

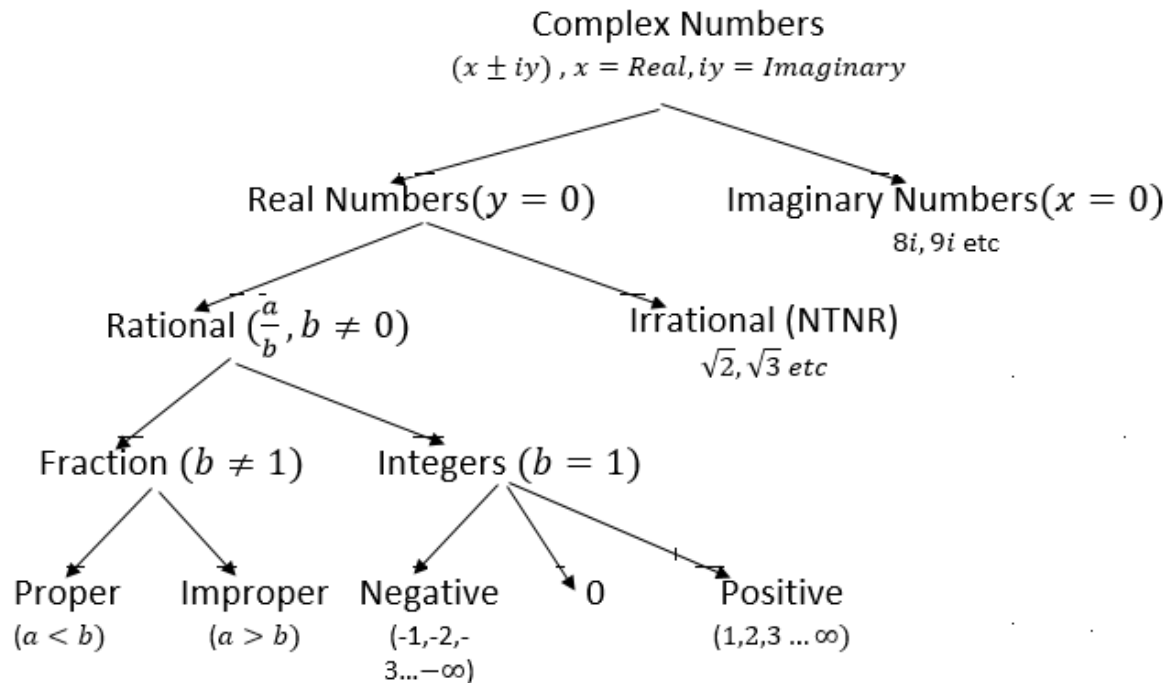
**A QUANTITATIVE
APPROACH TO
PROBLEM SOLVING**

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NUMBERS-1

Classification:



Complex Numbers: A number of the type $a + ib$ where a is real part and ib is the imaginary part. If $b = 0$, then the number is real. And if $a = 0$ and $b \neq 0$ then it is an imaginary number.

Note: Imaginary numbers cannot be compared. For example, we cannot compare between $8i$ and $9i$ since their values are unknown.

Real numbers: All numbers which can be represented using a number line system, are real numbers. All real numbers can be compared.

Rational Numbers: All real numbers which can be represented in the form of $\frac{a}{b}$ where $b \neq 0$. Rational numbers are subset of real numbers.

Integers: Those rational numbers whose denominator is 1 are integers.

Fractions: Those rational numbers whose denominator is $\neq 1$ are fractions. Fractions less than 1 are proper fractions and greater than 1 are improper fractions.

Natural: All positive integers are Natural numbers.

Note:

1. Sum of Even number of Odds is always Even else it's Odd.
2. Product of Even and Odd is always Even.
3. Product of Odd and Odd is always Odd.
4. An Even number may be divisible by both Even and Odd.
5. An Odd number is divisible by only Odd numbers.

Prime numbers: Any natural number greater than 1 with exactly two factors i.e., 1 and itself. Ex: 2, 3, 5, 7, 11, 13, 17, 19and so on.

Note: A prime number is of the form $6K + 1$ or $6K - 1$ or both.

There are 25 prime numbers less than 100.

3, 5 and 7 are the only three consecutive prime numbers with a common difference of 2.

Test of prime numbers(N):

Step 1: Find the nearest perfect square of N but less than N say x^2 .

Step 2: Apply divisibility of all prime numbers less than or equal to x on N.

Step 3: if not divisible by any in step 2, N is prime else not a prime.

Composite Numbers: All natural numbers greater than 1 with more than 2 factors.

Ex: 4, 6, 8, 9, 10, 12,.....and so on....

Co-Prime Pairs: Two natural number set (a, b) are said to be co-prime pairs if HCF of (a, b) is 1.

Example of co-prime pairs are (2, 9), (3, 8), (5, 7), (24, 77) etc.

Note: To form a co-prime pairs, the numbers must not necessarily be prime numbers.

Divisibility Rules:

Divisible by	Test	Example
2 or 5	Last digit divisible by 2 or 5	234 3 not divisible by 2 or 5
4	Last 2 digit divisible by 4	212 56 div by 4 as 56 is div by 4
8	Last 2 digit divisible by 4	512 984 div by 8 as 984 is div by 8
3 or 9	Sum of digits divisible by 3 or 9	49311->4+9+3+1+1=18 divisible 3 and 9
11	Difference of Sum of alternate digits divisible by 11	584232-> (5+4+2)-(8+2+2)=0 So div by 11
7	<p>Multiply the last digit by 2 and subtract from remaining till you get a number div by 7 else not divisible</p> <p>OR</p> <p>Make groups of 3 digits from right to left and alternately add and subtract. The result if divisible by 7 then entire number is divisible by 7 else not</p> <p>**The same method can also be applied for checking the divisibility by 11</p>	<p>51844-></p> $\begin{array}{r} 51844 \\ -8 \\ \hline 5176 \\ -12 \\ \hline 505 \\ -10 \\ \hline 40 \end{array}$ <p>40->not divisible by 7 So 51844 not div by 7</p> <p>Example: 100300200355 $\Rightarrow 100 300 200 355$ $\Rightarrow (300+355)-$ $(100+200)=355$ $\Rightarrow 355/7 \text{ Rem}=5$ $\Rightarrow \text{Hence not divisible by 7}$</p>
13	Same as Rule for 7	Try with the above example
17	Similar with the rule for 7 but with a group of 6 digits	Try with the above example

Divisibility for composite nos.

For any composite number composed of more than one type of prime as factors:

Step 1: Factorize the number into co-prime factors.

Step 2: Check the divisibility by those factors.

Divisible by those factors ensures divisibility by that number. The reason for selecting the co-prime factors is LCM of these numbers is the product of those numbers.

Example:

If we wish to check the divisibility of a number by 24, then first consider factors of 24 such as:-

$24 = 8 \times 3 \rightarrow$ co-prime
$24 = 6 \times 4 \rightarrow$ not co-prime
$24 = 12 \times 2 \rightarrow$ not co-prime
Only co prime pair=(8,3), $\text{LCM}(8,3) = 8 \times 3 = 24$

That is, when a number is divisible by 8 and 3, the number must be divisible by 24.

Simplification

In simplification of an expression, the various operations must be strictly performed in the following order.

1. Vinculum or bar () **V**
2. Removal of brackets in the order (), { }, [] **B**
3. Of **O**
4. Division **D**
5. Multiplication **M**
6. Addition **A**
7. Subtraction **S**

Remember it as **VBODMAS**.

Important Rules

1. Sum of 5 consecutive whole numbers will always be divisible by 5.
2. Squares of all numbers that are not divisible by 3 will leave remainder of 1 when divided by 3.
3. The square of an odd number will always leave remainder of 1 when divided by 8.
4. The product of 3 consecutive natural numbers i.e. $n^3 - n$ is divisible by 6.
5. The difference between two digit numbers xy and yx is divisible by 9.
6. $10n - 1$ is divisible by 9.
7. $(m + n)!$ is divisible by $m! n!$.
8. $an/(a + 1)$ leaves remainder of a if n is odd and leaves remainder 1 if n is even.
9. $(a + 1)n/a$ will always leave remainder of 1.
10. For any natural number n^5 has the same unit's digit as that of n .
11. For a given sum, the product is maximum if two numbers are equal.
12. If $a_1 \neq a_2 \neq a_3 \neq \dots \neq a_n$ then

Power Cycle:

Last Digit	Power Cycle
2, 3, 7 and 8	4
4 and 9	2
0,1,5 and 6	1

Example:

1. Find the unit digit of the product: $234 \times 347 \times 342$
2. Find the unit digit of the product: $212^{323} \times 47^{16} \times 219^{23}$
3. Find the unit digit of the product: $212^{323} + 47^{16} + 219^{23}$
4. Find the unit digit of the product: $212^{323} - 47^{16} - 219^{23}$

Last 2 digits: (Advanced)

1. A^k , A ends with 0 and k is a natural number
→ Last 2 digits will be 00
2. A^k , A ends with 5 and k is a natural number
→ Last 2 digits will be 25
3. $2m^{40k+1}$, m = odd not ending with 5, k is a natural number
→ Last 2 digits will be $(2m + 50)$
4. For all remaining cases i.e., A^{40k+r}
→ last 2 digits = last 2 digits of A^r

Making table of powers of a 2 digits' number:

47^2 : 47 is closer to 50 and the absolute difference is 3

$3^2 = 9$. So last two digits of 47^2 is 09

Similarly, last 2 digit of 57^2 is same as the last two of 7^2

Also, if the base is closer to 100, then we can take the absolute difference of 100 and the base.

88^2 has the last two digits same as the 12^2 i.e., 44.

Practice Problems:

- Find the last two digit of 34^{81} .
 Solution: $34 = 2 \times 17$, $81 = 2 \times 40 + 1$
 Last 2 digits = $2 \times 17 + 50 = 84$
- Find the last two digit of 58^{121}
 Solution: $58 = 2 \times 29$, $121 = 3 \times 40 + 1$
 Last 2 digits = $2 \times 29 + 50 = 108$ i.e., 08
- Find the last two digit of 34^{82}
 Solution: $82 = 2 \times 40 + 2$

Remainder:

Let us first know the binomial expansion of:

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}b^n$$

All terms are multiples of ' a ' except for the last term, which is a multiple of ' b ' only.

So, when $(a + b)^n$ is divided by a , the remainder is the last term.

Also note:

$a^{2n} - b^{2n}$ is always divisible by $a^2 - b^2$

$a^{3n} - b^{3n}$ is always divisible by $a^3 - b^3$

- ## LEVEL 2:

1. $4a56$ is a four-digit number divisible by 33. What is the value of a ?
a. 3 b. 4 c. 5 d. 6
2. What is the unit digit of the expression $1^{222} + 2^{222} + 3^{222} + \dots + 9^{222}$?
a. 5 b. 0 c. 1 d. 9
3. Find the respective values of x and y if $268x51y$ is divisible by 72.
a. 3, 4 b. 5, 4 c. 5, 2 d. 3, 2
4. A number consists of two digits. If the number formed by interchanging the digits is added to the original number, the resulting number is always divisible by
a. 11 b. 9 c. 5 d. 3

5. $N = 1! + 2! + 3! + \dots + 10!$. What is the unit digit of N^N ?
a. 1 b. 2 c. 3 d. 4
6. Find the unit digit of the product $2464^{1793} \times 515^{317} \times 131^{491}$.
a. 0 b. 1 c. 2 d. 3
7. Find the last two digits of the product $20025 \times 20026 \times 200027 \times 200035$.
a. 80 b. 70 c. 60 d. 50
8. $3^{x-y} = 27$, $2^{x+y} = 128$, find x .
a. 5 b. 4 c. 7 d. 8
9. Find the value of $x - y$, if $x^2 + y^2 = 25$ and $xy = 12$.
a. 1 b. 2 c. 3 d. 4
10. How many key strokes are required to type numbers from 1 to 1000?
a. 2890 b. 2892 c. 2889 d. 2893

NUMBERS-II

Factors:

Factors are those divisors which leaves no remainder. A natural number (except 1) must have two or more than two factors. Let us check the factors of some natural numbers:

Natural Number	Factors	No. of factors
1	1	1
2	1, 2	2
4	1, 2, 4	3
6	1, 2, 3, 6	4
12	1, 2, 3, 4, 6, 12	6
9	1, 3, 9	3
24	1, 2, 3, 4, 6, 8, 12, 24	8
16	1, 2, 4, 8, 16	5
36	1, 2, 3, 4, 6, 9, 12, 18, 36	9

Formula for finding the number of factors:

Let N be a natural number and p, q, r, ... are prime factors of N raised to the power x, y, z, ...i.e.,
 $N = p^x \times q^y \times r^z..$

Then number of factors = $(x + 1)(y + 1)(z + 1)..\$

Example: How many natural numbers can divide 60 exactly leaving no remainder.

Solution: First we express 60 in prime factors as

$$60 = 2^2 \times 3^1 \times 5^1 \text{ So, number of factors} = (2+1)(1+1)(1+1) = 12$$

Additional Analysis:

In the above table, the cells in the first column which are highlighted have values which are perfect squares. The number of factors in each case is odd. Following points can be concluded:

Number of factors of N is	N is
Odd but not prime	Perfect square with more than one type of prime factor. E.g., 36, 144 etc.
Prime (except 2)	Perfect Square with only one type of prime factor, e.g., 4, 9, 16 etc.

Example: A natural number N has 7 natural factors. How many factors does N^2 have?

Solution: No. of factors is 7, a prime number, N must be a perfect square of type a^6 , where a is a prime no. So, $N^2 = a^{12}$ No. of factors = $12+1=13$.

Additional Types on Factors:

Let $N = 2^4 \times 3^3 \times 5^4$

a. Find the sum of all the factors of N.

Solution: For these, let us express N as

$$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2 + 3^3)(5^0 + 5^1 + 5^2 + 5^3 + 5^4)$$

Factor 2 as a max power of 4, so 2 is raised to power from 0 till 4. Similar is the case for factor 3 and 5.

Sum of all factors=Simplified value of the above expression.

$$\begin{aligned} &= (1+2+4+8+16)(1+3+9+27)(1+5+25+125+625) \\ &= 31 \times 40 \times 781 = 968440 \end{aligned}$$

b. Find the number of odd factors.

Solution: From the above expression, consider only the odd factors i.e., $3^0, 3^1, 3^2, 3^3 \rightarrow 4$ factors

And $5^0, 5^1, 5^2, 5^3, 5^4 \rightarrow 5$ factors

$$\text{Number of odd factors} = 4 \times 5 = 20$$

Note: Number of Even factors= Total number of factors- Odd Factors

c. Find the number of factors which are perfect squares.

Solution: For perfect squares, consider only even powered factors

i.e., $2^0, 2^2, 2^4 \rightarrow 3$ factors

$3^0, 3^2 \rightarrow 2$ factors

$5^0, 5^2, 5^4 \rightarrow 3$ factors

$$\text{Number of factors} = 3 \times 2 \times 3 = 18 \text{ factors}$$

d. Find the number of factors which are perfect cubes.

Solution: For perfect squares, consider only those factors having powers as multiples of 3

i.e., $2^0, 2^3 \rightarrow 2$ factors

$3^0, 3^3 \rightarrow 2$ factors

$5^0, 5^3 \rightarrow 2$ factors

$$\text{Number of factors} = 2 \times 2 \times 2 = 8 \text{ factors}$$

e. In how many ways N can be expressed as a product of two numbers?

Solution: Number of factors/2=100/2=50

Note: If number of factors is odd, then $\rightarrow \frac{\text{No. of factors} + 1}{2}$

HCF-LCM

Prime Factorization

We can write any composite number as a product of prime factors. This is called prime factorization.

Least Common Multiple (LCM)

Least common multiple of two or more numbers is the least number which is divisible by each of these numbers without leaving a remainder. It is also known as Lowest Common Dividend.

Methods of Finding LCM of Given Numbers

There are two methods which are usually used to find LCM of two or more numbers.

1. Prime Factorization Method:

Resolve each one of the given numbers into prime factors. The LCM is the product of the highest power of all prime factors.

2. Division Method:

In this method, we divide the given numbers by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. We keep on repeating this process till no two numbers have a common factor. The product of the divisors and the remaining numbers is LCM of the given numbers.

Highest Common Factor (HCF)

Highest Common Factor of two or more given numbers is the largest common factor. It is also known as Greatest Common Factor (GCF) or Greatest Common Divisor (GCD) or Greatest Common Measure (GCM).

Methods of Finding HCF of Given Numbers

There are two methods that are usually used to find the HCF of given numbers.

1. Prime Factorization Method:

In this method, we write prime factors of each of the given numbers in the exponential form. HCF is the product of the common prime factors with least powers.

2. Long Division Method:

In this method, we divide the largest number by smaller number and get a remainder. Then we divide the first divisor by the remainder getting a new remainder and continue this process till the last number is zero. The last divisor in this process is the HCF of the given two numbers.

In order to find the HCF of three or more numbers, follow these steps:

- i. First find the HCF of any two numbers.
- ii. Then find the HCF of the third number and the HCF obtained above.
- iii. HCF obtained in step (ii) is the required HCF of the given three numbers.

for large numbers. when $a > b$. $HCF(a, b) = HCF(b, a - b)$

HCF and LCM of Fractions

- i) HCF of Fractions = HCF of Numerators/LCM of Denominators
- ii) LCM of Fractions = LCM of Numerators/HCF of Denominators

Product of Two Numbers

If two numbers are A and B, then $\text{HCF}(A, B) \times \text{LCM}(A, B) = A \times B$. This formula is applicable for two numbers only.

1. For any two digit numbers, HCF is a factor of LCM.
2. Any number which divides each of the two numbers also divides their sum, difference, and also difference of their multiples.

Important Results

1. Lowest number that is divisible by A, B, C leaving same remainder 'r' in each case is $\text{LCM}(A, B, C) + r$.
2. Greatest number that will divide A, B, C leaving remainders r_1, r_2 , and r_3 respectively is HCF of $(A - r_1), (B - r_2)$, and $(C - r_3)$.

Factorials:

Factorial of a natural number n is the product of first n natural numbers and it is denoted by $n!$

$$n! = 1 \times 2 \times 3 \times 4 \times \dots (n-3)(n-2)(n-1)n$$

$$1! = 1$$

$$2! = 1 \times 2 = 2$$

$$3! = 1 \times 2 \times 3 = 6$$

$$4! = 1 \times 2 \times 3 \times 4 = 24$$

And so on...

Note:

- a. $0! = 1$
- b. If p is a prime number, all factorials greater than or equal to p is always divisible by p .
(E.g., 7 can divide $n!$ only if $n \geq 7$)

Highest power of a prime factor that divides $n!$ exactly.

Example: Find the highest power of 5 that divides 50! Exactly.

Solution: The rule is applicable for all prime numbers.

$\frac{50}{5} + \frac{50}{5^2}$ [Divide 50 by 5 and increase the power of 5 till it divides 50 and add all the quotients]

$$\Rightarrow 10+12=22$$

Max power of 5 that divides 50! Is 12.

Similarly, we can find max power of three, in the following way

$$\frac{50}{3} + \frac{50}{3^2} + \frac{50}{3^3}$$

$$\Rightarrow 16+5+1=22$$

Max power of 3 that divides 50! Is 22.

