Multiple-Level Optimization

Virendra Singh
Professor
Computer Architecture and Dependable Systems Lab
Department of Electrical Engineering
Indian Institute of Technology Bombay
http://www.ee.iitb.ac.in/~viren/
E-mail: viren@ee.iitb.ac.in

EE-224: Digital Systems

Lecture 20-A: 22 October 2020

CADSL

Multiple-Level Optimization

- Multiple-level circuits are circuits that have more than two level (plus input and/or output inverters)
- For a given function, multiple-level circuits can have reduced gate input cost compared to two-level (SOP and POS) circuits
- Multiple-level optimization is performed by applying transformations to circuits represented by equations while evaluating cost

Transformations

- Factoring - finding a factored form from SOP or POS expression
  - Algebraic - No use of axioms specific to Boolean algebra such as complements or idempotence
  - Boolean - Uses axioms unique to Boolean algebra
- Decomposition - expression of a function as a set of new functions

Transformations (continued)

- Substitution of G into F - expression function F as a function of G and some or all of its original variables
- Extraction - decomposition applied to multiple functions simultaneously

Transformation Examples

- **Algebraic Factoring** ✓ 12+4

$$F = \bar{A}\bar{C}\bar{D} + \bar{A}B\bar{C} + ABC + AC\bar{D} \qquad G = 16$$

– **Factoring:**

$$F = \bar{A}(\bar{C}\bar{D} + B\bar{C}) + A(BC + C\bar{D}) \qquad G = 16$$

– **Factoring again:**

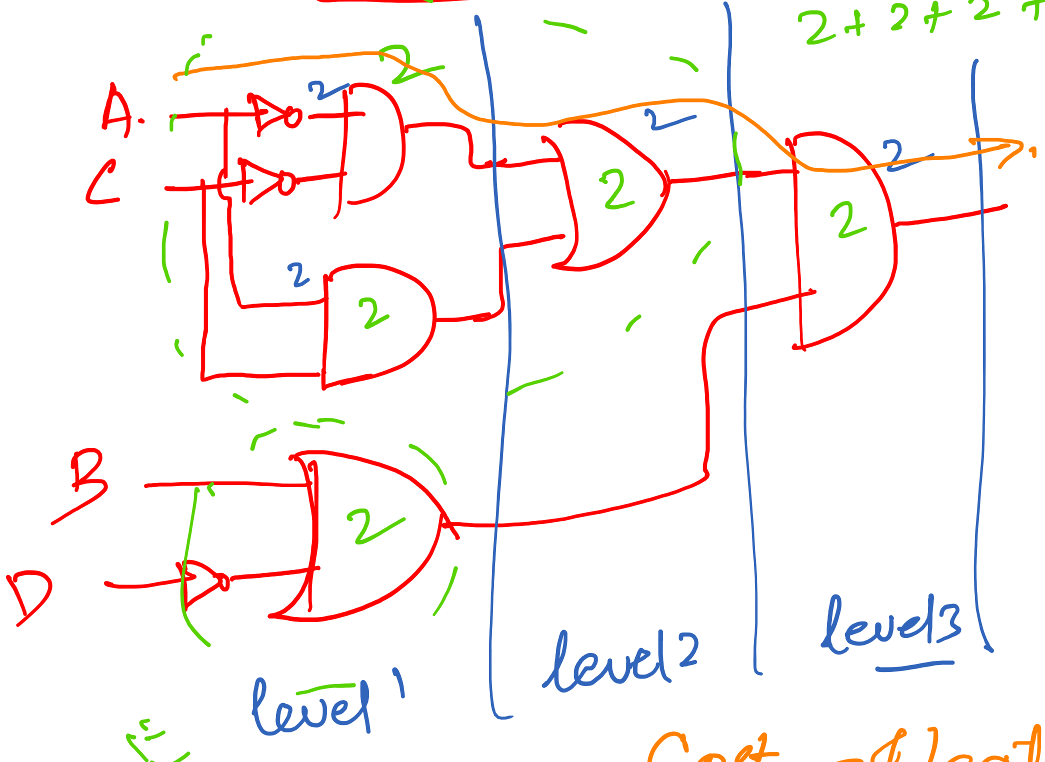
$$F = \bar{A}\bar{C}(B + \bar{D}) + AC(B + \bar{D}) \qquad G = 12$$

– **Factoring again:**

$$F = (\bar{A}\bar{C} + AC)(B + \bar{D}) \qquad G = 10$$

$$F = (\bar{A}\bar{C} + AC)(B + \bar{D})$$

$2+2+2+2+2 = 10$ cost



delay $1+2+2+2 = 7$

Cost effective solution may not be optimal for performance.

