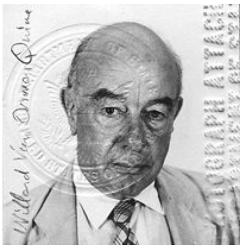
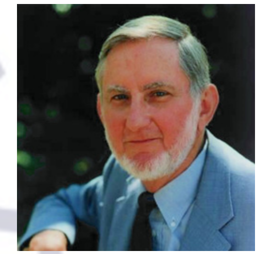
Logic Optimization:

Tabular Method (Quine-McCluskey)





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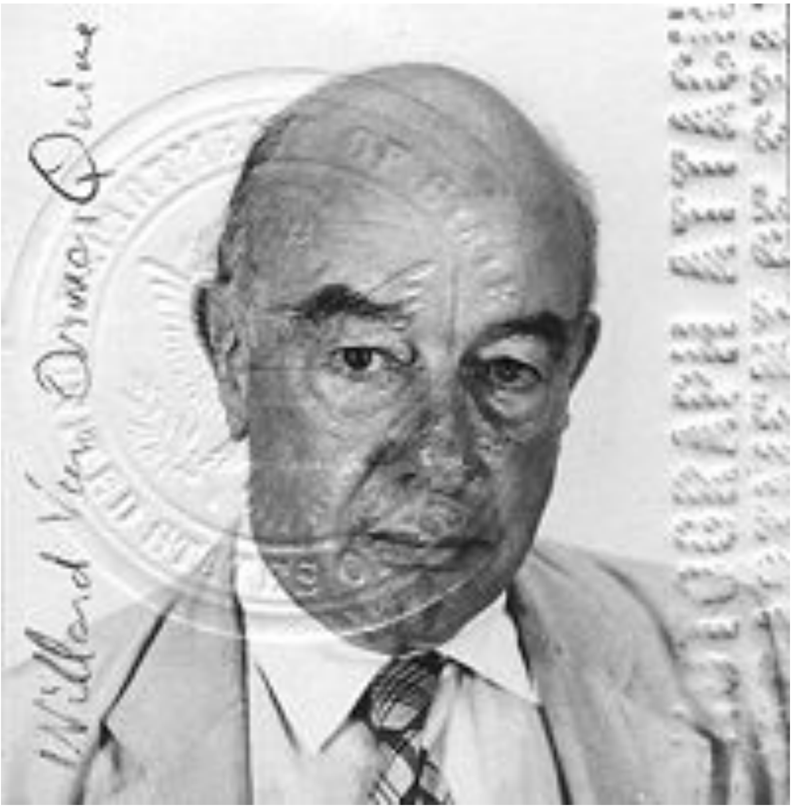
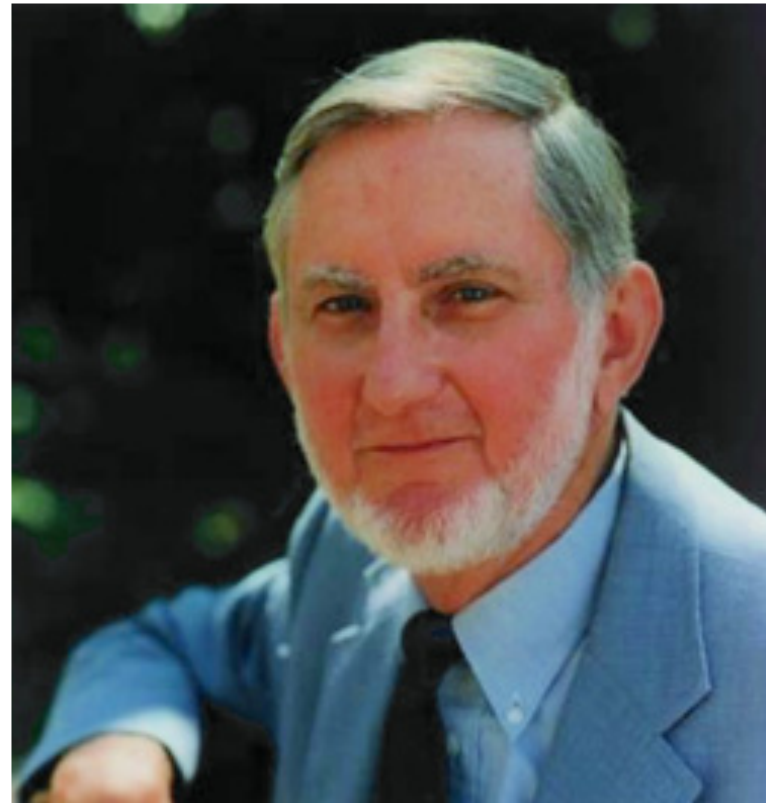
EE-224: Digital Systems

