EE 325: Probability and Random Processes

Modules B1 and M1

You will see two streams of videos and slides. Names starting with 'B' will indicate background material and those that start with 'M' will indicate lecture material.

Module B1 covered the following topics.

- **Module B1a** Three computation problems.
- **Module B1b** Statistics: definitions and motivating probability
- **Module B1c** Some famous problems, popular applications, and some not so famous problems. Further motivating examples in probability and statistics:

Module M1 will cover the following topics.

- **Module M1a:** Preliminaries
  - Set theory primer, set operations, DeMorgan's Cardinality of sets.
  - Sample space: Discrete, continuous, mixed.
  - Events and sigma algebra.
  - Countable additivity.
  - Probability measure: Axioms and definitions.
  - Properties of probability measures.
- **Module M1b:** Conditional probability.
  - Multiplication rule.
  - Total probability theorem.
  - Bayes theorem.
- **Module M1c:** Independent events.
  - Counting problems: Sampling: with and without replacement, with and without ordering.
  - Binomial and Multinomial.
  - Pairwise independence.
- **Module M1d** Borel Cantelli Lemma

Much of this material is based on Chapter 2 of the Papoulis and Pillai textbook. Additional material on combinatorial problems is collected from various sources.

Assignment 2

The following is the homework assignment from Chapter 2 of the text: 2-5, 2-10, 2-14, 2-16, 2-17, 2-19, 2-24, 2-25, 2-27. In addition also solve the following problems.

1. A course is taught by four instructors. Before every lecture, the instructors draw lots and one of them is randomly chosen to teach on that day. What is the probability that in N classes, all the lecturers would have taught at least once. Generalise to k instructors teaching the course.

2. Numbers 1, 2, 3, 4, 5, and 6 are randomly placed on a circle. What is the probability that they are placed in increasing order?

3. A and B play the following game of dice. Both roll their dice. If B rolls a one, then it rolls again and keeps whatever appears. The one with the highest value wins. If there is a tie A wins. What is the probability that A wins the game.

4. There are R brown balls and B black balls in an urn. Balls are drawn at random without replacement. Let A_k be the event that a brown ball is drawn for the first time on the k-th draw. Find p_k, the probability of A_k. Now consider the case when B and R are increased to ∞ while keeping $\alpha = R/(B+R)$. Find p_k as $B + R \to \infty$.

5. There are n of which the r-th urn contains $r-1$ brown balls and $n-r$ black balls. You pick an urn at random and pick two balls at random without replacement. What is the probability that the second ball is black. What is conditional probability that the second ball is black given that the first ball is black.

6. **Prosecutor's fallacy:** Let G be the probability that an accused is guilty, and T that the testimony of a witness is true. Many times it is argued that $\mathsf{Prob}(\mathsf{G} \mid \mathsf{T}) = \mathsf{Prob}(\mathsf{T} \mid \mathsf{G})$. Show that this is true iff $\mathsf{Prob}(\mathsf{G}) = \mathsf{Prob}(\mathsf{T})$.

7. **Extra credit:** 10% of the surface area of a sphere is white and the rest is black. There are no assumptions on how this white part is distributed on the surface. Prove that it is always possible to inscribe a cube with all its vertices black. Think of a randomly inscribed cube. Let A_i be the probability a random vertex is white. Now obtain an upper bound on the probability that at least one of the vertices is white. Show that this strictly less than one. This proves that there is at least one cube with all black vertices.