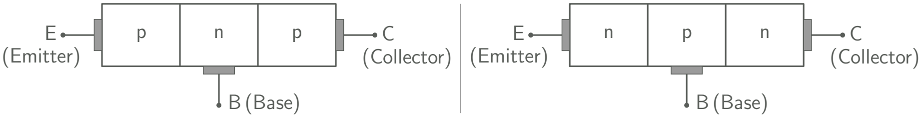
Semiconductor Devices

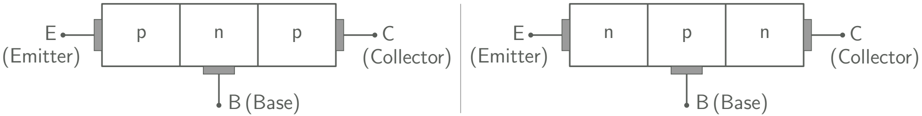
Bipolar Junction Transistors: Part 1

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay


E
(Emitter)
p
n
p
C
(Collector)
B (Base)
E
(Emitter)
n
p
n
C
(Collector)
B (Base)

Bipolar junction transistors



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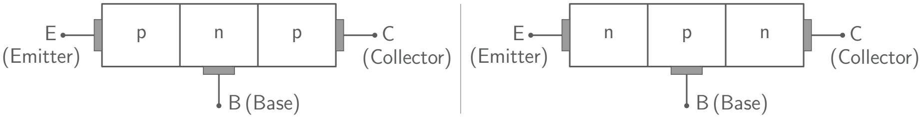

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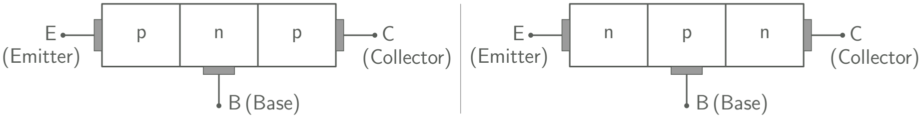

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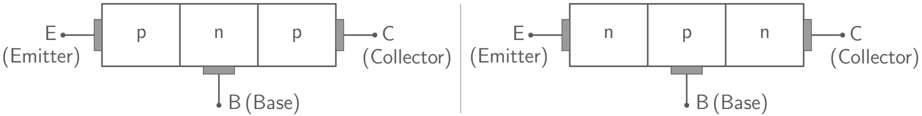

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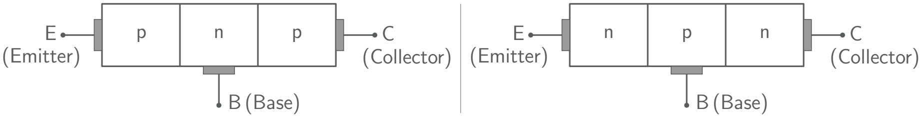

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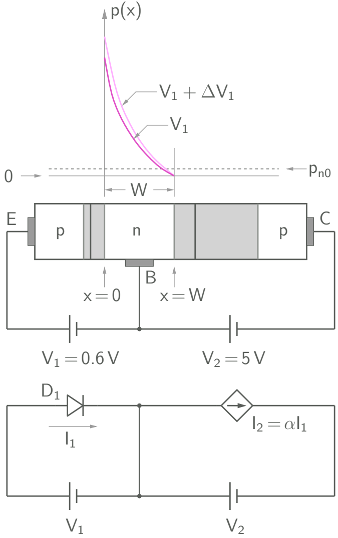
* The actual device construction is different than the above schematic diagram (to be discussed).
* For the device to work as a transistor (rather than two independent diodes), the two junctions must be "close."

Basic operation



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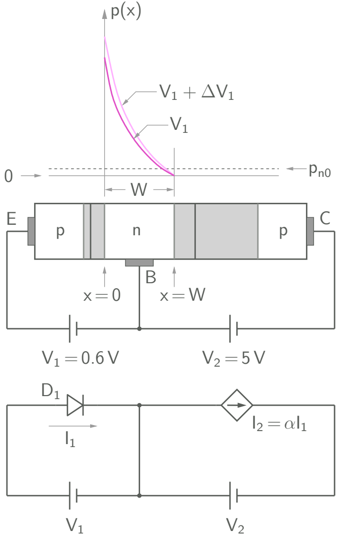

* If V_1 is varies, $p(x)$ varies $\rightarrow I_p(W) \propto \dfrac{dp}{dx}(W)$ varies, i.e., by changing V_1, I_2 can be controlled. This is the basic transistor action.

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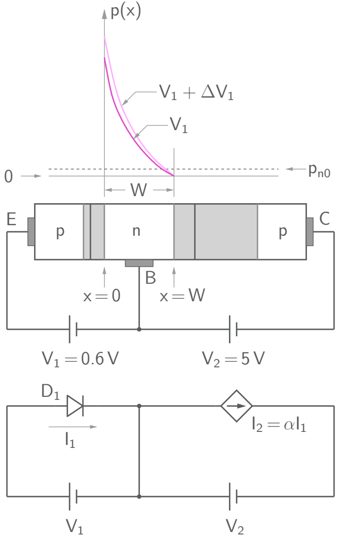

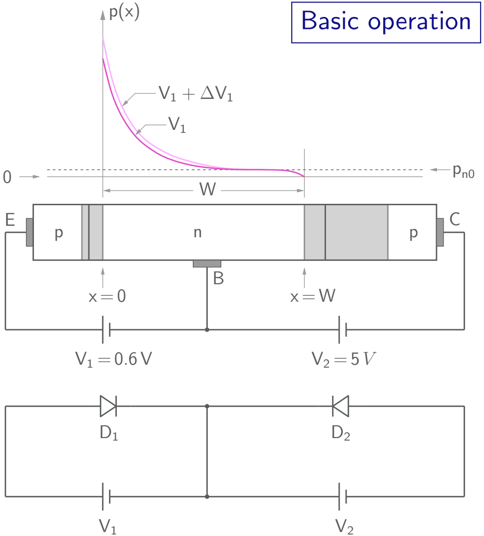

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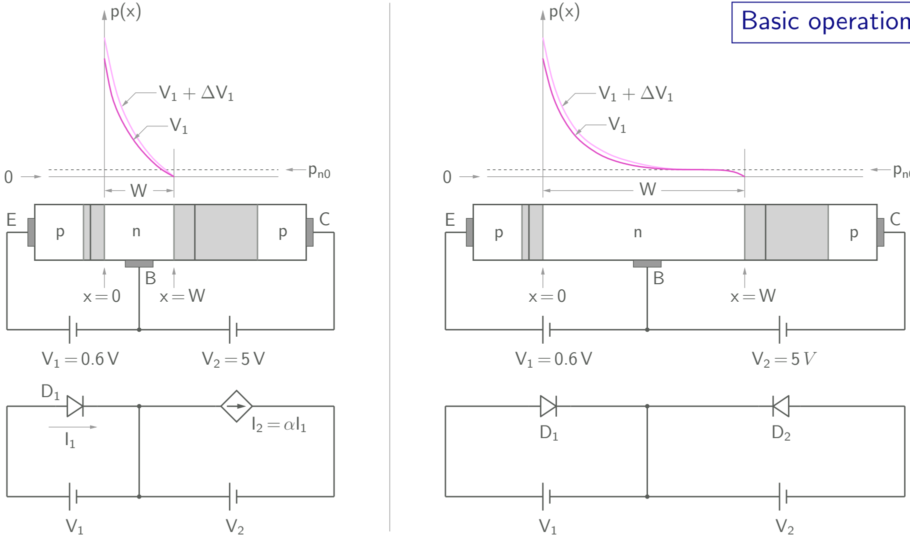
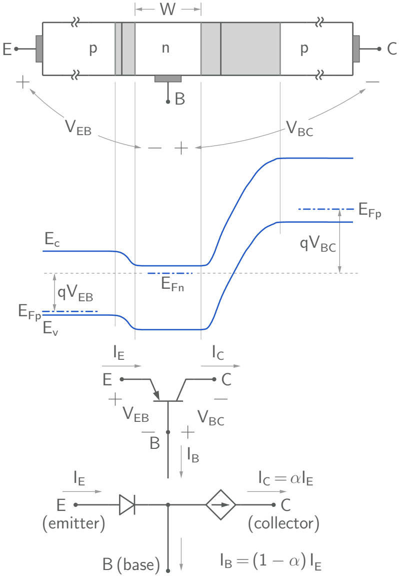
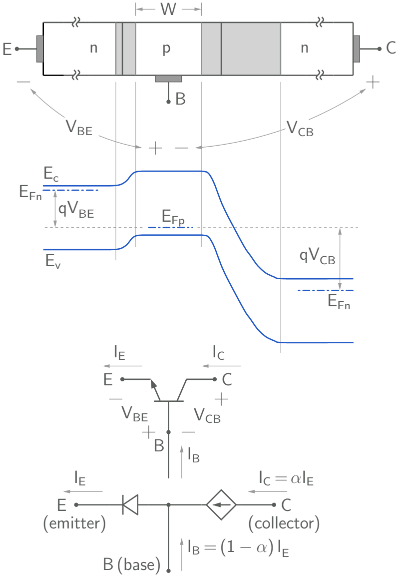


* If V_1 is varies, $p(x)$ varies $\rightarrow I_p(W) \propto \dfrac{dp}{dx}(W)$ varies, i.e., by changing V_1, I_2 can be controlled. This is the basic transistor action.
* If the two junctions are not sufficiently close, the device behaves like two *independent* diodes connected back-to-back, and there is no transistor action.

pnp and *npn* transistors





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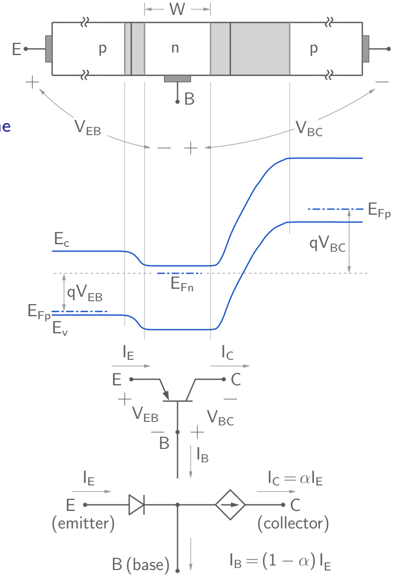

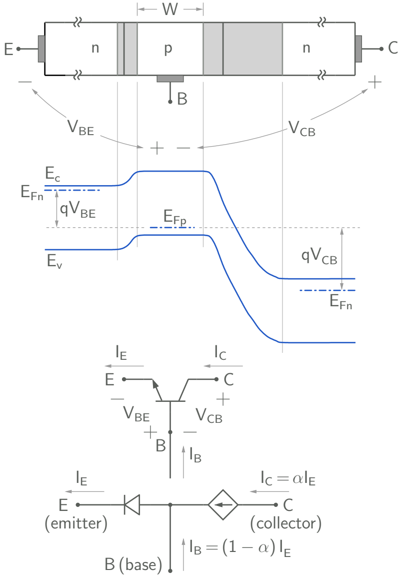

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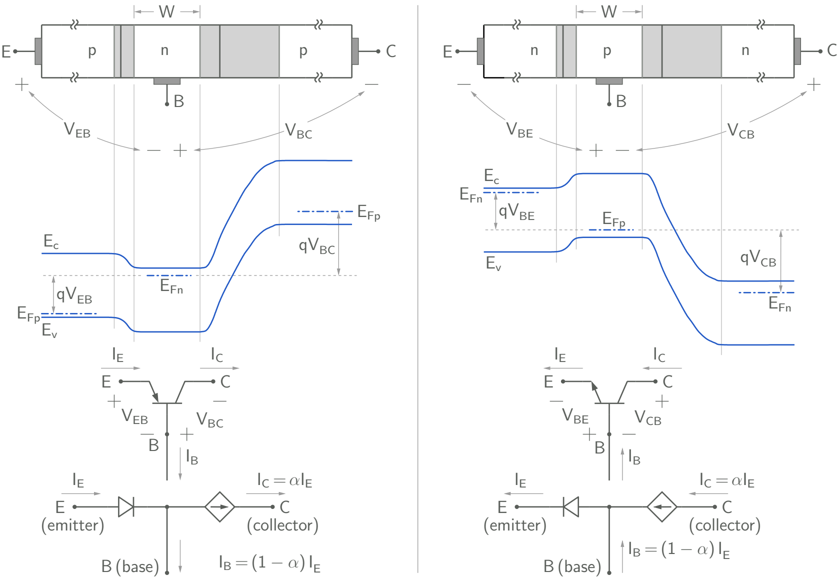

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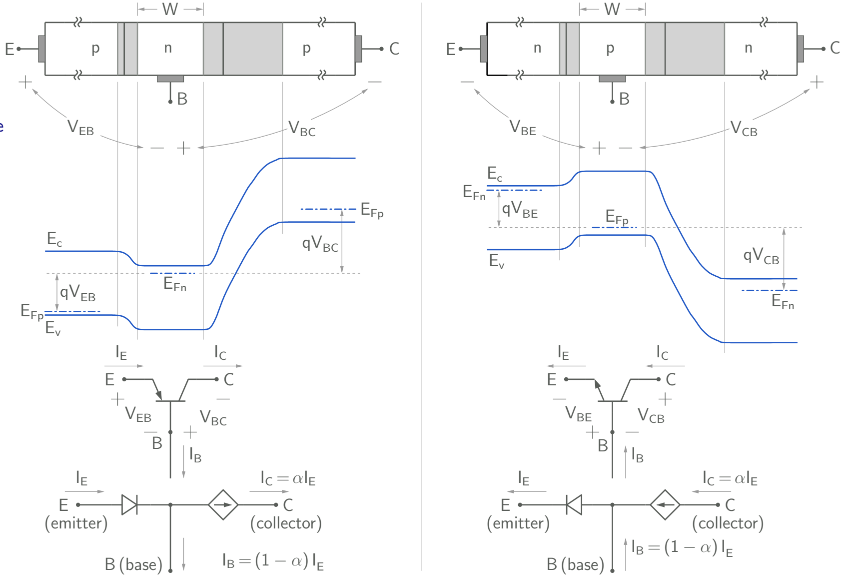

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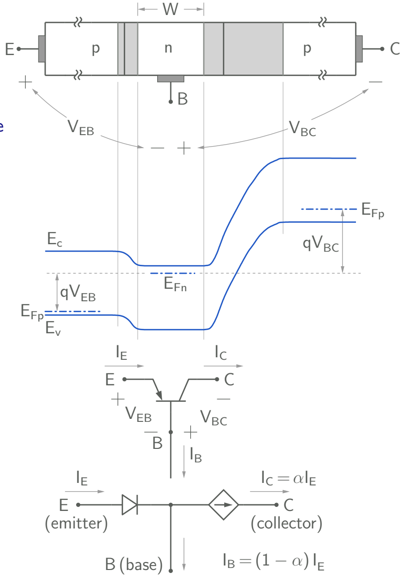

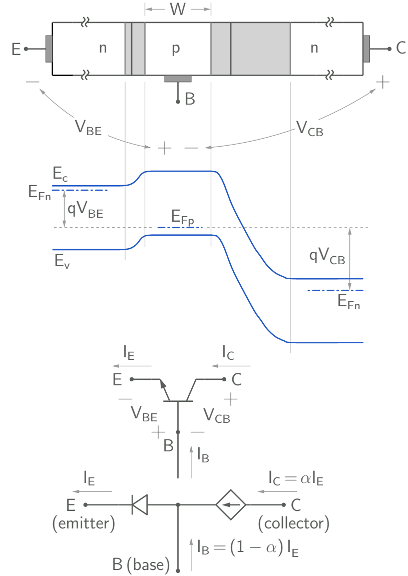

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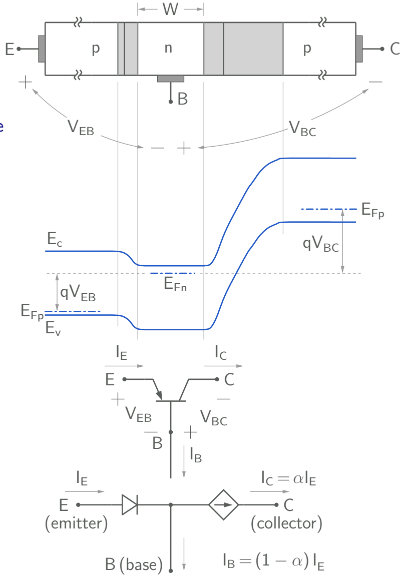

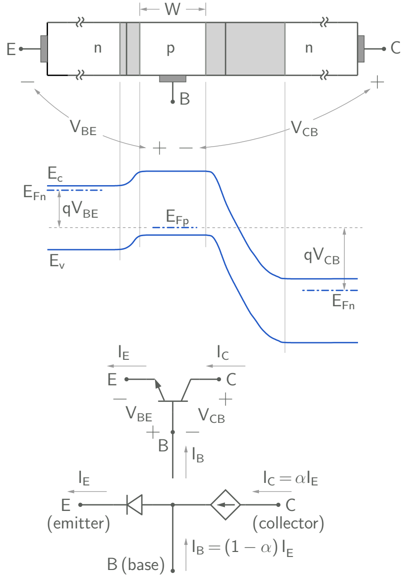

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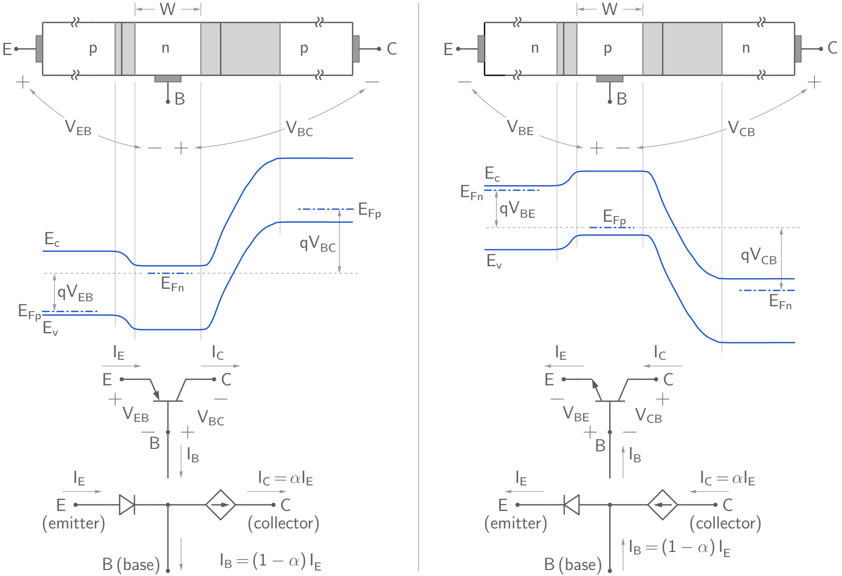

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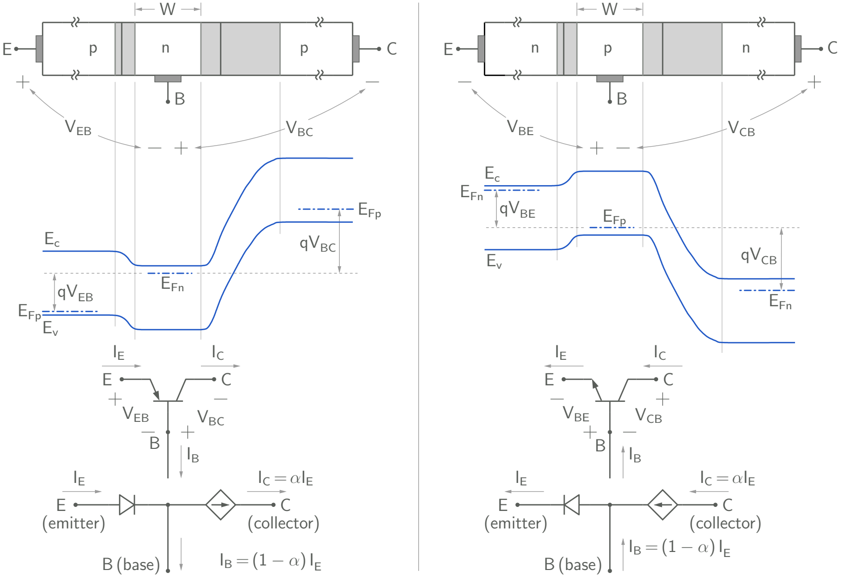

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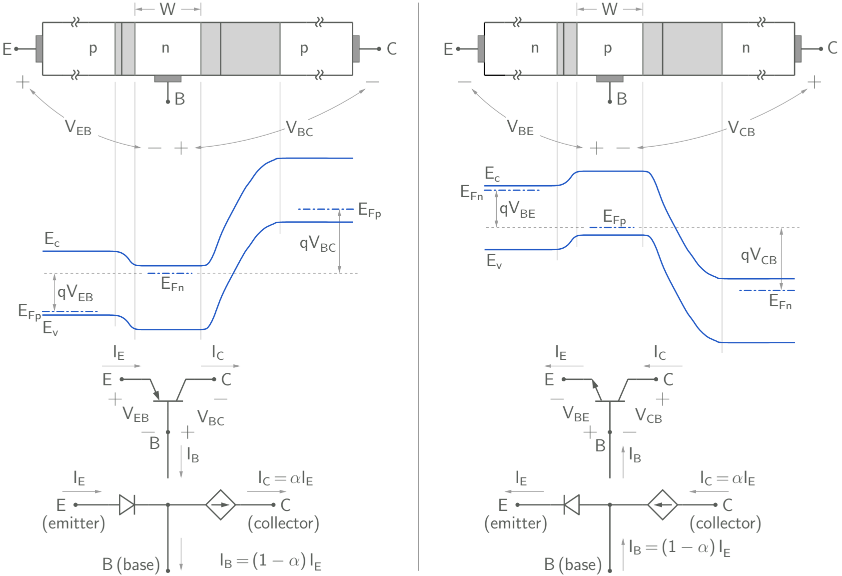
  For a good transistor, $\alpha \approx 1$.