Lecture 19-A: 22 October 2020

Quine-McCluskey



Willard V. O. Quine
1908 – 2000



Edward J. McCluskey
1929 -- 2016

Quine-McCluskey Tabular Minimization Method

- W. V. Quine, "The Problem of Simplifying Truth Functions," *American Mathematical Monthly*, vol. 59, no. 10, pp. 521-531, October 1952. ✓
- E. J. McCluskey, "Minimization of Boolean Functions," *Bell System Technical Journal*, vol. 35, no. 11, pp. 1417-1444, November 1956. ✓

$10 \quad 11' = 1- \rightarrow a$

$a\bar{b} + ab = a$

Q-M Tabular Minimization

- Minimizes functions with many variables.
- Begin with minterms:
  - Step 1: Tabulate minterms in groups of increasing number of true variables.
  - Step 2: Conduct linear searches to identify all prime implicants (PI).
  - Step 3: Tabulate PI's vs. minterms to identify EPI's.
  - Step 4: Tabulate non-essential PI's vs. minterms not covered by EPI's. *Select* minimum number of PI's to cover all minterms.
- MSOP contains all EPI's and *selected* non-EPI's.

$\bar{a}\bar{b}\bar{c}$ 000
$\bar{a}\bar{b}c$ 001
00−

$F(A,B,C,D) = \sum m(2,4,6,8,9,10,12,13,15)$

- Q-M Step 1: Group minterms with 1 true variable, 2 true variables, etc.

Minterm	ABCD	Groups
2	0010	1: single 1
4	0100	
8	1000	
6	0110	2: two 1's
9	1001	
10	1010	
12	1100	
13	1101	3: three 1's
15	1111	4: four 1's

Gp 1

Gp2

Gp3

Gp4

(2,6)
0010
0110
0-10

(2,9)
0010
1001

(2,10)
0010
1010
-010

(2,12)
0010
1100

(4,6)
0100
0110
01-0

Q-M Step 2

- Find all implicants by combining minterms, and then combining products that differ in a single variable: For example,
  - 2 and 6, or $\underbrace{\overset{0}{\bar{A}}\ \overset{0}{\bar{B}}\ \overset{1}{C}\ \overset{0}{\bar{D}}}$ and $\underbrace{\overset{0}{\bar{A}}\ \overset{1}{B}\ \overset{1}{C}\ \overset{0}{\bar{D}}} \rightarrow \bar{A}\ C\ \bar{D}$, written as 0 – 1 0. $\quad$ 0–10
- Try combining a minterm (or product) with all minterms (or products) listed below in the table.
- Include resulting products in the next list.
- If minterm (or product) does not combine with any other, mark it as PI. ✓
- Check the minterm (or product) and repeat for all other minterms (or products).

Step 2 Executed on Example

0- implicant · 1-Implicants · 2-Implicants

List 1			List 2			List 3		
Minterm	ABCD	PI?	Minterms	ABCD	PI?	Minterms	ABCD	PI?
2	0010	X	2, 6	0-10	PI_2	8,9,12,13	1-0-	PI_1
4	0100	X	2,10	-010	PI_3			
8	1000	X	4,6	01-0	PI_4			
6	0110	X	4,12	-100	PI_5			
9	1001	X	8,9	100-	X			
10	1010	X	8,10	10-0	PI_6			
12	1100	X	8,12	1-00	X			
13	1101	X	9,13	1-01	X			
15	1111	X	12,13	110-	X			
			13,15	11-1	PI_7			

Gp1 · Gp2 · Gp3 · Gp4

(8,9) 100-

(12,13) 110-

$\boxed{1-0-}$

-(8,9,12,13)

7 PIs

PI-1

$\downarrow$

PI-7

$\in$ PI

(8,12) | 1-0-

(9,13)

-(8,12,9,13)

minimize (# PI)

Step 3: Identify EPI's

Covered by EPI →				x	x		x	x	x
Minterms →	2	4	6	8	9	10	12	13	15
PI_1 is EPI				x	x		x	x	
PI_2	x		x						
PI_3	x					x			
PI_4		x	x						
PI_5		x					x		
PI_6				x		x			
PI_7 is EPI								x	x

EPI = { PI-1, PI-7 }

UC = { 2, 4, 6, 10 }

select min PIs to cover UC

Step 4: Cover Remaining Minterms

Remaining minterms →	2	4	6	10
x2 → PI_2	x		x	
x3 → PI_3	x			x
x4 → PI_4		x	x	
x5 → PI_5		x		
x6 → PI_6				x

Integer linear program (ILP), available from MATLAB and other sources: Define integer {0,1} variables, xk = 1, select PI_k; xk = 0, do not select PI_k.

