Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 7 EE 229 Signal Processing

Tutorial 4 Sep 25, 2020

Question 1) One sided Laplace transform is a very handy tool where we wish to analyze transient effects in a circuits. Transients are often associated with the immediate effect of some event happening in a system. In stable systems, the transient effects will die its natural death after a sufficiently long time. What remains afterward is called the **steady-state response**.

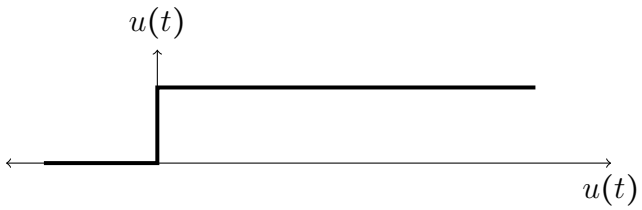
For example, consider a dc supply to a circuit switched ON at time $t = 0$. Do the currents and voltages in the circuit settle to some value immediately after the switch is turned on. You can imagine the answer to be NO, because of the possible intertia of capacitors and inductors.

The one sided Laplace Transform of signal $x(t)$ is defined as

$$X(s) = \int_{\mathbb{R}^+} x(t) \exp(-st) dt.$$

Notice that bilateral Laplace Transform is an integral over $\mathbb{R}$, whereas one sided has $t \in \mathbb{R}^+$. The latter property allows us to analyze transients. When we say Lapace Transform, the one-sided integral is meant.

(a) Find the Lapace transform of a unit step function $u(t)$, shown below.

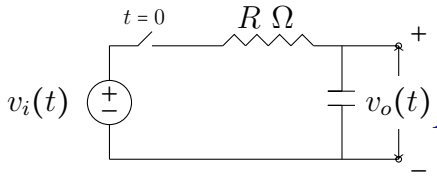


(b) State the conditions under which your evaluation is valid, in other words, the **Region of Covergence** (ROC).

(c) Find the Laplace Transform of $x(t) = u(t) \exp(-at)$, where $u(t)$ is the unit step function. Mention the ROC.

(d) Find the Lapace transform of $x'(t)$, i.e. the derivative of $x(t)$, if $x(t)$ has Fourier transform $X(s)$.

Question 2) For the circuit shown below, the dc supply used is of 5Volts. The capacitance shown is C Farads. Suppose the switch got closed at time $t = 0$.



$X(s) = \int_0^\infty x(t) \exp(-st) dt$ (*Ty'pp'o*)

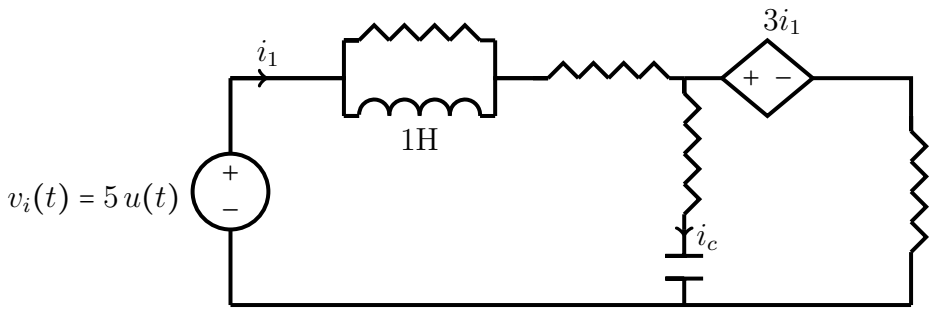
(a) Find the voltage function $v_0(t)$ across the capacitor for $t \geq 0$.

(b) Suppose the capacitor had an initial voltage of 1.5Volts. Find $v_0(t)$ as a function of time.

Question 3) Find the Laplace Transform of

$$x(t) = \exp(-at)\sin(\alpha t)u(t)$$

Question 4) In the following circuit, all resistances are 1Ω, inductance is 1H and capacitance is 1F. Find the voltage $v_c(t)$ across the capacitor.



Question 5) For you to think: What will happen if the dependent voltage source in the previous circuit is $3i_c$, instead of $3i_1$? Dependent sources appear frequently in modelling transistor circuits.