# Special Functions

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## EE-224: Digital Systems

# Dual Function

- To obtain ~~dual of~~ Dual of Boolean Expression, exchange AND’s and OR’s; and exchange 0’s and 1’s. The functional definition is:

*Dual of* $f(x_1, x_2, \ldots. xn) =$ *Complement of* $f(\bar{x}_1, \overline{x2}, \ldots, \bar{x}_n)$

$f(a,b,c) = ab + bc + ca$

$\text{dual of } f = \overline{(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})}$

$= \overline{(\bar{a}\cdot\bar{b})} \cdot \overline{(\bar{b}\cdot\bar{c})} \cdot \overline{(\bar{c}\cdot\bar{a})}$

$= (a+b)\cdot(b+c)\cdot(c+a)$

$f(a,b,c) = a\bar{b} + bc$ $\quad$ $\text{dual of } f = (a+\bar{b})\cdot(b+c)$

# Self Dual Function

- A function is dual of itself
- function $f$ is *self-dual iff* when complementing its input variables, the output becomes complement of $f$.

*Dual of* $f(x_1, x_2, \ldots . xn) = f(x_1, x_2, \ldots . xn)$

$$\underline{\text{dual}}\ f = \overline{f(\bar{x}_1, \bar{x}_2 \cdots \bar{x}_n)} = f(x_1, x_2 \cdots x_n)$$

$$f(\bar{x}_1, \bar{x}_2, \cdots \bar{x}_n) = \overline{f(x_1, x_2 \cdots x_n)}$$

# Self Dual Function

$f = ab + bc + ca$

dual ~~of~~ $f = (a+b) \cdot (b+c) \cdot (c+a)$

$= (ab + b + ac + bc)(c + a)$

$= (abc + bc + ac + bc + ab + ab + ac + abc)$

$= abc + bc + ac + ab$

$= bc + ac + ab$

# Symmetrical Function

- A Boolean function that does not change under any permutation of its input variables is called a Totally Symmetric Function

$$f(a,b,c) = ab + bc + ca = ba + ac + cb = ab + ac + bc$$

Symmetrical

$$f(a,b,c) = a \oplus b \oplus c = b \oplus a \oplus c = c \oplus b \oplus a$$

$$f(a,b) = a \cdot b = a + b = a \oplus b$$

# Symmetrical Function

- When a function does not change under any permutation of a subset of its variables is called a **Partially Symmetric Function** ✓

$$f(a,b,c) = \bar{a}\bar{b}c + \bar{a}b\bar{c} + abc \checkmark$$

partially symmetrical in $b \& c$

$$= \bar{a}\cdot\bar{c}b + \bar{a}c\bar{b} + acb$$

$$= \bar{a}\bar{b}c + \bar{a}b\bar{c} + abc$$

# Symmetry Theorem

- A function $f(x_1, x_2, \ldots, x_n)$ is totally symmetric *iff* it can be specified by a list of integers $A = \{a_1, a_2, \ldots, a_m\}$, $0 \leq a_i \leq n$ so that $f = 1$ *iff* exactly $a_i$ of the $n$ variables are 1.

$f = ab + bc + ca$ $\quad A = \{2, 3\}$ ✓

$f = a \oplus b \oplus c \oplus d$ $\quad A = \{1, 3\}$ ✓

$1 \oplus 0 \oplus 0 \oplus 0 = 1$

$1 \oplus 1 \oplus 1 \oplus 0 = 1$

$1 \oplus 1 \oplus 0 \oplus 1 = 1$

# Symmetry Theorem

# Unate Function

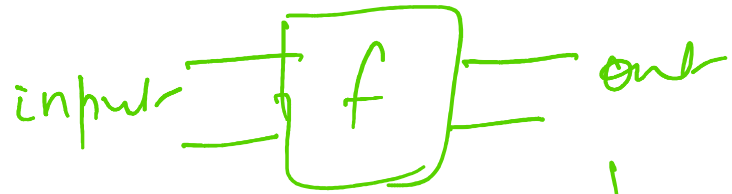
$f = ab + bc + ca$ — positive unate functio

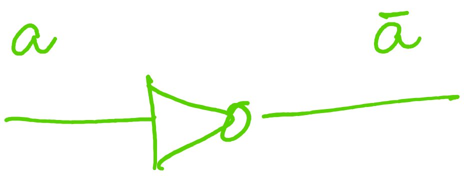
$f = \bar{a}\bar{b} + \bar{c}\bar{a}$ — -ve unate funct..

$f = \bar{a}b + b\bar{c}$ — unate

$f = \bar{a}b + \bar{b}\bar{c}$

# Reversible Function

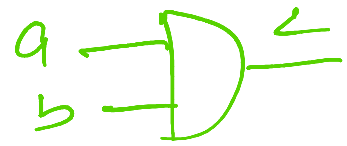



| | |
|---|---|
| 0 | 1 |
| 1 | 0 |

Reversible.

Quantum circuits




| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Irreversible

# Thank You