Bayesian estimation of global glacial isostatic adjustment for sea level rise re-evaluation

Zhe Sha[1], Maike Schumancher[1],
Jonathan Rougier[2] and Jonathan Bamber[1]

[1]School of Geographical Sciences,[2]School of Mathematics
University of Bristol

26 July 2017

GlobalMass

- combine satellite and in-situ data related to different aspects of the sea level budget,
- attribute global sea level rise to its component parts.

erc

European Research Council

Established by the European Commission

Funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 69418.

Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \underbrace{\Delta\text{barystatic}(t)}_{\text{mass}} + \underbrace{\Delta\text{steric}(t)}_{\text{density}} + \underbrace{\text{GIA}}_{\text{ocean basins}}$$

- GIA: glacial isostatic adjustment
- inconsistencies between the discipline-specific estimates

GlobalMass Aims

- simultaneous global estimates of all the components
- close the sea level budget

The Bayesian hierarchical model

Observation layer
(direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer
(latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS | LAND ICE | HYDROLOGY | SOLID-EARTH

Parameter layer
(prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

First step – modelling GIA

What is GIA?

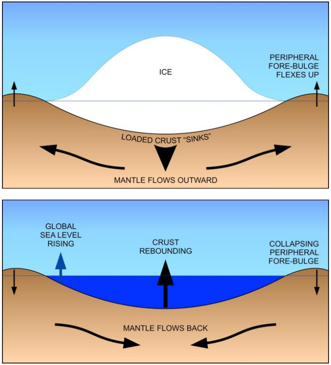


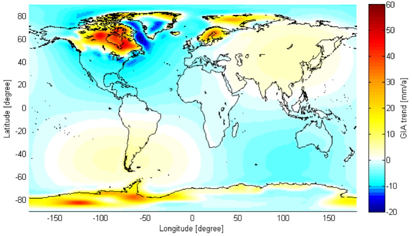
Figure courtesy of the Canadian Geodetic Survey, Natural Resources Canada.

GIA - glacial isostatic adjustment

- vertical displacement of the Earth's crust once burdened by ice-age glaciers (~20,000 yrs ago)
- constant over the time scale of this study (1981-2020)
- Regional discrepancies in estimates from physical models, e.g. ICE-6G (Peltier et al., 2015)

The GIA process

Denote the true GIA process by $\boldsymbol{Y} : \mathbb{S}^2 \mapsto \mathbb{R}$, and the prior mean trend by $\boldsymbol{\mu} : \mathbb{S}^2 \mapsto \mathbb{R}$.



GIA estimates from a ICE-6G model.

After de-trend, the residuals become stationary and assume

$$\boldsymbol{X} := \boldsymbol{Y} - \boldsymbol{\mu} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \qquad (1)$$

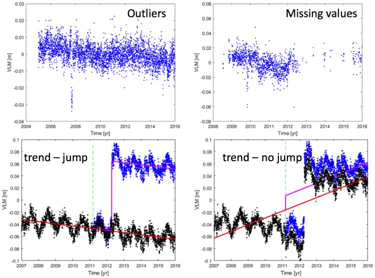
$\kappa(\boldsymbol{\theta})$ – Matérn covariance function with parameter $\boldsymbol{\theta} = (\rho, \sigma^2)$ for the length-scale and variance.

The GPS observations

Vertical movement in the Earth's surface

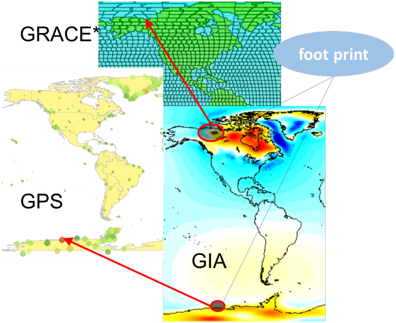
At a selected GPS station i, the time series are processed into an uplift rate Z_i (mm/yr) and a measurement error $\varepsilon_i \sim \mathcal{N}(0, e_i^2)$, where e_i is taken to be the standard error of the time series estimate.

A GPS station in Antarctic.



