Minimization of Logic Expression using Boolean Algebra

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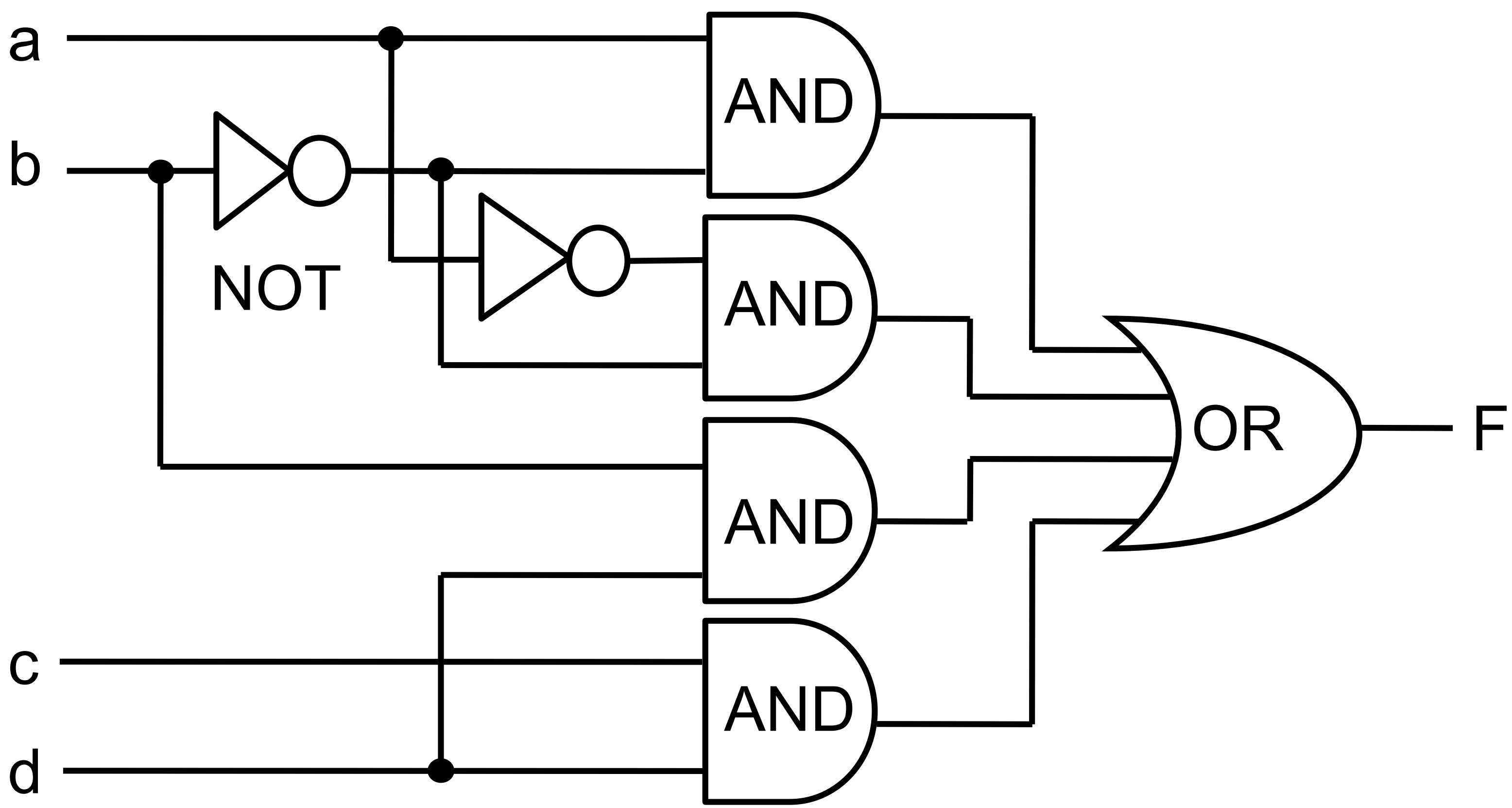
Understanding Minimization