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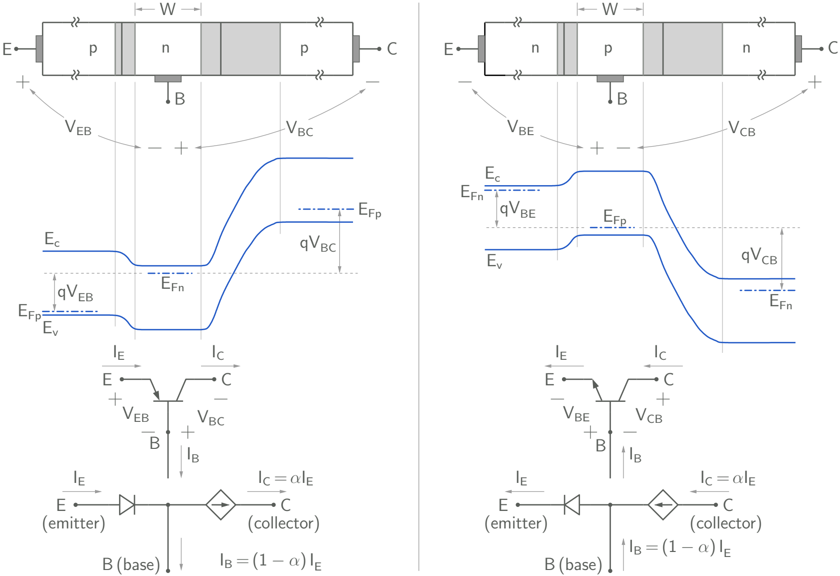
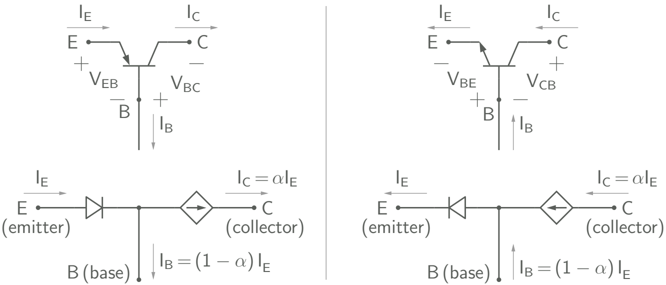
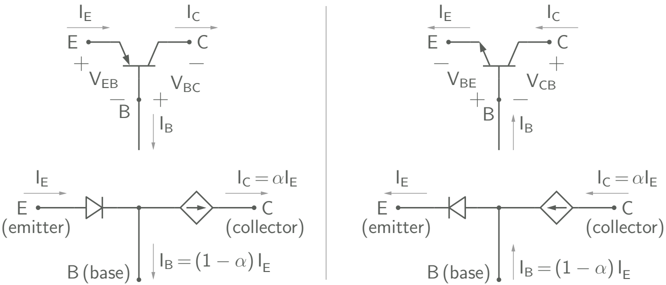
* The three currents satisfy KCL, i.e., $I_E = I_C + I_B$. Substituting for I_C, we get $I_B = (1-\alpha)\, I_E$.



BJT in active mode





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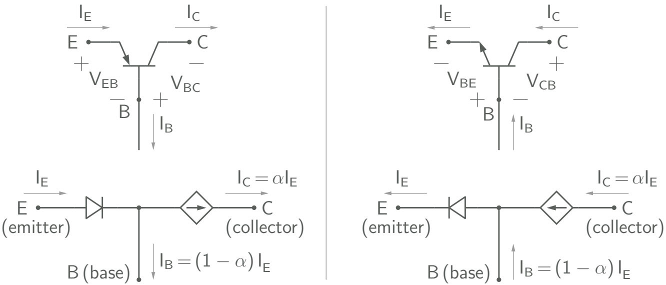

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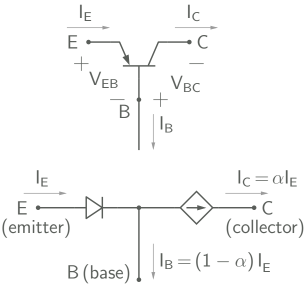

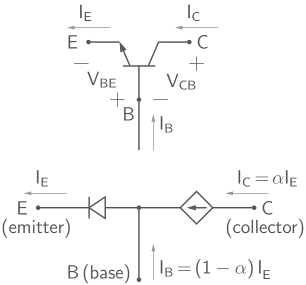

α	$1-\alpha$	$\beta=\alpha/(1-\alpha)$
0.9	0.1	9
0.95	0.05	19
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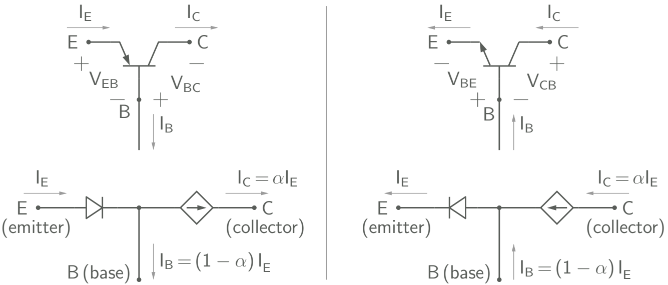

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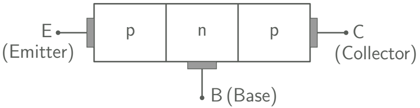
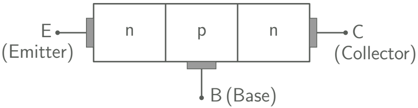
* For a typical discrete low-power transistor such as BC107A, β is in the range of 100 to 200.

BJT modes of operation





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Active (linear)	forward	reverse
Cutoff	reverse	reverse
Saturation	forward	forward
Inverse active	reverse	forward

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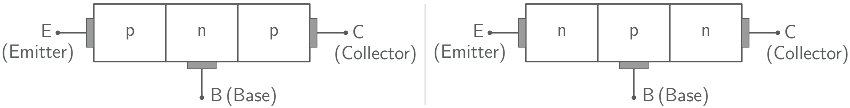

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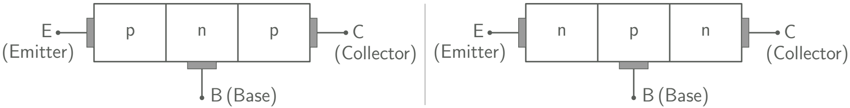

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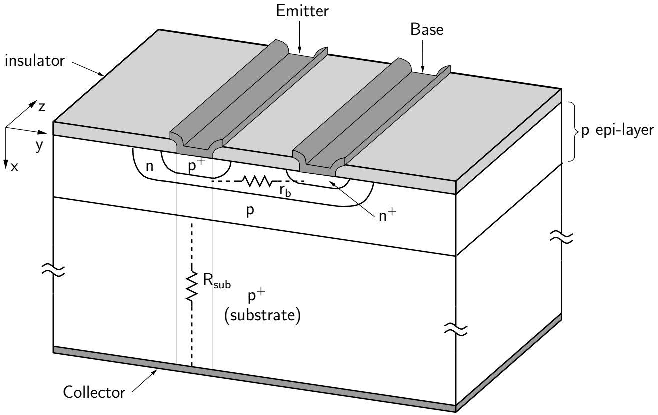
* BJT as a switch:
  - Closed: saturation mode
  - Open: cutoff mode

Device structure (*pnp* transistor)



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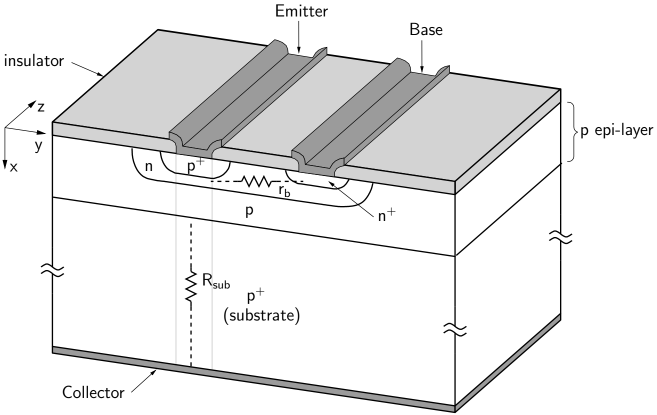
* The substrate thickness is hundreds of microns whereas the *p* epi-layer and the rest of the device structure is confined to a few microns.



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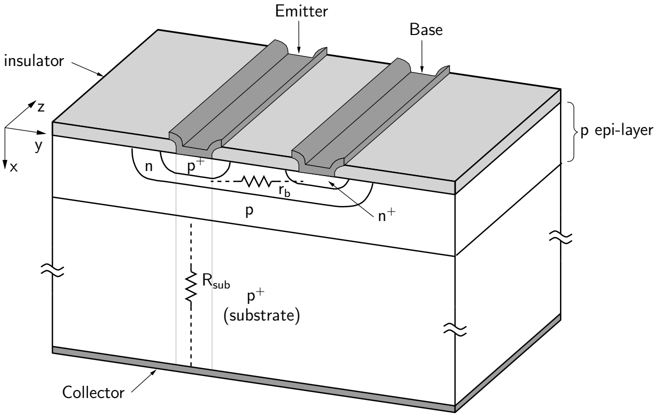

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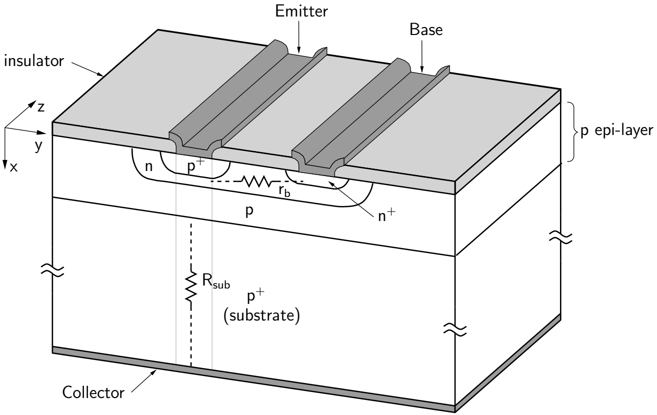

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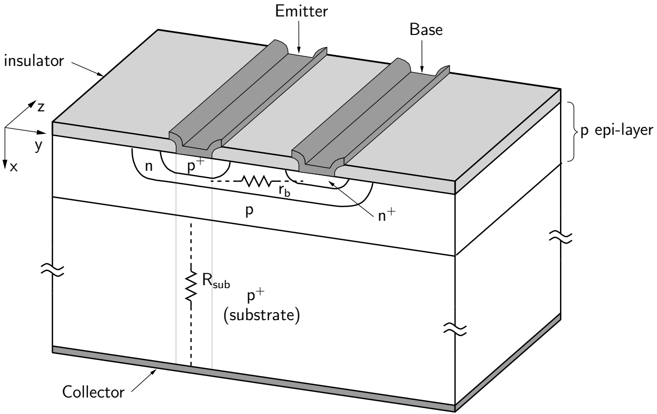

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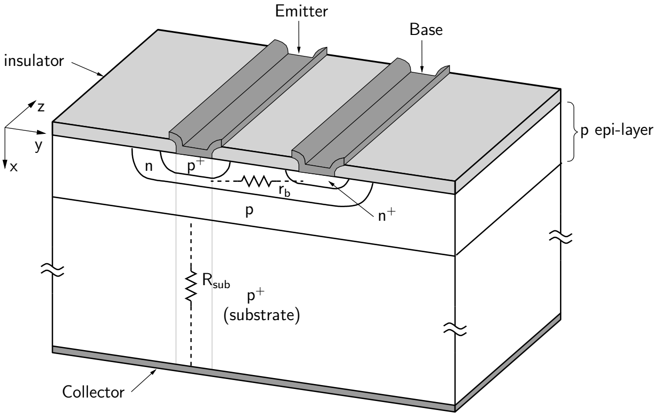

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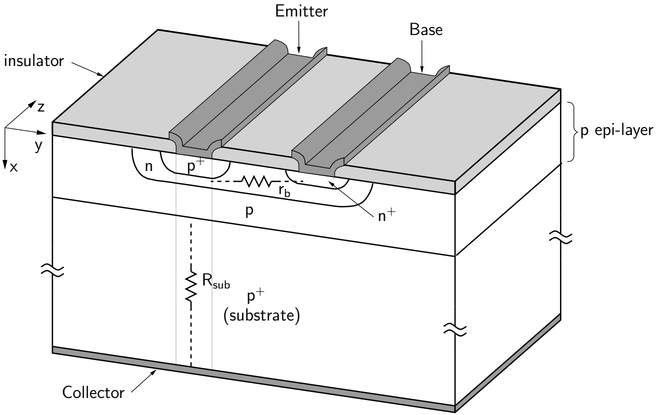
* A "base resistance" r_b exists between the base region and the base contact. To keep r_b small, the base contact is made close to the emitter.



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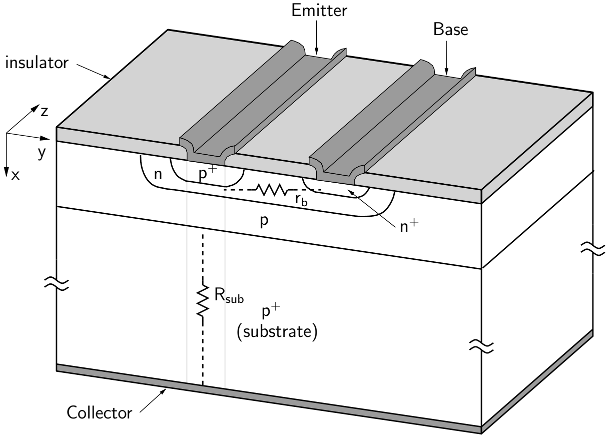

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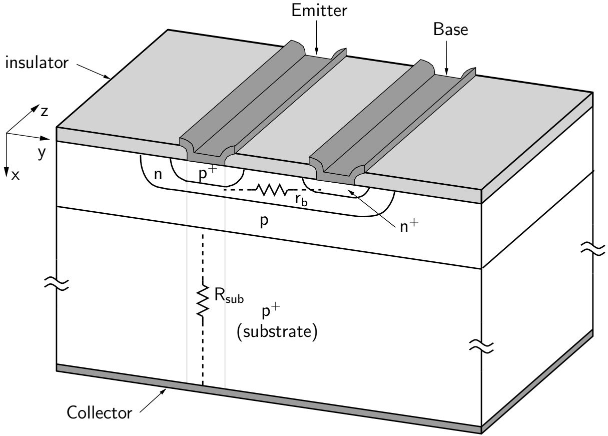

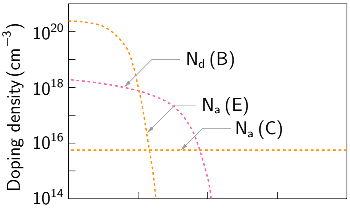

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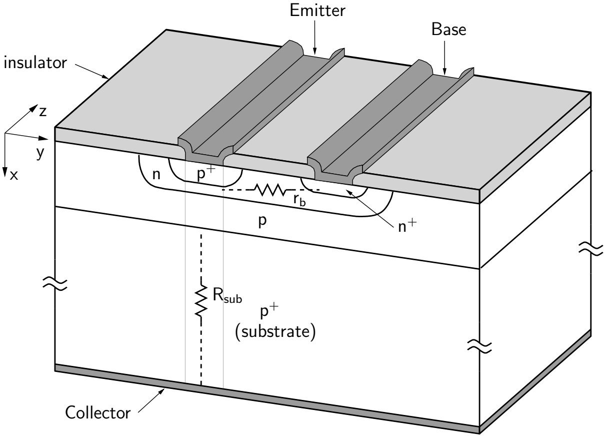

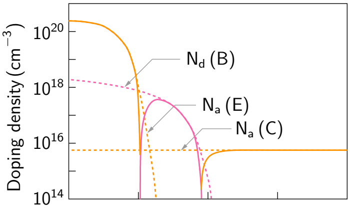

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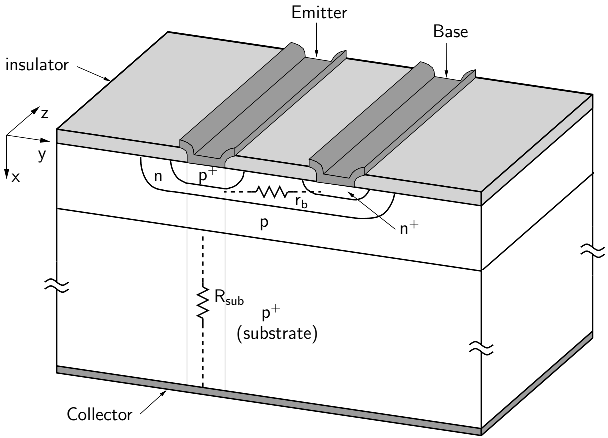

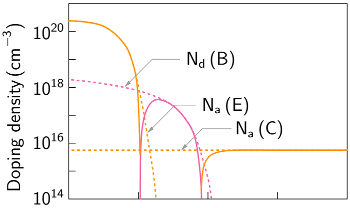

∗ For simplicity, we will assume the doping densities to be constant in the emitter, base, and collector regions.

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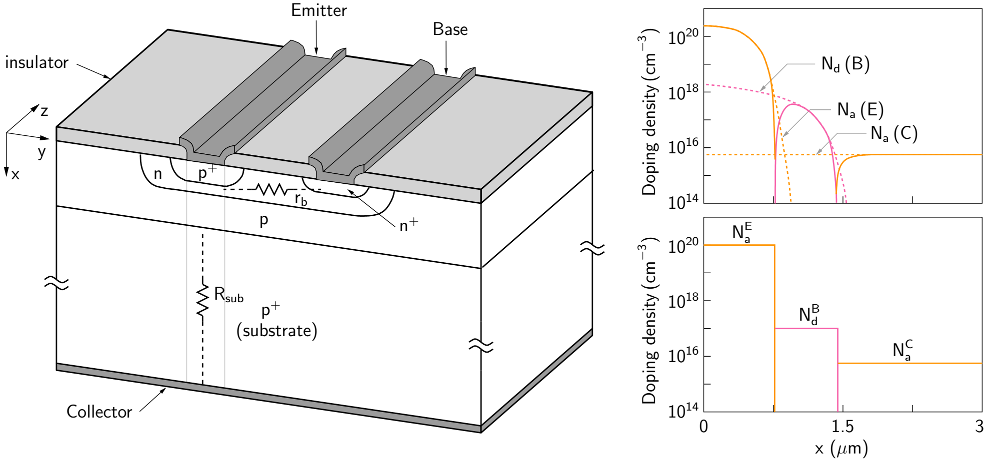

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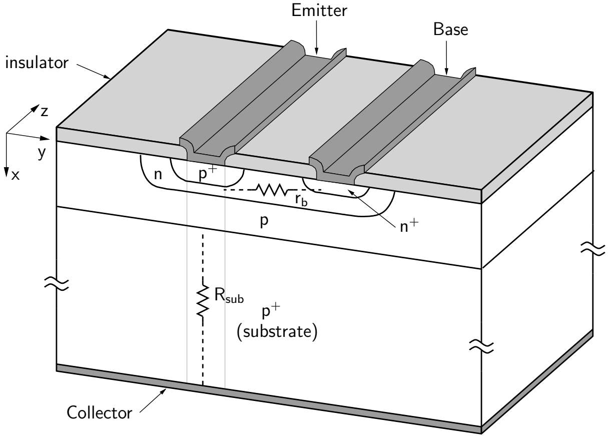
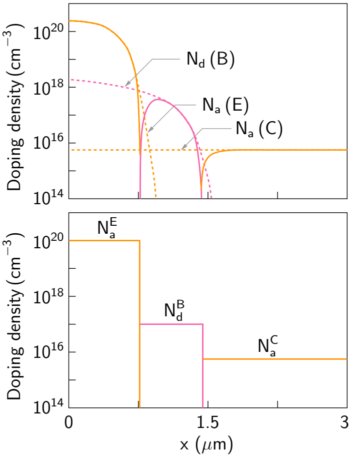
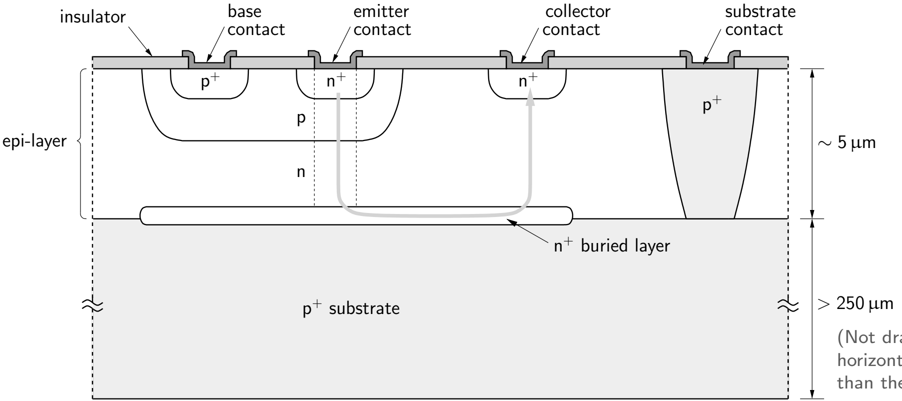




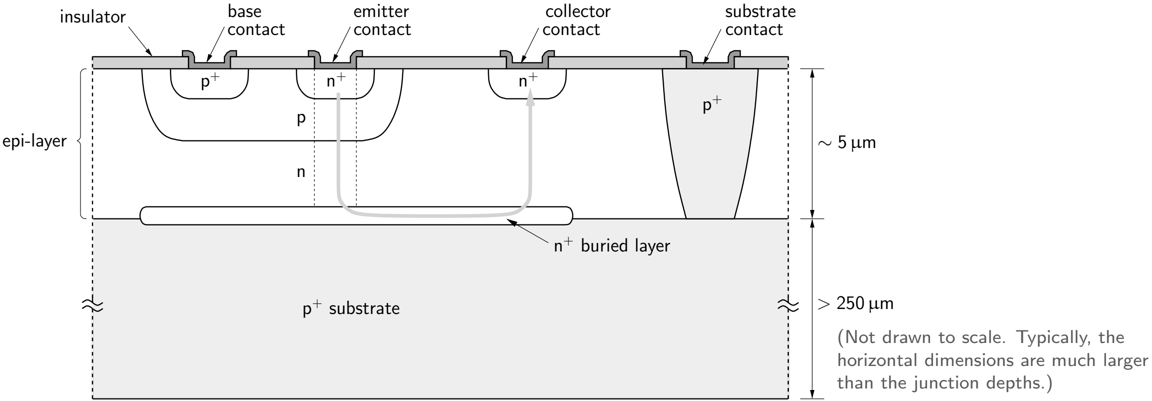
* For simplicity, we will assume the doping densities to be constant in the emitter, base, and collector regions.
* The relationship $N_a^E > N_d^B > N_a^C$, which is a consequence of the fabrication process, is also desirable from the device performance angle.

