Signal Processing - 1 by One

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Outline

- So Far: Sampling, Convolution, Interpolation
- Previous Week: Fourier Series and Fourier Transform
- Previous Class:Inverse Fourier Transform
- Dirac's Formalism, Time and Frequency

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Dirac's Formalisms

(1) Dirac Definition: Non-negative unit area operator s.t.

$$\int_{\mathbb{R}} x(t)\delta(t)dt = x(0), \text{ whenever } x(0^+) = x(0_-) = x(0).$$

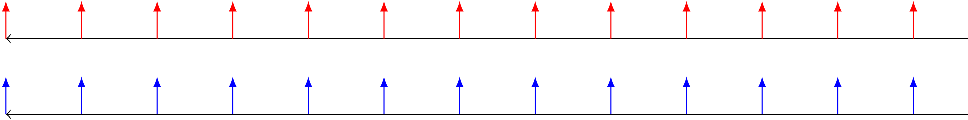
(2) Fourier Transform and Inverse for Diracs:

$$\delta(t) \overset{F.T}{\rightleftharpoons} \mathbb{I}_{\{f \in \mathbb{R}\}}$$

$$\mathbb{I}_{\{t \in \mathbb{R}\}} \overset{F.T}{\rightleftharpoons} \delta(f).$$

(3) Fourier Transform of an impulse train obeys,

$$\sum_{n \in \mathbb{Z}} \delta(t - nT) \overset{F.T.}{\rightleftharpoons} \sum_{m \in \mathbb{Z}} \frac{1}{T}\delta(f - \frac{m}{T})$$



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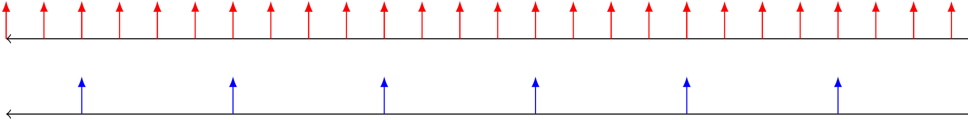

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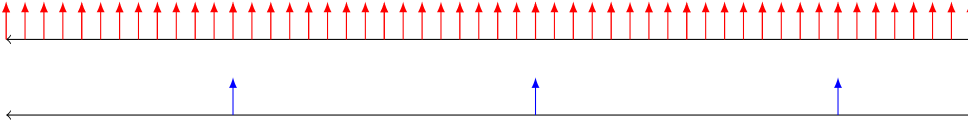

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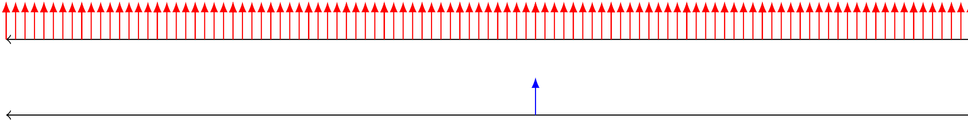
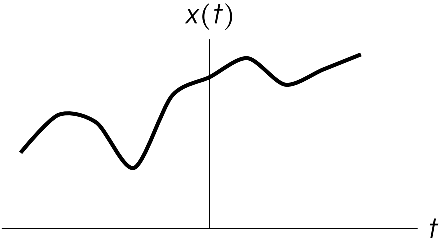
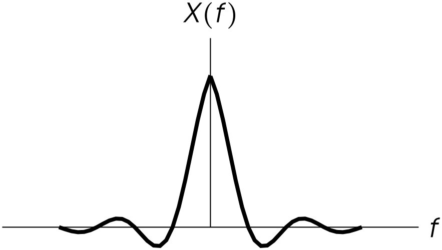


Sampling Revisited





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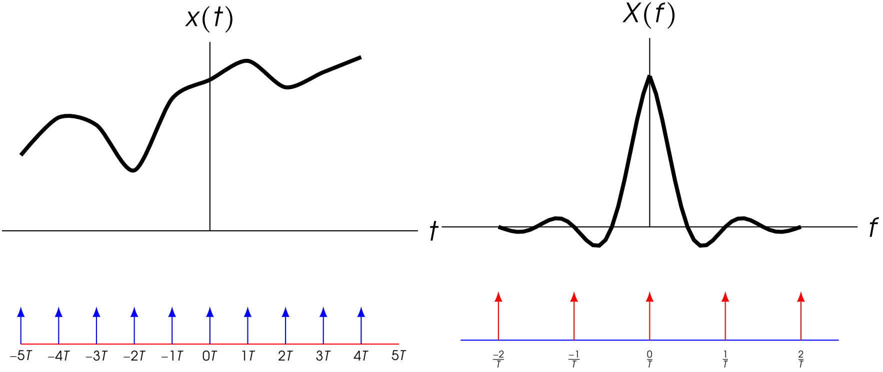

