Signal Processing - 1 by One

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Outline

- Digital-Analog-Digital
- Fourier Analysis, Series and Transform
- Previous Weeks: DTFT, DFT, FFT, Circular Convolutions
- Previous Class: Practical Example of 4G
- Today: Filter Design, Z-Transform

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Filter Design

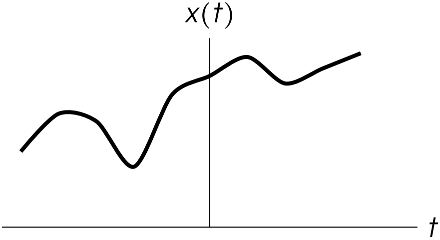
Question: A audio waveform $x(t)$ is sampled at 16kHz to obtain the discrete values $x[n], n \in \mathbb{Z}^+$. You have a *woofer* system operating at the audio rate of 16kHz, admitting samples intended for low frequences below f_l kHz.

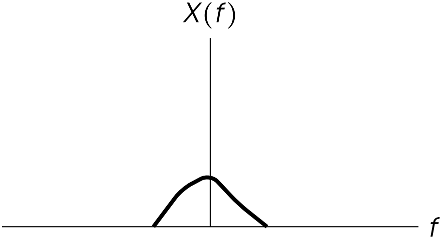
(a) What will be the *ideal* filter if $f_l = 4$kHz.

(b) If our computations only permit a filter length of L, design a filter $h_0, h_1, \cdots h_{L-1}$ when $L = 13$ **(FIR filter design)**.

(c) If the audio rate of the woofer is 32kHz (standard spec), and other parameters in question remain the same, how can your design accommodate the system demands?

DTFT and Low Pass Filtering





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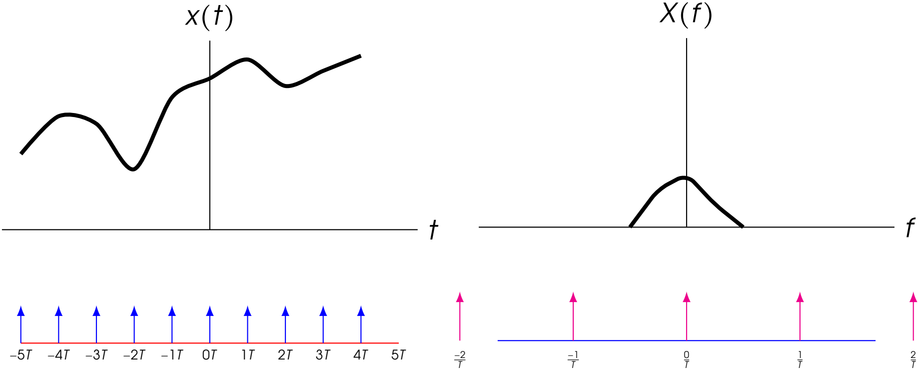

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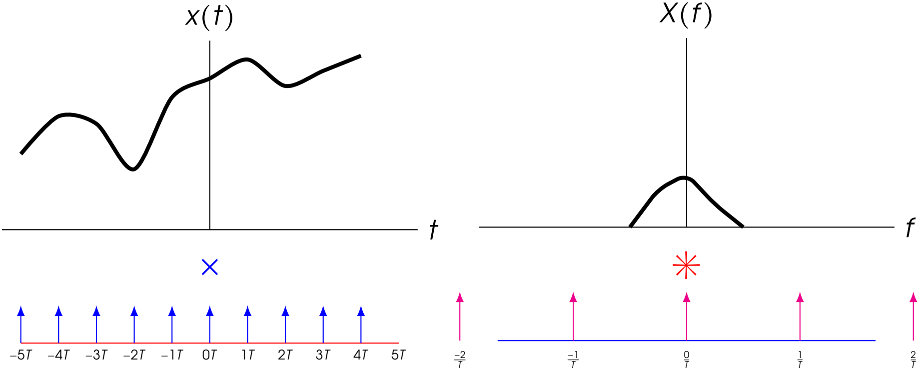