BJT structure in integrated circuits (*npn* transistor)



(Not drawn to scale. Typically, the horizontal dimensions are much larger than the junction depths.)

BJT structure in integrated circuits (*npn* transistor)



* To make an integrated circuit (IC), a large number of transistors are fabricated on a single silicon piece and interconnected as required.

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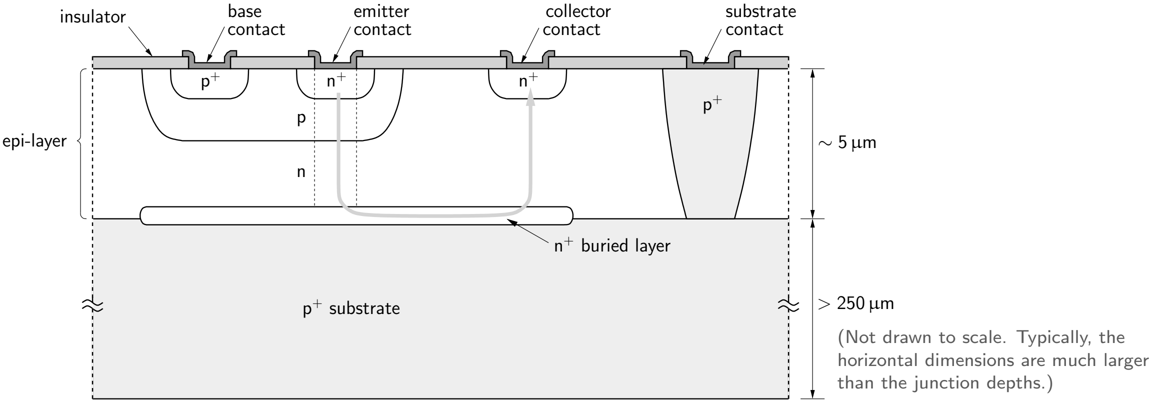

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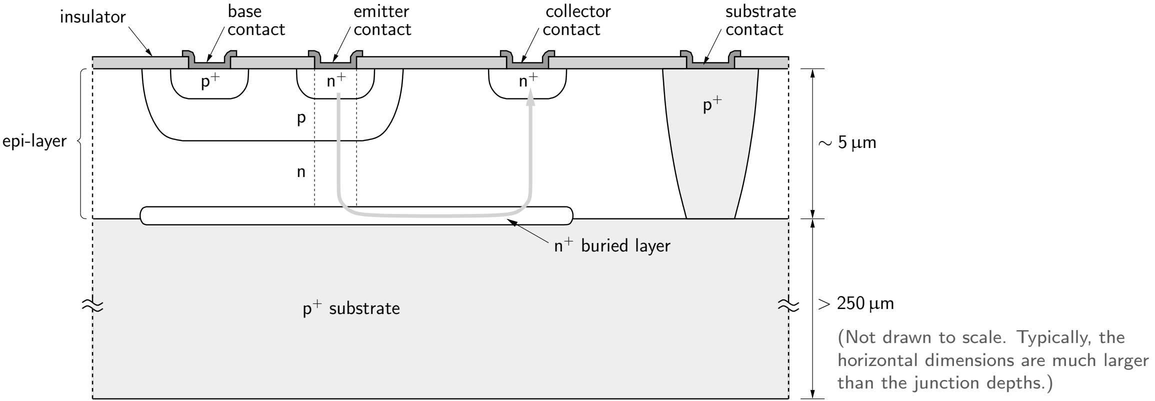

* To make an integrated circuit (IC), a large number of transistors are fabricated on a single silicon piece and interconnected as required.
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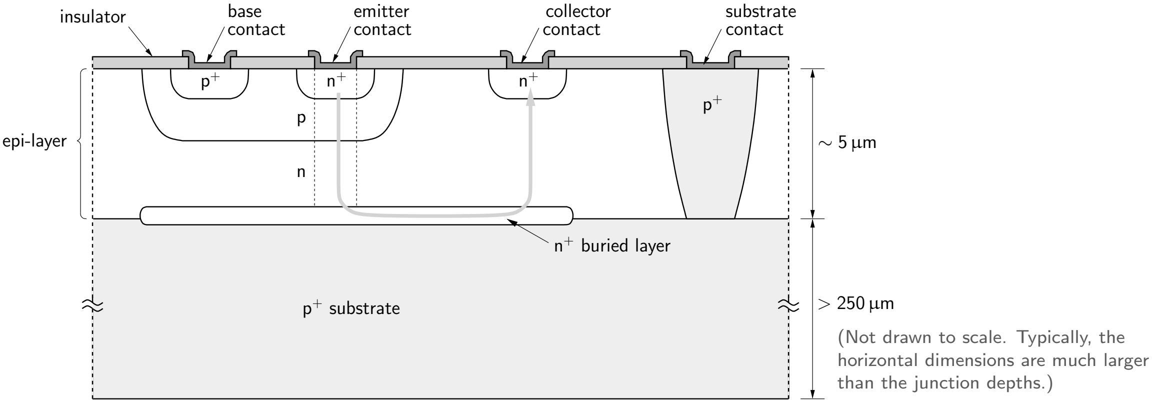
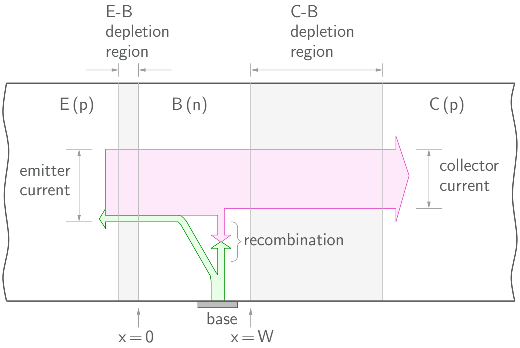
BJT structure in integrated circuits (*npn* transistor)



* To make an integrated circuit (IC), a large number of transistors are fabricated on a single silicon piece and interconnected as required.
* The BJTs are isolated from each other using reverse-biased *pn* junctions.
* Contacts (E, B, C) are made on the top surface for connecting to other transistors.
* An n^+ buried layer is used to provide a low-resistance path for the electron current.

Dependence of α on device parameters



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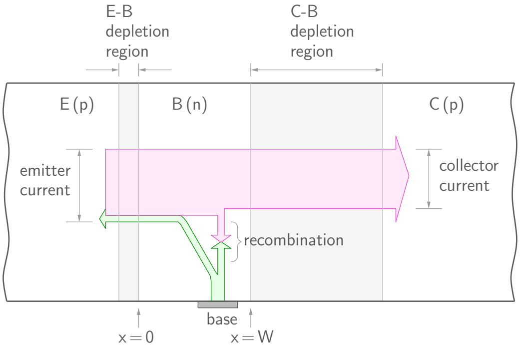

Consider a *pnp* transistor.

* I_E has a hole component and an electron component. Of these, only the hole component contributes to I_C.

  We define "emitter injection efficiency" (or simply "injection efficiency") as $\gamma = \dfrac{I_{pE}}{I_E} = \dfrac{I_{pE}}{I_{pE} + I_{nE}}$.

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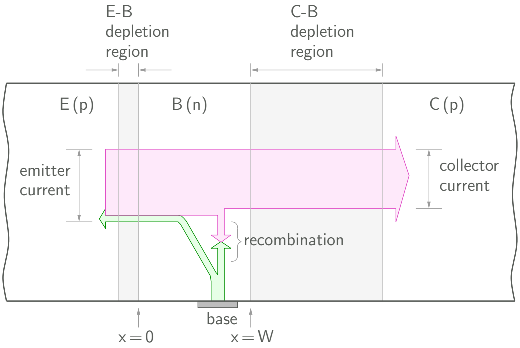

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  We define "base transport factor" as $\alpha_T = \dfrac{I_{pC}}{I_{pE}}$.

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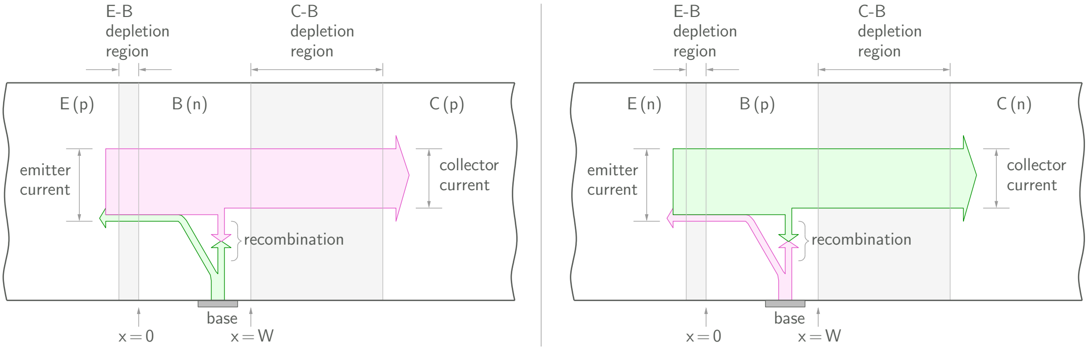

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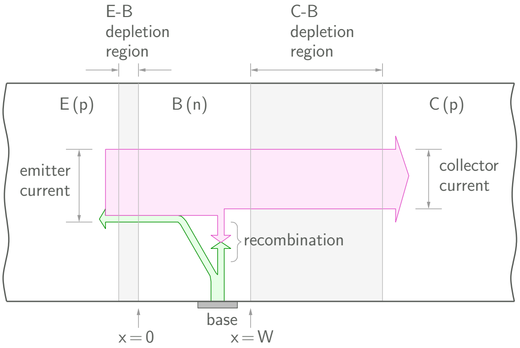

Since the C-B junction is reverse biased, the *pn* junction current arising because of V_{CB} is negligibly small.

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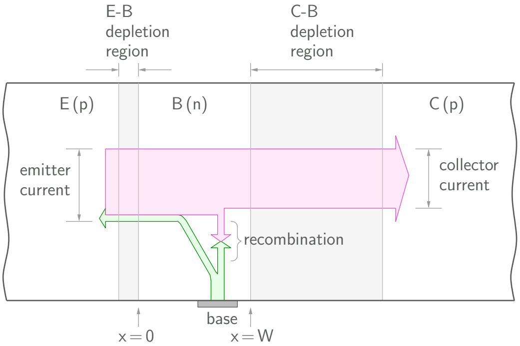

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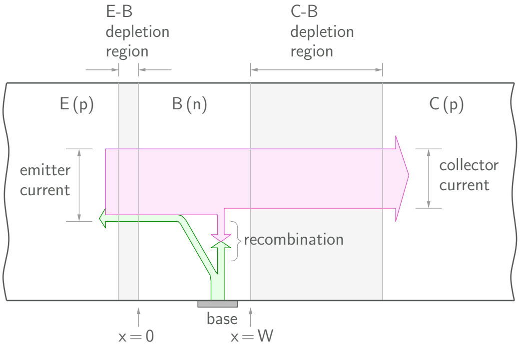
$\rightarrow$ I_C is entirely due to the holes injected by the emitter which make it to the C-B depletion boundary ($x = W$), i.e.,

$$I_C \approx I_{pC} = \alpha_T I_{pE} = \alpha_T (\gamma I_E) \rightarrow \alpha = \frac{I_C}{I_E} = \gamma \alpha_T .$$

Dependence of α on device parameters



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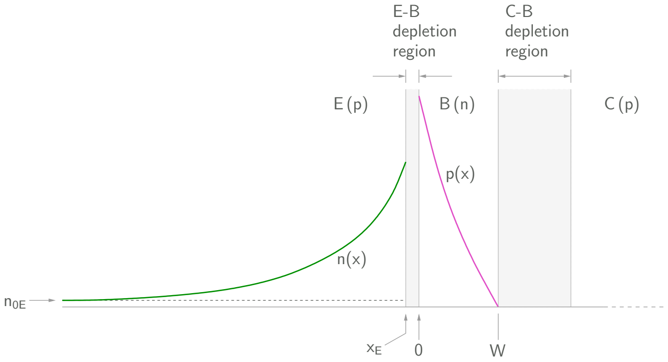
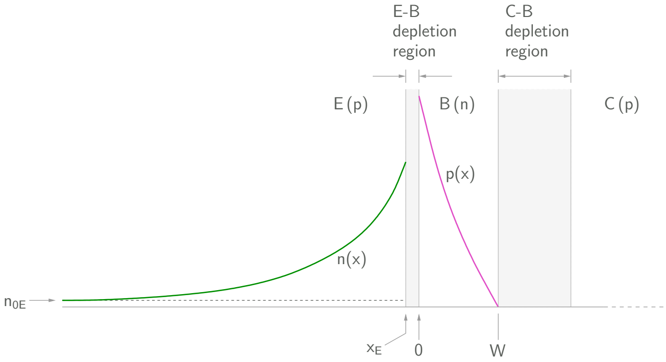
$\rightarrow$ For $\alpha \approx 1$, both γ and α_T must be close to 1.



We assume that the emitter width is greater than $5\,L_n$.



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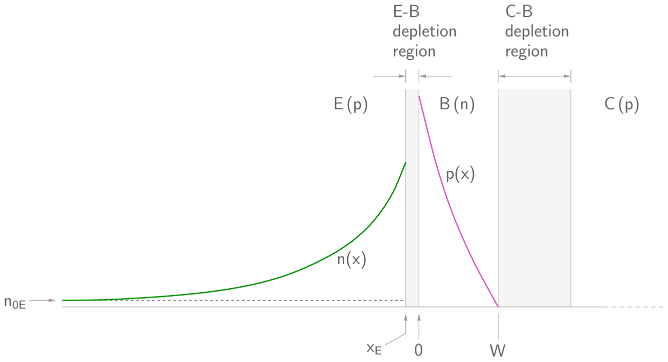
Neglecting the drift components for minority carriers in the emitter and base neutral regions, we get

$$D_{nE}\,\frac{d^2\Delta n}{dx^2} - \frac{\Delta n}{\tau_{nE}} = 0, \quad x < x_E, \text{ with}$$

$$\Delta n(x_E) = n_{0E}\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right],$$

$$\Delta n(-\infty) = 0.$$



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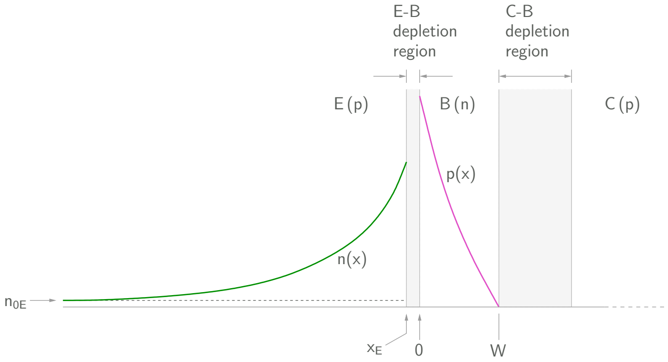
$$\Delta p(0) = p_{0B}\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right],$$

$$\Delta p(W) = p_{0B}\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right].$$

Dependence of α on device parameters



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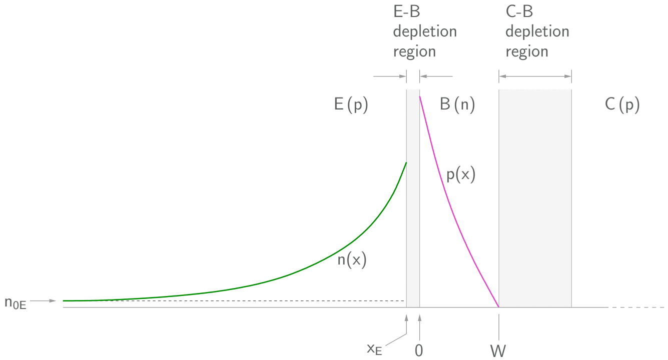
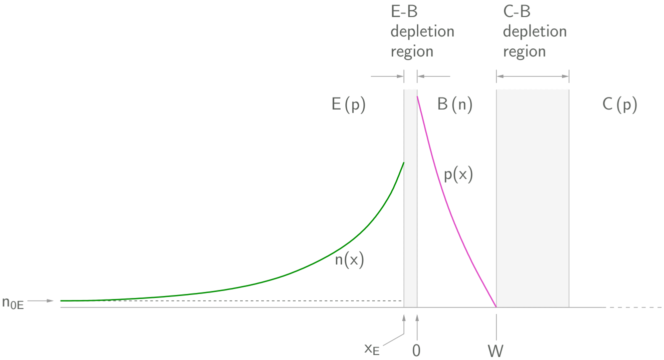


Solution: $\Delta n(x) = \Delta n(x_E)\, e^{-(x_E - x)/L_{nE}}, \qquad x < x_E,$

$$\Delta p(x) = A\, e^{-x/L_{pB}} + B\, e^{+x/L_{pB}}, \quad 0 < x < W.$$

Dependence of α on device parameters



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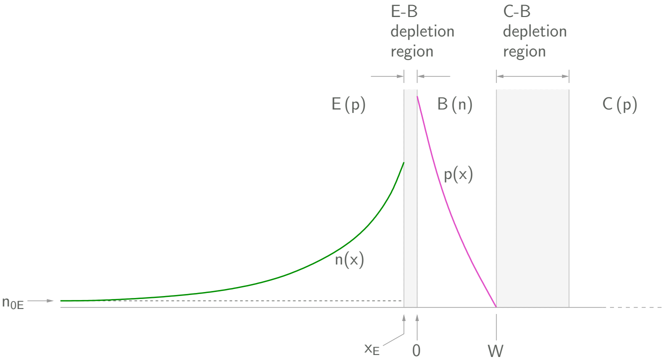
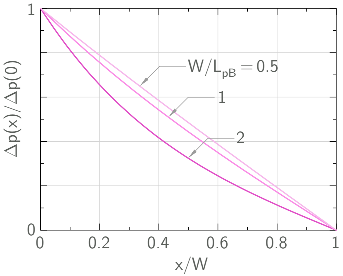
Using the boundary conditions (last slide), we get

$$\Delta p(x) = \Delta p(0) \frac{\sinh\left(\dfrac{W-x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)} + \Delta p(W) \frac{\sinh\left(\dfrac{x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)}.$$





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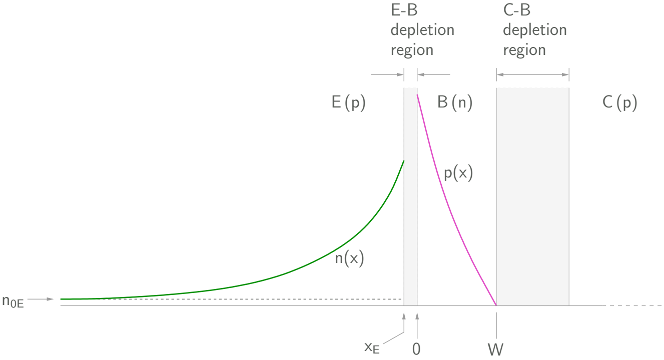
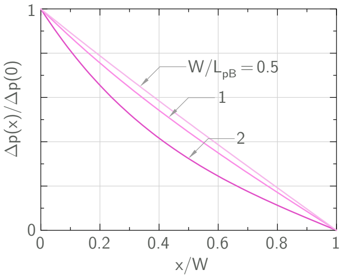
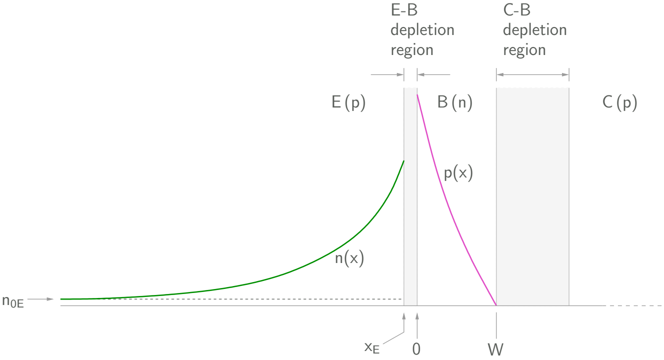
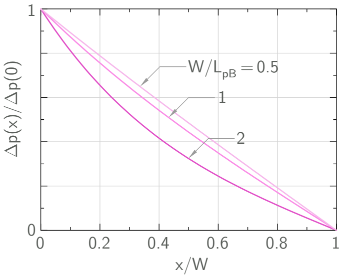




$$I_{nE} = qAD_{nE}\,\frac{dn}{dx}(x_E) = qAD_{nE}\,\frac{d\Delta n}{dx}(x_E)$$

$$= \frac{qAD_{nE}}{L_{nE}}\,\Delta n(x_E) = \frac{qAD_{nE}}{L_{nE}}\,n_{0E}\left(e^{V_{EB}/V_T}-1\right),$$





