

# Signal Processing - 1 by One

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- So Far: Sampling, Fourier Analysis
- Previous Week: DTFT, DFT and Circular Convolution
- Previous Class: Discrete Fourier Transform
- Today: Fast Fourier Transform (FFT)



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## 8-point FFT

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha^0 & \alpha^2 & \alpha^4 & \alpha^6 \\ 1 & \alpha^3 & \alpha^6 & \alpha^1 & \alpha^4 & \alpha^7 & \alpha^2 & \alpha^5 \\ 1 & \alpha^4 & \alpha^0 & \alpha^4 & \alpha^0 & \alpha^4 & \alpha^0 & \alpha^4 \\ 1 & \alpha^5 & \alpha^2 & \alpha^7 & \alpha^4 & \alpha^1 & \alpha^6 & \alpha^3 \\ 1 & \alpha^6 & \alpha^4 & \alpha^2 & \alpha^0 & \alpha^6 & \alpha^4 & \alpha^2 \\ 1 & \alpha^7 & \alpha^6 & \alpha^5 & \alpha^4 & \alpha^3 & \alpha^2 & \alpha^1 \end{bmatrix}$$

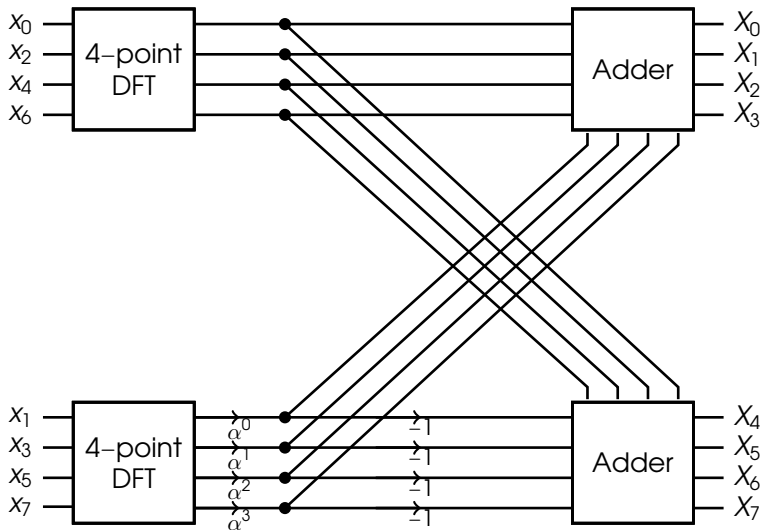


# Break it Up

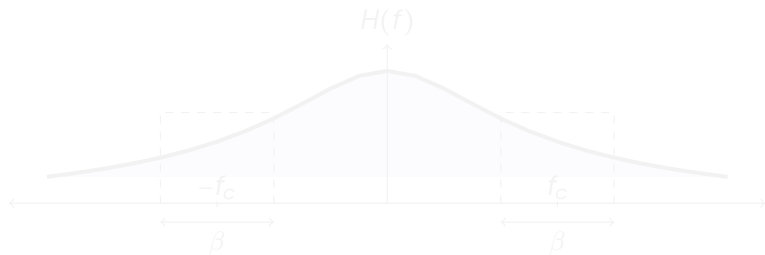
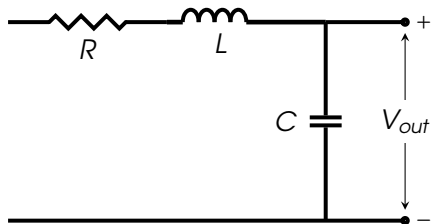
$$FX = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \alpha^4 & \alpha^2 & \alpha^6 \\ 1 & \alpha^0 & \alpha^4 & \alpha^4 \\ 1 & \alpha^4 & \alpha^6 & \alpha^2 \\ 1 & \alpha^0 & \alpha^0 & \alpha^0 \\ 1 & \alpha^4 & \alpha^2 & \alpha^6 \\ 1 & \alpha^0 & \alpha^4 & \alpha^4 \\ 1 & \alpha^4 & \alpha^6 & \alpha^2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_4 \\ x_2 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha^1 & \alpha^5 & \alpha^3 & \alpha^7 \\ \alpha^2 & \alpha^2 & \alpha^6 & \alpha^6 \\ \alpha^3 & \alpha^7 & \alpha^1 & \alpha^5 \\ \alpha^4 & \alpha^4 & \alpha^4 & \alpha^4 \\ \alpha^5 & \alpha^1 & \alpha^7 & \alpha^3 \\ \alpha^6 & \alpha^6 & \alpha^2 & \alpha^2 \\ \alpha^7 & \alpha^3 & \alpha^5 & \alpha^1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_5 \\ x_3 \\ x_7 \end{bmatrix}$$



