TUTORIAL 1

Exercises

1. A complex polynomial of degree n has exactly n roots. (Assuming fundamental theorem of algebra)

2. Show that a real polynomial that is irreducible has degree at most two. i.e., if

$$f(x) = a_0 + a_1 x + \ldots + a_n x^n, \ a_i \in \mathbb{R},$$

then there are non-constant real polynomials g and h such that $f(x) = g(x)h(x)$ if $n \geq 3$.

3. Show that if U is a path connected open set in $\mathbb{C}$, so is U minus finite set. (Try to prove it as rigorously as you can)

4. Check for real differentiability and holomorphicity:

   1. $f(z) = c$
   2. $f(z) = z$
   3. $f(z) = z^n, n \in \mathbb{Z}$
   4. $f(z) = \mathrm{Re}(z)$
   5. $f(z) = |z|$
   6. $f(z) = |z|^2$
   7. $f(z) = \bar{z}$
   8. $f(z) = \begin{cases} \frac{z}{\bar{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0. \end{cases}$

5. Show that the CR equations take the form

$$u_r = \frac{1}{r} v_\theta \ \& \ v_r = -\frac{1}{r} u_\theta$$

in polar coordinates.