EE-224: Digital Design

Minimization of Logic Expression using Boolean Algebra

Virendra Singh

Professor

Computer Architecture and Dependable Systems Lab

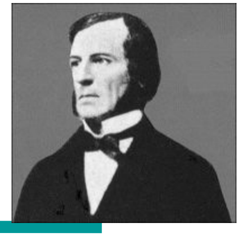
Department of Electrical Engineering

Indian Institute of Technology Bombay

http://www.ee.iitb.ac.in/~viren/

E-mail: viren@ee.iitb.ac.in

Boolean Algebra



- Boolean Algebra is defined as
  1. Set of elements {0, 1}
  2. Set of operators {+, ., ~}
  3. Number of postulates
- Boolean Algebra: 5-tuple

$$\{B, +, ., \sim, 0, 1\}$$

- Closure: If a and b are Boolean then $(a.b)$ and $(a+b)$ are also Boolean

Postulates

Postulate	Duals	
	Expression 1	Expression 2
0	$a, b, a + b \in B$	$a, b, a \cdot b \in B$
3	$a + 0 = a$	$a \cdot 1 = a$
1	$a + b = b + a$	$a \cdot b = b \cdot a$
2	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
4	$a + (b \cdot c)=(a + b) \cdot (a + c)$	$a \cdot (b + c)=(a \cdot b)+(a \cdot c)$
6	$a + \bar{a} = 1$	$a . \bar{a} = 0$

Theorems

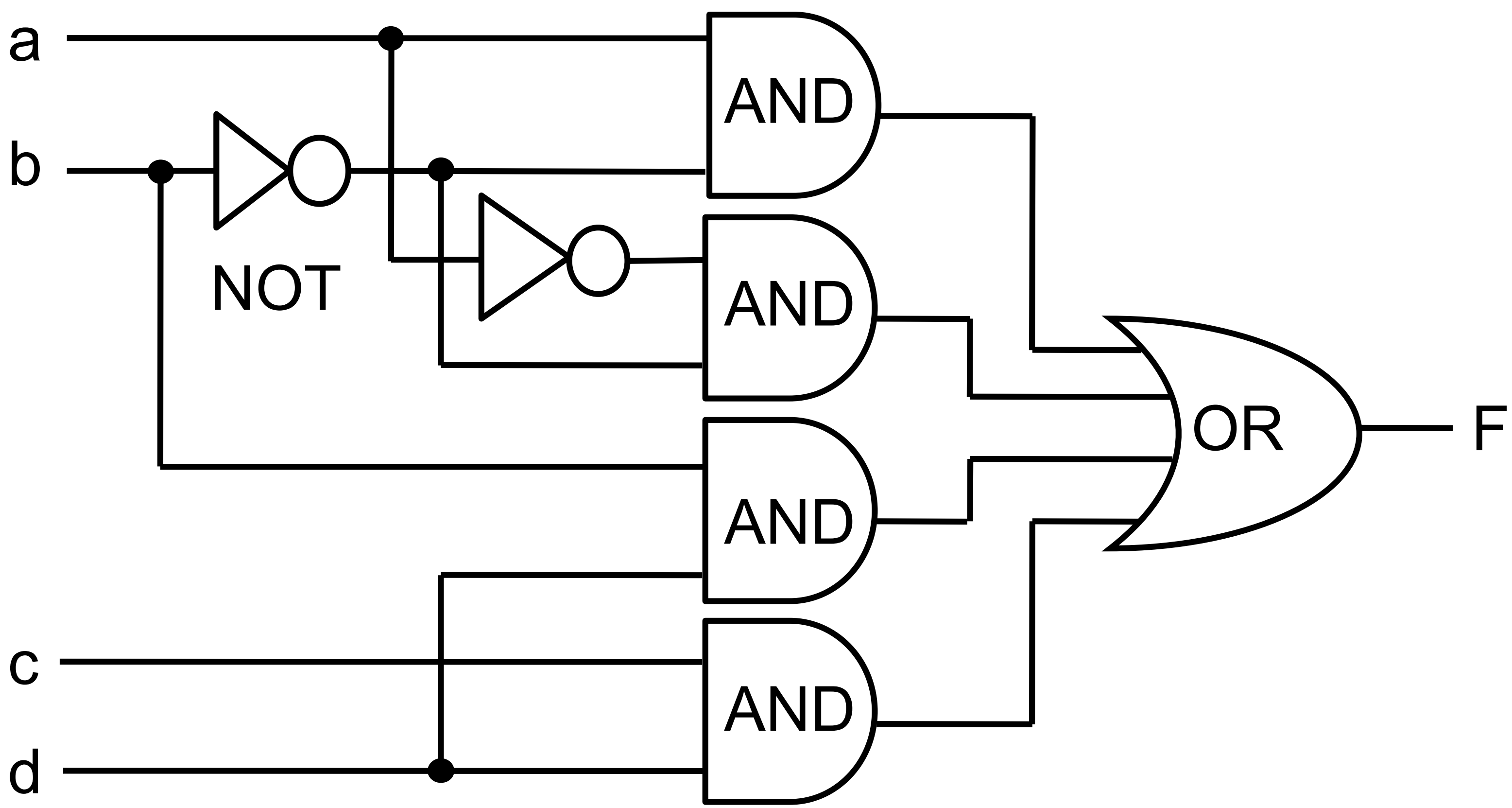
Theorem	Duals	
	Expression 1	Expression 2
Idempotency	$a + a = a$	$a \cdot a = a$
Null	$a + 1 = 1$	$a \cdot 0 = 0$
Involution	$\bar{\bar{a}} = a$	$\bar{\bar{a}} = a$
Absorption	$a + a.b = a$	$a \cdot (a + b) = a$
Adsorption	$a + \bar{a}.b = a + b$	$a \cdot (\bar{a} + b) = a.b$
Uniting	$a.b + a.\bar{b} = a$	$(a + b)(a + \bar{b}) = a$

