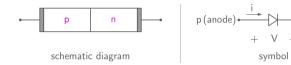
SEMICONDUCTOR DEVICES

p-n Junctions: Part 1



M.B.Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering Indian Institute of Technology Bombay





* A p-n junction is useful as a stand-alone device (the diode).



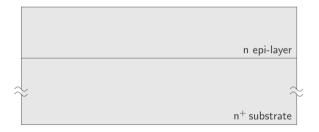
- * A p-n junction is useful as a stand-alone device (the diode).
- * It is also an integral part of devices such as transistors, IGBTs, thyristors, etc.



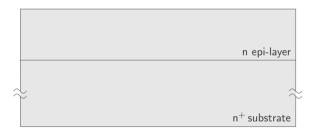
- * A p-n junction is useful as a stand-alone device (the diode).
- * It is also an integral part of devices such as transistors, IGBTs, thyristors, etc.
- * In integrated circuits, pn junctions are used to provide isolation between devices.



- * A p-n junction is useful as a stand-alone device (the diode).
- * It is also an integral part of devices such as transistors, IGBTs, thyristors, etc.
- * In integrated circuits, pn junctions are used to provide isolation between devices.
- We will focus on semiconductor p-n junctions first and look at metal-semiconductor junctions later.



Start with n^+ substrate, with n epitaxial layer grown on top.



 $\mathsf{Deposit}\ \mathsf{SiO}_2.$



 $\mathsf{Deposit}\ \mathsf{SiO}_2.$



Apply photoresist.



Apply photoresist.



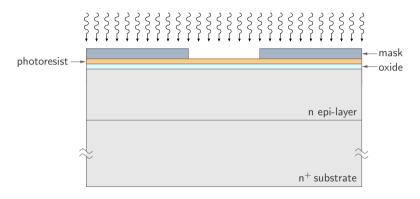
Place mask.



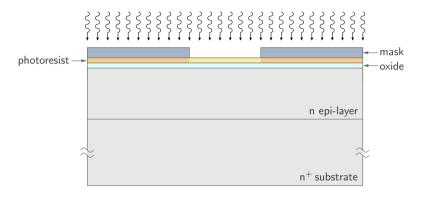
Place mask.



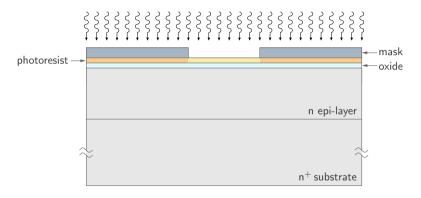
Expose to UV light.



Expose to UV light.



Expose to UV light.



Remove mask.



Remove mask.



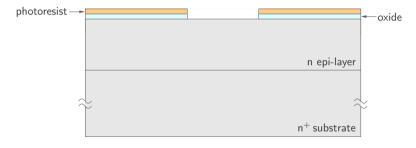
Develop photoresist.



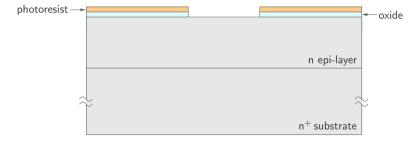
Develop photoresist.



Etch oxide (in HF).



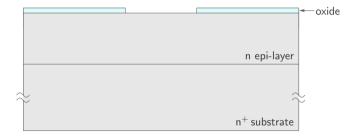
Etch oxide (in HF).



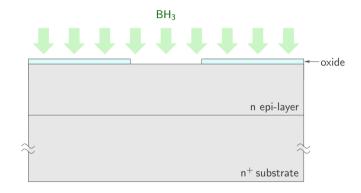
Remove photoresist.



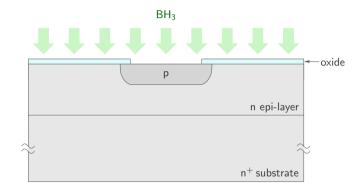
Remove photoresist.



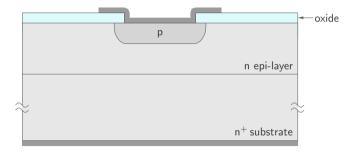
Perform diffusion of Boron.



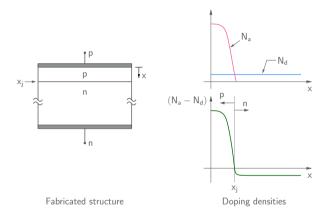
Perform diffusion of Boron.

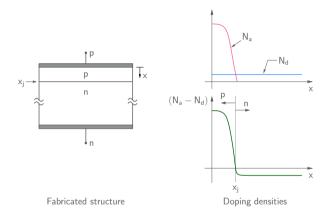


Perform diffusion of Boron.

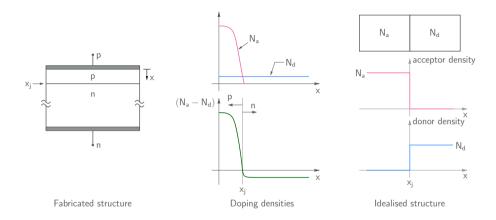


Add metal contacts (a few steps).

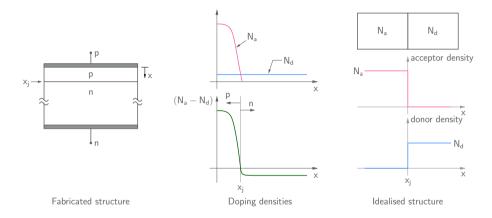




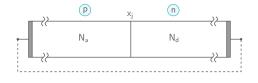
* For our analysis, we will consider a simplified structure with *p*-type doping on one side and *n*-type on the other.

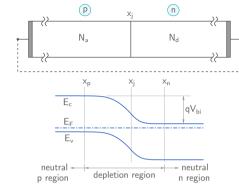


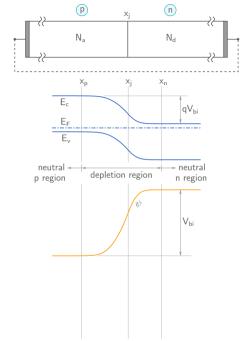
* For our analysis, we will consider a simplified structure with *p*-type doping on one side and *n*-type on the other.

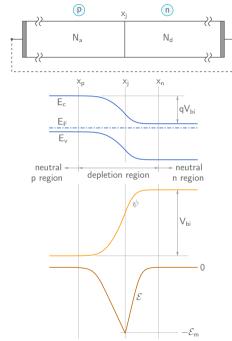


- * For our analysis, we will consider a simplified structure with *p*-type doping on one side and *n*-type on the other.
- * We will assume the doping densities to change abruptly at the junction \rightarrow "abrupt" pn junction.

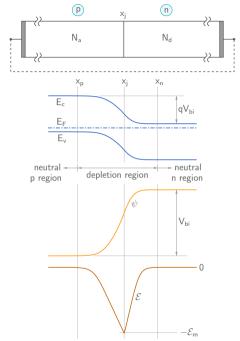




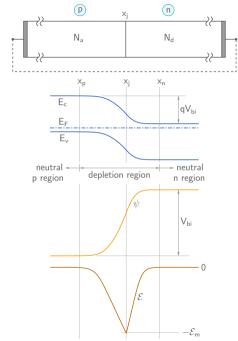




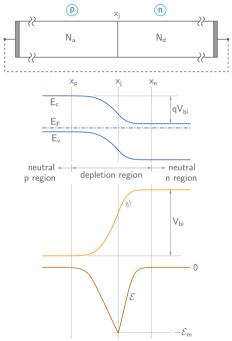
* There is a "depletion region" in which the potential ψ varies.



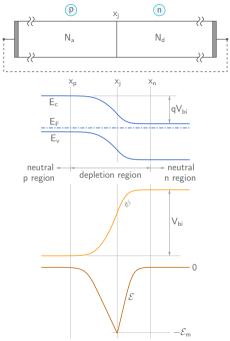
- * There is a "depletion region" in which the potential ψ varies.
- * Away from the depletion region, ψ is constant, and the electric field is zero.



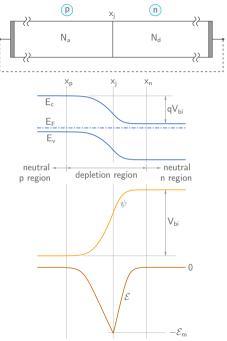
- * There is a "depletion region" in which the potential ψ varies.
- * Away from the depletion region, ψ is constant, and the electric field is zero.
- * There is a "built-in" voltage drop between the p and n sides, denoted by V_{bi} .

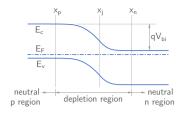


- * There is a "depletion region" in which the potential ψ varies.
- * Away from the depletion region, ψ is constant, and the electric field is zero.
- * There is a "built-in" voltage drop between the p and n sides, denoted by $V_{\rm bi}$.
- * Note that $\psi = -\frac{1}{q} \frac{dE_c}{dx}$, and $\mathcal{E} = -\frac{d\psi}{dx}$.



- * There is a "depletion region" in which the potential ψ varies.
- * Away from the depletion region, ψ is constant, and the electric field is zero.
- * There is a "built-in" voltage drop between the p and n sides, denoted by $V_{\rm bi}$.
- * Note that $\psi = -\frac{1}{a} \frac{dE_c}{dx}$, and $\mathcal{E} = -\frac{d\psi}{dx}$.
- * Let us check if this picture is consistent with Poisson's equation.



