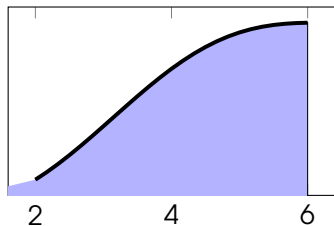


# Signal Processing - 1 by One

Sibi Raj B. Pillai  
Dept of Electrical Engineering  
IIT Bombay

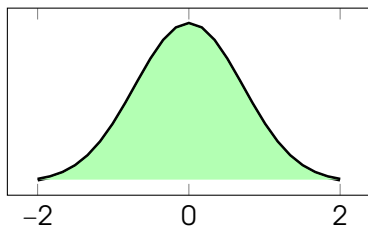


# Generalized Convolution



Function  $x(t)$

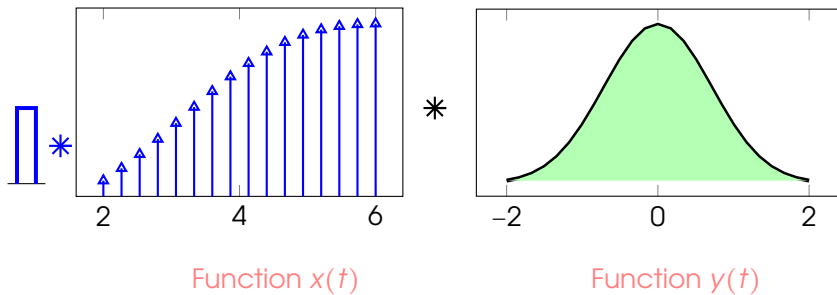
\*



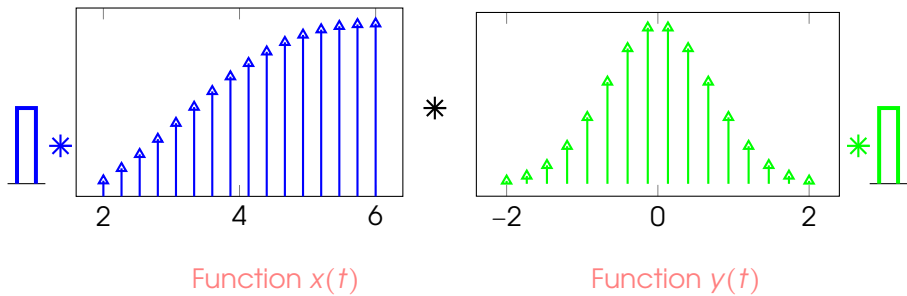
Function  $y(t)$



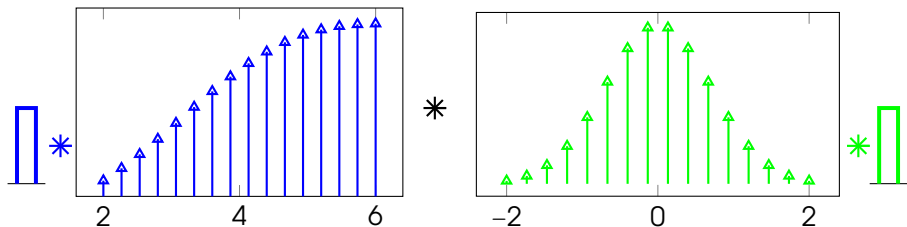
# Generalized Convolution



# Generalized Convolution



# Generalized Convolution



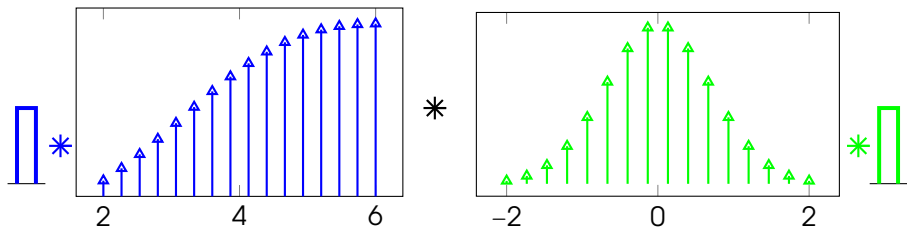
Function  $x(t)$

Function  $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT) \delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT) \delta(t - nT).$$



