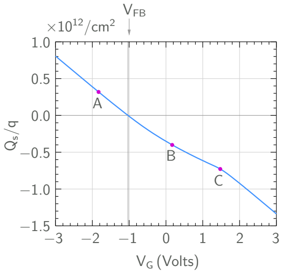
Semiconductor Devices

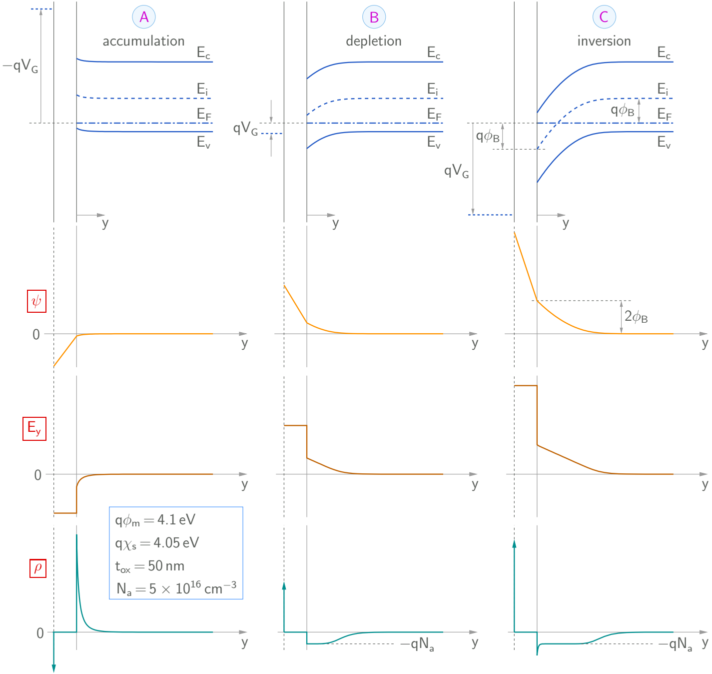
MOS Transistors: Part 2

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

MOS capacitor

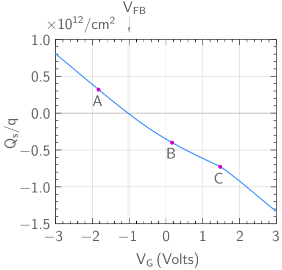


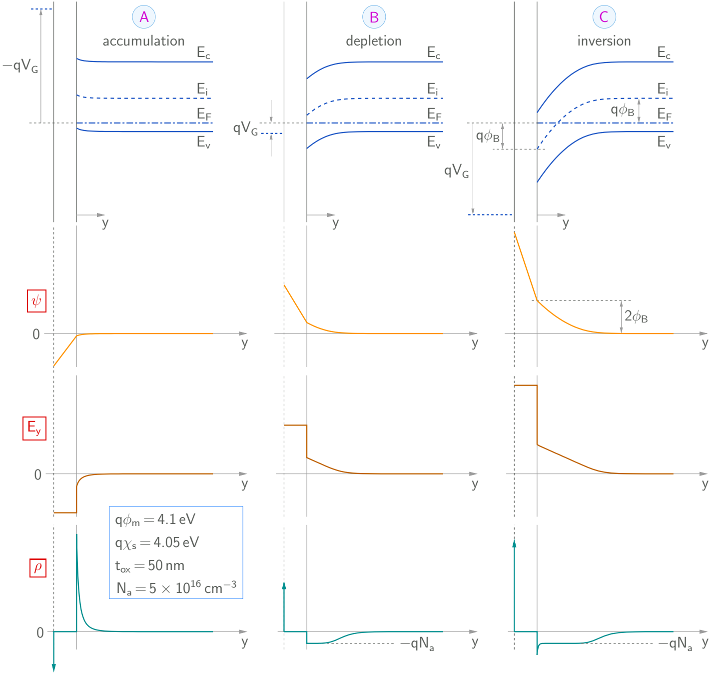


M. B. Patil, IIT Bombay

MOS capacitor

* $Q_s/q = \int_0^{\infty} (N_d^+ - N_a^- + p - n)\, dy.$

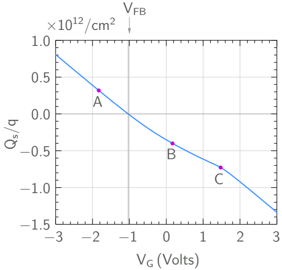


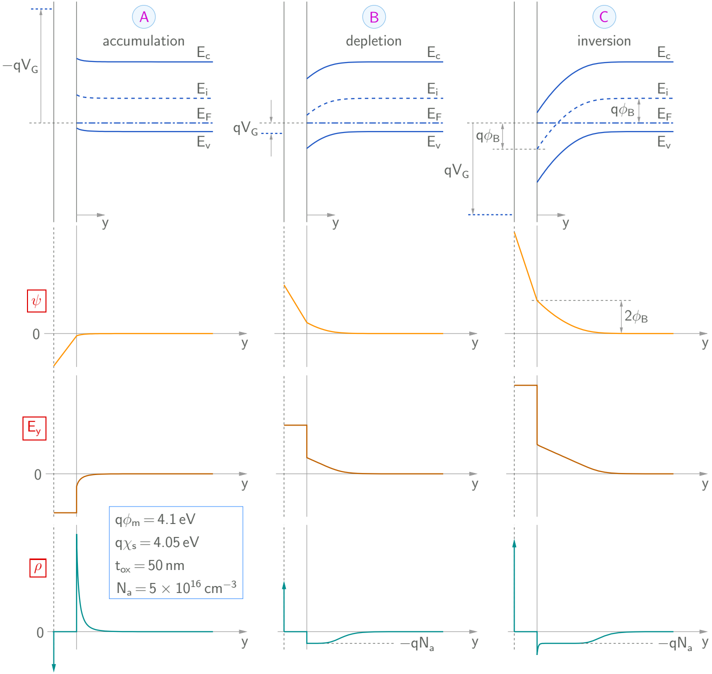


M. B. Patil, IIT Bombay

MOS capacitor

* $Q_s/q = \int_0^{\infty} (N_d^+ - N_a^- + p - n)\, dy.$
* $V_G = V_{FB} \rightarrow$ flad bands $\rightarrow Q_s = 0.$

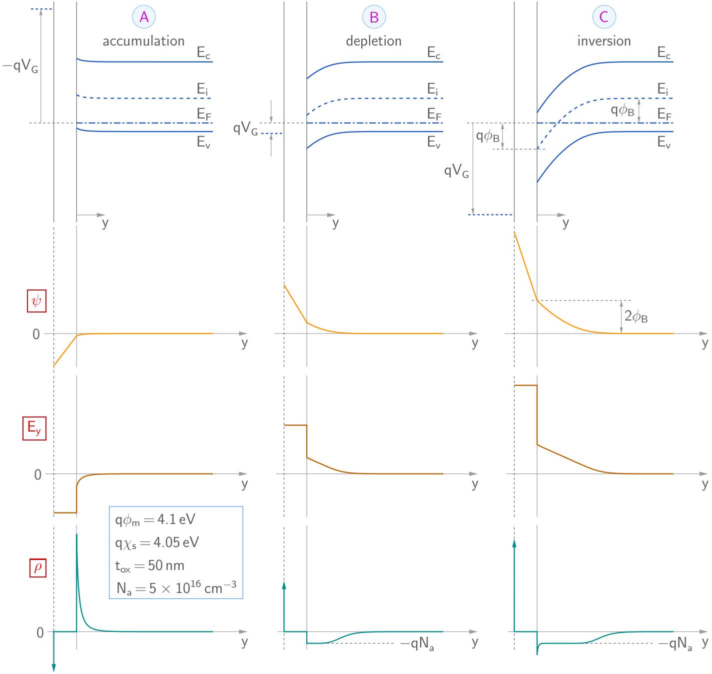


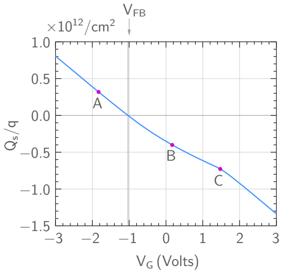


M. B. Patil, IIT Bombay

MOS capacitor

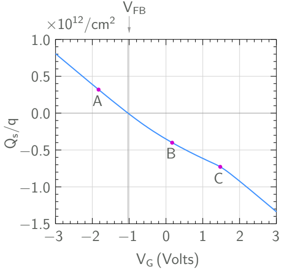
* $Q_s/q = \int_0^{\infty} (N_d^+ - N_a^- + p - n)\, dy.$
* $V_G = V_{FB} \rightarrow$ flad bands $\rightarrow Q_s = 0.$
* $V_G < V_{FB} \rightarrow$ accumulation
  $\rightarrow Q_s > 0.$

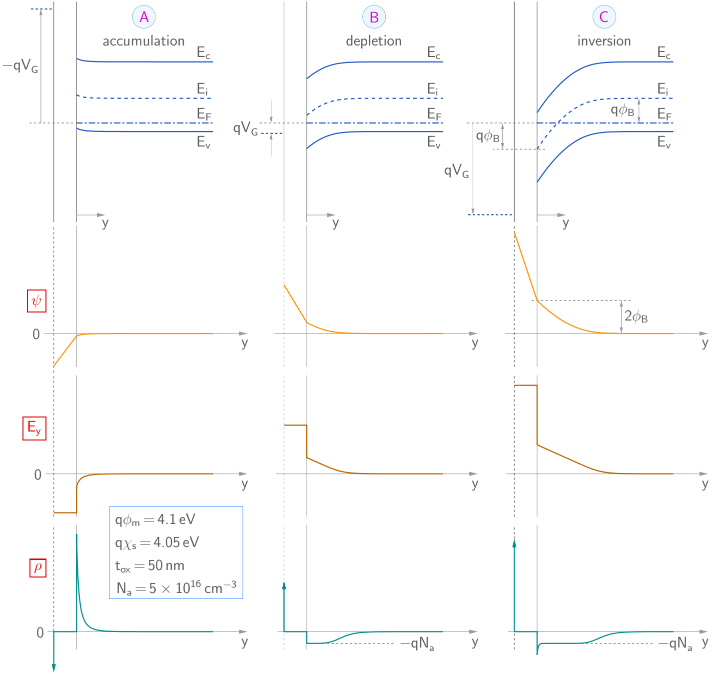




MOS capacitor

* $Q_s/q = \int_0^{\infty} (N_d^+ - N_a^- + p - n)\, dy.$
* $V_G = V_{FB}$ → flad bands → $Q_s = 0$.
* $V_G < V_{FB}$ → accumulation
  → $Q_s > 0$.
* $V_G > V_{FB}$ → depletion or inversion
  → $Q_s < 0$ in either case.

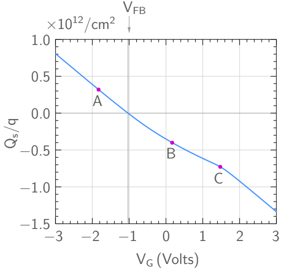


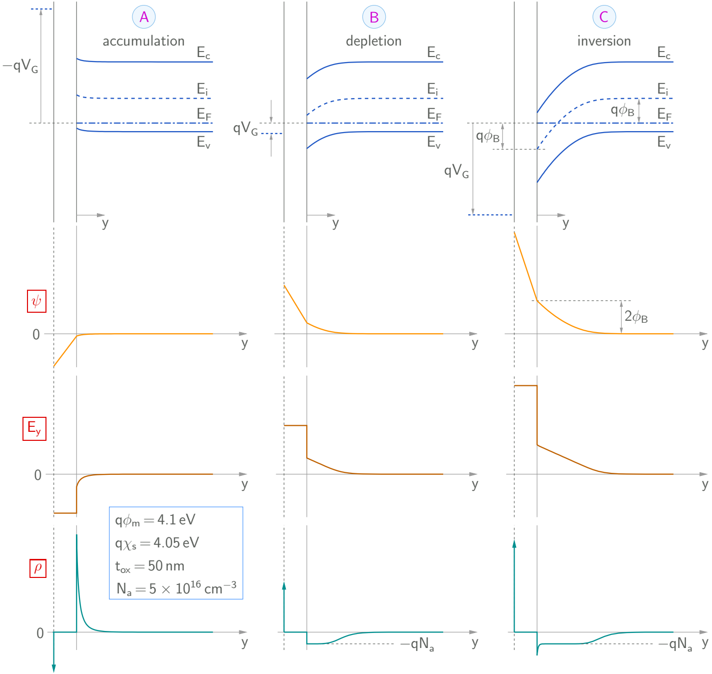


M. B. Patil, IIT Bombay

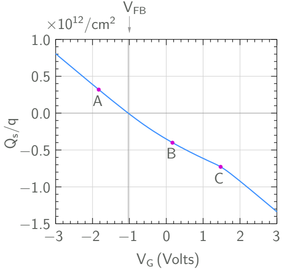
MOS capacitor

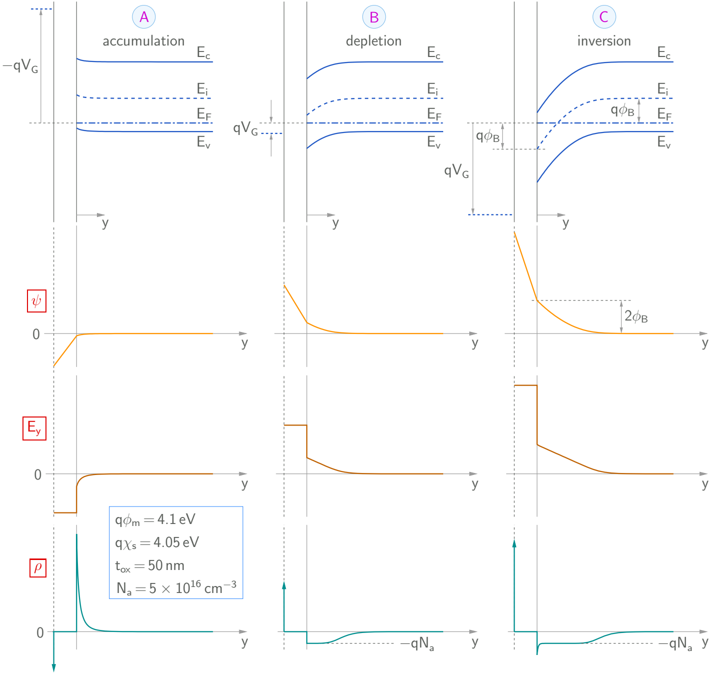
* $Q_s/q = \int_0^\infty (N_d^+ - N_a^- + p - n)\, dy.$
* $V_G = V_{FB} \rightarrow$ flad bands $\rightarrow Q_s = 0.$
* $V_G < V_{FB} \rightarrow$ accumulation
  $\rightarrow Q_s > 0.$
* $V_G > V_{FB} \rightarrow$ depletion or inversion
  $\rightarrow Q_s < 0$ in either case.
* Home work: Sketch ρ versus y for the case of an n-type substrate.





MOS capacitor

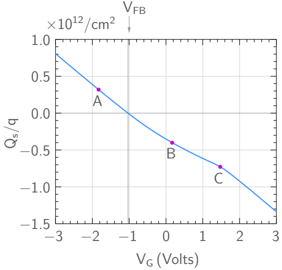


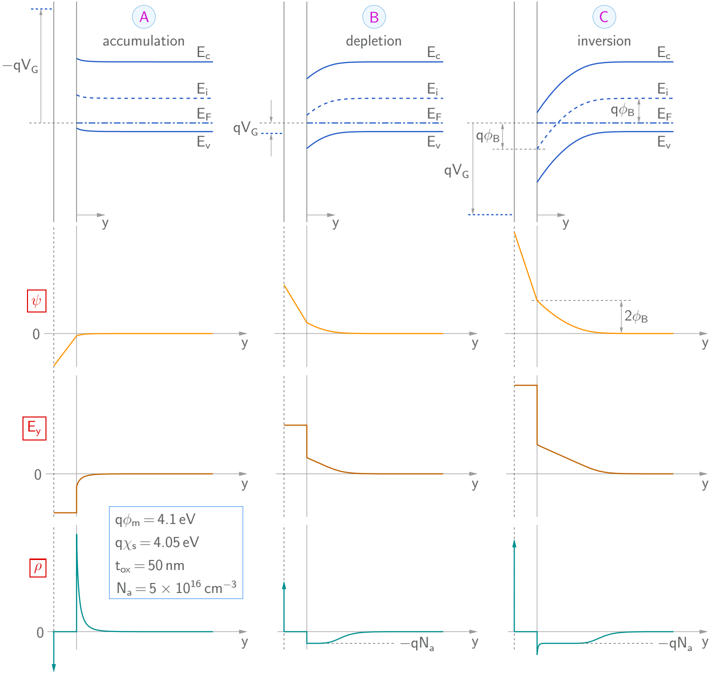


MOS capacitor

* The "inversion charge" Q_I is defined as

$$Q_I = -q\int_0^{\infty} n\,dy \rightarrow \frac{Q_I}{(-q)} = \int_0^{\infty} n\,dy.$$





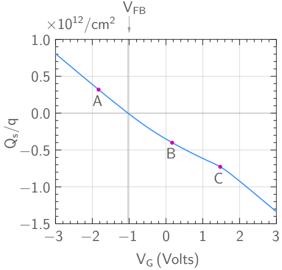
M. B. Patil, IIT Bombay

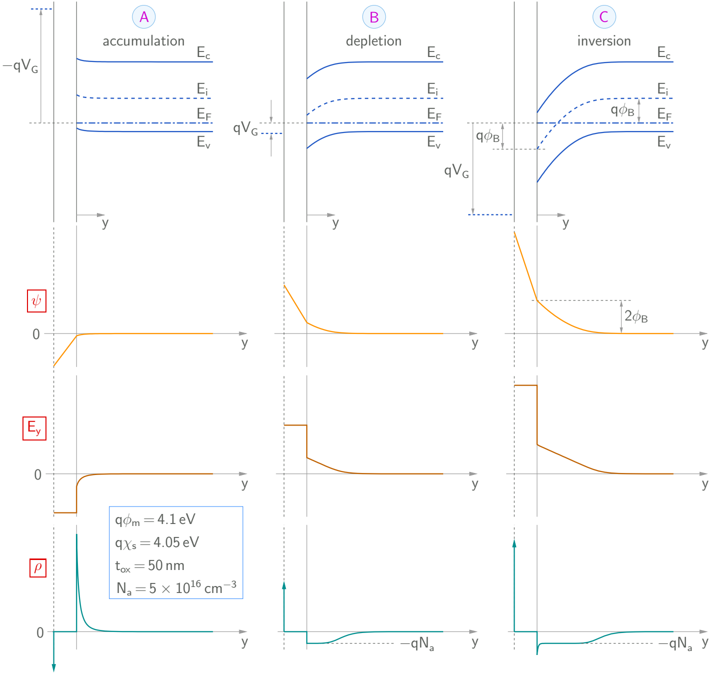
MOS capacitor

* The "inversion charge" Q_I is defined as

$$Q_I = -q \int_0^{\infty} n\,dy \rightarrow \frac{Q_I}{(-q)} = \int_0^{\infty} n\,dy.$$

* Q_I has a special significance in a MOS transistor since it represents the mobile charge (due to electrons in this case) which, unlike the charge due to the fixed acceptor and donor ions, can contribute to current.





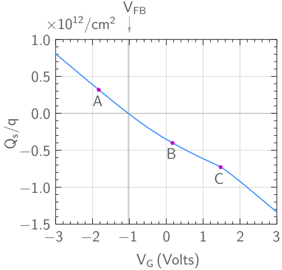
M. B. Patil, IIT Bombay

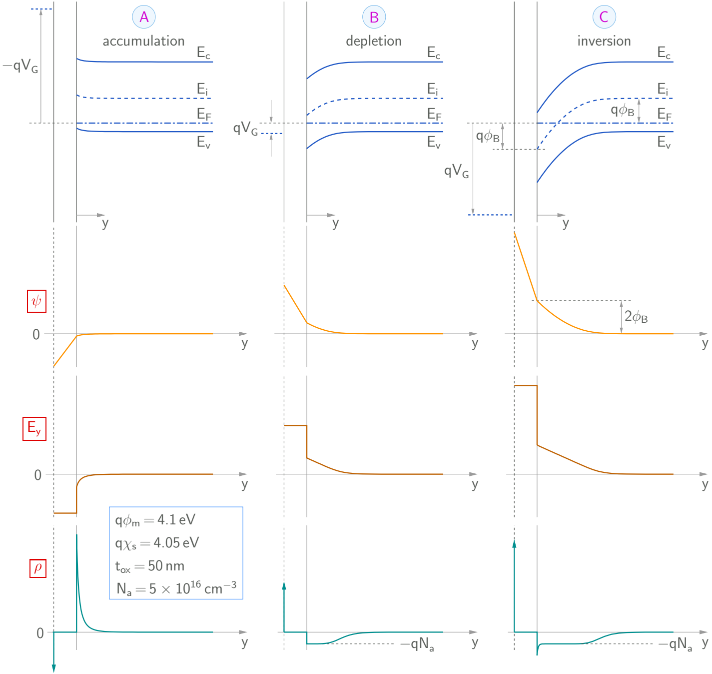
MOS capacitor

* The "inversion charge" Q_I is defined as

$$Q_I = -q \int_0^{\infty} n\, dy \rightarrow \frac{Q_I}{(-q)} = \int_0^{\infty} n\, dy.$$

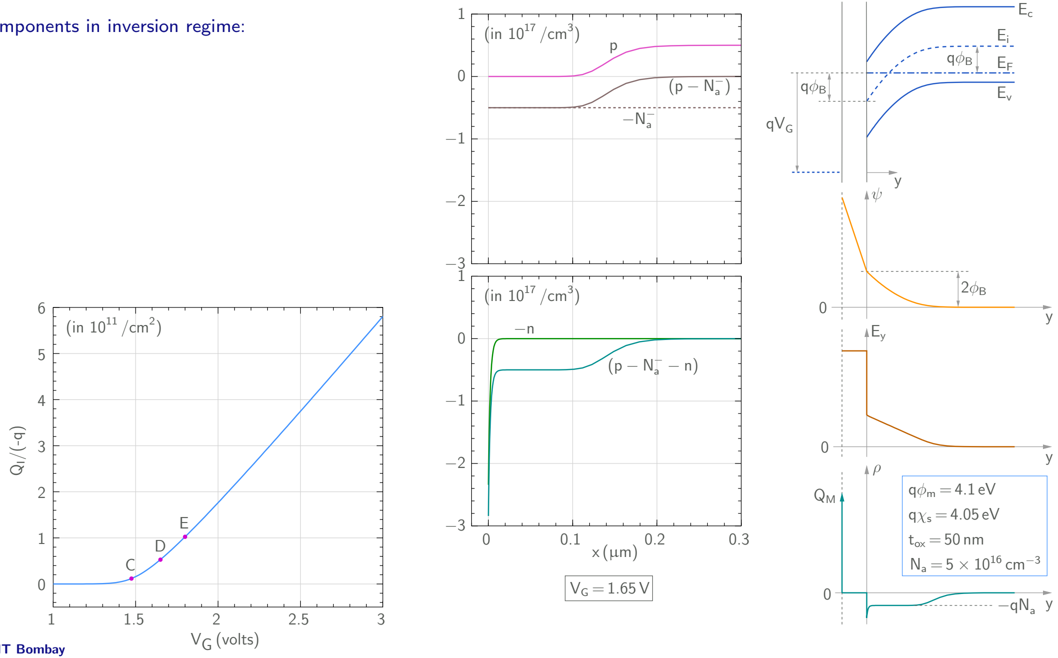
* Q_I has a special significance in a MOS transistor since it represents the mobile charge (due to electrons in this case) which, unlike the charge due to the fixed acceptor and donor ions, can contribute to current.
* $Q_I/(-q)$ gives the total number of electrons in the semiconductor per unit area (in the x-z plane).