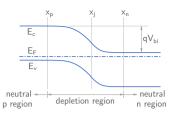


Charge density:

$$p(x) = N_v \exp - \left(\frac{E_F - E_v(x)}{kT}\right),$$

$$n(x) = N_c \exp - \left(\frac{E_c(x) - E_F}{kT}\right).$$

$$p(x) = N_c \exp -\left(\frac{E_c(x) - E_F}{kT}\right)$$



Charge density:

Charge density:

$$p(x) = N_v \exp - \left(\frac{E_F - E_v(x)}{kT}\right),$$

$$n(x) = N_c \exp{-\left(\frac{E_c(x) - E_F}{kT}\right)}.$$

In the neutral *p*-region (x <
$$x_p$$
), $p = N_a^- \approx N_a$.

In the neutral *n*-region $(x > x_n)$, $n = N_d^+ \approx N_d$.

$$E_c$$
 E_F
 V_p
 V_j
 V_n
 V_h
 V_h

 $N_a = N_v \exp - \left(\frac{E_F - E_v(x_p)}{kT}\right),$

 $N_d = N_c \exp - \left(\frac{E_c(x_n) - E_F}{kT}\right).$

Charge density:
$$(E_E - E_V(x))$$

 $p(x) = N_v \exp -\left(\frac{E_F - E_v(x)}{kT}\right),$

$$\frac{E_{\nu}(x)}{T}$$
,

$$= N_c \exp - \left(\frac{E_c(x) - E_F}{kT}\right).$$
 p regi

In the neutral p-region $(x < x_p)$, $p = N_a^- \approx N_a$.

ne neutral *p*-region
$$(x < x_p)$$
, $p = N_a^- \approx N_a$.

In the neutral *n*-region $(x > x_n)$, $n = N_d^+ \approx N_d$.

$$n(x) = N_c \exp{-\left(\frac{E_c(x) - E_F}{kT}\right)}.$$
neutral pregion depletion region n region n region

In the neutral p-region $(x < x_p), p = N_a^- \approx N_a$.

E.

 E_F

Ev

Charge density:

 $\rightarrow \frac{p(x)}{N} = \exp -\left(\frac{E_{\nu}(x_p) - E_{\nu}(x)}{\nu T}\right)$

 $\frac{n(x)}{N_d} = \exp -\left(\frac{E_c(x) - E_c(x_n)}{\nu \tau}\right).$

$$\begin{array}{c|c} E_c & qV_{bi} \\ \hline E_F & & \\ \hline E_v & & \\ \hline \\ neutral & \\ p \ region & depletion \ region & \\ \end{array}$$

$$n(x) = N_c \exp\left(\frac{E_c(x) - E_F}{kT}\right).$$
In the neutral *p*-region $(x < x_p)$, $p = N_a^- \approx N_a$.
In the neutral *n*-region $(x > x_n)$, $n = N_d^+ \approx N_d$.

$$N_a = N_v \exp\left(\frac{E_F - E_V(x_p)}{kT}\right).$$

$$N_d = N_c \exp\left(\frac{E_c(x_n) - E_F}{kT}\right).$$

In the neutral p-region $(x < x_p)$, $p = N_a^- \approx N_a$. In the neutral *n*-region $(x > x_n)$, $n = N_d^+ \approx N_d$.

$$n(x) = N_c \exp\left(\frac{kT}{kT}\right),$$
neutral pregion $n = N_c \exp\left(\frac{E_c(x) - E_F}{kT}\right)$
In the neutral pregion $n = N_d = N_d$
In the neutral pregion $n = N_d = N_d$

$$n = N_v \exp\left(\frac{E_F - E_v(x_p)}{kT}\right),$$

 $p(x) = N_v \exp -\left(\frac{E_F - E_v(x)}{vT}\right),$

Charge density:

$$p(x) = N_v \exp \left(-\frac{E_F - E_v(x)}{kT}\right),$$

$$n(x) = N_c \exp \left(\frac{kT}{kT}\right).$$
neutral depletion region depletion region

E.

EF

Ev

In the neutral p-region $(x < x_p)$, $p = N_a^- \approx N_a$.

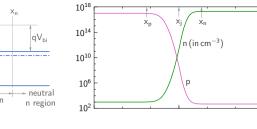
In the neutral *n*-region
$$(x > x_n)$$
, $n = N_d^+ \approx N_d$

In the neutral *n*-region
$$(x > x_n)$$
, $n = N_d^+ \approx N_d$.
 $(E_F - E_V(x_n))$

$$N_{a} = N_{v} \exp \left(-\frac{E_{F} - E_{v}(x_{p})}{kT}\right),$$

$$N_{d} = N_{c} \exp \left(-\frac{E_{c}(x_{n}) - E_{F}}{kT}\right).$$

$$\rightarrow \frac{p(x)}{N_{a}} = \exp \left(-\frac{E_{v}(x_{p}) - E_{v}(x)}{kT}\right).$$



Charge density:

$$p(x) = N_v \exp \left(-\left(\frac{E_F - E_v(x)}{kT}\right)\right),$$

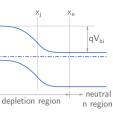
 $n(x) = N_c \exp - \left(\frac{E_c(x) - E_F}{kT}\right).$

In the neutral *p*-region
$$(x < x_p)$$
, $p = N_a^- \approx N_a$.

In the neutral *n*-region $(x > x_n)$, $n = N_d^+ \approx N_d$.

$$N_{a} = N_{v} \exp - \left(\frac{E_{F} - E_{v}(x_{p})}{kT}\right),$$

$$N_{d} = N_{c} \exp - \left(\frac{E_{c}(x_{n}) - E_{F}}{kT}\right).$$



E.

E.,

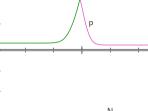
neutral

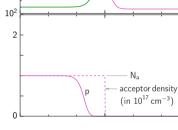
p region

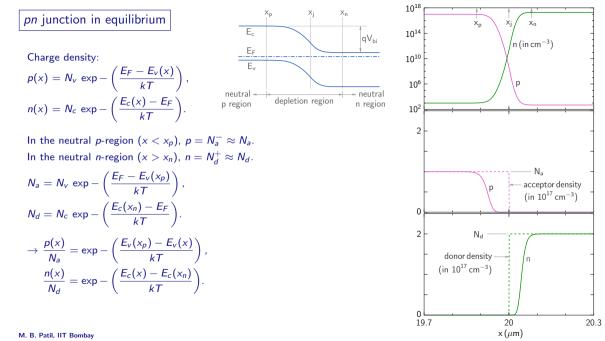


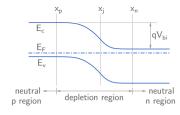
1014









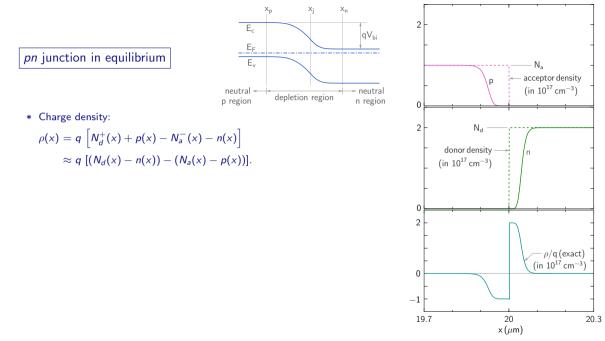


* Charge density:

$$\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$$

$$\approx q [(N_d(x) - n(x)) - (N_a(x) - p(x))].$$

- Na acceptor density $(in 10^{17} cm^{-3})$ N_d donor density $(in 10^{17} cm^{-3})$

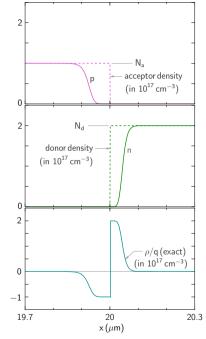


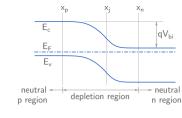
E qV_{bi} pn junction in equilibrium E_v neutral -- neutral depletion region p region n region * Charge density: $\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$

$$\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$$

$$\approx q \left[(N_d(x) - n(x)) - (N_a(x) - p(x)) \right].$$

* $\rho = 0$ in the neutral regions.

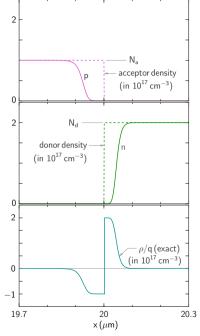


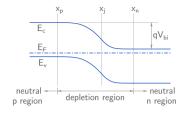


$$\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$$

$$\approx q [(N_d(x) - n(x)) - (N_a(x) - p(x))].$$

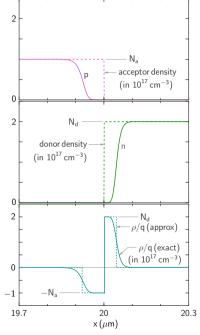
- * $\rho = 0$ in the neutral regions.
- * Within the depletion region, both n and p are small, i.e., this region is depleted of carriers \rightarrow "depletion region".

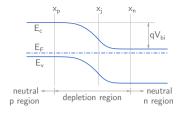




$$\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$$

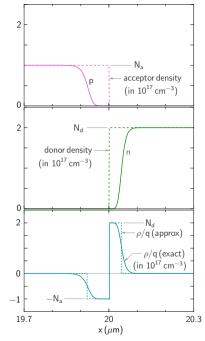
- $\approx q \left[\left(N_d(x) n(x) \right) \left(N_a(x) p(x) \right) \right].$
- * $\rho = 0$ in the neutral regions.
- Within the depletion region, both n and p are small, i.e., this region is depleted of carriers → "depletion region".