# Generalized Convolution



Function  $x(t)$

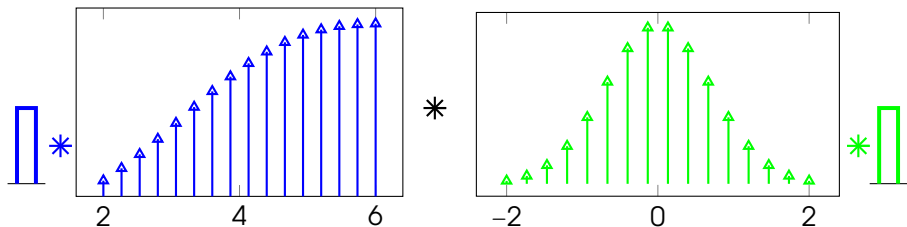
Function  $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT) \delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT) \delta(t - nT).$$

$$x(t) * y(t) \approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t)$$



# Generalized Convolution



Function  $x(t)$

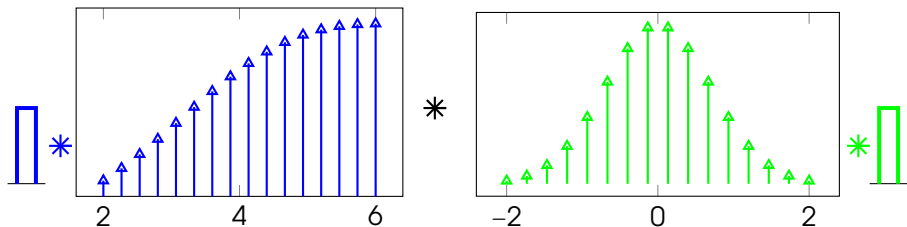
Function  $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT) \delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT) \delta(t - nT).$$

$$\begin{aligned} x(t) * y(t) &\approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t) \\ &= \bar{x}_T * \bar{y}_T * r_T(t) * r_T(t). \end{aligned}$$



# Generalized Convolution



Function  $x(t)$

Function  $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT) \delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT) \delta(t - nT).$$

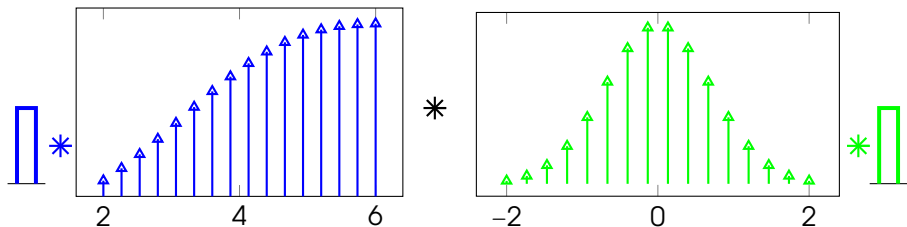
$$\begin{aligned} x(t) * y(t) &\approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t) \\ &= \boxed{\bar{x}_T * \bar{y}_T} * r_T(t) * r_T(t). \end{aligned}$$

Discrete Convolution





# Generalized Convolution



Function  $x(t)$

Function  $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT) \delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT) \delta(t - nT).$$

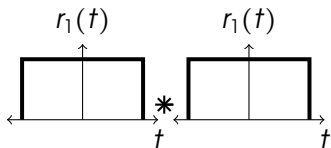
$$\begin{aligned}
 x(t) * y(t) &\approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t) \\
 &= \boxed{\bar{x}_T * \bar{y}_T} * \boxed{r_T(t) * r_T(t)}.
 \end{aligned}$$

