Signal Processing - 1 by One

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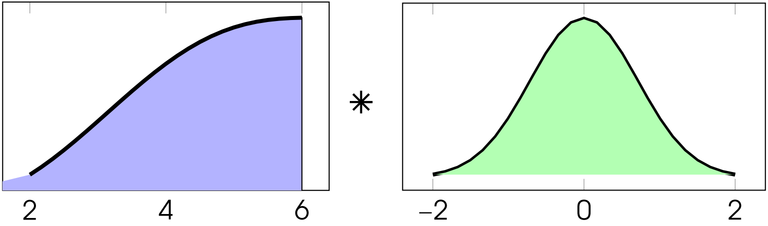
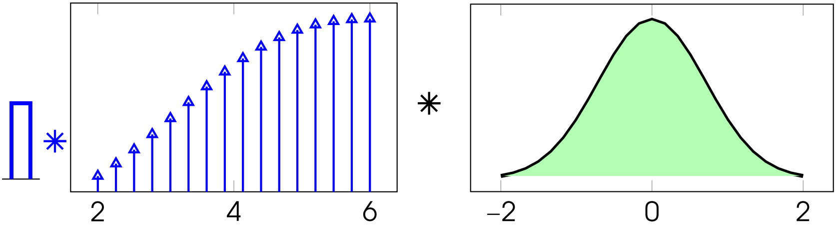
Generalized Convolution



Function $x(t)$ Function $y(t)$

Generalized Convolution



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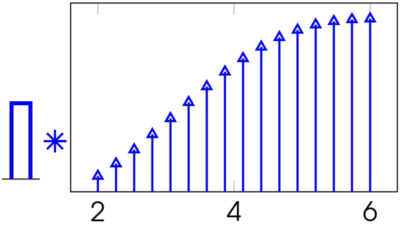

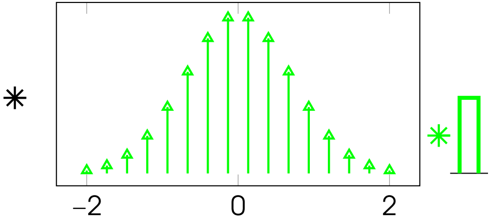

Function $x(t)$

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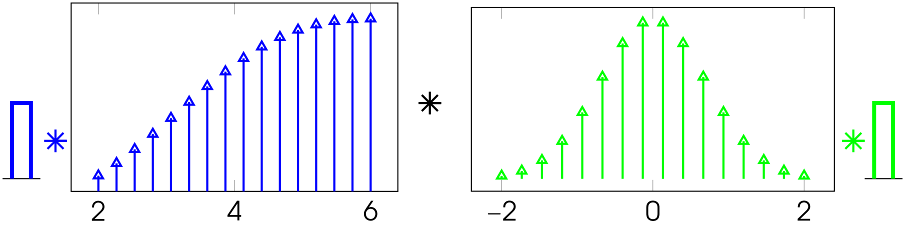

Function $x(t)$ Function $y(t)$

$$\bar{x}_T = \sum_{m \in I} x(mT)\delta(t - mT) \quad , \quad \bar{y}_T = \sum_{n \in J} y(nT)\delta(t - nT).$$

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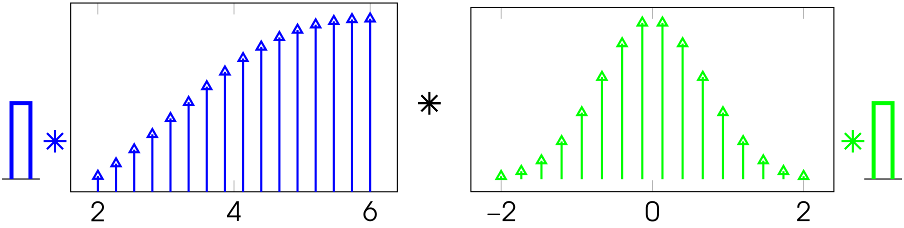

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$$x(t) * y(t) \approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t)$$

