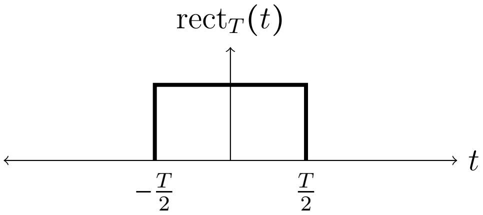
Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 1 EE 229 Signal Processing

Tutorial 1 Aug 24, 2020

Question 1) Find $\delta(t) * \mathrm{rect}_T(t+1)$, where $\mathrm{rect}_T(t)$ is a **rectangular** waveform of unit height and base T, centered at the origin.



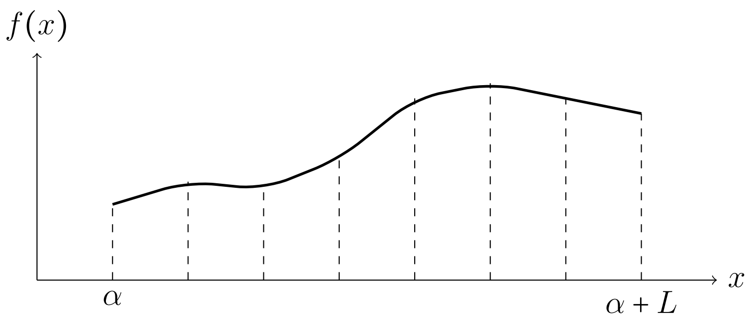
Question 2) A theme park in Las Vegas has 5 studios named Mickey, Donald, Pluto, Minney and Loui, each able to accommodate at most 20 people at a time for a funfilled drive/sail through. Each drive will take an hour, and costs 20, 28, 16, 32 and 8 dollars respectively. The park announced huge discounts for the X'mas break, and for logistical purposes made people follow the rides in the above order. Customers were let in every hour and had to leave at the end of 5 hours. They were asked to pay and collect tickets at the entrance of each studio. While people were given the option to miss a studio and wait in the cafeteria till their slot for the next studio arrives, it turned out that a 75% discount swayed all visitors on a particular day not to miss any ride.

(a) The park secretary wants to deposit all the money collected each hour, 10 minutes after the ride has started. If $11, 13, 13, 15, 20, 20, 20, 20, 19, 20, 20, 20, 20, 20, 13, 7, 0, 0, 0, 0$ were the number of customers arriving every hour that day after opening, find out how much money was deposited each time.

(b) What is the relation between the answer of the last part and the digital convolution we learnt in the class.

(c) How can you model this system if the management decides to give a cashback offer of 3 dollars to each visitor at the end of the visit.

Question 3) Piece-wise constants and Riemann's integral. The Riemann Integral is one of the first things you have learnt in Integral calculus (or integration).

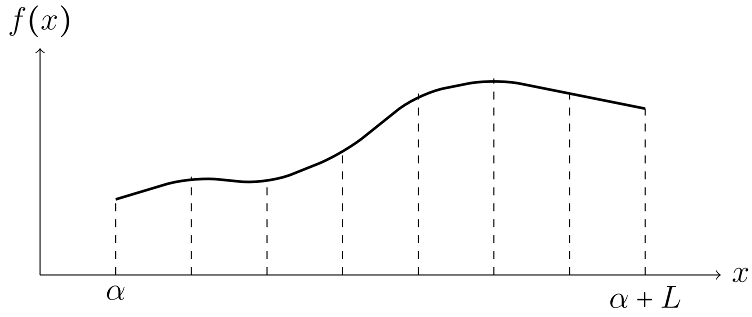


Let us integrate the function $f(x)$ from $x = \alpha$ to $x = \alpha + L$.

$$I = \int_{\alpha}^{\alpha+L} f(x)dx. \tag{1}$$

The idea of Riemann integral is to divide and conquer. As shown in the picture, we divide the $x-$axis to equispaced intervals. If there are m intervals, each will have a width of $\frac{L}{m}$. If we can find the integral in each of these intervals, we can put it together to get the total integral.

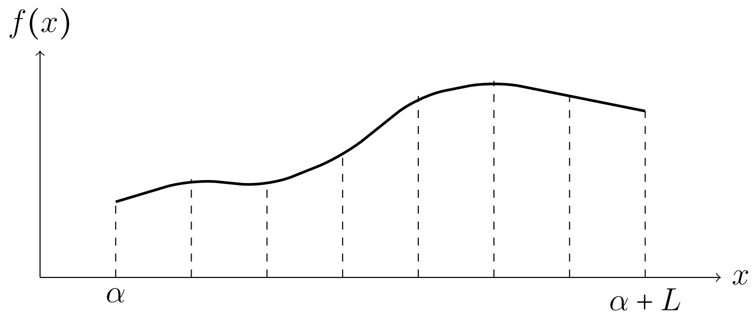
a) Assume for simplicity that the function is continuous. A continuous function will have a maxima and a minima in any bounded interval (why?). What is the biggest rectangular block that you can inscribe in each interval, draw it in the picture.



b) What is the total area covered by these rectangles, if $f_l(i)$ is the least value in interval i?.

$$S_{lower}(m) = \tag{2}$$

c) Draw the minimal rectangle for each sub-interval which completely contains the function in that interval.



d) What is the total area covered by these minimal rectangles in terms of $f_h(i)$, the highest value in interval i.

$$S_{upper}(m) = \tag{3}$$

e) Notice that $S_{lower}(m) \leq I \leq S_{upper}(m)$. If $S_{lower} = S_{upper}$, then we have the answer to our integral. Look at the picture and argue that they are not equal (though close).

f) The next step is to increase m, say to $m_1 = 2 * m$. Argue why the following is true.

$$S_{lower}(m) \leq S_{lower}(m_1) \leq I \leq S_{upper}(m_1) \leq S_{upper}(m). \tag{4}$$

g) Repeating this procedure with $m_i = 2 * m_{i-1}$, we can make the upper and lower bound arbitrary close (why?).

For $m = 100$ note that,

$$S_{lower}(100) = \sum_{i=1}^{m} f_l(i) \frac{L}{100} \tag{5}$$

For $m = 1000$ note that,

$$S_{lower}(100) = \sum_{i=1}^{m} f_l(i) \frac{L}{1000} \tag{6}$$

Denote $\Delta_m = \frac{L}{m}$, then,

$$S_{lower}(m) = \Delta_m \sum_{i=1}^{m} f_l(m) \tag{7}$$

$$= \sum_{i=1}^{m} f_l(m) \Delta_m \tag{8}$$

By taking m to infinity, we will have

$$\lim_{m \to \infty} \sum_{i=1}^{m} f_l(m) \Delta_m = \int_{\alpha}^{\alpha+L} f(x) dx \tag{9}$$

Notice that the role of Δ_m in the LHS is taken by dx on the RHS.
So when we use samples to make quick computations/approximations remember that we have to factor in for the Δ_m in the computations. Many a times, we will explicitly scale the sampled values so that their height indeed represents the area of the rectangular block.

Question 4) Find $x(t) * \text{triangle}_T(t)$, where $x(t) = \delta(t+T) + 2\delta(t) + \delta(t-T)$ and $\text{triangle}_T(t)$ is the triangle function of unit height and base $2T$, centered at origin (see figure below).