Note: Number of trailing zeroes implies highest power of 10 which is also same as the highest power of 5

Practice Problems:

1. What is the maximum power of 7 that divides 110!
 2. Find the maximum power of 5 that divides 70! exactly.
 3. Find the number of trailing zeroes in 55!
 4. Find the maximum power of 12 that divides 50! exactly.
 5. Find the number of trailing zeroes in 50! + 40!.
 6. Find the number of trailing zeroes in 70! – 40!.
 7. Find the number of trailing zeroes in 70! × 40!
-

Topics: HCF, LCM, Factors and Factorials

Number of Questions: Class Work: 25+10 Practice Sheet: 20

CLASS-SHEET

LEVEL 1:

1. Three numbers are in the ratio of 3: 4: 5 and their L.C.M. is 2400. Their H.C.F. is:
a. 40 b. 80 c. 120 d. 200
2. The ratio of two numbers is 3: 4 and their H.C.F is 4. Their L.C.M is
a. 12 b. 16 c. 24 d. 48
3. Product of two co-prime numbers is 117. Their L.C.M should be
a. 1 b. 117 c. Equal to their HCF d. Cannot be determined
4. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:
a. 1 b. 2 c. 3 d. 4
5. The H.C.F of two numbers is 11 and their L.C.M is 7700. If one of the numbers is 275, then the other is
a. 279 b. 283 c. 308 d. 318
6. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is:
a. 123 b. 127 c. 235 d. 305
7. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:
a. 74 b. 94 c. 184 d. 364
8. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is
a. 1008 b. 1015 c. 1022 d. 1032
9. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is:
a. 1677 b. 1683 c. 2523 d. 3363
10. Which of the following has the highest number of factors?
a. 60 b. 162 c. 172 d. 80
11. If $N = 2^7 \times 3^6 \times 5^3$
a. Find the total number of factors of N
b. Find the total number of factors of N which are odd
c. Find the total number of factors of N which are even
d. Find the total number of factors of N which are multiple of 3
e. Find the total number of factors of N which are multiple of 12
f. Find the total number of factors of N which are perfect square
g. Find the total number of factors of N which are perfect cube
h. Find the product of all the factors
i. In how many ways N can be written as a product of two numbers?
12. If $N = 150!$, find
a. The highest power of 2
b. The highest power of 3

- c. The highest power of 6
 - d. The highest power of 48
13. Find the number of trailing zeroes in the product $125 \times 332 \times 175 \times 333$.
- a. 1
 - b. 2
 - c. 3
 - d. 4
14. In how many ways, the number 36 can be expressed as a product of two natural numbers?
- a. 3
 - b. 5
 - c. 1
 - d. 6
15. Find the number of trailing zeroes in $72!$.
- a. 16
 - b. 17
 - c. 12
 - d. 10
16. A number has 72 factors. What can be the minimum and maximum number of prime factors of this number?
- a. 1, 71
 - b. 6, 2
 - c. 5, 2
 - d. None of these
17. Find the number of trailing zeroes in $70! \times 30!$.
- a. 23
 - b. 16
 - c. 6
 - d. 30
18. A number N has exactly 11 natural factors. How many factors will N^2 have?
- a. 12
 - b. 22
 - c. 21
 - d. 20
19. A rectangular courtyard 3.78 meters long 5.25 meters wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose?
- a. 14 cm
 - b. 21 cm
 - c. 42 cm
 - d. None of these
20. Three gold coins of weight 780gm, 840gm and 960gm are cut into small pieces, all of which have the equal weight. Each piece must be as heavy as possible. If one such piece is shared by two persons, then how many persons are needed to give all the pieces of gold coins?
- a. 86
 - b. 70
 - c. 43
 - d. 35
21. Six bells started to toll together and toll at an interval of 2, 3, 4, 5, 6 and 8 seconds respectively. How many times do they toll together in the next 60 minutes?
- a. 31
 - b. 30
 - c. 29
 - d. Never
22. Find the greatest number which will divide 215, 167 and 135 so as to leave the same remainder in each case?
- a. 64
 - b. 32
 - c. 24
 - d. 16
23. Square slabs are to be placed on a rectangular floor of size 4 m 37 cm in length and 3 m 23 cm in width. Find the minimum number of such slabs required?
- a. 391
 - b. 361
 - c. 19
 - d. 40
24. Three runners running around a circular race track can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point again?
- a. 22
 - b. 33
 - c. 11
 - d. 44
25. The least number which when divided by 3, 7 and 4 leaves a remainder of 3 but when divided by 9 leaves no remainder?
- a. 171
 - b. 81
 - c. 90
 - d. Cannot be determined

9. The sum of two numbers is 528 and their H.C.F is 33. The number of pairs of numbers satisfying the above condition is
 a. 4 b. 6 c. 8 d. 12
10. Find the number of trailing zeroes in $50! + 70! - 20!$.
 a. 4 b. 5 c. 6 d. 12

PRACTICE SHEET

- Find the remainder when 7^{777} is divided by 100?
 a. 1 b. 61 c. 41 d. 21
- If $N = 888\dots$ up to 100 digits, what is the remainder when N is divided by 625?
 a. 128 b. 138 c. 338 d. 388
- A natural number n is such that $120 \leq n \leq 240$. If HCF of n and 240 is 1, how many values of n are possible?
 a. 24 b. 32 c. 36 d. 40
- If a, b, c and d are integers such that $a + b + c + d = 30$, then find the minimum value of $(a - b)^2 + (a - c)^2 + (a - d)^2$.
 a. 4 b. 3 c. 2 d. 1
- The numbers 1, 2, 3...9 are arranged in a 3×3 square grid in such a way that each number occurs once and the entries along each column, each row and each diagonal add up to the same value. If the top left and top right entries are 6 and 2, then the bottom middle entry is
 a. 3 b. 4 c. 5 d. 7
- While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of the squares of the other two numbers is
 a. 50 b. 40 c. 26 d. 52
- Find the value of the sum $7 \times 11 + 11 \times 15 + 15 \times 19 + \dots + 95 \times 99$
 a. 80707 b. 80773 c. 80730 d. 80751
- How many two-digit numbers, with a non-zero digit in the unit's place, are there which are more than thrice the number formed by interchanging the positions of its digits?
 a. 5 b. 6 c. 7 d. 8
- If the sum of the squares of the two numbers is 97, then which of the following cannot be their product
 a. -32 b. 16 c. 64 d. 48
- What will be the remainder when $2222^{5555} + 5555^{2222}$ is divided by 7?
 a. 0 b. 2 c. 4 d. 5
- $3^n - 1$ is divisible by 2^{n+3} , for $n =$
 a. 1 b. 2 c. 3 d. none of these

12. If we divide the unknown two-digit number by the number consisting of the same digits written in reverse order, we get 4 as a quotient and 3 as a remainder. If we divide the required number by the sum of the digits, we get 8 as a quotient and 7 as a remainder. Find the number.
- a. 81 b. 91 c. 71 d. 72
13. Find the HCF of 111...11 hundred ones and 111.11 sixty ones.
- a. 111.. 40 ones b. 111... 25 ones c. 111... 20 ones d. 111... sixty ones
14. Find the remainder when $50^{51^{52}}$ is divided by 11.
- a. 6 b. 4 c. 7 d. 3
15. A number N has exactly 23 factors. Which of the following is necessarily true?
- a. N is a perfect square
b. N is an even perfect cube
c. N is a perfect square greater than 5000
d. N is odd but not a perfect square
16. What is the remainder when $1^2 + 2^2 + 3^2 + \dots + 100^2$ is divided by 12?
- a. 6 b. 7 c. 8 d. 0
17. If a is between 0 and 1, which of the following statements is (are) true?
- I. $a^2 - 1 > 0$
II. $a^2 + 1 > 0$
III. $a^2 - a > 0$
- a. Only II b. I & II c. III d. All the three
18. If A, B and C are even numbers whereas C, D and E are odd numbers, then which of the following statements is not true:
- a. $\frac{AB}{C} = XY$ b. $\frac{XY}{Z} = AC$ c. $\frac{A}{B} = XYZ$ d. $AC = \frac{BYZ}{X}$
19. Simplify : $\sqrt{5 + \sqrt{5} + \sqrt{3 + \sqrt{5} + \sqrt{14 + \sqrt{180}}}}$
- a. $1 + \sqrt{5}$ b. $2 + \sqrt{5}$ c. $3 + \sqrt{5}$ d. $4 + \sqrt{5}$
20. A number when divided by a divisor leaves a remainder of 24. When twice the original number is divided by the same divisor, the remainder is 11. What is the value of the divisor?
- a. 13 b. 59 c. 35 d. 37

PERCENTAGE

Definition

The word percent can be understood as follows:

Per cent \Rightarrow for every 100.

So, when percentage is calculated for any value, it means that you calculate the value for every 100 of the reference value. When you see the word "percent" or the symbol %, remember it means divide by 100.

Example:

$$20\% \text{ of } 1100 = \frac{20}{100} \times 1100 = 220$$

Some Important Results:

1. What is x% of y = $\frac{x}{100} \times y$
2. What is y% of x = $\frac{y}{100} \times x$
3. x is what % of y = $\frac{x}{y} \times 100$
4. x is what % greater than y = $\frac{x-y}{y} \times 100$
5. x is what % smaller than y = $\frac{y-x}{y} \times 100$
6. Percentage change = $\frac{\text{new value} - \text{old value}}{\text{old value}} \times 100$
 +ve answer indicates % increase
 -ve answer indicates % decrease
7. How to increase/decrease a number by x%?
 New value = Old value $\times \left(1 \pm \frac{x}{100}\right)$
8. How to increase/decrease a number by x% 'n' times successively?
 New value = Old value $\times \left(1 \pm \frac{x}{100}\right)^n$
 This equation can be used to solve questions based on population.
9. If the value of a number is first increased by a% and then increased by b%. Then the final increase (also known as successive percentage change) is given by $\left(a + b + \left(\frac{ab}{100}\right)\right)\%$.
 Note: In case of decrease we take -ve sign instead of +ve. And the sequence of increase or decrease doesn't affect the change.

Fraction and Percentage

To convert or to express any fraction $\frac{a}{b}$ into percent, multiply it by 100 and put a % sign. Similarly, to convert a percentage into a fraction, divide by 100 and remove the % sign.

For example, $\frac{1}{2}$ in terms of percentage is $= \frac{1}{2} \times 100\% = 50\%$.

For easy calculations, one needs to be familiar with the following fractions and their respective percentage values.

Fraction	Percentage	Fraction	Percentage	Fraction	Percentage
1/2	50%	1/10	10%	1/18	5.55%
1/3	33.33%	1/11	9.09%	1/19	5.26%
1/4	25%	1/12	8.33%	1/20	5%
1/5	20%	1/13	7.69%	1/25	4%
1/6	16.66%	1/14	7.14%	1/30	3.33%
1/7	14.28%	1/15	6.66%	1/40	2.50%
1/8	12.50%	1/16	6.25%	1/50	2%
1/9	11.11%	1/17	5.88%	1/100	1%

To understand the use of above theory, solve the following questions before moving ahead.

- Find 16.66% of 1080.
- Find 87.5% of 240.
- Find 37.5% of 3600.
- Find 42.84% of 70.
- Find 77.77% of 330.
- What is the difference between 20% of 80 and 80% of 20?
- 35 is what percent of 56?
- 63 is what percent more than 56?
- Difference between 48% of 3600 and 62.5% of 300?

Chapter: Percentage

Number of Questions: Class Work: 25+10 Practice Sheet: 20

CLASSSHEET

LEVEL 1:

1. Sixty-five percent of a number is 21 less than four fifth of that number. What is the number?
a) 120 b) 130 c) 140 d) 150
2. A is 20 % more than B and B is 20 % more than C. Then A is what % More than C
a) 40 b) 30 c) 44 d) None of these
3. What is 11.11% of 25% of 72
a) 40 b) 1 c) 36 d) 2
4. An inspector rejects 0.08% of the meters as defective. How many will he examine to reject 2?
a) 1000 b) 2500 c) 3000 d) 1500
5. If the numerator of a fraction be increased by 15% and its denominator be decreased by 8%, the value of the fraction is $\frac{15}{16}$. Find the original fraction.
a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{2}{3}$
6. If 25% of P = Q, then Q% of 25 is what percent of P
a) 25 b) 50 c) None of these d) Cannot be determined
7. A pipe X is 30 meters and 45% longer than another pipe Y. find the length of the pipe Y
a) 20.5 b) 20 c) 20.12 d) 20.68
8. If A's salary is 20% less than B's salary, by how much percent is B's salary more than A's?
a) 20% b) 16.66% c) 25% d) 10%
9. If A earns 33.33% more than B, how much percent does B earn less than A?
a) 20% b) 25% c) 33.33% d) 50%
10. If A earns 44% more than C, how much percent does C earn less than A?
a) 22% b) 35% c) 30.5% d) 25%
11. In the new budget, the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?
a) 25% b) 20% c) 16.66% d) 33.33%
12. Aman's salary was decreased by 50% and subsequently increased by 50%. How much percent does he lose?
a) 32% b) 30% c) 28% d) 25%
13. A batsman scored 120 runs which included 4 fours and 4 sixes. What percent of his total score did he make by running between the wickets?
a) 40% b) 80% c) 33.33% d) 66.66%
14. Two employees, X and Y are paid a total of Rs.550 per week by their employer. If X is paid 120 percent of the sum paid to Y, how much is Y paid per week?
a) 250 b) 200 c) 150 d) 300
15. A student has to obtain 38% of the total marks to pass. He got 138 marks and failed by 14 marks. The maximum marks are
a) 400 b) 438 c) 450 d) None of these

16. A student got 40% of the maximum Marks and failed by 20 marks another student got 55% marks and scored 25 marks more than the pass marks. Find the maximum marks
a) 300 b) 150 c) 360 d) 200
17. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.
a) 5000 b) 6000 c) 7000 d) 8000
18. A man gave 20% of his money to his wife, 25% of the remainder to his son and the remaining money equally to his four daughters. If each daughter gets Rs. 180K, what does the wife get?
a) 250k b) 240k c) 300k d) 360k
19. In a certain examination, there were 1500 students, out of which 1000 were boys and the rest were girls. If 55% of the boys and 45% of the girls passed, find the total percentage of failed students.
a) 51.66 b) 48.33 c) 47.33 d) 52.67
20. Akshay went to a shop and bought mobile phone worth Rs. 22670, including service tax of 13.35% on original price. What was the cost of the mobile phone without tax?
a) 21550 b) 20000 c) 21670 d) 20670
21. Ram and Leela were looking at the price of a car. The price of the car was 50% more than amount with Ram, and separately, the same price was 20% more than amount with Leela. Then by What percent is the amount with Ram less than Leela.
a) 25 b) 20 c) 30 d) 50
22. The Length of a Rectangle is increased by 30% then by what percent should its Breadth be decreased so that area increases by only 17%
a) 90 b) 10 c) 20 d) 13
23. In a museum, the entrance ticket costs Rs. 628.5. In vacations, the cost of the ticket is reduced thereby increasing the sale by 40%. But it was found that the collection is increased by only 12%. What percent deduction is done in the ticket price?
a) 28 b) 12 c) 20 d) 25
24. If the radius of circle increased by 10 Percent, then its area will increase by
a) 100 b) 10 c) 20 d) 21
25. If the price of sugar increased by 30% and Salman intends to spend only an additional 4% on sugar, by how much percent should he reduce the quantity of sugar purchased?
a) 30 b) 20 c) 26 d) 25

LEVEL 2:

- 1 At an election between two candidates, the candidate who got 54.5% of votes cast won by 180 votes. Find total number of voters on the voting list if 80% people cast their vote and there were no invalid votes.
a) 2000 b) 2500 c) 1550 d) 2550

- 2 The ratio of the populations of 3 cities A, B and C in 2013 was 4:5:8. From 2013 to 2014 the population of cities A, B and C increased by 25%, 20% and 15% respectively. Find the percentage increase in the total population of these 3 cities
a) 20 b) 16.66 c) 18.82 d) 20.41
- 3 A's salary first increased by 20% and then decreased by 15% whereas B's salary first increased by 20% and then decreased by 10%. If the salaries of A and B are equal after change then what will be the ratio of their original salaries?
a) 6:5 b) 1:1 c) 18:17 d) 17:18
- 4 If the radius of a cylinder increases by 10% and the height increases by 20% then find the percentage increase in its volume.
a) 40% b) 30% c) 37.2% d) 45.2%
- 5 In an examination, Raju got 25 marks less than Mohit. Mohit got 45 marks more than Moni. Mohan got 75 marks which is 10 marks more than Moni. Arvind's marks is 50 less than full marks. If Arvind got 34 marks more than Raju, then what is the approximate percentage of marks fetched by Arvind?
a) 90 b) 70 c) 80 d) 60
- 6 Balu's monthly salary is equal to 50% of Meelu's monthly salary. Meelu's monthly salary is Rs. 28000 less than Peelu's monthly salary. If Peelu's monthly salary is Rs. 58000. What is Balu's annual salary?
a) 1, 80,000 b) 1, 50, 000 c) 1, 60, 000 d) 1, 70, 000
- 7 A vendor sells 60 percent of apples he had and throws away 15 percent of the remainder. Next day he sells 50 percent of the remainder and throws away the rest. What percent of his apples does the vendor throw?
a) 17% b) 23% c) 77% d) 25%
- 8 The population of a town increases by 5% annually. If the current population is 84000, then find the population of the town after two years.
a) 92610 b) 91260 c) 96210 d) 96220
- 9 Hitesh lost 20% of his pocket money. He further spent 25% of the remainder and finally was left with Rs.480. What much money did he have initially?
a) Rs.800 b) Rs.720 c) Rs.840 d) Rs.880
- 10 The population of a town is 1,76,400. If it increases at the rate of 5% per annum, what will be its population 2 years ago?
a) 150000 b) 160000 c) 155000 d) 165000

PRACTICE SHEET

1. Three friends A, B and C donate 8%, 7% and 9% of their salary to a charitable trust in the given order. Salary of A and B is same and the difference of their donations is Rs 74. The total donation by A and B is Rs 525 more than C's donation. What percentage of their total salary does they together donate?
(a) 7.95% (b) 8.3%
(c) 6.34% (d) None of these

2. Chotu organized a party by taking contributions from the participants. 60% of the participants contributed 80% of the funds. The average contribution of all the people who attended is Rs.50. What is the average contribution of the remaining 40% of the people?
 - (a) Rs 100
 - (b) Rs 150
 - (c) Rs 250
 - (d) Rs 253.
3. Out of 51,600 candidates who appeared in an examination, 35% failed, while 15% passed with honors. What is the number of candidates who pass the examination, but failed to obtain honors, assuming that no student who has failed can obtain honors?
 - (a) 25,000
 - (b) 30,000
 - (c) 27,309
 - (d) 25,800
4. A's income is reduced from Rs 75,000 to Rs 60,000, while B's income for the same period is increased from Rs 60,000 to Rs 75,000. What percentage of decrease in A's income is the increase in B's income?
 - (a) 125%
 - (b) 75%
 - (c) 33%
 - (d) 100%
5. Lalu working hours per day were increased by 25% and his wages per hour were increased by 20%. By how much per cent was his daily earnings increased?
 - (a) 20%
 - (b) 25%
 - (c) 50%
 - (d) 45%
6. In a hospital 5% of all the childbirth cases result in twins. What is the approximate percentage of twins out of total children born?
 - (a) 5%
 - (b) 7.6%
 - (c) 9.5%
 - (d) 10.4%
7. In an examination, a total of 6,00,000 students appeared. 40% of them were females while the rest were males. Pass percentage among males 75% and the overall pass percentage is 70%. What is the pass percentage for females?
 - (a) 37.5%
 - (b) 50%
 - (c) 62.5%
 - (d) 70%
8. Pranav saves 10% of his total salary. Next year he increases his expenses by 20% but his percentage savings remain the same. What is the percentage increase in his salary next year?
 - (a) 10%
 - (b) 20%
 - (c) 40%
 - (d) 16.66%
9. X's income is 75% of Y's income, and X's expenditure is 80% of Y's expenditure. If X's income is 90% of Y's expenditure, find the ratio of X's savings to Y's savings.
 - (a) 1:2
 - (b) 2:1
 - (c) 1:4
 - (d) 2:3
10. The cost of painting a square with 10% greater dimensions by using 10% cheaper paint will be:
 - (a) 8% more
 - (b) same
 - (c) 8.9% more
 - (d) 10.8% more
11. A rainy day occurs once in every 10 days. Half of the rainy days produce rainbows. What in the percentage of all the days when we have no rainbows?
 - (a) 95%
 - (b) 10%
 - (c) 50%
 - (d) 5%
12. Soni went to a fruit market with a certain amount of money. With the money she has, she

can buy either 40 apples or 70 mangoes. She retains 15% of money for auto fare. If she buys 35 mangoes, how many more apples can she buy?

- (a) 15
- (b) 20
- (c) 14
- (d) None of these

13. There are 1,10,000 books at Laxmi Library, 40% of which are Science books. It was decided to add 20,000 new books to the library. How many of the new books should be Science books in order to bring the percentage of Science books in the library equal to 45%?

- (a) 15,000
- (b) 1,500
- (c) 1,450
- (d) 14,500

14. There were only two contestants A and B in the recent assembly election. B got 50% of the votes that A got. Had A got 200 votes less, there would have been a tie. What is the total number of votes polled?

- (a) 1,000
- (b) 800
- (c) 1,200
- (d) Cannot be determined

15. Every day, Om consumes 7 chapatis for lunch and 8 chapatis for dinner at hotel Sharada. Every Wednesday, he does not have to pay for 2 chapatis. Then in 4 weeks, his percentage saving is _____.

- a) 40/28%
- b) 40/21%
- c) 15/28%
- d) 15/21%

16. A man possessing Rs.12000 lent a part of it at 9% and the remaining at 3%. His total income after $1\frac{1}{3}$ year was Rs.800. Find the sums lent at 9% and 3% respectively.