Discrete Convolution

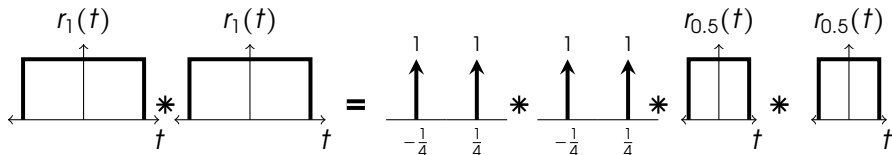
Analog Convolutn



# Convolving Rectangles



# Convolving Rectangles



$$= 2 \cdot \begin{array}{c} \begin{array}{ccc} 0.5 & 1 & 0.5 \\ \uparrow & \uparrow & \uparrow \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{array} \\ \begin{array}{ccc} r_{0.5}(t) & & r_{0.5}(t) \\ \uparrow & & \uparrow \\ \leftarrow & & \leftarrow \\ & t & t \\ \rightarrow & & \rightarrow \end{array} \end{array} * \begin{array}{c} \begin{array}{ccc} r_{0.5}(t) & & r_{0.5}(t) \\ \uparrow & & \uparrow \\ \leftarrow & & \leftarrow \\ & t & t \\ \rightarrow & & \rightarrow \end{array} \end{array}$$

# Convolving Rectangles

$$\begin{aligned}
 & r_1(t) * r_1(t) = \begin{array}{c} \uparrow 1 \quad \uparrow 1 \\ \text{---} \frac{1}{4} \quad \frac{1}{4} \text{---} \end{array} * \begin{array}{c} \uparrow 1 \quad \uparrow 1 \\ \text{---} \frac{1}{4} \quad \frac{1}{4} \text{---} \end{array} * \begin{array}{c} r_{0.5}(t) \\ \text{---} t \end{array} * \begin{array}{c} r_{0.5}(t) \\ \text{---} t \end{array} \\
 & = 2 \cdot \begin{array}{c} \uparrow 0.5 \quad \uparrow 1 \quad \uparrow 0.5 \\ \text{---} -\frac{1}{2} \quad 0 \quad \frac{1}{2} \text{---} \end{array} * \begin{array}{c} r_{0.5}(t) \\ \text{---} t \end{array} * \begin{array}{c} r_{0.5}(t) \\ \text{---} t \end{array} \\
 & = 4 \cdot \begin{array}{c} \uparrow 0.25 \quad \uparrow 0.5 \quad \uparrow 0.75 \quad \uparrow 1 \quad \uparrow 0.75 \quad \uparrow 0.5 \quad \uparrow 0.25 \\ \text{---} -\frac{3}{4} \quad -\frac{1}{2} \quad -\frac{1}{4} \quad 0 \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \text{---} \end{array} * \begin{array}{c} r_{0.25}(t) \\ \text{---} t \end{array} * \begin{array}{c} r_{0.25}(t) \\ \text{---} t \end{array}
 \end{aligned}$$



# Close Cousins

$$\begin{aligned}r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\&= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}).\end{aligned}$$



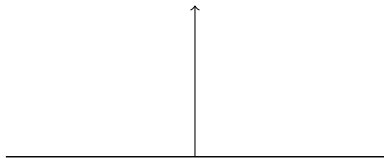
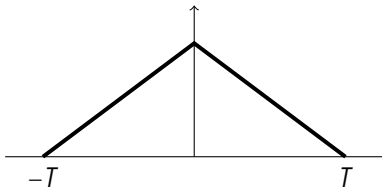
# Close Cousins

$$\begin{aligned} r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\ &= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}). \end{aligned}$$