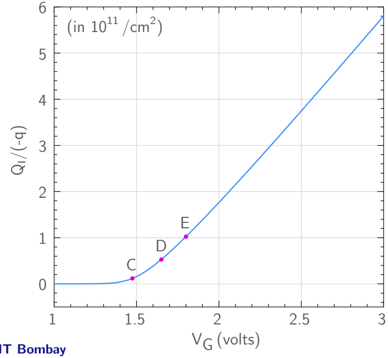
M. B. Patil, IIT Bombay

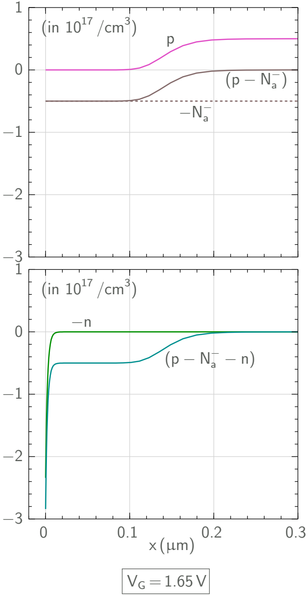
Charge components in inversion regime:

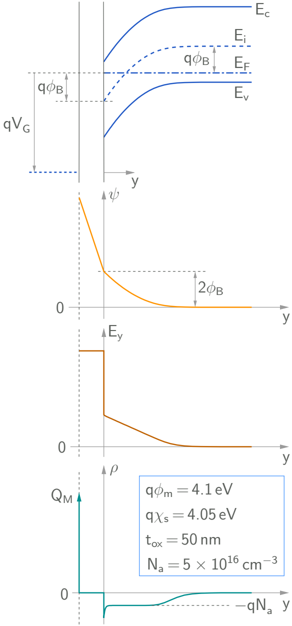


Charge components in inversion regime:

* Ionised acceptor density $N_a^- \approx N_a$ at $T = 300\,\mathrm{K}$.

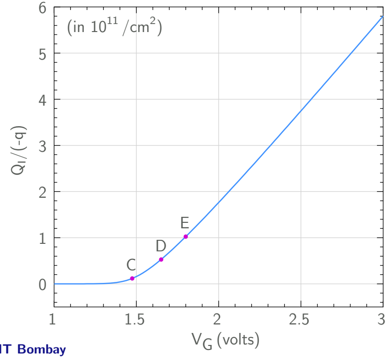


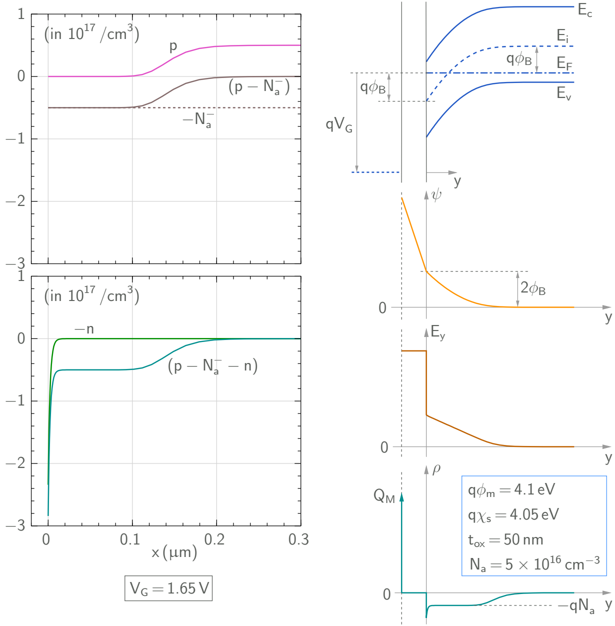




Charge components in inversion regime:

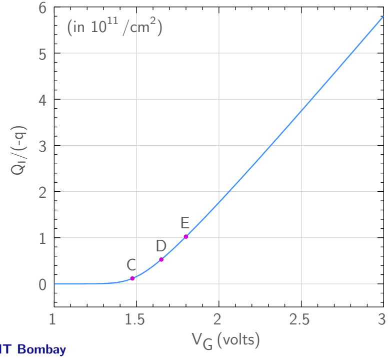
* Ionised acceptor density $N_a^- \approx N_a$ at $T = 300\,\text{K}$.
* Hole density $p(y)$ which is equal to p_0 $(\approx N_a)$ except near the surface.

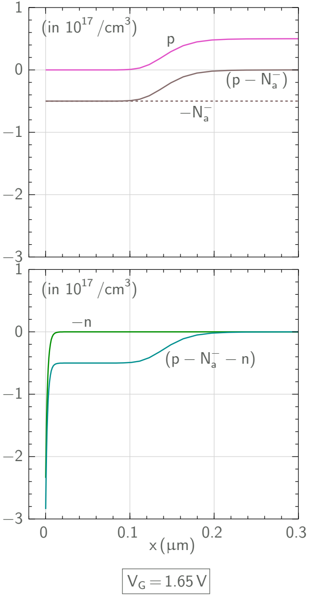


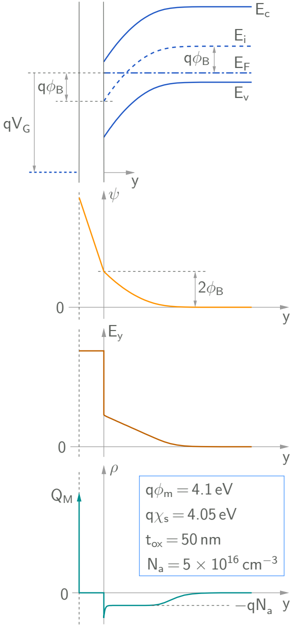


Charge components in inversion regime:

* Ionised acceptor density $N_a^- \approx N_a$ at $T = 300\,\mathrm{K}$.
* Hole density $p(y)$ which is equal to p_0 ($\approx N_a$) except near the surface.
* $p(y) - N_a^- \approx -N_a$ near the surface and becomes zero as we move into the bulk semiconductor region.

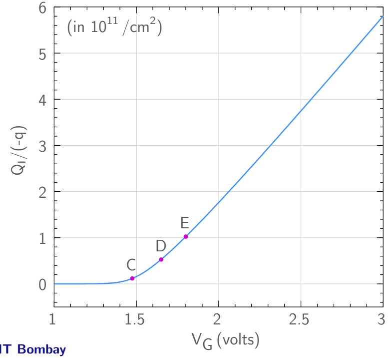


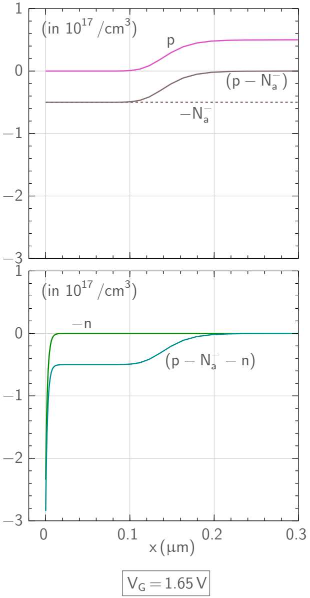


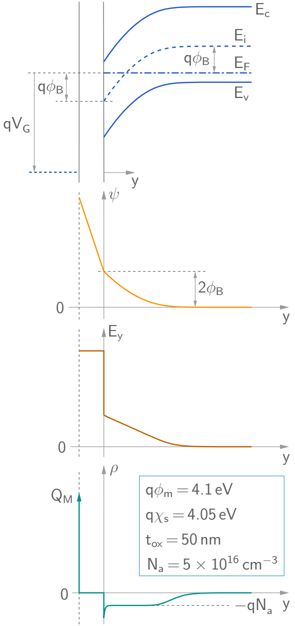


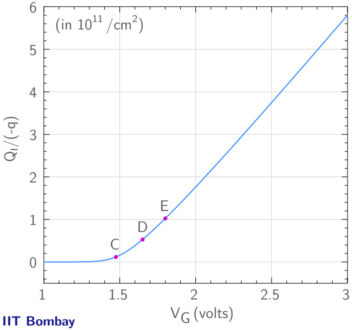
Charge components in inversion regime:

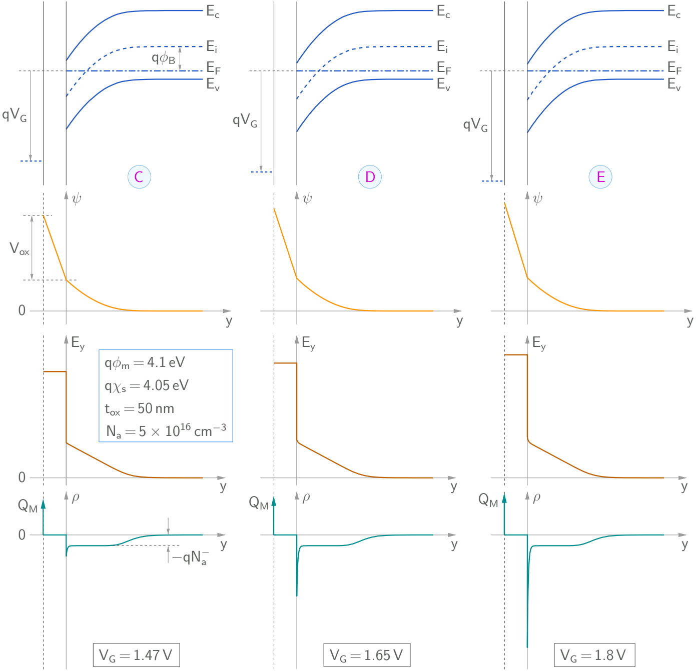
- Ionised acceptor density $N_a^- \approx N_a$ at $T = 300\,\text{K}$.
- Hole density $p(y)$ which is equal to p_0 ($\approx N_a$) except near the surface.
- $p(y) - N_a^- \approx -N_a$ near the surface and becomes zero as we move into the bulk semiconductor region.
- $n(y) = n_0 = \dfrac{n_i^2}{p_0}$ in most of the device except near the surface where it increases dramatically.





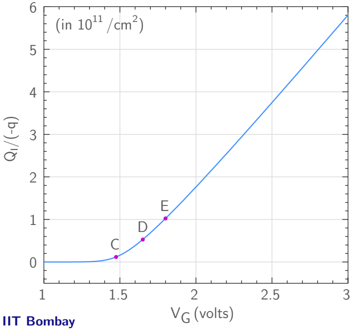


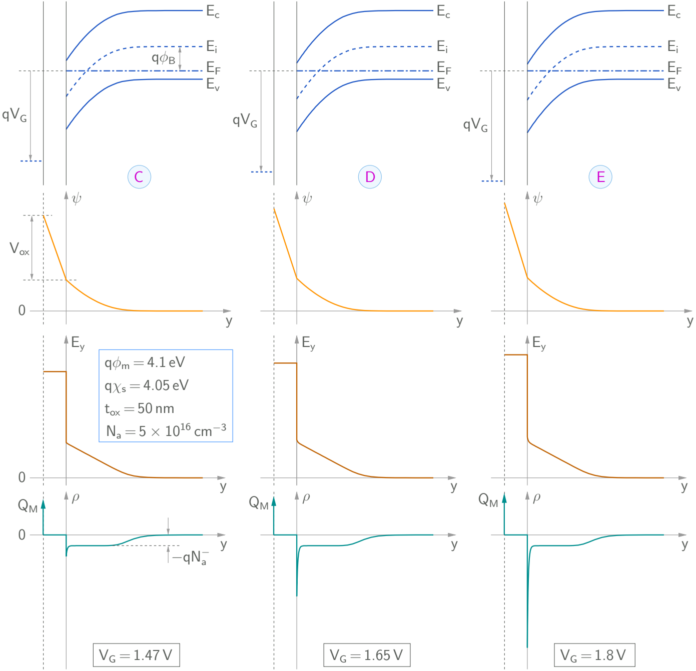




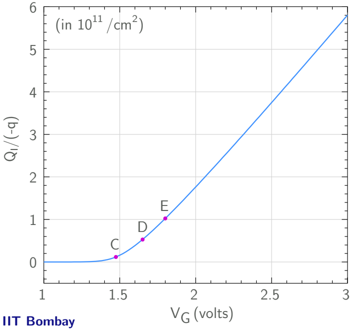
M. B. Patil, IIT Bombay

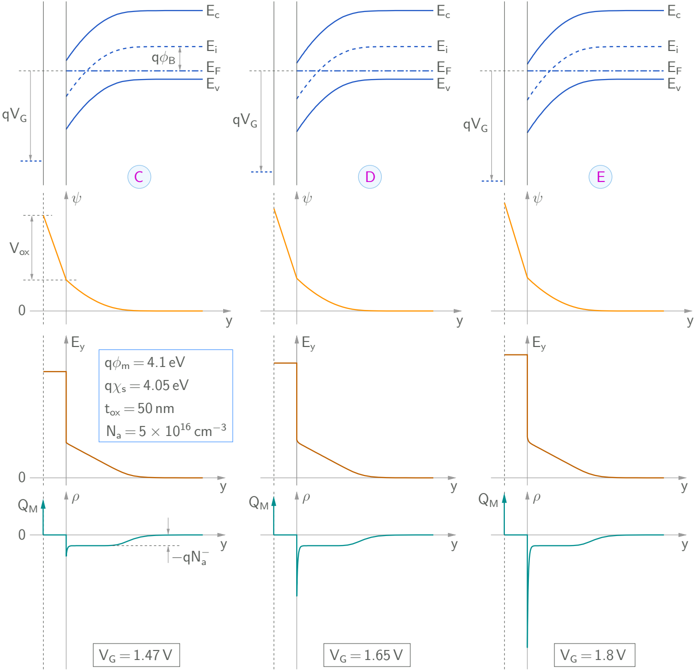
* As V_G is increased beyond the onset of inversion, almost the entire increase in Q_s is due to an increase in electron concentration n near the surface.





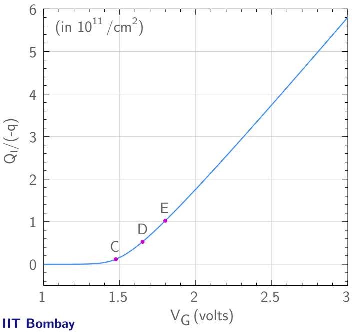
* As V_G is increased beyond the onset of inversion, almost the entire increase in Q_s is due to an increase in electron concentration n near the surface.
* The depletion layer width and the charge due to acceptor ions $\int(-qN_a^-)\,dy$ do not change significantly.

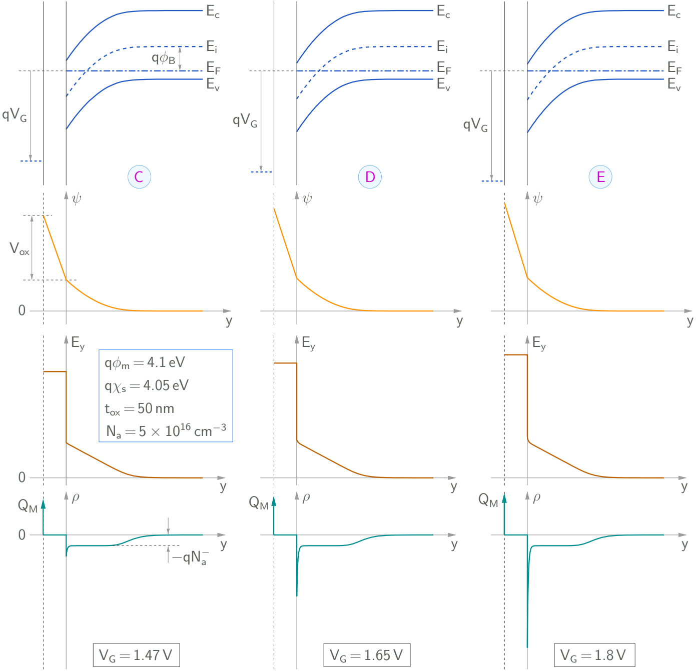




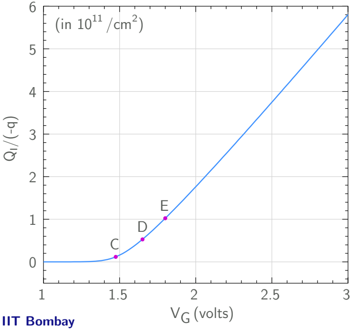
M. B. Patil, IIT Bombay

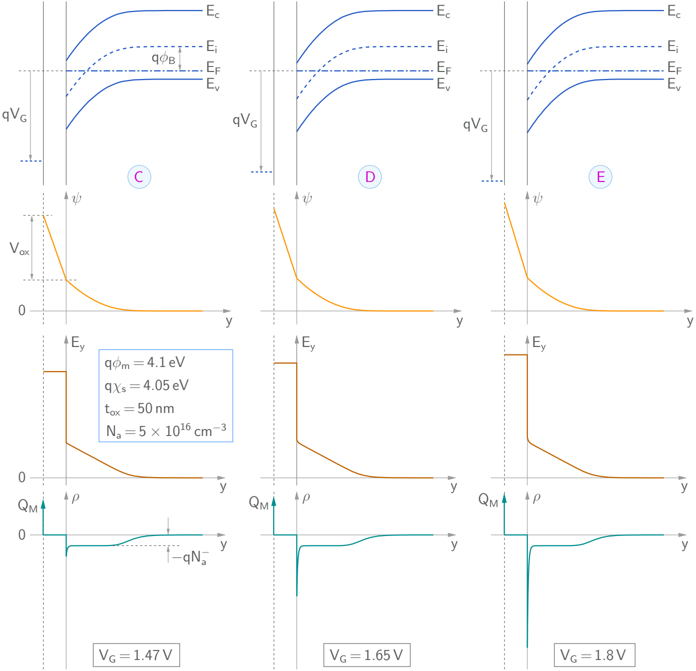
* As V_G is increased beyond the onset of inversion, almost the entire increase in Q_s is due to an increase in electron concentration n near the surface.
* The depletion layer width and the charge due to acceptor ions $\int(-qN_a^-)\,dy$ do not change significantly.
* The surface potential $\psi_s \equiv \psi(0^+)$ at the onset of inversion is $2\phi_B$. As V_G is increased, ψ_s changes only slightly, and the additional voltage drops almost entirely across the oxide layer.



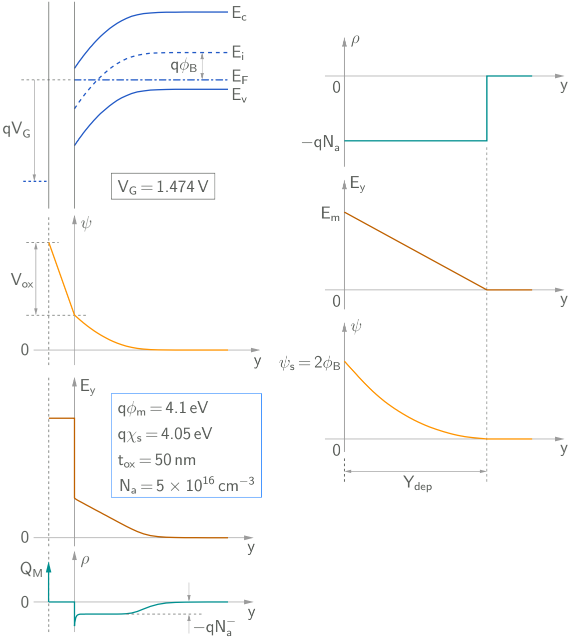


* As V_G is increased beyond the onset of inversion, almost the entire increase in Q_s is due to an increase in electron concentration n near the surface.
* The depletion layer width and the charge due to acceptor ions $\int(-qN_a^-)\,dy$ do not change significantly.
* The surface potential $\psi_s \equiv \psi(0^+)$ at the onset of inversion is $2\phi_B$. As V_G is increased, ψ_s changes only slightly, and the additional voltage drops almost entirely across the oxide layer.
* The inversion charge Q_I is confined to a narrow region near the Si-SiO$_2$ interface, typically about 10 nm (i.e., 100 Å).