- a. 3000, 9000
- b. 5000, 7000
- c) 10000, 2000
- d) 4000, 8000

17. In an office, the number of women wearing red sarees is twice that of those wearing blue sarees and the number of women wearing green sarees is $\frac{1}{3}$ rd of the number of women wearing blue sarees. If the total number of women is 125 out of which 25 wore mixed coloured sarees, then what percentage of women wore blue sarees?

- a) 48%
- b) 12%
- c) 24%
- d) 25%

18. In a test, Rohit secured 25% marks and failed by 50 marks while Susie secured 75% marks and passed by 100 marks. What are the passing marks?

19. A person buys watch worth Rs.750, mobile worth Rs.3600 and a television worth Rs.10500 and pays a duty of 4%, 7% and 9% respectively. Find the total duty paid by him.

- a. 1300
- b. 1197
- c) 1227
- d) 1327

20. In 2010, a library contained a total of 11500 books in two categories - fiction and non-fiction. In 2015, the library contained a total of 12760 books in these two categories. During this period, there was 10% increase in the fiction category while there was 12% increase in the non-fiction category. How many fiction books were in the library in 2015?

- a. 6600
- b. 5500
- c. 6000
- d. 6160

PROFIT AND LOSS

Definitions:

- **Cost Price:** The price at which an article is purchased is called cost price (abbreviated as C.P.)

- **Selling Price:** The price at which an article is sold is called selling price (abbreviated as S.P.)
- **Profit or Gain:** When an article is sold for more than what it costs, we say there is profit or gain.
Thus, Profit = S.P. - C.P.
- **Loss:** When an article is sold for less than what it costs we say there is loss. Thus,
Loss = C.P. - S.P.
- **Marked Price:** List price or the price printed on the article is known as marked price (abbreviated as M.P.)
- **Discount:** Sometimes dealers allow some reductions on list price or marked price. This reduction is known as discount.

NOTE- Profit or loss percent is always calculated over cost price unless mentioned.
Discount is calculated over marked price.

Important Formulae:

- $\text{PROFIT (GAIN)\%} = \frac{\text{PROFIT}}{\text{CP}} \times 100$
- $\text{LOSS\%} = \frac{\text{LOSS}}{\text{CP}} \times 100$
- $\text{DISCOUNT\%} = \frac{\text{DISCOUNT}}{\text{MP}} \times 100$
- $\text{SP} = \frac{100 \pm X}{100} \text{ CP}$; where 'X' is the profit or loss percent with +ve or -ve sign respectively.
- $\text{SUCCESSIVE PERCENTAGE CHANGE} = \left(a + b + \left(\frac{ab}{100} \right) \right) \%$; where 'a' and 'b' are +ve for profit and -ve for loss/discount.
- If CP of N articles is equal to SP of M articles then $\text{Profit\% or Loss\%} = \frac{N-M}{M} \times 100$. If the percentage value is negative then it is a loss, else it is a gain.
- If 'a' articles are purchased for Rs.b and sold at the price of 'c' articles for Rs.d, then $\text{Profit or Loss \%} = \left[\frac{ad-bc}{bc} \times 100 \right]$; if the percentage value is negative then it is a loss, else it is a gain.
- If SP of two articles is same and the loss percentage of one article is equal to gain percentage of the other, then in such a case there is always net loss. And the net loss is given by the below formula. $\text{Loss\%} = \frac{(\text{common loss or profit\%})^2}{100}$

Chapter: Profit and Loss

Number of Questions: Class Work: 25+10 Practice Sheet: 20

CLASSSHEET

LEVEL 1:

1. A book was purchased for Rs.750 and sold for Rs.675. Find the profit or loss percent.
a) 10% loss b) 10% profit c) 5% profit d) 5% loss
2. A bicycle was sold at Rs.99 at a loss of 10%. Find the cost price of the bicycle.
a) 110 b) 121 c) 88 d) 90
3. Selling price of 10 pens is same as the cost price of 12 pens. Find the profit or loss percent.
a) 21% profit b) 20% profit c) 10% loss d) 10% profit
4. Aakash purchased 11 chocolates for Rs.10 and sold 10 chocolates for Rs.11. Find the profit or loss percent.
a) 21% profit b) 11% profit c) 10% loss d) 10% profit
5. A person bought a horse and a carriage for Rs.3000. He then sold the horse at profit of 20% and the carriage at a loss of 10%, thereby gaining 2% in the whole transaction. Find the cost price of the horse.
a) 1100 b) 1200 c) 1300 d) 1400
6. John purchased a machine for Rs. 80,000. After spending Rs. 5000 on repair and Rs. 1000 on transport he sold it with 25% profit. What price did he sell the machine?
a) 107000 b) 107500 c) 108500 d) NOTA
7. I buy a houseboat for Rs 75,000/- and a paddle boat for Rs 15,000/-. I sell the houseboat at a profit of Rs 8,000/- and the paddle boat at a loss of Rs 5,000/-. What is my gain or loss in the whole transaction?
a) 3% b) $3\frac{1}{3}\%$ c) 30% d) $33\frac{1}{3}\%$
8. A grocery shop owner buys wheat in a large quantity. He sells half the wheat at 20% profit, half of the remaining at 20% loss and rest at cost price. Find his gain/ loss percentage in the whole transaction.
a) 5% loss b) 5% gain c) 10% gain d) Neither gain nor loss
9. Shagufta bought almonds worth Rs. 48000. She sold $\frac{1}{3}$ rd of them at a loss of 30%. What profit should she earn on the remaining part so as to make it a no profit, no loss deal?
a) 15% b) 30% c) 45% d) 50%
10. If an article is sold for Rs.178 at a loss of 11%, what should be its selling price in order to earn a profit of 11%?
a) Rs.222.50 b) Rs.267 c) Rs.222 d) Rs.220

11. A man sold an article at a loss of 20%. If he has sold that article for Rs.12 more he would have gained 10%. Find the cost price of that article:
a) 60 b) 40 c) 30 d) 22
12. An article is sold at a loss of 10%. Had it been sold for Rs.9 more, there would have been a gain of 25/2% on it. The cost price of the article is:
a) Rs.40 b) Rs.45 c) Rs.50 d) Rs.35
13. A person bought oranges at the rate of 7 oranges for a rupee. How many oranges should be sold for a rupee to earn 40% profit?
a) 3 b) 4 c) 5 d) 6
14. A hawker sells 100 candies for 5 Rupees. If he manages to make a profit of 20%, how many candies did he buy for a rupee?
a) 16 b) 24 c) 36 d) 40
15. A man sells an article at a profit of 25%. If he had bought it at 20% less and sold it for Rs.10.50 less, he would have gained 30%. Find the cost price of the article.
a) Rs.40 b) Rs.35 c) Rs.25 d) Rs.50
16. Amit bought equal quantities of two varieties of oranges, one variety at a rate of Rs.200 for 4 kilos and other variety at a rate of Rs.400 for 10 kilos. He mixed the two varieties and sold them at a rate of Rs.50 per kg. What is his profit or loss percentage?
a) 10% b) 9.09% c) 11.11% d) 100%
17. Rajesh earns a profit of 30% in one deal but incurs a loss of 30% in the second while selling 2 cycles for Rs. 4000 each. Find his total profit or loss in both the deals together?
a) Profit = 18% b) Loss = 18% c) Profit = 9% d) Loss = 9%
18. Anirudh bought 8 lemons for a rupee, but sells only 6 lemons for a rupee. Find his profit percentage.
a) 33.33% b) 3.33% c) 20% d) 66.66%
19. A trader marked the price of an article 35% above its cost price. The percentage of discount allowed in order to gain 8% profit?
a) 15% b) 25% c) 30% d) 20%
20. A trader gives 12% additional discount on the discounted price, after giving an initial discount of 20% on the labelled price of an item. The final sale price of the item is Rs. 704. Find out the labelled price?
a) Rs 1000 b) Rs 2000 c) Rs 1200 d) Rs 920
21. Arun bought a computer with 15% discount on the labelled price. He sold the computer for Rs.2880 with 20% profit on the labelled price. At what price did he buy the computer?
a) Rs.3000 b) Rs.2080 c) Rs.2040 d) Rs.2000
22. What is the single equivalent discount to a series of successive discounts of 20%, 10% and 5%?
a) 31.6% b) 33.1% c) 34.1% d) 34.65%
23. There are two shopkeepers who gives two successive discounts. The first one gives 30% and then 16% successive discounts whereas the second one gives 20% and then 26%

successive discounts. Which one of the two is profitable for the shopkeeper?

- a) First b) second c) none d) Can't be determined

24. What will be the selling price of 250 chairs marked at Rs.50 per chair if the shopkeeper offers successive discounts of 20%, 15% and 5%?

- a) Rs.8800 b) Rs.8950 c) Rs.8750 d) Rs.8075

25. A reduction of 10% in the price of a pen enabled a trader to purchase 9 more for Rs.540. What is the reduced price of the pen?

- a) Rs.8 b) Rs.6 c) Rs.5 d) Rs.4

LEVEL 2:

1 A car worth Rs.1,50,000 was sold by X to Y at 5% profit. Y sold the car back to X at 2% loss. In the entire transaction.

- a) X gained Rs. 4,350 b) Y lost Rs. 4,350 c) X gained Rs. 3,150 d) X lost Rs. 3,150

2 Ram buys some paper wind fans at 4 per rupee. He then buys same number of paper wind fans from another shop at 5 per rupee. He puts them all together and sells them at 4 per rupee. Will he make a profit or incur a loss and by how much?

- a) $115/9$ % Profit b) $115/9$ % Loss c) $100/9$ % Profit d) $100/9$ % Loss

3 An article is listed at a price that allows the shopkeeper to earn 25% profit. If he offers a certain discount, his profit reduces to 12.5%. Find the discount offered by him.

- a) 10% b) 15% c) 20% d) 25%

4 A man buys 2 flasks at a total price of Rs 900/- and then sells them making a profit of Rs 90/- in the whole deal. He sells one flask at $5/4$ of the CP while the other one at $4/5$ of the CP. Find the cost price of the flask with lower value.

- a) Rs. 300 b) Rs. 400 c) Rs. 500 d) Rs. 600

5 If selling price is doubled, then profit triples. Find the profit percent.

- a) 50% b) 100% c) 150% d) 200%

6 If a seller gives a discount of 15% on retail, she still makes a profit of 2%. Which of the following ensures that she makes a profit of 20%?

- a) Give a discount of 5% on retail
b) Give a discount of 2% on retail
c) Increase the retail price by 2%
d) Sell at retail price

7 In a market, the price of medium quality mangoes is half that of good mangoes. A shopkeeper buys 80 kg good mangoes and 40 kg medium mangoes from the market and then sells all these at a common price which is 10% less than the price at which he bought the good ones. His overall profit is

- a) 6% b) 8% c) 10% d) 12%

8 Two merchants sell, each an article for Rs.1000. If Merchant A computes his profit on cost price, while Merchant B computes his profit on selling price, they end up making profits of 25% respectively. By how much is the profit made by Merchant B greater than

that of Merchant A?

- a) Rs50 b) Rs66.67 c) Rs75 d) Rs150
- 9 A shopkeeper says he takes a profit of 1% on the cost price. But after a huge Diwali sale, he actually received a net profit of 10% using a faulty meter scale. Find the length of the meter scale.
- a) 91.81 cm b) 90.88 cm c) 91.88 cm d) 90.81 cm
- 10 A shopkeeper sells sugar at Rs.21 per kg. The cost price of sugar is Rs.28 per kg. The shopkeeper uses weight which is 20% less than a kg. Find his net gain or loss in the whole transaction.
- a) 6.25% loss b) 6.25% gain c) No gain or loss d) None of these

PRACTICE SHEET

1. A dealer incurs a loss of 5 % if he sells an article for Rs.1805. What price must he sell the article so as to gain 5 % on that article?
a) 1993 b) 1994 c) 1995 d) 1996
2. A merchant buys two articles for Rs.600. He sells one of them at a profit of 22% and the other at a loss of 8% and makes no profit or loss in the end. What is the selling price of the article that he sold at a loss?
a) Rs. 404.80 b) Rs, 440 c) Rs. 536.80 d) Rs. 160
3. A trader professes to sell his goods at a loss of 8% but weights 900 grams in place of a kg weight. Find his real loss or gain percent
a) 2% loss b) 2.22% gain c) 2% gain d) None of these
4. Rajiv sold an article for Rs.56 which cost him Rs. x. If he had gained x% on his outlay, what was his cost?
a) Rs. 40 b) Rs. 45 c) Rs. 36 d) Rs. 28
5. A man sold two watches at the same price, one at 10 % profit and other at 10 % loss. Find his overall gain or loss percent.
a) 1% b) 2% c) 3% d) 4%
6. If apples are bought at the rate of 30 for Rs.100. How many apples must be sold for Rs.100 so as to gain 20%?
a) 28 b) 25 c) 20 d) 22
7. One-year payment to the servant is Rs. 200 plus one shirt. The servant leaves after 9 months and receives Rs. 120 and a shirt. Then find the price of the shirt.
a) Rs. 80 b) Rs.100 c) Rs. 120 d) Cannot be determined
8. Two merchants sell, each an article for Rs.1000. If Merchant A computes his profit on cost price, while Merchant B computes his profit on selling price, they end up making profits of 25% respectively. By how much is the profit made by Merchant B greater than that of Merchant A?
a) Rs. 66.67 b) Rs. 150 c) Rs. 125 d) Rs. 200

9. A merchant marks his goods in such a way that the profit on sale of 50 articles is equal to the selling price of 25 articles. What is his profit margin?
a) 25% b) 50% c) 100% d) 66.67%
10. A merchant marks his goods up by 75% above his cost price. What is the maximum % amount that he can offer so that he ends up selling at no profit no loss?
a) 75% b) 46.67% c) 300% d) 42.85%
11. Ram goes from place P to Q to buy an article costing 20% less at Q. Although he spends Rs 200 on travelling, still he gains Rs.200 compared to buying at P. His profit percent is
a) 10% b) 20% c) 30% d) 40%
12. After giving a discount of 11.11% a trader still makes a profit of 14.28%. What is the percent mark up over Cost price?
a) 14.28% b) 28.56% c) 25% d) 50%
13. In selling a pen for Rs.11, as much percent of profit is gained as the C.P. of the pen. Find the CP of the pen.
a) Rs. 5 b) Rs. 10 c) Rs. 20 d) Rs. 8
14. If the selling price of the article is reduced by 40% and the cost price remains the same, the profit reduces by 50%. Find the original profit % of the articles
a) 400 b) 350 c) 200 d) 150
15. A shopkeeper gives marks the price of an article by 160% above the cost price of the article and then gives successive discounts of 10%, 15% and 20%. Find the profit percentage of the shopkeeper
a) 51 b) 59 c) 55 d) 49
16. A shopkeeper sells 1 kg rice to two customers Rani and Vani. For Rani he charges exactly the cost price but under weighs the quantity by 10%. For Vani he sells at 20% more than cost price but over weighs the quantity by 10%. What is his overall profit/ loss percentage?
a) 10% profit b) 10% loss c) 9.1% loss d) 9.1% profit
17. X goes to the shopkeeper P to purchase a plant for Rs 350 and gives him a 1000-rupee note. P does not have the change and hence goes to shopkeeper Q to get the change. He then gives X Rs 650. Later, Q realizes that the 1000-rupee note is a duplicate note and asks P to return his money. P returns the money. What is the loss incurred by P if it is given that P sold the plant at a profit of 25%?
a) 930 b) 1000 c) 1070 d) 1200
18. The price of a TV and a refrigerator set are in ratio 3:5. If the refrigerator cost Rs.4400 more than the TV set, then the price of the TV is
a) Rs. 6520 b) Rs. 5963 c) Rs.7260 d) Rs. 6600
19. Some articles were bought at 6 articles for Rs. 5 and sold at 5 articles for Rs. 6. Gain percent is
a) 30% b) 33.33% c) 35% d) 44%
20. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?
a) 36 b) 42 c) 54 d) 63

SIMPLE INTEREST AND COMPOUND INTEREST

Interest

Interest is the money paid by borrower to the lender for the use of the money lent.

The sum lent is called the principal. Usually denoted by P.

The sum of principal and interest is called amount. Usually denoted by A.

The interest is usually paid yearly, half yearly, quarterly etc. which is called time or period.

The interest that is calculated for every 100 rupees usually for a year is called Rate-percent per annum.

Interest is of two kinds - Simple and Compound.

Simple Interest

When the interest is calculated on the original principal for any length of time, it is called Simple Interest.

Simple Interest = (Principal x Rate x Time)/100.

Amount = Principal + Interest.

Notation or Symbols

The symbols used for these terms are as follows:

P - Principal,

A - Amount,

T - Time,

R - Rate Percent per annum

Useful Formulae

1. $P = (100 \times \text{S.I.}) / (R \times T)$.

2. $R = (100 \times \text{S.I.}) / (P \times T)$.

3. $T = (100 \times \text{S.I.}) / (P \times R)$.

4. If a sum of money becomes 'n' times itself in 'T' years at S.I., then the $R = 100(n - 1)/T$.

Compound Interest

When the simple interest (not paid as soon as it falls due) is added to the principal so that the amount becomes the principal for the next period, is called compound interest (abbreviated as C.I.).

Useful Formulae

Let P - Principal, n - Time or Period, r - Rate Percent p.a., A - Amount then

1. $A = P (1 + (r/100))^n$ when interest is compounded annually.

2. $A = P (1 + (r/2/100))^{2n}$ when interest is compounded half yearly.

3. $A = P (1 + (r/4/100))^{4n}$ when interest is compounded quarterly.

4. $A = P (1 + (r/12/100))^{12n}$ when interest is compounded monthly.

5. When rate of interest is $r_1\%$, $r_2\%$, and $r_3\%$ for first year, second year, and third year respectively, the $A = P (1 + (r_1/100)) (1 + (r_2/100)) (1 + (r_3/100))$.

6. $\text{C.I.} = P (1 + (r/100))^n - P = P [(1 + (r/100))^n - 1]$.

7. When the principal becomes k times itself at the end of n years then

Rate = $(k^{1/n} - 1)100$.

8. Difference of CI and SI for 2 years = $(P \times R \times R)/100 \times 100$.

9. Difference of CI and SI for 3 years = $(P \times R \times R) (300+r) / 100 \times 100 \times 100$.

Chapter: Simple Interest and Compound Interest

Number of Questions: Class Work: 25+10 Practice Sheet: 20

CLASS-SHEET

LEVEL 1:

- At what rate will a man get a simple interest of Rs. 1071 on a Principal of Rs. 2550 in 3 Years?
a) 12 % b) 14% c) 16% d) 18
- The simple interest accrued in 5 years on a principal of Rs. 24,000 is one-tenth the principal. What is the rate of simple interest per annum?
a) 5% b) 4 c) 6% d) 2%
- Rohan took a loan at simple interest rate of 6% per annum in the first year and the interest increased by 1.5% every year. If he pays Rs. 8190 as interest at the end of 3 years, what was his Loan Amount?
a) Rs. 36000 b) Rs. 36400 c) Rs. 36800 d) Cannot be determined
- There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest on Rs. 12,000 after 3 years at the same rate?
a) Rs. 2160 b) Rs. 3120 c) Rs. 3972 d) Rs. 6240
- If the simple interest on a sum of money for 2 years at 5% per annum is Rs. 50, what is the compound interest on the same at the same rate and for the same time?
a) Rs. 51.25 b) Rs. 52 c) Rs. 54.25 d) Rs. 60
- If the simple interest on a certain sum of money is $\frac{4}{25}$ of the sum and the rate per cent equals the number years, then the rate of interest per annum is:
a) 5% b) 4% c) 16% d) 3%
- A sum was put at simple interest at a certain rate for 5 years. Had it been put at 2% higher rate, it would have fetched Rs. 450 more. Find the sum?
a) Rs. 4500 b) Rs. 3200 c) Rs. 3800 d) Rs. 4200
- Amit borrowed some money on simple interest at 6% for the first 4 years, 8% for the next 6 years and 11% for the period beyond 2 years. If the total interest paid by him at

the end of eleven years is Rs 5640, how much money did he borrow?

- a) Rs. 10,000 b) Rs. 6,000 c) Rs. 8,000 d) Rs. 9,000

9. How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest?

- a) 3.5 b) 5 c) 4 d) 4.5

10. A man took loan from a bank at the rate of 12% p.a. Simple interest. After 3 years he had to pay Rs. 5400 interest only for the period. The principal amount borrowed by him was?

- a) 2000 b) 10000 c) 15000 d) 20000

11. A sum of Rs. 12,500 amounts to Rs. 15,500 in 4 years at the rate of simple interest. What is the rate of interest?

- a) 3% b) 4% c) 5% d) 6%

12. A borrows 5000 at simple interest. At the end of 3 years, he again borrows 3000 and finally pays 2340 as interest after 6 years from the time he made the first borrowing. Find the rate of interest per annum.

