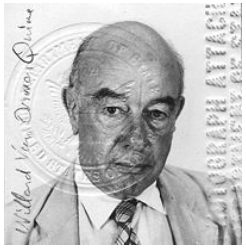
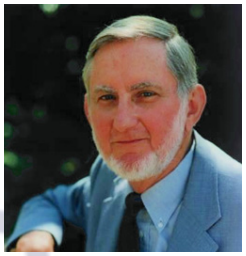
Logic Optimization:

Tabular Method (Quine-McCluskey)





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CADSL

Q-M Tabular Minimization

- Minimizes functions with many variables.
- Begin with minterms:
  - Step 1: Tabulate minterms in groups of increasing number of true variables.
  - Step 2: Conduct linear searches to identify all prime implicants (PI).
  - Step 3: Tabulate PI's vs. minterms to identify EPI's.
  - Step 4: Tabulate non-essential PI's vs. minterms not covered by EPI's. *Select* minimum number of PI's to cover all minterms.
- MSOP contains all EPI's and *selected* non-EPI's.

Function with Don't Cares

$F(A,B,C,D) = \sum m(4,6,8,9,10,12,13) + \sum d(2, 15)$

- Q-M Step 1: Group "all" minterms with 1 true variable, 2 true variables, etc.

	Minterm	ABCD	Groups
Gp1	2	0010	1: single 1
	4	0100	
	8	1000	
Gp2	6	0110	2: two 1's
	9	1001	
	10	1010	
	12	1100	
Gp3	13	1101	3: three 1's
Gp4	15	1111	4: four 1's

(2,6) 0010 / 0110 / 0-10

(2,9) 0010 / 1001 X

Step 2: Same As Before on "All" Minterms

0-implicant

1-implicant

List 1			List 2			List 3		
Minterm	ABCD	PI?	Minterms	ABCD	PI?	Minterms	ABCD	PI?
2	0010	X	2, 6	0-10	PI2	8,9,12,13	1-0-	PI1
4	0100	X	2,10	-010	PI3			
8	1000	X	4,6	01-0	PI4			
6	0110	X	4,12	-100	PI5			
9	1001	X	8,9	100-	X			
10	1010	X	8,10	10-0	PI6			
12	1100	X	8,12	1-00	X			
13	1101	X	9,13	1-01	X			
15	1111	X	12,13	110-	X			
			13,15	11-1	PI7			

Step 3: Identify EPI's Ignoring Don't Cares

Covered by EPI →			x	x		x	x
Minterms →	4	6	8	9	10	12	13
PI1 is EPI			x	x		x	x
PI2		x					
PI3					x		
PI4	x	x					
PI5	x					x	
PI6			x		x		
PI7							x

EPI

EPI = {PI1}

Step 4: Cover Remaining Minterms

Remaining minterms →	4	6	10
PI_2		x	
PI_3			x
PI_4	x	x	
PI_5	x		
PI_6			x

to select min no. of PI's to cover ∑ SPI all the min terms {2,6,10}

Integer linear program (ILP), available from Matlab and other sources: Define integer {0,1} variables, xk = 1, select PI_k; xk = 0, do not select PI_k.

Minimize $\sum_k$ xk, subject to constraints:

x4 + x5 ≥ 1 → at least once

x2 + x4 ≥ 1

x3 + x6 ≥ 1

#SPI Ax ≥ 1

A solution is x3 = x4 = 1, x2 = x5 = x6 = 0, or select PI_3, PI_4

EPI : PI1 SPI = PI3 & PI4

Step 4: Cover Remaining Minterms

Remaining minterms →	4	6	10
PI_2		x	
PI_3			x
PI_4	x	x	
PI_5	x		
PI_6			x

Patrick's Method

(POS) $= 1$ — convert to SOP

$(P_4+P_5)(P_2+P_4)(P_3+P_6)$

$= (P_2P_4 + P_4 + P_5P_2 + P_4P_5)(P_3+P_6)$

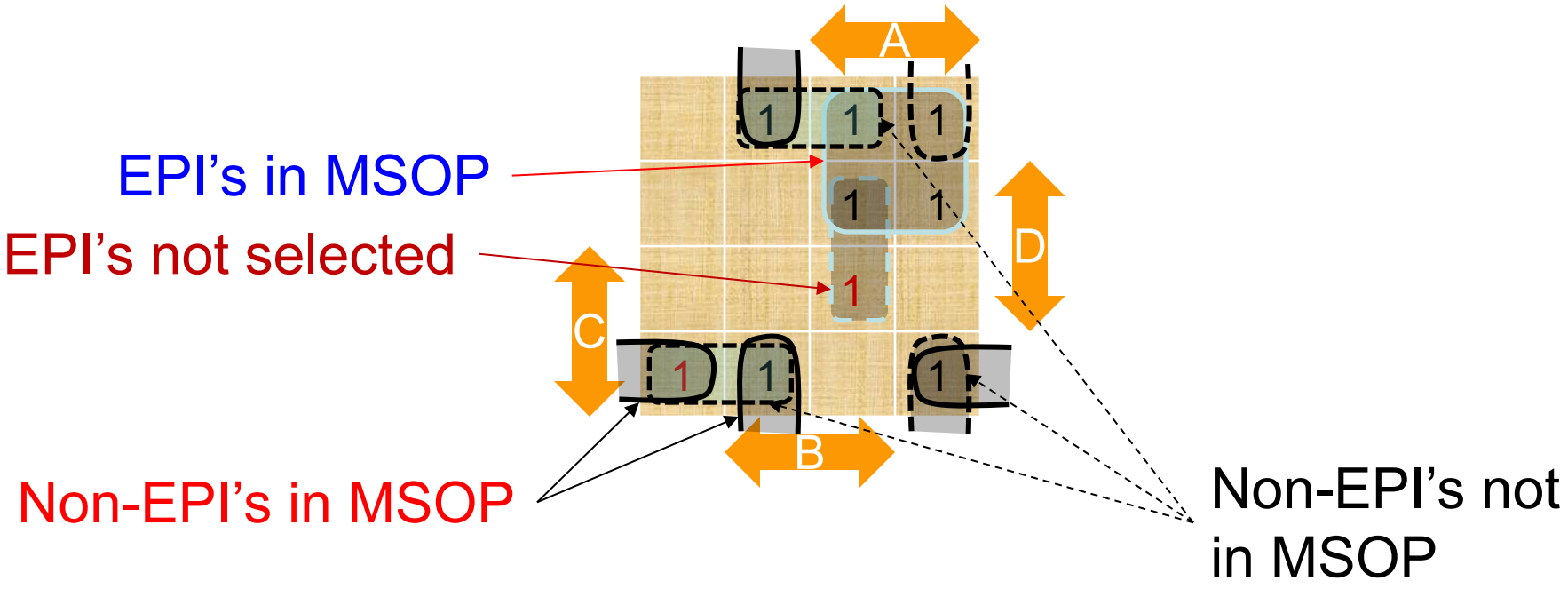
$= P_2P_3P_4 + P_3P_4 + P_2P_3P_5 + P_3P_4P_5 + P_2P_4P_6 + P_4P_6 + P_2P_5P_6 + P_4P_5P_6$ $= 1$