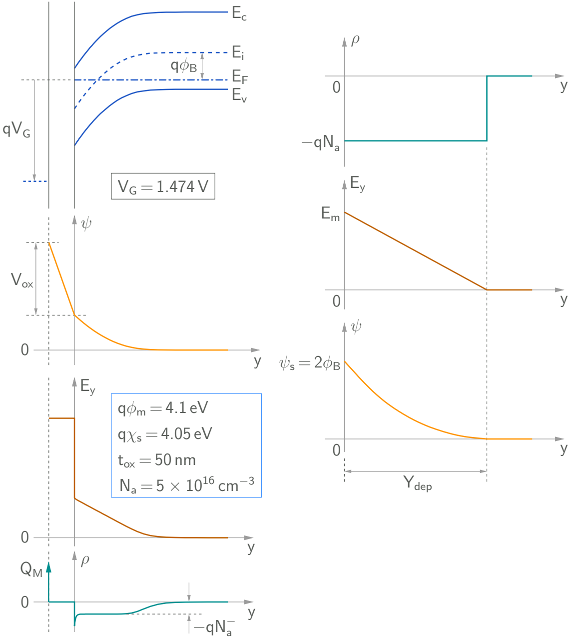


MOS capacitor: threshold voltage



MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow$ $n(y) = 0$.

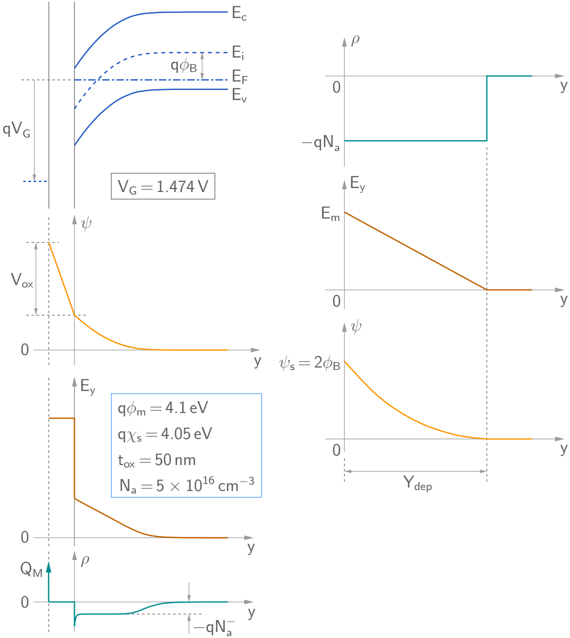


MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow$ $n(y) = 0$.

With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\mathrm{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\mathrm{Si}}} \int_{0^+}^{Y_{\mathrm{dep}}} \rho\, dy$$



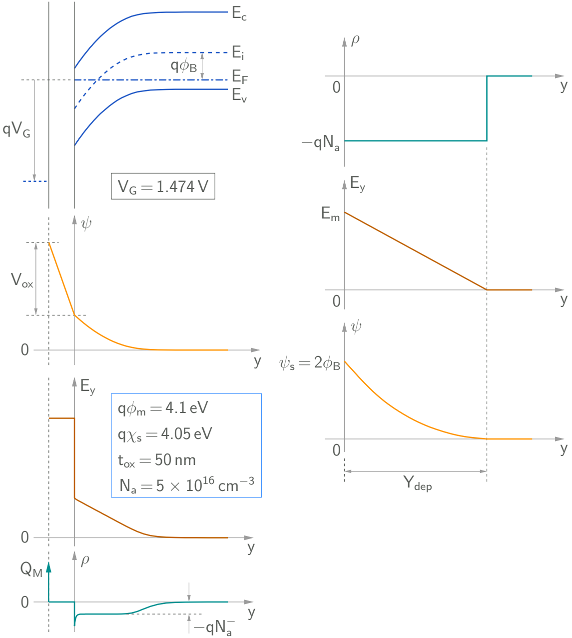
MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\mathrm{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\mathrm{Si}}} \int_{0^+}^{Y_{\mathrm{dep}}} \rho\, dy$$

$$\rightarrow \mathcal{E}_y(Y_{\mathrm{dep}}) - \mathcal{E}_y(0^+) = -\,\frac{qN_a Y_{\mathrm{dep}}}{\epsilon_{\mathrm{Si}}}$$



M. B. Patil, IIT Bombay

MOS capacitor: threshold voltage

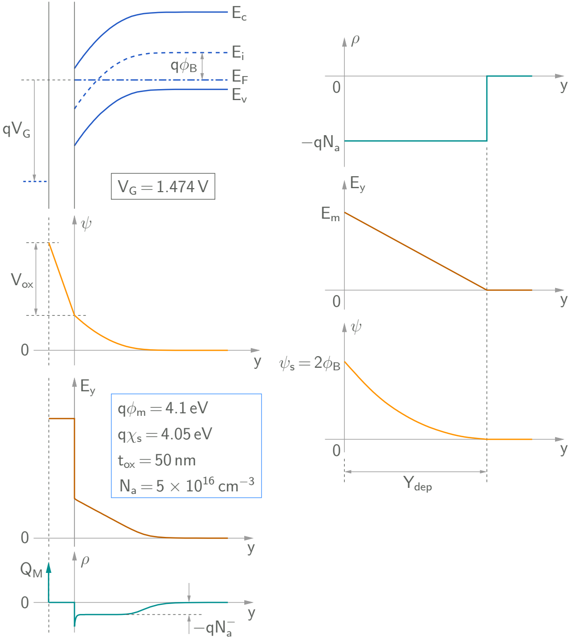
Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0+}^{Y_{dep}} d\mathcal{E}_y = \frac{1}{\epsilon_{Si}} \int_{0+}^{Y_{dep}} \rho\, dy$$

$$\rightarrow \mathcal{E}_y(Y_{dep}) - \mathcal{E}_y(0^+) = -\frac{qN_a Y_{dep}}{\epsilon_{Si}}$$

$$\rightarrow \mathcal{E}_y(0^+) = \frac{qN_a Y_{dep}}{\epsilon_{Si}}.$$



MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

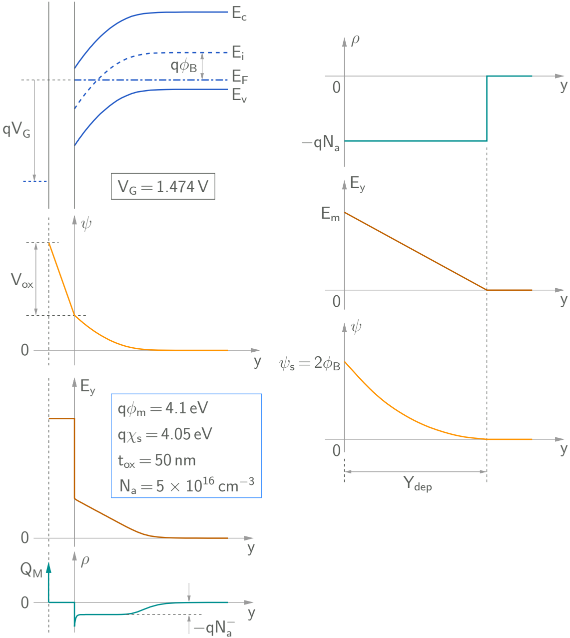
With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\mathrm{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\mathrm{Si}}} \int_{0^+}^{Y_{\mathrm{dep}}} \rho\, dy$$

$$\rightarrow \mathcal{E}_y(Y_{\mathrm{dep}}) - \mathcal{E}_y(0^+) = -\frac{qN_a Y_{\mathrm{dep}}}{\epsilon_{\mathrm{Si}}}$$

$$\rightarrow \mathcal{E}_y(0^+) = \frac{qN_a Y_{\mathrm{dep}}}{\epsilon_{\mathrm{Si}}}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow -\int_{0^+}^{Y_{\mathrm{dep}}} d\psi = \int_{0^+}^{Y_{\mathrm{dep}}} \mathcal{E}_y\, dy = \frac{qN_a}{2\epsilon_{\mathrm{Si}}} Y_{\mathrm{dep}}^2$$



M. B. Patil, IIT Bombay

MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

With the depletion approximation, and with $Y_{\rm dep}$ as the depletion width at the onset of inversion, we get

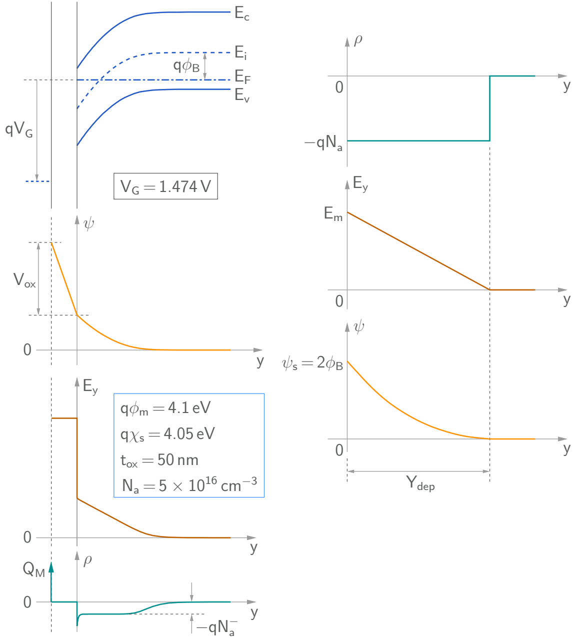
$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\rm dep}} d\mathcal{E}_y = \frac{1}{\epsilon_{\rm Si}} \int_{0^+}^{Y_{\rm dep}} \rho\, dy$$

$$\rightarrow \mathcal{E}_y(Y_{\rm dep}) - \mathcal{E}_y(0^+) = -\frac{qN_a Y_{\rm dep}}{\epsilon_{\rm Si}}$$

$$\rightarrow \mathcal{E}_y(0^+) = \frac{qN_a Y_{\rm dep}}{\epsilon_{\rm Si}}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow -\int_{0^+}^{Y_{\rm dep}} d\psi = \int_{0^+}^{Y_{\rm dep}} \mathcal{E}_y\, dy = \frac{qN_a}{2\epsilon_{\rm Si}} Y_{\rm dep}^2$$

$$\rightarrow \psi(0^+) = 2\phi_B = \frac{qN_a}{2\epsilon_{\rm Si}} Y_{\rm dep}^2$$



M. B. Patil, IIT Bombay

MOS capacitor: threshold voltage

Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

With the depletion approximation, and with $Y_{\rm dep}$ as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\rm dep}} d\mathcal{E}_y = \frac{1}{\epsilon_{\rm Si}} \int_{0^+}^{Y_{\rm dep}} \rho\, dy$$

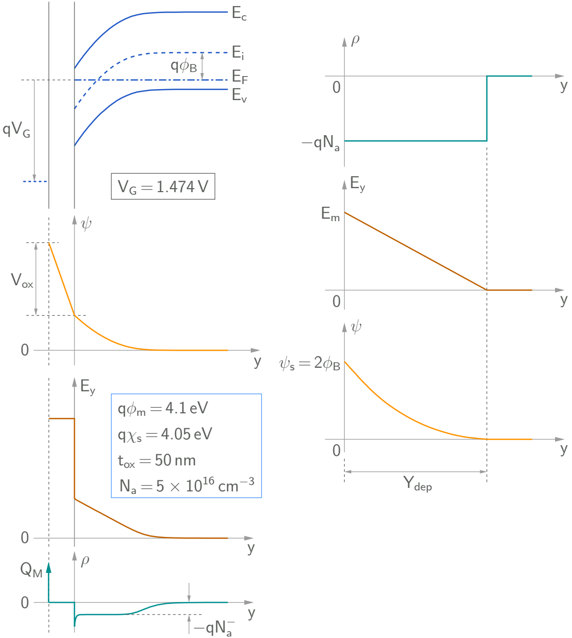
$$\rightarrow \mathcal{E}_y(Y_{\rm dep}) - \mathcal{E}_y(0^+) = -\frac{qN_a Y_{\rm dep}}{\epsilon_{\rm Si}}$$

$$\rightarrow \mathcal{E}_y(0^+) = \frac{qN_a Y_{\rm dep}}{\epsilon_{\rm Si}}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow -\int_{0^+}^{Y_{\rm dep}} d\psi = \int_{0^+}^{Y_{\rm dep}} \mathcal{E}_y\, dy = \frac{qN_a}{2\epsilon_{\rm Si}}\, Y_{\rm dep}^2$$

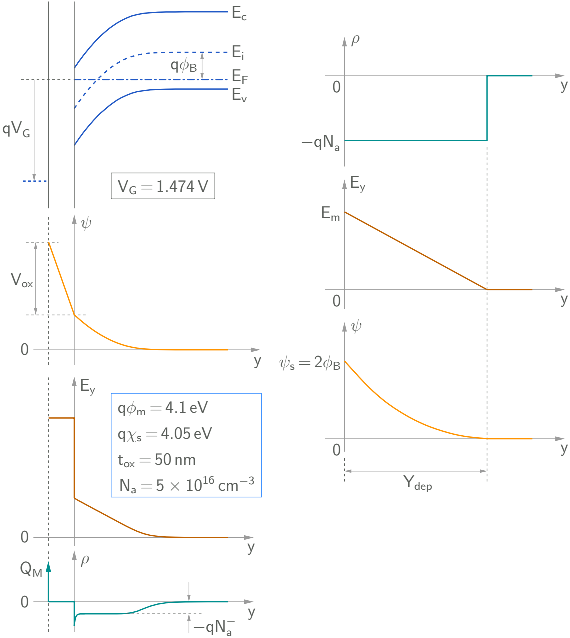
$$\rightarrow \psi(0^+) = 2\phi_B = \frac{qN_a}{2\epsilon_{\rm Si}}\, Y_{\rm dep}^2$$

$$\rightarrow Y_{\rm dep} = \sqrt{\frac{4\epsilon_{\rm Si}\phi_B}{qN_a}}.$$



M. B. Patil, IIT Bombay

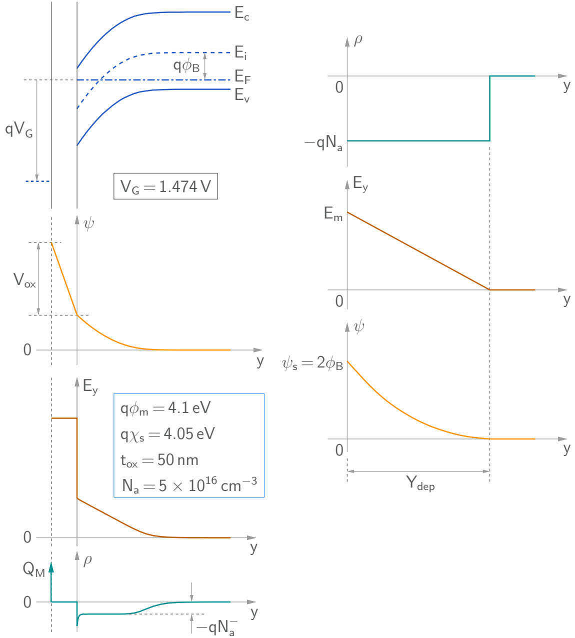
MOS capacitor: threshold voltage



MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO_2 interface, we have

$$\mathcal{E}_y^{\text{ox}} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}} = \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\,\frac{qN_aY_{\text{dep}}}{\epsilon_{\text{Si}}}$$

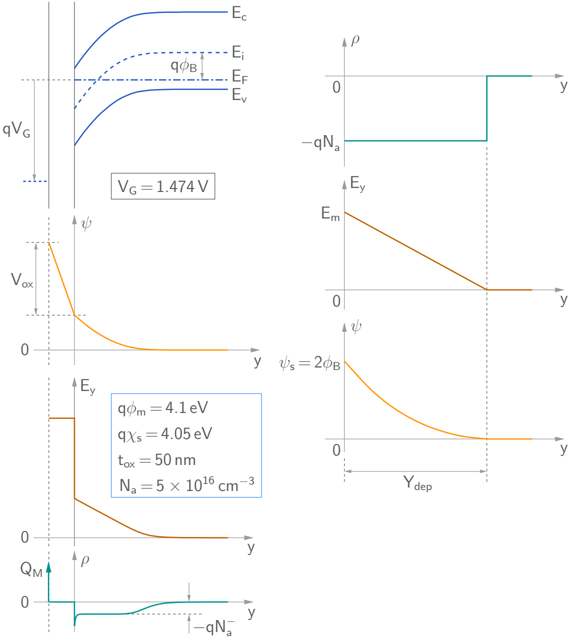


MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO_2 interface, we have

$$\mathcal{E}_y^{ox} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{Si}}{\epsilon_{ox}} = \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a Y_{dep}}{\epsilon_{Si}}$$

$$= \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a}{\epsilon_{Si}}\sqrt{\frac{4\epsilon_{Si}\phi_B}{qN_a}}$$

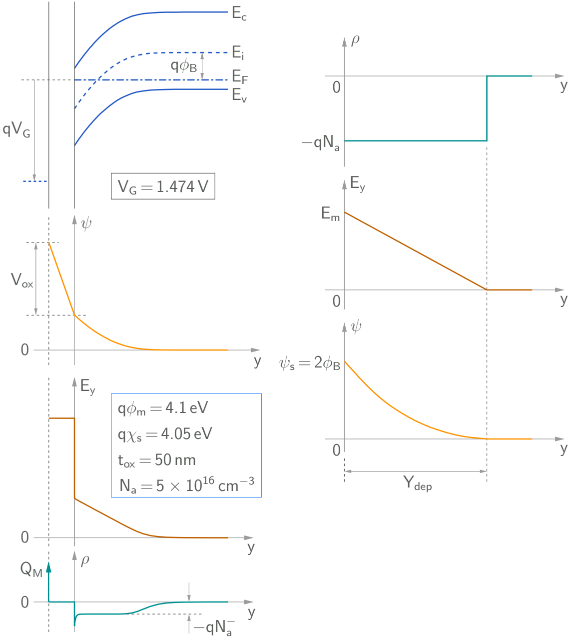


MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO_2 interface, we have

$$\mathcal{E}_y^{\text{ox}} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}} = \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\,\frac{qN_a Y_{\text{dep}}}{\epsilon_{\text{Si}}}$$

$$= \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\,\frac{qN_a}{\epsilon_{\text{Si}}}\sqrt{\frac{4\epsilon_{\text{Si}}\phi_B}{qN_a}} = \frac{1}{\epsilon_{\text{ox}}}\,\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}.$$



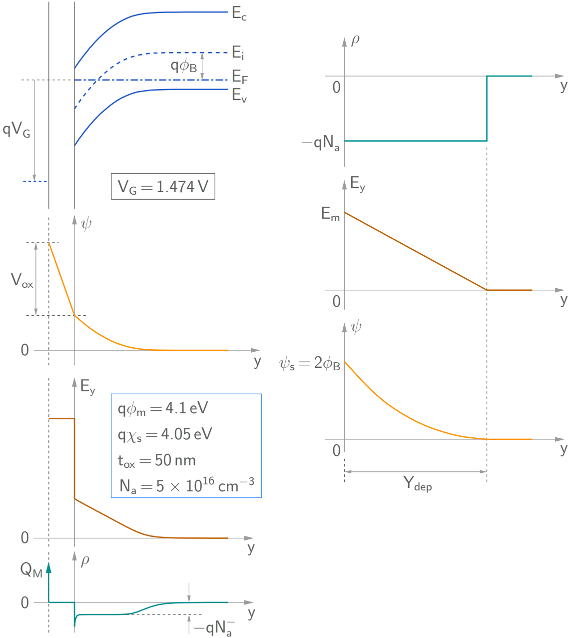
MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO$_2$ interface, we have

$$\mathcal{E}_y^{ox} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{Si}}{\epsilon_{ox}} = \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a Y_{dep}}{\epsilon_{Si}}$$

$$= \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a}{\epsilon_{Si}}\sqrt{\frac{4\epsilon_{Si}\phi_B}{qN_a}} = \frac{1}{\epsilon_{ox}}\sqrt{4qN_a\epsilon_{Si}\phi_B}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow \int_{-t_{ox}}^{0^-} d\psi = -\int_{-t_{ox}}^{0^-} \mathcal{E}_y^{ox}\,dy$$



