Signal Processing - 1 by One

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Outline

- So Far: Sampling, Fourier Analysis
- Previous Week: DTFT, DFT, FFT and Circular Convolution
- Previous Class: Digital Modelling
- Today: Digital Receivers

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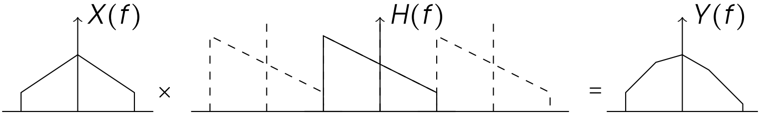
A Discrete World



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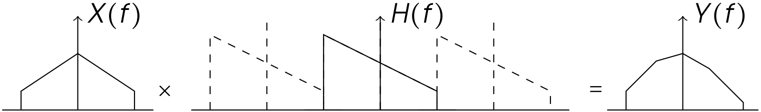
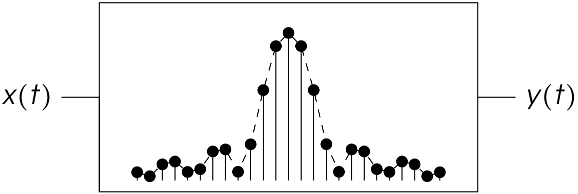
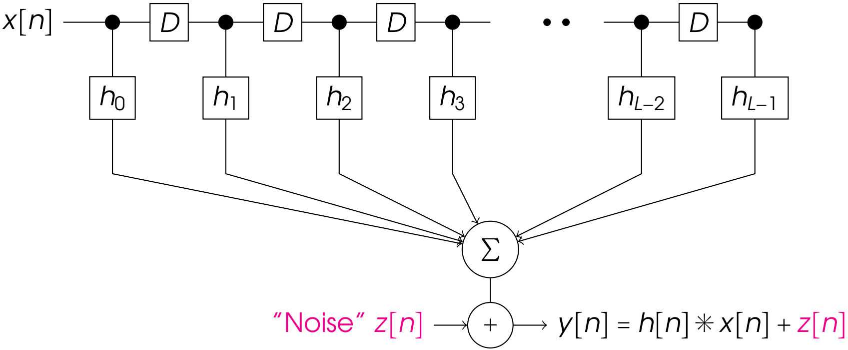




$$y(t) = h(t) * x(t) = \sum_{k \in \mathbb{Z}} \frac{1}{\beta} h\left(\frac{k}{\beta}\right) x(t - \frac{k}{\beta}).$$

Tapped Delay Line



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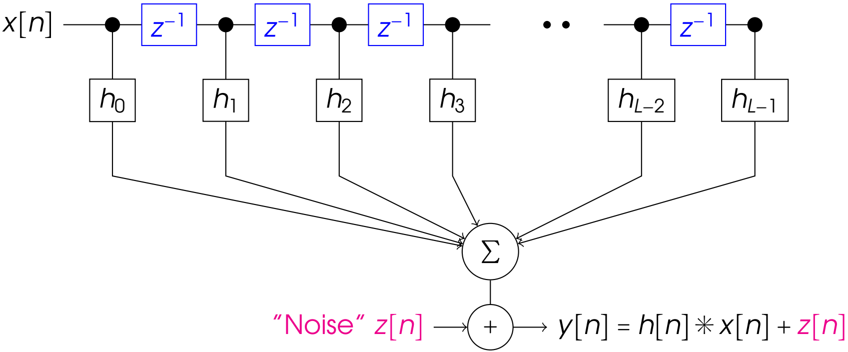

Linear to Circular

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For the green portion of samples

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Similarly for $\bar{h} = h_0, \cdots, h_{L-1}$, prefix $x_{N-L+1}, \cdots, x_{N-1}$ in each **frame**

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Prefix: a trick to convert **linear** convolution to **circular** convolution

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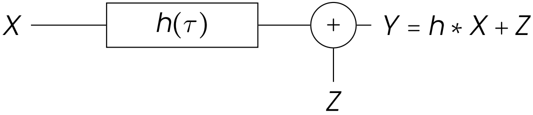
Each $D[k]$ can be recovered at the output if $H[k] \neq 0$.

Wideband and OFDM



OFDM converts this to N parallel **sub-carrier** channels.