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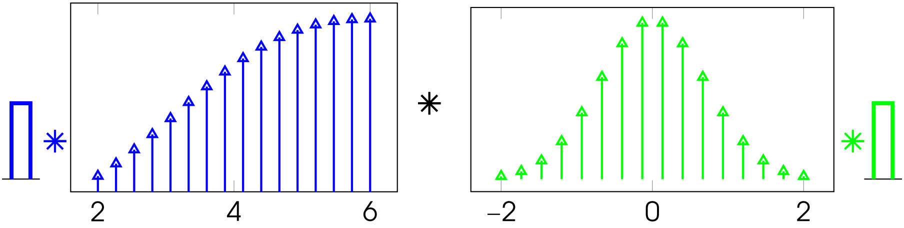

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$$\begin{aligned} x(t) * y(t) &\approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t) \\ &= \bar{x}_T * \bar{y}_T \ * r_T(t) * r_T(t). \end{aligned}$$

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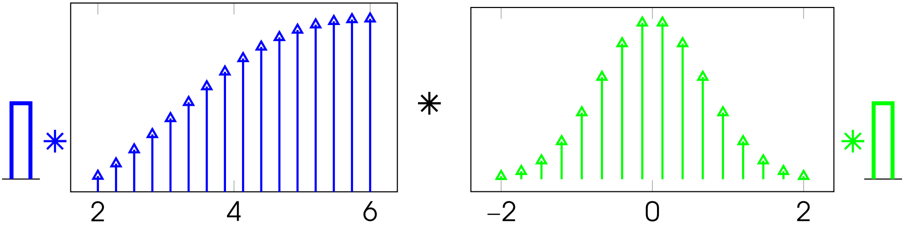

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Discrete Convolution

$$\begin{aligned} x(t) * y(t) &\approx r_T(t) * \bar{x}_T * \bar{y}_T * r_T(t) \\ &= \boxed{\bar{x}_T * \bar{y}_T} * r_T(t) * r_T(t). \end{aligned}$$

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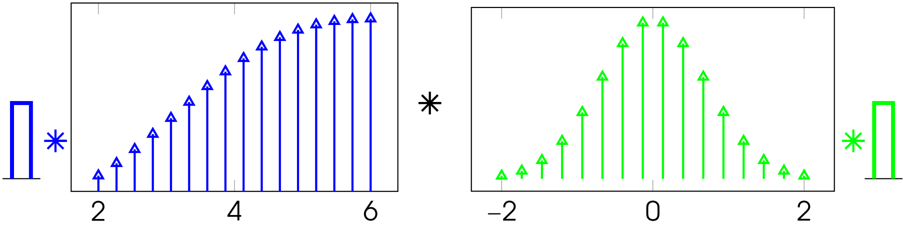

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Discrete Convolution

Analog Convoltn

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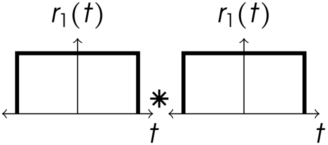
Convolving Rectangles



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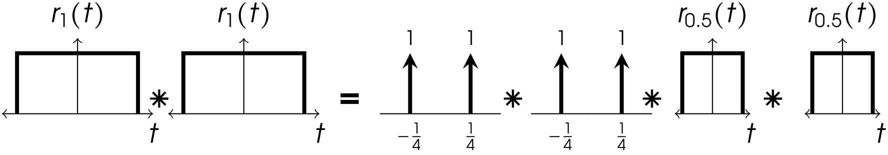

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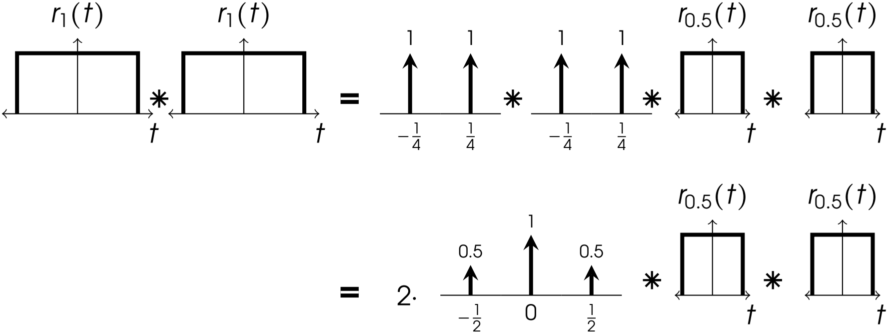