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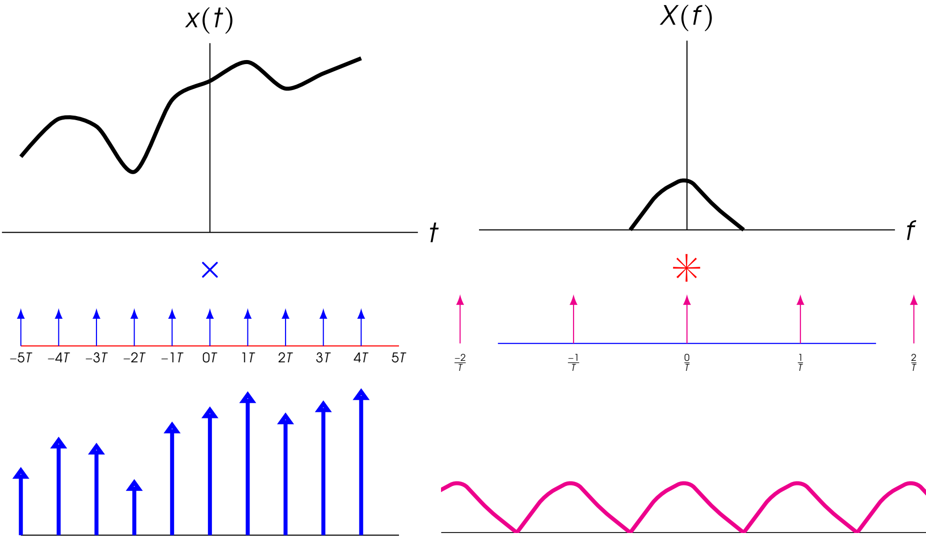

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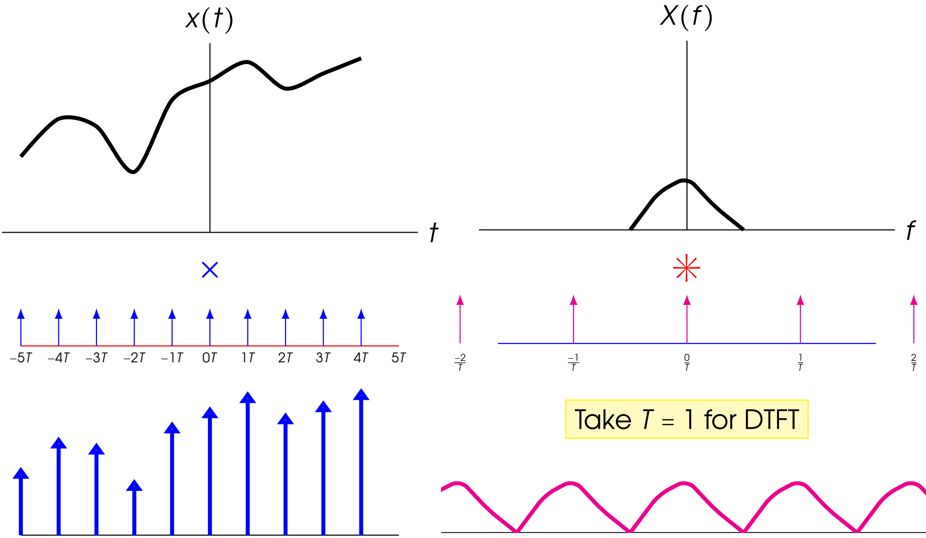

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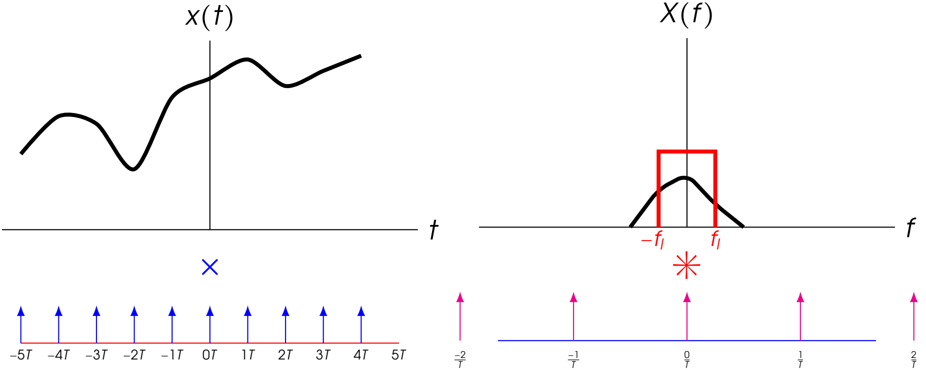
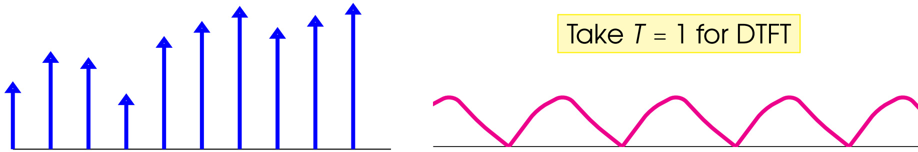


