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Tut 5

which eqn. is this ?

6. DE $\rightarrow$ $U_{tt} - c^2 U_{xx} = xe^{-t}$ $\quad 0<x<l \quad t>0$

BC $\rightarrow$ $U_x(0,t) = 0 \quad U_x(l,t) = e^{-t}$

IC $\rightarrow$ $U(x,0) = 0$

$\rightarrow$ BC - non-homogeneous $\Rightarrow$ we need to homogenise it $\rightarrow$

let $Z(x,t) = \sin t\left(1-\frac{x}{l}\right) + \frac{x}{l}$

let $v(x,t) = u(x,t) - Z(x,t)$

$\rightarrow$ $v(x,t)$ satisfies $\rightarrow$ $V_{tt} - c^2 V_{xx} = le^{-t}\left(\frac{x}{l}\right) + \sin t\left(1-\frac{x}{l}\right)$

BC $\rightarrow$ $V(0,t) = V(l,t) = 0$ $\rightarrow$ which BC is this ?

IC $\rightarrow$ $V(x,0) = -\frac{x}{l} \quad V_t(x,0) = 1 - \frac{x}{l}$

Solving for '__ __ __ __ __ __' BC,

$$V(x,t) = \sum_{n\geq 1} Y_n(t) \sin\left(\frac{n\pi x}{l}\right)$$

$$\frac{x}{l} = \sum_{n\geq 1} \frac{2}{n\pi}(-1)^{n+1} \sin\left(\frac{n\pi x}{l}\right)$$

$$1 - \frac{x}{l} = \sum_{n\geq 1} \frac{2}{n\pi} \sin\left(\frac{n\pi x}{l}\right)$$

$\rightarrow$ substituting in the DE of V $\rightarrow$

$$Y_n''(t) + \left(\frac{cn\pi}{l}\right)^2 Y_n(t) = \frac{2}{n\pi}\left(le^{-t}(-1)^{n+1} + \sin t\right)$$

$$Y_n(0) = \frac{2}{n\pi}(-1)^n$$

$$Y_n'(0) = \frac{2}{n\pi}$$

Now, $Y_n(t)$ can be solved using method of undetermined coefficients

$Y_n(t) \rightarrow U(x,t)$ - How?

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q. ii) Guess a function which satisfies the given BC (and also the DE)
$\rightarrow \phi(x,t)$ satisfies $\phi_x(0,t) = t$ and $\phi_x(L,t) = 0$

- ~~in both~~ let $\phi(x) = A(x) \times B(t)$

$A'(0) \times B(t) = t$

$A'(L) \times B(t) = 0$

- simplest function $\rightarrow B(t) = t$

$A'(x) \rightarrow$ linear $\Rightarrow A(x) = \ldots$ what?

$$\Rightarrow \phi(x) = -\frac{(x-L)^2}{2L} t$$

Now, $\phi(x,t)$ satisfies $\phi_{tt} - c^2\phi_{xx} = -c^2 t/L$

$\phi(x,0) = 0 \quad \phi_t(x,0) = -\frac{(x-L)^2}{2L}$

$\rightarrow$ the required soln. will be $\rightarrow u(x,t) = \phi(x,t) + w(x,t)$

DE $\rightarrow w_{tt} - c^2 w_{xx} = ?$

BC $\rightarrow w_x(0,t) = ? \quad w_x(L,t) = ?$

IC $\rightarrow w(x,0) = ? \quad w_t(x,0) = \frac{(x-L)^2}{2L}$

Now, consider $\rightarrow y'' + \lambda y = 0 \qquad 0 < x < L \qquad y \rightarrow y(x)$

$y'(0) = y'(L) = 0$

eigen values $\rightarrow \lambda_n = \left(\frac{n\pi}{L}\right)^2$

eigenfunctions $\rightarrow y_n = \cos\left(\frac{n\pi x}{L}\right)$

$$w(x,t) = \sum_{n \geq 0} a_n(t) y_n(x)$$

substituting in the DE $\longrightarrow a_n$ can be found

Final Answer $\rightarrow u(x,t) = \phi(x,t) + w(x,t)$

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10.i) c) DE $\to$ $U_t - U_{xx} = f$ $\quad x \in (0,l)$ $\quad t > 0$

$$U(x,0) = U_0$$

$$U(0,t) = U_x(l,t) = 0$$

$$U = X(x)\,T(t)$$

solutions coll $\to$ $X(x) = \sin\left(\frac{n\pi x}{l}\right)$

$$\Rightarrow U(x,t) = \sum_{n\geq 1} T_n(t) \sin\left(\frac{n\pi x}{l}\right)$$

substitute in DE $\to$

$$f(x,t) = U_t - U_{xx} = \sum_{n\geq 1} \left\{ T_n'(t) + \left(\frac{n\pi}{l}\right)^2 T_n(t) \right\} \sin\left(\frac{n\pi x}{l}\right)$$

Now, let $f(x,t) = \sum_{n\geq 1} b_n(t) \sin\left(\frac{n\pi x}{l}\right)$ $\to$ How? Where are the cos-terms?

let $\underline{U(x,0) = \sum_{n\geq 1} U_n \sin\left(\frac{n\pi x}{l}\right)}$

$$b_n(t) = \frac{2}{l}\int_0^l f(x,t) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$U_n = ?$$

now, $b_n(t) = T_n'(t) + \left(\frac{n\pi}{l}\right)^2 T_n(t)$

$$T_n(0) = U_n$$

$$\Rightarrow T_n(t) = U_n e^{-(n\pi/l)^2 t} + \int_0^t e^{-(n\pi/l)^2 (t-s)} b_n(s)\, ds$$

$$\Rightarrow U(x,t) = \sum_{n\geq 1} T_n(t) \sin\left(\frac{n\pi x}{l}\right)$$

$\downarrow$

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Tut 6

Q.1) $\hat{F}$ What is Fourier transform of a function u(x) ?

$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_0^1 x^2 e^{-wx} \, dx$$ — limits ?

$$= \frac{(iw^2 + 2w - 2i) e^{-iw} + 2i}{w^3}$$