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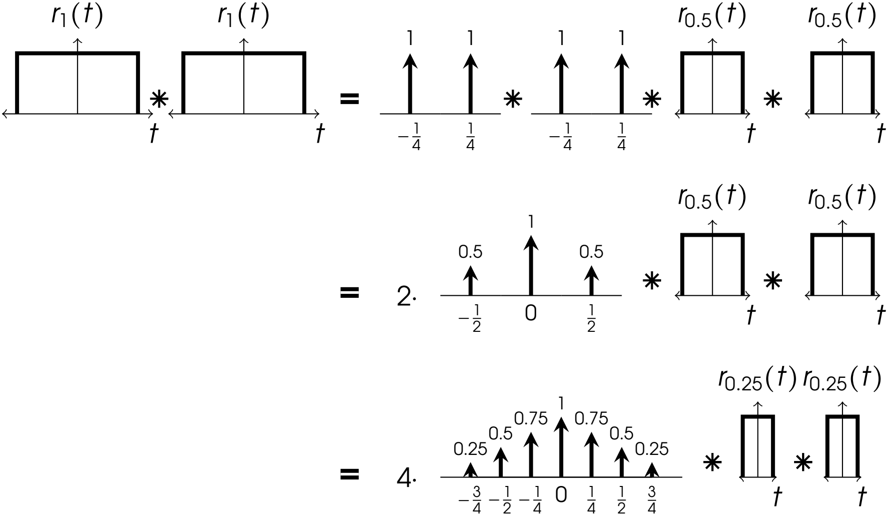

Close Cousins

$$r_T(t) * r_T(t) = [1\ 2\ 1] * r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t)$$

$$= r_{\frac{T}{2}}\left(t + \frac{T}{2}\right) * r_{\frac{T}{2}}\left(t + \frac{T}{2}\right) + 2r_{\frac{T}{2}}(t) * r_{\frac{T}{2}}(t) + r_{\frac{T}{2}}\left(t - \frac{T}{2}\right) * r_{\frac{T}{2}}\left(t - \frac{T}{2}\right).$$

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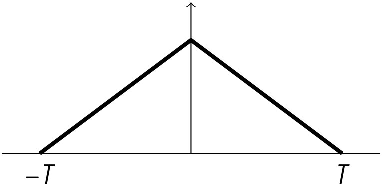

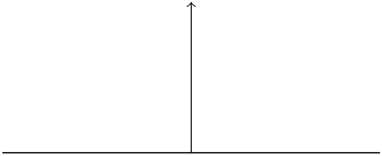

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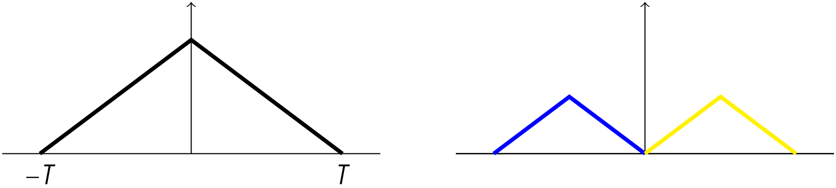

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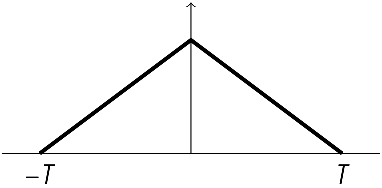

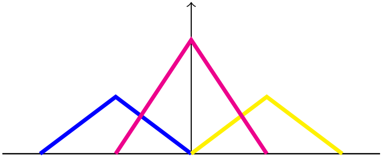

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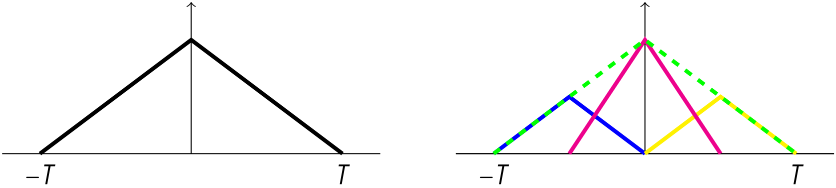