M. B. Patil, IIT Bombay

MOS capacitor: threshold voltage

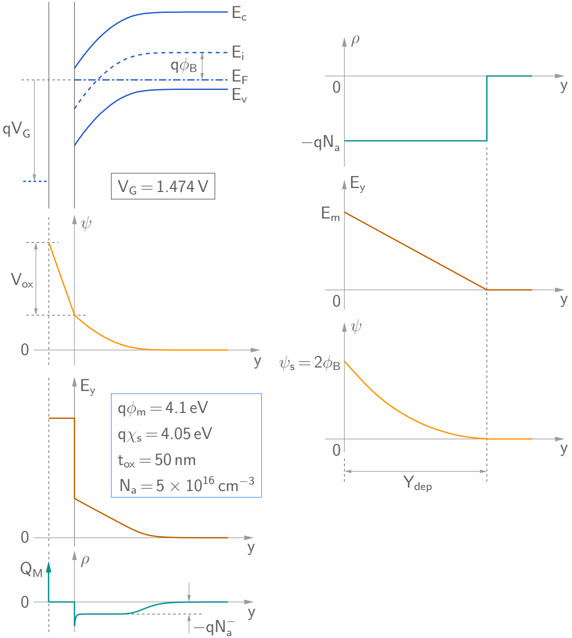
Assuming zero charge in the oxide and also at the Si-SiO_2 interface, we have

$$\mathcal{E}_y^{ox} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{Si}}{\epsilon_{ox}} = \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a Y_{dep}}{\epsilon_{Si}}$$

$$= \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a}{\epsilon_{Si}}\sqrt{\frac{4\epsilon_{Si}\phi_B}{qN_a}} = \frac{1}{\epsilon_{ox}}\sqrt{4qN_a\epsilon_{Si}\phi_B}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow \int_{-t_{ox}}^{0^-} d\psi = -\int_{-t_{ox}}^{0^-} \mathcal{E}_y^{ox}\,dy$$

$$\rightarrow \psi(0^-) - \psi(-t_{ox}) = -\mathcal{E}_y^{ox}\,t_{ox}.$$



MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO$_2$ interface, we have

$$\mathcal{E}_y^{\text{ox}} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}} = \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\,\frac{qN_a Y_{\text{dep}}}{\epsilon_{\text{Si}}}$$

$$= \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\,\frac{qN_a}{\epsilon_{\text{Si}}}\sqrt{\frac{4\epsilon_{\text{Si}}\phi_B}{qN_a}} = \frac{1}{\epsilon_{\text{ox}}}\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}.$$

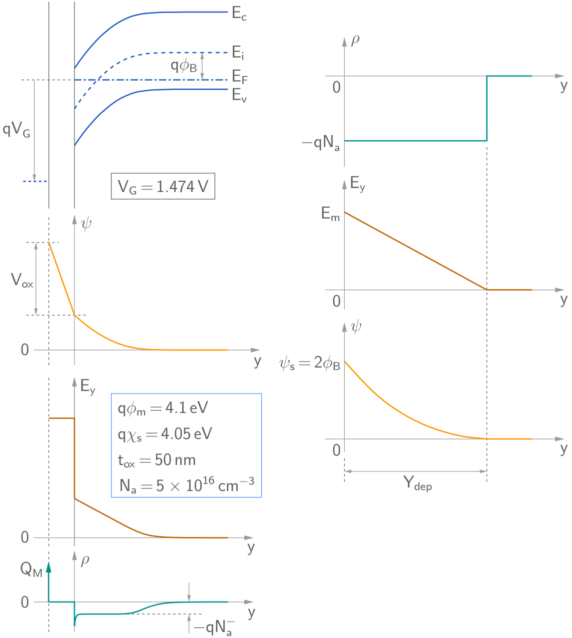
$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow \int_{-t_{\text{ox}}}^{0^-} d\psi = -\int_{-t_{\text{ox}}}^{0^-} \mathcal{E}_y^{\text{ox}}\, dy$$

$$\rightarrow \psi(0^-) - \psi(-t_{\text{ox}}) = -\mathcal{E}_y^{\text{ox}}\, t_{\text{ox}}.$$

The voltage drop across the oxide is

$$V_{\text{ox}} = \mathcal{E}_y^{\text{ox}} t_{\text{ox}} = \frac{t_{\text{ox}}}{\epsilon_{\text{ox}}}\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B} = \frac{\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}}{C_{\text{ox}}},$$

where $C_{\text{ox}} = \epsilon_{\text{ox}}/t_{\text{ox}}$ is the capacitance per unit area of a parallel plate capacitor with SiO$_2$ as the dielectric and a dielectric thickness t_{ox}.



MOS capacitor: threshold voltage

Assuming zero charge in the oxide and also at the Si-SiO_2 interface, we have

$$\mathcal{E}_y^{ox} = \mathcal{E}_y(0^-) = \mathcal{E}_y(0^+)\,\frac{\epsilon_{Si}}{\epsilon_{ox}} = \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a Y_{dep}}{\epsilon_{Si}}$$

$$= \frac{\epsilon_{Si}}{\epsilon_{ox}}\,\frac{qN_a}{\epsilon_{Si}}\sqrt{\frac{4\epsilon_{Si}\phi_B}{qN_a}} = \frac{1}{\epsilon_{ox}}\sqrt{4qN_a\epsilon_{Si}\phi_B}.$$

$$\mathcal{E}_y = -\frac{d\psi}{dy} \rightarrow \int_{-t_{ox}}^{0^-} d\psi = -\int_{-t_{ox}}^{0^-} \mathcal{E}_y^{ox}\,dy$$

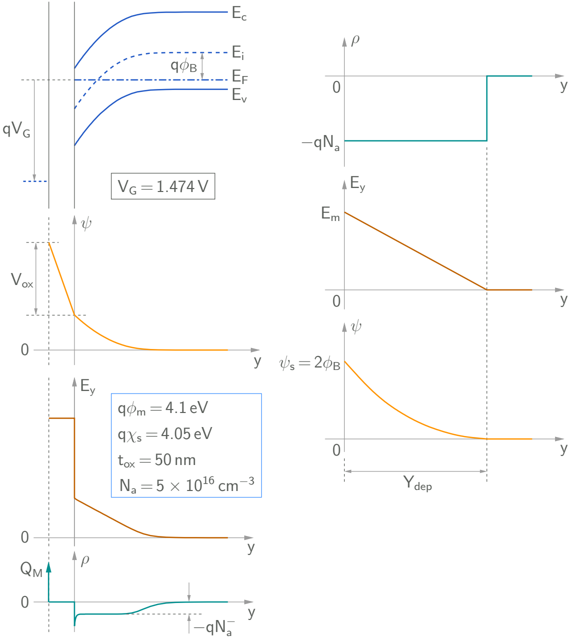
$$\rightarrow \psi(0^-) - \psi(-t_{ox}) = -\mathcal{E}_y^{ox}\,t_{ox}.$$

The voltage drop across the oxide is

$$V_{ox} = \mathcal{E}_y^{ox}\,t_{ox} = \frac{t_{ox}}{\epsilon_{ox}}\sqrt{4qN_a\epsilon_{Si}\phi_B} = \frac{\sqrt{4qN_a\epsilon_{Si}\phi_B}}{C_{ox}},$$

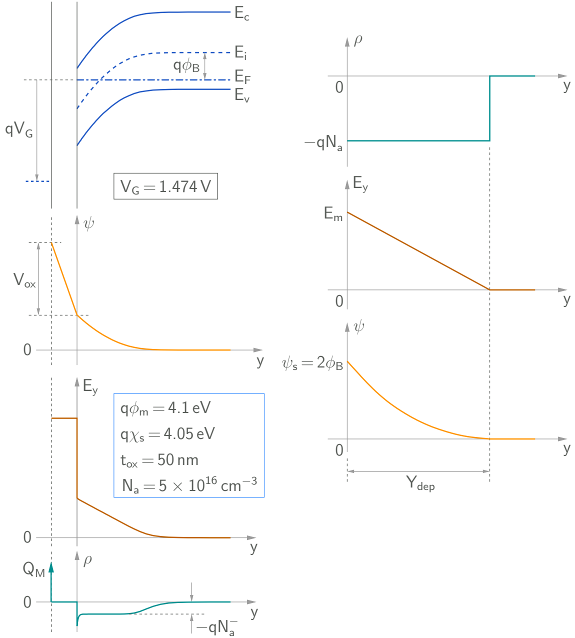
where $C_{ox} = \epsilon_{ox}/t_{ox}$ is the capacitance per unit area of a parallel plate capacitor with SiO_2 as the dielectric and a dielectric thickness t_{ox}.

Units of C_{ox}: $\dfrac{\text{F/cm}}{\text{cm}} = \dfrac{\text{F}}{\text{cm}^2}$.



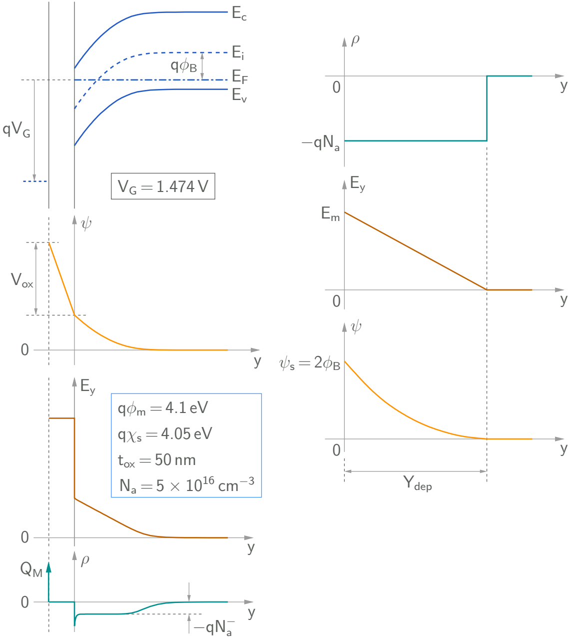
M. B. Patil, IIT Bombay

MOS capacitor: threshold voltage



MOS capacitor: threshold voltage

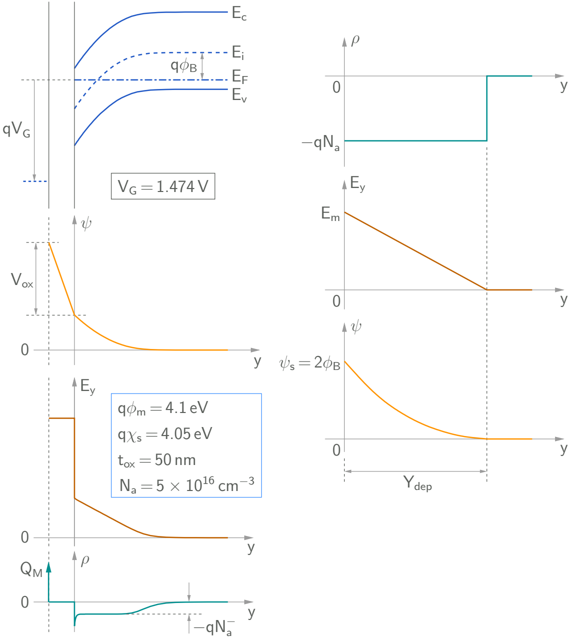
The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).



MOS capacitor: threshold voltage

The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).

With $V_G = V_{FB}$, $\psi_s = 0\,V$, and $V_{ox} = 0\,V$.

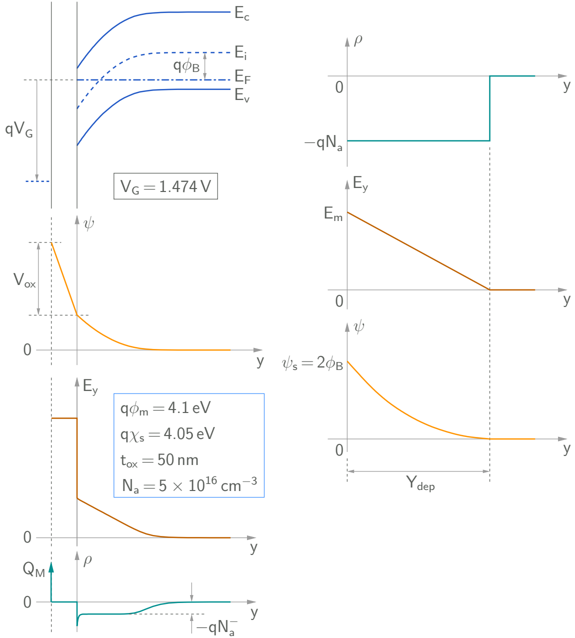


MOS capacitor: threshold voltage

The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).

With $V_G = V_{FB}$, $\psi_s = 0\,V$, and $V_{ox} = 0\,V$.

$\rightarrow$ *In addition to* V_{FB}, a voltage $\psi_s + V_{ox}$, i.e., $2\,\phi_B + V_{ox}$, is required for inversion.



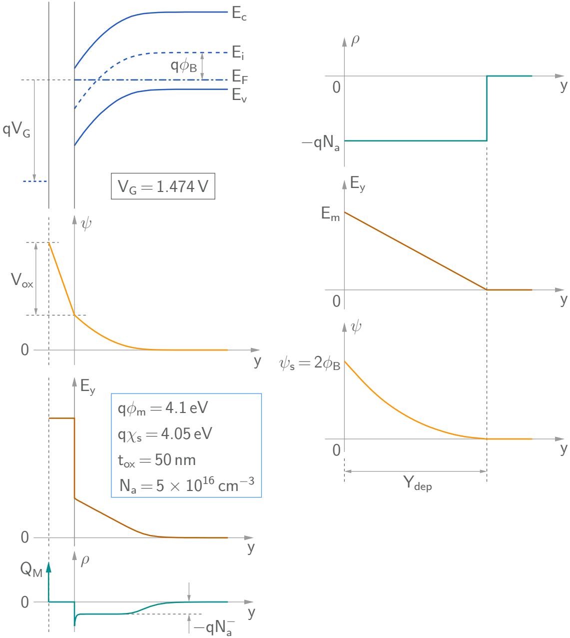
MOS capacitor: threshold voltage

The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).

With $V_G = V_{FB}$, $\psi_s = 0\,\text{V}$, and $V_{ox} = 0\,\text{V}$.

$\rightarrow$ *In addition to* V_{FB}, a voltage $\psi_s + V_{ox}$, i.e., $2\,\phi_B + V_{ox}$, is required for inversion.

$\rightarrow$ $V_{th} = V_{FB} + \psi_s + V_{ox}$



MOS capacitor: threshold voltage

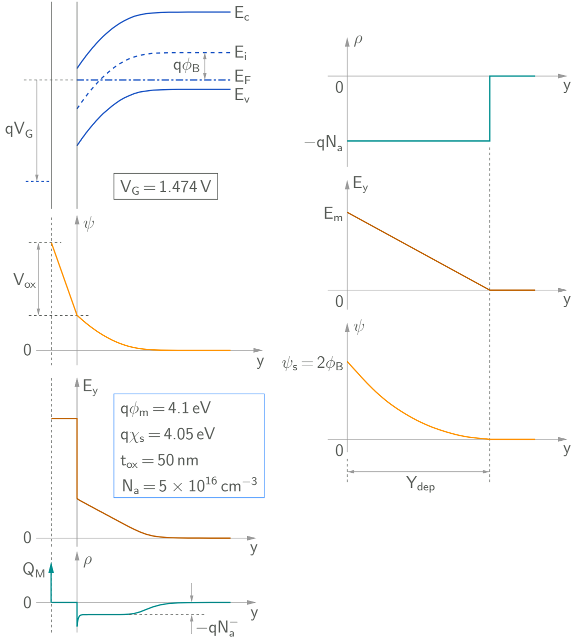
The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).

With $V_G = V_{FB}$, $\psi_s = 0\,\text{V}$, and $V_{ox} = 0\,\text{V}$.

→ *In addition to* V_{FB}, a voltage $\psi_s + V_{ox}$, i.e., $2\,\phi_B + V_{ox}$, is required for inversion.

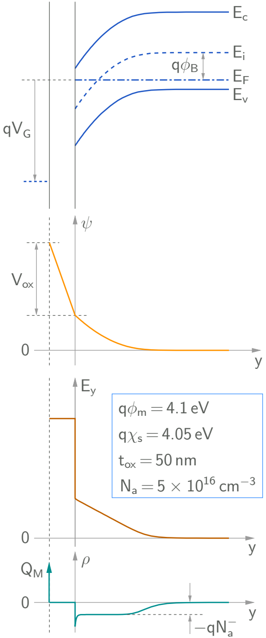
→ $V_{th} = V_{FB} + \psi_s + V_{ox}$

$$= V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{Si}\phi_B}}{C_{ox}}.$$



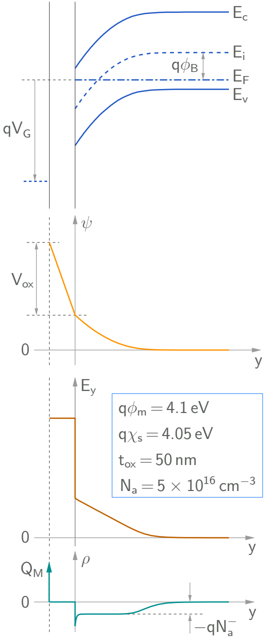
Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.



Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

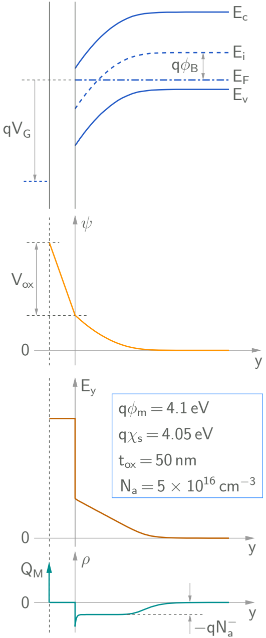


Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$



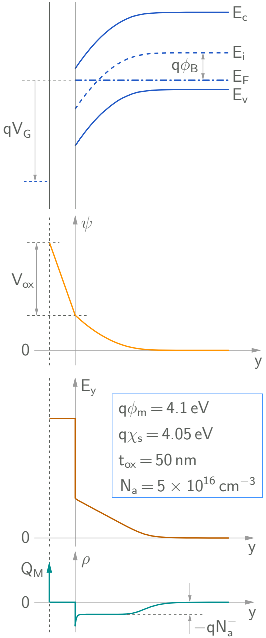
Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$



Example

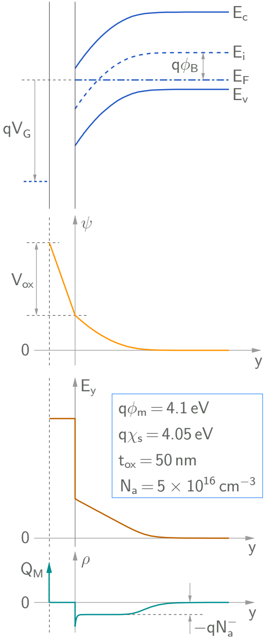
Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$



Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

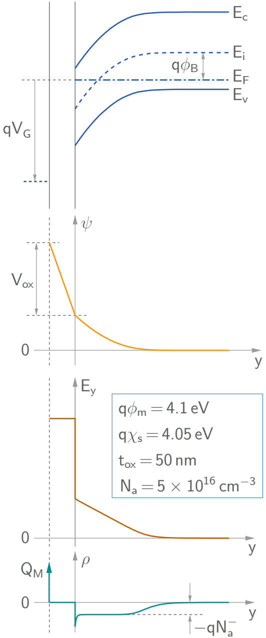
Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$

$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \rightarrow \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$



Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

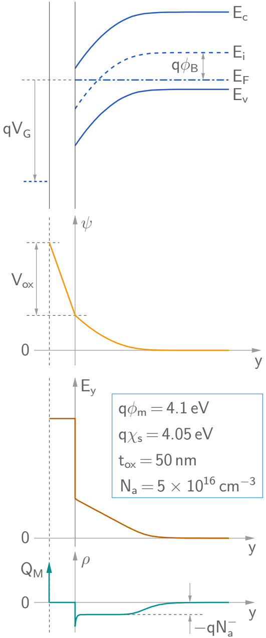
$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$

$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \rightarrow \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}\,\text{F/cm}}{50 \times 10^{-7}\,\text{cm}} = 69\,\text{nF/cm}^2.$$



Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

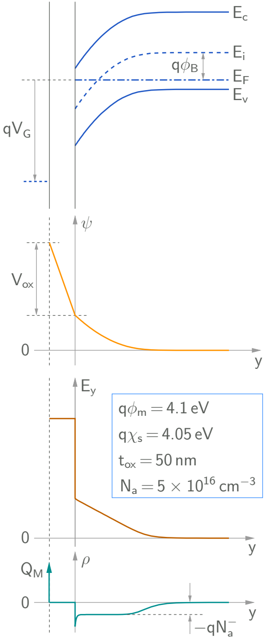
$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$
$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$

$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \rightarrow \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}\,\text{F/cm}}{50 \times 10^{-7}\,\text{cm}} = 69\,\text{nF/cm}^2.$$

$$\sqrt{4qN_a\epsilon_{Si}\phi_B} = \sqrt{4 \times 1.6 \times 10^{-19} \times 5 \times 10^{16} \times 11.7 \times 8.85 \times 10^{-14} \times 0.41}$$



Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.
What is the depletion width at the onset of inversion?

Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

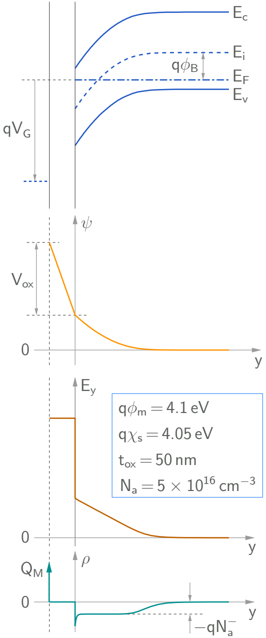
$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$

$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \rightarrow \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}\,\text{F/cm}}{50 \times 10^{-7}\,\text{cm}} = 69\,\text{nF/cm}^2.$$

$$\sqrt{4qN_a\epsilon_{Si}\phi_B} = \sqrt{4 \times 1.6 \times 10^{-19} \times 5 \times 10^{16} \times 11.7 \times 8.85 \times 10^{-14} \times 0.41}$$

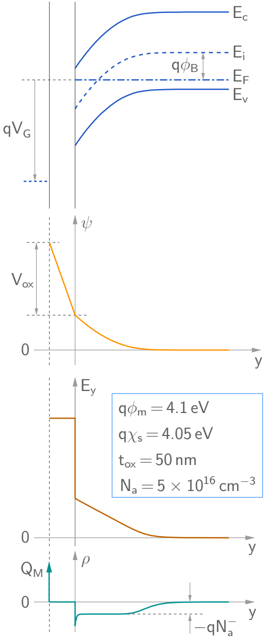
$$= 1.16 \times 10^{-7}\,\text{C/cm}^2.$$



Example (continued)

$$V_{\text{th}} = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}}{C_{\text{ox}}}$$

$$= -0.93 + 2 \times 0.41 + \frac{1.16 \times 10^{-7}}{69 \times 10^{-9}} = 1.57\,\text{V}.$$



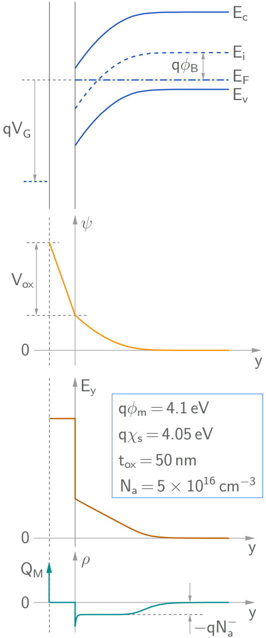
Example (continued)

$$V_{\rm th} = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\rm Si}\phi_B}}{C_{\rm ox}}$$

$$= -0.93 + 2 \times 0.41 + \frac{1.16 \times 10^{-7}}{69 \times 10^{-9}} = 1.57\,{\rm V}.$$

The depletion width at the onset of inversion is

$$Y_{\rm dep}^{\rm inv} = \sqrt{\frac{4\epsilon_{\rm Si}\phi_B}{qN_a}} = \sqrt{\frac{4 \times 11.7 \times 8.85 \times 10^{-14} \times 0.41}{1.6 \times 10^{-19} \times 5 \times 10^{16}}}$$



Example (continued)

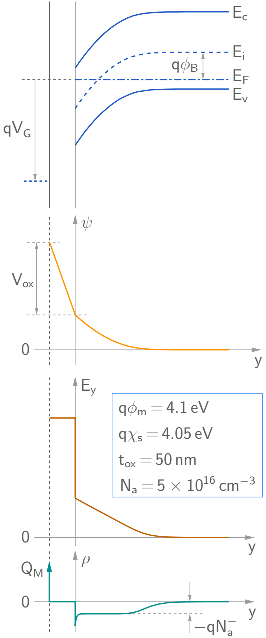
$$V_{\text{th}} = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}}{C_{\text{ox}}}$$

$$= -0.93 + 2 \times 0.41 + \frac{1.16 \times 10^{-7}}{69 \times 10^{-9}} = 1.57\,\text{V}.$$

The depletion width at the onset of inversion is

$$Y_{\text{dep}}^{\text{inv}} = \sqrt{\frac{4\epsilon_{\text{Si}}\phi_B}{qN_a}} = \sqrt{\frac{4 \times 11.7 \times 8.85 \times 10^{-14} \times 0.41}{1.6 \times 10^{-19} \times 5 \times 10^{16}}}$$

$$= 1.45 \times 10^{-5}\,\text{cm} = 0.145\,\mu\text{m}.$$



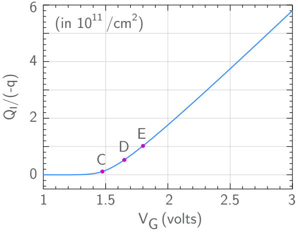
MOS capacitor: inversion charge

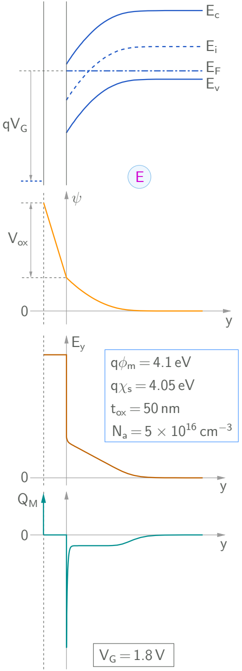
Consider $V_G > V_{\text{th}}$. We have $V_G = V_{FB} + \psi_s + V_{\text{ox}}$.

The surface potential ψ_s stays approximately constant $(= 2\phi_B)$ in inversion.

$\rightarrow$ The "excess" gate voltage (beyond V_{th}) can only appear as a change in V_{ox}.

$V_G \approx V_{FB} + 2\phi_B + V_{\text{ox}}$, where $V_{\text{ox}} = \mathcal{E}(0^-)\, t_{\text{ox}} = \mathcal{E}(0^+)\, \dfrac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\, t_{\text{ox}}$.





MOS capacitor: inversion charge

Consider $V_G > V_{\rm th}$. We have $V_G = V_{FB} + \psi_s + V_{\rm ox}$.

The surface potential ψ_s stays approximately constant ($= 2\phi_B$) in inversion.

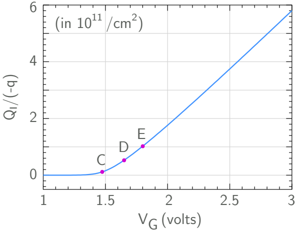
$\rightarrow$ The "excess" gate voltage (beyond $V_{\rm th}$) can only appear as a change in $V_{\rm ox}$.

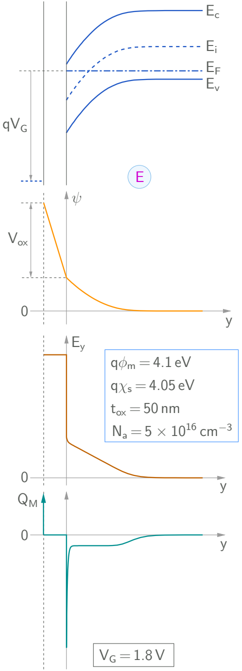
$V_G \approx V_{FB} + 2\phi_B + V_{\rm ox}$, where $V_{\rm ox} = \mathcal{E}(0^-)\, t_{\rm ox} = \mathcal{E}(0^+)\, \dfrac{\epsilon_{\rm Si}}{\epsilon_{\rm ox}}\, t_{\rm ox}$.

$$\int_{0^+}^{Y_{\rm dep}} d\mathcal{E}_y = \frac{1}{\epsilon_{\rm Si}} \int_{0^+}^{Y_{\rm dep}} \rho\, dy = \frac{1}{\epsilon_{\rm Si}} \int_{0^+}^{Y_{\rm dep}} q\,(-N_a^- - n)\, dy$$

$$\rightarrow \mathcal{E}(Y_{\rm dep}) - \mathcal{E}(0^+) = -\mathcal{E}(0^+) = \frac{1}{\epsilon_{\rm Si}}\left(-qN_a Y_{\rm dep} + Q_I\right) \rightarrow \mathcal{E}(0^+) = \frac{qN_a Y_{\rm dep}}{\epsilon_{\rm Si}} - \frac{Q_I}{\epsilon_{\rm Si}},\ \text{where}$$

$$Y_{\rm dep} \approx Y_{\rm dep}^{\rm inv} = \sqrt{\frac{4\epsilon_{\rm Si}\phi_B}{qN_a}}\ (=\text{depletion width at the onset of inversion}).$$





MOS capacitor: inversion charge

Consider $V_G > V_{\text{th}}$. We have $V_G = V_{FB} + \psi_s + V_{\text{ox}}$.

The surface potential ψ_s stays approximately constant ($= 2\phi_B$) in inversion.

$\rightarrow$ The "excess" gate voltage (beyond V_{th}) can only appear as a change in V_{ox}.

$V_G \approx V_{FB} + 2\phi_B + V_{\text{ox}}$, where $V_{\text{ox}} = \mathcal{E}(0^-)\, t_{\text{ox}} = \mathcal{E}(0^+)\, \dfrac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\, t_{\text{ox}}$.

$$\int_{0^+}^{Y_{\text{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\text{Si}}} \int_{0^+}^{Y_{\text{dep}}} \rho\, dy = \frac{1}{\epsilon_{\text{Si}}} \int_{0^+}^{Y_{\text{dep}}} q\,(-N_a^- - n)\, dy$$

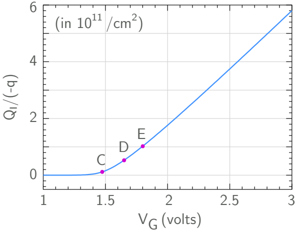
$\rightarrow \mathcal{E}(Y_{\text{dep}}) - \mathcal{E}(0^+) = -\mathcal{E}(0^+) = \dfrac{1}{\epsilon_{\text{Si}}}\,(-qN_a Y_{\text{dep}} + Q_I) \rightarrow \mathcal{E}(0^+) = \dfrac{qN_a Y_{\text{dep}}}{\epsilon_{\text{Si}}} - \dfrac{Q_I}{\epsilon_{\text{Si}}}$, where

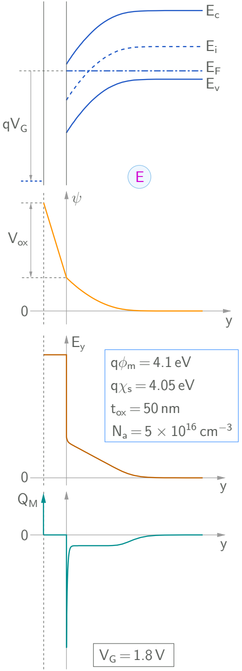
$Y_{\text{dep}} \approx Y_{\text{dep}}^{\text{inv}} = \sqrt{\dfrac{4\epsilon_{\text{Si}}\phi_B}{qN_a}}$ ($=$ depletion width at the onset of inversion).

Putting together the various terms, we get

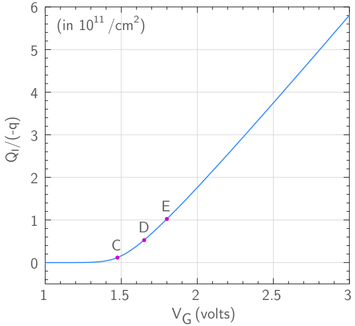
$$V_G = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}}{C_{\text{ox}}} - \frac{Q_I}{C_{\text{ox}}} = V_{\text{th}} - \frac{Q_I}{C_{\text{ox}}}.$$

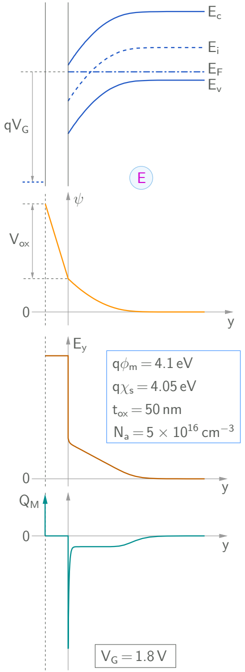
$\rightarrow Q_I = -C_{\text{ox}}(V_G - V_{\text{th}})$.



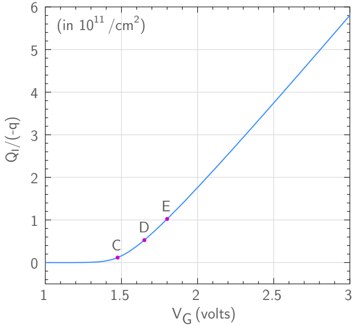


MOS capacitor: inversion charge



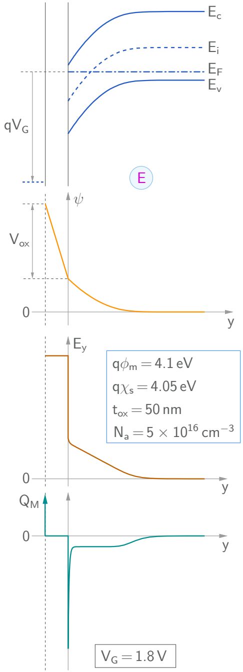


MOS capacitor: inversion charge

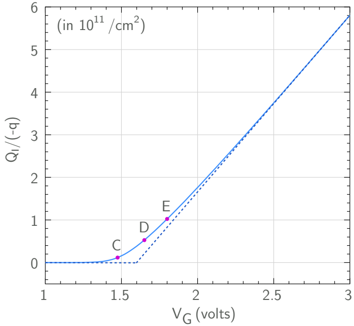


In a MOS capacitor with a uniformly doped *p*-type substrate, we can describe the inversion charge with the following approximate relationship.

$$
\begin{aligned}
Q_I &= 0, && V_G \leq V_{\text{th}}, \\
&= -C_{\text{ox}}(V_G - V_{\text{th}}), && V_G > V_{\text{th}}.
\end{aligned}
$$

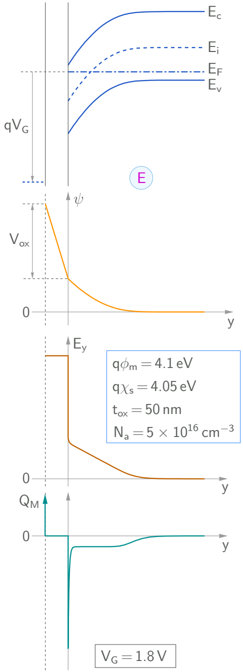


MOS capacitor: inversion charge

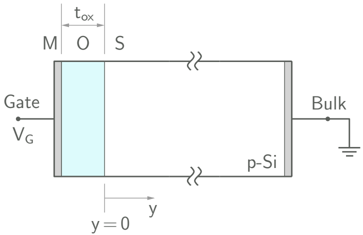


In a MOS capacitor with a uniformly doped *p*-type substrate, we can describe the inversion charge with the following approximate relationship.

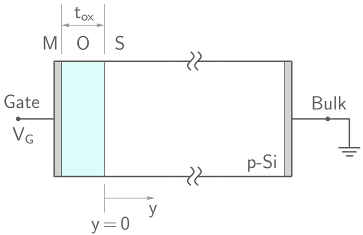
$$
\begin{aligned}
Q_I &= 0, && V_G \leq V_{\mathrm{th}}, \\
&= -C_{\mathrm{ox}}(V_G - V_{\mathrm{th}}), && V_G > V_{\mathrm{th}}.
\end{aligned}
$$



MOS capacitor: C-V relationship

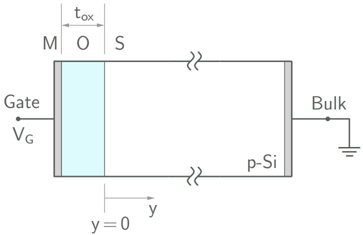


MOS capacitor: C-V relationship



* The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.

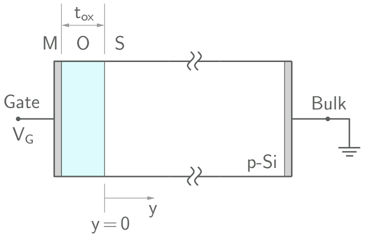
MOS capacitor: C-V relationship



* The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.

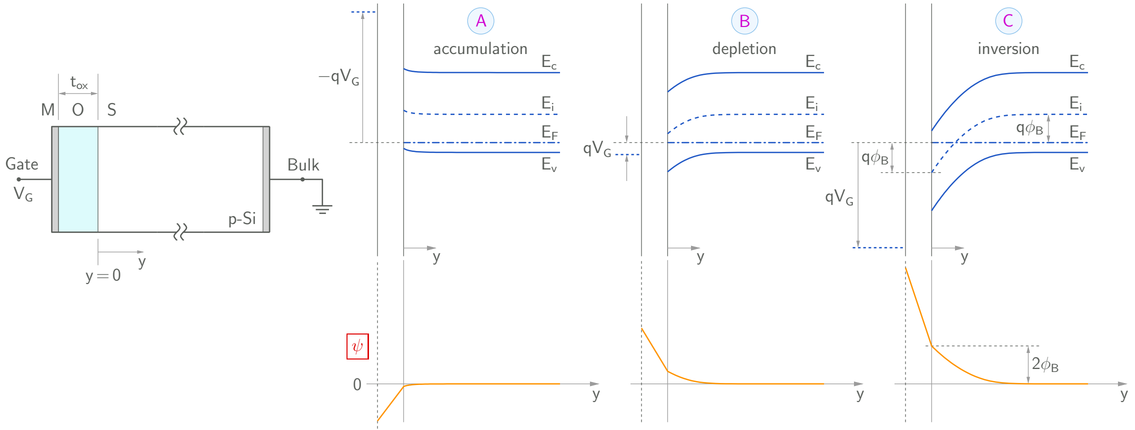
* The differential capacitance $C = \dfrac{dQ}{dV_G}$ is of great interest since it contains information about several important parameters, such as the oxide thickness, oxide charge, and doping density in the semiconductor.

MOS capacitor: C-V relationship

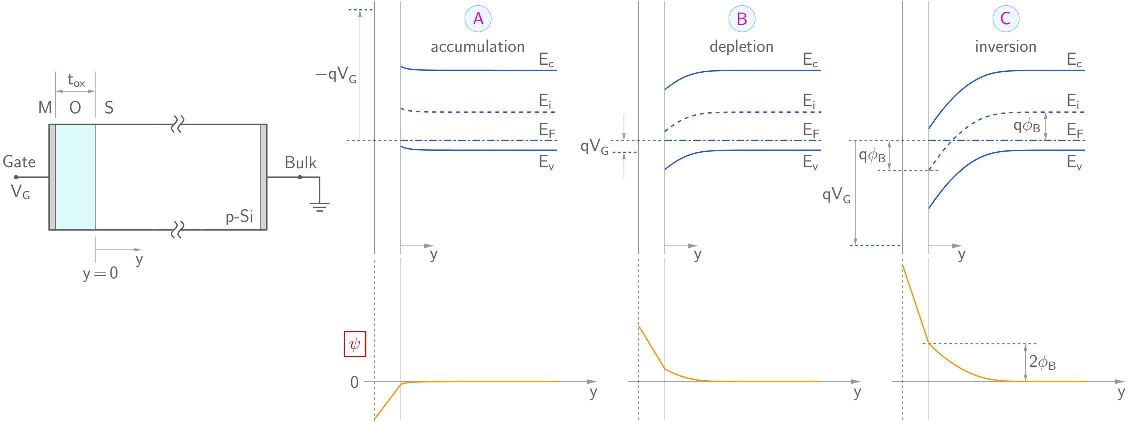


* The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.
* The differential capacitance $C = \dfrac{dQ}{dV_G}$ is of great interest since it contains information about several important parameters, such as the oxide thickness, oxide charge, and doping density in the semiconductor.
* C depends on the bias (DC) value of V_G. A plot of the capacitance C versus the bias voltage is known as the MOS C-V curve, and it serves as an important tool for process evaluation.

MOS capacitor: C-V relationship

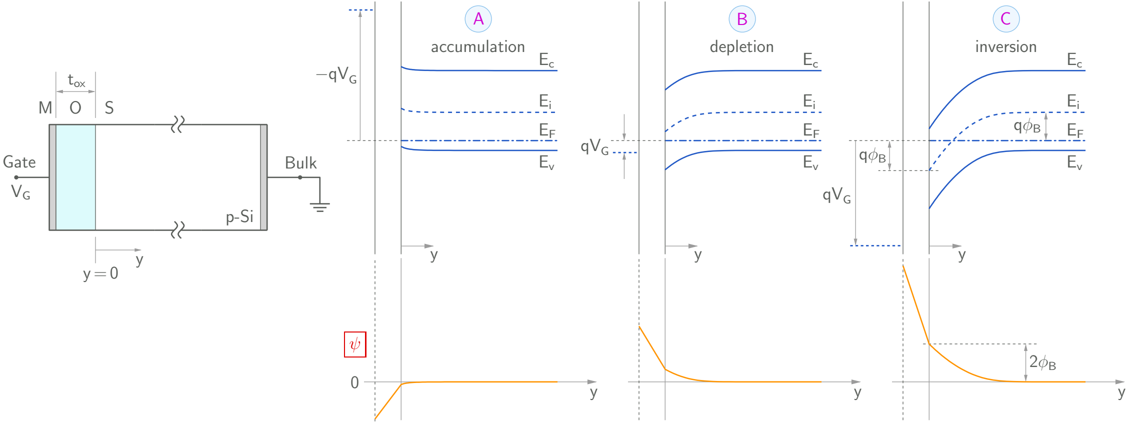


MOS capacitor: C-V relationship



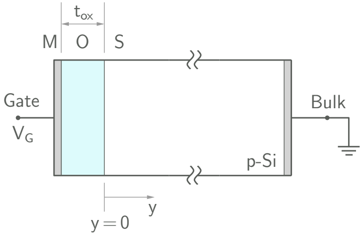
* $V_G = V_{FB} + V_{ox} + \psi_{Si}$.

MOS capacitor: C-V relationship

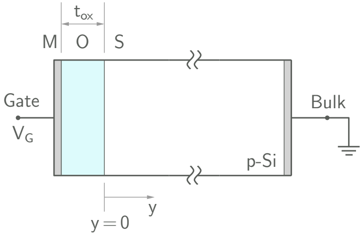


* $V_G = V_{FB} + V_{ox} + \psi_{Si}$.
* ψ_{Si}, the voltage drop across the semiconductor, is the same as the surface potential ψ_s if we take $\psi(\infty)$ as 0 V.

MOS capacitor: C-V relationship

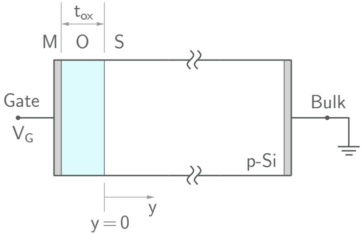


MOS capacitor: C-V relationship



Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^\infty \rho \, dy$.

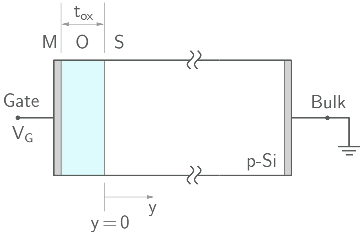
MOS capacitor: C-V relationship



Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^\infty \rho\, dy$.