Theorems

Theorem	Duals	
	Expression 1	Expression 2
DeMorgan	$\overline{a + b} = \bar{a} \,.\, \bar{b}$	$\overline{a \cdot b} = \bar{a} + \bar{b}$
Consensus	$a.b + \bar{a}.c + b.c$ $= a.b + \bar{a}.b$	$(a + b)(\bar{a} + c)\,(b + c)$ $= (a + b).(\bar{a} + c)$

Understanding Minimization

- Logic function: $\mathbf{F = a\bar{b} + \bar{a}\bar{b} + bd + cd}$



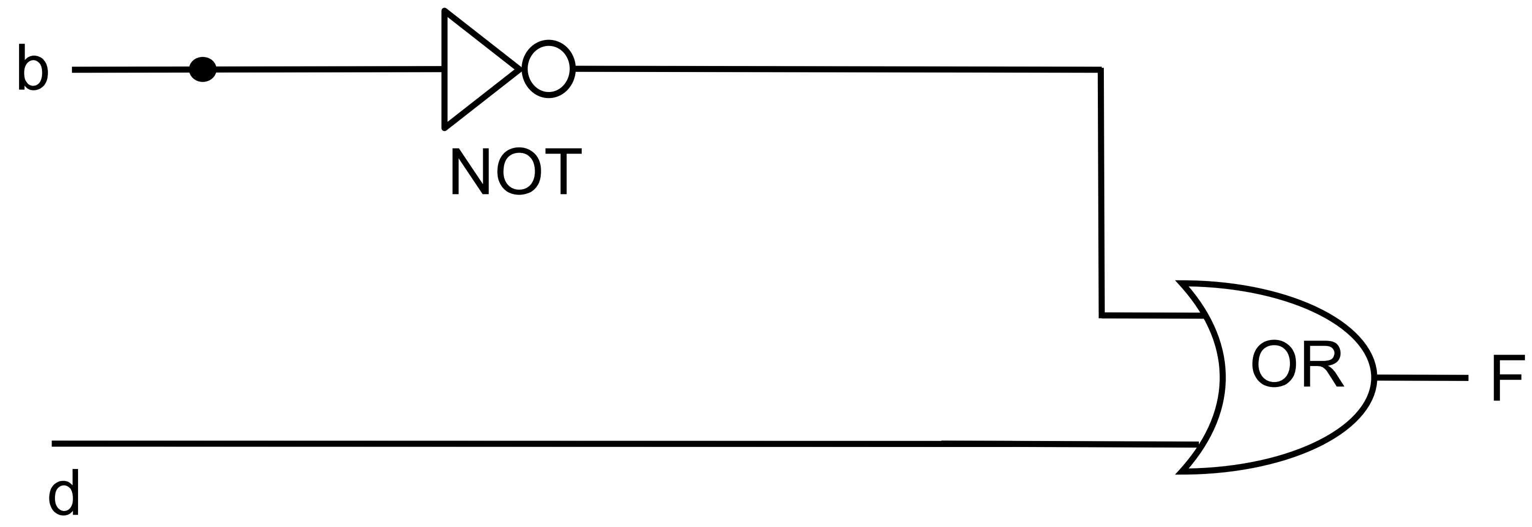
Logic Minimization

- Reducing products:

$F = a\bar{b} + \bar{a}\bar{b} + bd + cd$	Distributivity
$= \bar{b}(a + \bar{a}) + bd + cd$	Complementation
$= \bar{b}1 + bd + cd$	Identity
$= \bar{b}(c + \bar{c}) + bd + cd$	Complementation
$= bd + \bar{b}c + cd + \bar{b}\bar{c}$	Distribitivity
$= bd + \bar{b}c + \bar{b}\bar{c}$	Consensus theorem
$= bd + \bar{b}(c + \bar{c})$	Distributivity
$= bd + \bar{b}$	Complement, identity
$F = \bar{b} + d$	Adsorption

Logic Minimization

- Minimized expression: $F = \bar{b} + d$



Expression Simplification

- An application of Boolean algebra
- Simplify to contain the smallest number of literals (complemented and uncomplemented variables):

$$a.b + \bar{a}.c.d + \bar{a}.b.d + \bar{a}.c.\bar{d} + a.b.c.d$$
$$= a.b + a.b.c.d + \bar{a}.c.d + \bar{a}.c.\bar{d} + \bar{a}.b.d$$
