

- a) Firstly, transfer the cheetah image into 8\*8 matrices; and turn it into 1\*64 zigzag pattern.

Then, write the EM algorithm for the five random Guassian mixtures.

E-Step: write down the Q-function.

$$\begin{aligned} h_{ij} &= P_{Z|X}(e_j | \mathbf{x}_i; \Psi^{(n)}) \\ &= \frac{\mathcal{G}(\mathbf{x}_i, \mu_j^{(n)}, \sigma_j^{(n)}) \pi_j^{(n)}}{\sum_{k=1}^C \mathcal{G}(\mathbf{x}_i, \mu_k^{(n)}, \sigma_k^{(n)}) \pi_k^{(n)}} \end{aligned}$$

M-Step: Solve the maximization and derived a closed-form solution.

$$\begin{aligned} \mu_j^{(n+1)} &= \frac{\sum_i h_{ij} \mathbf{x}_i}{\sum_i h_{ij}} & \pi_j^{(n+1)} &= \frac{1}{n} \sum_i h_{ij} \\ \sigma_j^{2(n+1)} &= \frac{\sum_i h_{ij} (\mathbf{x}_i - \mu_j)^2}{\sum_i h_{ij}} \end{aligned}$$

Iterating 200 times until the changes are getting smaller.

Then, get EM estimation (mean, variance and probability) for Background and Foreground respectively.

Next, set the priors for BG/FG, which are between (0,1) and sum up to 1.

Applying the Bayes Decision rule to classify each block.

The figures are look similar to each other.

The result shows that the probability of error is independent of the initialization. Mainly, because the EM algorithm is converging to maximum likelihood.