If $V_G \to V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

MOS capacitor: C-V relationship

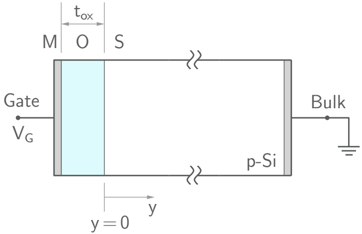


Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^{\infty} \rho \, dy$.

If $V_G \rightarrow V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{\mathrm{ox}} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{\mathrm{ox}}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q}$$

MOS capacitor: C-V relationship

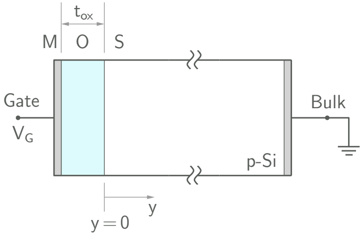


Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^{\infty} \rho \, dy$.

If $V_G \to V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{\mathrm{ox}} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{\mathrm{ox}}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q} \quad \to \quad \frac{1}{\left(\dfrac{dQ}{dV_G}\right)} = \frac{1}{\left(\dfrac{dQ}{dV_{\mathrm{ox}}}\right)} + \frac{1}{\left(\dfrac{dQ}{d\psi_s}\right)},$$

MOS capacitor: C-V relationship



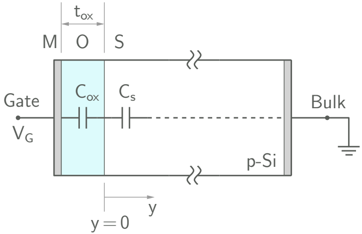
Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^{\infty} \rho \, dy$.

If $V_G \rightarrow V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{ox} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{ox}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q} \quad \rightarrow \quad \frac{1}{\left(\dfrac{dQ}{dV_G}\right)} = \frac{1}{\left(\dfrac{dQ}{dV_{ox}}\right)} + \frac{1}{\left(\dfrac{dQ}{d\psi_s}\right)},$$

i.e., $C = \dfrac{dQ}{dV_G}$ is given by $\dfrac{1}{C} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_s}$, a series connection of C_{ox} and C_s.

MOS capacitor: C-V relationship

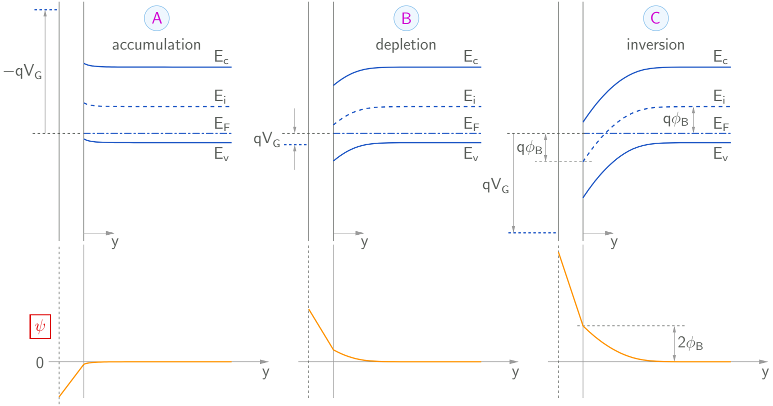


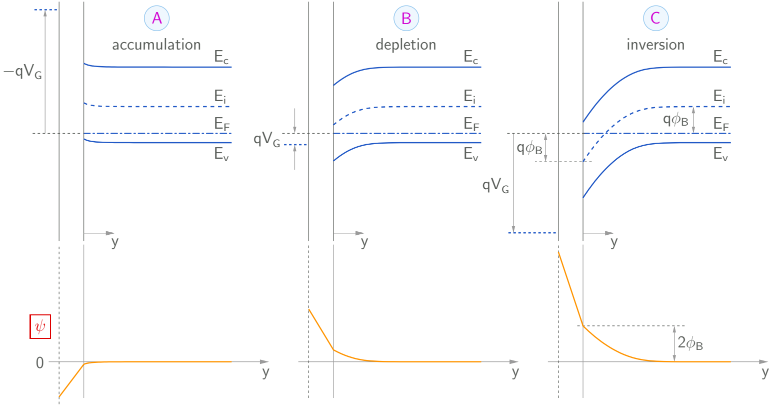
Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^{\infty} \rho\, dy$.

If $V_G \rightarrow V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

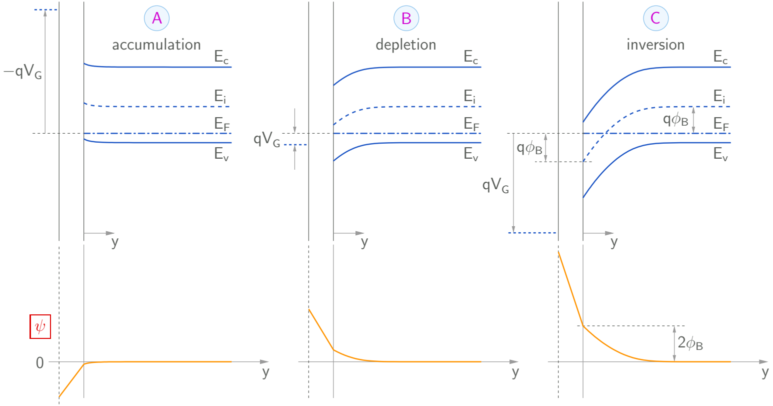
$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{ox} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{ox}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q} \quad \rightarrow \quad \frac{1}{\left(\dfrac{dQ}{dV_G}\right)} = \frac{1}{\left(\dfrac{dQ}{dV_{ox}}\right)} + \frac{1}{\left(\dfrac{dQ}{d\psi_s}\right)},$$

i.e., $C = \dfrac{dQ}{dV_G}$ is given by $\dfrac{1}{C} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_s}$, a series connection of C_{ox} and C_s.


A
accumulation
B
depletion
C
inversion
E_c
E_i
E_F
E_v
$-qV_G$
qV_G
$q\phi_B$
$2\phi_B$
ψ
0
y

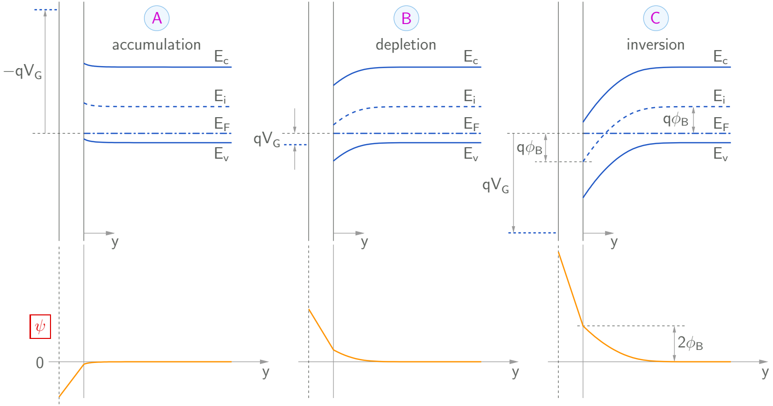


To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.



To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.

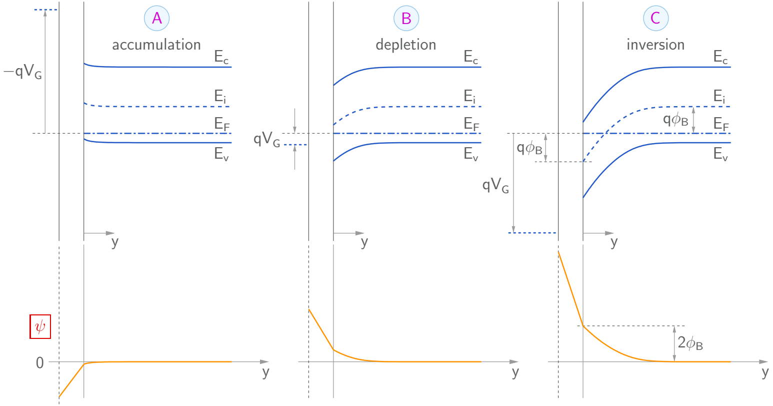
Sufficiently far from the interface, $\psi = 0\,\text{V}$, $n = n_0$, $p = p_0$, $\rho = p - n - N_a^- = 0$.



To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.

Sufficiently far from the interface, $\psi = 0\,\text{V}$, $n = n_0$, $p = p_0$, $\rho = p - n - N_a^- = 0$.

Poisson's equation: $\dfrac{d\mathcal{E}}{dx} = \dfrac{\rho}{\epsilon_{\text{Si}}} = \dfrac{q}{\epsilon_{\text{Si}}}\,(p - n - N_a^-)$.



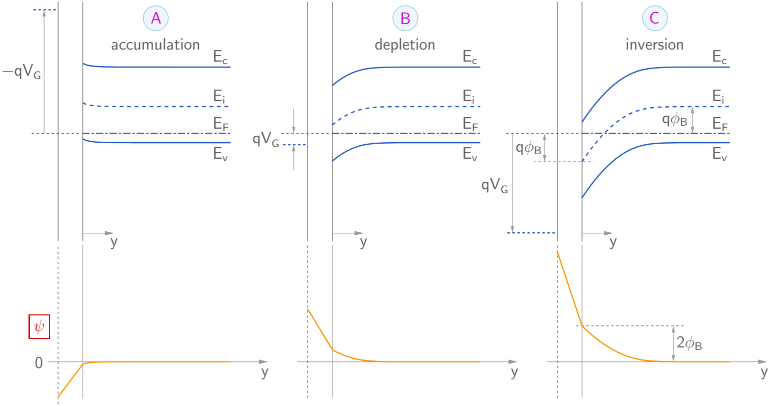
To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.

Sufficiently far from the interface, $\psi = 0\,\text{V}$, $n = n_0$, $p = p_0$, $\rho = p - n - N_a^- = 0$.

Poisson's equation: $\dfrac{d\mathcal{E}}{dx} = \dfrac{\rho}{\epsilon_{Si}} = \dfrac{q}{\epsilon_{Si}}\,(p - n - N_a^-)$.

$$\frac{d\mathcal{E}}{dx} = \frac{d\mathcal{E}}{d\psi}\,\frac{d\psi}{dx} = -\,\mathcal{E}\,\frac{d\mathcal{E}}{d\psi}$$

M. B. Patil, IIT Bombay

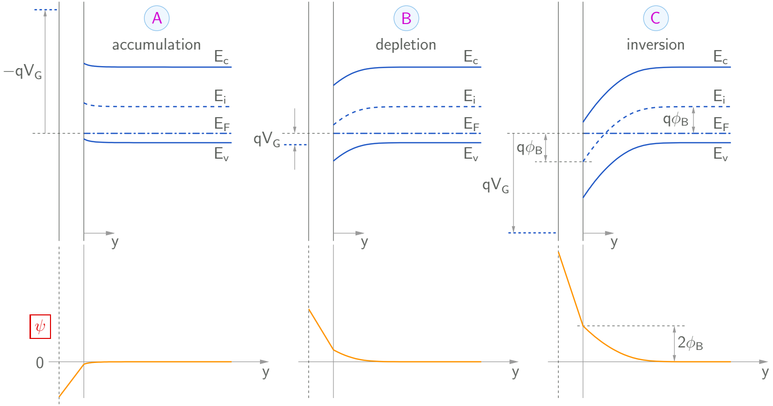


To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.

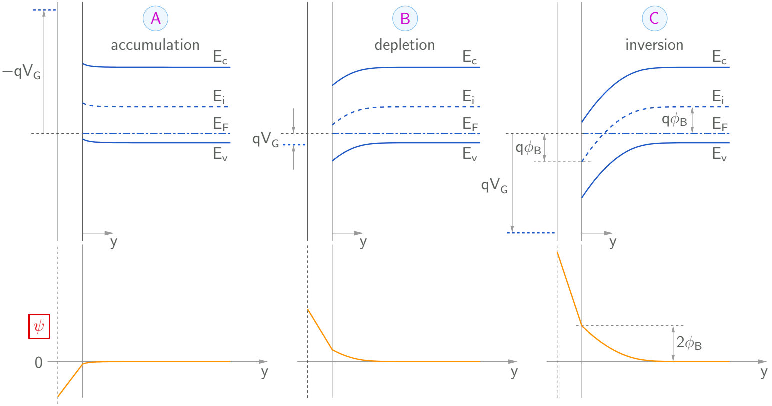
Sufficiently far from the interface, $\psi = 0\,\text{V}$, $n = n_0$, $p = p_0$, $\rho = p - n - N_a^- = 0$.

Poisson's equation: $\dfrac{d\mathcal{E}}{dx} = \dfrac{\rho}{\epsilon_{Si}} = \dfrac{q}{\epsilon_{Si}}\,(p - n - N_a^-)$.

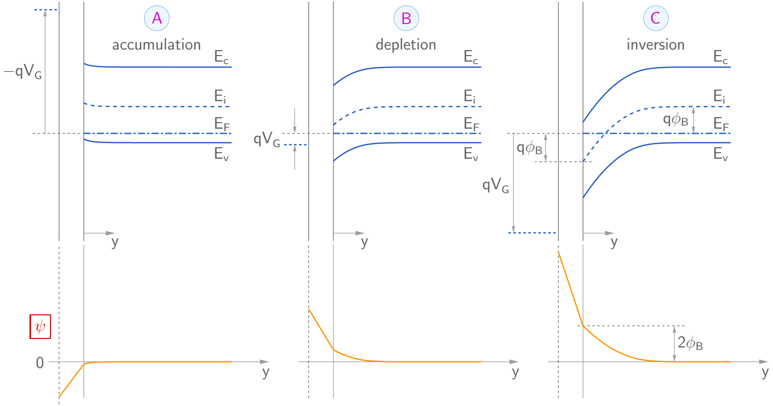
$$\frac{d\mathcal{E}}{dx} = \frac{d\mathcal{E}}{d\psi}\,\frac{d\psi}{dx} = -\,\mathcal{E}\,\frac{d\mathcal{E}}{d\psi} \quad \rightarrow \; \mathcal{E}\,d\mathcal{E} = \frac{q}{\epsilon_{Si}}\left[n_0\left(e^{\psi/V_T} - 1\right) - p_0\left(e^{-\psi/V_T} - 1\right)\right]d\psi.$$



A
accumulation
B
depletion
C
inversion
E_c
E_i
E_F
E_v
$-qV_G$
qV_G
$q\phi_B$
y
ψ
0
$2\phi_B$

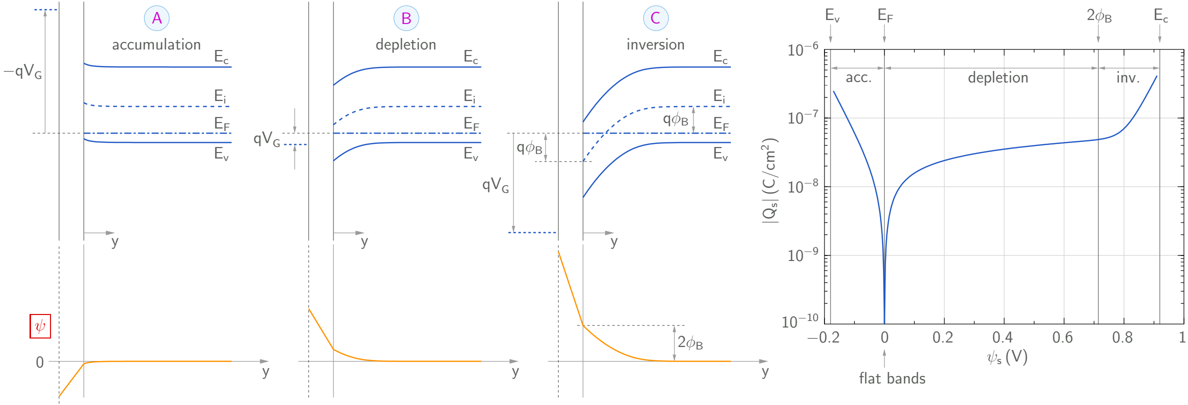


$$\int_{y=0^+}^{\infty} \mathcal{E} d\mathcal{E} = \frac{q}{\epsilon_{\mathrm{Si}}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$



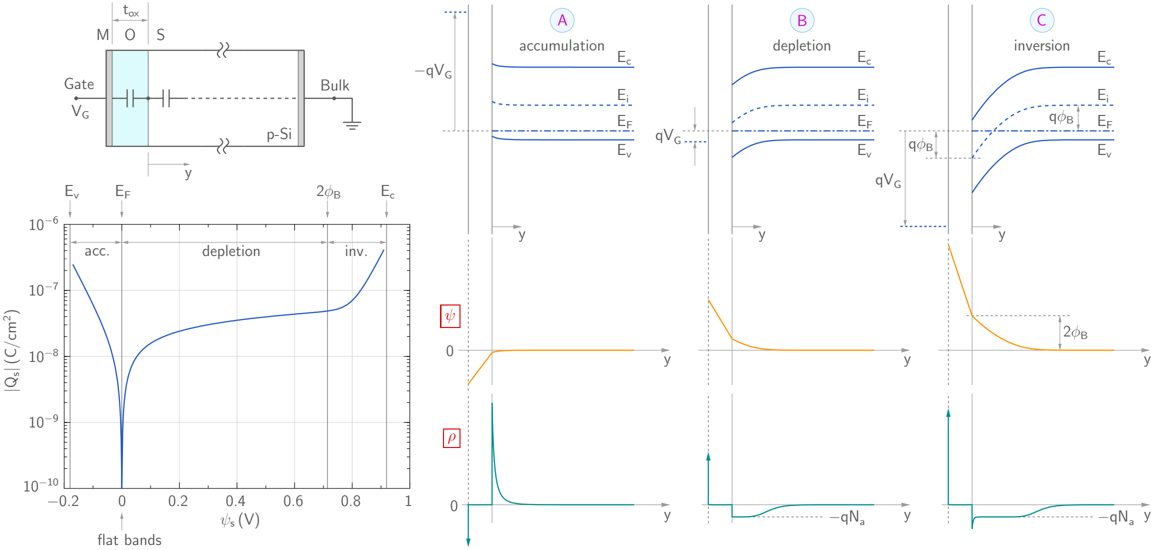
$$\int_{y=0^+}^{\infty} \mathcal{E}\, d\mathcal{E} = \frac{q}{\epsilon_{\mathrm{Si}}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$

We now obtain $\mathcal{E}(0^+)$ as a function of ψ_s and then $Q_s = -\epsilon_{\mathrm{Si}}\, \mathcal{E}(0^+)$ as the total charge in the semiconductor per unit area.

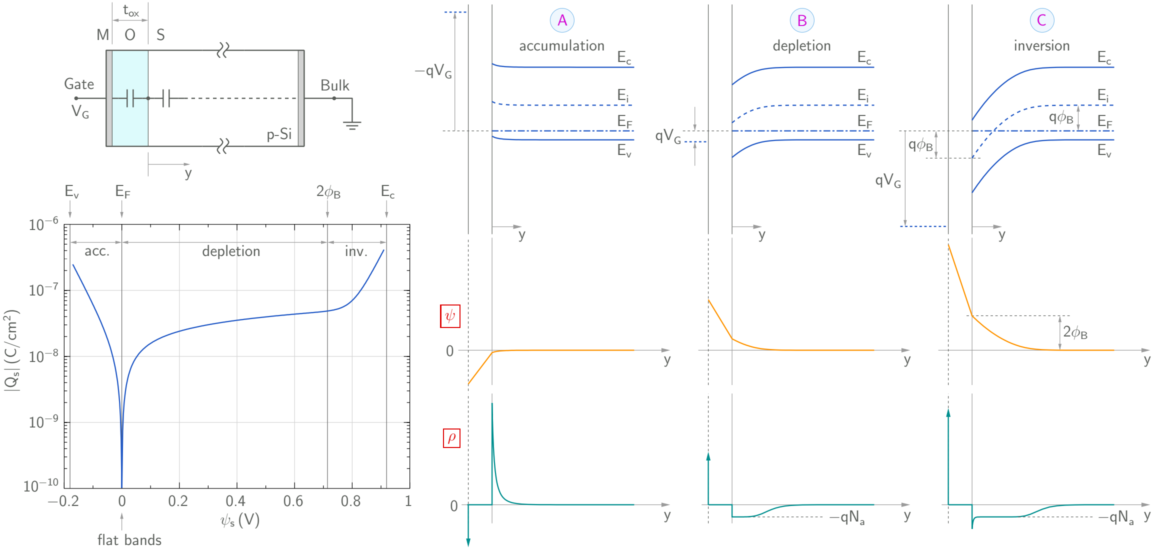


$$\int_{y=0^+}^{\infty} \mathcal{E}\, d\mathcal{E} = \frac{q}{\epsilon_{\mathrm{Si}}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$

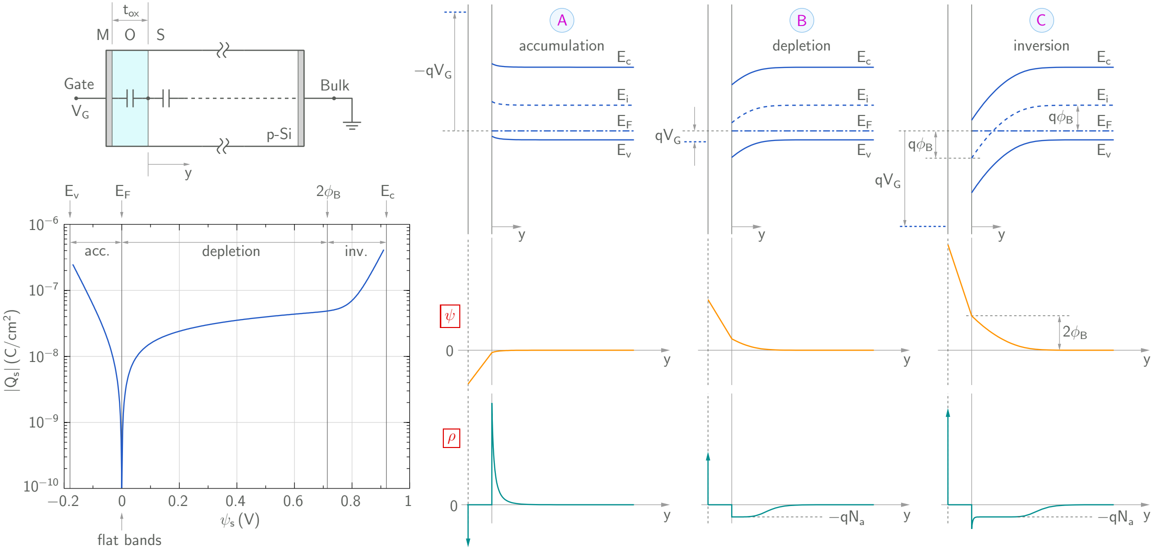
We now obtain $\mathcal{E}(0^+)$ as a function of ψ_s and then $Q_s = -\epsilon_{\mathrm{Si}}\, \mathcal{E}(0^+)$ as the total charge in the semiconductor per unit area.