- Logic function: $F = a\bar{b} + \bar{a}\bar{b} + bd + cd$



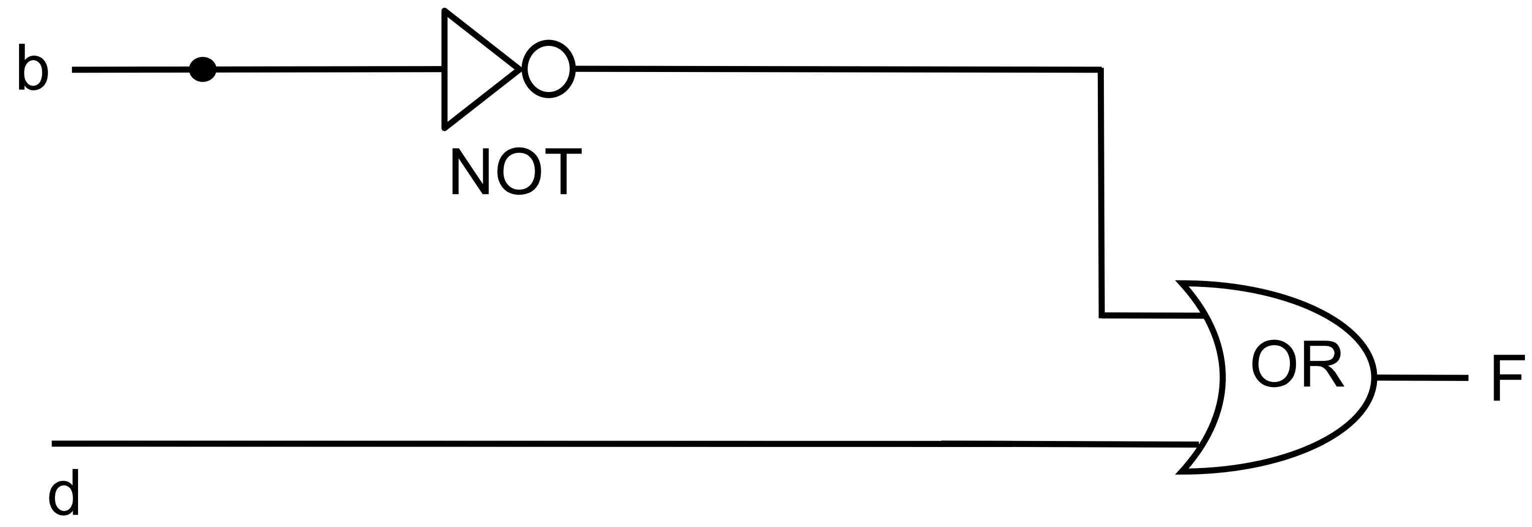
Logic Minimization

- Reducing products:

$F = a\overline{b} + \overline{a}\overline{b} + bd + cd$	Distributivity
$= \overline{b}(a + \overline{a}) + bd + cd$	Complementation
$= \overline{b}1 + bd + cd$	Identity
$= \overline{b}(c + \overline{c}) + bd + cd$	Complementation
$= bd + \overline{b}c + cd + \overline{b}\overline{c}$	Distribitivity
$= bd + \overline{b}c + \overline{b}\overline{c}$	Consensus theorem
$= bd + \overline{b}(c + \overline{c})$	Distributivity
$= bd + \overline{b}$	Complement, identity
$F = \overline{b} + d$	Adsorption

Logic Minimization

- Minimized expression: $F = \bar{b} + d$



DeMorgan's Theorem (1)

- $\overline{a + b} = \bar{a}\,.\,\bar{b}, \quad \forall a, b \in B$
- $\overline{\overline{a + b}} = a + b = \overline{\bar{a}\,.\,\bar{b}}, \quad \forall a, b \in B$

DeMorgan's Theorem (2)

- $\overline{a\,.\,b} = \bar{a} + \bar{b}, \quad \forall a, b \in B$
- $\overline{\overline{a\,.\,b}} = a.b = \overline{\bar{a} + \bar{b}}, \quad \forall a, b \in B$

Minimum Operator Set

- Minimum number of operators
- { ~, (+ or .)} / $\{\neg, (\wedge\ or\ \vee)\}$

Universal Operator: NAND

- NAND: Composite operator (AND and NOT)
- $\overline{a} = \overline{a\,.\,a}$
- $a\,.\,b = \overline{\overline{a\,.\,b}} = \overline{\overline{(a\,.\,b)}\,.\,\overline{(a\,.\,b)}}$
- $a + b = \overline{\overline{a}\,.\,\overline{b}} = \overline{\overline{(a.\,a)}\,.\,\overline{(b\,.\,b)}}$

Universal Operator: NOR

- NOR: Composite operator (OR and NOT)
- $\overline{a} = \overline{a + a}$
- $a + b = \overline{\overline{a + b}} = \overline{(a + b)} + \overline{(a + b)}$
- $a \,.\, b = \overline{\overline{a} + \overline{b}} = \overline{\overline{(a + a)} + \overline{(b + b)}}$

Complementing Functions

- Use DeMorgan's Theorem to complement a function:
  1. Interchange AND and OR operators
  2. Complement each constant value and literal
- Example: Complement $F = \bar{x}.y.\bar{z} + x.\bar{y}.\bar{z}$

$$\bar{F} = (x + \bar{y} + z)(\bar{x} + y + z)$$

Logic Expression (SOP)

- $F = a.b + c.d$
- $\overline{F} = \left(\overline{a} + \overline{b}\right).\left(\overline{c} + \overline{d}\right)$
- $\overline{\overline{F}} = F = \overline{\left(\overline{a} + \overline{b}\right).\left(\overline{c} + \overline{d}\right)}$

Logic Expression (SOP)

- $F = a.b + c.d$
- $F = \overline{(\overline{a} + \overline{b}).(\overline{c} + \overline{d})}$

Logic Expression (POS)

- $F = (a + b).(c + d)$
- $\overline{F} = \overline{(a + b)} + \overline{(c + d)}$
- $\overline{F} = (\overline{a}.\ \overline{b}) + (\overline{c}\ .\ \overline{d})$
- $\overline{\overline{F}} = F = \overline{(\overline{a}.\ \overline{b}) + (\overline{c}\ .\ \overline{d})}$

Logic Expression (POS)

- $F = (a + b).(c + d)$
- $F = \overline{(\overline{a}.\ \overline{b}) + (\overline{c}\ .\ \overline{d})}$

Representation

Canonical Forms

- It is useful to specify Boolean functions in a form that:
  - Allows comparison for equality.
  - Has a correspondence to the truth tables
- Canonical Forms in common usage:
  - Truth Table
  - Sum of Minterms (SOM)
  - Product of Maxterms (POM)

Thank You