HomeworkC-Functions

June 12, 2022

Due on Tuesday, June 21 at 5pm.

Please read the instructions *carefully*, and complete all the requested steps in a jupyter notebook. Include all the code you write as "Code" cells and any written responses as "Markdown/Raw NBConvert" cells. Name your notebook (up at the top) `homework#_{identikey}` (replacing `{identikey}` with your identikey and `#` with the appropriate letter for this week's homework).

In this homework you will practice writing and using functions and make some initial progress towards our final class project. The coding for this homework is relatively simple but you will need to spend some time thinking about the physics (vectors, magnitudes, unit vectors) of the problems to make sure you understand what your code is actually doing. Next week's homework will build on this and understanding this now will *really* help on the final project!

Be sure to use comments throughout your code to explain how it works, variables, etc. You must also include docstrings for every function you write.

1 Physics Background: Forces + Vectors

If we want to use forces to calculate the trajectories of particles, we need to know both the magnitude of the force acting on a particular particle and the direction in which it is acting. The easiest way to keep track of this information is using a force *vector*. The following math outlines the steps that go into calculating a gravitational force vector.

The magnitude of the gravitational force between two bodies is

$$F_{ij} = G\frac{m_i m_j}{r_{ij}^2}$$

where G is Newton's gravitational constant, m_i and m_j are the masses of bodies i and j, and r_{ij} is their center-to-center separation.

The gravitational force *vector* is obtained by multiplying the magnitude of the force by a unit vector (a vector with a magnitude of one) in the direction of that force. The gravitational force exerted on body i by body j points from i to j, so the appropriate unit vector to use is

$$\hat{r}_{ij} = \frac{\vec{r}_{ij}}{r_{ij}}$$

The "hat" on $\hat{r}_{ij}$ indicates a unit vector, r_{ij} is the magnitude of the vector, and the vector $\vec{r}_{ij}$ pointing from body i to body j is the difference between the two particle position vectors:

$$\vec{r}_{ij} = (x_j - x_i,\ y_j - y_i,\ z_j - z_i)$$

To obtain r_{ij} from $\vec{r}_{ij}$, recall that the magnitude of any arbitrary vector, $\vec{a} = (a_x,\ a_y,\ a_z)$, is just the total length of that vector:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Putting all this together, the vector force $\vec{F}_{ij}$ is the magnitude of the force F_{ij} multiplied by the force direction $\hat{r}_{ij}$ or

$$\vec{F}_{ij} = F_{ij}\ \hat{r}_{ij}\ = G\frac{m_i m_j}{r_{ij}^2}\hat{r}_{ij}$$

With these mathematical ingredients, you are ready to calculate the force vector from the gravity of one particle acting on another!

2 Coding Assignment: Forces + Vectors

In this assignment, your main task is to write a function that calculates the gravitational force vector between two objects. We will split up this task into multiple smaller functions, and test each one as we go. You ***must*** write docstrings for your functions and ***use comments*** to describe what is happening in your code to to receive full credit.

As you're writing, it will be helpful to have some real numbers to test each function individually. Let's say the Earth is particle "0" with $m_0 = 6.0 \times 10^{24}$ kg, and the Sun is particle "1" with $m_1 = 2.0 \times 10^{30}$ kg. Let's put these at initial (x, y, z) positions of $\vec{p}_0 = (1.37, -1.50, -1.30)$ AU for the Earth and $\vec{p}_1 = (0.50, -2.00, -1.30)$ AU for the Sun.

Throughout this assignment, use three-element `numpy` arrays to represent unit vectors, position vectors, separation vectors, and force vectors. In all cases, the three elements of each `numpy` array will represent the (x, y, z) components of the vector. ***Make sure you are taking advantage of power of `numpy` arrays!*** Remember to be very clear and careful with units throughout your code!

1. Create a Python function that accepts an array representation of a vector $\vec{a} = (a_x,\ a_y,\ a_z)$ and returns its magnitude a.
   - Test this function with a vector $\vec{a} = (1, 2, 3)$; comment on what the answer should be (show the calculation you expect your function to do) and check that your function returns that.
2. Create a Python function that accepts an array representation of a vector $\vec{a} = (a_x,\ a_y,\ a_z)$ and returns its unit vector $\hat{a}$. *(This function should use your previously-created function).*
   - Test this function with that same vector $\vec{a} = (1, 2, 3)$; comment on what the answer should be (show the calculation you expect your function to do) and check that your function returns that.
3. Create a Python function that accepts two position vectors $\vec{p}_i$ and $\vec{p}_j$, and returns the separation vector between them $\vec{r}_{ij} = \vec{p}_j - \vec{p}_i$.
   - Test this function by calculating the separation vector between the positions of the Earth and the Sun above. Use your magnitude function to calculate the magnitude of the distance r_{ij} between these bodies. For the Earth-Sun system, comment on what the

answer should be (you should be able to predict the answer for the magnitude without any calculations!) and check that your function returns that.

4. Create a Python function that accepts two masses and the magnitude of the separation between them as input and returns the magnitude of the force as a float. This is the force *magnitude*, not the force *vector*.
   - For the Earth-Sun system, comment (generally) on what the answer should be and check that your function returns something reasonable.
5. Create a Python function that accepts two masses and two position vectors, and returns the full gravitational force vector $\vec{F}_{ij}$ acting on a particle. *(This function should use some of your previously-created functions).*
   - Test your function by calculating the gravitational force on the Earth (particle 0) caused by the Sun (particle 1). Comment (generally) on what the answer should be and check that your function returns something reasonable. In particular, make sure you can explain the sign of each dimension of your force vector.

3 Turn in your Assignment

Your final version of your assignment should run from top to bottom without errors. (You should comment out or delete code blocks that didn't work unless you are submitting unfinished work for partial credit.) To create a clean version, rerun your notebook using "Kernel|Restart & Run All." Be sure to save this final version (with output). To submit, click "File|Download As >|HTML (html)" inside jupyter notebook to convert your notebook into an HTML web page file. It should have a name like `homework#_{identikey}.html` (where # is the appropriate letter for this week's homework). You can open this file in a browser to see what it looks like! Please upload this HTML file as your homework submission on `Canvas`.