

## Tutorial 02

### Matrix Algebra- Additional Questions

1. The sales figures for two car dealers during June show that Dealer A sold 100 compacts, 50 intermediates, and 40 full size cars, while Dealer B sold 120 compacts, 40 intermediates, and 35 full-size cars. During July, Dealer A sold 80 compacts, 30 intermediates, and 10 full-size cars, while Dealer B sold 70 compacts, 40 intermediates and 20 full-size cars. Total sales over the 3 month period of June – August revealed that Dealer A sold 300 compacts, 120 intermediates and 65 full-size cars. In the same 3 month period, dealer B sold 250 compacts, 100 intermediates, and 80 full-size cars.

- (a) Write 2 x 3 matrices summarizing sales data for June, July and the 3 month-period.
- (b) Find the sales over the 2-month period June and July.
- (c) Find the sales in August.

2. A company stock pots and pans in three different sizes. The numbers in stock are shown in the following table.

	Stock	
Size	Pots	Pans
Large	10	13
Medium	24	16
Small	17	9

A delivery of pots and pans arrives, with the numbers as shown in matrix D (which has rows showing sizes and columns showing pots or pans). In the following two weeks the numbers of pots and pans sold are described by the matrices W1 and W2.

$$D = \begin{bmatrix} 5 & 20 \\ 12 & 7 \\ 3 & 6 \end{bmatrix}$$

$$W1 = \begin{bmatrix} 3 & 16 \\ 10 & 15 \\ 4 & 2 \end{bmatrix}$$

$$W2 = \begin{bmatrix} 12 & 17 \\ 26 & 8 \\ 16 & 13 \end{bmatrix}$$

- (a) Describe the current stock-in-hand in matrix form.
- (b) What are the stocks after the delivery arrives?
- (c) What are the stocks remaining after each of the following week?

3. Use matrix inversion to solve the simultaneous equations:

(a)  $4x + 2y = 13$

$3x + 2y = 16$

(c)  $3x + y = -2$

$4x - 3y = -5$

(b)  $4x + 2y = 1$

$-2x - y = 2$

(d)  $-2x + 5y = 1$

$3x - 4y = 2$

4. In part of its business, a coffee blender uses two types of beans, T1 and T2, to make

two blends of coffee, American and Brazilian. The American blend uses 75% of the available beans of Type T1 and 10% of the available beans of Type T2. The Brazilian blend uses 20% of the available beans T1 and 60% of the available beans T2. One month the blender buys  $t_1$  and  $t_2$  kilograms of beans T1 and T2 respectively and makes  $Q_a$  and  $Q_b$  kilograms respectively of American and Brazilian blends.

- (a) Use matrices to describe this problem
  - (b) If  $t_1 = 200\text{kg}$  and  $t_2 = 300\text{kg}$ , how much of the American and Brazilian blends can the blender produce?
  - (c) If the blender wants to produce 400kg of the American blend and 600kg of the Brazilian blend, what beans should it buy?
5. A quadrilateral has vertices A, B, C, and D with coordinates (1,-1), (2,1), (2,0), and (-2,1) respectively.

$$V = \begin{bmatrix} 1 & 2 & 2 & -2 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

These points form the matrix **V** where:

Let **M** be a matrix which represents a geometric transformation. Then the matrix product **MV** has as its columns the images of A, B, C, and D after applying the geometric transformation.

For each of the following determine the matrix **M** representing the transformation and then find **MV**, the matrix of coordinates for the image of the quadrilateral under the transformation. In each case confirm your result for the final position of the quadrilateral using a sketch of the effect of the transformation on the original quadrilateral.

- (a) A rotation of  $90^\circ$  about the origin
- (b) A reflection in the x-axis
- (c) A stretch scale factor 4 parallel to the x axis followed by a rotation of  $90^\circ$  about the origin.

Confirm your answer to (c) by first applying a stretch scale factor 4 to **V** to give the co-ordinates of the image of the quadrilateral under this transformation. Then apply a rotation of  $90^\circ$  about the origin to the co-ordinates of the resulting image (as given in the matrix **MV**) to give the final co-ordinates of the quadrilateral after both transformations have been applied.

6. Consider  $V = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ , the matrix representation of a rotation of  $90^\circ$  about the origin.

- (a) What transformation would we need to apply after a rotation of  $90^\circ$  about the origin to cancel the effect of applying it, i.e. what is the inverse transformation to a rotation of  $90^\circ$  about the origin?
- (b) Let **T** be the matrix representation for the inverse transformation found in (a). What is **T**?

- (c) Find  $\mathbf{ST}$  and comment on your result.
  - (d) Find  $\mathbf{S^{-1}}$  and compare your result with  $\mathbf{T}$ . What do you observe?
  - (e) What can you conclude about the inverse of a transformation and the inverse of a matrix representing a transformation?
7. Determine the inverse transformation for each of the following and find the matrix representation for the inverse transformation. Confirm for each that the matrix representation of the transformation and the matrix representation of its inverse multiply to give  $\mathbf{I_2}$ .
- a) a rotation of  $270^\circ$  about the origin
  - (b) a reflection in the y axis
  - (c) a linear stretch scale factor 3 parallel to the y axis
- What do you notice about the transformation in (b) and its inverse?
8. By considering the images of the basis vectors  $\mathbf{i}$  and  $\mathbf{j}$  write down the  $2 \times 2$  matrices which represent the following transformations:
- (a) a rotation of  $-180^\circ$  about the origin
  - (b) a rotation of  $-270^\circ$  about the origin
  - (c) an enlargement, scale factor 0.5, centre the origin
  - (d) a linear stretch scale factor 3 parallel to the x axis
  - (e) a linear stretch scale factor 1.5 parallel to the y axis
  - (f) a shear scale factor 1.4 parallel to the x axis

The following will need some trigonometry or Pythagoras to answer:

- (g) a rotation of  $135^\circ$  about the origin
  - (h) a rotation of  $45^\circ$  about the origin
9. By multiplying the appropriate matrices representing the composite transformations together, find the single matrix representation for each of the following transformations:
- (a) A rotation of  $180^\circ$  about the origin followed by a reflection in the y axis.
  - (b) A reflection in the y axis followed by a rotation of  $180^\circ$  about the origin.
  - (c) A rotation of  $-90^\circ$  about the origin followed by a reflection in the x axis.
  - (d) A reflection in the x axis followed by a rotation of  $-90^\circ$  about the origin.