Forward modelling

From process to observations



*GRACE: the Gravity Recovery And Climate Experiment satellites

Define $\boldsymbol{\mathcal{A}_i}$ as the map from the latent process over the appropriate spatial ***foot print*** to the i^{th} observation,

$$Z_i = \boldsymbol{\mathcal{A}}_i \boldsymbol{Y} + \varepsilon_i, \qquad (2)$$

$\boldsymbol{\mathcal{A}_i}$ is usually a linear operator

- point $\rightarrow$ point
- area $\rightarrow$ point
- area $\rightarrow$ area

The BHM for GIA

Denote by $\boldsymbol{\mathcal{A}}^T = \left[\boldsymbol{\mathcal{A}}_1^T, \cdots, \boldsymbol{\mathcal{A}}_N^T\right]$, then we can have the vector form for equation (2)

$$\boldsymbol{Z} = \boldsymbol{\mathcal{A}} \boldsymbol{Y} + \boldsymbol{\varepsilon} \tag{3}$$

Bayesian hierarchical model for GIA

Combine the observation equation (3) and the process equation (1)

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}} \boldsymbol{X} + \boldsymbol{\varepsilon}, \; \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \boldsymbol{X} \sim \mathcal{GP}(\mathbf{0}, \kappa(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \tag{4}$$

Predicting GIA

The predictive distribution of the latent process

Predictive Distribution of GIA

The prediction is based on the posterior marginal distribution of the latent process

$$\pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \int_{\boldsymbol{\Theta}} \pi(\boldsymbol{X}, \theta|\tilde{\boldsymbol{Z}})\, \mathrm{d}\theta \tag{5}$$

– Integrating the uncertainty of the parameters.

Other choices include the "plug-in" predictive distribution $\pi_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}) = \pi_p(\boldsymbol{X}|\tilde{\boldsymbol{Z}}, \hat{\boldsymbol{\theta}})$, where $\hat{\boldsymbol{\theta}}$ are the estimated values of the parameters, e.g. posterior mean or mode.

Predicting GIA

Point-wise prediction of the latent process

Point-wise predicted mean and uncertainty

For point-wise update at X_i on a fine grid, we use

$$\text{predicted mean:} X_i^* = \mathbb{E}(X_i|\tilde{\boldsymbol{Z}})$$
$$\text{predicted uncertainty:} u_i^* = \text{s.d.}(X_i|\tilde{\boldsymbol{Z}})$$

where $\pi(X_i|\tilde{\boldsymbol{Z}}) = \int_{\mathcal{X}_{-i}} \pi(\boldsymbol{X}|\tilde{\boldsymbol{Z}})\,\mathrm{d}\boldsymbol{X}_{-\boldsymbol{i}}$.

When using the "plug-in" predictive distribution, $\pi_p(X_i|\tilde{\boldsymbol{Z}})$ is Gaussian and the Bayesian update can be written in closed form.

The GMRF approximation

- Bayesian update of the GP on a grid of m points scales as $\mathcal{O}(m^3)$, $m \sim 10^5$ for a 1° global grid.
- Gaussian Markov random field (GMRF) with sparse precision matrix has a much better scaling property.

The SPDE approach (Lindgren et al., 2011)

Denote by $\mathbf{x}$ the GMRF approximation of $\boldsymbol{X}$ on a triangulation of m vertices with basis functions $\{\phi_i\}_{i \in \mathbb{N}}$, then $s \in \mathbb{S}^2$

$$\boldsymbol{X}(s) \approx \phi_i(s)^T \mathbf{x} \tag{6}$$

For a grid $\boldsymbol{S} \subset \mathbb{S}$ of locations, we find the weight matrix $\boldsymbol{C}$ and write $\boldsymbol{X}(\boldsymbol{S}) \approx \boldsymbol{C}\mathbf{x}$.

The GMRF approximation

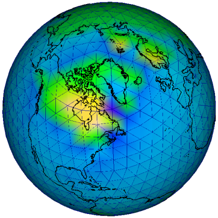
Now the BHM in equation (4) becomes

$$\begin{cases} \tilde{\boldsymbol{Z}} = \boldsymbol{\mathcal{A}}\boldsymbol{C}\mathbf{x} + \varepsilon, \ \varepsilon \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(e_1^2, e_2^2, \ldots, e_N^2)) \\ \mathbf{x} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}^{-1}(\boldsymbol{\theta})) \\ \boldsymbol{\theta} \sim \boldsymbol{\pi}(\boldsymbol{\theta}) \end{cases} \quad (7)$$

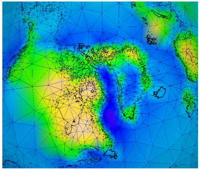
$\mathbf{x}$ is GMRF approximation defined by the precision matrix $\boldsymbol{Q}$.

