### Signal Processing - 1 by One

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## Outline

- Digital-Analog-Digital
- Fourier Analysis, Series and Transform
- Previous Weeks: DTFT, DFT, FFT, Circular Convolutions
- Previous Class: Filter Design, Stability
- Today: Problems from Digital Filters

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 for  $|z| > |a|$ .

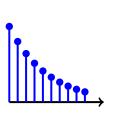


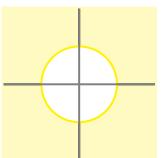


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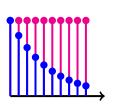


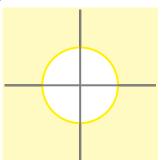


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### State (Block) Diagram

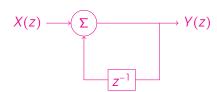
$$H(z)=\frac{1}{1-\alpha z^{-1}}.$$

$$Y(z)(1-\alpha z^{-1}) = X(z)$$

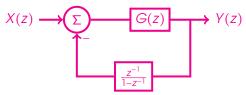
$$y[n] - ay[n-1] = x[n]$$

At a = 1,

$$y[n] = x[n] + x[n-1] \Rightarrow \text{Causal Sum/Integrator!}$$



#### **Transfer Functions**



Find the Transfer Function  $\frac{Y(z)}{X(z)}$ .

Is the system stable if  $G(z) = \alpha(1 - z^{-1})$  with  $\alpha = 0.9$ ?

### Stability of IIR

Is the transfer function given by

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1} + \frac{3}{4}z^{-2})}$$

BIBO stable? Also sketch its standard block diagram.