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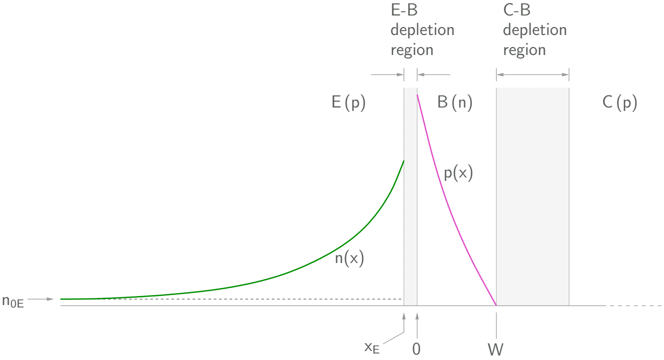
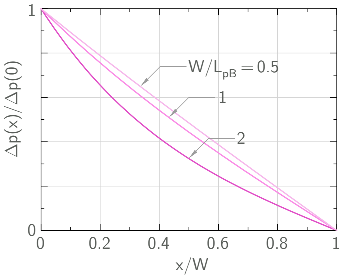
$$I_{pE} = -qAD_{pB}\,\frac{dp}{dx}(0) = -qAD_{pB}\,\frac{d\Delta p}{dx}(0)$$

$$= \frac{qAD_{pB}}{L_{pB}}\,p_{0B}\left(e^{V_{EB}/V_T} - 1\right)\frac{\cosh(W/L_{pB})}{\sinh(W/L_{pB})}.$$





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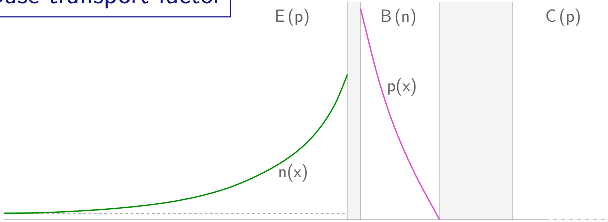
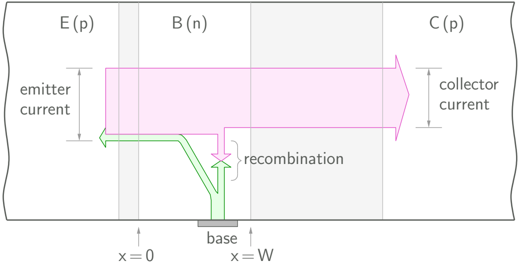
$$I_{pE} = -qAD_{pB}\,\frac{dp}{dx}(0) = -qAD_{pB}\,\frac{d\Delta p}{dx}(0)$$
$$= \frac{qAD_{pB}}{L_{pB}}\,p_{0B}\left(e^{V_{EB}/V_T} - 1\right)\frac{\cosh(W/L_{pB})}{\sinh(W/L_{pB})}.$$

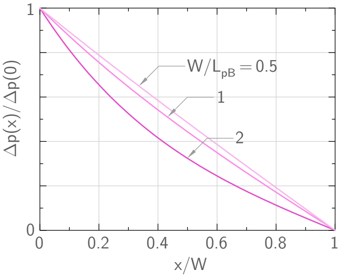
$$\gamma = \frac{I_{pE}}{I_{pE} + I_{nE}} = \frac{1}{1 + (I_{nE}/I_{pE})}$$
$$= \frac{1}{1 + \left(\dfrac{D_{nE}}{D_{pB}}\,\dfrac{L_{pB}}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}}\right)\dfrac{\sinh(W/L_B)}{\cosh(W/L_B)}},$$
since $\dfrac{n_{0E}}{p_{0B}} = \dfrac{n_i^2}{N_{aE}} \times \dfrac{N_{dB}}{n_i^2} = \dfrac{N_{dB}}{N_{aE}}$.

Base transport factor



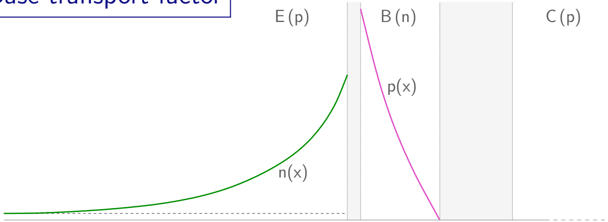


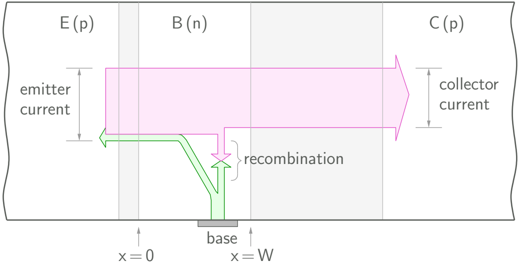
$$\Delta p(x) = \Delta p(0)\,\frac{\sinh\left(\dfrac{W-x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)} + \Delta p(W)\,\frac{\sinh\left(\dfrac{x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)}.$$



M. B. Patil, IIT Bombay

Base transport factor





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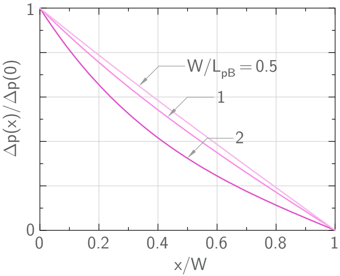
$$I_C \approx I_{pC} = -qAD_{pB}\,\frac{d\Delta p}{dx}(W)$$



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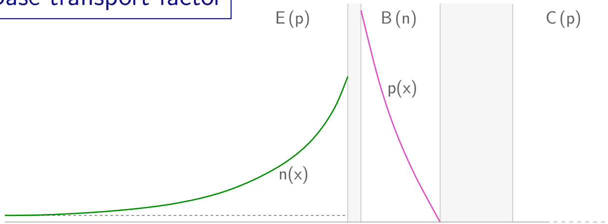

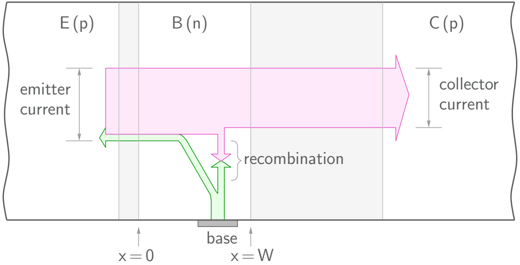

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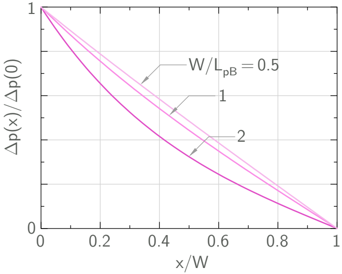
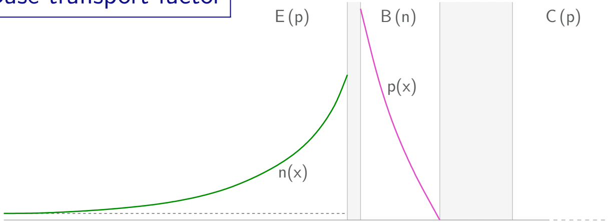
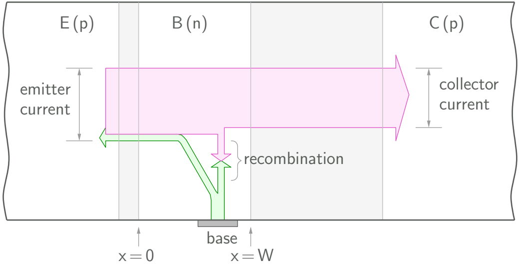
$$I_C \approx I_{pC} = -qAD_{pB}\,\frac{d\Delta p}{dx}(W)$$

$$= \frac{qAD_{pB}}{L_{pB}}\,p_{0B}\left(e^{V_{EB}/V_T} - 1\right)\frac{1}{\sinh(W/L_B)}.$$



Base transport factor





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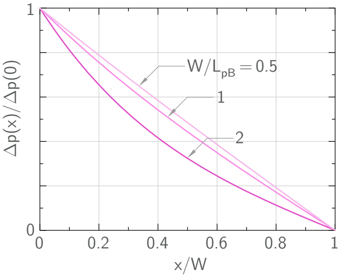
$$I_C \approx I_{pC} = -qAD_{pB}\,\frac{d\Delta p}{dx}(W)$$

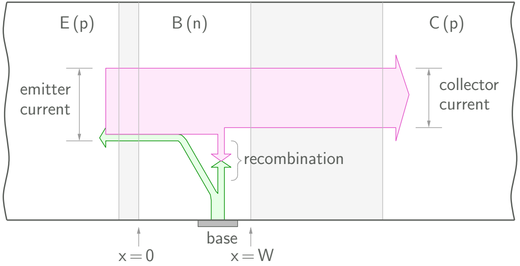
$$= \frac{qAD_{pB}}{L_{pB}}\,p_{0B}\left(e^{V_{EB}/V_T}-1\right)\frac{1}{\sinh(W/L_B)}.$$

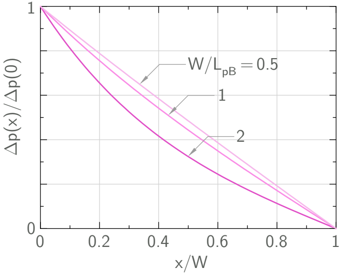
The base transport factor is (using I_{pE} from the last slide),

$$\alpha_T = \frac{I_{pC}}{I_{pE}} = \frac{1}{\cosh(W/L_{pB})}.$$

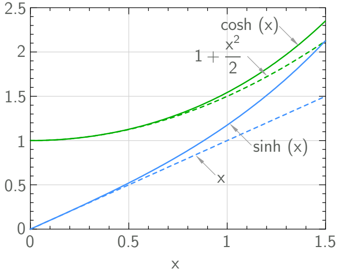


α_T with $W \ll L_{pB}$

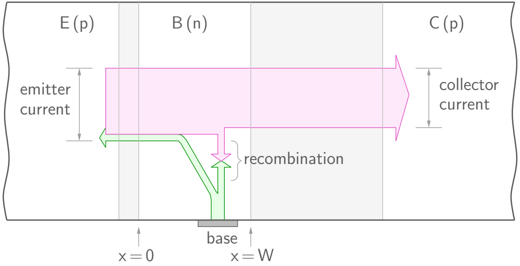


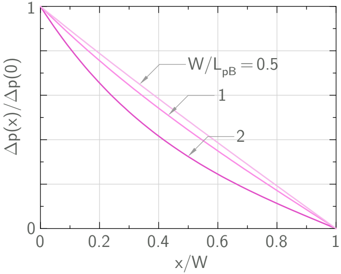


$$\Delta p(x) = \Delta p(0)\,\frac{\sinh\left(\dfrac{W-x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)} + \Delta p(W)\,\frac{\sinh\left(\dfrac{x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)}.$$



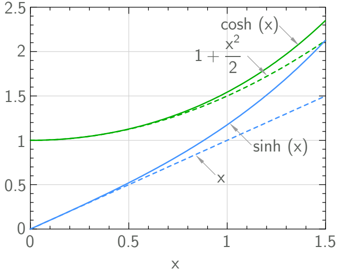
α_T with $W \ll L_{pB}$



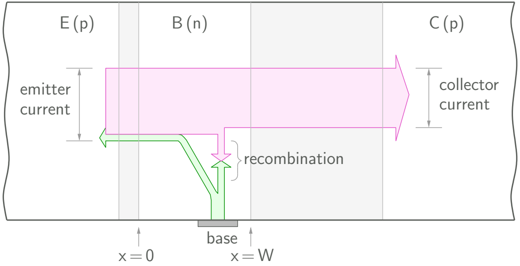


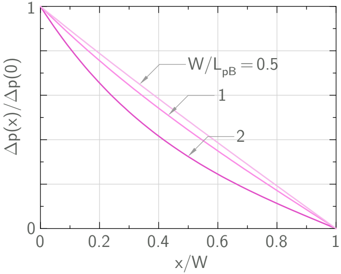
$$\Delta p(x) = \Delta p(0) \frac{\sinh\left(\dfrac{W-x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)} + \Delta p(W) \frac{\sinh\left(\dfrac{x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)}.$$

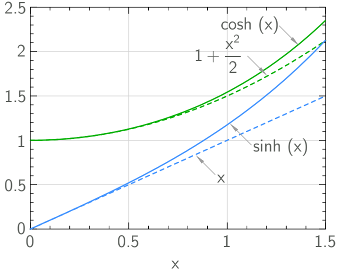
$$\alpha_T = \frac{I_{pC}}{I_{pE}} = \frac{1}{\cosh(W/L_{pB})} \approx \frac{1}{1+\dfrac{1}{2}\left(\dfrac{W}{L_{pB}}\right)^2}.$$



α_T with $W \ll L_{pB}$





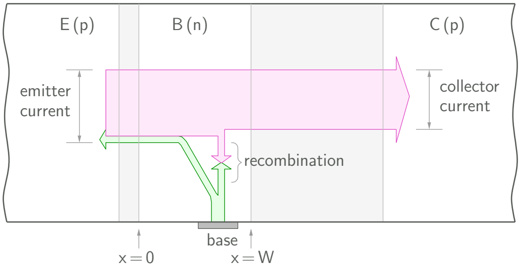


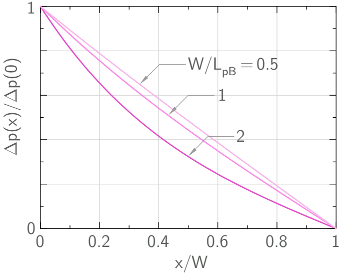
$$\Delta p(x) = \Delta p(0)\,\frac{\sinh\left(\dfrac{W-x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)} + \Delta p(W)\,\frac{\sinh\left(\dfrac{x}{L_{pB}}\right)}{\sinh\left(\dfrac{W}{L_{pB}}\right)}.$$

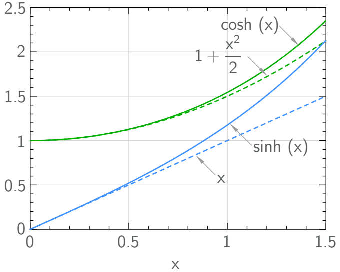
$$\alpha_T = \frac{I_{pC}}{I_{pE}} = \frac{1}{\cosh(W/L_{pB})} \approx \frac{1}{1+\dfrac{1}{2}\left(\dfrac{W}{L_{pB}}\right)^2}.$$

Remark: $\alpha_T \rightarrow 1$ if the base width W is made small compared to L_{pB}.

γ with $W \ll L_{pB}$

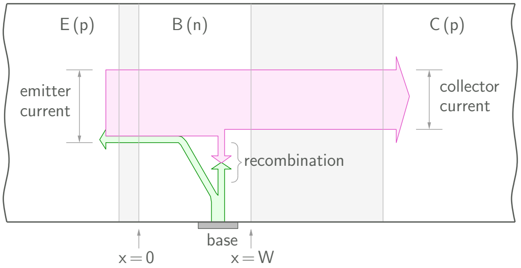


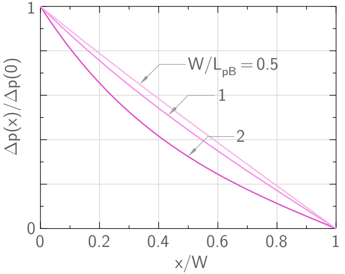




$$\gamma = \frac{I_{pE}}{I_{pE} + I_{nE}} = \frac{1}{1 + (I_{nE}/I_{pE})} = \frac{1}{1 + \left(\dfrac{D_{nE}}{D_{pB}}\,\dfrac{L_{pB}}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}}\right) \dfrac{\sinh(W/L_B)}{\cosh(W/L_B)}}$$

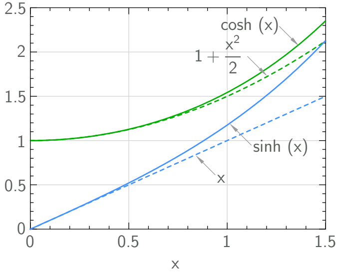
γ with $W \ll L_{pB}$



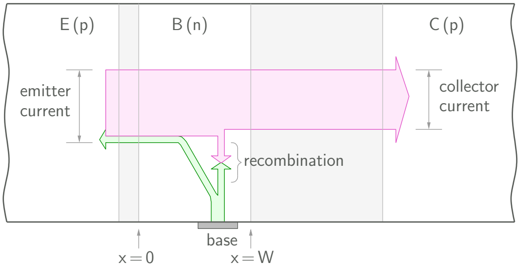


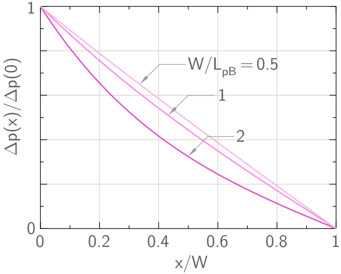
$$\gamma = \frac{I_{pE}}{I_{pE}+I_{nE}} = \frac{1}{1+(I_{nE}/I_{pE})} = \frac{1}{1+\left(\dfrac{D_{nE}}{D_{pB}}\,\dfrac{L_{pB}}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}}\right)\dfrac{\sinh(W/L_B)}{\cosh(W/L_B)}}$$

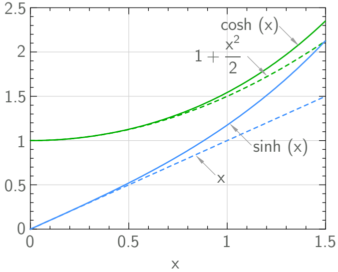
$$\approx \frac{1}{1+\left(\dfrac{D_{nE}}{D_{pB}}\,\dfrac{L_{pB}}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}}\right)\dfrac{W/L_{pB}}{1+\frac{1}{2}(W/L_{pB})^2}}$$



γ with $W \ll L_{pB}$





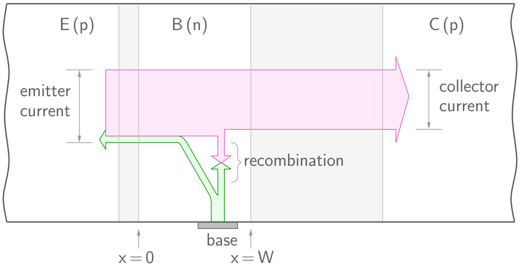


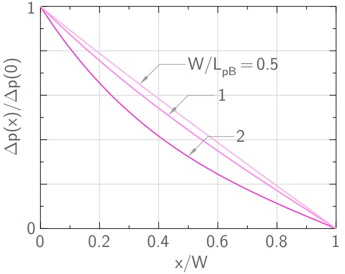
$$\gamma = \frac{I_{pE}}{I_{pE} + I_{nE}} = \frac{1}{1 + (I_{nE}/I_{pE})} = \frac{1}{1 + \left(\frac{D_{nE}}{D_{pB}}\frac{L_{pB}}{L_{nE}}\frac{N_{dB}}{N_{aE}}\right)\frac{\sinh(W/L_B)}{\cosh(W/L_B)}}$$

$$\approx \frac{1}{1 + \left(\frac{D_{nE}}{D_{pB}}\frac{L_{pB}}{L_{nE}}\frac{N_{dB}}{N_{aE}}\right)\frac{W/L_{pB}}{1 + \frac{1}{2}(W/L_{pB})^2}}$$

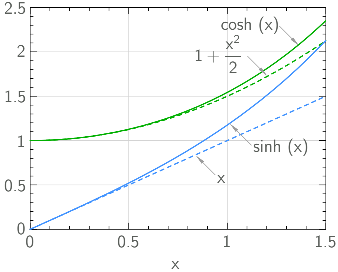
$$\approx \frac{1}{1 + \left(\frac{D_{nE}}{D_{pB}}\frac{W}{L_{nE}}\frac{N_{dB}}{N_{aE}}\right)}$$

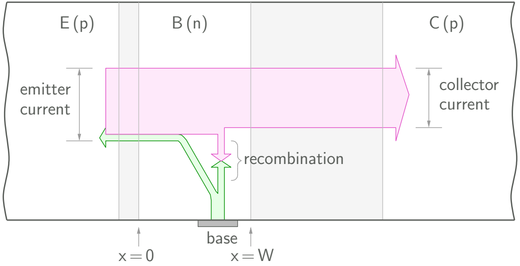
γ with $W \ll L_{pB}$

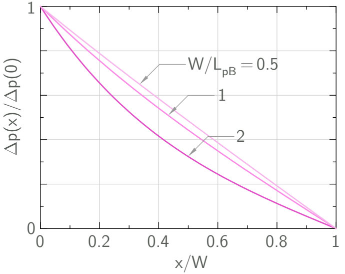


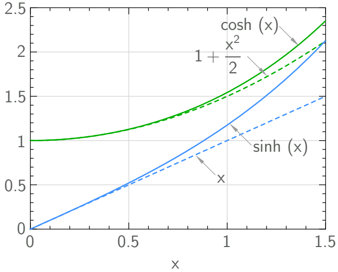


$$\gamma \approx \frac{1}{1 + \left(\dfrac{D_{nE}}{D_{pB}} \, \dfrac{W}{L_{nE}} \, \dfrac{N_{dB}}{N_{aE}} \right)}.$$



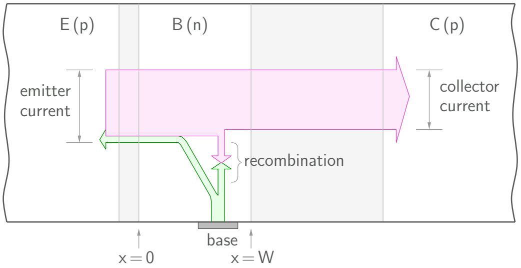


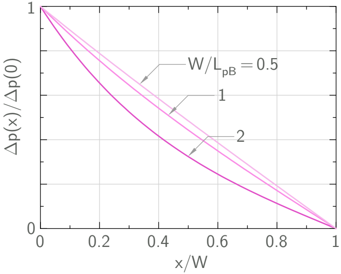


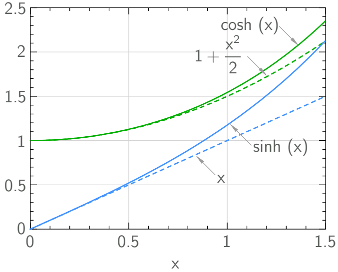


$$\gamma \approx \frac{1}{1 + \left(\dfrac{D_{nE}}{D_{pB}} \, \dfrac{W}{L_{nE}} \, \dfrac{N_{dB}}{N_{aE}} \right)} .$$

$* \ \gamma \rightarrow 1$ if $N_{aE} \gg N_{dB}$.