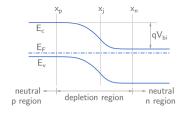




$$\rho(x) = q \left[N_d^+(x) + p(x) - N_a^-(x) - n(x) \right]$$

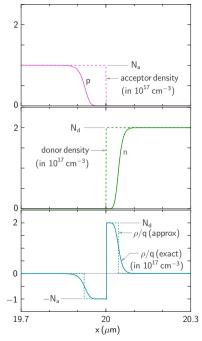
- $\approx q [(N_d(x) n(x)) (N_a(x) p(x))].$
- * ρ = 0 in the neutral regions.
 * Within the depletion region, both n and p are small, i.e., this region is depleted of carriers → "depletion region".
- * To proceed further analytically, we make the "depletion approximation," i.e., we assume that the transistions between the neutral regions and the depletion region are abrupt.

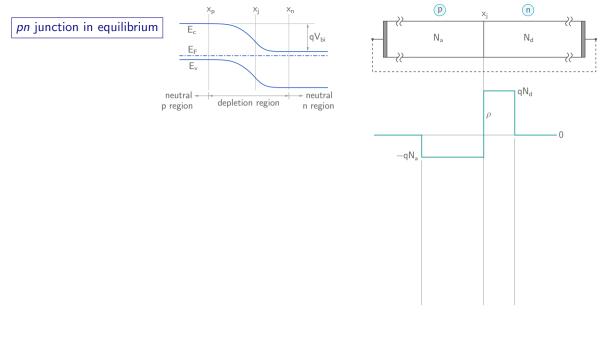


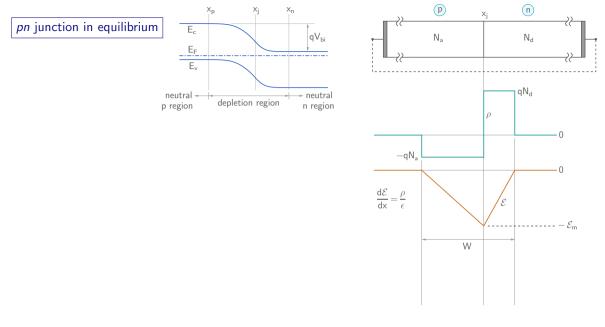


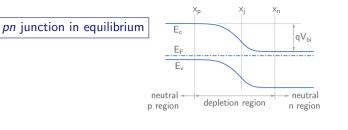
$$\rho(x) = q \left[N_d^{+}(x) + p(x) - N_a^{-}(x) - n(x) \right]$$

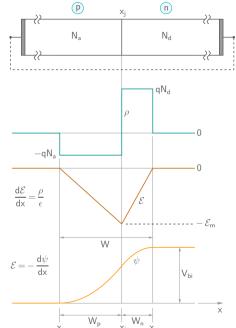
- $\approx q \left[(N_d(x) n(x)) (N_a(x) p(x)) \right].$ * $\rho = 0$ in the neutral regions.
- Within the depletion region, both n and p are small, i.e., this region is depleted of carriers \rightarrow "depletion region".
- * To proceed further analytically, we make the "depletion approximation," i.e., we assume that the transistions between the neutral regions and the depletion region are abrupt.
- Since the depletion region has non-zero charge density, it is also called "space charge region."

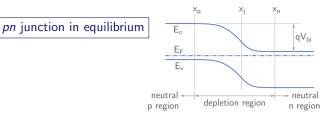




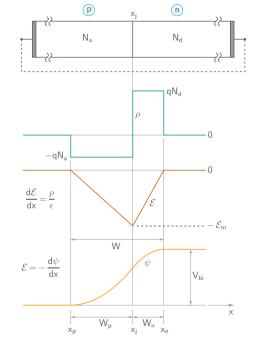


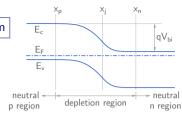






* Built-in voltage $V_{\rm bi}$:



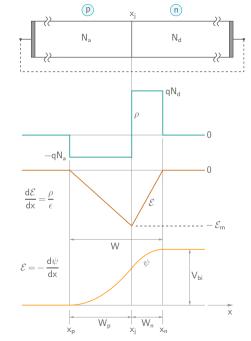


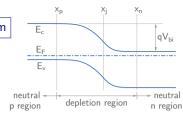
* Built-in voltage $V_{\rm bi}$:

$$p(x) = N_v \exp \left[-\frac{E_F - E_v(x)}{kT} \right], \quad n(x) = N_c \exp \left[-\frac{E_c(x) - E_F}{kT} \right].$$

$$\rightarrow \frac{p(x_n)}{p(x_p)} = \exp\left[-\frac{E_{\nu}(x_p) - E_{\nu}(x_n)}{kT}\right].$$

$$-E_{\nu}(x_n)$$



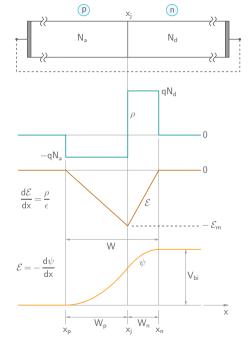


* Built-in voltage $V_{\rm bi}$:

$$p(x) = N_v \exp\left[-\frac{E_F - E_v(x)}{kT}\right], \quad n(x) = N_c \exp\left[-\frac{E_c(x) - E_F}{kT}\right].$$

$$\rightarrow \frac{\rho(x_n)}{\rho(x_p)} = \exp\left[-\frac{E_{\nu}(x_p) - E_{\nu}(x_n)}{kT}\right].$$

$$\rightarrow \frac{p(x_n)n(x_n)}{p(x_n)n(x_n)} = \frac{n_i^2}{N_a N_d} = \exp\left(-\frac{qV_{bi}}{kT}\right).$$



 qV_{bi}

n region

Built-in voltage $V_{\rm bi}$:

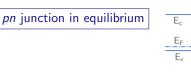
$$p(x) = N_v \exp\left[-\frac{E_F - E_v(x)}{kT}\right], \quad n(x) = N_c \exp\left[-\frac{E_c(x) - E_F}{kT}\right].$$

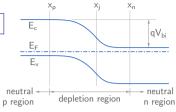
The built-in voltage $V_{\rm bi}$ is therefore given by

$$V_{\text{bi}} = \frac{kT}{q} \log \left(\frac{N_a N_d}{n_i^2} \right)$$

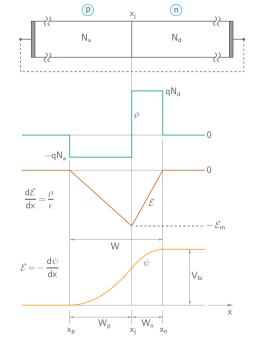
N_a Na qN_d $-qN_{a}$ $\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$ W $\mathcal{E} = -\frac{d\psi}{dx}$ V_{bi} W_n

(n)

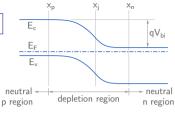




For a silicon pn junction with $N_a=5\times10^{17}\,\mathrm{cm}^{-3}$, $N_d=10^{17}\,\mathrm{cm}^{-3}$, compute V_{bi} at $T=300\,\mathrm{K}$. ($n_i=1.5\times10^{10}\,\mathrm{cm}^{-3}$ at $300\,\mathrm{K}$.)



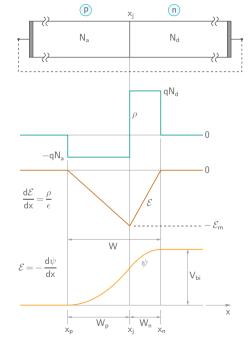


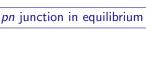


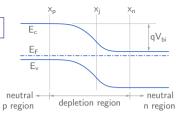
For a silicon pn junction with $N_a=5\times10^{17}\,\rm cm^{-3},~N_d=10^{17}\,cm^{-3},$ compute $V_{\rm bi}$ at $T=300\,\rm K.~(n_i=1.5\times10^{10}\,cm^{-3}$ at $300\,\rm K.)$

Solution:

$$V_{\rm bi} = \frac{kT}{q} \log \frac{N_a N_d}{n_i^2}$$





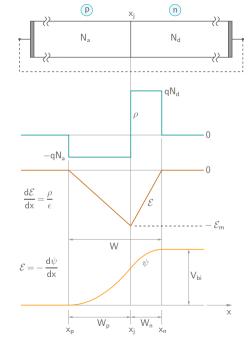


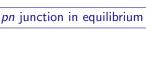
For a silicon pn junction with $N_a = 5 \times 10^{17} \, \mathrm{cm}^{-3}$, $N_d = 10^{17} \, \mathrm{cm}^{-3}$, compute V_{bi} at $T = 300 \, \mathrm{K}$. ($n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$ at $300 \, \mathrm{K}$.)

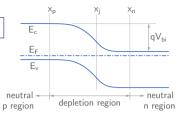
Solution:

$$V_{\text{bi}} = rac{kT}{q} \log rac{N_a N_d}{n_i^2}$$

= $(0.0259 \, \text{V}) \log rac{(5 imes 10^{17})(10^{17})}{(1.5 imes 10^{10})^2}$





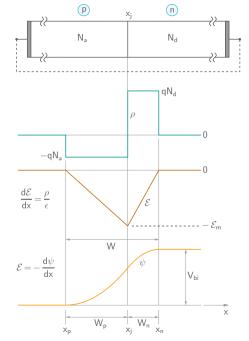


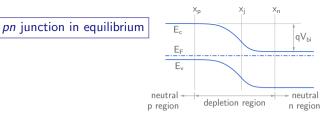
For a silicon pn junction with $N_a = 5 \times 10^{17} \, \mathrm{cm}^{-3}$, $N_d = 10^{17} \, \mathrm{cm}^{-3}$, compute V_{bi} at $T = 300 \, \mathrm{K}$. ($n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$ at $300 \, \mathrm{K}$.)