# Close Cousins

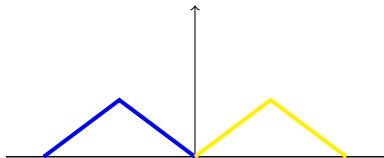
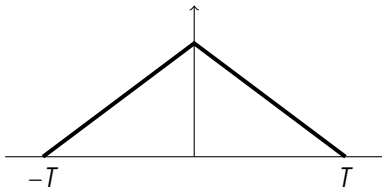
$$\begin{aligned} r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\ &= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}). \end{aligned}$$





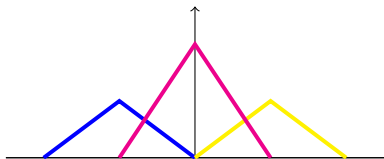
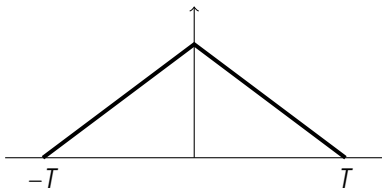
# Close Cousins

$$\begin{aligned} r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\ &= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}). \end{aligned}$$



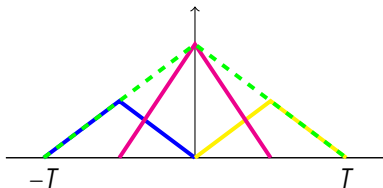
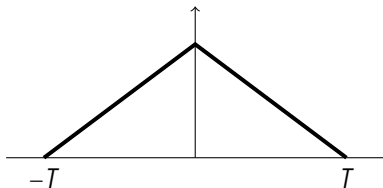
# Close Cousins

$$\begin{aligned} r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\ &= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}). \end{aligned}$$



# Close Cousins

$$\begin{aligned}
 r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\
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 \end{aligned}$$

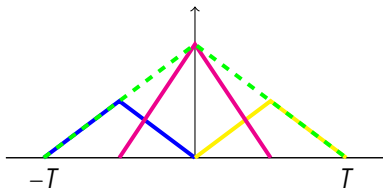
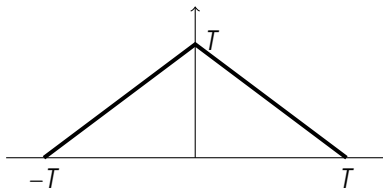


$$x(t) * y(t) = \bar{x} * \bar{y} * \text{Triangle}_T(t).$$



# Close Cousins

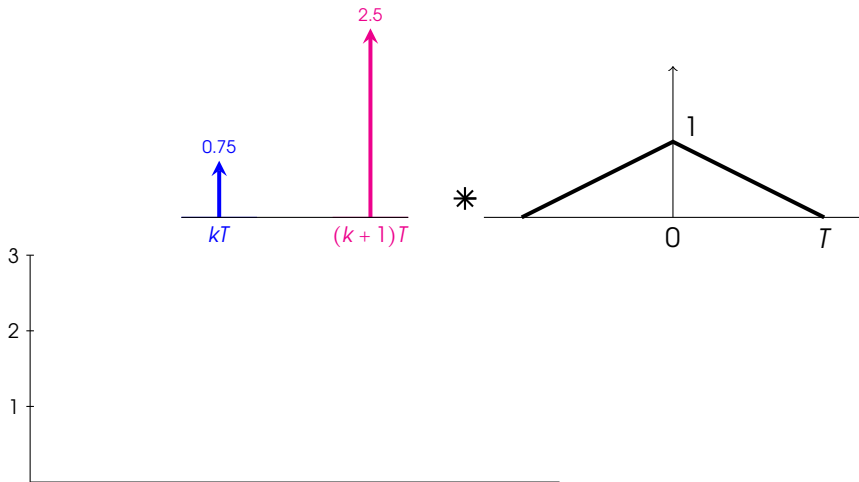
$$\begin{aligned}
 r_T(t) * r_T(t) &= [1 \ 2 \ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) \\
 &= r_{\frac{T}{2}}(t + \frac{T}{2}) * r_{\frac{T}{2}}(t + \frac{T}{2}) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}(t - \frac{T}{2}) * r_{\frac{T}{2}}(t - \frac{T}{2}).
 \end{aligned}$$