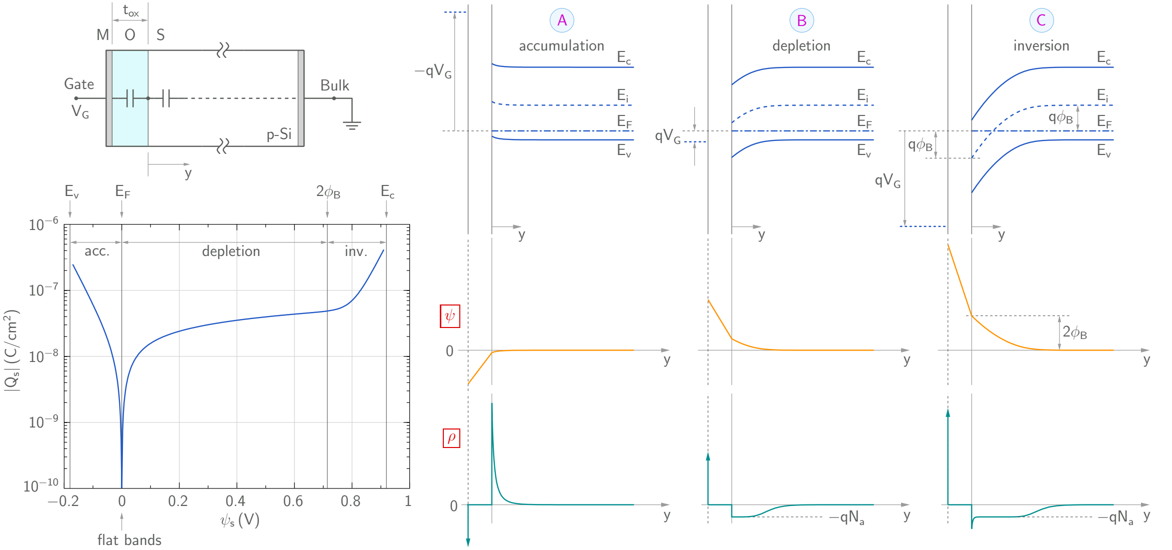
M. B. Patil, IIT Bombay



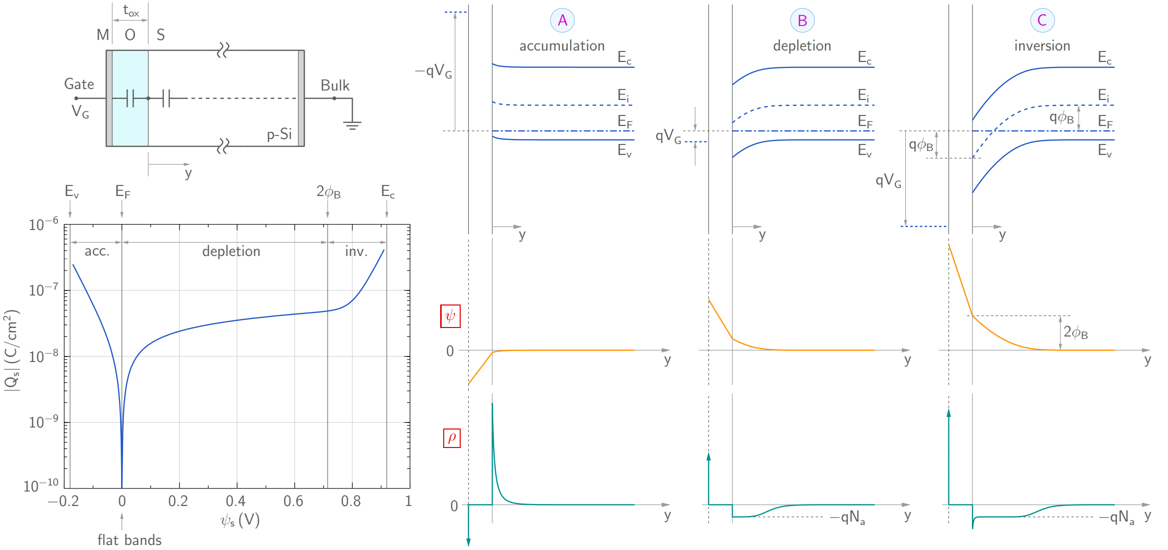
* In accumulation ($\psi_s < 0\,\text{V}$) and inversion ($\psi_s > 2\phi_B$), Q_s changes rapidly with ψ_s because $p \propto e^{-(E_F - E_v)/kT}$, $n \propto e^{-(E_c - E_F)/kT}$.



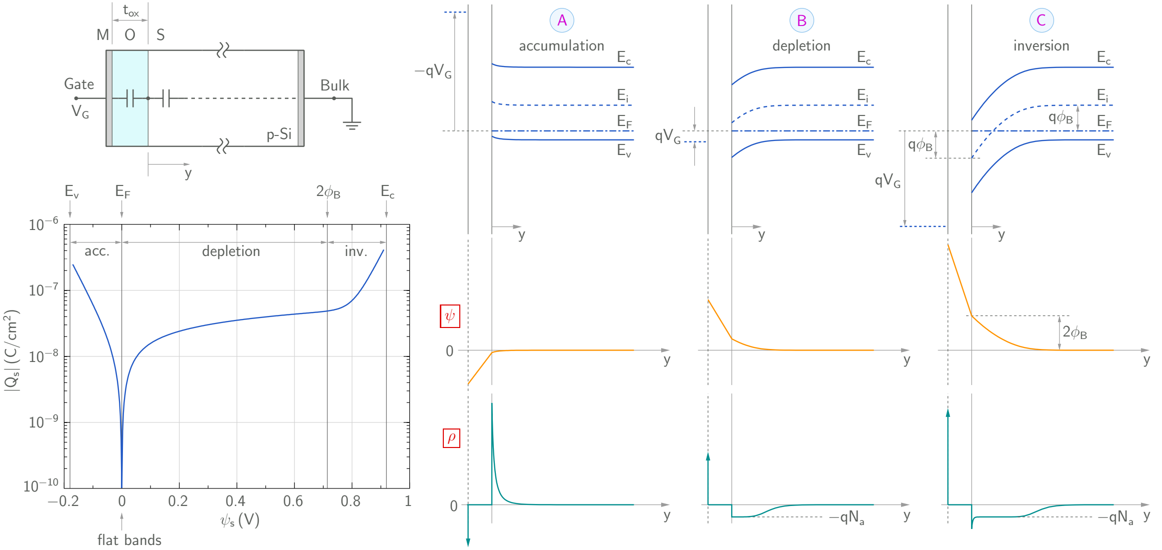
t_ox
M O S
Gate
V_G
Bulk
p-Si
y
E_v E_F 2φ_B E_c
acc.
depletion
inv.
|Q_s| (C/cm²)
ψ_s (V)
flat bands
A
accumulation
B
depletion
C
inversion
E_c
E_i
E_F
E_v
−qV_G
qV_G
qφ_B
2φ_B
ψ
ρ
0
y
−qN_a



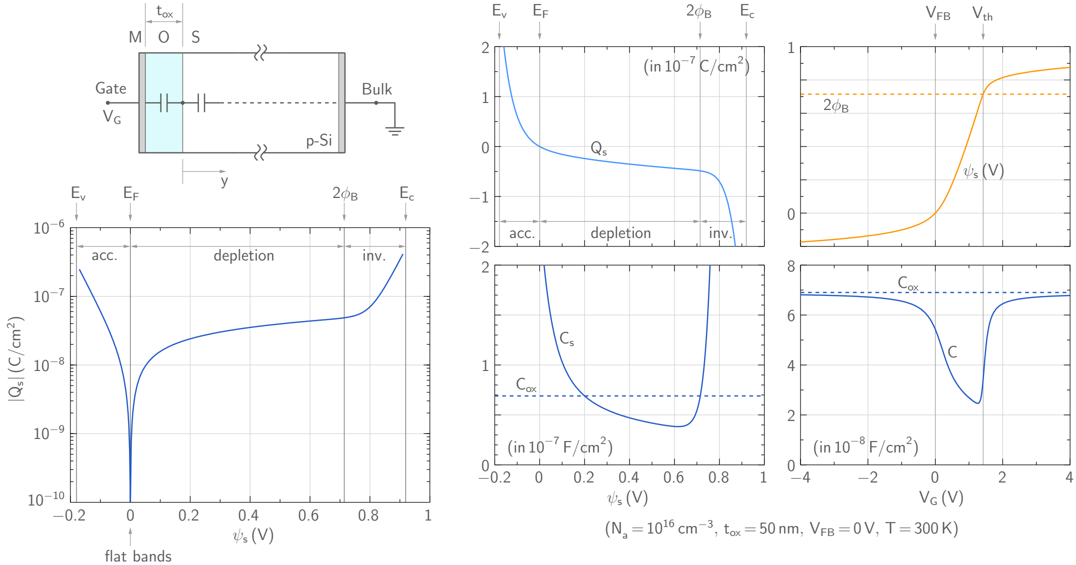
* In the depletion regime, the region near the surface is depleted of electrons and holes, and the variation of Q_s with ψ_s comes from the change in the ionised acceptor charge, i.e., the change in the depletion width with ψ_s.



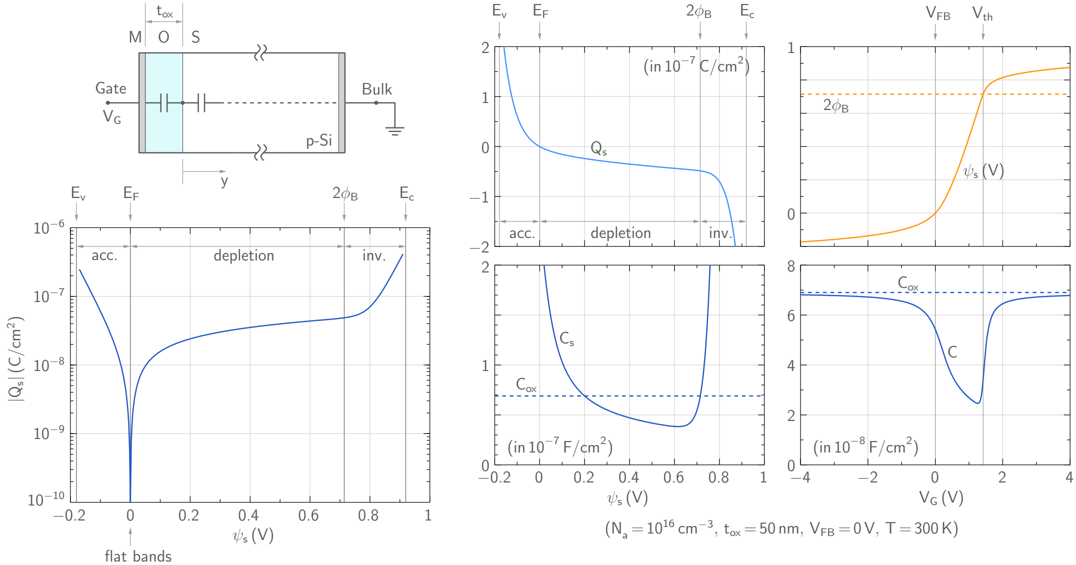
M. B. Patil, IIT Bombay



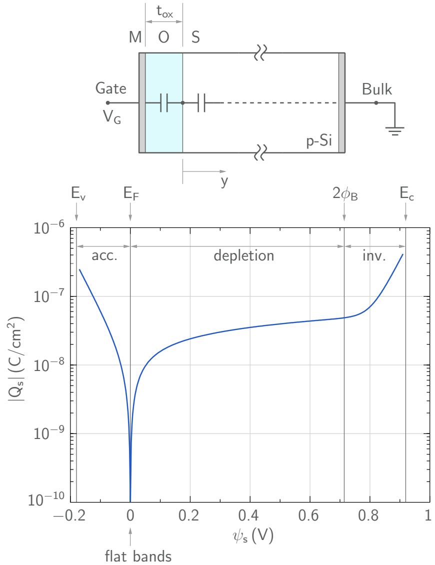
∗ Since the depletion width varies relatively slowly with ψ_s (as $\sqrt{\psi_s}$), $\dfrac{dQ}{d\psi_s}$ is relatively small in the depletion regime.

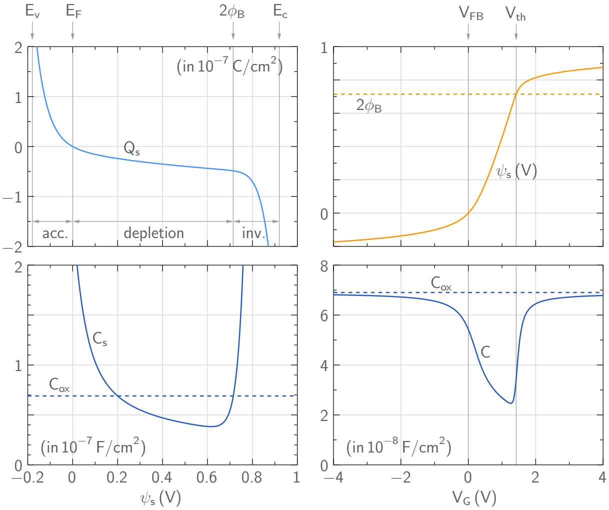


$(N_a = 10^{16}\ \mathrm{cm}^{-3},\ t_{ox} = 50\ \mathrm{nm},\ V_{FB} = 0\ \mathrm{V},\ T = 300\ \mathrm{K})$

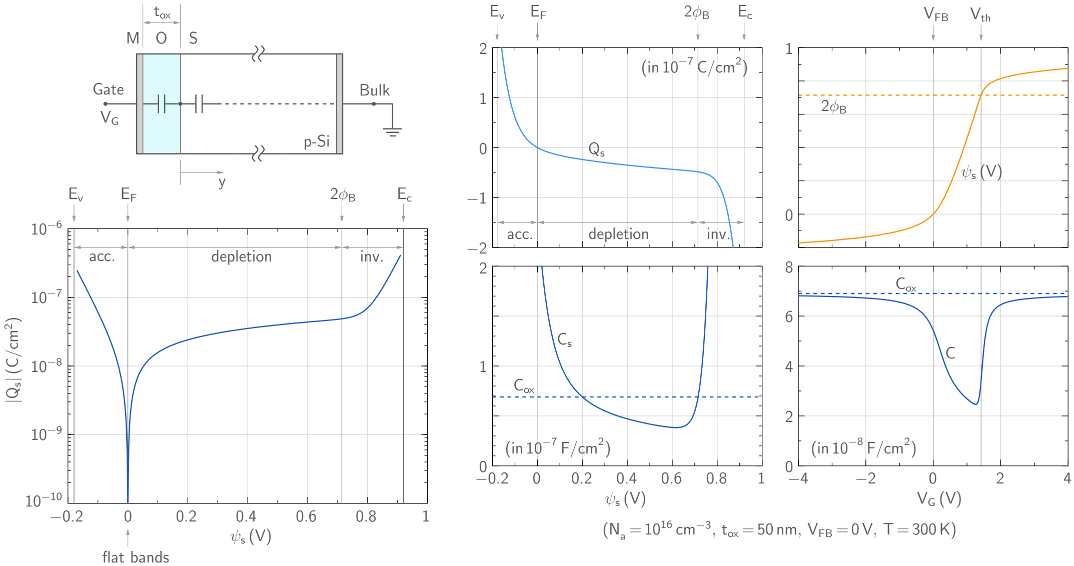


$$* \quad Q_M = -Q_s \;\rightarrow\; C_s \equiv \frac{dQ_M}{d\psi_s} = -\frac{dQ_s}{d\psi_s}.$$

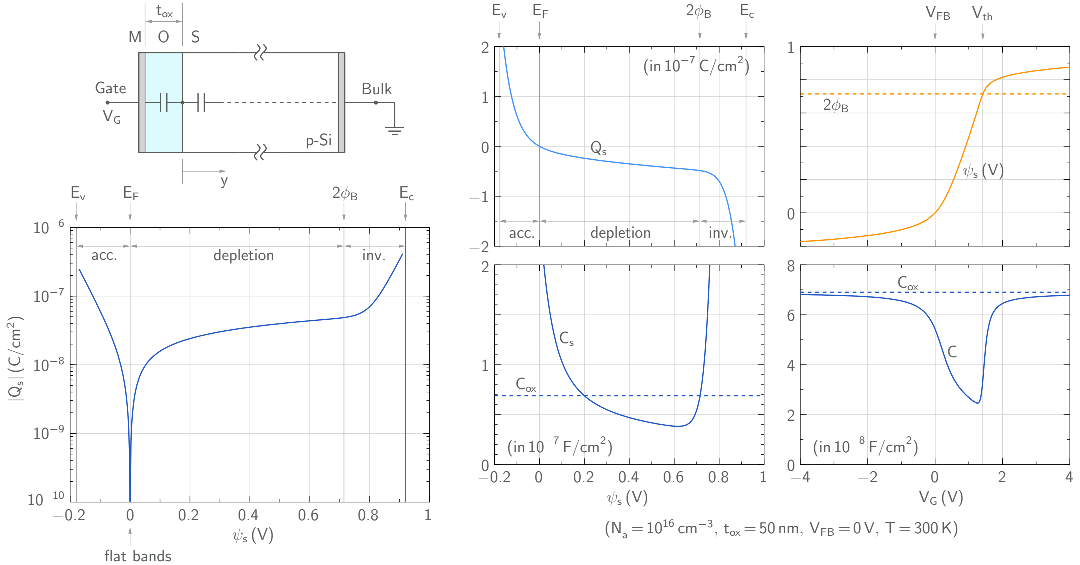




($N_a = 10^{16}$ cm^{-3}, $t_{ox} = 50$ nm, $V_{FB} = 0$ V, $T = 300$ K)

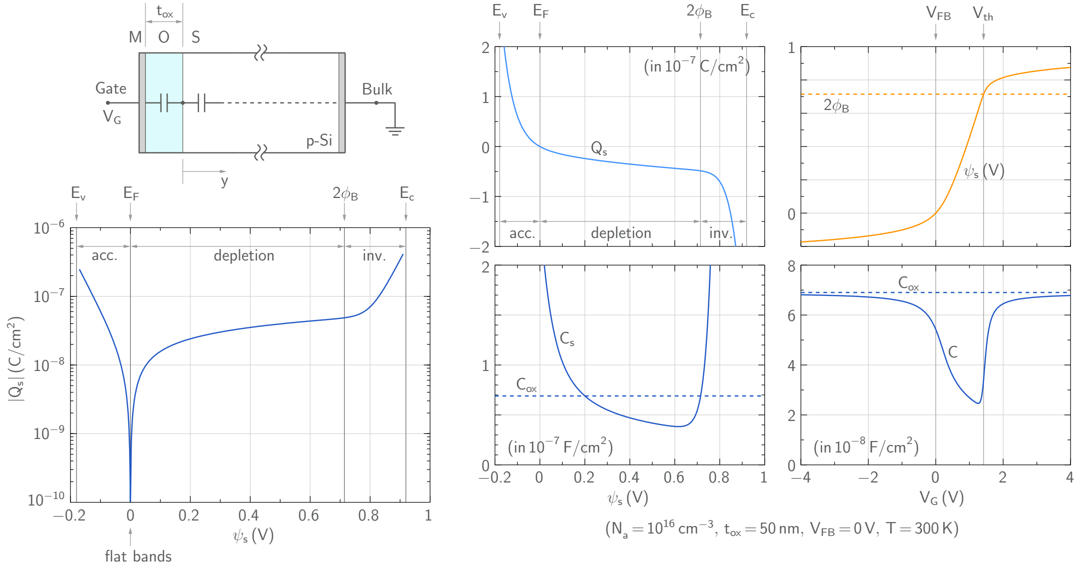


* Accumulation and inversion: C_s is large compared to C_{ox}. Since $\dfrac{1}{C} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_s}$, $C \rightarrow C_{ox}$.