Solution:

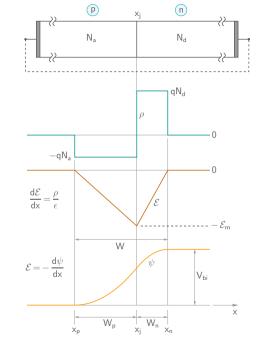
$$V_{\text{bi}} = \frac{kT}{q} \log \frac{N_a N_d}{n_i^2}$$

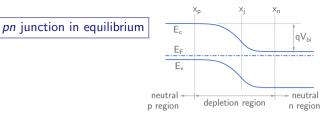
= $(0.0259 \,\text{V}) \log \frac{(5 \times 10^{17})(10^{17})}{(1.5 \times 10^{10})^2}$
= $0.86 \,\text{V}$





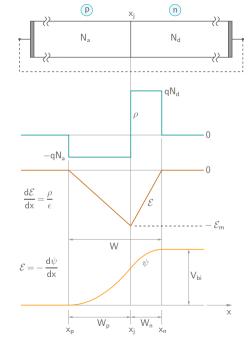
Electric field $\mathcal{E}(x)$:

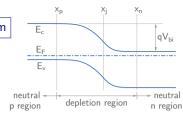




Electric field $\mathcal{E}(x)$:

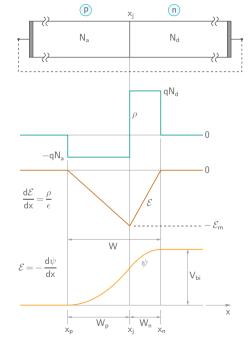
*
$$\mathcal{E} = \frac{d\psi}{dx} = 0$$
 in the neutral regions.

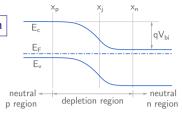




Electric field $\mathcal{E}(x)$:

- * $\mathcal{E} = \frac{d\psi}{dx} = 0$ in the neutral regions.
- * In the depletion region, $\int_{x_0}^{x_n} d\mathcal{E} = \int_{x_0}^{x_n} \frac{\rho}{\epsilon} dx$.



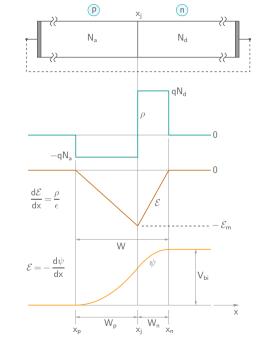


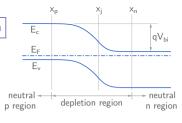
Electric field $\mathcal{E}(x)$:

- * $\mathcal{E} = \frac{d\psi}{dx} = 0$ in the neutral regions.
- * In the depletion region, $\int_{x_0}^{x_n} d\mathcal{E} = \int_{x_0}^{x_n} \frac{\rho}{\epsilon} dx$.

Since
$$\mathcal{E}(x_p)=\mathcal{E}(x_n)=0$$
, we must have $\int_{x_p}^{x_n} \frac{\rho}{\epsilon} dx=0$,

which means the area under the ρ versus x curve must be zero.





Electric field $\mathcal{E}(x)$:

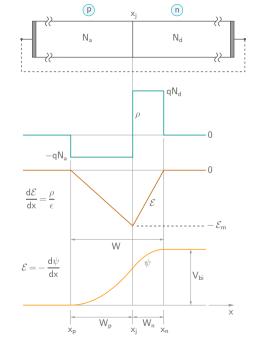
- * $\mathcal{E} = \frac{d\psi}{dx} = 0$ in the neutral regions.
- * In the depletion region, $\int_{x}^{x_n} d\mathcal{E} = \int_{x}^{x_n} \frac{\rho}{\epsilon} dx$.

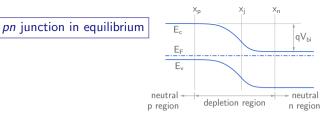
Since $\mathcal{E}(x_p) = \mathcal{E}(x_n) = 0$, we must have $\int_{x_n}^{x_n} \frac{\rho}{\epsilon} dx = 0$,

 $J_{x_p} \quad \epsilon$ which means the area under the ρ versus x curve must be zero.

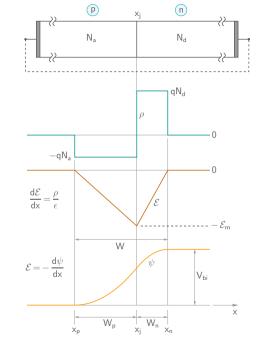
i.e., $N_a W_p = N_d W_n \rightarrow \frac{W_p}{W_p} = \frac{N_d}{N_a}$.

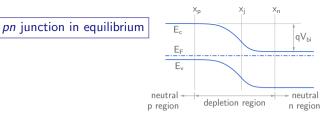
→ The depletion width is larger on the lightly doped side.





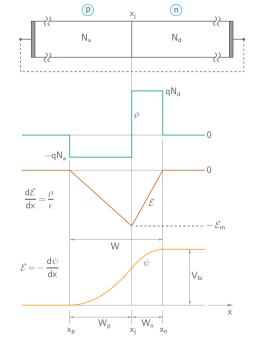
Electric field $\mathcal{E}(x)$:

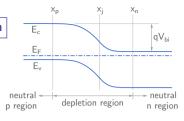




Electric field $\mathcal{E}(x)$:

* Since ρ is piecewise constant, ${\mathcal E}$ must be piecewise linear.





Electric field $\mathcal{E}(x)$:

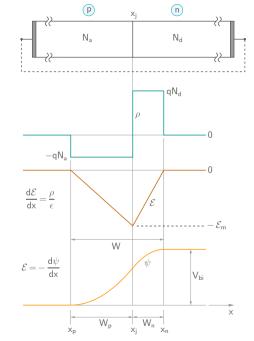
- * Since ρ is piecewise constant, \mathcal{E} must be piecewise linear.
- * The maximum value (magnitude) of \mathcal{E} occurs at $x = x_i$.

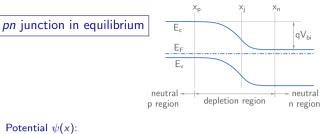
The maximum value (magnitude) of
$$\mathcal{E}$$
 occurs at x

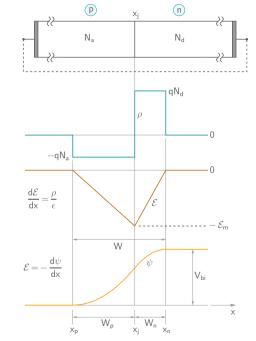
$$\int_{x_p}^{x_j} d\mathcal{E} = \frac{1}{\epsilon} \int_{x_p}^{x_j} \rho dx \to -\mathcal{E}_m - 0 = \frac{1}{\epsilon} (-qN_aW_p)$$

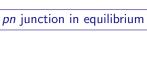
$$\to \mathcal{E}_m = \frac{qN_aW_p}{\epsilon} = \frac{qN_dW_n}{\epsilon} \quad \because N_aW_p = N_dW_n.$$

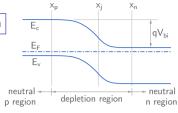
$$\rightarrow \mathcal{E}_m = \frac{q N_a W_p}{} = \frac{q N_d W_n}{} \quad \because N_a W_p = N_d W_n.$$



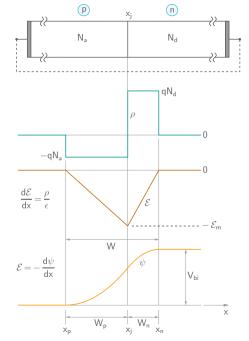


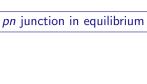


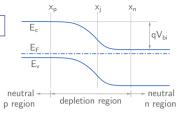




*
$$x_p < x < x_j$$
: $\frac{d\mathcal{E}}{dx} = -\frac{qN_a^-}{\epsilon} \approx -\frac{qN_a}{\epsilon} \to \mathcal{E}(x) = -\frac{qN_a}{\epsilon}x + k_1$.

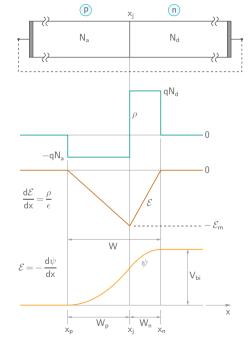


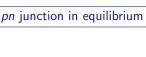


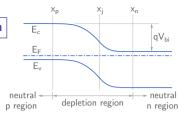


*
$$x_p < x < x_j$$
: $\frac{d\mathcal{E}}{dx} = -\frac{qN_a^-}{\epsilon} \approx -\frac{qN_a}{\epsilon} \to \mathcal{E}(x) = -\frac{qN_a}{\epsilon}x + k_1$.
Since $\mathcal{E} = 0$ at $x = x_p$, we get $\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x - x_p)$.

Since
$$\mathcal{E} = 0$$
 at $x = x_p$, we get $\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x - x_p)$.



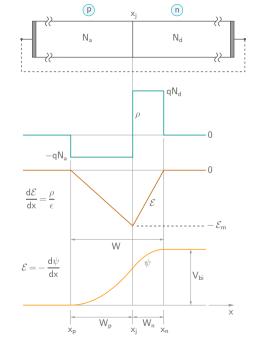


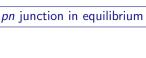


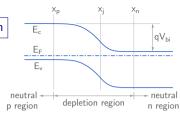
*
$$x_p < x < x_j$$
: $\frac{d\mathcal{E}}{dx} = -\frac{qN_a^-}{\epsilon} \approx -\frac{qN_a}{\epsilon} \to \mathcal{E}(x) = -\frac{qN_a}{\epsilon}x + k_1$.