- a) 4% b) 5.5% c) 6% d) 4.5%

13. What would be the compound interest obtained on an amount of Rs. 6,000 at the rate of 7% for 2 years?

- a) 765.5 b) 846.2 c) 769.4 d) 860.4 e) None of the above

14. If a sum of Rs.8000 being lend for 20% per annum at compound interest then, the sum of the amount will be Rs.13824 in how many years?

- a) 2 b) 1 c) 3 d) 4

15. A sum of money invested at compound interest amounts to Rs. 650 at the end of first year and Rs. 676 at the end of second year. The sum of money is:

- a) 600 b) 540 c) 625 d) 560

16. What will be the amount if a sum of Rs. 10000 is placed at compound interest for 3 years while rate of interest for the first, second and third years is 2, 5 and 10 percent, respectively?

- a) 11781 b) 11244 c) 11231 d) 11658

17. Rahul borrowed Rs.800 at rate of interest 10%. He repaid Rs.400 at the end of first year. What is the amount required to repay at the end of second year to discharge his loan which was calculated at compound interest?

- a) 650 b) 528 c) 490 d) 780

18. Find the C.I on Rs 20,000 at 10% rate of interest in 2 years if compounded half yearly.
a) 4210 b) 4310 c) 4410 d) 4510
19. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is:
a) 2 b) 2.5 c) 3 d) 4
20. The compound interest on a certain sum for 2 years at 10% per annum is Rs. 525. The simple interest on the same sum for double the time at half the rate percent per annum is:
a) Rs. 400 b) Rs. 500 c) Rs. 600 d) Rs. 800
21. If Rs. 60000 amounts to Rs. 68694 in 2 years at Compound Interest, then find the rate of interest.
a) 5% b) 7% c) 9% d) 11%
22. What is the difference between CI and SI ,if sum is Rs.10,000 for 3 years at the rate of 3%?
a) 42 b) 30 c) 27.27 d) 35
23. If the difference between Simple Interest and Compound Interest at 20% rate of Interest in 3 years is 5120, then find the sum?
a) 40000 b) 50000 c) 60000 d) 30000
24. The difference in simple and compound interest on a certain sum for 2 years at 5% per annum compounded annually is Rs. 75. Find the sum?
a) Rs. 40,000 b) Rs. 20,000 c) Rs. 50,000 d) Rs. 30,000
25. The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 4% per annum is Re. 1. The sum (in Rs.) is:
a) 625 b) 630 c) 640 d) 650

LEVEL 2

1. A lent Rs. 5000 to B for 2 Years and Rs. 3000 to C for 4 years on Simple Interest at the same rate of interest and received Rs. 2200 in all from both as interest. The rate of Interest per annum is ?
A. 7% B. 5% C. $7\frac{1}{8}\%$ D. 10% E. None of These
2. A sum of money at simple interest amounts to Rs. 815 in 3 years and to Rs. 854 in 4 years. The sum is:
A. Rs. 650 B. Rs. 690 C. Rs. 698 D. Rs. 700 E. None of These

3. A bank offers 5% compound interest calculated on half-yearly basis. A customer deposits Rs. 1600 each on 1 st January and 1 st July of a year. At the end of the year, the amount he would have gained by way of interest is:
A. Rs. 120 B. Rs. 121 C. Rs. 122 D. Rs. 123 E. None of These
4. Simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is:
A. Rs. 1550 B. Rs. 1650 C. Rs. 1750 D. Rs. 2000 E. none of These
5. The difference between simple interest and compound on Rs. 1200 for one year at 10% per annum reckoned half-yearly is:
A. Rs. 2.50 B. Rs. 3 C. Rs. 3.75 D. Rs. 4 E. None of these
6. Out of a sum of Rs 850, a part was lent at 6% SI and the other at 12% SI. If the interest earned on the first part after 2 years is equal to the interest earned on the second part after 4 years, then the second sum is?
A. Rs. 350 B. Rs. 280 C. 170 D. Rs. 220 E. None of These
7. A sum of Rs. 550 was taken as a loan. This is to be paid back in two equal installments. If the rate of interest be 20% compounded annually, then the value of each installment is:
A. Rs. 360 B. Rs. 280 C. Rs. 250 D. Rs. 320 E. None of These
8. A Woman took a loan of Rs. 15,000 to purchase a mobile. She promised to make the payment after three years. The company charges CI at 20% per annum for the same. But, suddenly the company announces the rate of interest as 25% per annum for the last one year of the loan period. What extra amount she has to pay due to the announcement of new rate of interest?
A. Rs.1230 B. Rs.1135 C. Rs.1080 D. Rs.1100 E. None of These
9. A sum becomes triple in 6 years at S.I. The same sum will become 19 times in how many years?
A.50 B. Rs.48 C.54 D. Rs.57 E. None of These
10. Rs. 20,000 was invested by Mahesh in a FD @ 10% pa at CI. However, every year he has to pay 20% tax on the CI. How much money does Mahesh have after 3 years approximately?
A. Rs.25694 B. Rs.25594 C. Rs.25394 D. Rs.25194 E. None of These

PRACTICE SHEET

1. David invested certain amount in three different schemes A, B and C with the rate of interest per annum 10%,12%, and 15% respectively. If the total interest accrued in one

year was 3200 and the amount invested in Scheme C was 150% of the amount invested in Scheme A and 240% of the amount invested in Scheme B, what was the amount invested in Scheme B?

- A. Rs.5000 B. Rs.3000 C. Rs.2000 D. Rs.6000 E. None of These
2. A sum of ₹1550 was lent partly at 5% and partly at 8% per annum simple interest. The total interest received after 3 years was ₹300. The ratio of the money lent at 5% to that lent at 8% is:
- A. 15:16 B. 16:15 C. 15:8 D. 8:15 E. None of These
3. A sum of money doubles in 12 years. In how many years, will it treble (assume simple interest)?
- A. 24 B. 8 C. 6 D. 12 E. None of These
4. A man invests a certain sum of money at 6% per annum simple interest and another sum at 7% per annum simple interest. His income from interest after 2 years was ₹354. One-fourth of the first sum is equal to one-fifth of the second sum. What was the total sum invested?
- A. Rs. 2200 B. Rs. 3100 C. Rs. 2700 D. Rs. 1800 E. None of These
5. Simple interest on a certain deposit at 5% per annum in one year is ₹101.20. How much will be the additional simple interest on the same deposit at 6% per annum in one year?
- A. 20.24 B. 19.74 C. 20.8 D. 19.5 E. None of These
6. A sum was put with simple interest at a certain rate for 2 years. Had it been put at 4% higher rate, it would have fetched ₹60 more. What is the sum?
- A. Rs. 940 B. Rs. 820 C. Rs. 700 D. Rs. 750 E. None of These
7. The simple interest on a sum at $x\%$ for x years is x . What is the sum?
- A. $100/x$ B. $100/x^2$ C. x^2 D. x E. None of These
8. A sum of ₹7700 is to be divided among three brothers Vikas, Vijay and Viraj in such a way that simple interest on each part at 5% per annum after 1, 2 and 3 years respectively remains equal. The Share of Vikas is more than that of Viraj by?
- A. Rs. 2800 B. Rs. 2100 C. Rs. 1400 D. Rs. 2200 E. None of These
9. Find the simple interest on ₹7500 at 11% for 2 years and 5 months.
- A. Rs. 1994.25 B. Rs. 1993.75 C. Rs. 1991.25 D. Rs. 1992.25 E. None of These
10. At what rate percent of simple interest will a sum of money double itself in 20 years?
- A. 6% B. 8% C. 5% D. 4% E. None of These
11. If the compound interest on a certain sum for 2 years is Rs. 80.80 and the simple interest Rs. 80; then the rate of interest per annum is:
- A. 3% B. 2% C. 1% D. 4% E. None of These
12. The compound interest on a sum for 2 years is Rs. 832 and the simple interest on the same sum for the same period is Rs. 800. The difference between the compound and

simple interest for 3 years will be:

- A. Rs. 48 B. Rs. 98.56 C. 66.56 D. 44 E. None of These
13. On a certain sum of money, the simple interest for 2 years is Rs. 200 at the rate of 7% per annum. Find the difference in C.I. and S.I.
A. Rs. 11 B. Rs. 9 C. 7 D. 4 E. None of These
14. A sum of money on compound interest amounts to Rs. 8240 in 2 years and Rs. 9888 in 3 years. The rate of interest is:
A. 12% B. 25% C. 10% D. 20% E. None of These
15. A bank offers 10% interest rate compounded annually. A person deposits Rs. 20,000 every year in his account. If he does not withdraw any amount, then how much balance will his account show after four years?
A. 104202 B. 102220 C. 102102 D. 104222 E. None of These
16. What annual payment will discharge a debt of Rs. 1025 due in 2 years at the rate of 5% compound interest?
A. Rs. 551.25 B. Rs. 560.75 C. Rs. 560 D. Rs. 550 E. None of These
17. A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to 8 times?
A. Rs. 6 B. Rs. 10 C. 12 D. 8 E. None of These
18. A man borrows Rs. 20,000 at 10% compound interest. At the end of every year he pays Rs. 2000 as part repayment. How much does he still owe after three such installments?
A. 24000 B. 20000 C. 15000 D. 10000 E. None of These
19. The present worth of Rs. 242 due in 2 years at 10% per annum compound interest is:
A. Rs. 200 B. Rs. 220 C. 180 D. 240 E. None of These
20. If in a certain number of years Rs. 10000 amount to Rs. 160000 at compound interest, in half that time Rs. 10000 will amount to:
A. 40000 B. 50000 C. 60000 D. 80000 E. None of These

RATIO PROPORTION AND VARIATION

Definitions:

- **Ratio:** Comparison of two numbers or quantities having the same units is known as Ratio. The ratio of a to b is written as $a:b = a/b$. In the ratio $a:b$, a is known as antecedent and b is known as consequent.
If n numbers $a_1, a_2, a_3, \dots, a_n$ are in the ratio $b_1:b_2:b_3:\dots:b_n$ then the numbers can be written as $kb_1, kb_2, kb_3, \dots, kb_n$, where k is a natural number.
- **Proportion:** Four quantities are said to be proportional, when the ratio of the first to the second is equal to the ratio of the third to the fourth. Thus a, b, c, d are proportional if $a:b = c:d$. This is often expressed as $a:b::c:d$. The terms a and d are called extremes and

the terms b and c are called means. The term d is called fourth proportional to a , b , and c .

- **Continued Proportion:** Three or more quantities are said to be in continued proportion when the first is to the second as second is to third, as the third is to fourth; and so on. Thus a, b, c, d are in continued proportion when $a:b = b:c = c:d$. If three quantities a, b, c are in continued proportion ($a:b::b:c$) then b is called the mean proportional between a and c i.e. $a/b = b/c$ implies $b = \sqrt{ac}$ and c is called third proportional to a and b.
- **Variation:** When two or more quantities are so related that if one of them be changed the other is changed with respect to it, the relation is called variation.

Direct Variation: One quantity is said to vary directly as another when the two quantities are so related that if one of them be increased (or decreased) the other increases (or decreases) in the same ratio i.e. if x varies directly as y, then, $x \propto y$ or $x = ky$, where k is any constant. After equating 'k' we can get $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

Inverse Variation: One quantity is said to vary inversely as another when the two quantities are so related that if one of them be increased (or decreased) the other is decreased (or increased) in the same ratio. i.e. if x varies inversely as y, then $x \propto 1/y$ or $x = k/y$, where k is any constant. After equating 'k' we get $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

The sign \propto is called the sign of variation.

Chapter: Ratio, Proportion and Variation

Number of Questions: Class Work: 25+10

CLASS SHEET

Level 1:

1. Two numbers are in the ratio of 3:5 and the sum of these numbers is 72. Find the difference between two numbers.
a) 27 b) 45 c) 36 d) 18

2. A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets Rs. 1000 more than D, what is B's share?
a) 500 b) 1500 c) 2000 d) None of these
3. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is
a) 2:5 b) 3:5 c) 4:5 d) 6:5
4. $x:y = 2:3$ find $\frac{5x+3y}{3x+5y}$
a) 1 b) $\frac{19}{21}$ c) $\frac{21}{19}$ d) Can't be determined
5. Find the fourth proportional to the numbers 3, 4, 6
a) 8 b) 6 c) 12 d) 24
6. Find a third proportional to the numbers 1.6, 0.4
a) 0.4 b) 1 c) 0.1 d) 0.25
7. If $A:B = 2:5$ and $B:C = 3:4$ and $D:C = 5:8$, find $A:D$
a) 16:3 b) 12:25 c) 9:16 d) 3:16
8. $4A=3B=5C$ what is the ratio of A: B:C
a) 4:3:5 b) 3:4:5 c) 12:20:15 d) 15:20:12
9. $\frac{A}{4} = \frac{B}{5} = \frac{C}{7}$ then what will be the value of $\frac{A+B}{B+C}$?
a) 4:3 b) 3:4 c) 1:1 d) None of these
10. $A: B = 3:5$ and $B:C = 4:5$ and if C is 39 more than A then what is the value of B
a) 60 b) 78 c) 72 d) 80
11. If $A:B = 4:5$, $B:C = 4:5$, find $A:B:C$
a) 4:5:4 b) 5:4:5 c) 16:20:25 d) 16:25:20
12. x varies directly with cube of y. when y is 2, x is 7 what will be x if y is 4
a) 14 b) 343 c) 56 d) 28
13. x varies inversely with square root of y. when y is 16, x is 12 what will be x if y is 36
a) 8 b) 24 c) 36 d) 18
14. The sum of three numbers is 98. If the ratio of the first to the second is 2:3 and that of the second to the third is 5:8, then the second number is
a) 10 b) 17 c) 25 d) 30
15. Six years ago, A was half of B's age. If the ratio of their present ages is 2:3, what is the total of their present ages?
a) 18 b) 20 c) 30 d) 36
16. Four years ago, the ratio of the ages of Akash and Beena was 3:4. Seventeen years hence, the ratio of their ages will be 6:7. What is the present age of Akash?
a) 21 b) 28 c) 25 d) 32
17. Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs.4000, the new ratio becomes 40:57. What is Sumit's present salary.
a) 32000 b) 34000 c) 38000 d) 40000

18. The salaries of A, B and C are of ratio 2:3:5. If the increments of 15%, 10% and 20% are done to their respective salaries, then find the new ratio of their salaries.
a) 20:33:60 b) 21:33:60 c) 22:30:60 d) 23:33:60
19. A and B together have Rs.1210. If $\frac{4}{15}$ th of A's amount is equal to $\frac{2}{5}$ th of B's amount.
How much amount B have
a) 484 b) 478 c) 470 d) 478
20. Rs.1050 is divided among P, Q and R. The share of P is $\frac{2}{5}$ of the combined share of Q and R. Thus, P gets
a) 200 b) 300 c) 320 d) 420
21. A bag contains Rs.120 in the form of One-Rupee, Two- Rupees and 5-Rupees coins in the ratio of 4:3:1. The number of 2-rupees coins is
a) 45 b) 48 c) 24 d) 20
22. The present ages of A, B and C are in proportions 3:4: 5. Seven years ago, the sum of their ages was 75. Then B is older than A by?
a) 8 b) 9 c) 10 d) 12
23. The incomes of two persons are in ratio of 3:4 and the ratio of their expenditures is 2:3 find the ratio of their savings
a) 6:1 b) 12:1 c) 3:1 d) Can't be determined
24. A sum of Rs.312 was divided among 100 boys and girls in such a way that the boy gets Rs.3.60 and each girl Rs.2.40 the number of girls is
a) 35 b) 40 c) 45 d) 50
25. The value of diamond is proportional to square of its weight. One such diamond is broken in to 3 pieces, weights of whose are in proportion of 1:2:3. If price of original diamond was 54K find the loss due to breakage
a) 33k b) 21k c) 17k d) 25k

LEVEL 2:

- 1 A wheel named "A" with 16 cogs is meshed with another wheel B with 24 cogs which in turn meshed with another wheel "C" with 8 cogs. If wheel 'A' makes 100 revolutions, how many revolutions will wheel "C" make?
a) 50 b) 100 c) 150 d) 200
- 2 If the monthly salaries of two persons are in the ratio 4:7 and each receives an increment of Rs 25 in the salary then the ratio becomes 3:5. Find their respective salaries.
a) 120, 210 b) 80, 140 c) 180,300 d) 200,350
- 3 The incomes of two persons are in ratio of 3:4 and the ratio of their expenditures is 8:11 if each of them saves 4000 rupees find their total income
a) 84000 b) 56000 c) 36000 d) 76000

- 4 A bag contains Rs.120 in the form of One-Rupee, Two- Rupees and 5-Rupees coins in the ratio of 4:3:1. The value of 2-rupees coins is
a) 45 b) 48 c) 24 d) 20
- 5 A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.
a) 360,160,200 b) 160,360,200 c) 200,360,160 d) 200,160,360
- 6 Divide Rs.600 among A, B and C so that Rs.40 more than $\frac{2}{5}$ th of A's share, Rs.20 more than $\frac{2}{7}$ of B's share and Rs.10 more than $\frac{9}{17}$ th of C's share may all be equal. What is A's share?
a) 280 b) 150 c) 170 d) 200
- 7 The ratio of a two-digit natural number to a number formed by reversing its digits is 4:7. Which of the following is the sum of all such numbers?
a) 99 b) 198 c) 120 d) 330
- 8 An outgoing batch of students wants to gift a PA system worth Rs 4200 to their school. If the teachers offer to pay 50% more than the students and an external benefactor gives three times the teacher's contribution, then how much should the teachers donate?
a) 600 b) 800 c) 900 d) 1200
- 9 Rs 4830 is divided among Abhishek, Dishant and Prashant such that if Abhishek's share diminishes by Rs 5, Dishant's share diminishes by Rs 10 and Prashant's share diminishes by Rs 15, their shares will be in the ratio 5:4:3. Find the Dishant's original share
a) 1610 b) 1600 c) 1590 d) 2410
- 10 Seven years ago the ratio of ages of A and B was 3:2 which of the following cannot be the ratio of their ages after four years
a) 7:5 b) 9:8 c) 13:8 d) 16:13

AVERAGE

The sum of all the quantities of the same kind divided by their number is referred to as average of those quantities.

Average = $\frac{\text{Sum of all the quantities}}{\text{Number of quantities}}$

The average is also called Arithmetic Mean.

Thus, the sum of all the quantities = Number of quantities x Average

Weighted Average

Let $w_1, w_2, w_3, \dots, w_n$ be the weights assigned to the quantities $x_1, x_2, x_3, \dots, x_n$ respectively, then their weighted average \bar{x} is defined as:

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n}{w_1 + w_2 + w_3 + \dots + w_n} = \frac{\sum w_ix_i}{\sum w_i}$$

Points to Remember

1. If the value of each item in a group is increased by the same value p , then the average of the group of items will also increase by p .
2. If the value of each item in a group is decreased by the same value p , then the average of the group of items will also decrease by p .
3. If the value of each item in a group is multiplied by the same value p , then the average of the group of items will also be multiplied by p .
4. If the value of each item in a group is divided by the same value p (p is not equal to zero), then the average of the group of items will also be divided by p .
5. The average of a group of items will always lie between the smallest value in the group and the largest value in the group.

Median

If we arrange the given data in ascending order or descending order, then the data which lies exactly in the middle is called median. Median is a value which divides the series into two equal parts, value of each term in first group is less than median and that of the second is more than median.

Let the number of terms in the given distribution be N .

1. If N is odd then, Median = $\left[\frac{(N + 1)}{2}\right]$ th term.
2. If N is even then, Median = $\left\{\frac{(N/2) \text{th term} + [(N + 1)/2] \text{th term}}{2}\right\}$.

Mode

It is defined as the most common value found in a given distribution i.e. the value with highest frequency in the distribution.

Relation among Mean, Median, and Mode

In a *moderately symmetrical distribution*, the following relationship exists.

$$\text{Mode} = 3\text{Median} - 2\text{Mean}.$$

Important Results

1. The average of first ' n ' natural numbers is $(n+1)/2$
2. The average of ' n ' consecutive numbers is equal to the middle term if n is odd or equal to the average of the two middle terms if n is even.

