Consider a portfolio comprising of a $100,000 par value 3-year zero coupon bond and a $100,000 par value 20-year zero coupon bond.

US Treasury Yield Curve Rates

Term (Year)	1	2	3	5	7	10	20	30
Spot Rate (%)	0.15	0.48	0.75	1.18	1.44	1.55	1.98	1.93

By mapping the cash flows to vertices at 3 and 20 years, the PV01 of the two bonds are calculated.

PV01 of the 3-Year Coupon Bond

Years to Maturity	Cash Flow	Spot Rate (%)	Present Value	Spot Rate (%)	Present Value	PV01
3	100,000	0.75	97,783.33	0.74	97,812.46	29.13

PV01 of the 20-Year Coupon Bond

Years to Maturity	Cash Flow	Spot Rate (%)	Present Value	Spot Rate (%)	Present Value	PV01
20	100,000	1.98	67,561.59	1.97	67,694.22	132.63

Principal component analysis (PCA) is performed on the correlation matrix of daily changes in spot rates, using data between January 2010 and December 2020.

Results of the PCA

Principal Component (PC)	Eigenvalue	Variance Explained (%)	Cumulative Variance Explained (%)
1	6.3248	79.06	79.06
2	1.0033	12.54	91.60
3	0.4305	5.38	96.98
4	0.1366	1.71	98.69
5	0.0548	0.68	99.37
6	0.0216	0.27	99.64
7	0.0153	0.19	99.83
8	0.0131	0.16	99.99

The first 3 principal components are retained for parsimony reason as they explain more than 95% of the total variance. The 3-factor model aligns with Litterman and Scheinkman (1991). They identified that there are 3 common factors influencing the yield curve: level, slope, and curvature.

Eigenvectors for the first 3 Principal Components

	PC1	PC2	PC3
DGS1	0.2283	-0.6862	0.6726
DGS2	0.3342	-0.3995	-0.3726
DGS3	0.3657	-0.2371	-0.3708
DGS5	0.3848	-0.0198	-0.2556
DGS7	0.3877	0.1194	-0.1017
DGS10	0.3835	0.2240	0.0647
DGS20	0.3642	0.3375	0.2683
DGS30	0.3526	0.3668	0.3457

Net Sensitivities on PC Risk Factors

PC	1	2	3
Beta	58.9567	37.8559	24.7832

The portfolio volatility is

$$\sigma_{daily} = \sqrt{58.9567^2(6.3248) + 37.8559^2(1.0033) + 24.7832^2(0.4305)} = \$153.9043$$

$$\sigma_{monthly} = 153.9043 \times \sqrt{20} = \$688.2809$$

The 5% 1-month VaR of the portfolio is $1135.66.