Signal Processing - 1 by One

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Outline

- So Far: Sampling and Convolution
- Fourier Series and Fourier Transform
- Previous Session: Laplace Transform, Shannon Sampling
- Today: Discrete-time Fourier Transform (DTFT)

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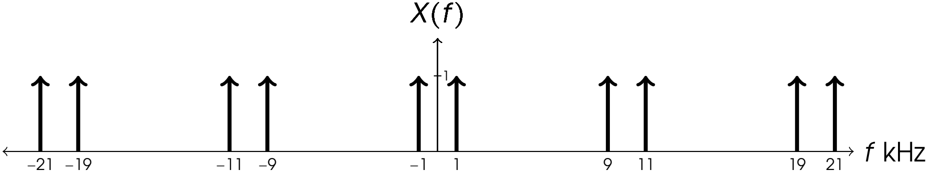

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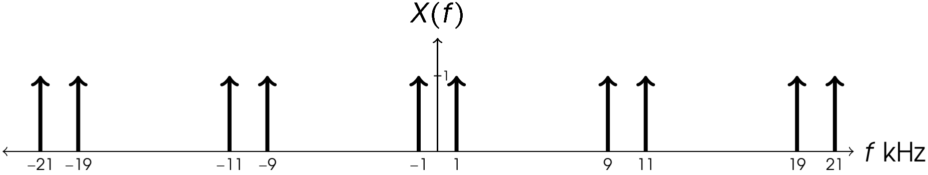

Discrete-time Signal

$$x_s(t) = \frac{2}{10^4} \sum_{n \in \mathbb{Z}} \cos(2\pi 1000 t)\, \delta\left(t - \frac{n}{10^4}\right).$$

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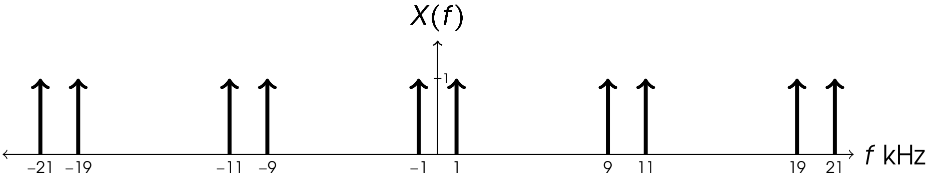

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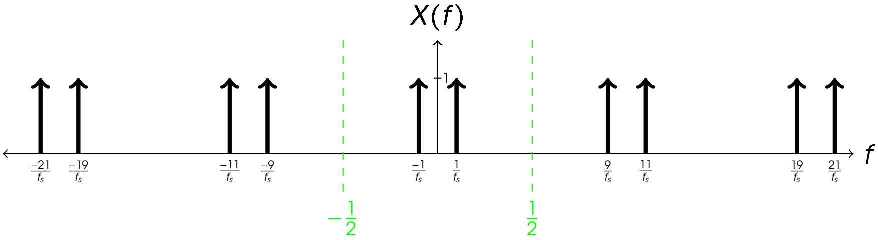

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DTFT

For uniform time samples $x[n]$:

$$\hat{X}(\nu) = \sum_{n \in \mathbb{Z}} x[n] \exp(-j2\pi \nu n), \quad -\frac{1}{2} \leq \nu \leq +\frac{1}{2}.$$

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Often we have finite number of samples $x[n], 0 \le n \le N-1$.

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Generate a signal $x(t)$ of bandwidth β, from samples $x[n], n \in \mathbb{Z}$.

Wireless Communication

Spectrum is a costly resource, centrally allocated usually.

Application	Bandwidth
AM Radio	10kHz
2G	200kHz - 1MHz
3G	5MHz
4G	10 – 20MHz
5G	$\approx$ 100MHz

Data (video/audio/file) should be sent *within* the bandwidth.

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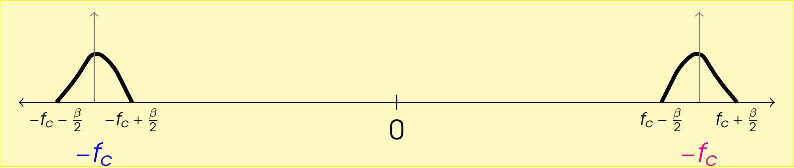
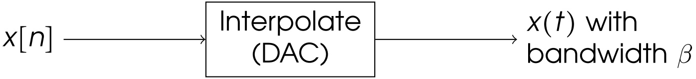
Data (video/audio/file) should be sent *within* the bandwidth.

$$X(f) = 0 \text{ if } \left\{|f| \leq f_c - \frac{\beta}{2}\right\} \text{ OR } \left\{|f| \geq f_c + \frac{\beta}{2}\right\}.$$



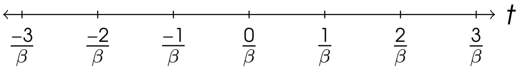
Digital-to-Analog



Digital-to-Analog

$x[n] \longrightarrow$ Interpolate (DAC) $\longrightarrow x(t)$ with bandwidth β

Discrete-time Input and Response:



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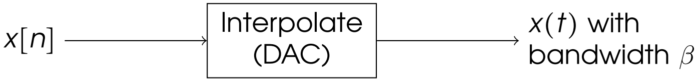

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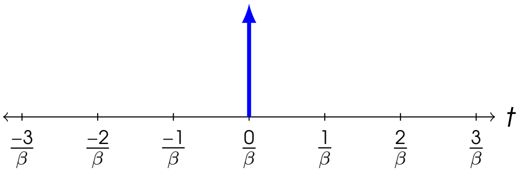

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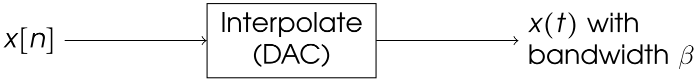

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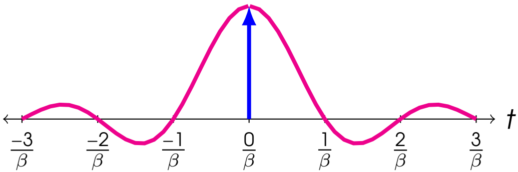

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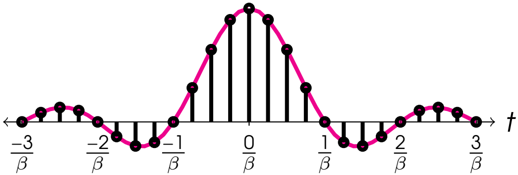

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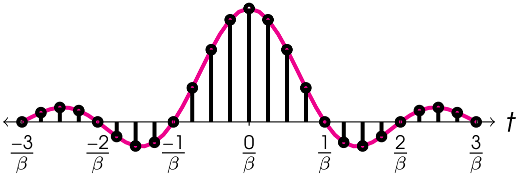

GNURADIO: Generate a baseband signal with bandwidth 10kHz