldsymbol{\theta}|\tilde{\boldsymbol{Z}})\, \mathrm{d}\boldsymbol{\theta} \tag{5}$$

– Integrating the uncertainty of the parameters.

Other choices include the "plug-in" predictive distribution $\boldsymbol{\pi}_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \boldsymbol{\pi}_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}, \hat{\boldsymbol{\theta}})$, where $\hat{\boldsymbol{\theta}}$ are the estimated values of the parameters, e.g. posterior mean or mode.

Predicting GIA

Point-wise prediction of the latent process

Point-wise predicted mean and uncertainty

For point-wise update at X_i on a fine grid, we use

$$\text{predicted mean:} X_i^* = \mathbb{E}(X_i|\tilde{\boldsymbol{Z}})$$
$$\text{predicted uncertainty:} u_i^* = \text{s.d.}(X_i|\tilde{\boldsymbol{Z}})$$

where $\pi(X_i|\tilde{\boldsymbol{Z}}) = \int_{\mathcal{X}_{-i}} \pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}})\, \mathrm{d}\boldsymbol{X}_{-\boldsymbol{i}}$.

When using the "plug-in" predictive distribution, $\pi_p(X_i|\tilde{\boldsymbol{Z}})$ is Gaussian and the Bayesian update can be written in closed form.

The GMRF approximation

- Bayesian update of the GP on a grid of m points scales as $\mathcal{O}(m^3)$, $m \sim 10^5$ for a 1° global grid.
- Gaussian Markov random field (GMRF) with sparse precision matrix has a much better scaling property.

The SPDE approach (Lindgren et al., 2011)

Denote by $\mathbf{x}$ the GMRF approximation of $\boldsymbol{X}$ on a triangulation of m vertices with basis functions $\{\phi_i\}_{i\in\mathbb{N}}$, then for $s \in \mathbb{S}^2$

$$\boldsymbol{X}(s) \approx \phi_i(s)^T \mathbf{x} \tag{6}$$

For a grid $\boldsymbol{S} \subset \mathbb{S}$ of locations, we find the basis matrix $\boldsymbol{C}$ and write $\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}\mathbf{x}$.

The GMRF approximation

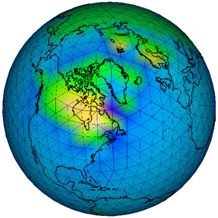
Now the BHM in equation (4) becomes

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{C}\mathbf{x} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \mathbf{x} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}^{-1}(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \tag{7}$$

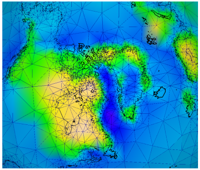
$\mathbf{x}$ is GMRF approximation defined by the precision matrix $\boldsymbol{Q}$.

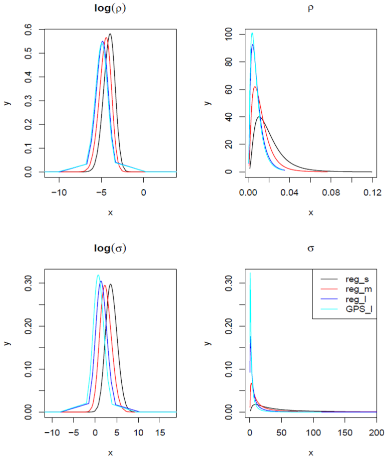
Choosing the mesh on a sphere

semi-regular mesh



irregular mesh

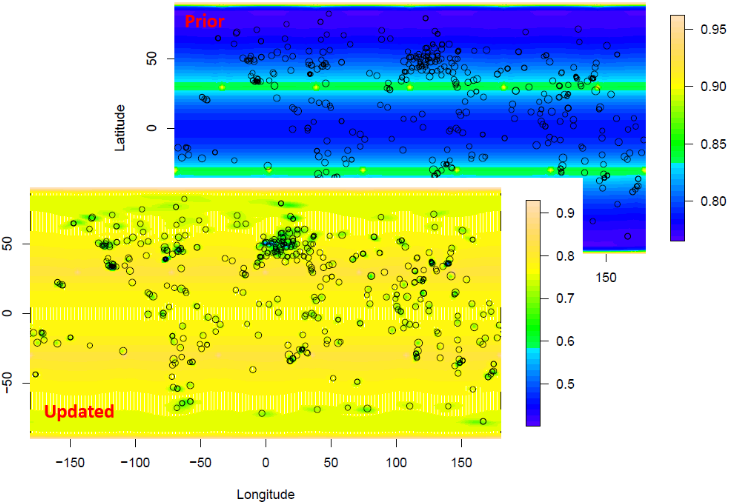




Parameter estimation using different meshes.

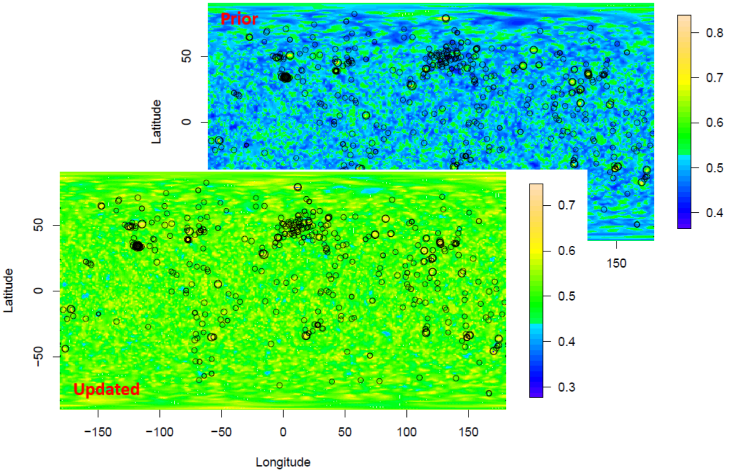
Mesh effect

The predicted uncertainties using regular mesh



Mesh effect

The predicted uncertainties using irregular mesh



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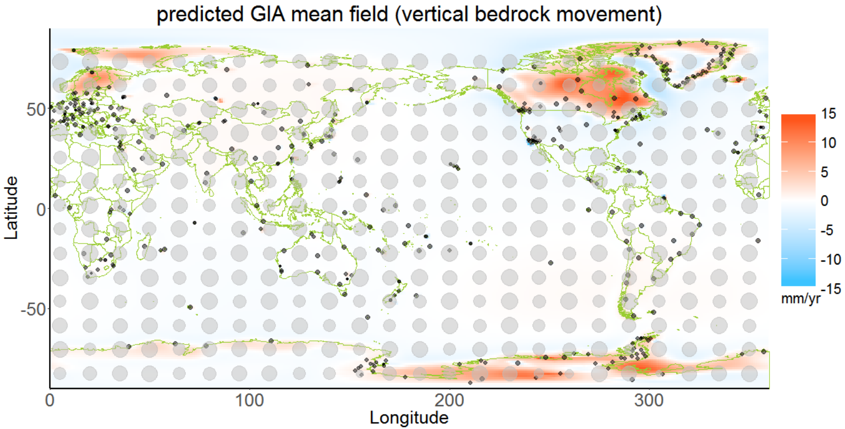
First solution for GIA



GIA disc size (grey) proportional to predicted uncertainty

Conclusions and outlook

- Testing the statistical framework for the GIA.
- Requiring improved mesh.
    - Adaptive – fast computation
    - Regular – reliable approximation
- Improving GPS data quality.
- Extending to the full system.

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Questions

Thank you!