Minimize $\sum_k$ xk, subject to constraints:

$$x2 + x3 \geq 1$$
$$x4 + x5 \geq 1$$
$$x2 + x4 \geq 1$$
$$x3 + x6 \geq 1$$

PI-3 PI-4

A solution is x3 = x4 = 1, x2 = x5 = x6 = 0, or select PI_3, PI_4

PI_4 & PI_3 ⇐ EPI

Linear Programming (LP)

- A mathematical optimization method for problems where some "cost" depends on a large number of variables.
- An easy to understand introduction is:
  - S. I. Gass, *An Illustrated Guide to Linear Programming*, New York: Dover Publications, 1970.
- Very useful tool for a variety of engineering design problems.
- Available in software packages like MATLAB.

Step 4: Cover Remaining Minterms

Remaining minterms →	2	4	6	10
α → PI_2	x		x	
β → PI_3	x			x
γ → PI_4		x	x	
δ → PI_5		x		
η → PI_6				x

Patrick's Method

(All min terms must be covered)

2 $\alpha+\beta$

4 $\gamma+\delta$

6 $\alpha+\gamma$

10 $\beta+\eta$

$$\underbrace{(\alpha+\beta)\cdot(\gamma+\delta)(\alpha+\gamma)(\beta+\eta)}_{\Downarrow} = 1$$

