Signal Processing - 1 by One

Sibi Raj B. Pillai
Dept of Electrical Engineering
IIT Bombay

Outline

- So Far: Impulse, Sampling, Convolution and Interpolatio
- Previous Week: Fourier Series
- Previous Class: Uniqueness of Fourier Series
- Today: Fourier Transform

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Analog LTI Systems

input		LTI System		output
$x(t)$	$\longrightarrow$	$h(t)$	$\longrightarrow$	$y(t) = x(t) * h(t)$

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$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) \ast h(t)$$

$$y(t) = \int_{\tau \in \mathbb{R}} h(\tau) x(t-\tau) d\tau = \int_{\tau \in \mathbb{R}} x(\tau) h(t-\tau) d\tau.$$

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Causal Systems $\Rightarrow h(t) = 0, \forall t < 0.$

Sinusoidal Inputs to LTI

Definition:
We say that a function $h(t)$ is integrable if $\int_{\mathbb{R}} |h(t)| dt < \infty$.

For $h(t)$ integrable, $x(t) \circledast h(t)$ is well defined for **bounded** $x(t)$.

The second term depends only on f, denote it as $H(f)$.

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From Series to Transform

Fourier Transform:

$$H(f) := \int_{t \in \mathbb{R}} h(t) \exp(-j2\pi f t) dt.$$

$$H(f) = |H(f)| \exp\left[j\theta(f)\right].$$

Thus for a pure sinusoidal input $\exp(j2\pi f t)$ to the LTI system $h(t)$:

1. The amplitude will be scaled by $|H(f)|$.
2. Phase will be shifted by $\theta(f) \in [-\pi, \pi]$.
3. But frequency is unchanged, unless $H(f) = 0$.

Closely related to **Laplace Transform** for solving ODEs.

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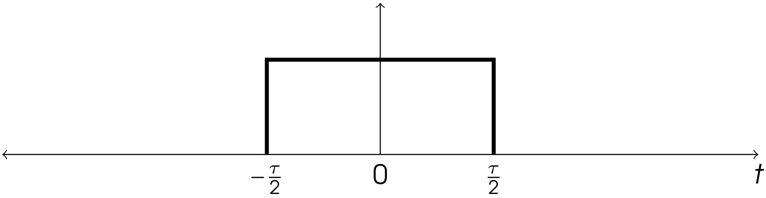
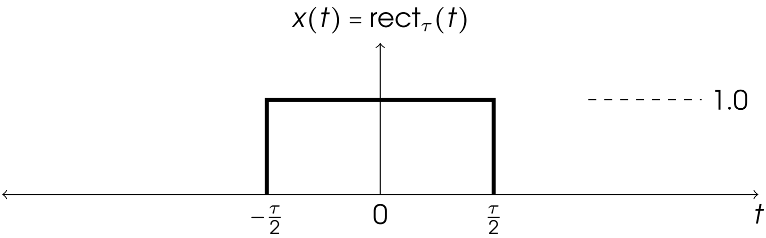
Example-1

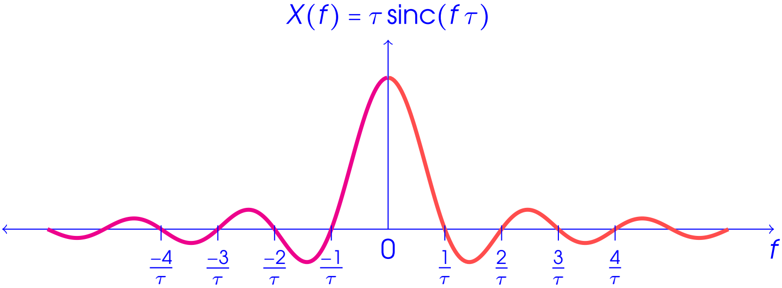
$$x(t) = \mathrm{rect}_\tau(t)$$



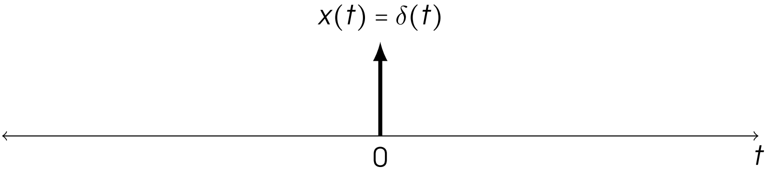
$$
\begin{aligned}
X(f) &= \int_{\mathbb{R}} x(t)\exp(-j2\pi f\,t)\,dt \\
&= \frac{1}{-j2\pi f}\left(\exp(-j2\pi f\frac{\tau}{2}) - \exp(j2\pi f\frac{\tau}{2})\right) \\
&= \frac{\sin(\pi f \tau)}{\pi f} \\
&= \tau\,\mathrm{sinc}(f\tau).
\end{aligned}
$$

Example-1:Plot





Example-2



$$
\begin{aligned}
X(f) &= \int_{\mathbb{R}} x(t)\exp(-j2\pi f\,t)dt \\
&= \int_{\mathbb{R}} \delta(t)\exp(-j2\pi f\,t)dt \\
&= \exp(-j2\pi f 0) = \text{1.0 everywhere}
\end{aligned}
$$