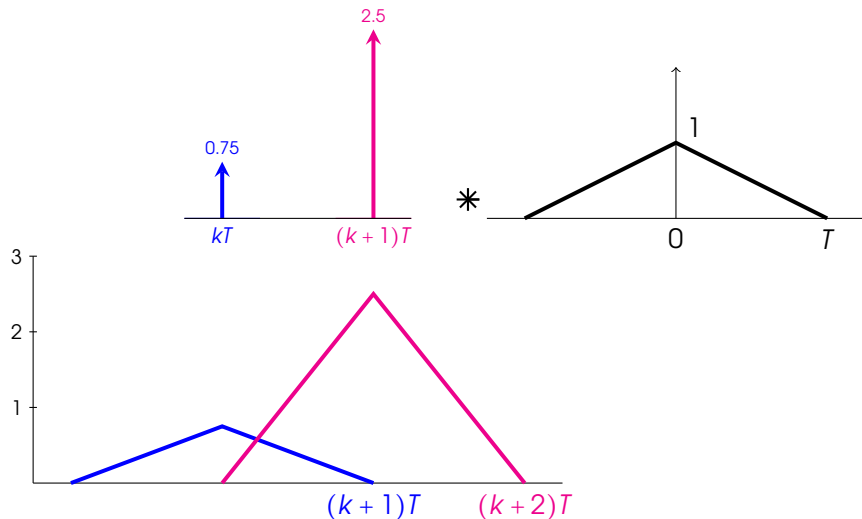
$$x(t) * y(t) = \bar{x} * \bar{y} * \text{Triangle}_T(t).$$