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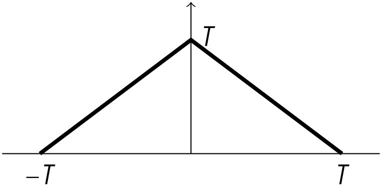

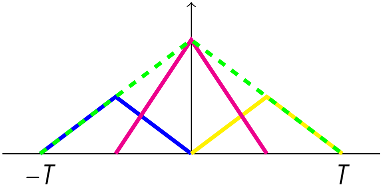

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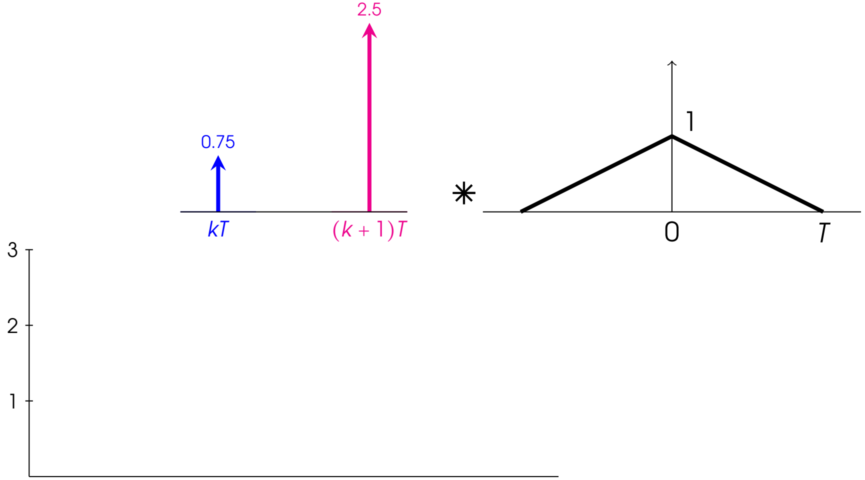
Linear Interpolation



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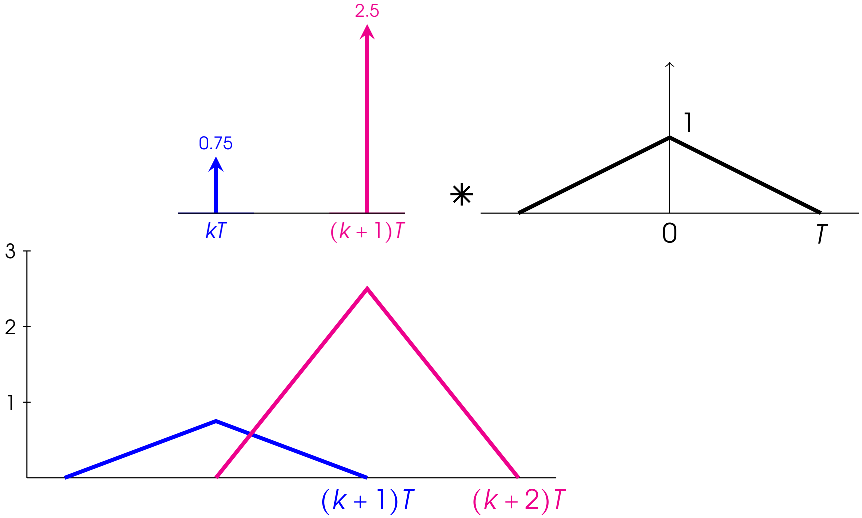