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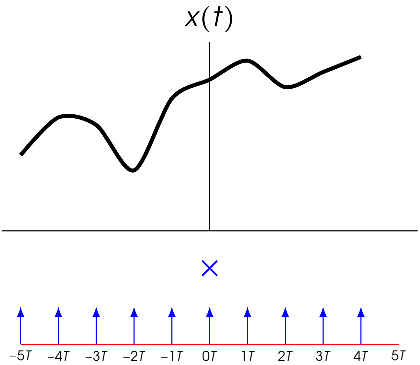

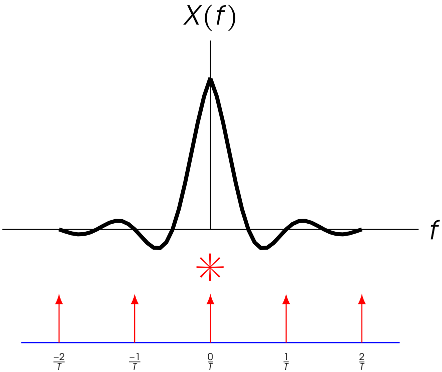

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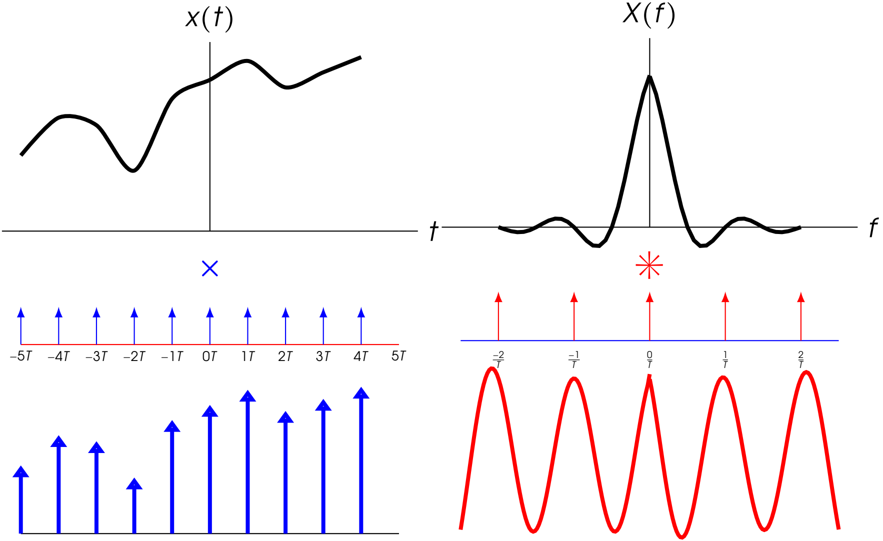
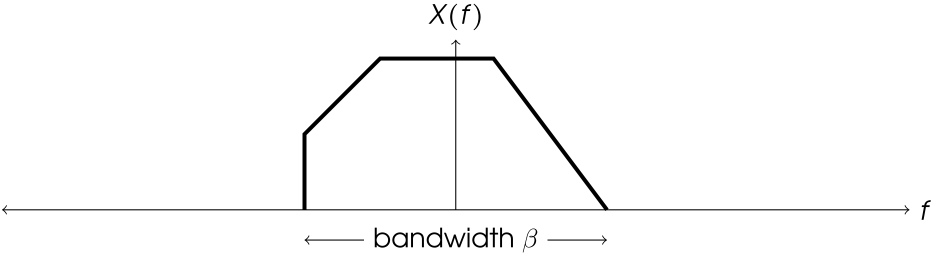


Band Limited Signals

A waveform with $x(t) \overset{F.T}{\rightleftharpoons} X(f)$ s.t. $|X(f)| = 0, \forall |f| \geq f_0$.