Transformation Examples

- Decomposition
  - F = A'C'D' + A'BC' + ABC + ACD'   G = 16
  - The terms A'C' + AC and B + D' can be defined as new functions H and E respectively, decomposing F = (A'C' + AC)(B + D'):

  F = H E, H = A'C' + AC, E = B + D'   G = 10

  H E

- This series of transformations has reduced G from 16 to 10, a substantial savings.
- The resulting circuit has three levels plus input inverters.

Transformation Examples

- **Substitution of E into F**
  - **Returning to F just before the final factoring step:**

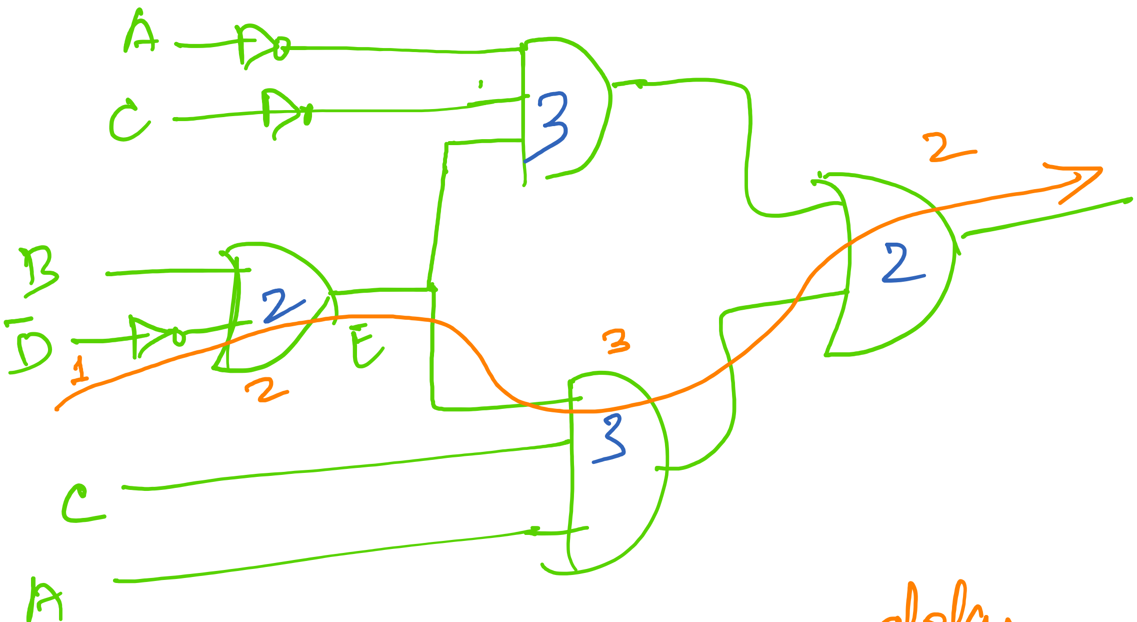
$$F = \bar{A}\,\bar{C}\,(B + \bar{D}) + AC\,(B + \bar{D}) \qquad G = 12$$

  - **Defining $E = B + \bar{D}$, and substituting in F:**

$$F = \bar{A}\,\bar{C}\,E + ACE \qquad G = 10$$

  - **This substitution has resulted in the same cost as the decomposition**

Transformation Examples



cost $\boxed{10}$

Including Inverters

$10 + 3 = 13$

delay $= 1 + 2 + 3 + 2$

$= 8$

Summary

- Multi-level Optimization
- Transformations
  - Factoring - find a factored form from SOP or POS expression
  - Decomposition - express a function as a set of new functions
  - Substitution - express function F as a function of G and some or all of its original variables

Thank You