Take $T = 1$ for DTFT



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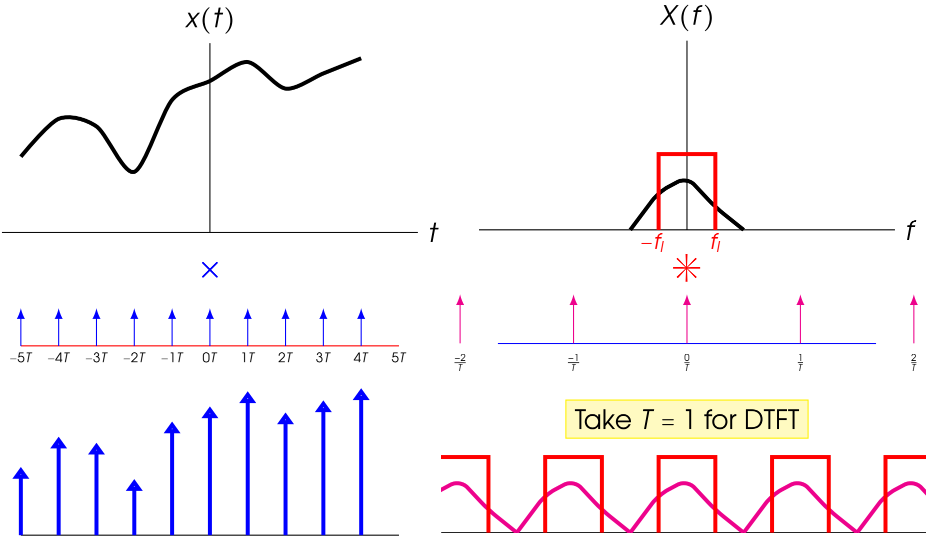

Ideal/Desired Digital Filter

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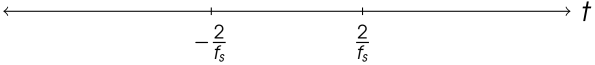
Desired Filter Response $h_d[n]$:



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$$x[n] \longrightarrow \boxed{\text{Interpolate (DAC)}} \longrightarrow x_I(t)$$

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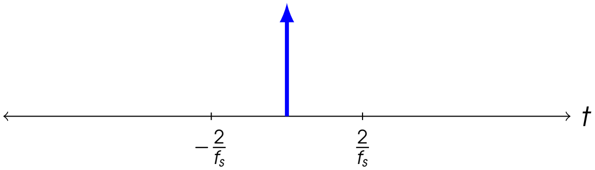

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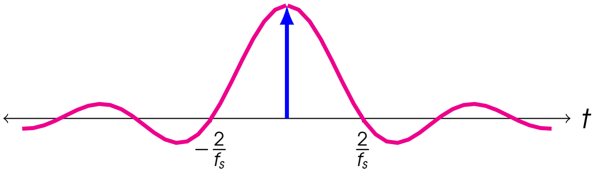

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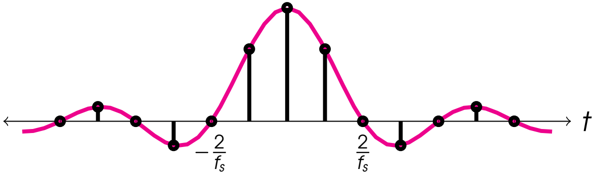

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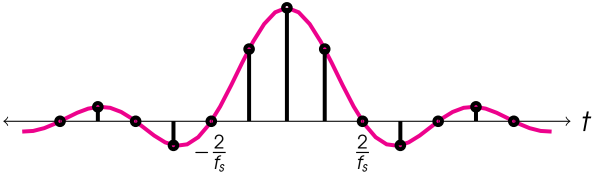