Chapter: Average

Number of Questions: Class Work: 25+10

CLASS-SHEET

LEVEL 1:

- Find the average of 13, 18, 13, 14, 13, 16, 14, 21, 13.
a. 13 b. 14 c. 15 d. 16
- Find the average of 97, 101, 109, 99, 105, 88, 103, 110.
a. 101.5 b. 100.5 c. 102 d. 99.5
- Find the average first 80 natural numbers.
a. 41 b. 41.5 c. 40 d. 40.5
- Find the average of squares of first 41 natural numbers.
a. 581 b. 381 c. 400 d. 500
- In a test conducted recently, it was found that the average marks obtained by 40 boys is 65 while the average marks obtained by 60 girls is 55. Find the overall marks per student.
a. 59 b. 55 c. 60 d. 59.5
- In an organization, 7 workers were paid an average wage of 300, 5 supervisors were paid 270 in average and 3 executives were paid an average of 500. Find the average wage of all the employees.
a. 300 b. 310 c. 320 d. 330
- The average age of father, wife and son 3 years ago was 27 years. 5 years ago from now, the average age of wife and son was 20. Find the present age of the father.
a. 30 b. 40 c. 30.5 d. 50
- The average rainfall in a week is 3 cm. It rained as much on Wednesday as on all other days combined. Find the net rainfall on Wednesday.
a. 10.5 b. 11 c. 3 d. 10
- The average temperature from Monday till Thursday is 48 degrees and the average from Tuesday till Friday is 52 degrees. If the temperature on Monday was 42, what was the temperature on Friday?
a. 54 b. 52 c. 56 d. 58
- The average of 4 consecutive even numbers is 20, what are the numbers?
a. 18, 20, 22, 24 b. 16, 18, 20, 22 c. 20, 22, 24, 26 d. None of these
- The average of first 7 consecutive even numbers is A. If the next 4 consecutive even numbers are also included, then find the average of all the 11 numbers.
a. A+11 b. A+4 c. A-4 d. Cannot be determined

12. The average first 7 numbers are 5 and the average of first 6 numbers are 4. Find the seventh number.
 a. 8 b. 9 c. 10 d. 11
13. The average of 11 numbers is 24. If 108 is added find the new average.
 a. 31 b. 32 c. 33 d. 34
14. The average weight of 18 students is 60 and if the teacher is included the average becomes 61. Find the weight of the teacher.
 a. 78 a. 79 c. 80 d. 79.5
15. A batsman has certain average in his 12 innings. He scored 96 runs in his 13th inning, and thereby increasing the average by 5 runs. What is his new average?
 a. 36 b. 31 c. 30 d. 32
16. The average weight of 25 persons increases by 2 when one of them weighing 40 is replaced by a new person. Find the weight of the new person.
 a. 60 b. 80 c. 90 d. 100
17. Consider a class of 40 students whose average weight is 40 kgs. m new students join this class whose average weight is n kgs. If it is known that $m + n = 50$, what is the maximum possible average weight of the class now?
 a. 40.18 b. 40.56 c. 40.67 d. 40.49
18. Suppose there are 7 positive numbers and their average is Z . Now if we take any 4 numbers at a time and calculate their average. Considering all such possible groups, we take average of all these averages. Will this average be equal to the average of seven numbers?
 a. Yes b. No c. Cannot be determined d. None of these
19. The average score in an examination of 10 students of a class is 60. If the scores of the top five students are not considered, the average score of the remaining students falls by 5. The pass mark was 40 and the maximum mark was 100. It is also known that none of the students failed. If each of the top five scorers had distinct integral scores, the maximum possible score of the topper is.
 a. 100 b. 99 c. 80 d. 98
20. A batsman has certain average for 35 innings. In the next inning he scored zero runs so his average decreased by 0.5 runs per inning. Find his new average.
 a. 18 b. 17 c. 17.5 d. Cannot be determined
21. The average amount with the group of 10 members is 65 another 4 members joined with each having 55 Rs. What is the new average of amount with them?
 a. 62.14 b. 65.28 c. 58.25 d. 59
22. Find the average of $(15+15+15+15+\dots+200 \text{ times})$ and $(21+21+21+21+\dots+400 \text{ times})$
 a. 19 b. 18 c. 333.33 d. None of These
23. If $27A+27B=810$ then, Find the average of A & B
 a. 30 b. 3 c. 15 d. None of These

24. The average weight of a class of 41 students is 41 years. When the weight of the teacher is also included, the average weight increases by 0.5kg. What is the weight of the teacher?
- a. 62 b. 61 c. 63.5 d. None of These
25. If the average marks of three batches of 55, 60 and 45 students respectively is 50, 55, 60, what is the average marks of all the students?
- a. 53.23 b. 54.68 c. 51.33 d. 50

LEVEL 2:

1. The average of 71 results is 48. If the average of first 59 results is 46 and that of the last 11 is 52. Find the 60th result.
a. 132 b. 122 c. 134 d. 128
2. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach the target of 282 runs?
a. 6.25 b. 5.5 c. 7.4 d. 5
3. A grocer has a sale of Rs. 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs. 6500?
a. 4800 b. 4991 c. 5004 d. 5000
4. The average of 20 numbers is zero. Of them, How many of them may be greater than zero, at the most?
a. 1 b. 20 c. 0 d. 19
5. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. Find out the average age of the team.
a. 23 b. 20 c. 21 d. 24
6. The average monthly income of A and B is Rs. 5050. The average monthly income of B and C is Rs. 6250 and the average monthly income of A and C is Rs. 5200. What is the monthly income of A?
a. 2000 b. 3000 c. 4000 d. 5000
7. In Kiran's opinion, his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Kiran and he thinks that Kiran's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all are correct in their estimation, what is the average of different probable weights of Kiran?
a. 66 b. 65 c. 69 d. 67
8. A library has an average of 510 visitors on Sundays and 240 on other days. What is the average number of visitors per day in a month of 30 days beginning with a Sunday?
a. 290 b. 304 c. 285 d. 270

9. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. What is the present age of the husband?
a. 40 b. 32 c. 28 d. 30
10. Suresh drives his car to a place 150 km away at an average speed of 50 km/hr and returns at 30 km/hr. What is his average speed for the whole journey?
a. 32.5 b. 35 c. 37.5 d. 40

AGES

SOLVED EXAMPLES

1. A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old B is?

A.7

B.8

C.9

D.10

Ans.: Let the age of C be x .

\therefore Age of B = $2x$ and the age of A = $2x + 2$

$\therefore x + 2x + 2x + 2 = 27 \Rightarrow x = 5$

\therefore B is 10 years old.

2. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age?
- A) 2 B) 3 C) 4 D) 5

Ans.: Let Ronit's present age be x years. Then, father's present age = $(x + 3x)$ years = $4x$ years.

$$\therefore 4x + 8 = \frac{5}{2}(x + 8) \Rightarrow 8x + 16 = 5x + 40 \Rightarrow x = 8.$$

$$\therefore \text{The required ratio} = \frac{4x + 8}{x + 8} = \frac{48}{24} = 2$$

3. At present, the respective ratio between the ages of A and B is 3:4 and that between A and C is 1:2. six years hence, the sum of A, B and C will be 96 years. What is the present age of A?
- A. 12 years B. 24 years C. 19 years D. 18 years

Ans.: Given: A: B = 3 : 4 and A : C = 1 : 2

$$\therefore A : B : C = 3 : 4 : 6$$

$$\therefore \text{Ages of A, B and C are } 3x, 4x \text{ and } 6x$$

$$\text{By the given condition, } 3x + 6 + 4x + 6 + 6x + 6 = 96 \Rightarrow x = 6$$

$$\therefore \text{Present age of A is } 3 \times 6 = 18 \text{ years.}$$

4. A father said to his son, "I was as old as you are at the present at the time of your birth". If the father's age is 38 years now, what was the son's age five years back?
- A) 12 years B) 14 years C) 15 years D) 18 years

Ans.: Let the son's present age be x years.

$$\text{Then, } (38 - x) = x \Rightarrow x = 19$$

$$\therefore \text{Son's age 5 years back} = 19 - 5 = 14 \text{ years.}$$

5. The difference between the ages of two persons is 10 years. Fifteen years ago, the elder one was twice as old as younger one. The present age of the elder person is:

A) 25 years

B) 35 years

C) 45 years

D) 55 years

Ans.: Let their ages be x years and $(x + 10)$ years respectively.

$$\text{Then, } (x + 10) - 15 = 2(x - 15) \Leftrightarrow x - 5 = 2x - 30 \Leftrightarrow x = 25.$$

Present age of the elder person = $(x + 10) = 35$ years.

Chapter: Ages

Number of Questions: Class Work: 25+10

CLASS-SHEET

LEVEL 1:

- The present ages of A and B are in the ratio 6 : 4. Five years ago their ages were in the ratio 5:3. Find their present ages.
a. 42,28 b. 36,24 c. 30,20 d. 25,15
- The sum of the present ages of a father and his son is 60 years. Five years ago, father's age was four times the age of the son. What is the present age of the son?
a. 5 b. 10 c. 15 d. 20
- The age of a man is 4 times of his son. Five years ago, the man was nine times old as his son was at that time. The present age of the man is?
a. 32 b. 36 c. 40 d. 42
- The ages of Krish and Vaibhav are in the proportion of 3:5. After 9 years, the proportion of their ages will be 3:4. Then the current age of Vaibhav is:
a. 10 b. 13 c. 15 d. 18
- If two times of the daughter's age in years is included to the mother's age, the total is 70 and if two times of the mother's age is included to the daughter's age, the total is 95. So the Mother's age is,
a. 30 b. 38 c. 40 d. 41
- The present average age of a family of five members is 26 years. If the present age of the youngest member in the family is ten years, then what was the average age of the family at the time of the birth of the youngest member? (Assume no death occurred in the family since the birth of the youngest)
a. 19 years b. 16 years c. 18 years d. 20 years
- If 6 years are subtracted from the present age of Arun and the remainder is divided by 18, then the present age of his grandson Gokul is obtained. If Gokul is 2 years younger to Madan whose age is 5 years, then what is the age of Arun?
a. 72 years b. 54 years c. 60 years d. 47 years

8. Eight years ago, Ajay's age was $\frac{4}{3}$ times that of Vijay. Eight years hence, Ajay's age will be $\frac{6}{5}$ times that of Vijay. What is the present age of Ajay?
a. 41 years b. 40 years c. 37 years d. 33 years
9. Ten years ago, A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages?
a. 15 b. 25 c. 35 d. 45
10. Proportion of Yuvaraj's age to Ganguly's age is 4:3. Yuvaraj will be 26 years old after 6 years. What is the age of Ganguly now?
a. 17 years b. 15 years c. 19 years d. 12 years
11. The ages of two persons differ by 16 years. 6 years ago, the elder one was 3 times as old as the younger one. What is the present age of the elder person?
a) 10 b) 20 c) 30 d) 40
12. Present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. What is father's present age?
a) 30yrs b) 31yrs c) 32yrs d) 33yrs
13. Kamal was 4 times as old as his son, 8 years ago. After 8 years, Kamal will be twice as old as his son. Find out the present age of Kamal.
a) 40yrs b) 38yrs c) 42yrs d) 36yrs
14. The sum of ages of 5 children born at the intervals of 3 years each is 50 years. Find out the age of the youngest child?
a) 6yr b) 4yr c) 5yr d) 3yr
15. The product of the ages of Syam and Sunil is 240. If twice the age of Sunil is more than Syam's age by 4 years, what is Sunil's age?
a) 16 b) 14 c) 12 d) 10
16. One year ago, the ratio of Sooraj's and Vimal's age was 6:7 respectively. Four years hence, this ratio would become 7:8. How old is Vimal at present ?
a) 32 b) 34 c) 36 d) 38
17. Sachin's age after 15 years will be 5 times his age 5 years back. Find out the present age of Sachin?
a) 10 b) 11 c) 12 d) 13
18. Sandeep's age after six years will be three-seventh of his father's age. Ten years ago the ratio of their ages was 1:5. What is Sandeep's father's age at present?
a) 30yrs b) 40yrs c) 50yrs d) 60yrs
19. The present ages of A,B and C are in proportions 4:7:9. Eight years ago, the sum of their ages was 56. What are their present ages (in years)?
a) 16,30,40 b) 16,28,40 c) 16,28,36 d) Data insufficient
20. A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. What is the present age of the mother?
a) 60 b) 50 c) 40 d) 30

21. A is as much younger than B and he is older than C. If the sum of the ages of B and C is 50 years, what is definitely the difference between B and A's age?
a) 2yrs b) 3yrs c) 5yrs d) Data Insufficient
22. The age of father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. What is the ratio of their present ages?
a) 7:3 b) 9:4 c) 4:9 d) 3:7
23. A man is 24 years older than his son. In two years, his age will be twice the age of his son. The present age of his son is:
a) 14yrs b) 18yrs c) 20yrs d) 22yrs
24. Six years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present?
a) 16yrs b) 18yrs c) 20yrs d) None of These
25. Kiran is younger than Binesh by 7 years and their ages are in the respective ratio of 7:9. How old is Kiran?
a) 24yrs b) 23.5yrs c) 24.5yrs d) 25yrs

LEVEL 2:

1. Raju got married 8 years ago. His present age is $\frac{6}{5}$ times his age at the time of his marriage, Raju's sister was 10 years younger to him at the time of his marriage. The present age of Raju's sister is?
a. 32 b. 30 c. 38 d. None of these
2. The Average age of 12 men is decreased by one year when two of them having ages 28 years and 32 years are replaced by two women of same age. The age of a women is?
a. 24 b. 26 c. 28 d. 30
3. Ratio of the ages of Mahesh and Nilesh is 5:X. Mahesh is 18 years younger to Ramesh. After nine years Ramesh will be 47 years old. If the difference between the ages of Mahesh and Nilesh is same as the age of Ramesh, what is the value of X?
a. 11.8 b. 12.9 c. 13.7 d. 14.5
4. Sobha's father was 38 years of age when she was born while her mother was 36 years old when her brother (four years younger to her) was born. What is the difference between the ages of her parents?
a) 6yrs b) 5yrs c) 12yrs d) 10yrs
5. The total age of A and B is 12 years more than the total age of B and C. C is how many year younger than A?
a) 10 b) 11 c) 12 d) 13
6. The age of a person is thrice the total ages of his 2 daughters. 0.5 decades hence, his age will be twice of the total ages of his daughters. Then what is the father's current age?
[0.5 Decades = 5 Years]
a. 35 years b. 40 years c. 45 years d. 47 year

7. A man's age is 125% of what it was 10 years ago, but $83\frac{1}{3}\%$ of what it will be after 10 years. What is his present age?
a) 60yrs b) 70yrs c) 50yrs d) 40yrs
8. The ratio of the age of a man and his wife is 4:3. At the time of marriage the ratio was 5:3 and after 4 years this ratio will become 9:7. How many years ago were they married?
a) 8yrs b) 10ys c) 11yrs d) 12yrs
9. Ayisha's age is $\frac{1}{6}$ th of her father's age. Ayisha's father's age will be twice Shankar's age after 10 years. If Shankar's eighth birthday was celebrated two years before, then what is Ayisha's present age?
a) 8yr b) 10yr c) 12yr d) 5yr
10. My brother is 3 years elder to me. My father was 28 years of age when my sister was born while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then what was the age of my father when my brother was born?
a) 35yr b) 32yr c) 33yr d) 34yr

MIXTURES AND ALLIGATION

1. Alligation

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

2. Mean Price

The cost price of a unit quantity of the mixture is called the mean price.

3. Basic Formula

If two ingredients **A** and **B** of price **x** and **y** respectively are mixed and the price of resultant mixture is **M** (mean price) then the ratio (**R**) in which ingredients are mixed is given by, the rule of allegation

$$R = (M - y) / (x - M)$$

We can use the allegation rule in following manners:

1. To find the cost price(s) of mixture (mean price) when two ingredients A and B are mixed with

quantity q_1 and q_2 and cost C_1 and C_2 respectively.

$$C = \text{cost of mixture} = \frac{C_1q_1 + C_2q_2}{q_1 + q_2}$$

When more than two ingredients are mixed

$$C = \text{cost of mixture} = \frac{C_1q_1 + C_2q_2 + C_3q_3 + \dots + C_nq_n}{q_1 + q_2 + q_3 + \dots + q_n}$$

2. To find the ratio: - If two ingredients A and B of price x and y respectively are mixed and the price of resultant mixture is M (mean price) then the ratio (R) in which ingredients are mixed is given by, the rule of allegation

$$R = \frac{M - y}{x - M}$$

3. Replacement of Part of Solution Formula

Suppose a container contains a solution from which some quantity of solution is taken out and replaced with one of the ingredients. This process is repeated n times then,

$$\text{Final Amount of ingredient not replaced} = \text{Initial Amount} \left(1 - \frac{\text{Amount Replaced}}{\text{total Volume}}\right)^n$$

Note: Above formula is not only true for absolute amounts but for ratios as well.

11. A Trader has 2000Kg of rice. He sells a part at 20% profit and the rest at 28% profit. If he gains 25% on the whole, find the quantity sold (in Kg) at 20% profit
 - a. 750
 - b. 1250
 - c. 1500
 - d. NOTA
12. Some amount out of Rs.8000 was lent at 11% per annum and the remaining was lent at 15% per annum. If the total simple interest from both the fractions in 2 years was Rs.2000, then sum lent of 11% per annum was
 - a. 3000
 - b. 5000
 - c. 2750
 - d. 4000
13. Prakash bought 17 kg of wheat at the rate of Rs.13 per kg and 85 kg at the rate of Rs.14.2 per kg. He mixed the two. at what price per kg should he sell the mixture to make 20% profit?
 - a. 14
 - b. 17.2
 - c. 16.6
 - d. 16.8
14. A jar full of mixture of milk and water contains 60% milk. A part of this mixture is replaced by another containing 35% milk and now the percentage of milk was found to be 45%. The quantity of mixture replaced is
 - a. $\frac{2}{3}$
 - b. $\frac{3}{2}$
 - c. $\frac{3}{5}$
 - d. $\frac{2}{5}$
15. Two solutions of 90% and 97% purity are mixed resulting in 21 liters of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?
 - a. 15 liters
 - b. 12 liters
 - c. 9 liters
 - d. 6 liters
16. 13 liter of water is added to 52 liter of a solution containing 40% of alcohol in the water. The percentage of alcohol in the new mixture is
 - a. 32
 - b. 20
 - c. 28.5
 - d. NOTA
17. A container contains 60 liters of water. From this container 6 liters was taken out and replaced by alcohol. This process was repeated further two times. How much liters of water is now left in the container?
 - a. 42
 - b. 18
 - c. 48.6
 - d. 43.74
18. 15 liters are drawn from a cask full of pure milk and is then replaced with water. This operation is performed one more time. The ratio of the quantity of milk now left in cask to that of the water is 16:9. How much milk did the cask originally hold?
 - a. 80
 - b. 100
 - c. 60
 - d. 75
19. A vessel is filled with liquid, 2 parts of which are water and 1 part of milk. How much of the mixture must be drawn off and replaced with pure milk so that the mixture will have 50% milk in it?
 - a. $\frac{1}{3}$
 - b. $\frac{1}{4}$
 - c. $\frac{2}{3}$
 - d. $\frac{1}{2}$
20. Metal A, B, and C are melted together in the ratio 3:2:1 and formed into a mass of an alloy. Cost of 1 kg of metal A is Rs. 20, of a kg of metal B is 25% more than that of metal A, If the quantity melted was sold at Rs. 24.15 for profit of 15%, what is the price of metal C in rupees per kg.?
 - A. 15
 - b. 21
 - c. 16
 - d. 18.5
21. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety of tea in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, what is the price of the third variety per kg?
 - a. Rs.182.50
 - b. Rs.170.50
 - c. Rs.175.50
 - d. Rs.180
22. A merchant has 1000 kg of sugar part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is

- a. 300 b. 400 c. 600 d. 500
23. Some amount out of Rs.7000 was lent at 6% per annum and the remaining was lent at 4% per annum. If the total simple interest from both the fractions in 5 years was Rs.1600, the sum lent at 6% per annum was
- a. 2400 b. 2200 c. 2000 d. 1800
24. John bought 20 kg of wheat at the rate of Rs.8.50 per kg and 35 kg at the rate of Rs.8.75 per kg. He mixed the two. Approximately at what price per kg should he sell the mixture to make 40% profit at the cost price?
- a. Rs.12 b. Rs.8 c. Rs.16 d. Rs.20
25. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?
- a. 80% b. 83.33% c. 85.71% d. 88.88%

LEVEL 2:

1. Two alloys A and B are composed of two basic elements. The ratios of the compositions of the two basic elements in the two alloys are 5:3 and 1:2, respectively. A new alloy X is formed by mixing the two alloys A and B in the ratio 4:3. What is the ratio of the composition of the two basic elements in alloy X?
- a. 1:1 b. 2:3 c. 5:2 d. 4:3
2. A 20 liter mixture of milk and water contains milk and water in the ratio 3 : 2. 10 liters of the mixture is removed and replaced with pure milk and the operation is repeated once more. At the end of the two removals and replacement, what is the ratio of milk and water in the resultant mixture?
- a. 17:3 b. 9:1 c. 3:17 d. 5:3
3. A 20% ethanol solution is mixed with another ethanol solution, say, S of unknown concentration in the proportion 1:3 by volume. This mixture is then mixed with an equal volume of 20% ethanol solution. If the resultant mixture is a 31.25% ethanol solution, then the unknown concentration of S is
- a. 52% b. 50% c. 55% d. 48%
4. The strength of a salt solution is p% if 100 ml of the solution contains p grams of salt. If three salt solutions A, B, C are mixed in the proportion 1 : 2 : 3, then the resulting solution has strength 20%. If instead the proportion is 3 : 2 : 1, then the resulting solution has strength 30%. A fourth solution, D, is produced by mixing B and C in the ratio 2 : 7. The ratio of the strength of D to that of A is
- a. 2:5 b. 1:3 c. 1:4 d. 3:10
5. The strength of a salt solution is p% if 100 ml of the solution contains p grams of salt. Each of three vessels A, B, C contains 500 ml of salt solution of strengths 10%, 22%, and 32%, respectively. Now, 100 ml of the solution in vessel A is transferred to vessel B. Then, 100 ml of the solution in vessel B is transferred to vessel C. Finally, 100 ml of the solution in vessel C is transferred to vessel A. The strength, in percentage, of the resulting solution in vessel A is
- a. 14 b. 12 c. 15 d. 13

6. A man buys milk at Rs.8.5 per liter and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every liter of milk.
 - a. 0.111 liters
 - b. 0.909 liters
 - c. 0.1 liters
 - d. 0.125 liters
7. A 20% gain is made by selling the mixture of two types of ghee at Rs.480 per kg. if the type costing 610 per kg was mixed with 126 kg of the other, how many kilograms of the former was mixed?
 - a. 138 kg
 - b. 34.5 kg
 - c. 69 kg
 - d. Can't be determined
8. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in the three vessels of equal capacity in the ratios 4:1, 5:2 and 6:1 respectively?
 - a. 166:22
 - b. 83:222
 - c. 83:44
 - d. None of these
9. A milkman mixed 1 : 4 solution of milk and water with another 1 : 2 solution of milk and water in the volume of ratio 3 : 2. If the profit earned by selling the first solution was 20% and the mixture was sold at the same price, what is the profit or loss percentage? You have to assume that water comes free of cost.
 - a. 5.26%
 - b. 5.25%
 - c. 6.25%
 - d. None of these
10. A vessel contains 'K' liters of pure alcohol. 10% of it is replaced with water. This process was repeated once more. The difference in the quantity of alcohol in the vessel and the quantity of water in the vessel now is 186 liters. How much the vessel originally holds?
 - a. 100 lit
 - b. 260 lit
 - c. 280 lit
 - d. 300 lit

PRACTICE SHEET

1. How many litres of a 12 litre mixture containing milk and water in the ratio of 2:3 be replaced with pure milk so that the resultant mixture contains milk and water in equal proportion?
 - a. 1 lit
 - b. 1.5 lit
 - c. 2 lit
 - d. 4 lit
2. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price?
 - a. 1 : 6
 - b. 6 : 1
 - c. 2 : 3
 - d. 4 : 3
3. 6000 is lent out in two parts. On part is lent at 7% SI and other is lent at 10% SI. Total interest at the end of the year was 450. Find the ratio of the amount lent at 7% to 10%.
 - a. 5:1
 - b. 4:1
 - c. 3:2
 - d. 2:1
4. A vessel contains 10 litres of milk and 2.5 litres of water. After 20% of the contents of the vessel is taken out and x litres of water is added to it, the ratio of milk to water is reversed. The value of x is then
 - a. 30ltr
 - b. 32ltr
 - c. 40ltr
 - d. 36ltr

5. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?
 a. 4 litres, 8 litres b. 6 litres, 6 litres c. 5 litres, 7 litres d. 7 litres, 5 litres
6. To gain 10% on selling sample of milk at the cost price of pure milk, the quantity of water to be mixed with 50 kg. of pure milk is
 a. 2.5 Kg b. 5 kg c. 7.5 Kg d. 10 Kg
7. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of water is 16 : 65. How much wine did the cask hold originally?
 a. 18 litres b. 24 litres c. 32 litres d. 42 litres
8. A vessel is full of mixture of kerosene and petrol in which there is 18% kerosene. Eight liters are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the vessel hold?
 a. 32L b) 36L c) 40L d) 48L
9. Three vessels contain equal mixtures of milk and water in the ratio 6 : 1, 5 : 2 and 3 : 1 respectively. If all the solutions are mixed together, the ratio of milk to water in the final mixture will be
 a. 64:65 b. 65:64 c. 19:65 d. 65:19
10. A dairy man pays Rs. 6.4 per litre of milk. He adds water and sells the mixture at Rs. 8 per litre, thereby making 37.5% profit. Find the proportion of the water to that of the milk received by the customers.
 a. 1:15 b. 1:20 c. 1:10 d. 1:12
11. A vessel contains a mixture of milk and water in the ratio of 14 : 3. Now, 25.5 liters of the mixture is taken out from the vessel and 2.5 litres of pure water and 5 litres of pure milk is added to the mixture. If the resultant mixture contains 20% water, what was the initial quantity of mixture in the vessel before the replacement? (in litres)
 a. 51 b. 102 c. 68 d. 85
12. There is a vessel holding 40 litre of milk. 4 litre of milk is initially taken out from the vessel and 4 litre of water is then poured in. After this, 5 litre of mixture is replaced with 5 litre of water. And finally, 6 litres of the mixture is replaced with 6 litre of water. How much milk (in litres) is there in the vessel?
 a. 26.775 b. 29.16 c. 24.72 d. 27.42
13. Two varieties of wheat M and N are mixed in the ratio 4:3. Cost of M is more than the cost of N by Rs. 7 per kg. If the cost of the mixture is Rs. 23 per kg, the cost of N is (in Rs. per kg),
 a. 18 b. 20 c. 19 d. 22

14. 20 litres of milk is taken out from a vessel containing 200 litres of pure milk and replaced with water. This process of replacement was repeated x number of times to leave 145.8 litres of pure milk in the mixture. Find the value of x
- a. 3 b. 2 c. 5 d. 4
15. Concentration of alcohol in three containers P, Q and R are 30%, 25% and 45% respectively. If 5 litres from container P, 6 litres from container Q and 4 litres from container R are mixed together, the alcohol concentration in the mixture would be,
- a. 25% b. 32% c. 37.5% d. 31.25%
16. How many kgs of Basmati rice costing Rs.42/kg should a shopkeeper mix with 25 kgs of ordinary rice costing Rs.24 per kg so that he makes a profit of 25% on selling the mixture at Rs.40/kg?
- a. 20 b. 12.5 c. 16 d. 200
17. A sample of x litres from a container having a 60 litre mixture of milk and water containing milk and water in the ratio of 2 : 3 is replaced with pure milk so that the container will have milk and water in equal proportions. What is the value of x ?
- a. 6lit b. 10lit c. 30lit d. None
18. A zookeeper counted the heads of the animals in a zoo and found it to be 80. When he counted the legs of the animals he found it to be 260. If the zoo had either pigeons or horses, how many horses were there in the zoo?
- a. 30 b. 40 c. 50 d. 60
19. A merchant mixes three varieties of rice costing Rs.20/kg, Rs.24/kg and Rs.30/kg and sells the mixture at a profit of 20% at Rs.30 / kg,. How many kgs of the second variety will be in the mixture if 2 kgs of the third variety is there in the mixture?
- a. 1kg b. 3kg c. 5kg d. 6kg
20. In what ratio must a person mix three kinds of tea costing Rs.60/kg, Rs.75/kg and Rs.100 /kg so that the resultant mixture when sold at Rs.96/kg yields a profit of 20%?
- a. 1:2:4 b. 3:7:6 c. 1:4:2 d. None

PARTNERSHIP

If **two or more persons** invest their money (capital) in a joint business, their **association** is called **partners**

Partnership is of two kinds: **Simple** and **Compound**

1. **Simple Partnership**: If the capitals of several partners are invested for the **same period**, then it is called simple partnership.

2. **Compound Partnership**: If the capitals are invested for **different periods**, then it is called compound partnership.

A partner who simply invests money but does not attend to the business is called a **sleeping partner**. One who invests money as well as attends business is a **working partner**.

Generally, the gain or loss is divided among the partners in a partnership on the basis of following rules:

1. In a partnership, the gain or loss is distributed among the partners in the ratio of their capital investments when the investments of all the partners are for the same period.

2. In a partnership, the gain or loss is distributed among the partners in the ratio of their equivalent capital investments for a unit time. The equivalent capital investments for a unit of time are calculated by taking [**Capital × Number of units of time**].

If three persons A, B and C invests I_a , I_b and I_c amounts respectively for T_a , T_b and T_c time respectively and if total profit of the business is P , then

$\text{A's share in profit} = \frac{I_a \times T_a}{[(I_a \times T_a) + (I_b \times T_b) + (I_c \times T_c)]} P$
$\text{B's share in profit} = \frac{I_b \times T_b}{[(I_a \times T_a) + (I_b \times T_b) + (I_c \times T_c)]} P$
$\text{C's share in profit} = \frac{I_c \times T_c}{[(I_a \times T_a) + (I_b \times T_b) + (I_c \times T_c)]} P$

Chapter: Partnership

Number of Questions: Class Work: 25+10 Practice Sheet: 20

CLASS-SHEET

LEVEL 1:

1. A and B are partners in a business. A invests Rs 6000 for complete year & B invests Rs 4000 for 6 months. What is B's share if they earn Rs 60K as profit?
 a. 20K b) 15K c) 45K d) 30K
2. Three partners A, B and C invests Rs8000, Rs9000 and Rs9500 respectively in a business. How should they divide a profit of Rs27440?
 a. 8:9:10 b) 16:18:21 c) 16:18:19 d) 8:9:9
3. A, B and C invested capitals in the ratio 3:4:6; the timing of their investments being in the ratio 4:3:6. In what ratio would their profit be distributed?
 a) 1:1:3 b) 6:8:5 c) 6:8:9 d) 3:3:5
4. A, B and C invested capitals in the ratio 2:4:3; they finally shared profits among them in the ratio of 3:8:9. Then what must have been their time periods of investments?
 a) 6:32:27 b) 6:4:9 c) 6:8:9 d) 3:4:6
5. A starts a business with Rs15000. B joins him after 4 months with Rs20000. C puts a sum of Rs30000 in the business for 2 months only. At the end of the year the business gave a profit of Rs85500. How should the profit be divided among them?
 a) 10:9:3 b) 8:7:6 c) 11:8:9 d) 9:8:3
6. X and Y invest Rs.21000 and Rs.17500 respectively in a business. At the end of the year, they make a profit of Rs.26400. What is the share of X in the profit?
 a. 14000 b. Rs.26400 c. Rs.12000 d. Rs.12500
7. X starts a business with Rs.45000. Y joins in the business after 3 months with Rs.30000. What will be the ratio in which they should share the profit at the end of the year?
 a. 1:2 b. 2:1 c. 1:3 d. 3:1
8. Suresh started a business with Rs. 20,000. Kiran joined him after 4 months with Rs. 30,000. After 2 months, Suresh withdrew Rs. 5,000 of his capital and 2 more months later, Kiran brought in Rs. 20,000 more. What should be the ratio in which they should share their profits at the end of the year?
 a. 21:32 b. 32:21 c. 12:17 d. 17:12
9. Kamal started a business with Rs.25000 and after 4 months, Kiran joined him with Rs.60000. Kamal received Rs.58000 including 10% of profit as commission for managing the business. What amount did Kiran receive?
 a. 75000 b. 70000 c. 72000 d. 78000
10. A and B started a partnership business investing Rs. 20,000 and Rs. 15,000 respectively. C joined them with Rs. 20,000 After six months. Calculate B's share in total profit of Rs. 25,000 earned at the end of 2 years from the starting of the business?
 a. 7500 b. 8500 c. 9000 d. 8000
11. A starts a business with a capital of Rs. 85,000. B joins in the business with Rs.42500 after some time. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3 : 1?

- a. 5 months b. 6 months c. 7 months d. 8 months
12. A starts a business with Rs. 40,000. After 2 months, B joined him with Rs. 60,000. C joined them after some more time with Rs. 1,20,000. At the end of the year, out of a total profit of Rs. 3,75,000, C gets Rs. 1,50,000 as his share. How many months after B joined the business, did C join?
- a. 4 months b. 5 months c. 6 months d. 7 months
13. A and B invest in a business in the ratio 3: 2. Assume that 5% of the total profit goes to charity. If A's share is Rs. 855, what is the total profit?
- a. 1400 b. 1500 c. 1600 d. 1200
14. A, B and C invest in a partnership in the ratio: $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share 50%. If the total profit at the end of one year be Rs. 21,600, then what is B's share in the profit?
- a. Rs. 2000 b. Rs. 3000 c. Rs. 4000 d. Rs. 5000
15. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Rs. 6500 for 6 months, B, Rs. 8400 for 5 months and C, Rs. 10,000 for 3 months. A wants to be the working member for which, he was to receive 5% of the profits. The profit earned was Rs. 7400. What is the share of B in the profit.
- a. Rs. 2660 b. Rs. 1000 c. Rs. 2300 d. Rs. 4000
16. A, B, C subscribe Rs. 50,000 for a business. If A subscribes Rs. 4000 more than B and B Rs. 5000 more than C, out of a total profit of Rs. 35,000, what will be the amount A receives?
- a. Rs. 14200 b. Rs. 14700 c. Rs. 14800 d. Rs. 14500
17. A, B, C rent a pasture. If A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing and the rent of the pasture is Rs. 175, then how much amount should C pay as his share of rent?
- a. 45 b. 35 c. 55 d. 60
18. A and B entered into partnership with capitals in the ratio 4 : 5. After 3 months, A withdrew $\frac{1}{4}$ of his capital and B withdrew $\frac{1}{5}$ of his capital. At the end of 10 months, the gain was Rs. 760. What is A's share in the profit?
- a. 310 b. 330 c. 370 d. 350
19. A starts a business with Rs. 3500. After 5 months, B joins with A as his partner. After a year, the profit is divided in the ratio 2 : 3. B's contribution in the capital is
- a. 7000 b. 8000 c. 9000 d. 10000
20. A, B and C shared the profit in a business in the ratio 5 : 7 : 8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investments?
- a. 10:12:14 b. 12:24:28 c. 20:22:12 d. 20:49:64

21. A and B started a partnership business investing capital in the ratio of 3 : 5. C joined in the partnership after six months with an amount equal to that of B. At the end of one year, the profit should be distributed among A, B and C in --- proportion.
- a. 10 : 5 : 4 b. 5 : 3 : 4 c. 3 : 4 : 5 d. 6 : 10 : 5
22. A & B partner in a business, A contribute $\frac{1}{4}$ of the capital for 15 months & B received $\frac{2}{3}$ of the profit. For how long B's money was used
- a. 12 months b. 10 months c. 14 months d. 16 months
23. A , B , C started a partnership business by investing Rs 27000 , 72000 , 81000 respectively. At the end of the year, the profit were distributed among them. If C's share of profit is 36000, what is the total profit?
- a. 80000 b. 90000 c. 70000 d. 120000
24. A & B started a partnership business. A's investment was thrice the investment of B and the period of his investment was two times the period of investments of B. If B received Rs 4000 as profit, what is their total profit?
- a. 28000 b. 30000 c. 32000 d. 34000
25. P and Q invested in a business. The profit earned was divided in the ratio 2 : 3. If P invested Rs 40000, the amount invested by Q is
- a. 40000 b. 50000 c. 60000 d. 70000

LEVEL 2:

1. Akanksha started a business by investing Rs. 48,000. Later she was joined by Kamini with Rs. 36,000. At the end of the first year, Kamini received $\frac{1}{5}$ th of the total profit. After how many months did Kamini join?
- a)4 b) 6 c) 8 d)3
2. A, B, C hired a car for Rs. 840 and used it for 4, 8 and 9 hours respectively. Hire charges paid by B were
- a)240 b) 320 c) 360 d)160
3. A and B enter in to a partnership with their capitals in the ratio 3: 4. At the end of 9 months, A withdraws his capital. If they receive the profits in the ratio 9:4, find how long B's capital was used in months?
- a)4 b) 9 c) 12 d)3
4. A, B and C signed up a partnership. A invested Rs. 3000/- . B invested Rs. 5000/- and C invested Rs. 4000/-. At the end of 6 months A withdrawn Rs. 1000/- of his capital and at the end of another 2 months B increased his capital by Rs. 1000/-. First year profit is Rs 25400. What is B's share of total profit?
- a)10200 b) 12000 c) 12800 d)9600
5. Akbar and Birbal invested Rs. 80,000 and Rs. 60,000 in a business. They decided to divide 60% of their profit equal between them and balance in the ratio of their

investment. If the difference in their individual profits is Rs. 400, find total profit of the business?

- a) 10500 b) 6400 c) 8400 d) 7000
6. A, B and C invested some amount in a business in the ratio of 4:6:9 respectively. In the next year, their investments are increased by 50%, 40% and 20% respectively. In what ratio the profit earned during the second year should be distributed among them?
a) 5:7:10 b) 5:7:9 c) 11:8:9 d) 9:8:3
7. A, B and C invested some amount in a business in the ratio of 4:6:9 respectively. In the next year, the investments of all of them increased by 29% each. In what ratio the profit earned during the first two years should be distributed among them?
a) 33:35:38 b) 5:6:8 c) 4:6:9 d) NOTA
8. A and B invest in a business in the ratio 4:5. If 5% of the total profit goes to charity and A's share is Rs. 800, then total profit is
a) 1800 b) 1850 c) 2000 d) NOTA
9. A and B entered in to partnership with the capitals in the ratio 3:4. After 4 months, A withdrew $\frac{1}{3}$ rd of his capital and B withdrew $\frac{1}{4}$ th of his capital. The gain at the end of year was Rs. 6300. Find A's share in the profit if B gets Rs. 100 per month salary for running the business.
a) 4000 b) 3000 c) 2100 d) NOTA
10. A and B entered in to a partnership and A invests Rs. 10,000 in the partnership. At the end of 4 months he withdraws Rs. 2000. At the end of another 5 months, he withdraws another Rs. 3000. If B invests a certain sum in the partnership at the beginning of the year and leaves it intact and receives Rs. 9600 as his share of the total profit of Rs. 19,100 for the year, how much did B invest in the company?
a) 6000 b) 8000 c) 9000 d) 8500

TIME AND WORK, PIPES AND CISTERNS

Theory

Work is always taken as one unit. The unit time in which the work is being done may be in minutes, hours, days, weeks, or months.

Basic Concepts

1. If a man can do a piece of work in 5 days, then he will finish $\frac{1}{5}th$ of the work in one day.
2. If a man can finish $\frac{1}{5}th$ of the work in one day, then he will take 5 days to complete the work.
3. The basic relation between total work done (W), rate of work (R), and time required to perform a work (t) is $W = R \cdot t$

Concept of Variation

1. More the number of persons employed, more the work done.
2. More the number of days for which a work was done, more shall be the amount of work done.
3. More the number of persons employed, less will be the time required to finish the work.

Rule 1: This rule can be used in almost every problem. If M_1 persons can do W_1 work in D_1 days and M_2 persons can do W_2 works in D_2 days then we can say,

$$\frac{M_1 \times D_1}{W_1} = \frac{M_2 \times D_2}{W_2}$$

If the persons work T_1 and T_2 hours per day respectively then the equation gets modified to

$$\frac{M_1 \times D_1 \times T_1}{W_1} = \frac{M_2 \times D_2 \times T_2}{W_2}$$

If the persons have efficiency of E_1 and E_2 respectively then,

$$\frac{M_1 \times D_1 \times T_1}{W_1} = \frac{M_2 \times D_2 \times T_2}{W_2}$$

Rule 2: If **A** can do a piece of work in n days, then The work done by **A** in one day $= \frac{1}{n}$

Rule 3: If **A** can do a work in D_1 days and **B** can do the same work in D_2 days then **A** and **B** together can do the same work in

$$\frac{D_1 \times D_2}{D_1 + D_2} \text{ days}$$

Rule 4: If **A** is twice as good a workman as **B**, then **A** will take half of the time taken by **B** to complete a piece of work.