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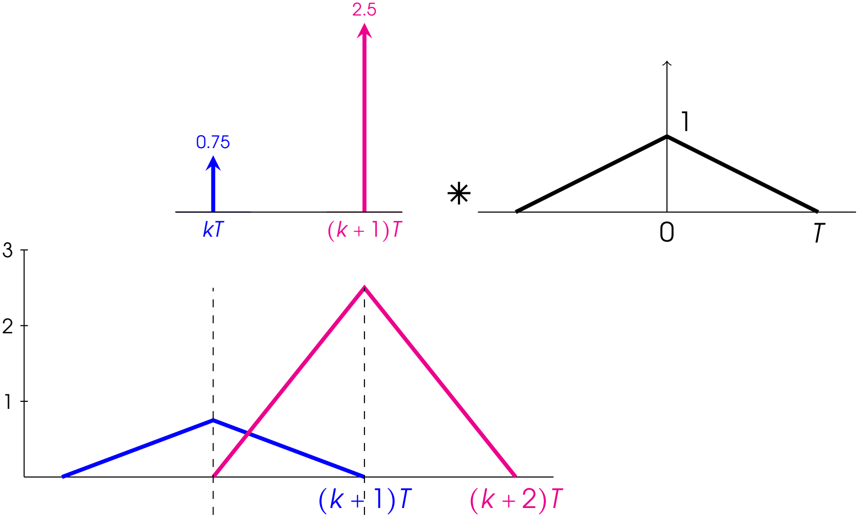

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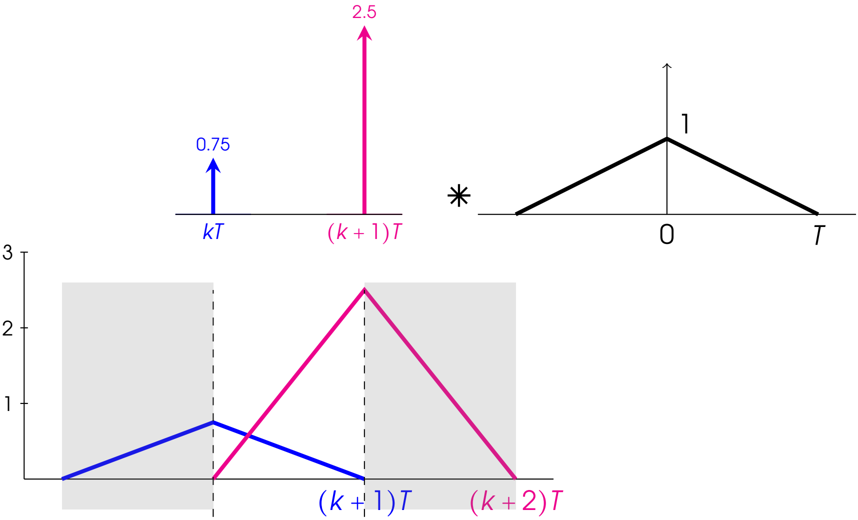

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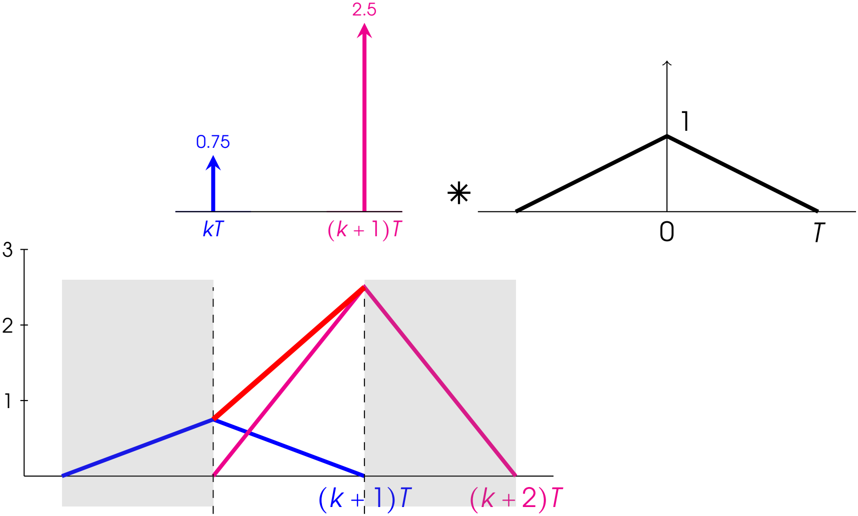

Digital Interpolation

Upsampling followed by Digital Convolution

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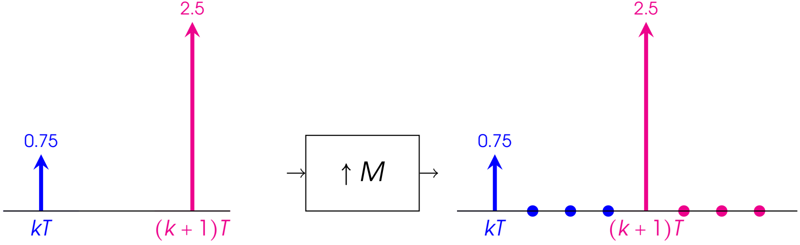