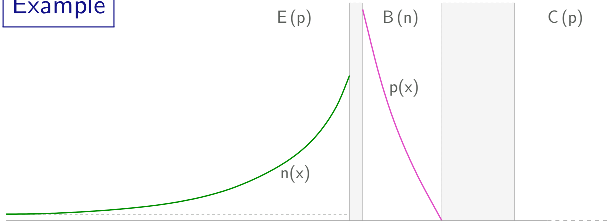


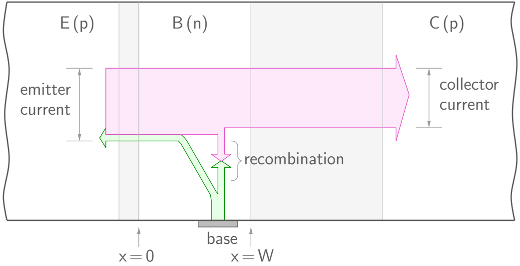


$$\gamma \approx \frac{1}{1 + \left(\frac{D_{nE}}{D_{pB}} \frac{W}{L_{nE}} \frac{N_{dB}}{N_{aE}} \right)}.$$

* $\gamma \to 1$ if $N_{aE} \gg N_{dB}$.
* It is now clear why a higher doping density in the emitter region (compared to the base doping density) is desirable.

Example





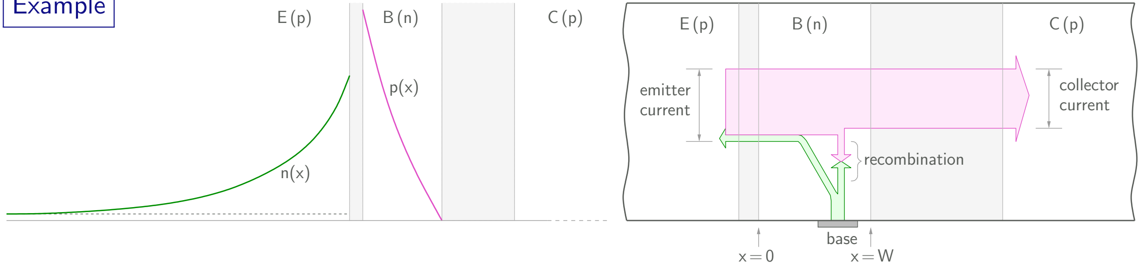
Consider a *pnp* BJT with $N_{aE} = 10^{18}\,\text{cm}^{-3}$, $N_{dB} = 5 \times 10^{16}\,\text{cm}^{-3}$, $N_{aC} = 10^{15}\,\text{cm}^{-3}$, and with a base width $W = 2\,\mu\text{m}$ ($T = 300\,\text{K}$).

(a) Calculate α_T, γ, α, and β, using the following parameters.

$\mu_{nE} = 250\,\text{cm}^2/\text{V-s}$, $\mu_{pB} = 500\,\text{cm}^2/\text{V-s}$, $\mu_{nC} = 1500\,\text{cm}^2/\text{V-s}$,

$\tau_{nE} = 0.2\,\mu\text{s}$, $\tau_{pB} = 1\,\mu\text{s}$, $\tau_{nC} = 1\,\mu\text{s}$.

Example



Consider a *pnp* BJT with $N_{aE} = 10^{18}\,\text{cm}^{-3}$, $N_{dB} = 5 \times 10^{16}\,\text{cm}^{-3}$, $N_{aC} = 10^{15}\,\text{cm}^{-3}$, and with a base width $W = 2\,\mu\text{m}$ ($T = 300\,\text{K}$).

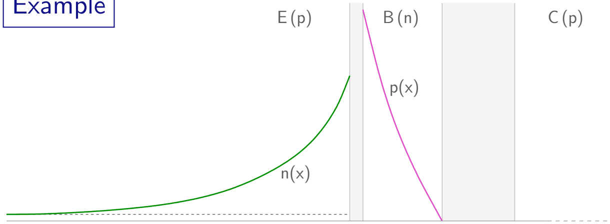
(a) Calculate α_T, γ, α, and β, using the following parameters.

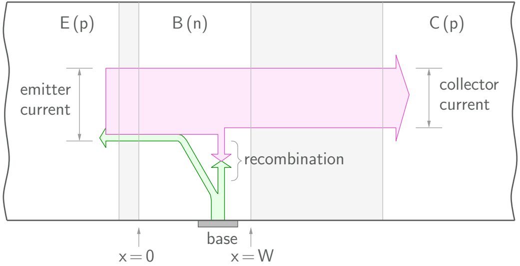
$\mu_{nE} = 250\,\text{cm}^2/\text{V-s}$, $\mu_{pB} = 500\,\text{cm}^2/\text{V-s}$, $\mu_{nC} = 1500\,\text{cm}^2/\text{V-s}$,

$\tau_{nE} = 0.2\,\mu\text{s}$, $\tau_{pB} = 1\,\mu\text{s}$, $\tau_{nC} = 1\,\mu\text{s}$.

(b) Repeat (a) for the BJT operating in the reverse active mode.

Example





Solution:

The minority carrier diffusion lengths are

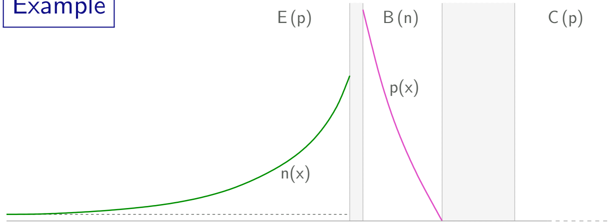
$$L_{nE} = \sqrt{D_{nE}\tau_{nE}} = \sqrt{V_T\,\mu_{nE}\tau_{nE}} = \sqrt{0.0258 \times 250 \times 0.2 \times 10^{-6}} = 1.14 \times 10^{-3}\,\text{cm} = 11.4\,\mu\text{m}.$$

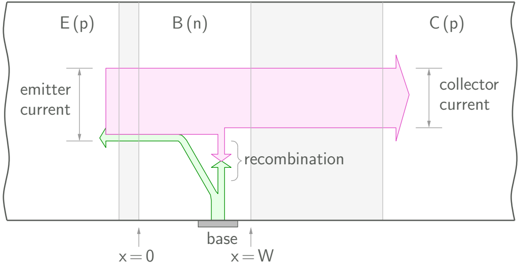
$$L_{pB} = \sqrt{D_{pB}\tau_{pB}} = \sqrt{V_T\,\mu_{pB}\tau_{pB}} = \sqrt{0.0258 \times 500 \times 1 \times 10^{-6}} = 3.59 \times 10^{-3}\,\text{cm} = 35.9\,\mu\text{m}.$$

$$L_{nC} = \sqrt{D_{nC}\tau_{nC}} = \sqrt{V_T\,\mu_{nC}\tau_{nC}} = \sqrt{0.0258 \times 1500 \times 1 \times 10^{-6}} = 6.22 \times 10^{-3}\,\text{cm} = 62.2\,\mu\text{m}.$$

Note that $L_{pB} \gg W$ $(2\,\mu\text{m})$.

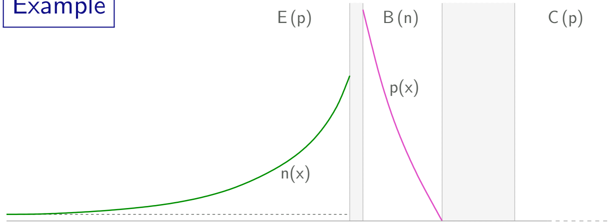
Example

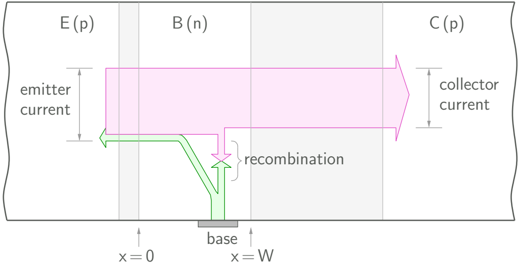




(a) $$\frac{D_{nE}}{D_{pB}}\,\frac{W}{L_{nE}}\,\frac{N_{dB}}{N_{aE}} = \frac{\mu_{nE}}{\mu_{pB}}\,\frac{W}{L_{nE}}\,\frac{N_{dB}}{N_{aE}} = \frac{250}{500}\,\frac{2\times10^{-4}}{1.14\times10^{-3}}\,\frac{5\times10^{16}}{10^{18}} = 4.386\times10^{-3}.$$

Example

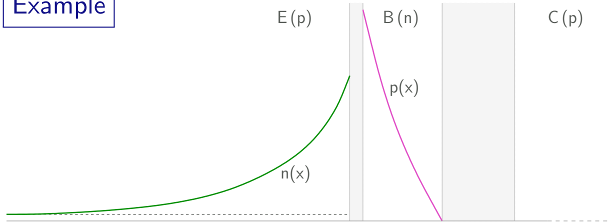


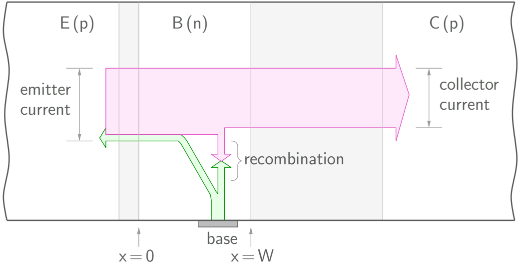


(a) $\dfrac{D_{nE}}{D_{pB}}\,\dfrac{W}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}} = \dfrac{\mu_{nE}}{\mu_{pB}}\,\dfrac{W}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}} = \dfrac{250}{500}\,\dfrac{2\times10^{-4}}{1.14\times10^{-3}}\,\dfrac{5\times10^{16}}{10^{18}} = 4.386\times10^{-3}.$

$$\gamma = \frac{1}{1+4.386\times10^{-3}} = 0.9956.$$

Example



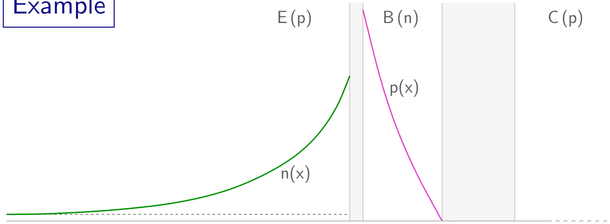


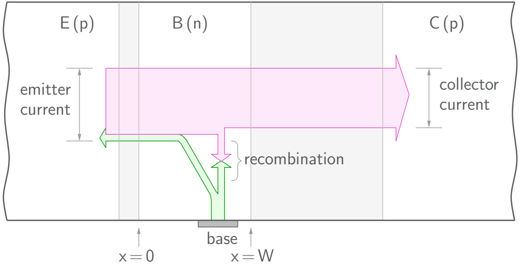
(a) $\dfrac{D_{nE}}{D_{pB}}\,\dfrac{W}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}} = \dfrac{\mu_{nE}}{\mu_{pB}}\,\dfrac{W}{L_{nE}}\,\dfrac{N_{dB}}{N_{aE}} = \dfrac{250}{500}\,\dfrac{2\times10^{-4}}{1.14\times10^{-3}}\,\dfrac{5\times10^{16}}{10^{18}} = 4.386\times10^{-3}.$

$$\gamma = \frac{1}{1+4.386\times10^{-3}} = 0.9956.$$

$$\alpha_T = \frac{1}{1+\frac{1}{2}(W/L_{pB})^2} = \frac{1}{1+\frac{1}{2}(2.0/35.9)^2} = 0.9985.$$

Example





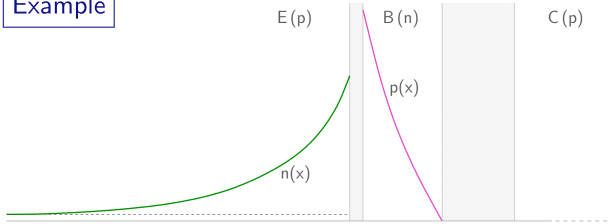
(a) $$\frac{D_{nE}}{D_{pB}}\,\frac{W}{L_{nE}}\,\frac{N_{dB}}{N_{aE}} = \frac{\mu_{nE}}{\mu_{pB}}\,\frac{W}{L_{nE}}\,\frac{N_{dB}}{N_{aE}} = \frac{250}{500}\,\frac{2\times 10^{-4}}{1.14\times 10^{-3}}\,\frac{5\times 10^{16}}{10^{18}} = 4.386\times 10^{-3}.$$

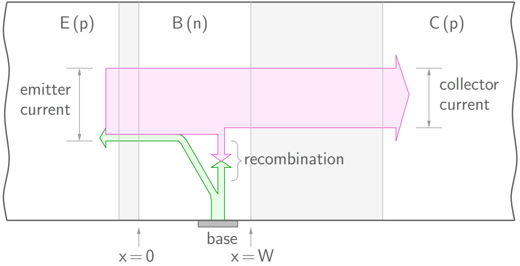
$$\gamma = \frac{1}{1+4.386\times 10^{-3}} = 0.9956.$$

$$\alpha_T = \frac{1}{1+\frac{1}{2}(W/L_{pB})^2} = \frac{1}{1+\frac{1}{2}(2.0/35.9)^2} = 0.9985.$$

$$\alpha = \gamma\alpha_T = 0.9940 \rightarrow \beta = \frac{\alpha}{1-\alpha} = 166.$$

Example





(b) With $E \leftrightarrow C$,

$$\frac{D_{nE}}{D_{pB}}\,\frac{W}{L_{nE}}\,\frac{N_{dB}}{N_{aE}} \rightarrow \frac{D_{nC}}{D_{pB}}\,\frac{W}{L_{nC}}\,\frac{N_{dB}}{N_{aC}} = \frac{\mu_{nC}}{\mu_{pB}}\,\frac{W}{L_{nC}}\,\frac{N_{dB}}{N_{aC}} = \frac{1500}{500}\,\frac{2\times 10^{-4}}{6.22\times 10^{-3}}\,\frac{5\times 10^{16}}{10^{15}} = 4.823.$$

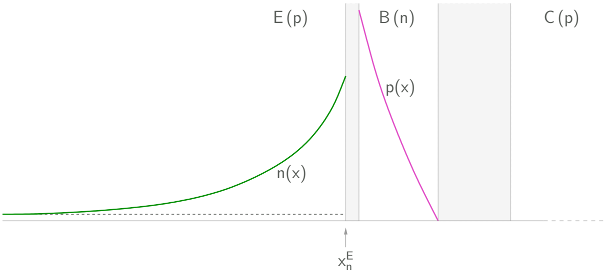
$$\gamma = \frac{1}{1+4.823} = 0.1717,\ \alpha_T = \frac{1}{1+\frac{1}{2}(2/35.9)^2} = 0.9985.$$

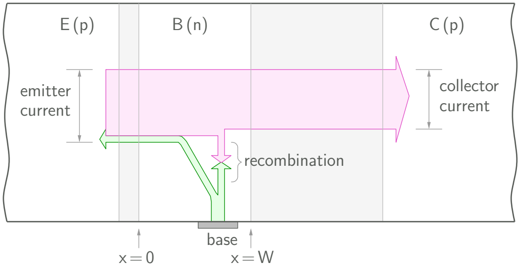
$$\rightarrow \alpha = \gamma\,\alpha_T = 0.1714 \rightarrow \beta = 0.2,\ \text{a disaster.}$$

Conclusion: $N_{aE} \gg N_{dB}$ is crucial.

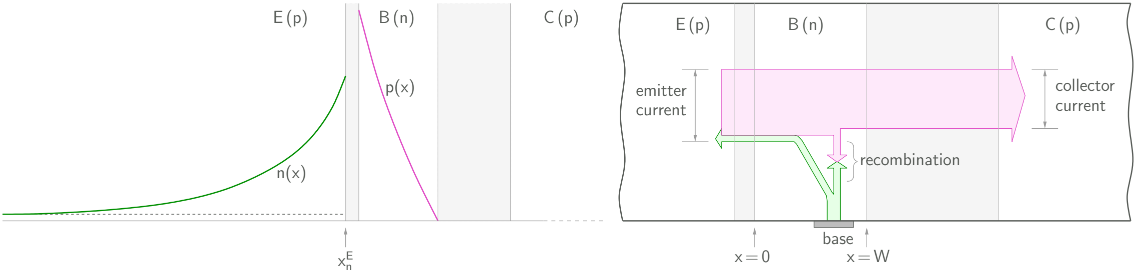
(Note that we have treated W as a constant, but it would vary with bias conditions.)

γ with $W << L_{pB}$



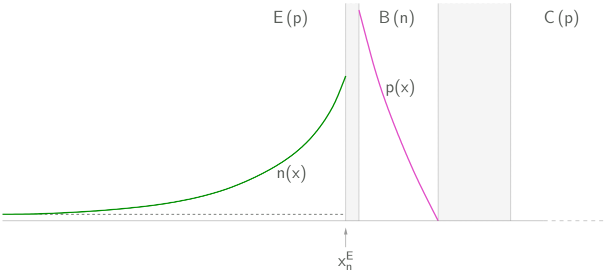


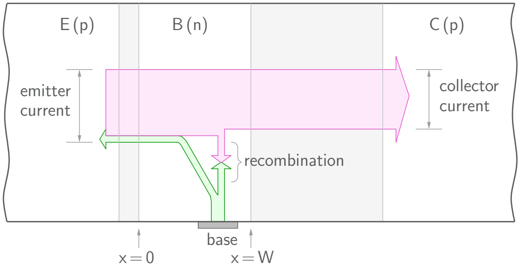
γ with $W << L_{pB}$



$$I_{nE} = -q\,A\,D_{nE}\,\frac{d}{dx}\left[n_{0E}\exp\left(\frac{V_{EB}}{V_T}\right)\exp\left(-\frac{x_n^E - x}{L_{nE}}\right)\right] \text{ at } x = x_n^E$$

γ with $W << L_{pB}$

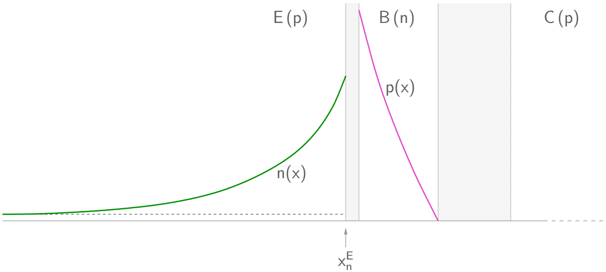


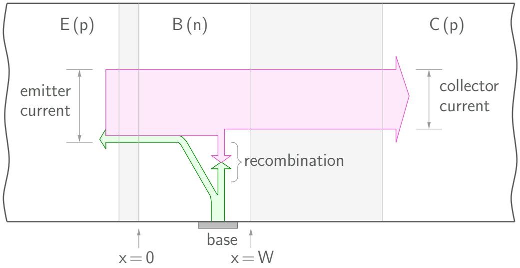


$$I_{nE} = -q\,A\,D_{nE}\,\frac{d}{dx}\left[n_{0E}\exp\left(\frac{V_{EB}}{V_T}\right)\exp\left(-\frac{x_n^E - x}{L_{nE}}\right)\right] \quad \text{at } x = x_n^E$$

$$= q\,A\,D_{nE}\,n_{0E}\exp\left(\frac{V_{EB}}{V_T}\right) \times \frac{1}{L_{nE}}$$

γ with $W << L_{pB}$





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$$= q\,A\,D_{nE}\,n_{0E}\exp\left(\frac{V_{EB}}{V_T}\right) \times \frac{1}{L_{nE}}$$

$$I_{pE} \approx q\,A\,D_{pB}\,p_{0B}\exp\left(\frac{V_{EB}}{V_T}\right) \times \frac{1}{W}$$

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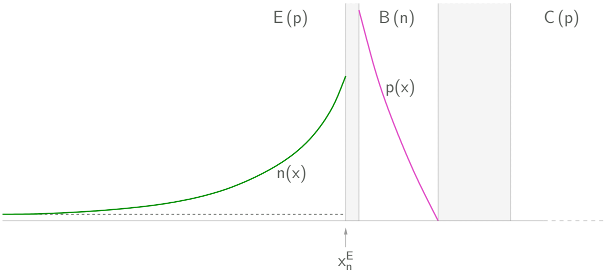
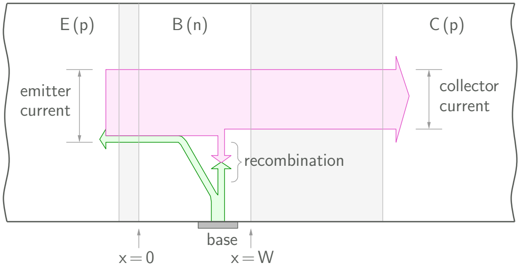
γ with $W << L_{pB}$





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$$I_{pE} \approx q\,A\,D_{pB}\,p_{0B}\exp\left(\frac{V_{EB}}{V_T}\right) \times \frac{1}{W}$$

$$\frac{I_{nE}}{I_{pE}} \approx \frac{D_{nE}}{D_{pB}}\,\frac{n_{0E}}{p_{0B}}\,\frac{W}{L_{nE}}$$

