

EE 325: Probability and Random Processes

Homework for Module 3

Instructions

- There will be viva on these on 30 September.

Questions

1. There are several solved examples in the textbook in Chapter 5. Make sure you have studied these problems and the solutions.
2. This module has also covered a small part of Chapter 6. Look into the relevant solved problems in that chapter.
3. Solve problems 5.8, 5.10, 5.21, 5.27, 6.3, 6.5, 6.8, 6.20. You may be quizzed on these.
4. Submit solutions to 5.40, 5.41, 5.50, 6.25, 6.46 from the text and the following problems.
 - (a) The probability that a family has n children is p_n for $n = 0, 1, \dots$. You meet a random person on the street. Let X be the number of siblings that this person has. Let Y be the number of elder siblings that this person has. Obtain the marginal and the joint expectations of X and Y . Also obtain the expectations of X and Y . Make any reasonable assumptions that you may need.
 - (b) A graph, or a network, is represented by the set of vertices V and the set of edges E . An edge is a 2-tuple (unordered) (v_1, v_2) . You can visualise an edge as a connection between the two vertices. A graph is connected if there is a path, a sequence of edges, between every pair of nodes. Construct a random graph as follows. Choose points uniformly and independently in $(0, 1)$ for the vertices. Thus X_i is a uniform random variable in $(0, 1)$. Nodes i and j are connected if $|X_i - X_j| \leq r$.
For example, for a two node network let X_1 and X_2 be the ‘random’ locations of nodes 1 and 2 chosen as above. They are connected if $|X_1 - X_2| < r$ for a given r . What is the probability that this 2-node graph is connected. Repeat this for a three node graph.
 - (c) Consider two random variables X and Y . Define $\text{VAR}(X|Y)$. Show that $\text{VAR}(X) = \text{E}(\text{VAR}(X|Y)) + \text{VAR}(\text{E}(X|Y))$.