Signal Processing - 1 by One

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Outline

- Digital-Analog-Digital
- Fourier Analysis, Series and Transform
- Previous Weeks: DTFT, DFT, FFT, Circular Convolutions
- Previous Class: Practical Example of 4G
- Today: Filter Design, Z-Transform

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Z-Transform

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Polynomial in z, which takes complex values.

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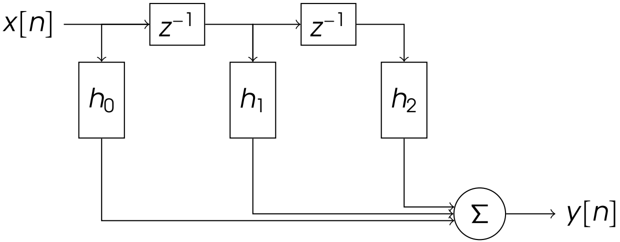

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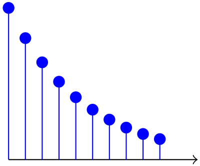

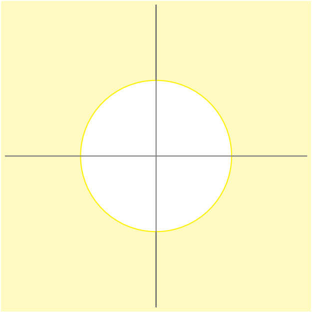

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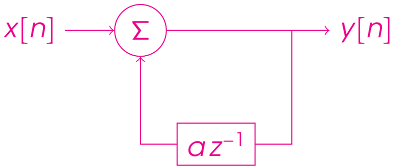

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Stability: Poles inside the unit circle.