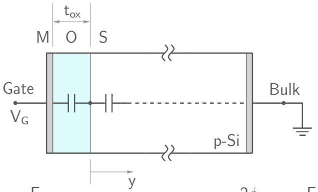
$(N_a = 10^{16}\,\mathrm{cm}^{-3},\ t_{ox} = 50\,\mathrm{nm},\ V_{FB} = 0\,\mathrm{V},\ T = 300\,\mathrm{K})$

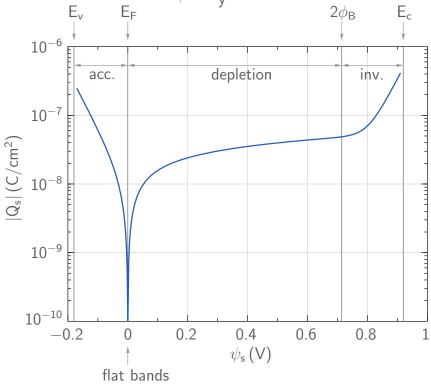
M. B. Patil, IIT Bombay

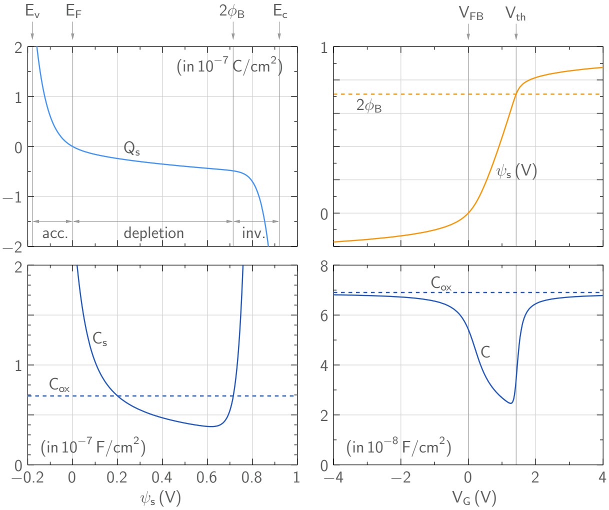


* To map the surface potential ψ_s to the gate voltage V_G, we use

$$V_G = V_{FB} + \psi_s + \mathcal{E}_{ox} t_{ox} = V_{FB} + \psi_s + \frac{(-Q_s)}{\epsilon_{ox}}\, t_{ox} = V_{FB} + \psi_s + \frac{(-Q_s)}{C_{ox}}.$$

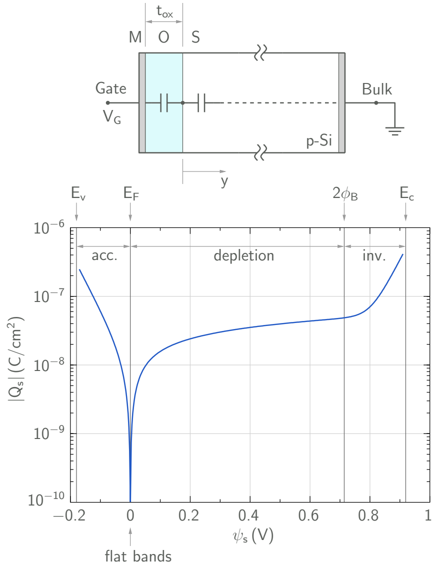


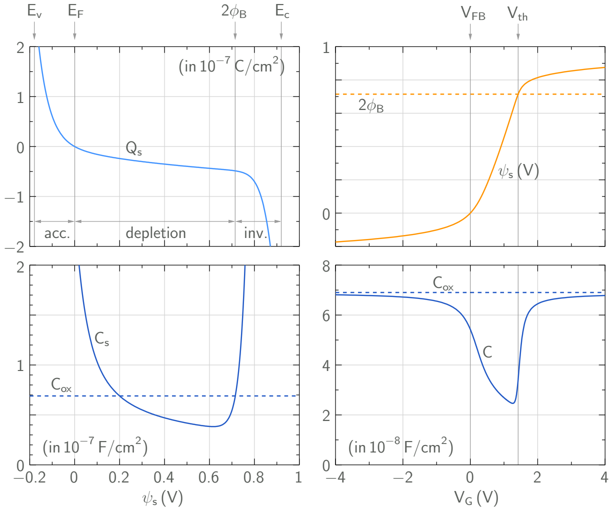




($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

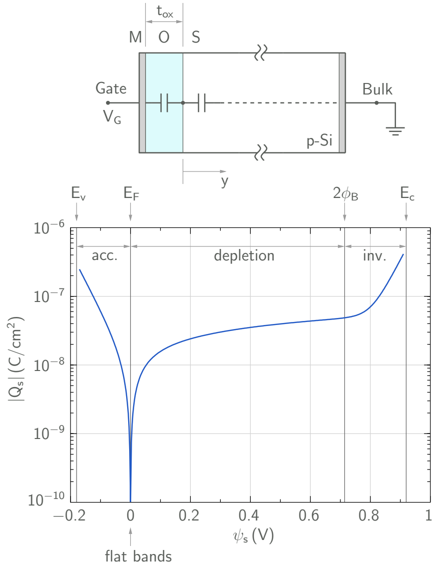
M. B. Patil, IIT Bombay

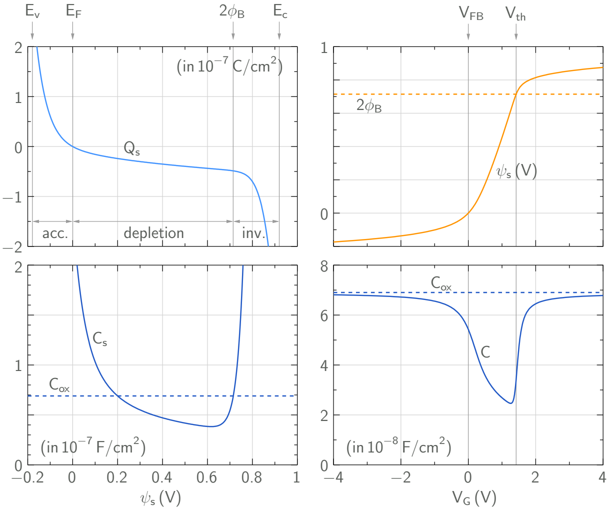




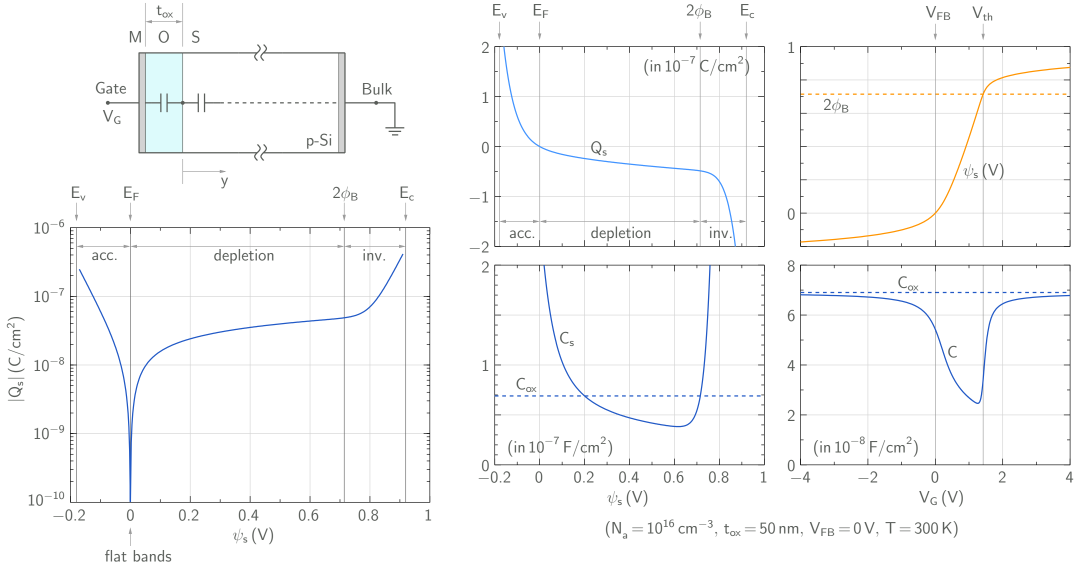
$(N_a = 10^{16}\,\mathrm{cm}^{-3},\ t_{ox} = 50\,\mathrm{nm},\ V_{FB} = 0\,\mathrm{V},\ T = 300\,\mathrm{K})$

* In accumulation and inversion, $C \to C_{ox}$.

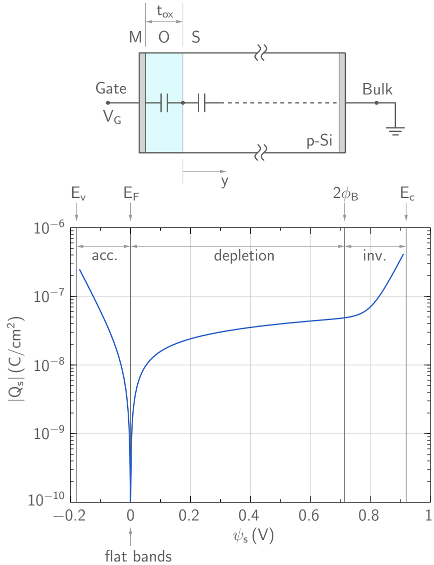


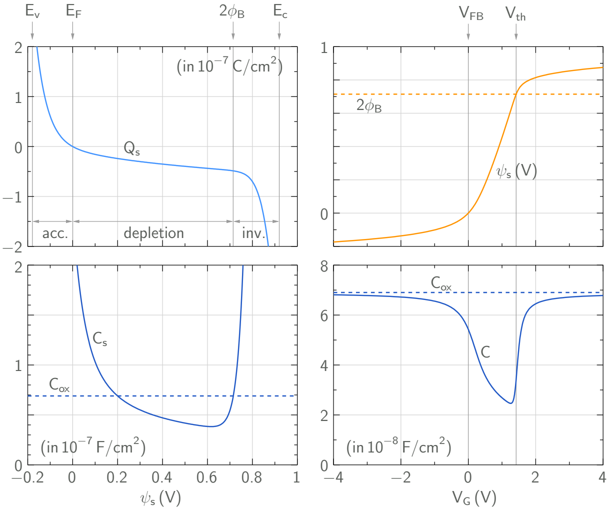


($N_a = 10^{16}\ \mathrm{cm^{-3}}$, $t_{ox} = 50\ \mathrm{nm}$, $V_{FB} = 0\ \mathrm{V}$, $T = 300\ \mathrm{K}$)



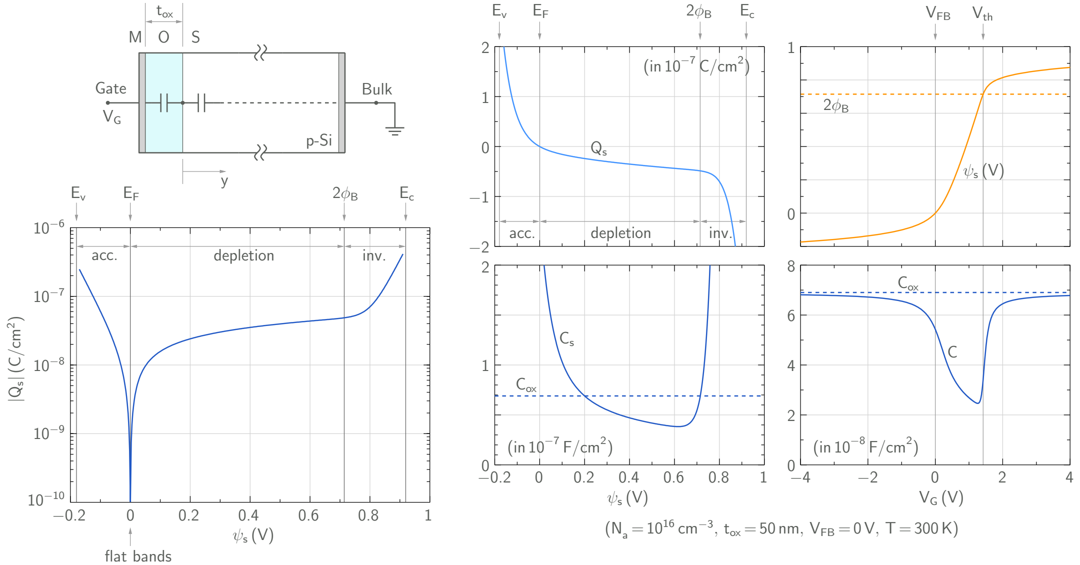
* In depletion, C is smaller than C_{ox} and is minimum when C_s is minimum. This corresponds to the situation where there is no inversion charge yet, but the depletion width has reached its maximum value which happens at the onset of inversion, i.e., $V_G \approx V_{th}$.



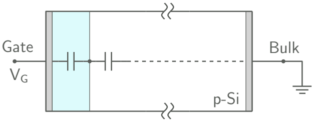


($N_a = 10^{16}\ \mathrm{cm}^{-3}$, $t_{ox} = 50\ \mathrm{nm}$, $V_{FB} = 0\ \mathrm{V}$, $T = 300\ \mathrm{K}$)

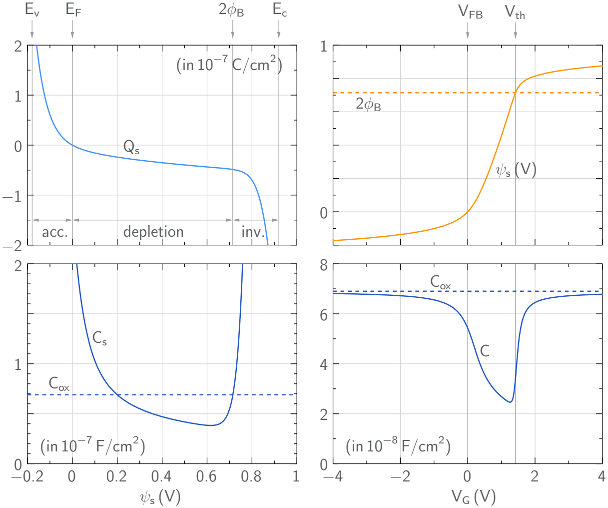
M. B. Patil, IIT Bombay



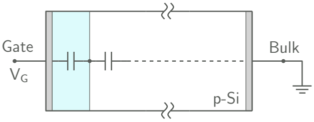
* Since $V_G = V_{FB} + \psi_s + \mathcal{E}_{ox} t_{ox}$, a change in V_{FB} by ΔV_{FB} causes the C-V curve to shift horizontally by ΔV_{FB}.



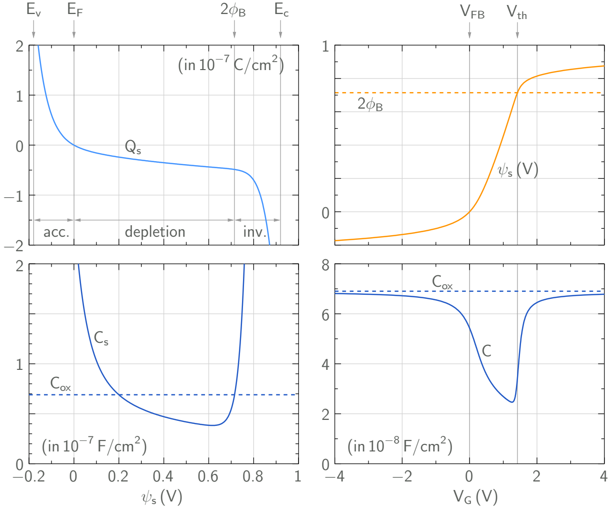
Gate
V_G
Bulk
p-Si

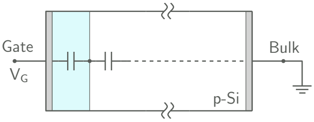


E_v
E_F
$2\phi_B$
E_c
V_{FB}
V_{th}
(in 10^{-7} C/cm^2)
Q_s
acc.
depletion
inv.
$2\phi_B$
ψ_s (V)
C_s
C_{ox}
(in 10^{-7} F/cm^2)
C_{ox}
C
(in 10^{-8} F/cm^2)
ψ_s (V)
V_G (V)

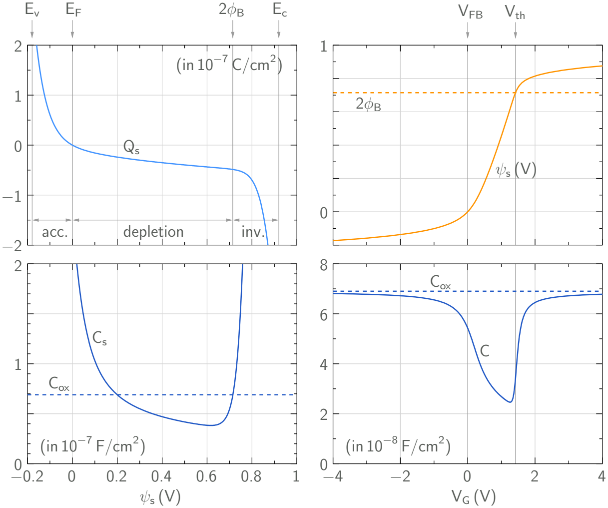


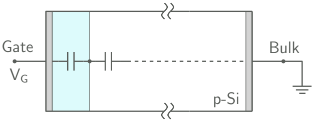
* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.



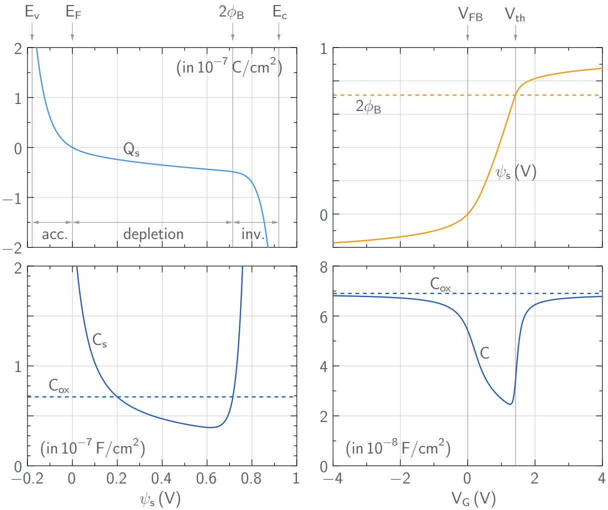


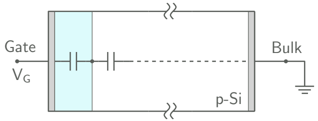
* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
* The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.



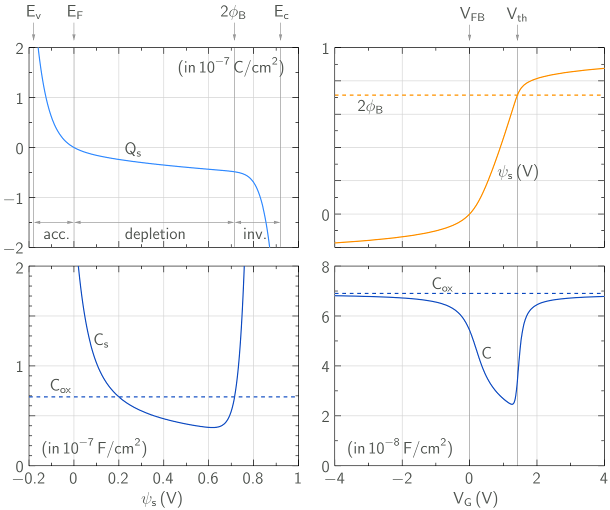


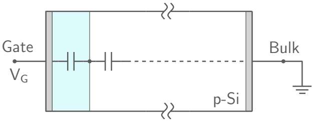
* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
* The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.
* At high frequencies, the C–V curve in the accumulation region remains unaffected since it involves the readjustment of the majority carriers, a fast process.



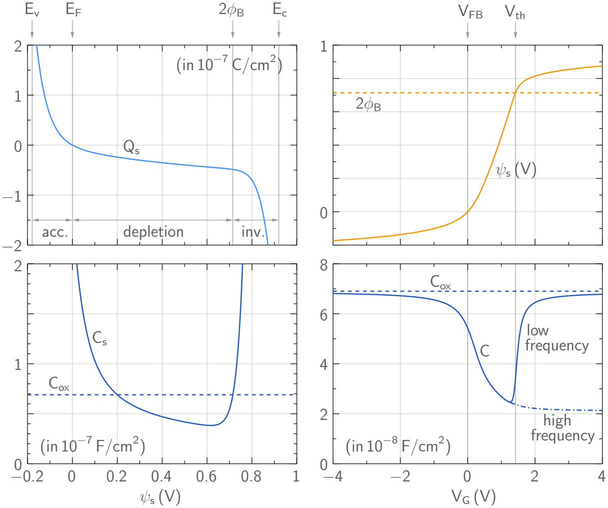


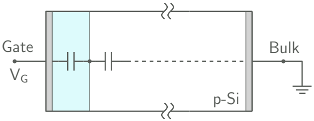
* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
* The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.
* At high frequencies, the C–V curve in the accumulation region remains unaffected since it involves the readjustment of the majority carriers, a fast process.
  However, the inversion charge — which is made up of minority carriers — cannot follow the changes in the gate voltage because the minority carriers must come from the generation-recombination process in the bulk. As a result, C_s stays as its minimum value which occurs when the depletion width is maximum, corresponding to $\psi_s = 2\phi_B$.





* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
* The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.
* At high frequencies, the C–V curve in the accumulation region remains unaffected since it involves the readjustment of the majority carriers, a fast process.
  However, the inversion charge — which is made up of minority carriers — cannot follow the changes in the gate voltage because the minority carriers must come from the generation-recombination process in the bulk. As a result, C_s stays as its minimum value which occurs when the depletion width is maximum, corresponding to $\psi_s = 2\phi_B$.





* We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
* The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.
* At high frequencies, the C–V curve in the accumulation region remains unaffected since it involves the readjustment of the majority carriers, a fast process.
  However, the inversion charge — which is made up of minority carriers — cannot follow the changes in the gate voltage because the minority carriers must come from the generation-recombination process in the bulk. As a result, C_s stays as its minimum value which occurs when the depletion width is maximum, corresponding to $\psi_s = 2\phi_B$.
* The low-frequency ($f < 100\,\text{Hz}$) and high-freq ($f > 1\,\text{MHz}$) C-V curves offer an excellent "diagnostic" tool during processing since they can be used to find the oxide thickness, flat-band voltage, etc.