Satisfiability Problem

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CADSL

SAT Problem Definition

Given a CNF formula, f :

- A set of variables, V — (a,b,c)
- Conjunction of clauses — (C_1, C_2, C_3)
- Each clause: disjunction of literals over V

Does there exist an assignment of Boolean values to the variables, V which sets at least one literal in each clause to '1' ?

Example : $(a + b + \bar{c})(\bar{a} + c)(a + \bar{b} + c)$

C_1 C_2 C_3

$a = b = c = 1$

DPLL Algorithm for SAT

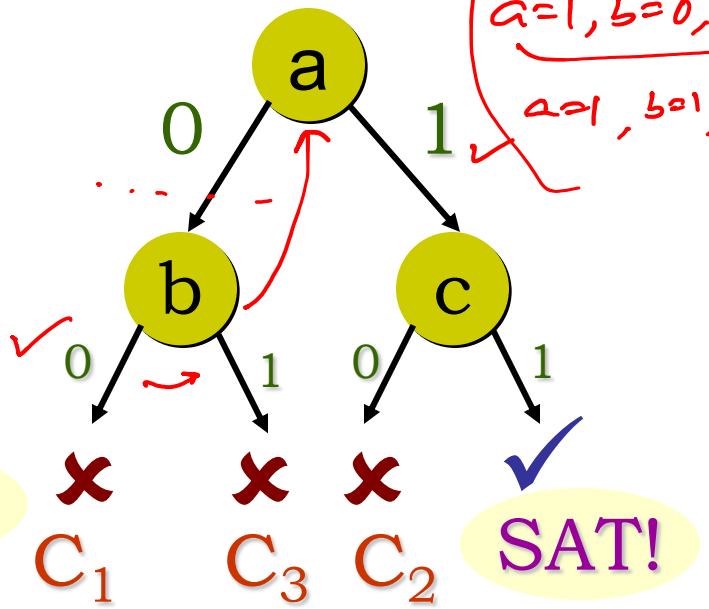
[Davis, Putnam, Logemann, Loveland 1960,62]

Given : CNF formula $f(v_1,v_2,..,v_k)$, and an ordering function Next_Variable

Example :

$$\phi = (a + b)(\bar{a} + c)(a + \bar{b})$$

C_1 C_2 C_3



2^n possibilities

DPLL Algorithm

Simple algorithm

Not scalable

1960

DPLL Algorithm: Unit Clause Rule

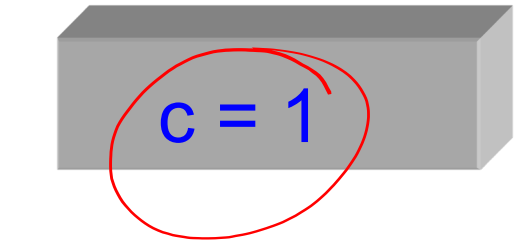
Rule: Assign to *true* any single literal clauses.

$$(a + b + c)$$

$$0 \quad 0$$

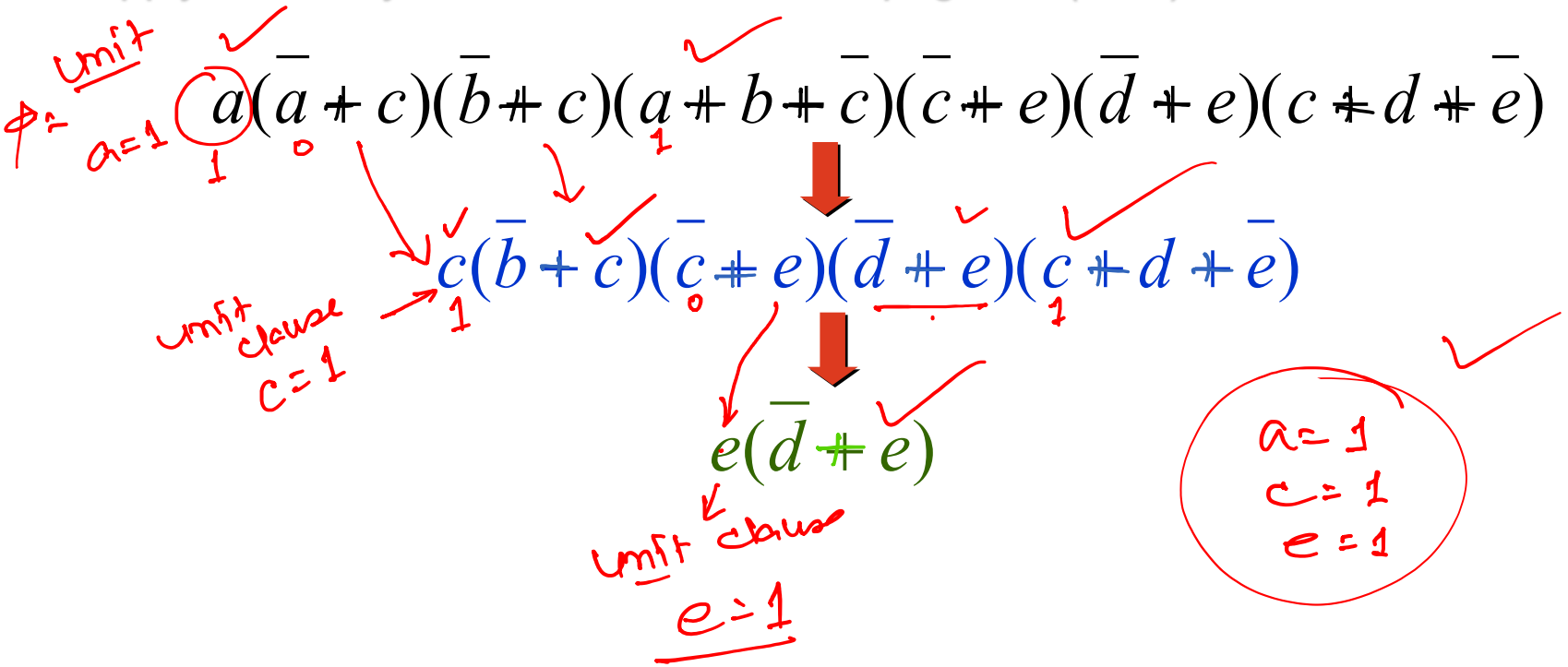
(c)