Since $\mathcal{E} = 0$ at $x = x_p$, we get $\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x - x_p)$.

$$\phi = -\int \mathcal{E} dx = rac{qN_{ heta}}{\epsilon} \left[rac{x^2}{2} - x_p x
ight] + k_2.$$







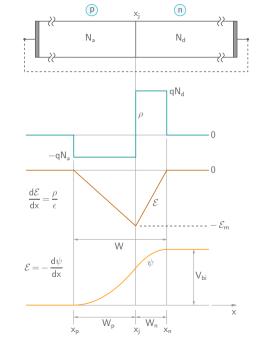
*
$$x_p < x < x_j$$
: $\frac{d\mathcal{E}}{dx} = -\frac{qN_a^-}{\epsilon} \approx -\frac{qN_a}{\epsilon} \to \mathcal{E}(x) = -\frac{qN_a}{\epsilon}x + k_1$.

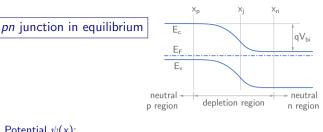
Since
$$\mathcal{E} = 0$$
 at $x = x_p$, we get $\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x - x_p)$.

$$\phi + \psi(x) = -\int \mathcal{E} dx = \frac{qN_a}{\epsilon} \left[\frac{x^2}{2} - x_p x \right] + k_2.$$

Taking $\psi(x_p) = 0$, we can find k_2 .

$$\to \psi(x) = \frac{qN_a}{2\pi}(x - x_p)^2.$$





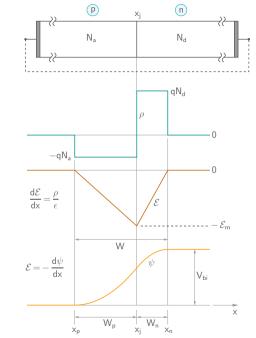
*
$$x_p < x < x_j$$
: $\frac{d\mathcal{E}}{dx} = -\frac{qN_a^-}{\epsilon} \approx -\frac{qN_a}{\epsilon} \to \mathcal{E}(x) = -\frac{qN_a}{\epsilon}x + k_1$.
Since $\mathcal{E} = 0$ at $x = x_p$, we get $\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x - x_p)$.

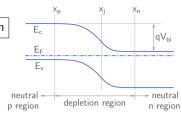
$$\rightarrow \psi(x) = -\int \mathcal{E} dx = \frac{qN_a}{\epsilon} \left[\frac{x^2}{2} - x_p x \right] + k_2.$$

Taking
$$\psi(x_p) = 0$$
, we can find k_2 .

 $\rightarrow \psi(x) = \frac{qN_a}{2}(x-x_p)^2$.

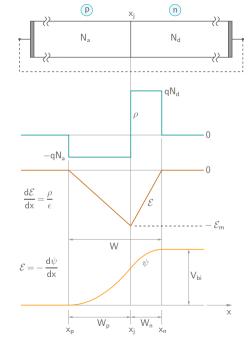
If
$$x_j$$
 is taken as 0, i.e., $x \leftarrow (x-x_j)$, we get
$$\psi(x) = \frac{qN_a}{2\epsilon}(x+W_p)^2.$$

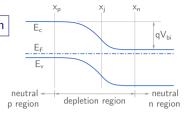




Potential $\psi(x)$:

* $x_j < x < x_n$:



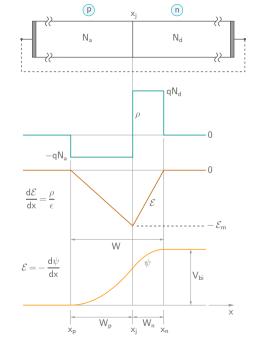


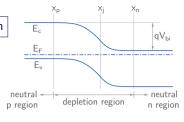
Potential $\psi(x)$:

*
$$x_i < x < x_n$$
:

For convenience, let us take $x_j = 0 \rightarrow x_p = -W_p$, $x_n = W_n$.

$$\frac{d\mathcal{E}}{dx} = \frac{qN_d^+}{\epsilon} \approx \frac{qN_d}{\epsilon} \to \mathcal{E}(x) = \frac{qN_d}{\epsilon}x + k_3.$$





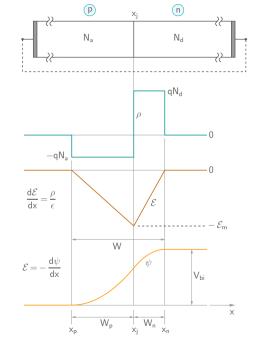
Potential $\psi(x)$:

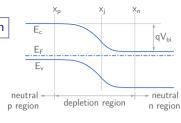
*
$$x_i < x < x_n$$
:

For convenience, let us take
$$x_j = 0 \rightarrow x_p = -W_p$$
, $x_n = W_n$.

$$\frac{d\mathcal{E}}{dx} = \frac{qN_d^+}{\epsilon} \approx \frac{qN_d}{\epsilon} \to \mathcal{E}(x) = \frac{qN_d}{\epsilon}x + k_3.$$

Since
$$\mathcal{E} = 0$$
 at $x = W_n$, we get $\mathcal{E}(x) = \frac{qN_d}{\epsilon}(x - W_n)$.





Potential $\psi(x)$:

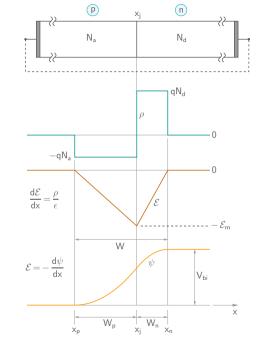
*
$$x_i < x < x_n$$
:

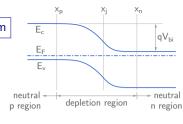
For convenience, let us take
$$x_j = 0 \rightarrow x_p = -W_p$$
, $x_n = W_n$.

$$\frac{d\mathcal{E}}{dx} = \frac{qN_d^+}{\epsilon} \approx \frac{qN_d}{\epsilon} \to \mathcal{E}(x) = \frac{qN_d}{\epsilon}x + k_3.$$

Since
$$\mathcal{E} = 0$$
 at $x = W_n$, we get $\mathcal{E}(x) = \frac{qN_d}{\epsilon}(x - W_n)$.

$$\rightarrow \psi(x) = -\int \mathcal{E} dx = -\frac{qN_d}{\epsilon} \left[\frac{x^2}{2} - W_n x \right] + k_4.$$





Potential $\psi(x)$:

*
$$x_i < x < x_n$$
:

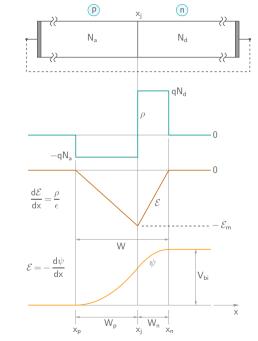
For convenience, let us take
$$x_j = 0 \rightarrow x_p = -W_p$$
, $x_n = W_n$.
$$\frac{d\mathcal{E}}{dx} = \frac{qN_d^+}{\epsilon} \approx \frac{qN_d}{\epsilon} \rightarrow \mathcal{E}(x) = \frac{qN_d}{\epsilon} x + k_3.$$

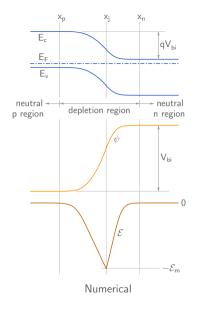
dx
$$\epsilon$$
 ϵ ϵ Since $\mathcal{E} = 0$ at $x = W_n$, we get $\mathcal{E}(x) = \frac{qN_d}{\epsilon}(x - W_n)$.

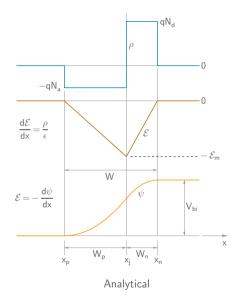
$$\rightarrow \psi(x) = -\int \mathcal{E} dx = -\frac{qN_d}{\epsilon} \left[\frac{x^2}{2} - W_n x \right] + k_4.$$

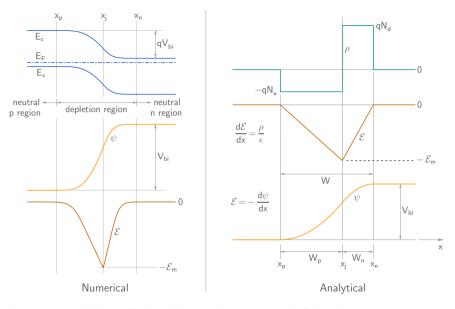
We can find k_4 using continuity of ψ at x = 0.

$$\rightarrow \psi(x) = \frac{qN_d}{\epsilon} \left[W_n x - \frac{x^2}{2} \right] + \frac{qN_a}{2\epsilon} W_p^2.$$

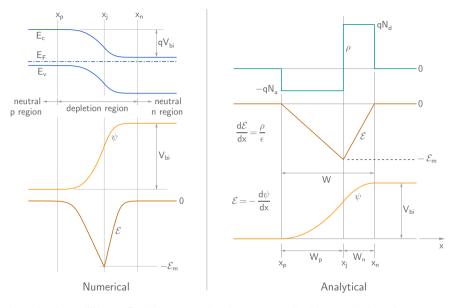




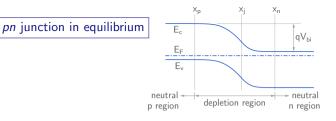




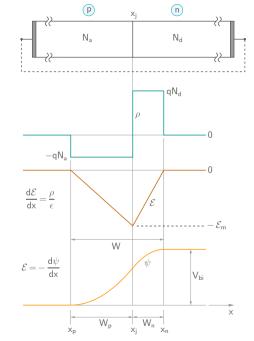
 $st\ pn$ junction in equilibrium: The band diagram is consistent with Poisson's equation.

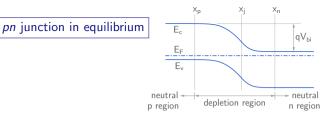


* pn junction in equilibrium: Depletion approximation agrees well with numerical results.