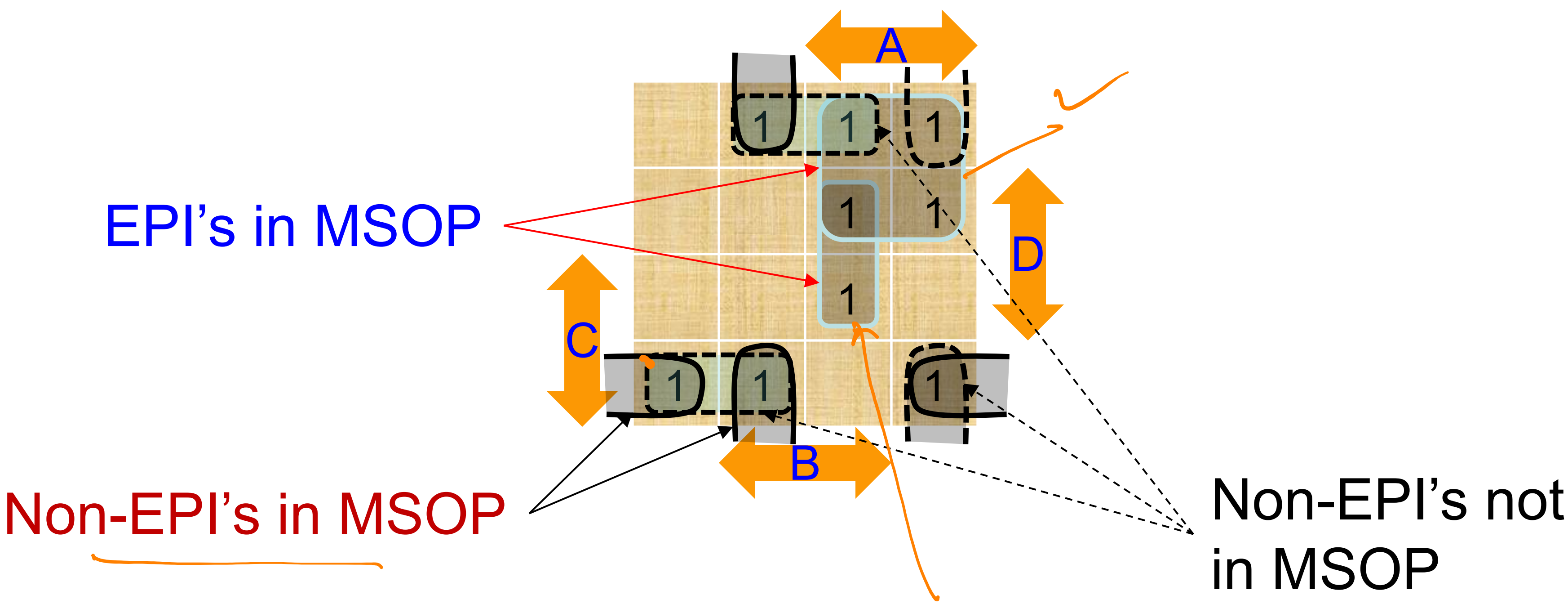
SOP terms

Q-M MSOP Solution and Verification

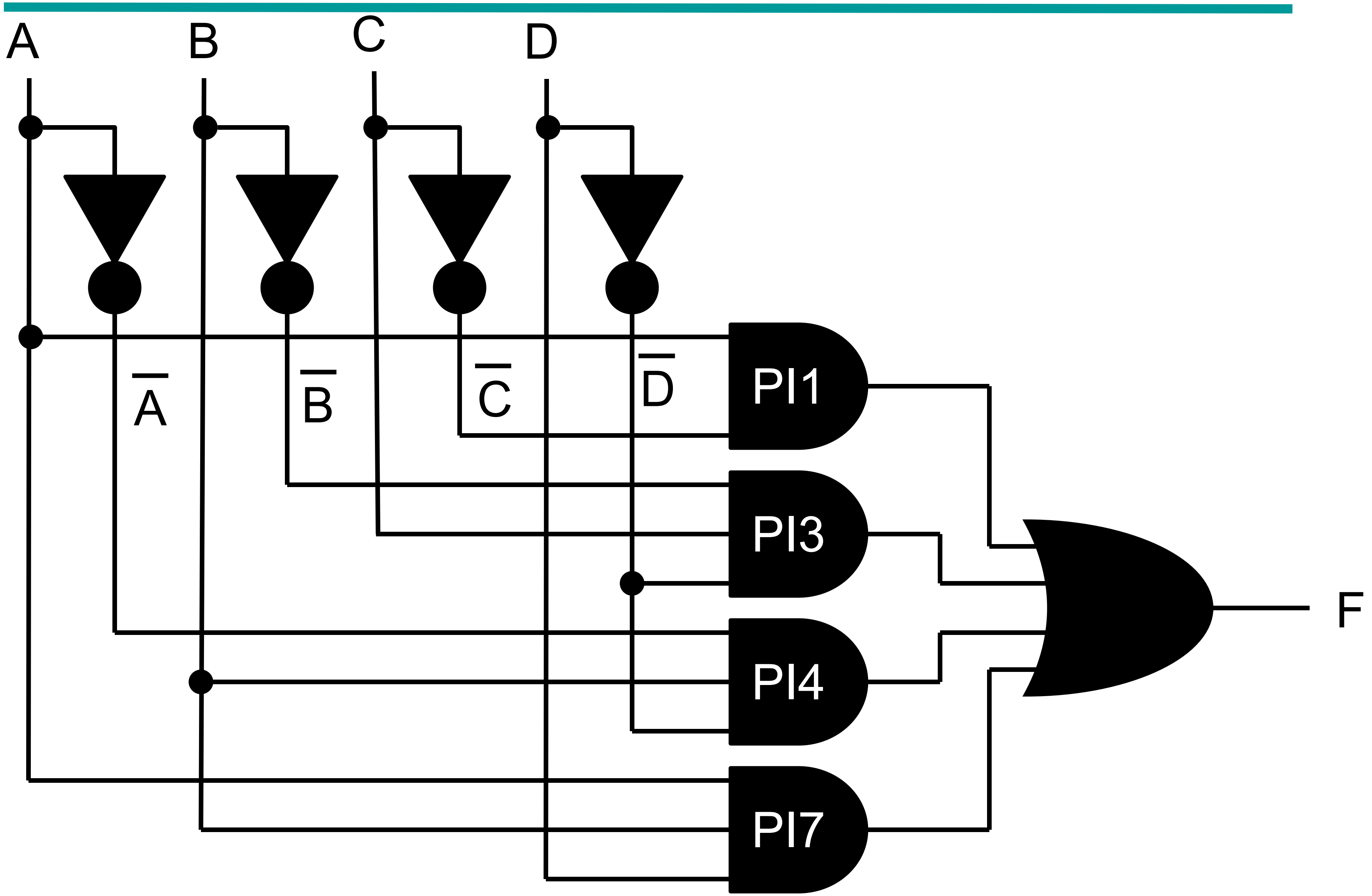
- $F(A,B,C,D) \quad = \quad PI\_1 + PI\_3 + PI\_4 + PI\_7$

  $= \quad 1\text{-}0\text{-} + \text{-}010 + 01\text{-}0 + 11\text{-}1$

  $= \quad A\,\overline{C} + \overline{B}\,C\,\overline{D} + \overline{A}\,B\,\overline{D} + A\,B\,D$
- See Karnaugh map.



Minimized Circuit



QM Minimizer on the Web

- http://quinemccluskey.com/

Thank You