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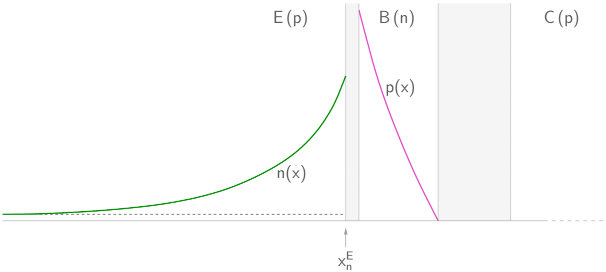

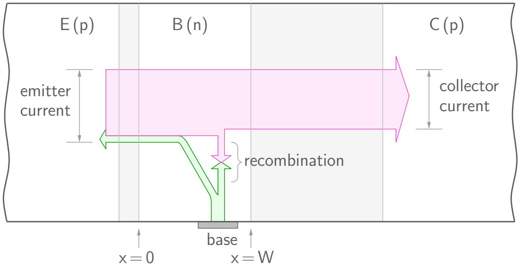

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$$= q\,A\,D_{nE}\,n_{0E}\exp\left(\frac{V_{EB}}{V_T}\right)\times\frac{1}{L_{nE}}$$

$$I_{pE} \approx q\,A\,D_{pB}\,p_{0B}\exp\left(\frac{V_{EB}}{V_T}\right)\times\frac{1}{W}$$

$$\frac{I_{nE}}{I_{pE}} \approx \frac{D_{nE}}{D_{pB}}\,\frac{n_{0E}}{p_{0B}}\,\frac{W}{L_{nE}} = \frac{D_{nE}}{D_{pB}}\,\frac{n_i^2}{N_{aE}}\,\frac{N_{dB}}{n_i^2}\,\frac{W}{L_{nE}}$$

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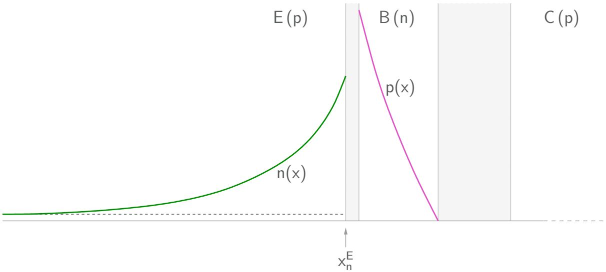
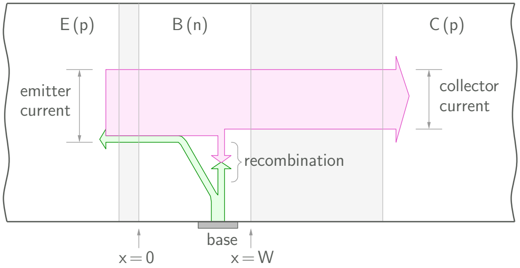
γ with $W << L_{pB}$





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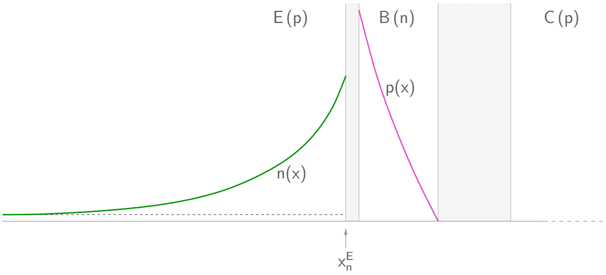
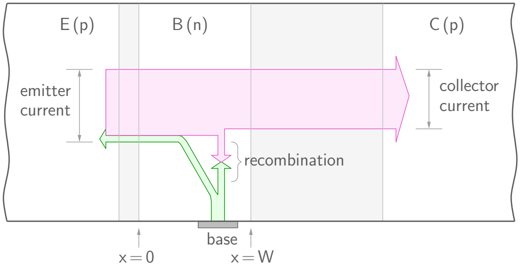
γ with $W << L_{pB}$





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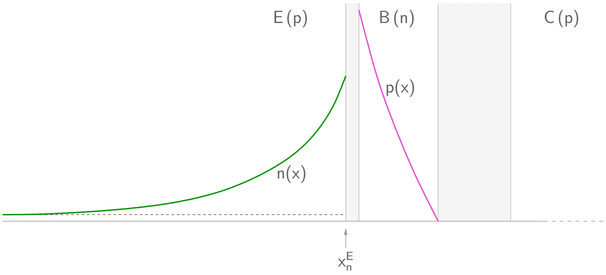
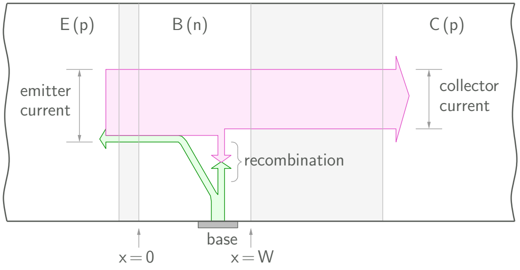
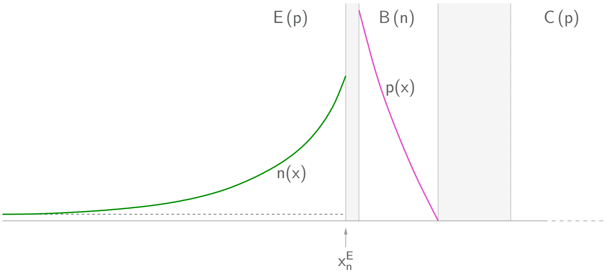
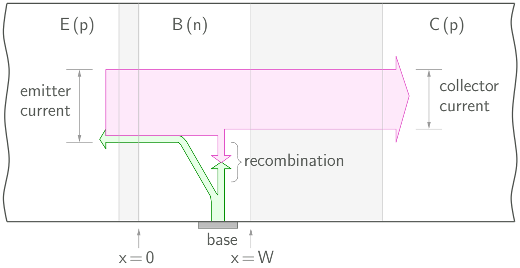




$$\frac{I_{nE}}{I_{pE}} \approx \frac{D_{nE}}{D_{pB}} \frac{N_{dB}}{N_{aE}} \frac{W}{L_{nE}}$$

γ with $W << L_{pB}$

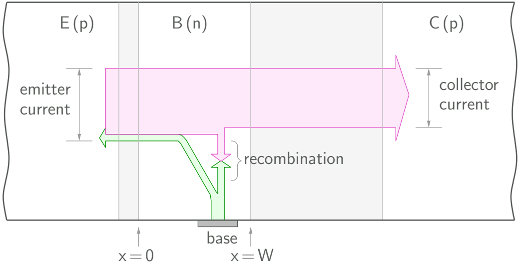


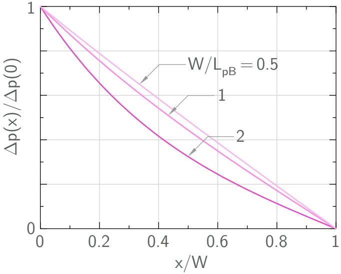


$$\frac{I_{nE}}{I_{pE}} \approx \frac{D_{nE}}{D_{pB}} \frac{N_{dB}}{N_{aE}} \frac{W}{L_{nE}}$$

$$\rightarrow \gamma = \frac{I_{pE}}{I_{pE} + I_{nE}} = \frac{1}{1 + \dfrac{I_{nE}}{I_{pE}}} \approx \frac{1}{1 + \left(\dfrac{D_{nE}}{D_{pB}} \dfrac{W}{L_{nE}} \dfrac{N_{dB}}{N_{aE}} \right)}.$$

α_T with $W \ll L_{pB}$

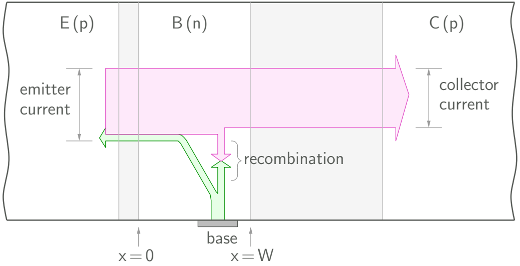


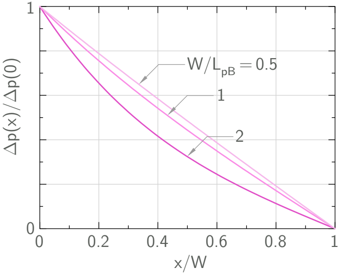


When $W \ll L_{pB}$, $\Delta p(x)$ is linear.

$$\alpha_T = \frac{I_{pC}}{I_{pE}} = \frac{I_{pC}}{I_{pC} + I_{pB}} = \frac{1}{1 + \dfrac{I_{pB}}{I_{pC}}}.$$

α_T with $W \ll L_{pB}$





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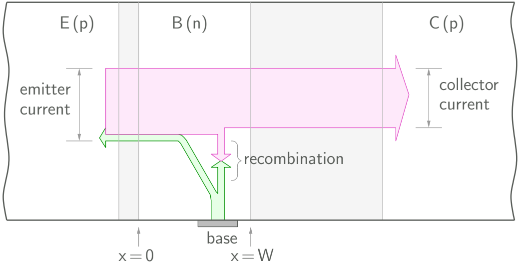
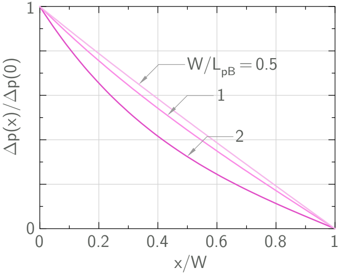
$$I_{pC} = -q\,A\,D_{pB}\,\frac{dp}{dx}(W) \approx q\,A\,D_{pB}\,\frac{\Delta p(0)}{W}.$$

α_T with $W \ll L_{pB}$





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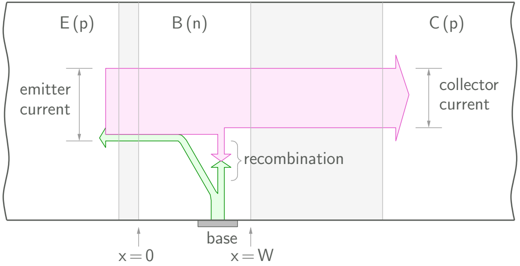
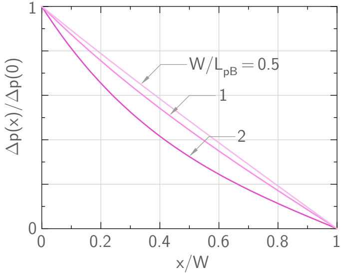
$$I_{pB} = \frac{Q_p}{\tau_{pB}} = \frac{q\,A\,\frac{1}{2}\,\Delta p(0)\,W}{\tau_{pB}}$$

α_T with $W \ll L_{pB}$





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$$I_{pB} = \frac{Q_p}{\tau_{pB}} = \frac{q\,A\,\frac{1}{2}\,\Delta p(0)\,W}{\tau_{pB}} \rightarrow \alpha_T \approx \frac{1}{1 + \dfrac{1}{2}\left(\dfrac{W}{L_{pB}}\right)^2}.$$

* We have considered a BJT in the active mode (B-E junction under forward bias, B-C junction under reverse bias) and obtained α.

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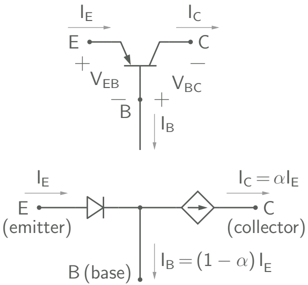
  The BJT can now be replaced with its equivalent circuit.



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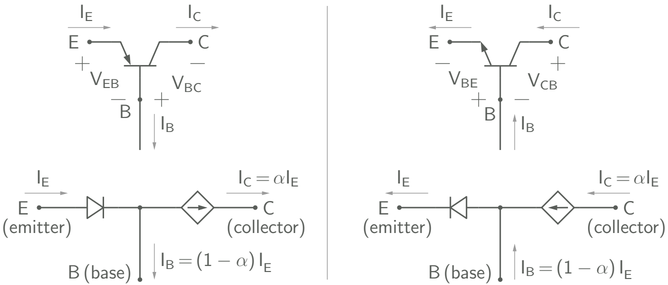
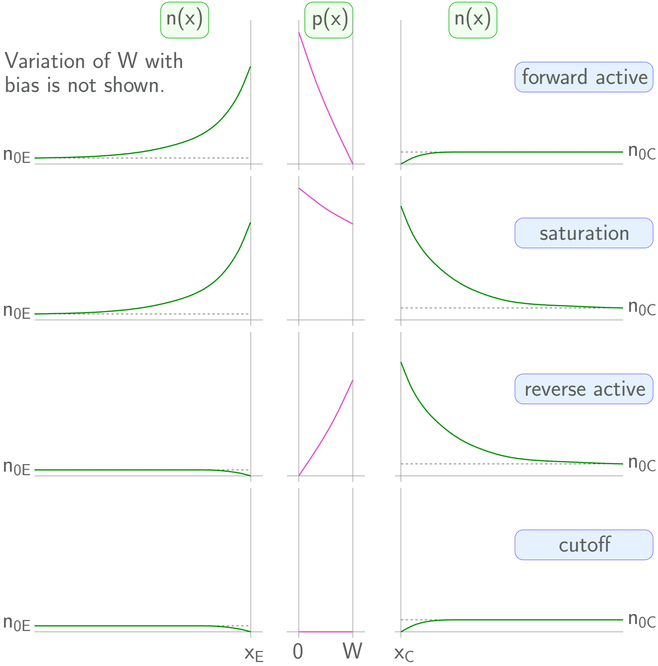


* A generalised model valid in all modes can be obtained by removing the conditions of a forward bias across the *E-B* junction and a reverse bias across the *C-B* junction → Ebers-Moll model.

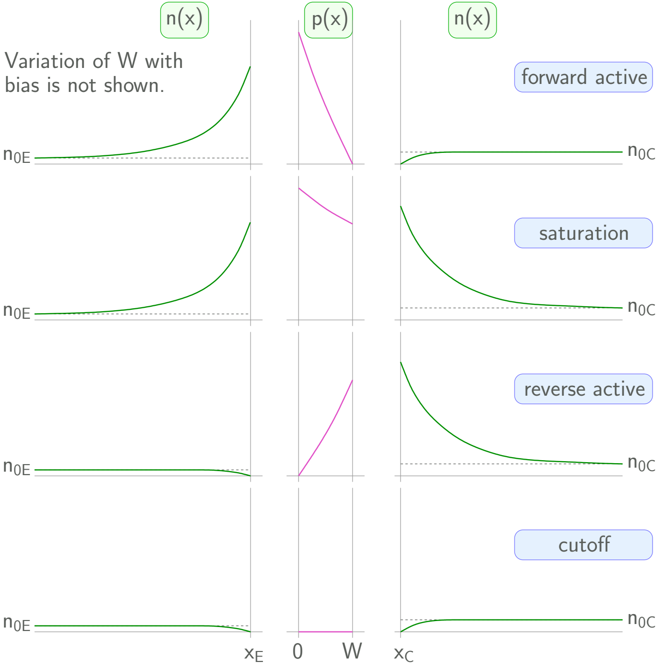
Ebers-Moll model:

Outline of derivation for a *pnp* BJT



Ebers-Moll model:
Outline of derivation for a *pnp* BJT

* Boundary conditions:



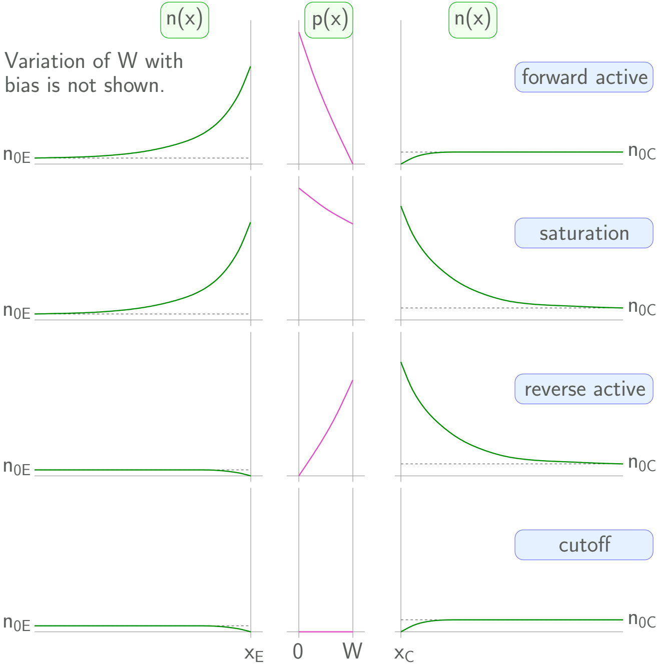
Ebers-Moll model:

Outline of derivation for a *pnp* BJT

* Boundary conditions:

$$\Delta n(x_E) \quad = n_{0E}\left[\exp\left(\frac{V_{EB}}{V_T}\right)-1\right]$$

$$\Delta n(-\infty) = 0$$



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$$\Delta n(x_E) = n_{0E}\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right]$$

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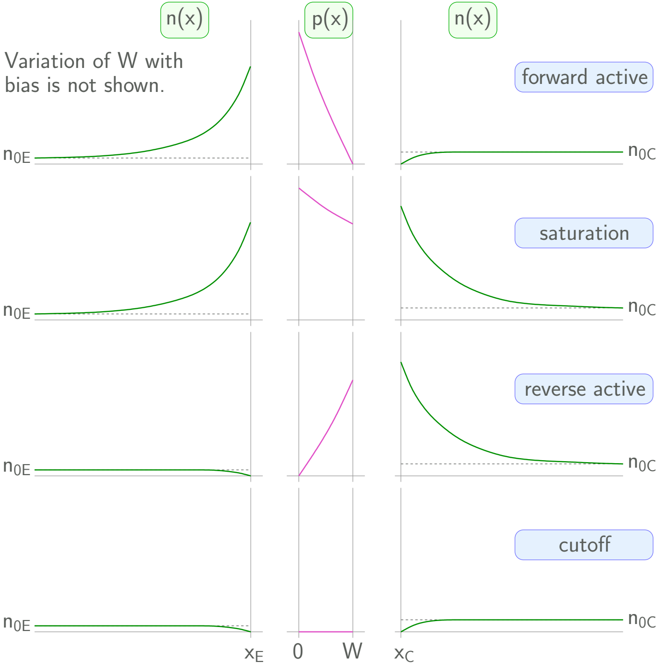
$$\Delta p(0) = p_{0B}\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right]$$

$$\Delta p(W) = p_{0B}\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]$$



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Outline of derivation for a *pnp* BJT

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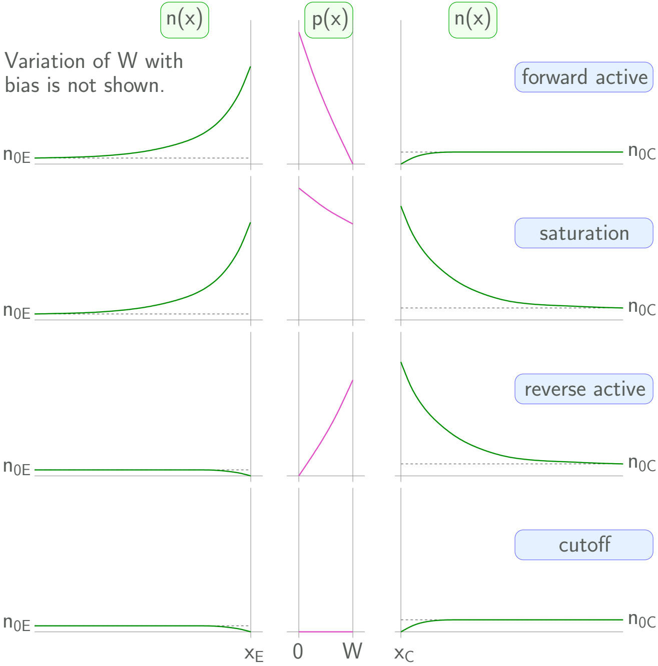
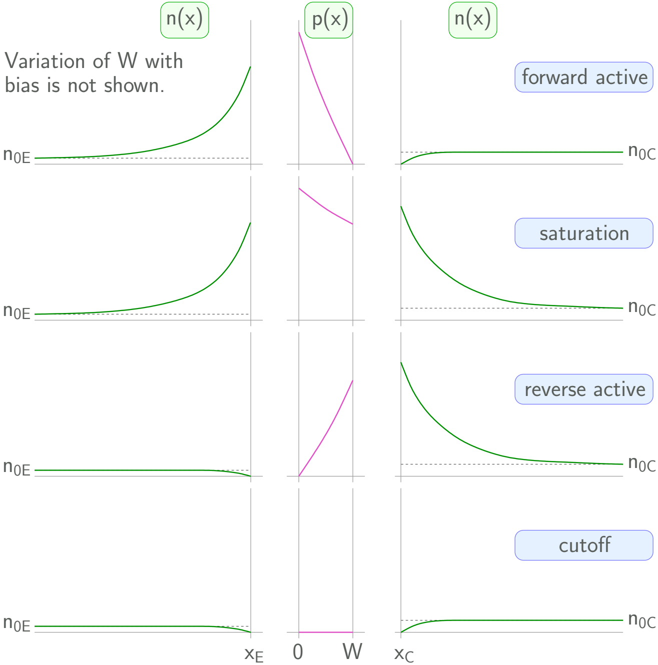
$$\Delta n(x_C) = n_{0C}\left[\exp\left(\frac{V_{CB}}{V_T}\right)-1\right]$$

$$\Delta n(\infty) = 0$$



Ebers-Moll model:
Outline of derivation for a *pnp* BJT

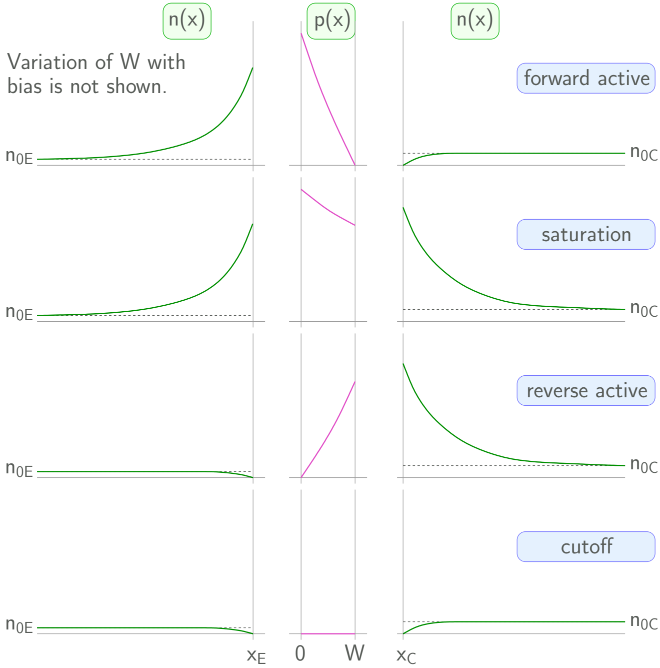
Variation of W with bias is not shown.