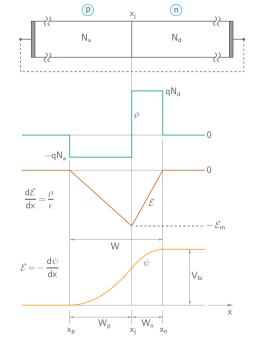
Depletion region width W:

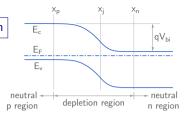




Depletion region width W:

The built-in voltage V_{bi} is given by the area under the $\mathcal{E}(\mathsf{x})$ curve.

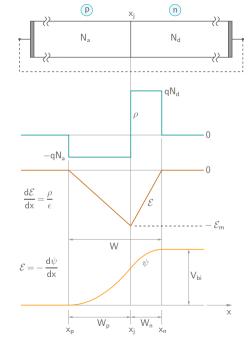


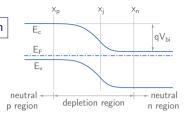


Depletion region width W:

The built-in voltage $V_{\rm bi}$ is given by the area under the $\mathcal{E}(x)$ curve.

$$V_{\text{bi}} = \frac{1}{2} \mathcal{E}_m W_p + \frac{1}{2} \mathcal{E}_m W_n = \frac{1}{2} \mathcal{E}_m W = \frac{1}{2} \frac{q N_a W_p}{\epsilon} W.$$





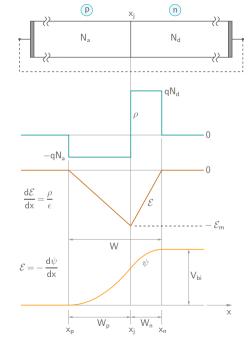
Depletion region width W:

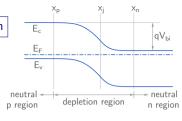
The built-in voltage $V_{\rm bi}$ is given by the area under the $\mathcal{E}(x)$ curve.

$$V_{bi} = \frac{1}{2}\,\mathcal{E}_m W_\rho + \frac{1}{2}\,\mathcal{E}_m W_n = \frac{1}{2}\,\mathcal{E}_m W = \frac{1}{2}\,\frac{q N_a W_\rho}{\epsilon}\,W.$$

Since $W_n + W_p = W$ and $W_n N_d = W_p N_a$, we get

$$W_n = \frac{N_a}{N_a + N_d} W, \quad W_p = \frac{N_d}{N_a + N_d} W.$$





Depletion region width W:

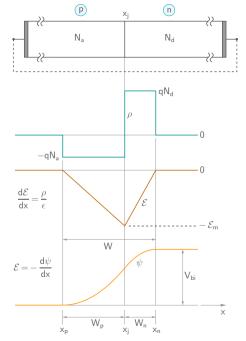
The built-in voltage $V_{\rm bi}$ is given by the area under the $\mathcal{E}(x)$ curve.

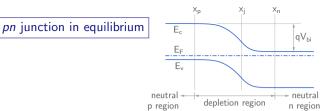
$$V_{\mathrm{bi}} = \frac{1}{2} \mathcal{E}_m W_p + \frac{1}{2} \mathcal{E}_m W_n = \frac{1}{2} \mathcal{E}_m W = \frac{1}{2} \frac{q N_a W_p}{\epsilon} W.$$

Since $W_p + W_p = W$ and $W_p N_d = W_p N_a$, we get

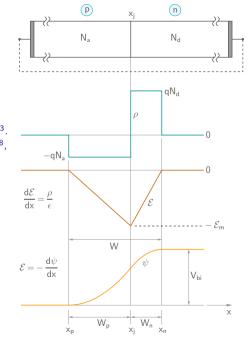
$$W_n = \frac{N_a}{N_a + N_d} W, \quad W_p = \frac{N_d}{N_a + N_d} W.$$

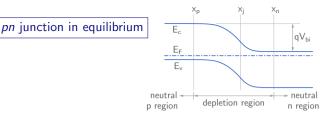
$$\rightarrow V_{\rm bi} = \frac{1}{2} \, \frac{q}{\epsilon} \, \frac{N_{\rm a} N_{\rm d}}{N_{\rm a} + N_{\rm d}} \, W^2, \quad {\rm i.e.,} \ W = \sqrt{\frac{2\epsilon}{q} \left(\frac{N_{\rm a} + N_{\rm d}}{N_{\rm a} N_{\rm d}}\right) \, V_{\rm bi}}.$$





For an abrupt, uniformly doped silicon pn junction, $N_a=5\times 10^{17}~\rm cm^{-3}$. Compute $V_{\rm bi}$, W, W_n , W_p , and \mathcal{E}_m for $N_d=10^{16}$, 10^{17} , 5×10^{17} , 10^{18} , and $5\times 10^{18}~\rm cm^{-3}$ ($T=300~\rm K$).





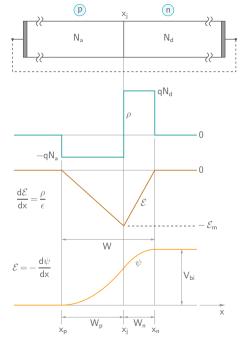
For an abrupt, uniformly doped silicon pn junction, $N_a = 5 \times 10^{17}$ cm⁻³. Compute V_{bi} , W, W_n , W_p , and \mathcal{E}_m for $N_d = 10^{16}$, 10^{17} , 5×10^{17} , 10^{18} , and 5×10^{18} cm⁻³ ($T = 200 \, \text{K}$)

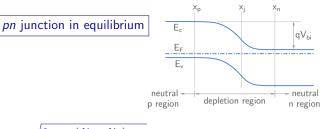
and $5 \times 10^{18} \, \text{cm}^{-3}$ ($T = 300 \, \text{K}$).

Solution:
$$V_{\rm bi} = V_T \log \frac{N_a N_d}{n_i^2}$$

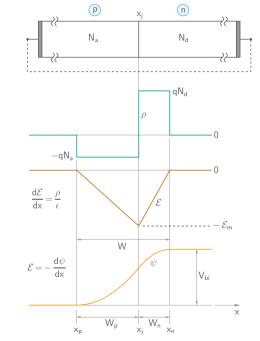
$$= 0.0259 \,\mathsf{V} \times \log \frac{(5 \times 10^{17})(1 \times 10^{16})}{(1.5 \times 10^{10})^2}$$

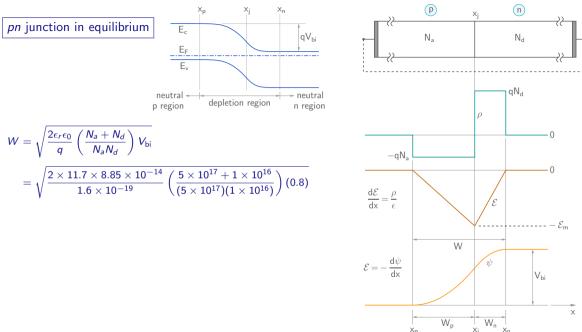
$$= 0.8 \, V.$$

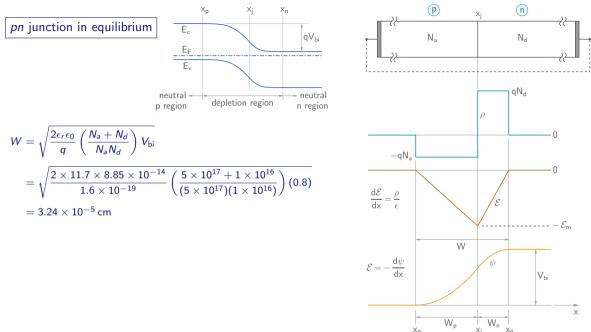


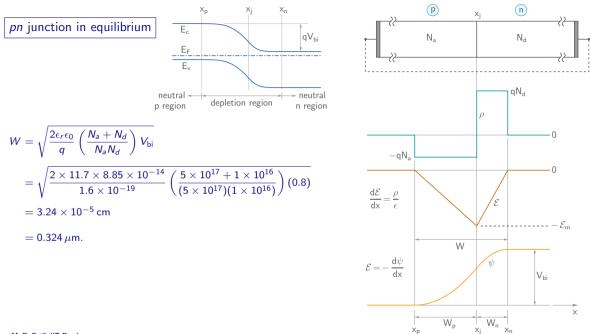


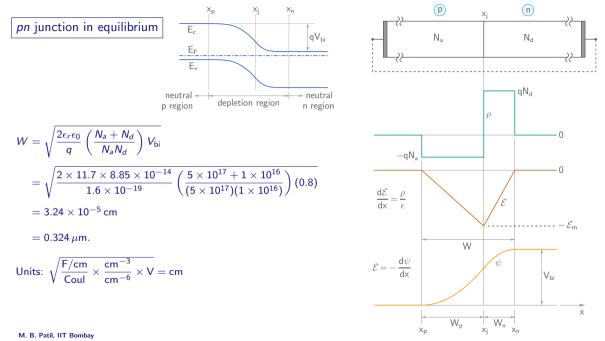
$$W = \sqrt{\frac{2\epsilon_r \epsilon_0}{q} \left(\frac{N_a + N_d}{N_a N_d}\right) V_{bi}}$$