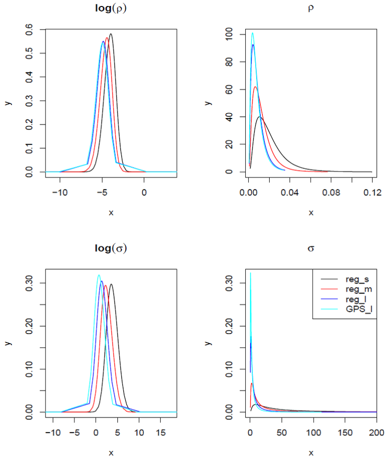
Choosing the mesh on a sphere

semi-regular mesh



irregular mesh

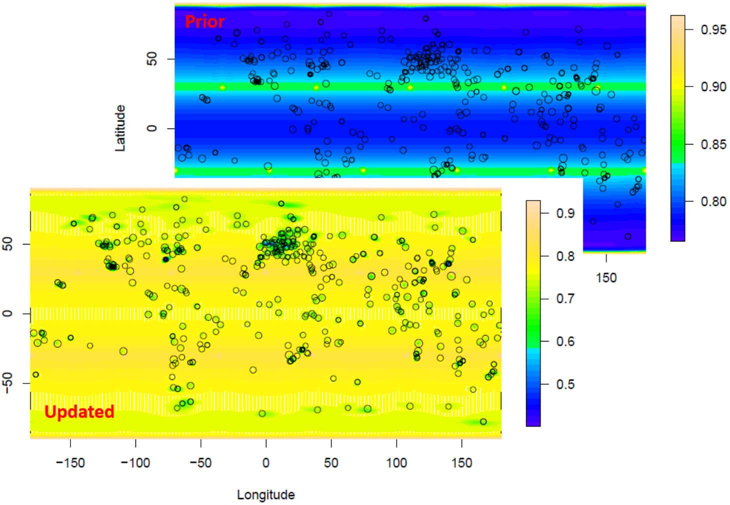




Parameter estimation using different meshes.

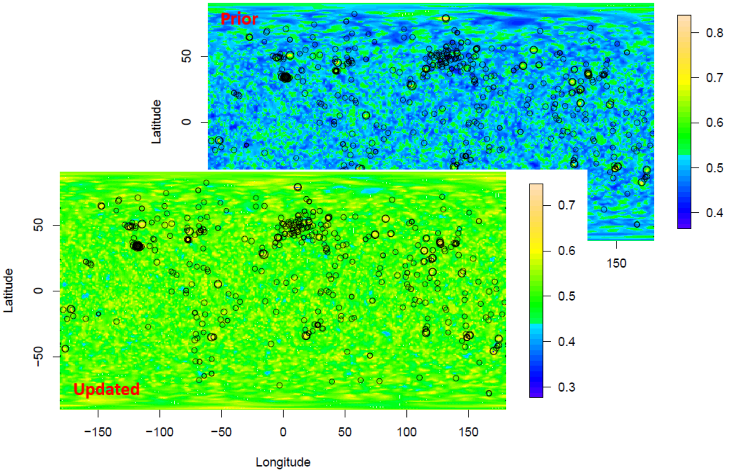
Mesh effect

The predicted uncertainties using regular mesh

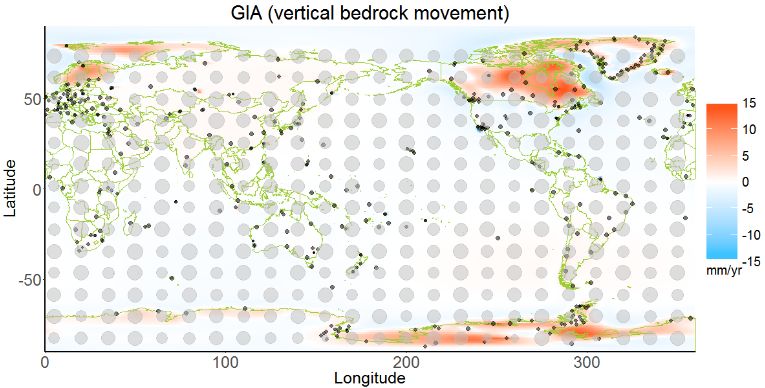


Mesh effect

The predicted uncertainties using irregular mesh



First solution for GIA



Predicted GIA mean field overlaid with uncertainty discs on a sparse grid and at the GPS locations. The disc size is proportional to the predicted uncertainty.

Conclusion

- Tested the statistical framework on the GIA.
- Need a better mesh.
  - Adaptive – fast computation
  - Regularity – stable approximation
- Improve GPS data quality.
- Extend to the full system.

References

Lindgren, F., Rue, H., and Lindström, J. (2011). An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 73(4):423–498.

Peltier, W. R., Argus, D. F., and Drummond, R. (2015). Space geodesy constrains ice age terminal deglaciation: The global ICE-6G_C (VM5a) model. *Journal of Geophysical Research: Solid Earth*, 120(1):450–487.

Questions

Thank you!