Rule 5: If **A** and **B** together can do a piece of work in x days, **B** and **C** together can do in y days and **C** and **A** together can do in z days, then the same work can be done in

$$\frac{2xyz}{xy + yz + zx} \text{ days}$$

Rule 6: If **A** can do a piece of work in D_1 days, **B** can do in D_2 days and **C** can do in D_3 days then they together can do the same work in

$$\frac{xyz}{xy + yz + zx} \text{ days}$$

Rule 7: If **A** and **B** together can do a piece of work in D_1 days and **A** alone can do it in D_2 days, then **B** alone can do the work in

$$\frac{D_1 \times D_2}{D_2 - D_1} \text{ days}$$

Pipes and Cisterns

The principle used to solve problems on Pipes and Cisterns is same as in Time and Work. Here the work done is in terms of filling or emptying a cistern.

Time taken to fill a tank is taken as positive and the time taken to empty a tank is taken as negative.

1. If a pipe can fill a cistern in ' h ' hours, then in 1 hour $\frac{1}{h}$ of the tank will be filled.
2. If a pipe can empty a filled tank in ' h ' hours, then in 1 hour $\frac{1}{h}$ of the tank will be emptied.

If **A** does a % work in one day and **B** does b % of the same work in one day then **A** and **B** together will take $\frac{100}{a+b}$ days to complete the work.

Relation for building a wall of a certain length, breadth, and height is

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{m_1 t_1 d_1}{m_2 t_2 d_2}$$

where **m** stands for number of men, **t** stands for amount of time for which the work is done per day, and **d** stands for number of days taken to build the wall of given dimensions.

Relation between number of men and number of days taken to complete the work is
number of men x number of days taken to complete the work = constant.

Alternative method (LCM method)

Let's say A takes ' a ' days, B takes ' b ' days and C takes ' c ' days to do same work individually. Assume total work as LCM (a, b, c) = L unit

Time→	A(a days)	B(b days)	C(c days)
Rate→unit/day	$\frac{L}{a}$	$\frac{L}{b}$	$\frac{L}{c}$
Work→unit	L	L	L

Chapter: Time and Work, Pipes and Cisterns

Number of Questions: Class Work: 25+10 Practice Sheet: 20

Class Sheet Level 1:

1. A man takes 5 hours to complete a task. What fraction of the work is completed in 1 hour?

- a. 1 b. $\frac{1}{2}$ c. $\frac{1}{5}$ d. $\frac{1}{4}$
2. A man can complete a work in 12 days. What fraction of the work is completed in 4 days?
- a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{5}$
3. If A can make 25 candles in 5 hours, what is his efficiency of making candles?
- a. 5 b. 25 c. 20 d. 15
4. If A can complete a task in 12 days where as B can finish the same work in 15 days. What is the ratio of their working efficiency?
- a. 4:5 b. 5:4 c. 3:5 d. 5:3
5. A is 40% more efficient than B. what is the ratio of their working rate?
- a. 7:5 b. 5:7 c. 2:7 d. 7:2
6. John takes twice the time taken by Harry to complete a task. If John can make 30 candles in a day, then how many candles can Harry make in a day?
- a. 60 b. 30 c. 15 d. 45
7. X is thrice as fast as Y to complete a task. If X takes 40 days less than Y to complete a task, then find the taken by them working together to complete the task.
- a. 12 b. 13 c. 14 d. 15
8. If A takes 20 days to complete a task while B takes 30 days to complete the same task. If they started working together and after 7 days, A left the work and the remaining working was completed by B working alone. Find the time taken to complete the work.
- a. 19.5 b. 17.5 c. 12.5 d. None of these
9. A, B and C can do a job in 20, 30 and 60 days working individually. In how many days A and B can complete the job if they are assisted by C on every third day?
- a. $7\frac{1}{8}$ b. $8\frac{8}{11}$ c. $9\frac{8}{11}$ d. None of these
10. A can complete a work in 6 days. While B can complete the same work in 12 days. If they work composed and complete it, the portion of the work done by A is
- a. $\frac{1}{3}$ b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. $\frac{2}{3}$
11. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day?
- a. $\frac{1}{6}$ b. $\frac{1}{9}$ c. $\frac{2}{5}$ d. $\frac{2}{7}$
12. Reena, Aastha and Shloka can independently complete a piece of work in 6 hours, 4 hours and 12 hours respectively. If they work together, how much time will they take to complete that piece of work?
- a. 2 hours b. 5 hours c. 6 hours d. 8 hours
13. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled?
- a. 3 hours b. 2 hours c. 4 hours d. 5 hours
14. Two taps can fill a tank in 20 minutes and 30 minutes respectively. There is an outlet tap at exactly half level of that rectangular tank which can pump out 50 liters of water per

minute. If the outlet tap is open, then it takes 24 minutes to fill an empty tank. What is the volume of the tank?

- a. 900 liters b. 600 liters c. 180 liters d. 240 liters
15. Rosa can eat 32 rosogollas in one hour. Her sister Lila needs three hours to eat the same number. How much time will they take to eat 32 rosogollas together?
- a. 45 minutes b. 75 minutes c. 90 minutes d. None of these
16. A, B and C can separately do a work in 12, 15 and 20 days respectively. They started to work together but C left after 2 days. The remaining work will be finished in
- a. 4 days b. 5 days c. 6 days d. 15 days
17. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last?
- a. 6 days b. 10 days c. 15 days d. 20 days
18. The daily wages of a worker are Rs.100. Five workers can do a work in 10 days. If you pay Rs.20 more daily, they agree to do 25% more work daily. If the proposal is accepted, then the total amount that could be saved is
- a. Rs.200 b. Rs.250 c. Rs.300 d. Rs.350
19. Twenty – four men can complete a work in sixteen days. Thirty- two women can complete the same work in twenty- four day. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days?
- a. 16 b. 24 c. 36 d. 48
20. A started a work and left after working for 2 days. Then B was called and he finished the work in 9 days. Had A left the work after working for 3 days, B would have finished the remaining work in 6 days. In how many days can each of them, working alone, finish the whole work?
- a. 2.5 days, 7.5 days b. 5 days, 8.5 days c. 5 days, 15 days d. None of these
21. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man alone to complete the same work. How many days will be required for one woman alone to complete the same work?
- a. 90 b. 225 c. 145 d. 150
22. Ayesha can complete a piece of work in 16 days. Amita can complete the same piece of work in 8 days. If both of them work together in how many days can they complete the same piece of work?
- a. $4\frac{2}{5}$ days b. $5\frac{1}{3}$ days c. 6 days d. 12 days
23. A, B and C together earn 300 per day, while A and C together earn Rs.188 and B and C together earn Rs.152. The daily earning of C is:
- a. Rs.40 b. Rs.68 c. Rs.112 d. Rs.150
24. At a constant rate, 10 men can build 18 bricks in 12 minutes. How much time will it take for 6 men to build 9 bricks?

- a. 10 b. 6 c. 9 d. 12
25. David and Michael together can finish a job in 4 days 19hrs 12min. If David works at two – thirds Michael's speed, how long does it take Michael alone to finish the same job?
- a. 8 days b. 12 days c. 15 days d. None of these

LEVEL 2

1. Madhu takes twice as much time as Uma to complete a work and Rahul does it in the same time as Madhu and Uma together. If all three working together can finish the work in 6 days, then the time taken by Madhu to finish the work is

a. 12 days b. 14 days c. 36 days d. 40 days
2. Two spinning machines A and B can together produce 3,00,000 meters of cloth in 10 hours. If machine B alone can produce the same amount of cloth in 15 hours, then how much cloth can machine A produce alone in 10 hours?

a. 50,000 meters b. 1,00,000 meters c. 1,50,000 meters d. 2,00,000 meters
3. A does 20% less work than B. If A can complete a piece of work in $7\frac{1}{2}$ hours, then B can do it in

a. 5 hours b. 5.5 hours c. 6 hours d. 6.5 hours
4. A is 50% as efficient as B. C does half of the work done by A and B together. If C alone does the work in 40 days, then A, B and C together can do the work in

a. $13\frac{1}{3}$ days b. 15 days c. 20 days d. 30 days
5. 4 men and 10 women were put on a work. They completed $\frac{1}{3}$ of the work in 4 days. After this, 2 men and 2 women were increased. They completed $\frac{2}{9}$ more of the work in 2 days. If the remaining work is to be completed in 3 days, then how many more women must be increased?

a. 8 b. 32 c. 50 d. 55
6. A leak in the bottom of a cistern can empty the tank in 12 hrs. An inlet pipe fills water at the rate of 5 liters a minute. When the tank is full, the inlet is opened and due to the leak, the tank is emptied in 15 hrs. How many liters does the cistern hold?

a. 82600 liters b. 12000 liters c. 15000 liters d. 18000 liters
7. Working on an assignment together, Pam and Ken can finish it in 12 days. Ken and Ron need 4 days more than Pam and Ken to finish the assignment while Ron and Pam together would take 3 weeks and 3 days to finish the same assignment. If all of them are allowed to work together on the assignment, how much time would they take to finish the assignment?

a. $9\frac{3}{4}$ days b. 10 days c. $10\frac{2}{3}$ days d. $11\frac{1}{4}$ days
8. 20 women can finish a job in 20 days. After each day, a woman is replaced by a man and a man is twice as efficient as a woman. On which day does the job get completed?

- a. 16th day b. 15th day c. 17th day d. 18th day
9. A manufacturer builds a machine which will address 500 envelopes in 8 minutes. He wishes to build another machine so that when both are operating together they will address 500 envelopes in 2 minutes. The equation used to find how many minutes' x it would require the second machine to address 500 envelopes alone, is
- a. $8 - x = 2$ b. $\frac{1}{8} + \frac{1}{x} = \frac{1}{2}$ c. $\frac{500}{8} + \frac{500}{x} = 500$ d. $\frac{x}{2} + \frac{x}{8} = 1$
10. 16 men can finish a piece of work in 49 days. 14 men started working and in 8 days they could finish certain amount of work. If it is required to finish the remaining work in 24 days. How many more men should be added to the existing workforce?
- a. 21 b. 28 c. 16 d. 14

PRACTICE SHEET

- 1) 3 men and 6 women finish a job in 9 days, while 2 men and 8 women finish it in 12 days. In how many days will 12 women working alone finish the same job?
- 2) In a day if 18 men and 10 boys can do as much work as 10 men and 22 boys, then how much should a man be paid per day if a boy is to get Rs.5 a day?
- a) 6.50 b) 7.50 c) 8.00 d) 8.50
- 3) 4 pipes can fill a reservoir in 15, 20, 30 and 60 hours respectively. The first was opened at 6 am, second at 7 am, third at 8 am and the fourth at 9 am. When will the reservoir be filled up completely?
- a) 2 pm b) 3 pm c) 1 pm d) 2.30 pm
- 4) If the first pipe takes 30 min to fill a tank and second takes 45 min to empty it, then in how much time will the tank be full, if the second pipe is opened fifteen min earlier than the first pipe and the tank is empty at first?
- a) 75min b) 90min c) 65min d) never
- 5) Three pipes can fill a reservoir in 10, 15 and 20 hrs respectively. If the three taps are opened one after the other in the given order with a certain fixed time gap between them, the reservoir fills in 5 hours. Find the time gap?
- a) 45min b) 25min c) 30min d) 35min
- 6) Kapil and Yogesh alone are able to finish a project assigned to them in a span of 30 days, in spite of taking leave for 12 days and 15 days respectively. If both of them decide to do the same project together, how many minimum days will be required by them, if they don't abstain from work at all?
- 7) A and B can finish a piece of work in 24 days; B and C can do it in 36 days; C and A can do it in 30 days. In how many days will A, B and C together finish it?

- 8) P does the job in 11 days, Q does it in 12 days while R takes 22 days for the same job. P and Q started working together. But after two days they left and R had to complete the remaining job. Find the total number of days required to complete the job?
- 9) A barrel contains 56 litres of kerosene. It has two taps. One tap draws 500 ml in every 6 minutes. After first 5 litres are drawn from barrel, the second tap starts. It draws 1 litre in every 5 minutes. How many hours will be taken by both taps to empty the tank?
a) 7 hrs b) 3 hrs c) 4 hrs d) 11 hrs
- 10) A can do a piece of work in 7 days working 9 hours each and B can do it in 6 days working 7 hours each. How long will they take to do the work together, working $8\frac{2}{5}$ hours a day? (3 days)
- 11) Two pipes A and B can fill a cistern in 2 and 4 hours respectively, while pipe C can empty the cistern in 12 hours. All the three pipes are opened together at 4 pm and after 1 hour pipe C is closed. At what time will the tank be full?
a) 4.15pm b) 5.27 pm c) 5 pm d) 4.30 pm
- 12) Two taps can fill a tank in 10 minutes and 15 minutes respectively. There is an outlet tap at exactly half level of that rectangular tank which can pump out 25 liters of water per minute. If the outlet tap is open, then it takes 12 minutes to fill an empty tank. Find the volume of tank.
a) 300 b) 225 c) 150 d) 550
- 13) Two pipes can fill a cistern in 8 hours and 12 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 12 minutes more to fill the cistern. If the cistern is full, in what time will the leak empty it?
a) 130 hrs b) 120 hrs c) 125 hrs d) 112 hrs
- 14) If 10 boys or 6 men can do a piece of work in 8 days, then find the number of days that 15 boys and 11 men will take to finish the work.
- 15) A and B together finish a work in 24 days. They worked together for 16 days and then B left. A finished the remaining work in 15 days. In how many days can A alone finish the job?
a) 5 days b) 28 days c) 30 days d) 45 days
- 16) There is a certain amount of food in a pet shop. It is known that this food will last 7 dogs and 4 cats for 3 days, while it would last 8 dogs and 9 cats for 2 days. What is the ratio of amount of food eaten by a dog to that of a cat?
- 17) Three persons A, B, C finished a piece of work, A working on it for 5 days, B for 7 days and C for 9 days. Their daily wages were in a ratio 4:3:2, and the total earning amounted to Rs.118. What was the daily earnings of C?
(a) Rs.2 (b) Rs.2.5 (c) Rs.3.50 (d) Rs.4
- 18) An alloy contains Gold and Nickel in ratio 5:1 and other contains them in a ratio 7:2 respectively. What weights of the two must be melted so as to make a 5 kg mass with 80% Gold?
(a) 3, 2 (b) 2, 3 (c) 4, 1 (d) 1, 4
- 19) 30 men, working 8 hours a day dig up a trench 200 x 6 x 2 in 6 days. How many days

will be required for 45 men to dig a trench 300 x 3 x 4 working 6 hours a day?

(a) 6 (b) 8 (c) 9 (d) 12

20) An empty tank can be filled by a tap in 12 mins. The full tank can be emptied by another tap in 8 Minutes. If the tank is now three-fourth full, and both taps are opened together, after how many minutes will the tank be empty?

a) 12 b) 18 c) 24 d) 30

TIME SPEED DISTANCE-1

We define speed as distance divided by time,

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

but once we have the equation, we can use any of its variations, i.e.,

$$\text{speed} = \text{distance} / \text{time},$$

OR

$$\text{distance} = \text{speed} * \text{time}$$

OR

$$\text{time} = \text{distance} / \text{speed}$$

to compute any one of the quantities when we happen to know the other two.

For example, suppose we drive for 2 hours at 30 miles per hour, for a total of 60 miles. If we know the time and the speed, we can find the distance: 2 hours * 30 miles/hour = 60 miles. If we know the time and the distance, we can find the speed: 60 miles / 2 hours = 30 miles/hour

Conversion of units:

$$1 \text{ kmph} = \frac{5}{18} \text{ mps} , \quad 1 \text{ mps} = \frac{18}{5} \text{ kmph}$$

Note: Unit conversion should not be neglected.

Average Speed

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

1. **Average Speed** = $\frac{2ab}{a+b}$ Applicable when one travels at speed a for half the distance and speed b for other half of the distance. In this case, average speed is the harmonic mean of the two speeds. On similar lines, you can modify this formula for one-third distance.

2. **Average Speed** = $\frac{a+b}{2}$ Applicable when one travels at speed a for half the time and speed b for other half of the time. In this case, average speed is the arithmetic mean of the two speeds.

Speed is inversely proportional to time if distance is kept constant

$$\frac{s_1}{s_2} = \frac{t_2}{t_1}, \text{ Keeping distance constant}$$

Or,

$$s_1 t_1 = s_2 t_2$$

Applied when one object covers same distance with different speeds

Time, Speed Distance-1

Number of Questions: Class Work: 15+5

Class Sheet

Level 1:

- A car is travelling at a speed of 54 km/h. Find its speed in meter per second.
a. 15 m/s b. 20 m/s c. 25 m/s d. 30 m/s
- A man goes from A to B at 20 km/h and return at 25 km/h. What is the ratio of time taken from A to B to the time taken from B to A?
a. 5:4 b. 4:5 c. 1:5 d. 5:1

3. A car covers a distance of 40 km at a speed of 20 m/s. Find the time taken to cover that distance.
a. 33 m 20 s b. 30 m 24 s c. 30 m 20 s d. 33 m 24 s
4. A man travels from city P to city Q at a speed of 30 km/h and returned at 20 km/h. What is the average speed of the whole journey?
a. 25 km/h b. 24 km/h c. 21 km/h d. 23 km/h
5. A car travels 30 km for 30 minutes and the next 45 km for 60 minutes. Find the average speed of the car.
a. 5 km/h b. 15 km/h c. 50 km/h d. 25 km/h
6. A car travels from P to Q at a speed of 15 km/h and returns at a speed of 30 km/h. He completed the whole journey in 5 hours. What is the distance between P and Q?
a. 30 km b. 40 km c. 50 km d. 60 km
7. Walking at the rate of 4 km/h a man covers certain distance in 2 hours 45 min. Running at a speed of 16.5 km/h the man will cover the same distance in
a. 12 min b. 25 min c. 40 min d. 60 min
8. Sachin can cover a distance in 1 hour 24 min by covering $\frac{2}{3}$ of the distance at 4 km/h and the rest at 5 km/h. The total distance is?
a. 5 km b. 6 km c. 7 km d. 8 km
9. A passenger train takes two hours less for a journey of 300 km if its speed is increased by 5 km/h from its normal speed. The normal speed is
a. 35 km/h b. 50 km/h c. 25 km/h d. 30 km/h
10. A train covers a distance in 50 min, if it runs at a speed of 48 km/h on an average. The speed at which the train must run to reduce the time of journey to 40 min will be
a. 45 km/h b. 60 km/h c. 75 km/h d. None of these
11. Jay started cycling along the boundaries of a square field from corner point A. After half an hour he reached the corner point C, diagonally opposite to A. If his speed was 8 km/h, the area of the field in square km is:
a. 6 b. 1 c. 9 d. 4
12. A car travels first half distance between two places with a speed of 40 km/h and rest of the half distance with a speed of 60 km/hr. The average speed of the car is:
a. 48 km/h b. 37 km/h c. 44 km/h d. 45 km/h
13. A train covered a certain distance at a uniform speed. If the train had been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, the train would have taken 6 hours more than the scheduled time. The length of the journey is:
a. 700 km b. 740 km c. 720 km d. 760 km
14. Sohan left his home for school at 7 am at a speed of 2 km/h. He found the school to be closed and immediately returned at a speed of 3 km/h and reached home at 10 a.m. Find the distance between his home and the school.
a. 7.2 km b. 3.6 km c. 5.4 km d. 1.8 km

15. Travelling at three-fourth of the usual speed, a man is late to his office by 6 minutes. What is his usual time to cover the distance?
- a. 18 min d. 24 min c. 30 min d. 12 min

LEVEL 2:

1. A jogger wants to save one-fourth of his jogging time, find the percentage increase in his speed?
a. 25% d. 66.66% c. 33.33% d. 20%
2. Excluding stoppages, the speed of a bus is 54 km/h and including stoppages, it is 45 km/h. For how many minutes does the bus stop per hour?
a. 4 b. 6 c. 8 d. None of these
3. A monkey tries to ascend a greased pole 14 m high. He ascends 2 m in first minute and slips 1 m in the next minute. If he continues to ascend in this fashion, how long does he take to reach the top?
a. 26 min b. 24 min c. 22 min d. 25 min
4. A train covers a distance in 100 min, if it runs at a speed of 48kmph on an average. The speed at which the train must run to reduce the time of journey to 40 min will be:
a. 30 km/h b. 50 km/h c. 80 km/h d. 120 km/h
5. After meeting with an accident the speed of the goods train was reduced by $\frac{2}{5}$ th of its actual speed and thereby reached the destination 30 minutes after the scheduled time. Had the accident occurred 21 km further, it would have reached 15 minutes after the scheduled time. What was the actual speed of the train?
a. 23 km/h b. 56 k/h c. 30 k/h d. Cannot be determined

TIME SPEED DISTANCE-2

Distance is directly proportional to speed when time is constant

$$\frac{s_1}{s_2} = \frac{d_1}{d_2}, \text{ Keeping time constant}$$

(Relative Speed)

Note: Applied when two objects are moving simultaneously

RELATIVE SPEED

Case1: Two bodies are moving in opposite directions at speed s_1 & s_2 respectively. The relative speed is defined as $s_r = s_1 + s_2$

Case2: Two bodies are moving in same directions at speed s_1 & s_2 respectively. The relative speed is defined as $s_r = s_1 - s_2$

Train Problems:

The basic equation in train problem is the same $S=VT$. The following things need to be kept in mind while solving the train related problems.