$$= a.b + a.b.(c.d) + \bar{a}.c.(d + \bar{d}) + \bar{a}.b.d$$
$$= a.b + \bar{a}.c + \bar{a}.b.d = b\,(a + \bar{a}.d) + \bar{a}.c$$
$$= b.(a + d) + \bar{a}.c$$

5 literals

Expression Simplification

- Logic minimization

$$F = \bar{x}.\bar{y}.z + x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.\bar{z} + x.y.z$$

$$F = \bar{x}.\bar{y}.z + x.\bar{y}.(\bar{z} + z) + x.y.(\bar{z} + z)$$

$$F = \bar{x}.\bar{y}.z + x.\bar{y} + x.y = \bar{x}.\bar{y}.z + x.(\bar{y} + y)$$

$$F = \bar{x}.\bar{y}.z + x = \bar{y}.z + x$$

Theorem 7: DeMorgan's Theorem

- $\overline{a + b} = \bar{a} \,.\, \bar{b}, \quad \forall a, b \in B$
- $\overline{a \,.\, b} = \bar{a} + \bar{b}, \quad \forall a, b \in B$

1806 - 1871

Generalization of DeMorgan's Theorem:

$$\overline{a + b + \cdots + z} = \bar{a} \cdot \bar{b} \cdots \bar{z}$$

$$\overline{a \cdot b \cdots z} = \bar{a} + \bar{b} + \cdots + \bar{z}$$

Complementing Functions

- Use DeMorgan's Theorem to complement a function:
  1. Interchange AND and OR operators
  2. Complement each constant value and literal
- Example: Complement $F = \bar{x}.y.\bar{z} + x.\bar{y}.\bar{z}$

$\bar{F} = (x + \bar{y} + z)(\bar{x} + y + z)$

Thank You