Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 4 EE 229 Signal Processing

Tutorial 2 Sep 07, 2020

Question 1) Find a polynomial $p(x)$ which passes through the five points $(x_0, y_0), \cdots, (x_4, y_4)$. *Hint: Superpose appropriate polynomials*

Question 2) Recall that we came up with an approximate formula for convolution of two analog functions $x(t)$ and $h(t)$, in terms of discrete convolutions and linear interpolation.

(a) For $y(t) = h(t) \ast x(t)$, argue that the approximation

$$y(t) \approx T \sum_{k \in \mathbb{Z}} h_k x_{n-k}, \; -\frac{nT}{2} \leq t \leq \frac{nT}{2}, \;\; n \in \mathbb{Z},$$

with $h_k = h(kT)$ and $x_k = x(kT)$, is close enough for sufficiently small values of T (whenever the convolution is well defined).

(b) Considering the limiting case as T gets smaller, let us define

$$y(t) \coloneqq \int_{\tau \in \mathbb{R}} h(\tau) x(t - \tau) d\tau,$$

as *the formula for convolution*. Verify the formula by computing

$$\mathrm{rect}_T(t) \ast \mathrm{rect}_T(t).$$

(c) Let $x(t)$ be a periodic ramp function having time period T and unit height. Sketch the functions $x(-t)$ and $2\,x\big(-(t - 2.5T)\big)$.

(d) Consider two real functions $h(t)$ and $x(t)$. Let $h(t)$ be a function limited to $[0, T_d]$. Given that $y(t) = x(t) \ast h(t)$, argue that the signal $y(\cdot)$ at instant t is nothing but $\langle h(\tau), \tilde{x}(\tau) \rangle$, where $\tilde{x}(\tau) = x(-\tau + t)$ (i.e. $x(\cdot)$ time reversed and shifted appropriately). Notice that $h(\tau)$ is defined for $\tau \in [0, T_d]$, and so is the inner product integral. Visualize these operations geometrically for each output instant.

Question 3) Find $x(t) \ast h(t)$, when $x(t)$ is a triangle with base-width $2T$, having height α, and $h(t) = \mathrm{rect}_T(t)$.

Question 4) Consider a periodic train of impulses given by $s(t) = \sum_{n \in \mathbb{Z}} \delta(t - nT)$. Let $x(t)$ be a triangle of base-width $2T$ and height α, and $h(t)$ be one cycle of a cosine wave of period T. Find $s(t) \ast x(t) \ast h(t)$. (Feel free to appeal to the integral expression of convolution wherever suitable)

Question 6) A mathematician took

$$w(t) = \sum_{m=-13}^{13} \frac{1}{4} \mathrm{sinc}^2\left(\frac{m}{2}\right) \exp(j \frac{2\pi}{T} mt),$$

instead of a triangle function of base-width $2T$ and height 1. Comment whether she is on the right track or not. Recall that $\mathrm{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}, \forall x \neq 0$.

Question 7) Learning about Frequency Sink in GNURADIO. Generate a real square waveform with unit height, having time-period 1 milli-seconds and duty cycle 0.25. Connect a QT frequency sink. The frequency sink in this case is giving you some ideas about the Fourier Series, we expect a line spectrum (why?).

(a) Is the line spectrum symmetric (why).

(b) What does the negative frequencies signify?

(c) Change the amplitude of the square wave and notice how much the line-spectra magnitude changes (relatively). From this can you guess what quantity is plotted in the screen.

(d) Given a line spectra, how will you construct a time waveform having the given spectra.

(e) Is it possible to have an one-sided line spectra. Also, can you identify a pure sinusoidal waveform at 50Hz.

(g) Let us now generate the convolution of a square waveform having duty-cycle half, with another signal which is nothing but one cycle of the square wave itself. What will be observed difference in amplitudes at the QT frequency sink.