Audio Rate 16kHz, Lowpass Cut-off f_l = 4kHz, Ideal Filter

FIR: Window Method

Among all $h_0, \cdots, h_{L-1}$, find

$$\min \int_{-\frac{1}{2}}^{\frac{1}{2}} |\hat{H}_d(f) - \hat{H}(f)|^2 df.$$

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$$= \sum_{n=0}^{L-1} |h_d[n] - h[n]|^2 + \sum_{n<0} |h_d[n] - h[n]|^2$$

$$+ \sum_{n>L} |h_d[n] - h[n]|^2$$

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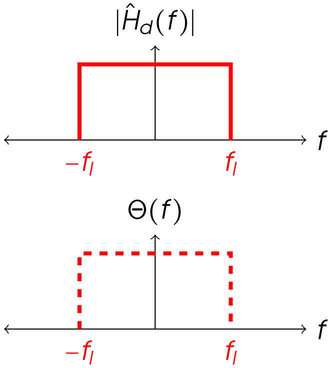
&\geq \sum_{n \notin \{0, \cdots, L-1\}} |h_d[n]|^2.
\end{aligned}$$

Take $h[n] = h_d[n]$ for $0 \leq n \leq L-1$ to get RHS! (Window)

Causality



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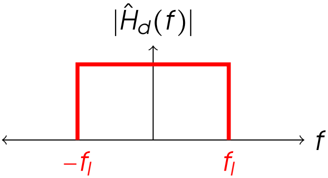

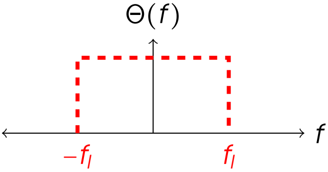

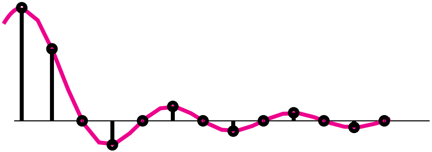

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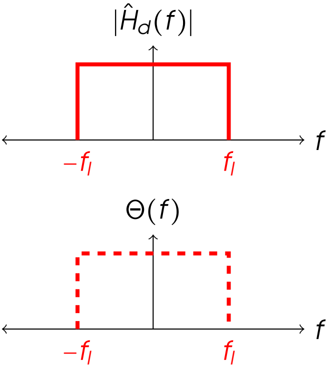

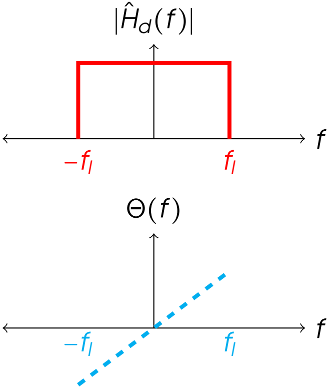

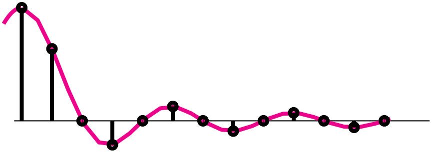

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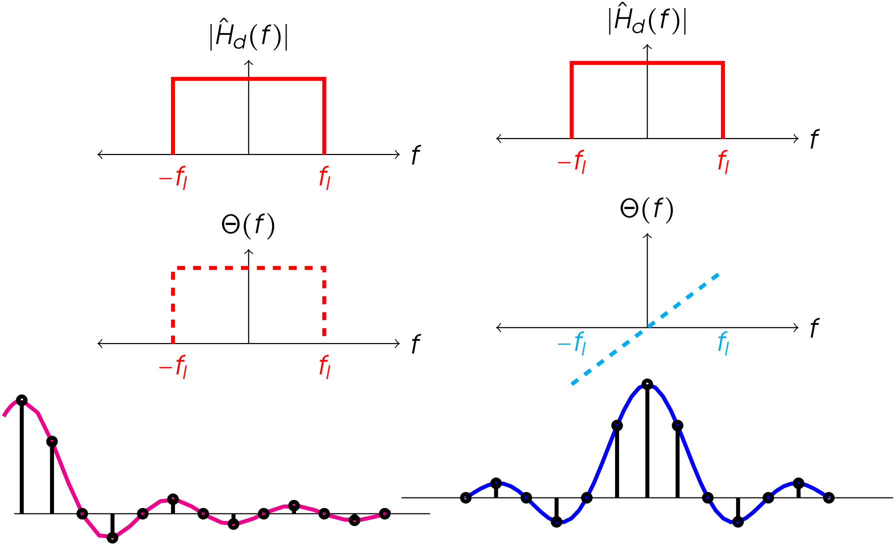

Z-Transform

$$H(z) = \sum_{n \in \mathbb{Z}} h[n] z^{-n}. \tag{1}$$

Polynomial in z, which takes complex values.

Example: $(h_0, h_1, h_2) = (2, 7, 1 - j1)$:

$$H(z) = 2 + 7z^{-1} + (1 - j1)z^{-2}.$$