Ebers-Moll model:

Outline of derivation for a *pnp* BJT

* Solve the minority-carrier continuity equations in the neutral emitter, base, and collector regions.



Ebers-Moll model:

Outline of derivation for a *pnp* BJT

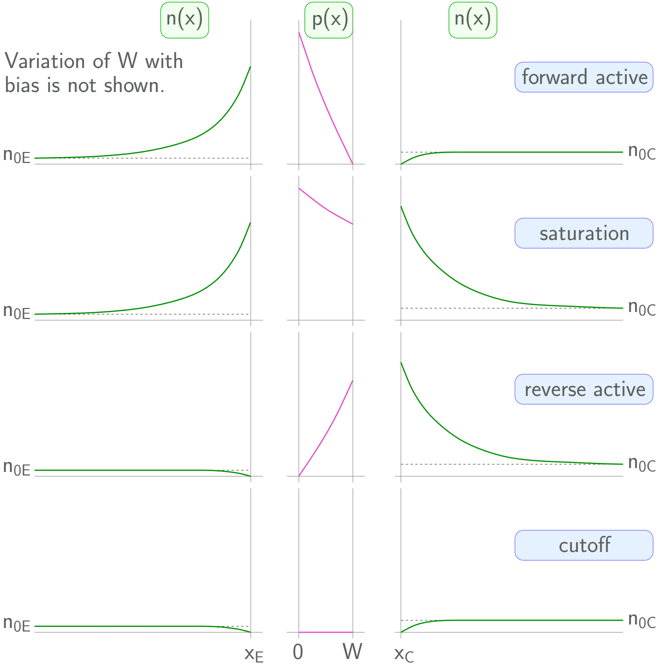
* Solve the minority-carrier continuity equations in the neutral emitter, base, and collector regions.
* From the solutions, obtain the following currents.

$$I_{nE}(x_E) = \quad qAD_{nE}\,\frac{dn}{dx}(x_E).$$

$$I_{pB}(0) \quad = -qAD_{pB}\,\frac{dp}{dx}(0).$$

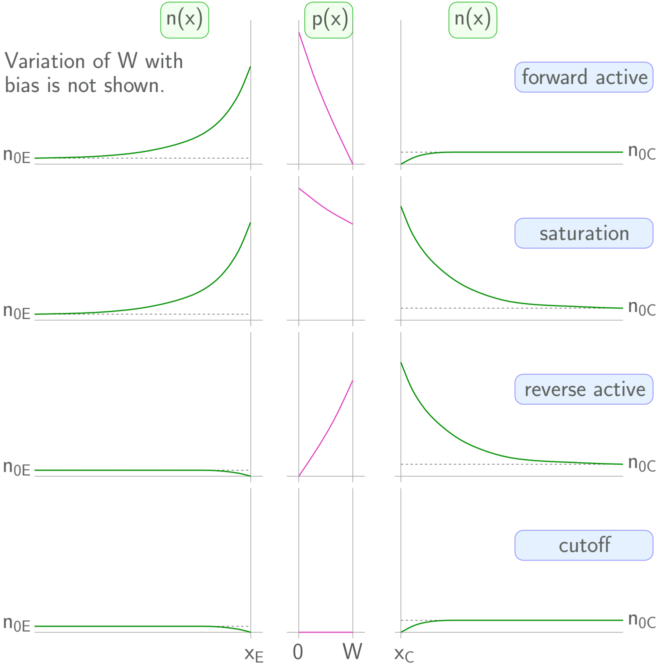
$$I_{pB}(W) = -qAD_{pB}\,\frac{dp}{dx}(W).$$

$$I_{nC}(x_C) = \quad qAD_{nC}\,\frac{dn}{dx}(x_C).$$



Ebers-Moll model:

Outline of derivation for a *pnp* BJT



Ebers-Moll model:

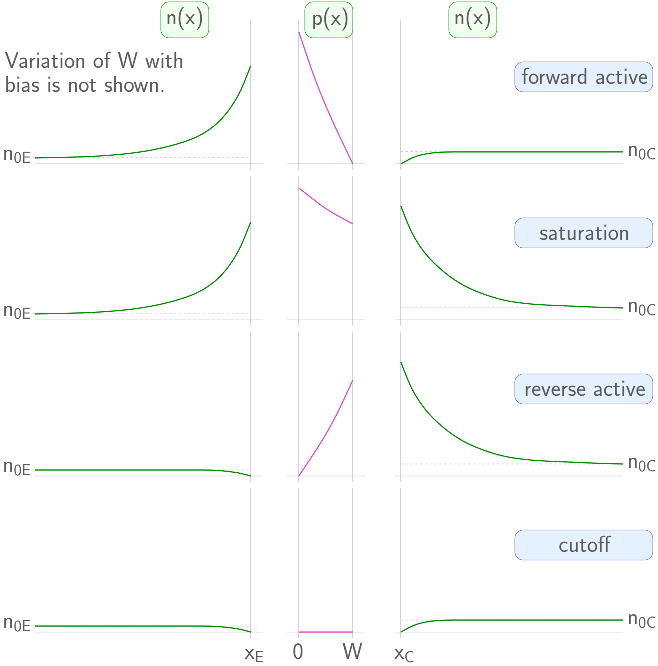
Outline of derivation for a *pnp* BJT

* Obtain the terminal currents, ignoring G-R in the depletion regions.

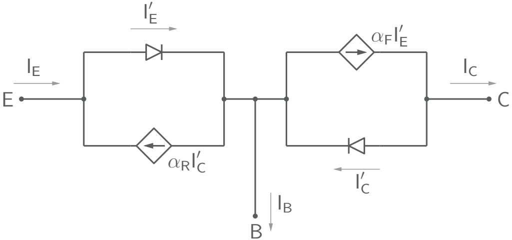
  $I_E = I_{nE}(x_E) + I_{pB}(0)$.

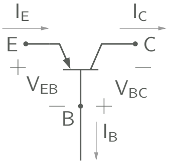
  $I_C = I_{nC}(x_C) + I_{pB}(W)$.

  $I_B = I_E - I_C$.



Bipolar junction transistors: Ebers-Moll model

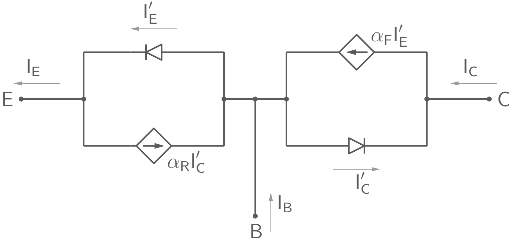


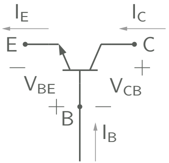


pnp transistor

$$I'_E = I_{ES}\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right]$$

$$I'_C = I_{CS}\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]$$





npn transistor

$$I'_E = I_{ES}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right]$$

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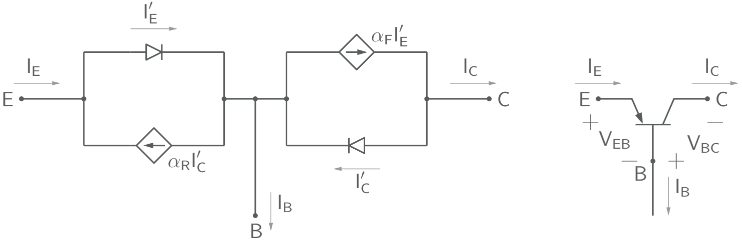
Bipolar junction transistors: Ebers-Moll model



pnp transistor

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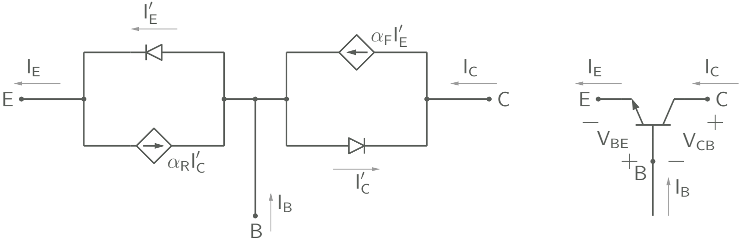

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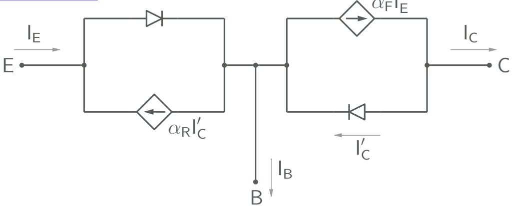
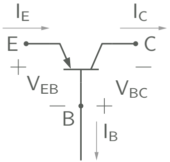
* Current directions are assigned such that I_C, I_E, I_B are all positive if the BJT operates in the active mode.

Ebers-Moll model





pnp transistor

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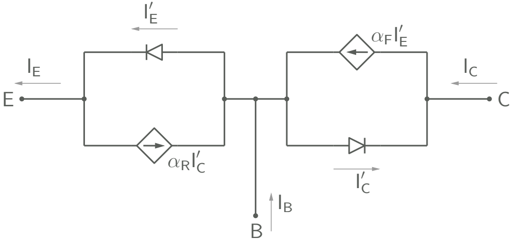

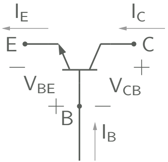

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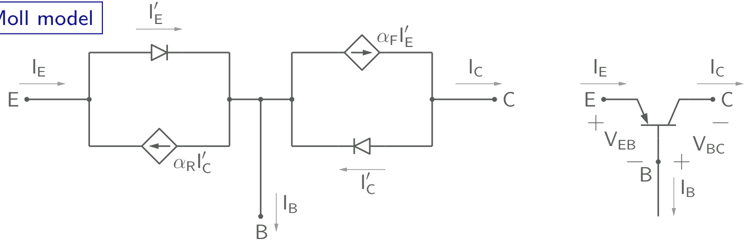
Ebers-Moll model



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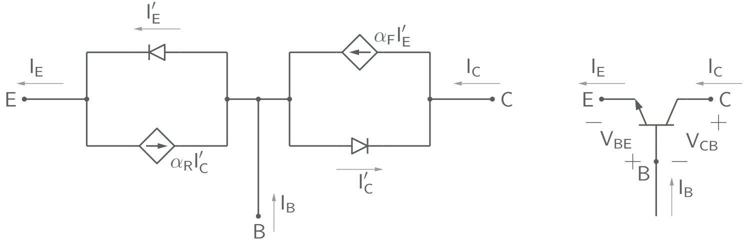

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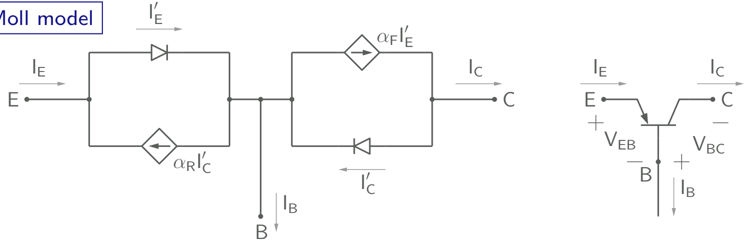

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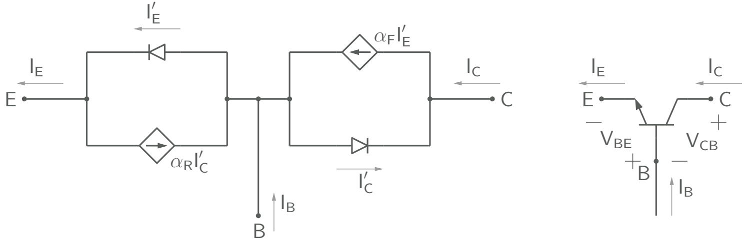

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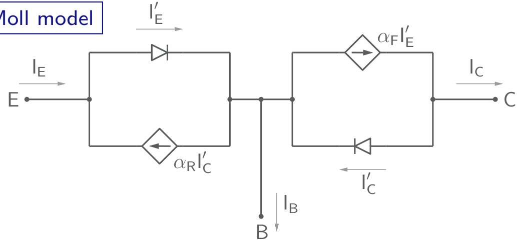
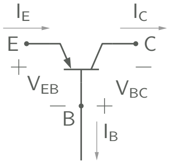
* The forward transistor is represented by the *E-B* diode and the corresponding dependent source (the upper branches), and the reverse transistor by the *C-B* diode and the corresponding dependent source (the lower branches).

Ebers-Moll model





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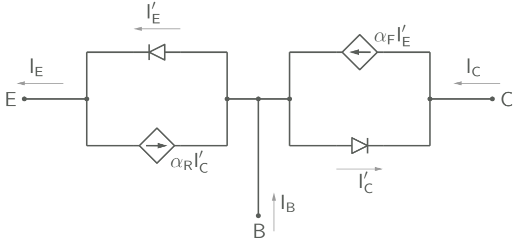

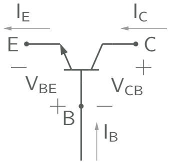

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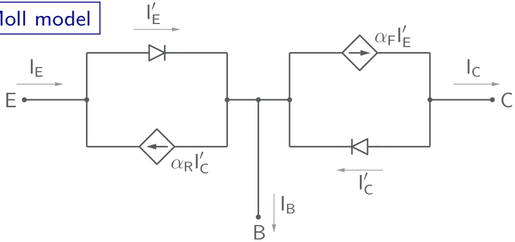

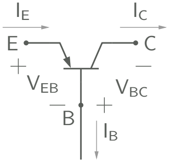

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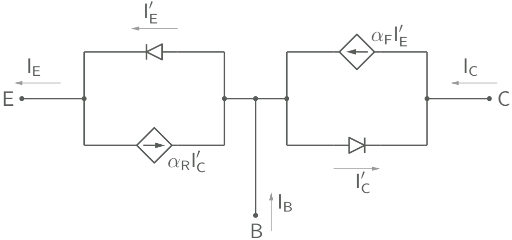

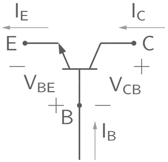

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* The model has four parameters: I_{ES}, I_{CS}, α_F, α_R (F for forward, R for reverse) which can be related to the geometry (base width, device area) and material parameters (doping densities, diffusion coefficients, lifetimes) of the transistor.[1]

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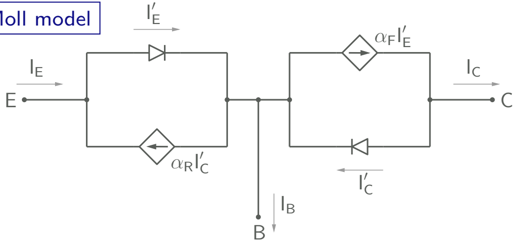

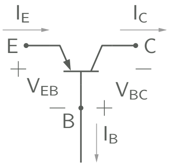

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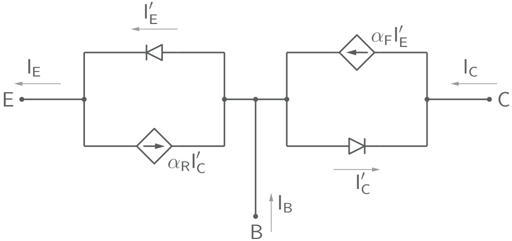

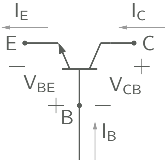

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Ebers-Moll model

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* The Ebers-Moll model can still be used as a "phenomenological" description of the device if model parameters are suitably extracted using measured data.
* More advanced BJT models are available (e.g., the SPICE model[2]) and are used for circuit simulation.

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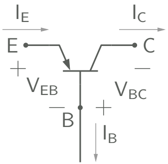
[2] P. Antognetti and G. Massobrio, *Semiconductor Device Modeling with SPICE*. New York: McGraw-Hill, 1988.

BJT I-V description

pnp transistor



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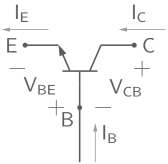

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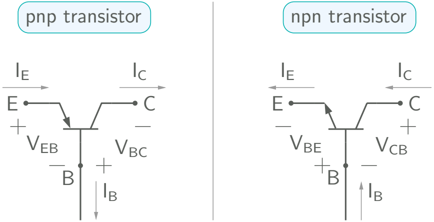

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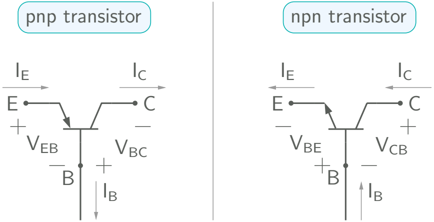

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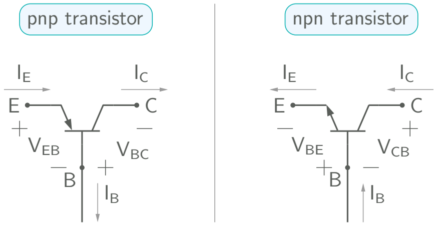

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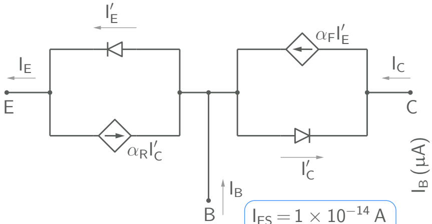
* The current-voltage relationship is described in the form of a "family" of curves, with a current selected as the y variable, a voltage as the x variable, and a third variable as a quantity to be held constant for each I-V curve.
* Two descriptions, which are related to the "common-base" and "common-emitter" configurations, are commonly used.

Common-emitter configuration



$I_{ES} = 1 \times 10^{-14}$ A

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$\alpha_F = 0.99$

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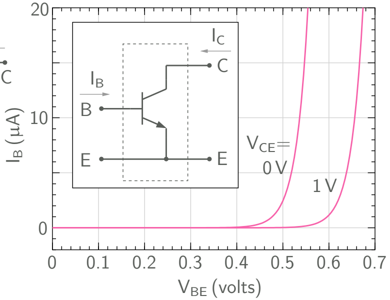

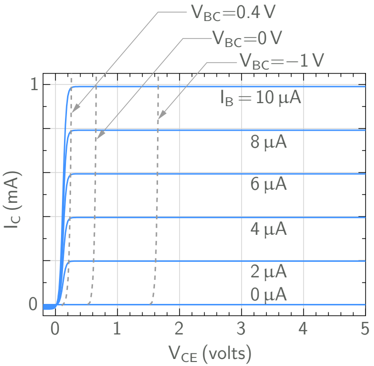

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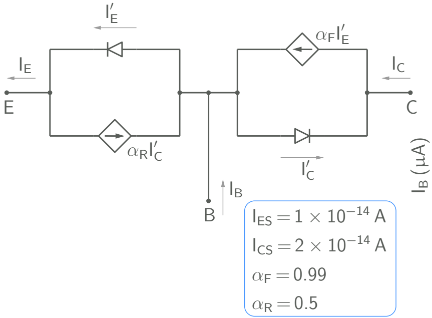

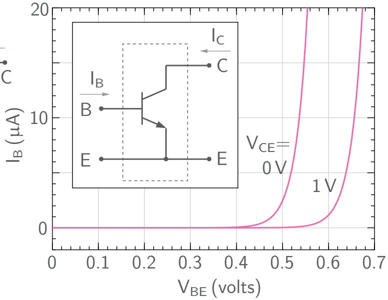

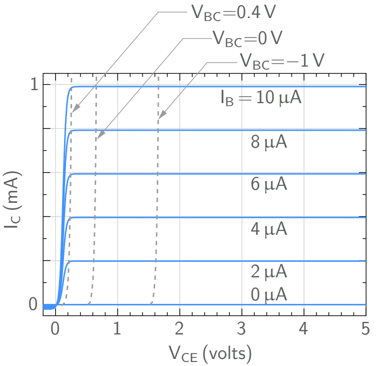

* In the active region (where I_C is constant for a given I_B), the B-C junction is reverse biased.
  $\rightarrow I'_C \approx 0 \rightarrow I_C = \alpha_F I_E = \beta I_B$, irrespective of V_{CE}.