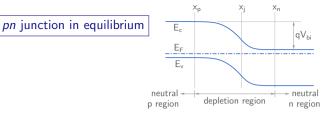




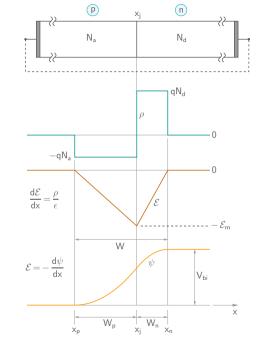


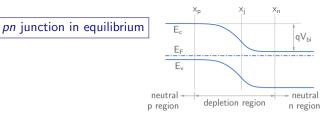






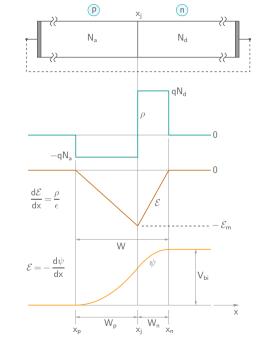
$$W_n = \frac{N_a}{N_a + N_d} \ W = 0.318 \, \mu \text{m}, \quad W_p = \frac{N_d}{N_a + N_d} \ W = 0.006 \, \mu \text{m}.$$

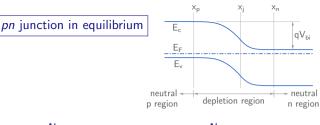




$$W_n = \frac{N_a}{N_a + N_d} W = 0.318 \,\mu\text{m}, \quad W_p = \frac{N_d}{N_a + N_d} W = 0.006 \,\mu\text{m}.$$

$$\mathcal{E}_m = \frac{qN_d}{\epsilon} W_n \text{ or } \frac{qN_a}{\epsilon} W_p$$



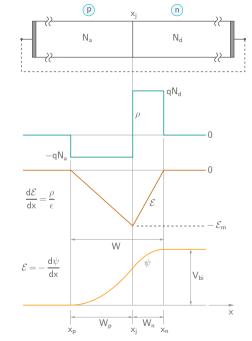


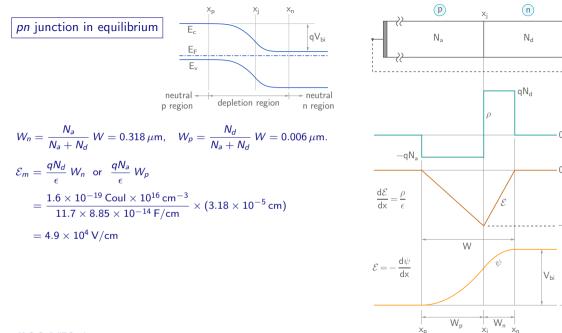
$$W_n = \frac{N_a}{N_a + N_d} W = 0.318 \,\mu\text{m}, \quad W_p = \frac{N_d}{N_a + N_d} W = 0.006 \,\mu\text{m}.$$

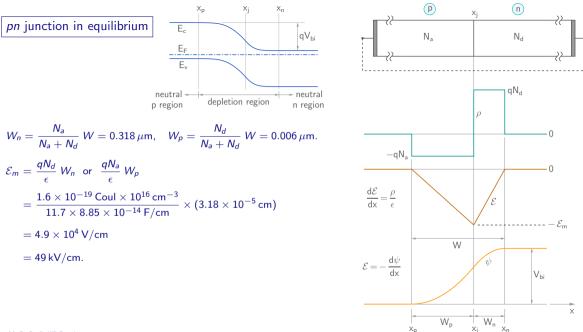
$$\mathcal{E}_m = \frac{qN_d}{\epsilon} W_n \quad \text{or} \quad \frac{qN_a}{\epsilon} W_p$$

$$= \frac{1.6 \times 10^{-19} \,\text{Coul} \times 10^{16} \,\text{cm}^{-3}}{11.7 \times 9.85 \times 10^{-14} \,\text{F/cm}} \times (3.18 \times 10^{-5} \,\text{cm})$$

$$= \frac{1.6 \times 10^{-19}\, \mathsf{Coul} \times 10^{16}\, \mathsf{cm}^{-3}}{11.7 \times 8.85 \times 10^{-14}\, \mathsf{F/cm}} \times (3.18 \times 10^{-5}\, \mathsf{cm})$$

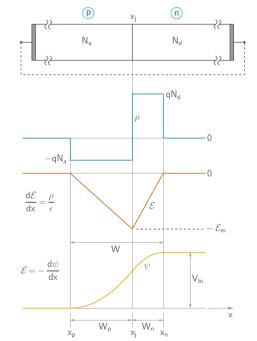






Effect of N_d , with $N_a = 5 \times 10^{17} \, \mathrm{cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in $\mu \mathrm{m}$, \mathcal{E}_m in kV/cm.)

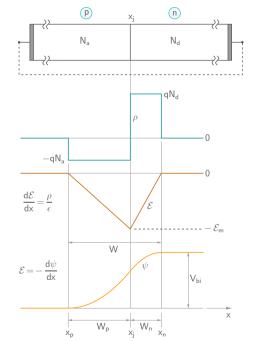
N_d (cm ⁻³)	$V_{ m bi}$	W	W _n	W_p	\mathcal{E}_{m}
$1.0 imes 10^{16}$	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366



Effect of N_d , with $N_a = 5 \times 10^{17} \, \text{cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in μ m, \mathcal{E}_m in kV/cm.)

N_d (cm ⁻³)	$V_{ m bi}$	W	W _n	W_p	\mathcal{E}_{m}
$1.0 imes 10^{16}$	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366

*
$$V_{\text{bi}} = V_T \log \frac{N_a N_d}{n_i^2}$$
, $W = \sqrt{\frac{2\epsilon}{q} \left(\frac{N_a + N_d}{N_a N_d}\right) V_{\text{bi}}}$, $W_n = \frac{N_a}{N_a + N_d} W$, $W_p = \frac{N_d}{N_a + N_d} W$.



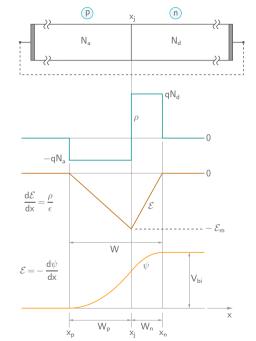
Effect of N_d , with $N_a = 5 \times 10^{17} \, \mathrm{cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in $\mu \mathrm{m}$, \mathcal{E}_m in kV/cm.)

N_d (cm ⁻³)	V_{bi}	W	W _n	W_p	\mathcal{E}_{m}
$1.0 imes 10^{16}$	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366

*
$$V_{\text{bi}} = V_T \log \frac{N_a N_d}{n_i^2}$$
, $W = \sqrt{\frac{2\epsilon}{q} \left(\frac{N_a + N_d}{N_a N_d}\right) V_{\text{bi}}}$, $W_n = \frac{N_a}{N_a + N_d} W$, $W_p = \frac{N_d}{N_a + N_d} W$.

 $N_d \ll N_a \ (p^+ n \ \text{junction})$:

 $W_n \approx W$, and W is determined mainly by N_d .



Effect of N_d , with $N_a = 5 \times 10^{17} \text{ cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in μ m, \mathcal{E}_m in kV/cm.)

N_d (cm ⁻³)	$V_{ m bi}$	W	W _n	W_p	\mathcal{E}_{m}
1.0×10^{16}	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366

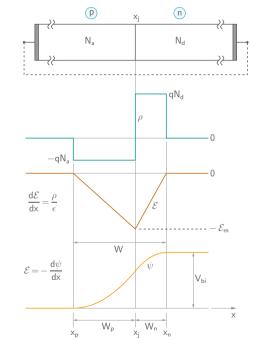
*
$$V_{\text{bi}} = V_T \log \frac{N_a N_d}{n_i^2}$$
, $W = \sqrt{\frac{2\epsilon}{q} \left(\frac{N_a + N_d}{N_a N_d}\right)} V_{\text{bi}}$, $W_n = \frac{N_a}{N_2 + N_d} W$, $W_p = \frac{N_d}{N_2 + N_d} W$.

 $N_d \ll N_a \ (p^+ n \ \text{junction})$:

 $W_n \approx W$, and W is determined mainly by N_d .

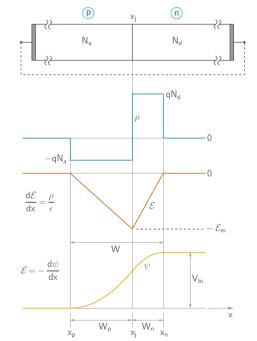
$$N_a \ll N_d \ (n^+ p \ \text{junction})$$
:

$$W_p \approx W$$
, and W is determined mainly by N_a .



Effect of N_d , with $N_a = 5 \times 10^{17} \text{ cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in μ m, \mathcal{E}_m in kV/cm.)

N_d (cm ⁻³)	$V_{ m bi}$	W	W _n	W_p	\mathcal{E}_{m}
$1.0 imes 10^{16}$	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366

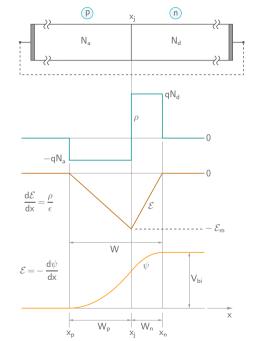


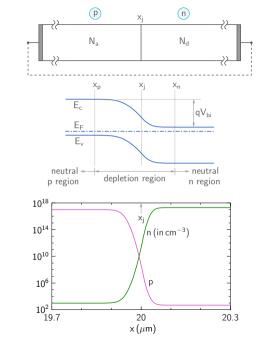
Effect of N_d , with $N_a = 5 \times 10^{17} \, \text{cm}^{-3}$ held fixed. (V_{bi} in Volts, W, W_n , W_p in μm , \mathcal{E}_m in kV/cm.)