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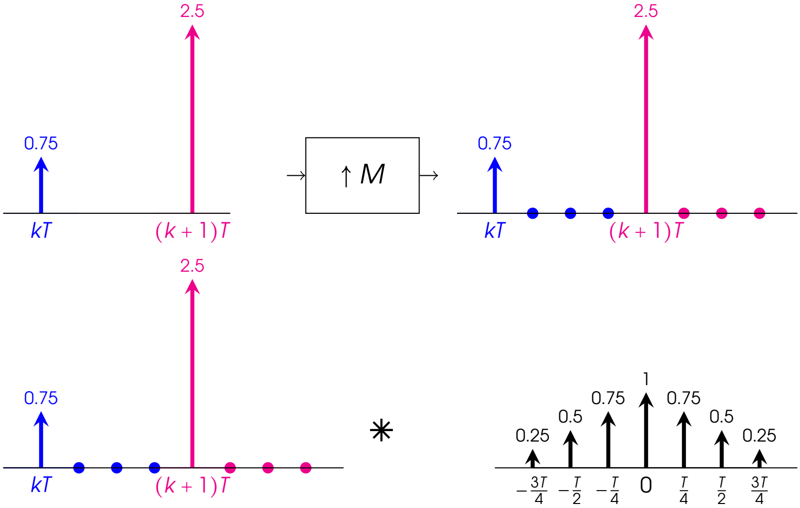
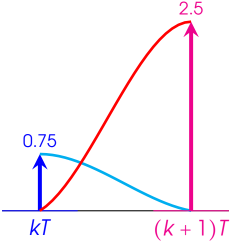


Cubic Interpolation

Let us interpolate in the interval $[0, 1]$ between two samples.



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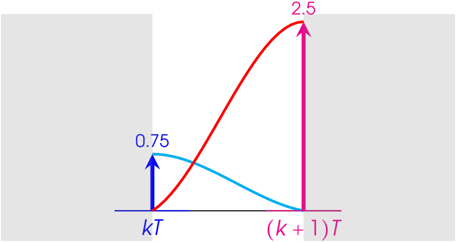

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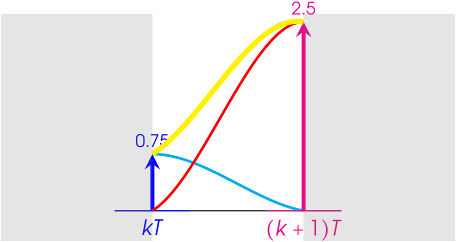

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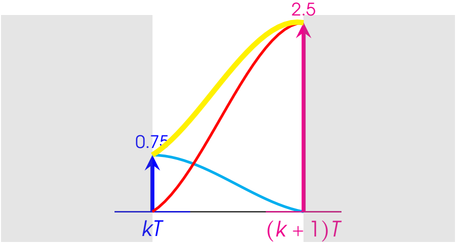

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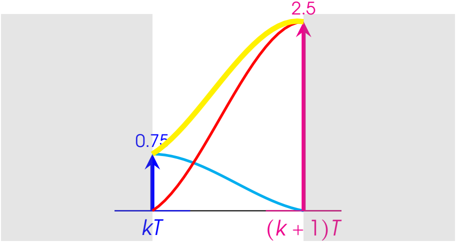

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Eg: For $\delta(t)$ as input, the output in $t \in [0, 1]$

$$p(t) = \frac{3}{2}t^3 - \frac{5}{2}t^2 + 1.$$

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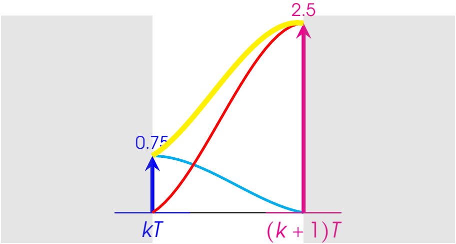

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We need to do slightly more work to finish this!