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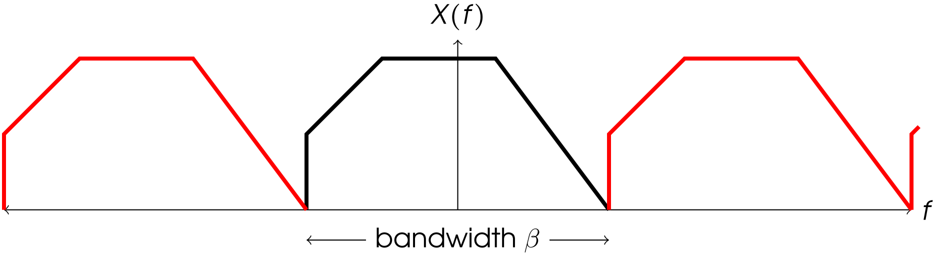

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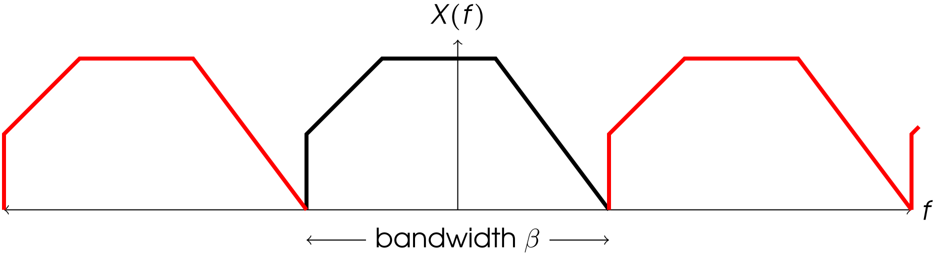

$$\sum_{n\in\mathbb{Z}} X(f+n\beta) \overset{I.F.T}{\rightleftharpoons} \sum_{m\in\mathbb{Z}} \frac{1}{\beta} x(\frac{m}{\beta})\delta(t-\frac{m}{\beta}).$$

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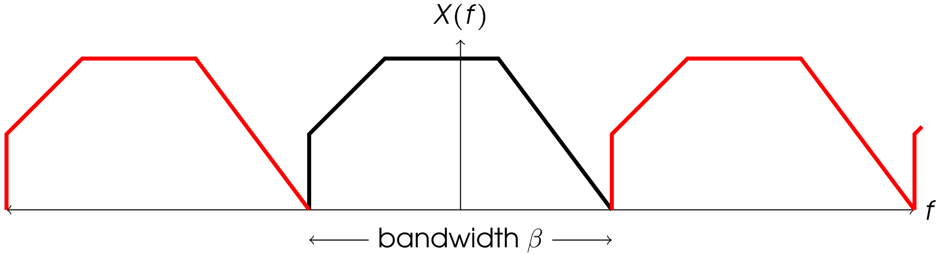
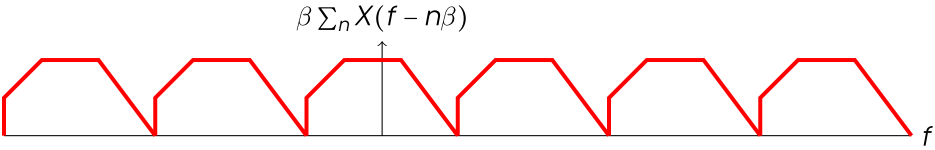


$$\sum_{n \in \mathbb{Z}} X(f + n\beta) \overset{I.F.T}{\rightleftharpoons} \sum_{m \in \mathbb{Z}} \frac{1}{\beta} x\left(\frac{m}{\beta}\right) \delta\left(t - \frac{m}{\beta}\right).$$

Uniform sampling at the rate of β samples per second suffices.

Reconstruction Formula



$$X(f) = \beta \sum_{n \in \mathbb{Z}} X(f - n\beta) \times \left(\frac{1}{\beta} \mathrm{rect}_{\beta}(f) \right).$$ (If Non-overlapping)

$$x(t) = \sum_{m \in \mathbb{Z}} x\left(\frac{m}{\beta}\right) \delta\left(t - \frac{m}{\beta}\right) * \mathrm{sinc}(\beta t).$$ (Convolution-Multipln)

Shannon Reconstruction Formula

$$x(t) = \sum_{m \in \mathbb{Z}} x\left(\frac{m}{\beta}\right) \mathrm{sinc}(\beta t - m)$$