1. When the train is crossing a moving object, the speed has to be taken as the relative speed of the train with respect to the object.
2. The distance to be covered when crossing an object, whenever train crosses an object will be equal to: **Length of the train + Length of the object**
3. Length of man/car/pole are to be considered negligible i.e., numerically zero in calculation.

Boats and Stream

These problems revolve around the movement of bodies in still and moving fluids. If a body swims in still water e.g. in a pond or a swimming pool, then the speed with which he moves is the speed of that boy in still water. Now imagine the same boy swimming in a river, then he can either swim with the flow of water or against the flow of water. The movement of the boy in the river with the flow of water is called **downstream movement** whereas his movement against the flow of water is called as an **upstream movement**.

If the speed of the boy is known to be 'x' and that of the flow of water is known to be 'y' then

Upstream Speed (u) = x - y

Downstream Speed (v) = x + y.

Upstream downstream average speed (for a same distance both ways) = $\frac{x^2 - y^2}{x}$

Circular races:

Two persons A and B are running around a circular track of length L meters. Their speeds are 'x' and 'y' meters per second respectively.

1. When will they meet at the starting point for the first time?

Time to complete one round = $\frac{L}{x}$ and $\frac{L}{y}$

First time meeting at the starting point = $\text{LCM} \left(\frac{L}{x}, \frac{L}{y} \right)$

2. When will they meet for the first time on the track (not necessarily at the starting point)?

Time to meet for the first time after the start

$$= \frac{L}{x+y} \quad \text{.....if they run in opposite directions}$$

$$= \frac{L}{x-y} \quad \text{.....if they run in same direction}$$

3. Number of points they will cross each other.

If Ratio of speeds = **a : b**

Number of crossings = **a + b** if running in opposite directions.

Number of crossings = **|a – b|** if running in same directions.

Chapter: Time, Speed and Distance-2

Number of Questions: Class Work: 15+10 Practice Sheet: 20

Class Sheet

Level 1:

- Two trains starting at the same time from 2 stations 200 km apart and going in opposite direction cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?
a. 11:9 b. 7:3 c. 18:4 d. None of these
- A man can row a boat upstream to a certain place in 8 hours and return to the starting point in 6 hours. If the river flows at a speed of 4 km/h, then find the speed of the boat in still water.
a. 32 km/h b. 34 km/h c. 28 km/h d. 30 km/h
- The jogging track in a sports complex is 825 m in circumference. Suresh and his wife start from the same point and walk in opposite direction at 4.5 km/h and 3.75 km/h respectively. They will meet for the first time in:
a. 5.5 minutes b. 6 minutes c. 4.9 minutes d. 5.28 minutes
- Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 am and travel towards B at 20 km/h speed. Another train starts from B at 8 am and travel towards A at 25 km/h speed. At what time will they meet?
a. 9 am b. 10 am c. 11 am d. None of these
- A thief steals a car and drives it at 15 km/hr. The theft has been discovered after one hour and the owner of the car sets off in another car at 25 km/hr. When will the owner overtake the thief from the starting point?

- a. 1 hour b. 1.5 hour c. 2 hour d. 2.5 hour
6. A car travels from P to Q in 1 hour and another car travels from Q to P in 1.5 hours. If both of them started at the same time, when will they cross each other?
- a. 36 min b. 30 min c. 24 min d. 48 min
7. X and Y start walking towards each other at 10 am at speeds of 3 km/h and 4 km/h respectively. They were initially 17.5 km apart. At what time do they meet?
- a. 2:30 pm b. 11:30 pm c. 1:30 pm d. 12:30 pm
8. In a race, the speeds of A and B are in the ratio 3:4. A takes 30 minutes more than B to reach the destination. The time taken by A to reach the destination is:
- a. 1 hour b. 2 hours c. 1.5 hour d. 5 hours
9. A train 120 m in length passes a pole in 12sec and another train of length 100 m travelling in opposite direction in 10 sec. Find the speed of the second train in km per hour.
- a. 43.2 km/h b. 43 km/h c. 44 km/h d. 43.5 km/h
10. A train starts from Delhi at 6:00 am and reaches Ambala cantonment at 10am. The other train starts from Ambala cantonment. at 8am and reached Delhi at 11:30 am, If the distance between Delhi and Ambala cantonment is 200 km, then at what time did the two trains meet each other?
- a. 8:46 am b. 8.30 am c. 8:56 am d. 8:50 pm
11. Two identical trains A and B running in opposite direction at same speed take 2 min to cross each other completely. The number of bogies of A are increased from 12 to 16. How much more time would they now require to cross each other?
- a. 40 sec b. 50 sec c. 60 sec d. 20 sec
12. Two friends A and B simultaneously start running around a circular track. They run in the same direction. A travels at 6 m/s and B runs at b m/s. If they cross each other at exactly two points on the circular track and b is a natural number less than 30, how many values can b take?
- a. 3 b. 4 c. 5 d. 7
13. A car of length 4m wants to overtake a trailer truck of length 20m travelling at 36km/h within 10 seconds. At what speed should the car travel?
- a. 12 m/s b. 14.8 m/s c. 12.4 m/s d. 7.6 m/s
14. A man sitting in train travelling at the rate of 50 km/h observes that it takes 9 seconds for a goods train travelling in the opposite direction to pass him. If the goods train is 187.5m long. Find its speed.
- a. 40 km/h b. 30 km/h c. 24 km/h d. 25 km/h
15. A man can row 4.5 km/h in still water and he finds that it takes him twice as long to row up as to row down the river. Find the rate of the stream.
- a. 2 km/h b. 2.5 km/h c. 1.5 km/h d. 1.75 km/h

LEVEL 2:

1. A tower is 61.25 m high. A rigid body is dropped from its top and at the same instant another body is thrown up-wards from the bottom of the tower with such a velocity that they meet in the middle of the tower. The velocity of projection of the second body is:
a. 24.5 m/s b. 20 m/s c. 25 m/s d. 22 m/s
2. Point P lies between points A and B such that the length of BP is thrice of AP. Car 1 starts from A and moves towards B. Simultaneously, car 2 starts from B and moves towards A. Car 2 reaches P one hour after car 1 reaches P. If the speed of car 2 is half that of car 1, then the time, in minutes, taken by car 1 in reaching P from A is
a. 12 min b. 13 min c. 14 min d. 15 min
3. Train T leaves station X for station Y at 3 pm. Train S, travelling at three quarters of the speed of T, leaves Y for X at 4 pm. The two trains pass each other at a station Z, where the distance between X and Z is three-fifths of that between X and Y. How many hours does train T take for its journey from X to Y?
a. 16 b. 15 c. 14 d. 12
4. On a long stretch of east-west road, A and B are two points such that B is 350 km west of A. One car starts from A and another from B at the same time. If they move towards each other, then they meet after 1 hour. If they both move towards east, then they meet in 7 hrs. The difference between their speeds, in km per hour, is
a. 50 b. 60 c. 70 d. 80
5. Points A and B are 150 km apart. Cars 1 and 2 travel from A to B, but car 2 starts from A when car 1 is already 20 km away from A. Each car travels at a speed of 100 km/h for the first 50 km, at 50 km/h for the next 50 km, and at 25 km/h for the last 50 km. The distance, in km, between car 2 and B when car 1 reaches B is
a. 3 b. 4 c. 5 d. 6
6. A bus overtakes an auto-rickshaw at the 10 am on the way to Mumbai. The bus reaches Mumbai at 12:30 pm. The bus started for its return journey at 1:30 pm and crosses the same auto at 2 pm. When will the auto reach Mumbai?
a. 2:30 pm b. 3 pm c. 3:30 pm d. 4 pm
7. Roy was exactly in the middle coach of a 200 meters long train running at 4 km/h when he noticed Max running at 6 km/h was trying to board the train but was 125 meters behind the train. So, Roy tried to move towards the last door of the train so that he could pull up Max, but due to congestion he could move only at 1 km/h. Find the time taken by Max to board the train if Roy pulls up Max onto train.
a. 3 min b. 15 min c. 1.33 min d. 1.5 min
8. A car travels from A to B and back takes 3 hours one way and another car travels from B to A and back takes 2 hours one way. If both the cars start at the same time, when will they cross for the seventh time?
a. 14 hours 24 minutes b. 7 hours c. 15 hours 36 minutes d. 14 minutes

9. In a 10 km race, A, B and C, each running at uniform speed, get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then by how many meters does A beat C?
- a. 1900 b. 1800 c. 1960 d. 2000
10. A motorbike leaves point A at 1 pm and moves towards point B at a uniform speed. A car leaves point B at 2 pm and move towards point A at a uniform speed which is double that of the motorbike. They meet at 3:40 pm at a point which is 168 km away from A. What is the distance, in km, between A and B?
- a. 378 b. 364 c. 380 d. 416

PRACTICE SHEET

- 1) Ram and Shyam are 200 m apart. They start walking towards each other with speeds of 6 m/sec and 4 m/sec respectively, they cross each other and then after reaching each other's initial positions they turn and start walking back, maintaining their speeds throughout. Find the distance of their 2nd meeting point from Ram's initial position (in meters).
- 2) A thief steals a car at 5:00 pm and drives it away at 60 kmph. The theft is discovered at 5:30 pm. The owner sets off in pursuit immediately in another car at 65 kmph. He will catch up with thief in how many hours?
- 3) A boat travels at a speed of 15 kmph. It travels between points A and B, which are 100 km apart. The boat generally goes downstream from A to B in 5 hours. But suddenly the speed of river doubles itself. How long will it take now for a round trip (in hours)?
- 4) Meera and Meena got a punishment of running along the 400m long border of their school ground, till they meet for the 1st time. They start running in the same direction from the same place with speeds of 2 m/sec and 1 m/sec respectively. How long will it take for them to meet?
- 5) A motorist covers a distance of 78 km in 45 minutes by moving at a speed of 'x' kmph for the first 15 minutes, then moving at twice his original speed for the next 20 minutes and then again moving at his original speed for the rest of the journey. Find the value of 'x'.
- 6) A man takes total 6 hours 30 minutes by walking to a certain place and come back by cycle. He would have gained 1 hour 30 minutes by cycling both ways. The time he would take to walk both ways is:
- 7) Time taken by P and Q to complete a race of 1000 metres is 50 seconds and 80 seconds respectively. If P and Q decided to run a race of 600 metres, then by what distance should P Give Q a head start so that both of them finish the race at the same time?
- 8) The wheel together with a new tyre attached to my car has 50 cm radius. Due to usage, the radius reduces by one fifth of a centimeter. For the same distance travelled, the number of revolutions will approximately:

- a) Increase by 40% b) decrease by 4% c) increase by 0.4% d) decrease by 0.4%
- 9) Two boys running in opposite directions meet each other after 10 minutes on travelling a distance of 2.4 km. Their speeds are in the ratio 3 : 5. Find the time required for both of them to meet each other on a circular track of 2.4 km if they run in the same direction.
- 10) A man can row 8 kmph in still water. When the river is running at 1.4 kmph, it takes him 2 hours to row to a place and to come back. If that man increases his speed of rowing by 25%, then how long will it take him to row to a place and to come back?
- 11) It takes 9 hours to travel a certain distance by a train. If on the return trip, another train which runs 9 kmph faster than the first took three fourth of the time taken by the first train, then how fast is the first train?
- 12) A boat takes 11 hours for travelling downstream from point A to point B and coming back to a point C midway between A and B. If the velocity of the stream is 4 kmph and the speed of the boat in still water is 16 kmph, what is the distance between A and B?
- 13) Red and black ants are running on a rectangular frame of length 7 cm and breadth 3 cm. Red ant is running with a speed of 60 cm/min and black ant is running with a speed of 40 cm/min. How much time will it take for both of them to meet at starting point if both of them start running in the same direction from the same point?
- a) 20 sec b) 30 sec c) 60 sec d) 100 sec
- 14) A boat takes 'X' hours to travel a distance of '2Y' km downstream and the same boat takes '2X' hours to travel a distance of 'Y' km upstream. What is the ratio of the speed of the boat in still water and the speed of the stream?
- a) 5:4 b) 5:3 c) 3:5 d) c:b
- 15) Two trains of 400 m and 475 m in length run at the speeds of '45' km/hr and 'X' km/hr respectively in opposite directions on parallel tracks. If the time taken by them to cross each other is 42 seconds, then find 'X'.
- a) 28km/hr b) 30km/hr c) 32m/sec d) 30m/sec
- 16) Every day a person P cycles from A towards B at a fixed time with a constant speed 'p'. Along the same route Q travels from B towards A, also at a fixed time with a constant speed 'q'. They usually meet at noon at point C. One day P started at hour later than usual and hence met at point D at 12:10 pm. What is the ratio of speeds of P and Q?
- a) 1:6 b) 1:5 c) 2:3 d) 2:5
- 17) Two people run around a circular track in the same direction. The speed of A is 7m/s and that of B is 4m/s. At how many distinct points they will meet?
- a) 3 b) 4 c) 5 d) 6
- 18) There is a pool PQ of length 'l' meters. A starts swimming from P and B simultaneously starts from Q and they meet for the first time 13m from P. They complete one lap and turn back and then met for the second time 9m from Q. If swim at constant speed what can be said about 'l'.
- a) 30 b) 40 c) 50 d) 60

- 19) Excluding stoppages, the speed of the bus is 54kmph and including stoppages, it is 45kmph. For how many minutes did the bus stop per hour?
 a) 15min b) 25min c) 10min d) 20min
- 20) A car covers 4 successive 3km stretches at a speed of 10kmph, 20kmph, 30kmph and 60 kmph resp. Its average speed is
 a) 15kmph b) 10kmph c) 20kmph d) 30kmph

ANSWER KEYS

Numbers-1

Level 1

1. b	2. a	3. a	4. a	5. b
6. a	7. a	8. b	9. d	10. a
11. b	12. a	13. d	14. b	15. c
16. a	17. a	18. c	19. c	20. b
21. c	22. b	23. b	24. d	25. a

Level 2

1. a	2. a	3. d	4. a	5. a
6. a	7. d	8. a	9. a	10. d

Numbers-2

Level 1

1. a	2. d	3. b	4. b	5. c
6. b	7. d	8. b	9. c	10. B
11. DIY	12. DIY	13. B	14. B	15. A
16.	17. A	18. C	19. B	20. a
21. a	22. d	23. a	24. d	25. a

Level-2

1. a	2. c	3. b	4. a	5. a
6. c	7. c	8. c	9. a	10. a

Practice Sheet

1. A	2. B	3. B	4. C	5. A
6. B	7. A	8. B	9. D	10. A
11. D	12. C	13. C	14. A	15. A
16. D	17. A	18. B	19. A	20. D

Percentage

LEVEL 1:

1. C	2. C	3. D	4. B	5. C
6. C	7. D	8. C	9. B	10. C
11. B	12. D	13. D	14. A	15. A
16. A	17. B	18. B	19. B	20. B
21. B	22. B	23. C	24. D	25. B

LEVEL 2:

1. B	2. C	3. C	4. D	5. B
6. A	7. B	8. A	9. A	10. B

Practice Sheet:

1.A	2.D	3.D	4.A	5.C
6.C	7.C	8.B	9.A	10.C
11.A	12.C	13.D	14.C	15.C
16.B	17.C	18. 125	19.C	20. A

Profit & Loss

LEVEL 1:

1. A	2. A	3. B	4. A	5. B
6. B	7. C	8. B	9. A	10. C
11. B	12. A	13. C	14. B	15. D
16. C	17. D	18. A	19. D	20. A
21. C	22. A	23. B	24. D	25. B

LEVEL 2:

1. A	2. C	3. A	4. A	5. B
6. D	7. B	8. A	9. A	10. A

Practice Sheet

1. C	5. A	9. C	13. B	17. A
2. A	6. B	10. D	14. A	18. D
3. B	7. C	11. A	15. B	19. D
4. A	8. B	12. B	16. A	20. D

Simple and Compound Interest

Level 1:

1.B	6.B	11.D	16.A	21.B
2.D	7.A	12.C	17.B	22.C
3.B	8.B	13.E	18.B	23.A
4.C	9.C	14.C	19.A	24.D
5.A	10.C	15.C	20.B	25.A

Level 2:

1.D	2.C	3.B	4.C	5.B
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6.C	7.A	8.C	9.C	10.D
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Practice Sheet

1.A	5.A	9.B	13.C	17.C
2.B	6.D	10.C	14.D	18.B
3.A	7.A	11.B	15.C	19.A
4.C	8.A	12.B	16.A	20.A

Ratio Proportion

LEVEL 1:

1. D	2. C	3. C	4. B	5. A
6. C	7. B	8. D	9. B	10. A
11. C	12. C	13. A	14. D	15. C
16. C	17. C	18. D	19. A	20. B
21. C	22. A	23. D	24. B	25. B

LEVEL 2:

1. D	2. D	3. A	4. C	5. C
6. A	7. C	8. C	9. A	10. C

Averages

Level 1:

1.C	6.D	11.B	16.C	21.A
2.A	7.B	12.D	17.B	22.A
3.D	8.A	13.A	18.A	23.C
4.A	9.D	14.B	19.B	24.A
5.A	10.D	15.A	20.C	25.B

Level 2:

1.B	2.C	3.B	4.D	5.A
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6.C	7.D	8.D	9.A	10.C
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Ages

Level 1:

1.C	6.D	11.C	16.C	21.D
2.C	7.C	12.D	17.A	22.A
3.A	8.B	13.A	18.C	23.D
4.C	9.C	14.B	19.C	24.A
5.C	10.B	15.C	20.A	25.C

Level 2:

1.C	2.A	3.D	4.A	5.C
6.C	7.C	8.D	9.D	10.B

Mixtures and Alligation

Level 1:

1. a	2. c	3. b	4. b	5. c
6. b	7. a	8. b	9. a	10. c
11. a	12. b	13. d	14. c	15. c
16. a	17. d	18. d	19. b	20. c
21. c	22. c	23. c	24. a	25. b

Level 2:

1. d	2. b	3. b	4. b	5. a
6. c	7. d	8. d	9. b	10. d

Practice Sheet

1. C	2. A	3. A	4. A	5. B
6. B	7. B	8. D	9. D	10. C
11. C	12. A	13. C	14. A	15. B
16. A	17. B	18. C	19. C	20. C

Partnership

LEVEL 1:

1. A	2. C	3. A	4. D	5. D
6. A	7. B	8. A	9. C	10. A
11. D	12. C	13. B	14. C	15. A
16. B	17. A	18. B	19. C	20. D
21. D	22. B	23. A	24. A	25. C

LEVEL 2:

1. C	2. B	3. D	4. C	5. D
6. B	7. C	8. D	9. B	10. B

Time and Work

Level 1:

1. C	2. B.	3. A	4. B	5. A
6. A	7. D	8. A	9. C	10. D
11. A	12. A	13. A	14. A	15. A
16. C	17. B	18. A	19. B	20. C
21. B	22. B	23. A	24. A	25. A

Level 2:

1. A	2. B	3. C	4. a	5. a
6. d	7. c	8. b	9. b	10. d

Practice Sheet

21. 36	22. B	23. C	24. B	25. C
26. 9	27. 720/37	28. 16 1/3	29. C	30. 3
31. B	32. B	33. B	34. 2 2/5	35. D
36. 6:5	37. D	38. C	39. B	40. B

Time Speed Distance-1

Level 1:

1. a	2. a	3. a	4. b	5. c
6. c	7. c	8. b	9. c	10. b
11. d	12. a	13. c	14. b	15. a

Level 2:

1. c	2. b	3. d	4. d	5. b
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Time Speed Distance-2

Level 1:

1. a	2. c	3. b	4. b	5. b
6. a	7. d	8. b	9. a	10. c
11. d	12. a	13. c	14. d	15. c

Level 2:

1. a	2. a	3. b	4. a	5. c
6. b	7. d	8. c	9. a	10. a

Practice Sheet

1.40	5.72	9.40 min	13.C	17.A
2.6	6.8	10.96	14.B	18.A
3.24	7.225	11.27	15.D	19.C
4.6 min 40 sec	8.C	12.120	16.B	20.C