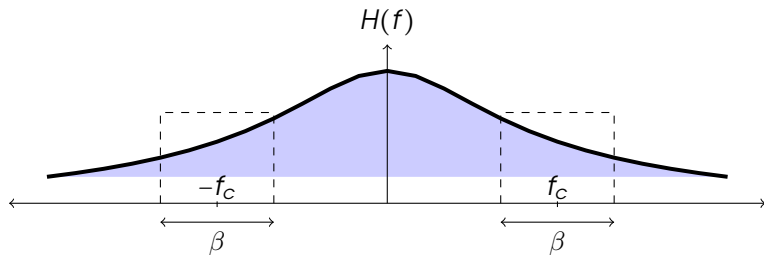
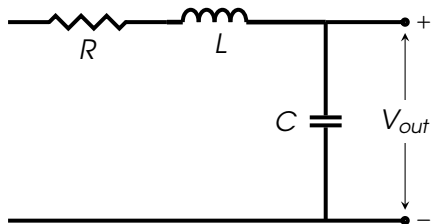
# Butterflies in the Belly



# Transmission Media and Bandwidth



# Transmission Media and Bandwidth





# Wireless Communication

Spectrum is a costly resource, centrally allocated usually.

Application	Bandwidth
<b>AM Radio</b>	10kHz
<b>2G</b>	200kHz - 1MHz
<b>3G</b>	5MHz
<b>4G</b>	10 – 20MHz
<b>5G</b>	≈ 100MHz

Data (video/audio/file) should be sent *within* the bandwidth.



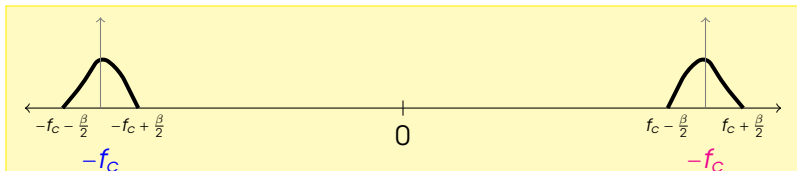
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Data (video/audio/file) should be sent *within* the bandwidth.

$$X(f) = 0 \text{ if } \{|f| \leq f_c - \frac{\beta}{2}\} \text{ OR } \{|f| \geq f_c + \frac{\beta}{2}\}.$$



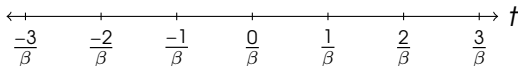
# Digital-to-Analog



# Digital-to-Analog



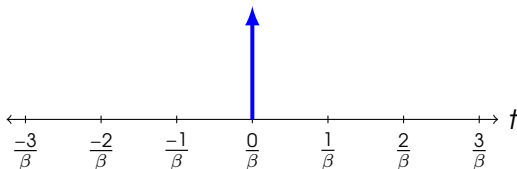
Discrete-time Input and Response:



# Digital-to-Analog



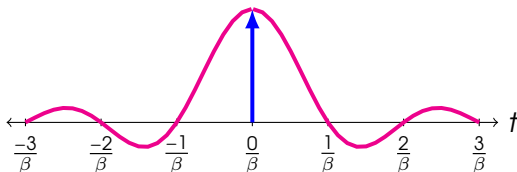
Discrete-time Input and Response:



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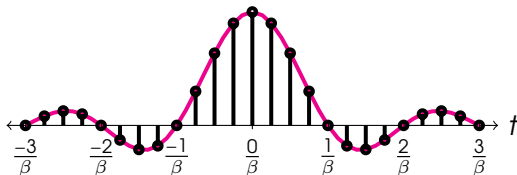
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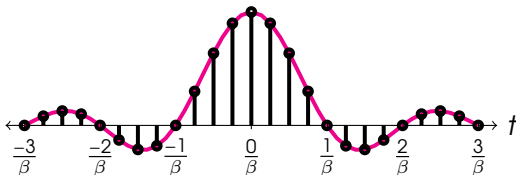
Discrete-time Input and Response:



# Digital-to-Analog



Discrete-time Input and Response:

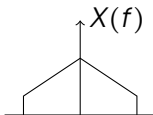
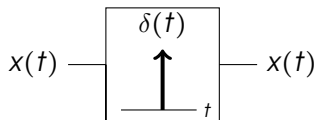


GNURADIO: Generate a baseband signal with bandwidth 10kHz

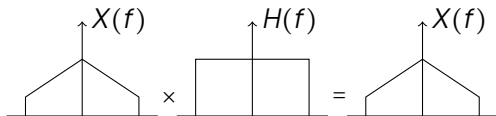
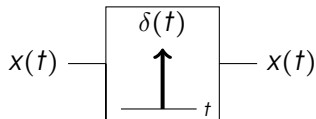