N_d (cm ⁻³)	V_{bi}	W	W _n	W_p	\mathcal{E}_{m}
$1.0 imes 10^{16}$	0.80	0.324	0.318	0.006	49
$1.0 imes 10^{17}$	0.86	0.115	0.096	0.019	148
$5.0 imes 10^{17}$	0.90	0.068	0.034	0.034	263
$1.0 imes 10^{18}$	0.92	0.060	0.020	0.040	307
$5.0 imes 10^{18}$	0.96	0.052	0.004	0.047	366

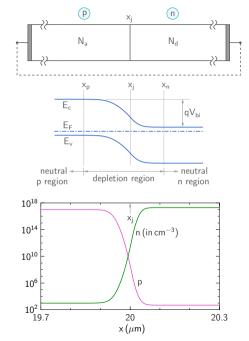
* For high doping densities such as $10^{18}\,\mathrm{cm^{-3}}$, degenerate statistics should be used for higher accuracy, i.e.,

$$n=N_c\,rac{2}{\sqrt{\pi}}\,\mathcal{F}_{1/2}(\eta_c), \; ext{with} \; \eta_c=rac{E_F-E_c}{kT}, \; ext{and} \
onumber \ p=N_v\,rac{2}{\sqrt{\pi}}\,\mathcal{F}_{1/2}(\eta_v), \; ext{with} \; \eta_v=rac{E_v-E_F}{kT}.$$



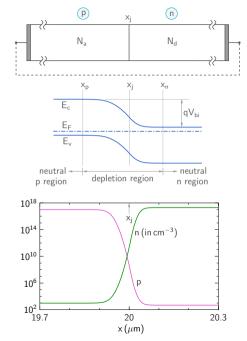


* The diffusion currents can be expected to be substantial since there is a large change in *n* or *p* between the *p*-side and the *n*-side.

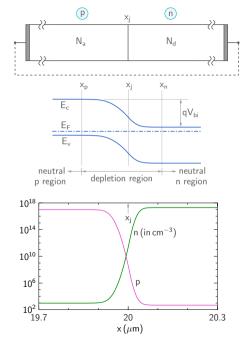


- * The diffusion currents can be expected to be substantial since there is a large change in *n* or *p* between the *p*-side and the *n*-side.
- In equilibrium, the drift and diffusion currents are equal and opposite for eletrons as well as holes, i.e.,

$$J_n^{\text{diff}} = -J_n^{\text{drift}}, \quad J_p^{\text{diff}} = -J_p^{\text{drift}}.$$



- * The diffusion currents can be expected to be substantial since there is a large change in *n* or *p* between the *p*-side and the *n*-side.
- In equilibrium, the drift and diffusion currents are equal and opposite for eletrons as well as holes, i.e.,
 J_n^{diff} = -J_n^{drift},
 J_n^{diff} = -J_n^{drift}.
- * Qualitatively, we can see that the diffusion and drift currents will be in opposite directions:



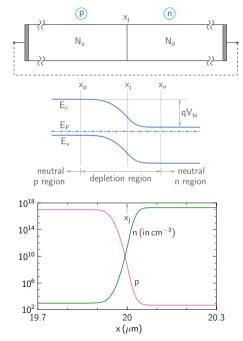
- * The diffusion currents can be expected to be substantial since there is a large change in *n* or *p* between the *p*-side and the *n*-side.
- In equilibrium, the drift and diffusion currents are equal and opposite for eletrons as well as holes, i.e.,

$$J_{p}^{\text{diff}} = -J_{p}^{\text{drift}}, \quad J_{p}^{\text{diff}} = -J_{p}^{\text{drift}}.$$

* Qualitatively, we can see that the diffusion and drift currents will be in opposite directions:

Electrons:

$$\mathcal{F}_n^{\text{diff}}: \longleftarrow, \mathcal{E}: \longleftarrow, \mathcal{F}_n^{\text{drift}}: \longrightarrow.$$



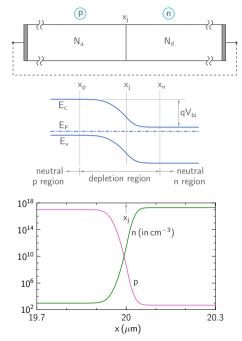
- * The diffusion currents can be expected to be substantial since there is a large change in *n* or *p* between the *p*-side and the *n*-side.
- In equilibrium, the drift and diffusion currents are equal and opposite for eletrons as well as holes, i.e.,
 Jaiff = -Jarift, Jaiff = -Jarift.

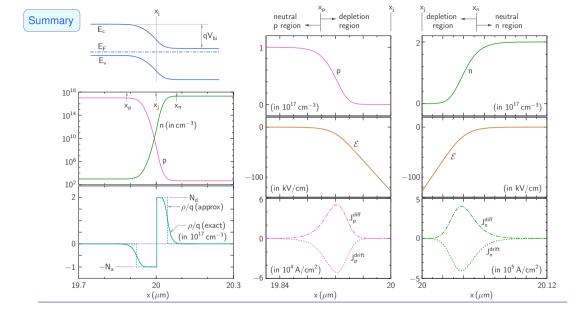
Electrons:

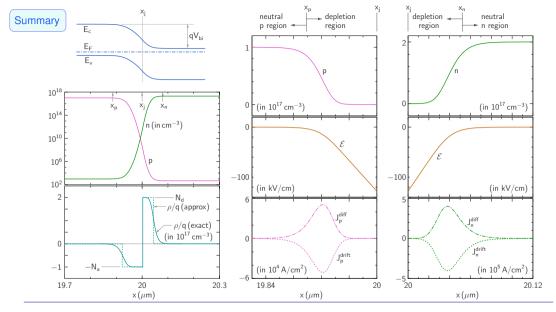
$$\mathcal{F}_n^{\text{diff}}: \longleftarrow, \mathcal{E}: \longleftarrow, \mathcal{F}_n^{\text{drift}}: \longrightarrow.$$

Holes:

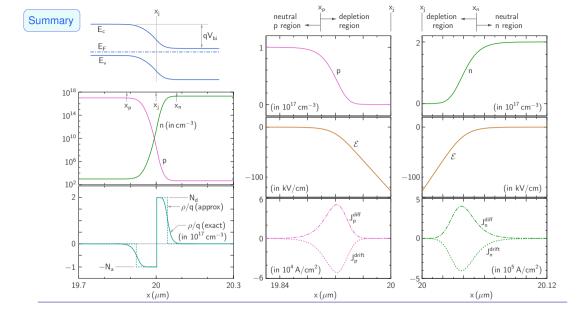
$$\mathcal{F}_p^{\mathsf{diff}}: \longrightarrow$$
 , $\mathcal{E}: \longleftarrow$, $\mathcal{F}_p^{\mathsf{drift}}: \longleftarrow$.

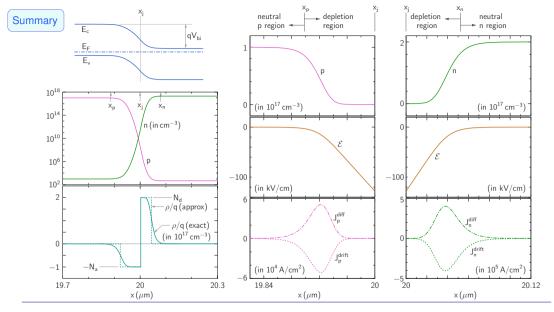




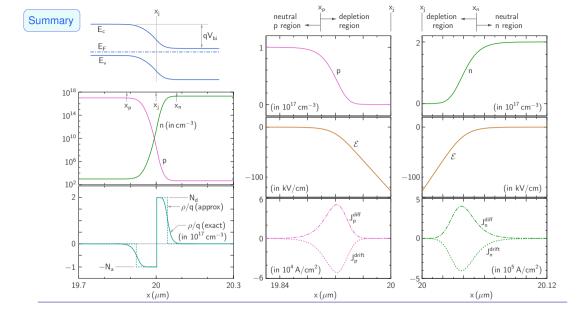


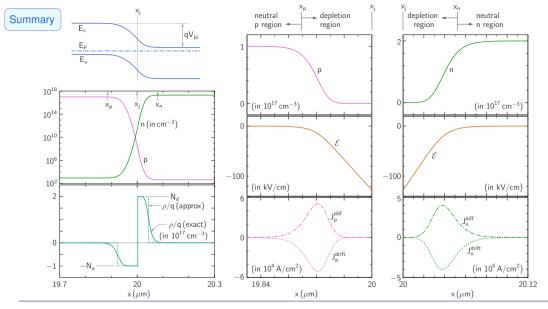
* There are three regions: p neutral region, n neutral region, and depletion region.



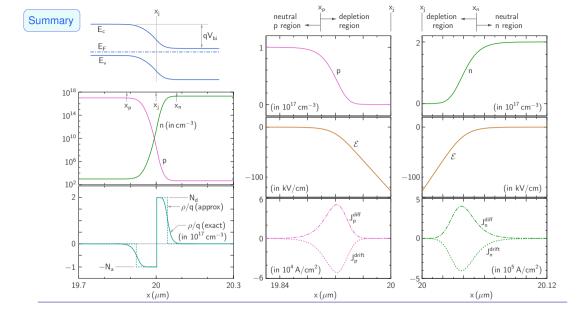


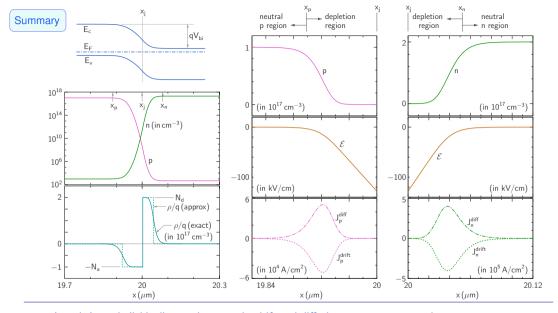
* The electric field is zero in the neutral regions and maximum (in magnitude) at the junction.





* There is a potential difference – the built-in voltage $V_{\rm bi}$ – between the neutral p and neutral n sides.





* J_n and J_p are individually zero because the drift and diffusion components cancel out.