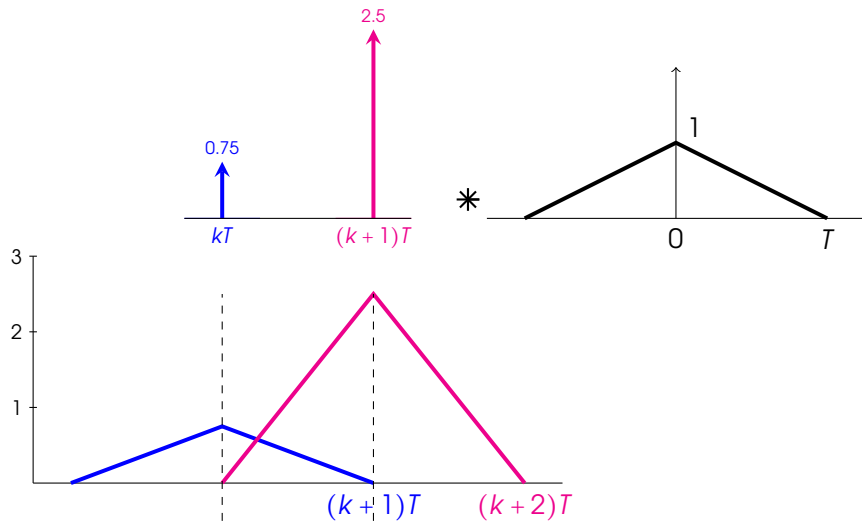
# Linear Interpolation



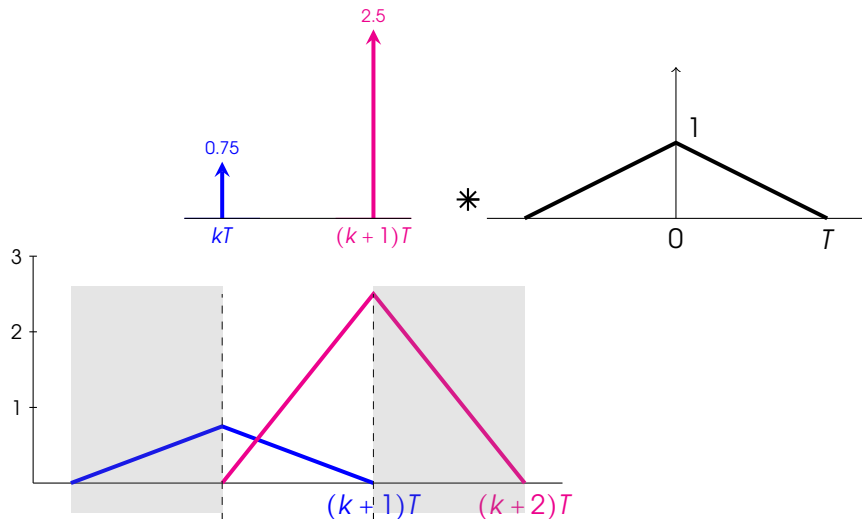
# Linear Interpolation



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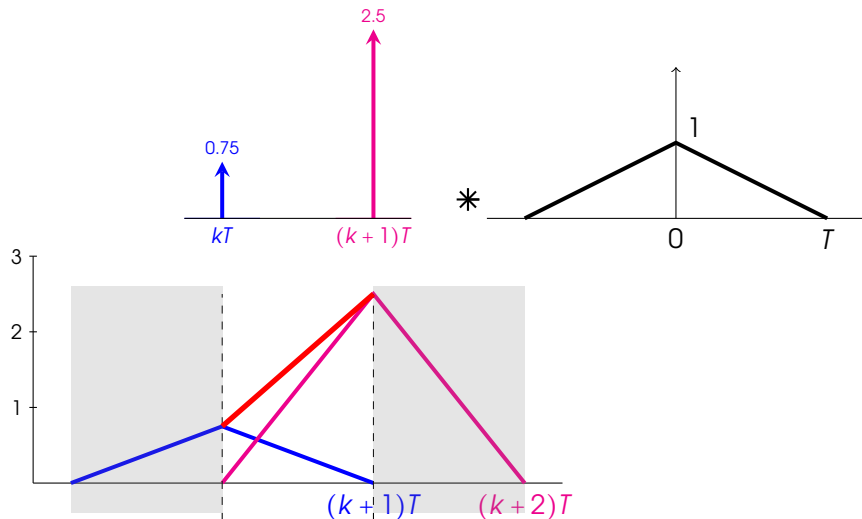