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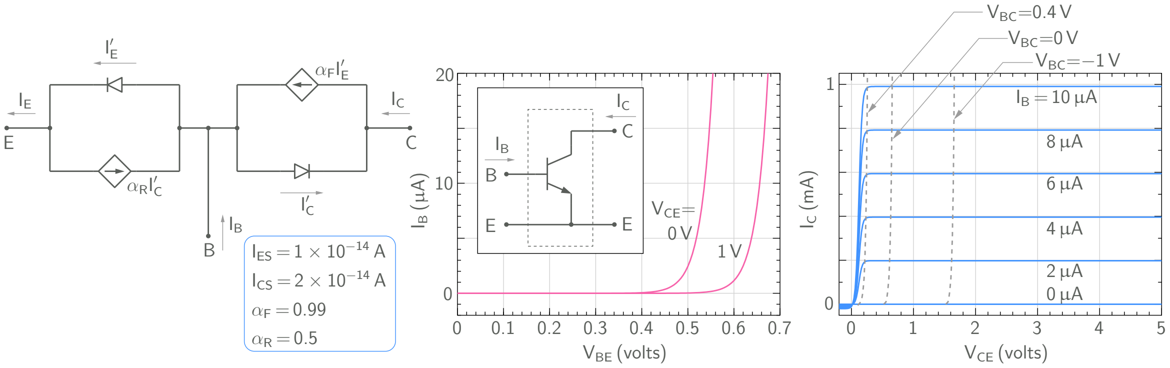

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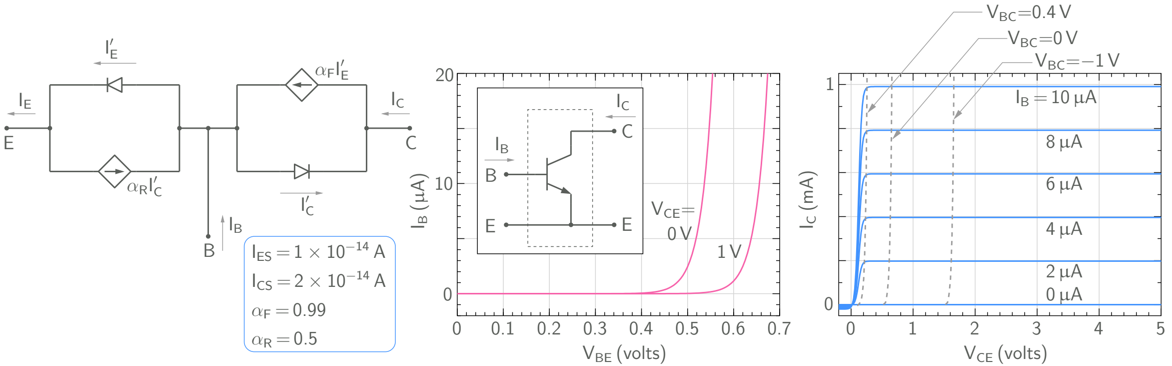

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* When V_{BC} becomes greater than about 0.4 V, I'_C becomes significant, and $I_C = \alpha_F I'_E - I'_C$ decreases $\rightarrow I_C < \beta I_B$.
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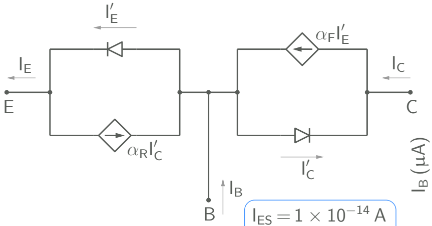

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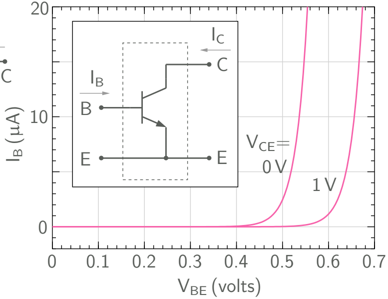

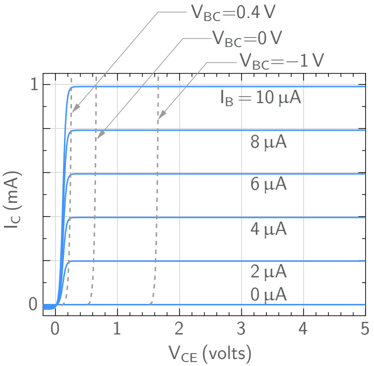

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* In the saturtion region, V_{CE} is 0.2 V or smaller. This is generally true for all low-power BJTs.

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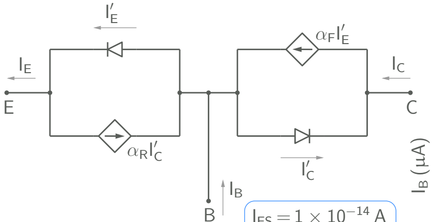

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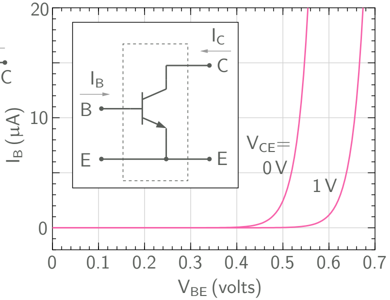

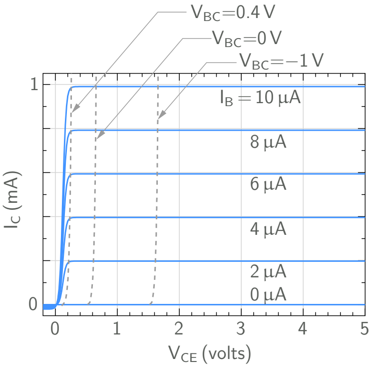

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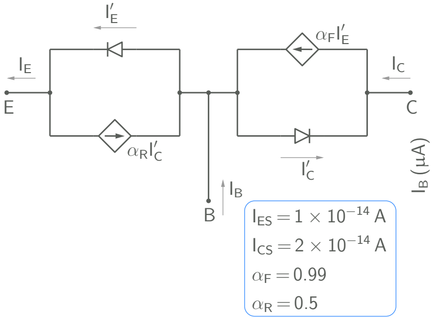

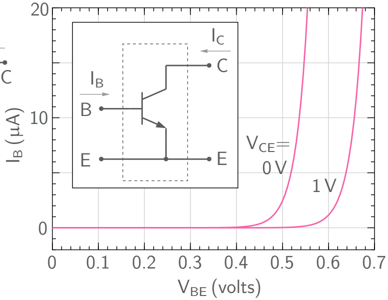

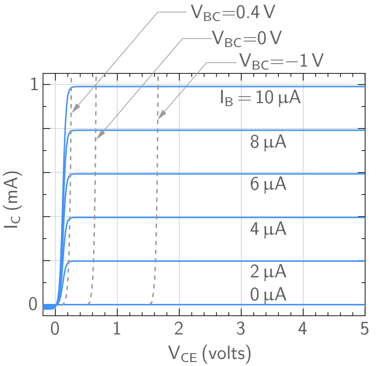

* Comparison of I_B versus V_{BE} for $V_{CE} = 0$ V and $V_{CE} = 1$ V:

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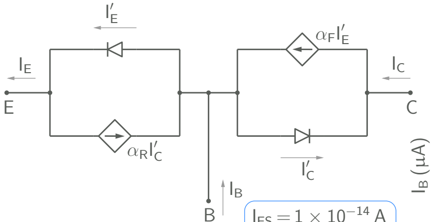

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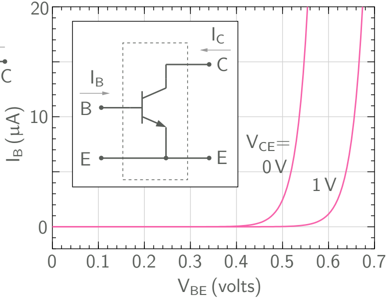

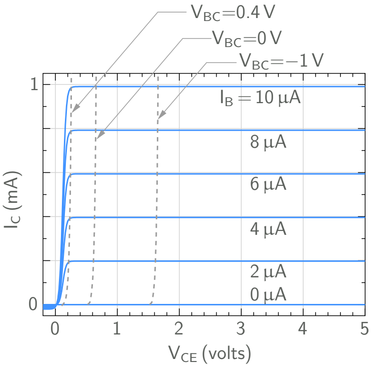

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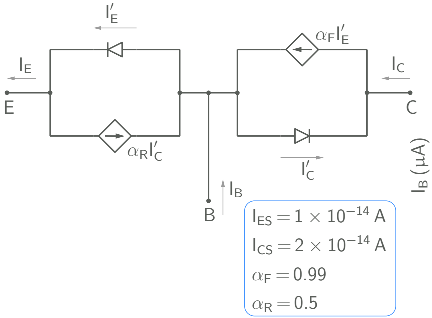
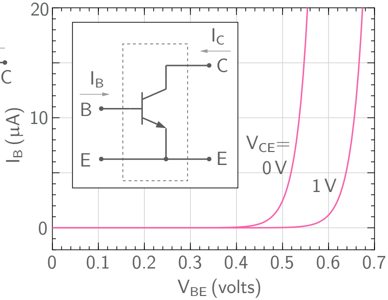
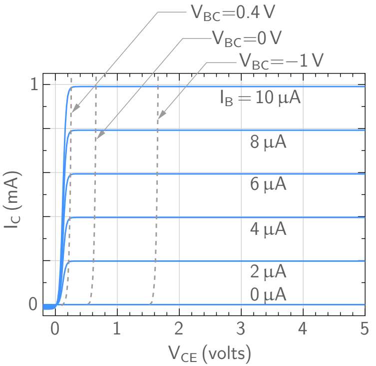
  - $V_{CE} = 1$ V (active region):

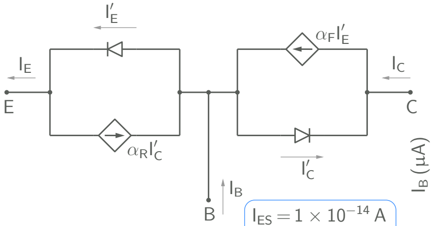




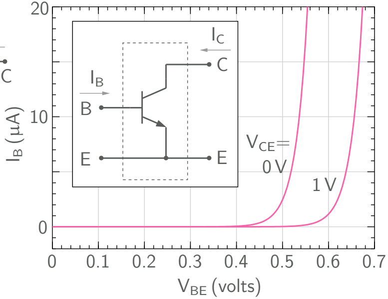


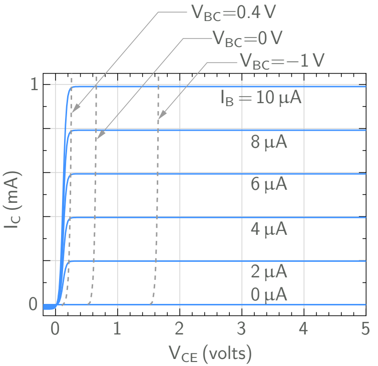
* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V}$

Common-emitter configuration



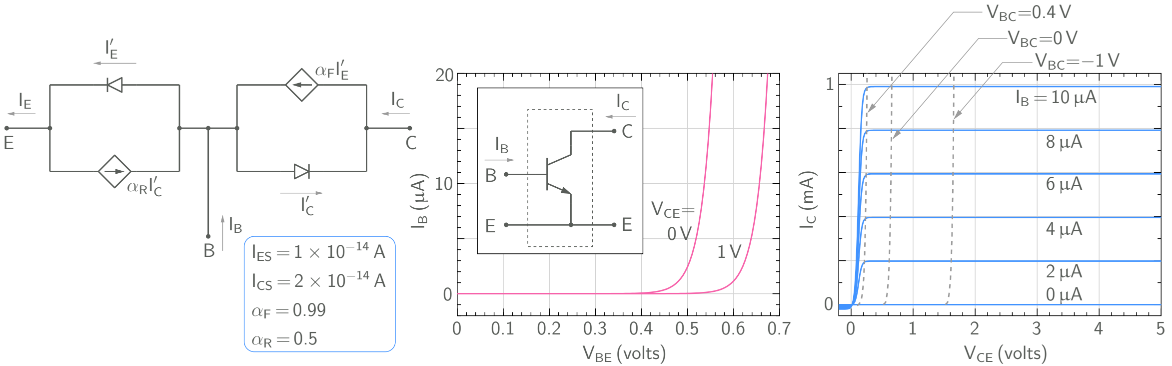
$I_{ES} = 1 \times 10^{-14}$ A
$I_{CS} = 2 \times 10^{-14}$ A
$\alpha_F = 0.99$
$\alpha_R = 0.5$



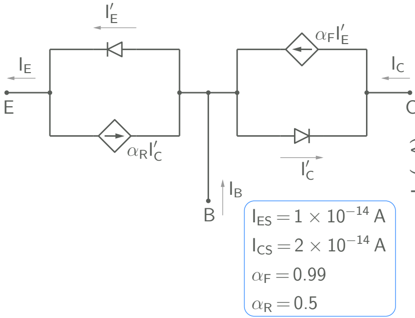


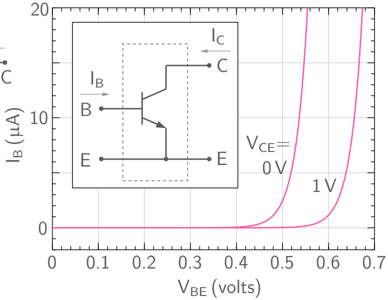
* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0$

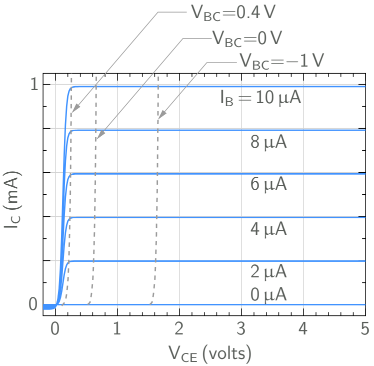
Common-emitter configuration



* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E$



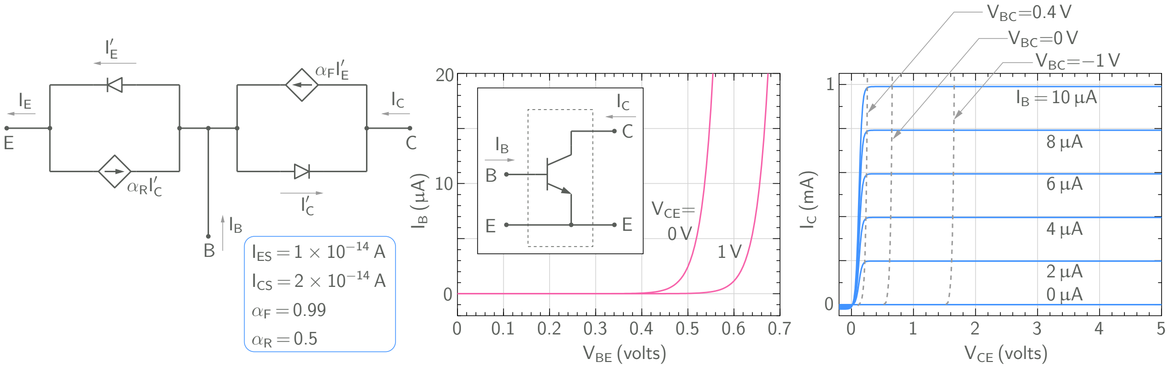




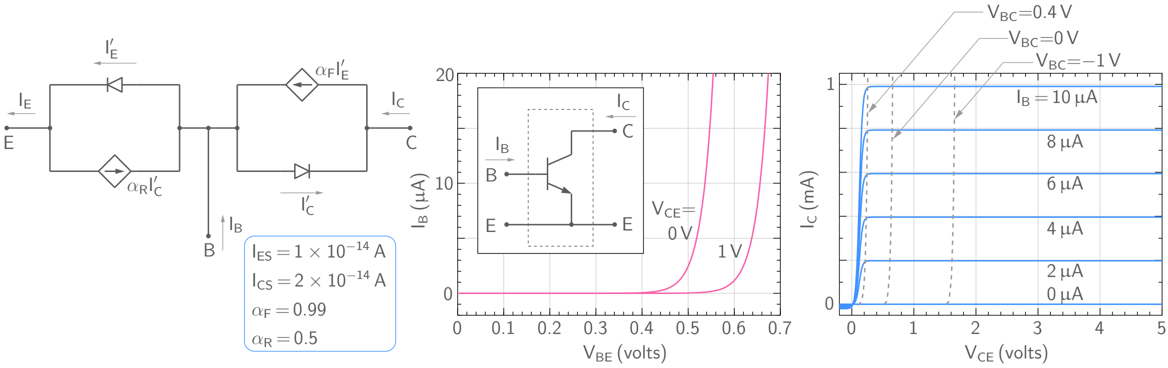
∗ Comparison of I_B versus V_{BE} for $V_{CE} = 0$ V and $V_{CE} = 1$ V:

- $V_{CE} = 1$ V (active region):
  $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E \approx (1 - \alpha_F)\, I_{ES}\, e^{V_{BE}/V_T}$.

Common-emitter configuration

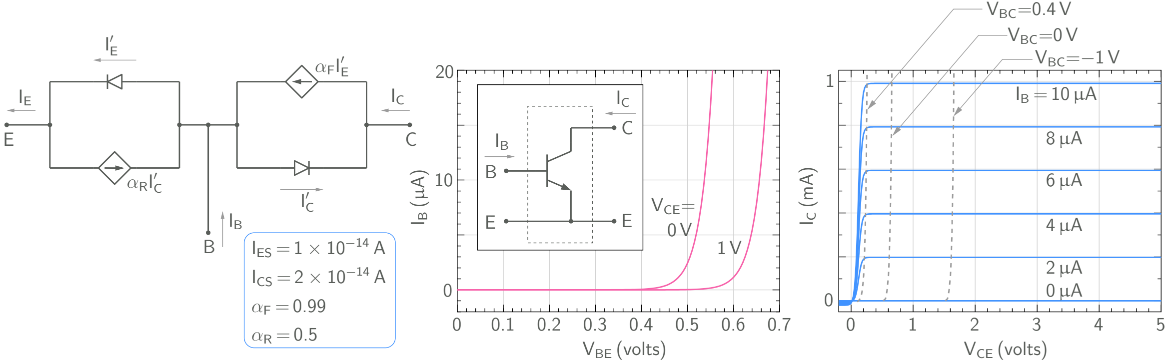


* Comparison of I_B versus V_{BE} for $V_{CE} = 0$ V and $V_{CE} = 1$ V:
  - $V_{CE} = 1$ V (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E \approx (1 - \alpha_F)\, I_{ES}\, e^{V_{BE}/V_T}$.
  - $V_{CE} = 0$ V (saturation region):

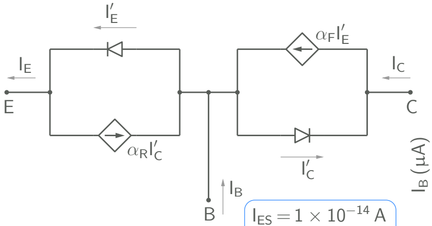


* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E \approx (1 - \alpha_F)\, I_{ES}\, e^{V_{BE}/V_T}$.
  - $V_{CE} = 0\,\text{V}$ (saturation region):
    $V_{BC} = V_{BE} - V_{CE} = V_{BE}$

Common-emitter configuration



* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E \approx (1 - \alpha_F)\, I_{ES} e^{V_{BE}/V_T}$.
  - $V_{CE} = 0\,\text{V}$ (saturation region):
    $V_{BC} = V_{BE} - V_{CE} = V_{BE} \rightarrow I'_C$ is comparable to I'_E.



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$I_{CS} = 2 \times 10^{-14}$ A
$\alpha_F = 0.99$
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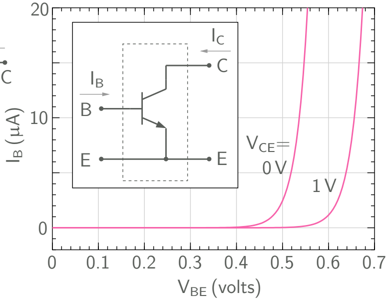

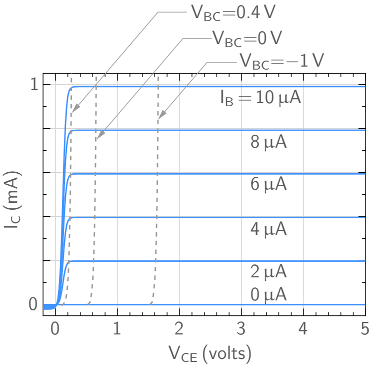

* Comparison of I_B versus V_{BE} for $V_{CE} = 0\,\text{V}$ and $V_{CE} = 1\,\text{V}$:
  - $V_{CE} = 1\,\text{V}$ (active region):
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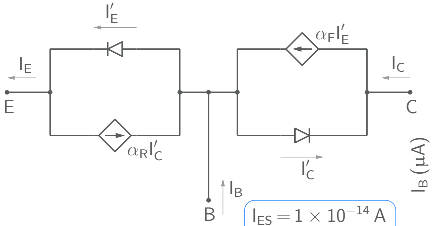
  - $V_{CE} = 0\,\text{V}$ (saturation region):
    $V_{BC} = V_{BE} - V_{CE} = V_{BE} \rightarrow I'_C$ is comparable to I'_E.
    $\rightarrow I_B = (1-\alpha_F) I'_E + (1-\alpha_R) I'_C$

Common-emitter configuration



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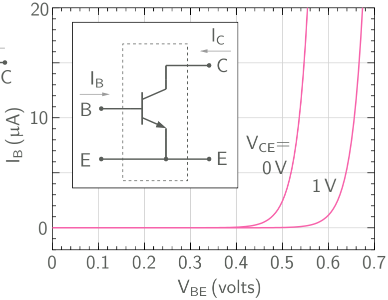

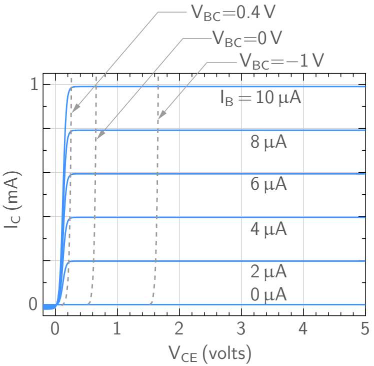

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    $V_{BC} = V_{BE} - V_{CE} \approx 0.7 - 1 = -0.3\,\text{V} \rightarrow I'_C \approx 0 \rightarrow I_B = I'_E - \alpha_F I'_E \approx (1-\alpha_F)\, I_{ES}\, e^{V_{BE}/V_T}$.
  - $V_{CE} = 0\,\text{V}$ (saturation region):
    $V_{BC} = V_{BE} - V_{CE} = V_{BE} \rightarrow I'_C$ is comparable to I'_E.
    $\rightarrow I_B = (1-\alpha_F)I'_E + (1-\alpha_R)I'_C \rightarrow I_B$-$V_{BE}$ curve shifts left.