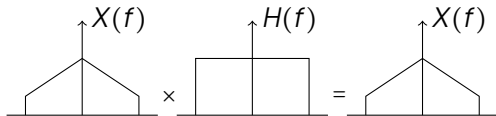
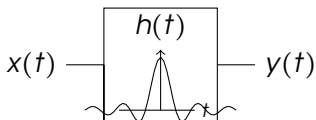
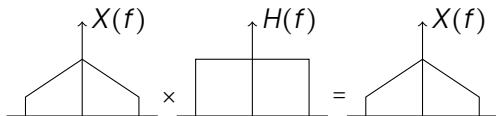
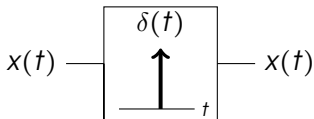
# Reigning Impulses



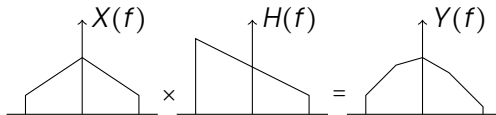
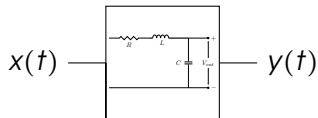
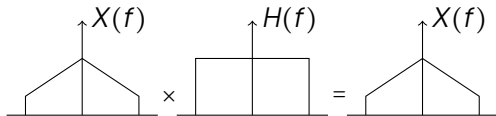
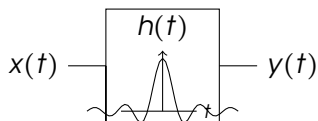
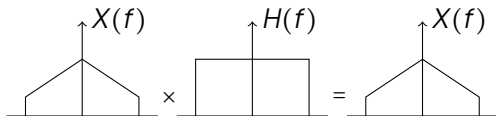
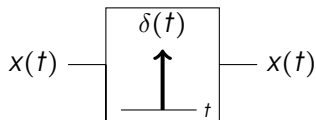
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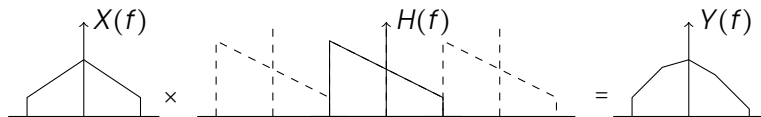
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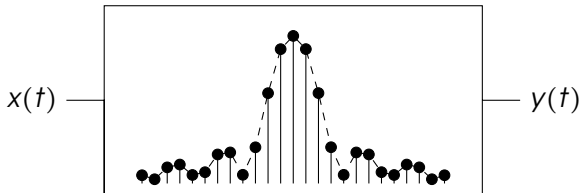
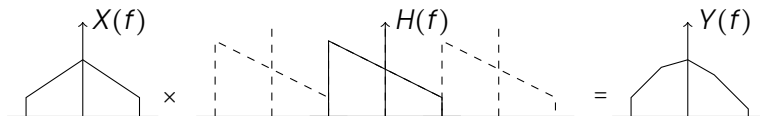
# Reigning Impulses



# A Discrete World



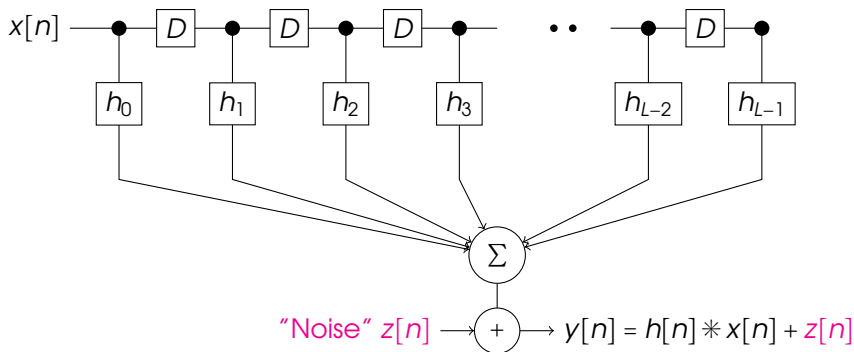
# A Discrete World



$$y(t) = h(t) * x(t) = \sum_{k \in \mathbb{Z}} \frac{1}{\beta} h\left(\frac{k}{\beta}\right) x\left(t - \frac{k}{\beta}\right).$$



# Tappped Delay Line



# TappeD Delay Line