# Linear Interpolation





# Linear Interpolation



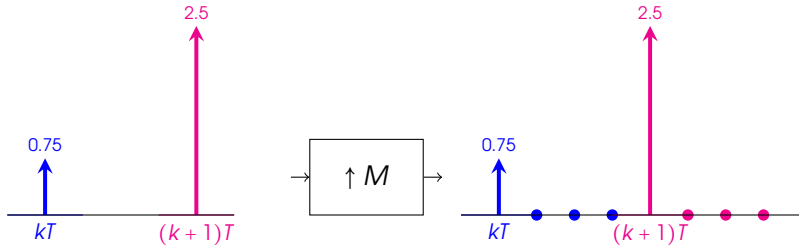
# Digital Interpolation

Upsampling followed by Digital Convolution



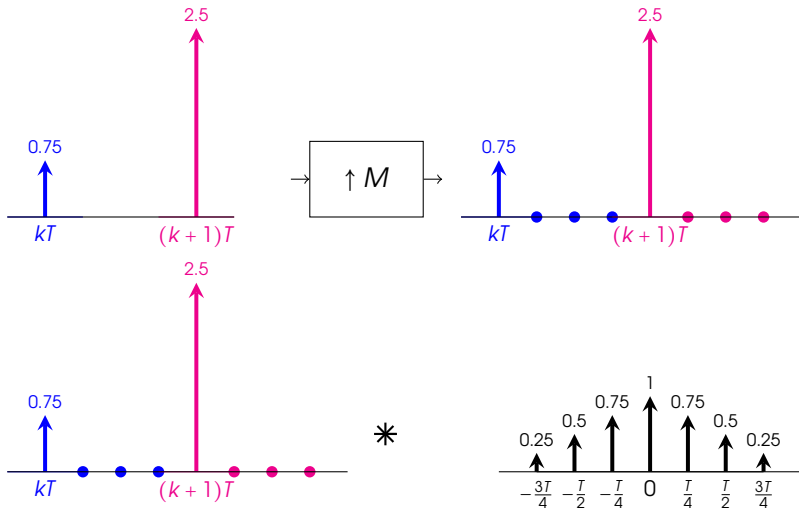
# Digital Interpolation

Upsampling followed by Digital Convolution



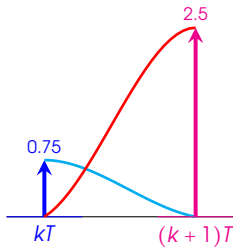
# Digital Interpolation

## Upsampling followed by Digital Convolution



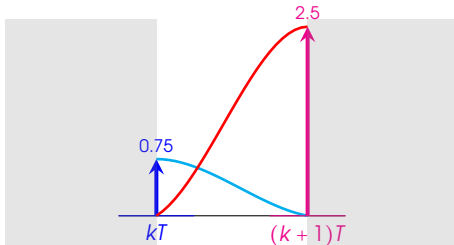
# Cubic Interpolation

Let us interpolate in the interval  $[0, 1]$  between two samples.



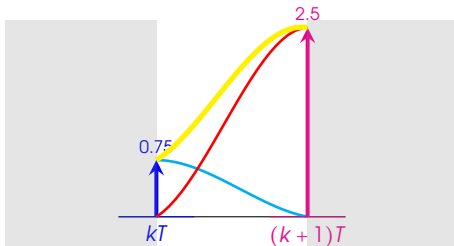
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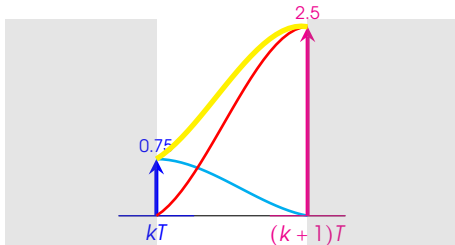
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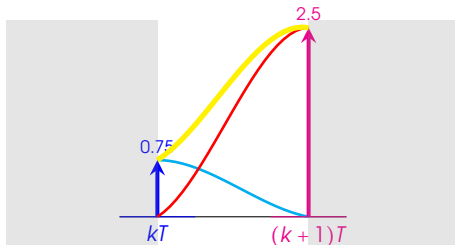
Key: Find the *fitting* polynomial  $p(t) = at^3 + bt^2 + ct + d$  in  $[0, 1]$ .





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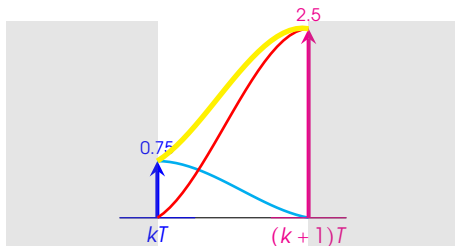
Eg: For  $\delta(t)$  as input, the output in  $t \in [0, 1]$

$$p(t) = \frac{3}{2}t^3 - \frac{5}{2}t^2 + 1.$$



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Key: Find the *fitting* polynomial  $p(t) = at^3 + bt^2 + ct + d$  in  $[0, 1]$ .

Eg: For  $\delta(t)$  as input, the output in  $t \in [0, 1]$

$$p(t) = \frac{3}{2}t^3 - \frac{5}{2}t^2 + 1.$$

We need to do slightly more work to finish this!