unit clause.



DPLL Algorithm: BCP

Apply Iteratively: *Boolean Constraint Propagation (BCP)*



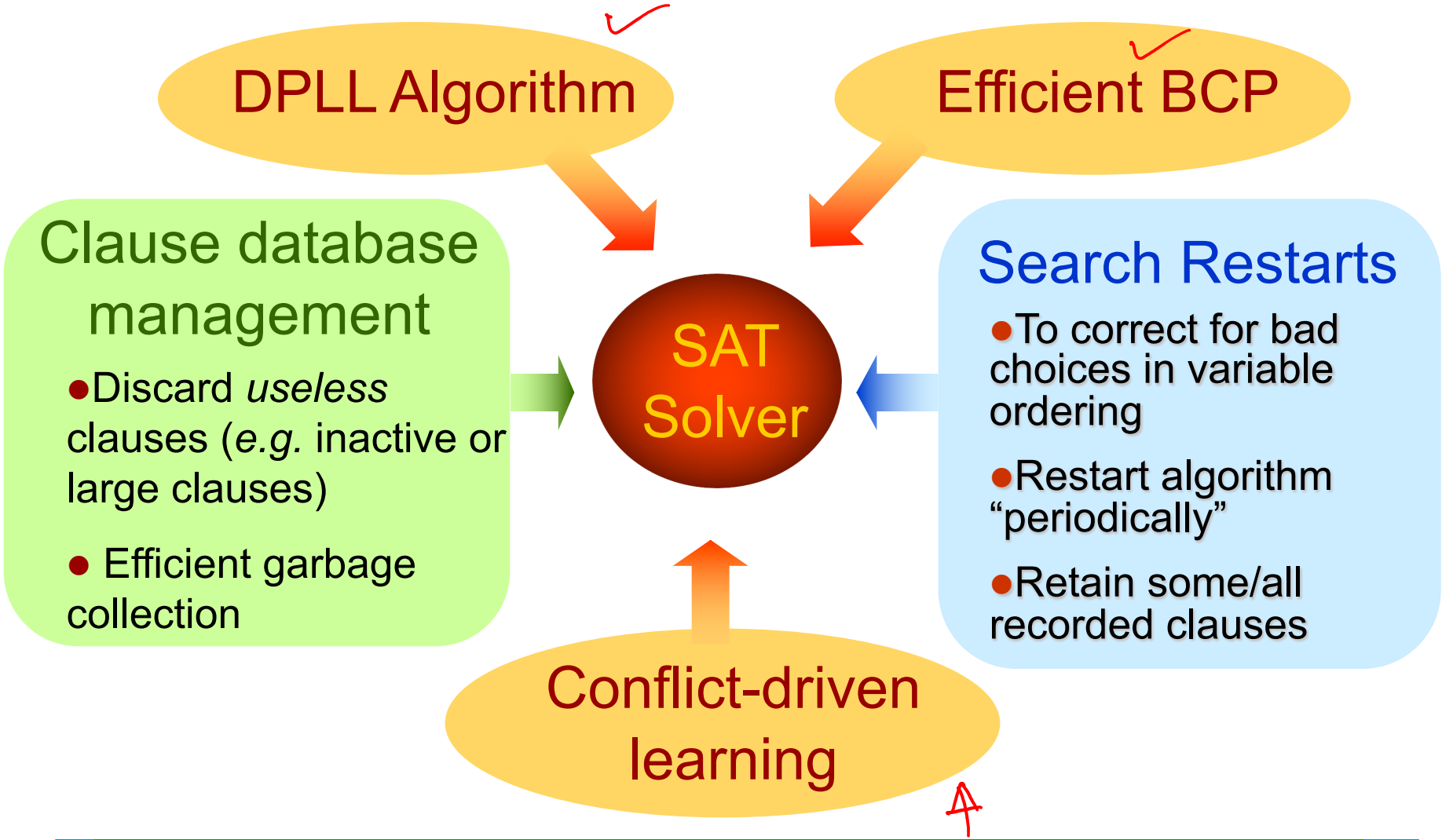
Pure Literal Rule

- A variable is *pure* if its literals are either all positive or all negative
- Satisfiability of a formula is unaffected by assigning pure variables the values that satisfy all the clauses containing them

$$\varphi = (a + c)(b + c)(b + \neg d)(\neg a + \neg b + d)$$

- Set c to 1; if φ becomes unsatisfiable, then it is also unsatisfiable when c is set to 0.

Anatomy of a Modern SAT Solver



Modern SAT Solvers

SAT ϕ - $f(a,b,c)$ <= POS

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Thank You