$P_3 = 1 \quad P_4 = 1$

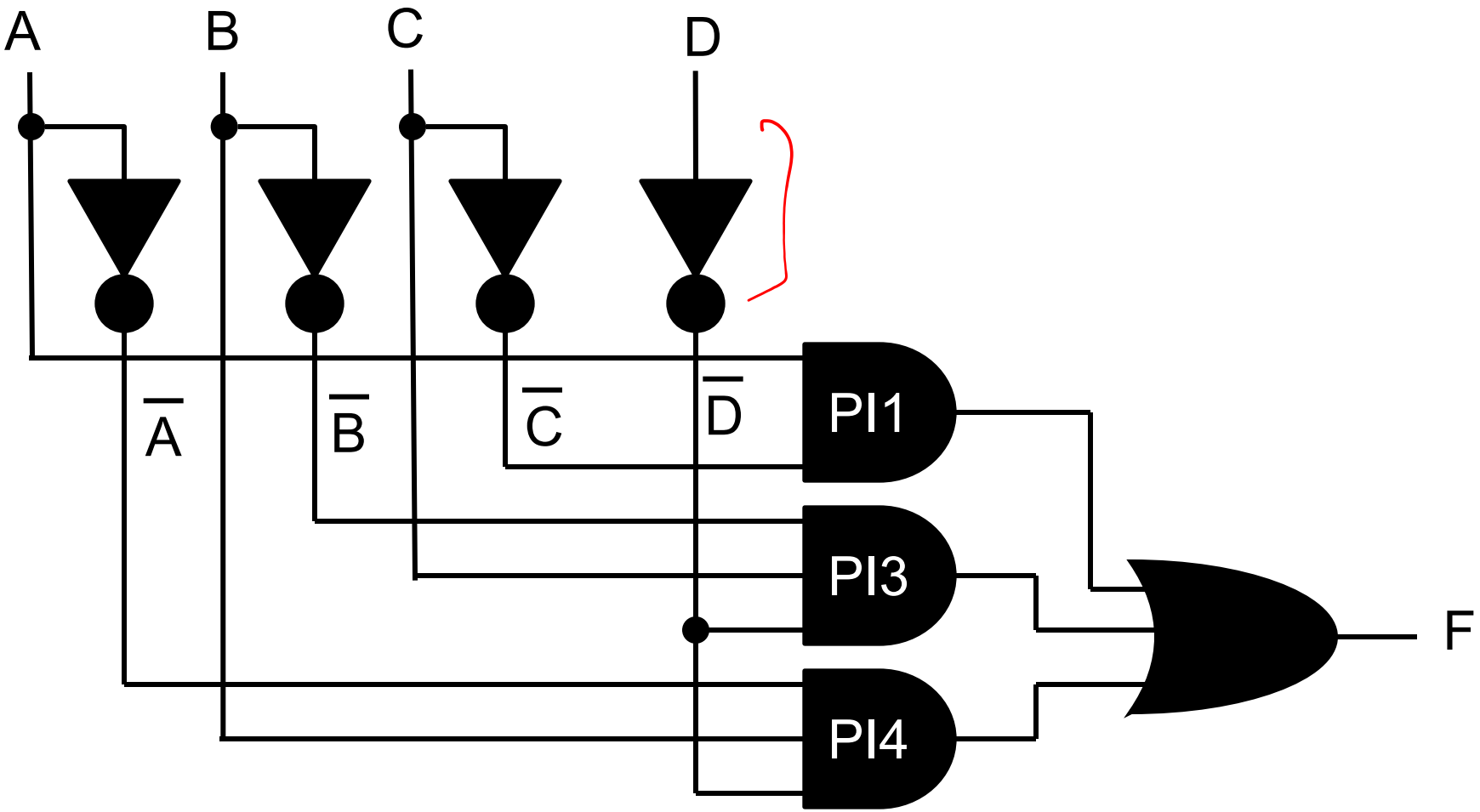
EPI = PI1 SPI = {PI3, PI4}

Q-M MSOP Solution and Verification

- F(A,B,C,D) = PI_1 + PI_3 + PI_4
  = 1-0- + -010 + 01-0
  = $A\ \bar{C} + \bar{B}\ C\ \bar{D} + \bar{A}\ B\ \bar{D}$
- See Karnaugh map.



Minimized Circuit



Thank You