Stability : Formal characterization $\boxed{\text{Version-I}}$

$$\Delta_1(\omega) = \|W^*(S \cup k) - w^*(S)\| \text{ is bounded.}$$

: Formal charactrization $\boxed{\text{Version II}}$

$$S = \{z_1, z_2, z_i, z_n\}$$

$$\Delta_2(\omega) = \|W^*(S) - w^*(S_i')\| \text{ is bounded.}$$

$$S_i' = \{z_1, z_2, z_i', z_n\}$$

for all choice of S and S_i'

$$\Delta_1(\omega) = O\left(\frac{\omega_{max} + \cdots}{\lambda |S|}\right)$$

$$\Delta_2(\omega) = \frac{2B}{\lambda |S|}$$

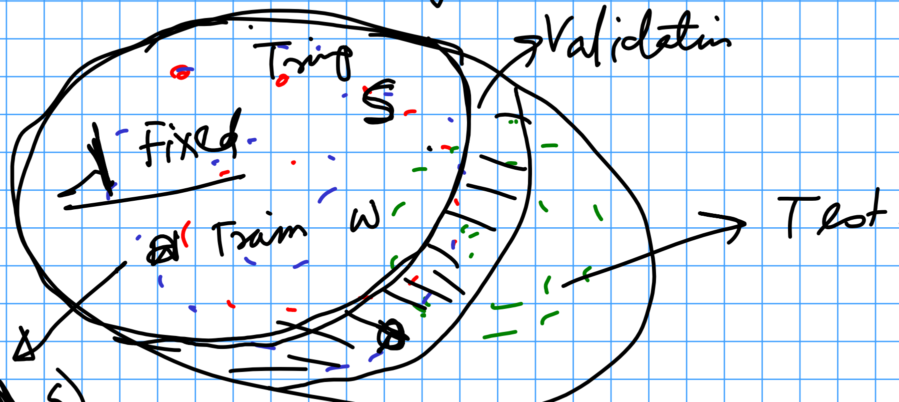
$$B = \left[\frac{\partial l}{\partial w}\right]_{\text{max-element wise}}$$

Recall $l(w, z)$ is the point wise loss

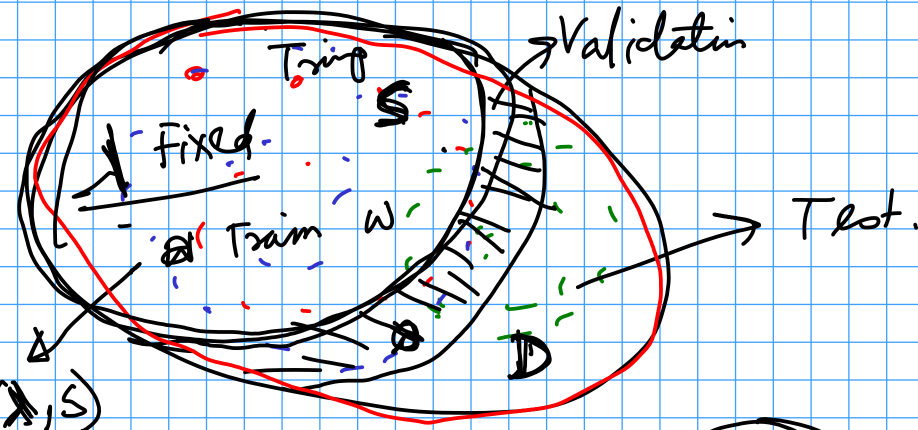
and the obj was

$$F_S(w) = \sum_{i \in S} \left[\lambda \|w\|^2 + l(w, z_i)\right]$$

— Until now — loss, Regression, Classification → Quantitative aspect of performance; → formal aspect of training properties



$$\lambda \rightarrow W(\lambda, S)$$



$\lambda \to \underline{w(\lambda, S)}$

Regression/
Classification

λ, w

$K(x_1, x_2)$

$\lambda, K \longrightarrow w$

$$\underbrace{\sum_{i\in S} \lambda\|w\|^2 + \ell(w,z_i)} = F_S(w)$$

$$\boxed{\frac{\sum_{i\in S} \ell(w,z_i)}{|S|}} = \underline{L_S(w)}$$

$$\frac{\sum_{j\in D} \ell(w,z_j)}{|D|} = L_D(w)$$

Test set is D

Traing set is S

$\xrightarrow{\text{argmin}} \boxed{w^*(S)}$

$L_D(w^*(S))$ = Test error after training w by minimizing $F_S(w)$

$$L_D(w^*(S)) = \underbrace{L_S(w^*(S))} + \left[L_D(w^*(S)) - L_S(w^*(S))\right]$$

Avg-Test-error = Avg-Traing-error + (Test err. − Av.Traing error)

$$L_D(w^*(S)) = L_S(w^*(S)) + \left[L_D(w^*(S)) - L_S(w^*(S))\right]$$

① Traing error possible

② Test error possible.

$$L_S(w^*(S)) = \frac{1}{|S|}\sum_{i\in S} \ell(w^*(S), z_i) \leq \underbrace{\frac{1}{|S|}\sum_{i\in S}\ell(0, z_i)}_{\text{Thanks to Ankan}}$$

$$L_S(w^*(S)) = \frac{1}{|S|}\sum_{i\in S} \ell(w^*(S), z_i) \leq \left[\lambda\|w\|^2 + \frac{1}{|S|}\sum_{i\in S}\ell(w^*(S), z_i)\right]$$

$$\leq \frac{1}{|S|}\sum_{i\in S}\ell(0, z_i)$$

$$\boxed{w^*(S) = \arg\min_w \sum_{i\in S} \lambda\|w\|^2 + \ell(w, z_i) = \arg\min \sum_{i\in S}\frac{1}{|S|}\left[\lambda\|w\|^2 + \ell(w, z_i)\right]}$$

$$\frac{1}{|S|}\sum_{i\in S}\left[\lambda\|w^*\|^2 + \ell(w^*, z_i)\right] \leq \frac{1}{|S|}\sum_{i\in S}\ell(0, z_i)$$

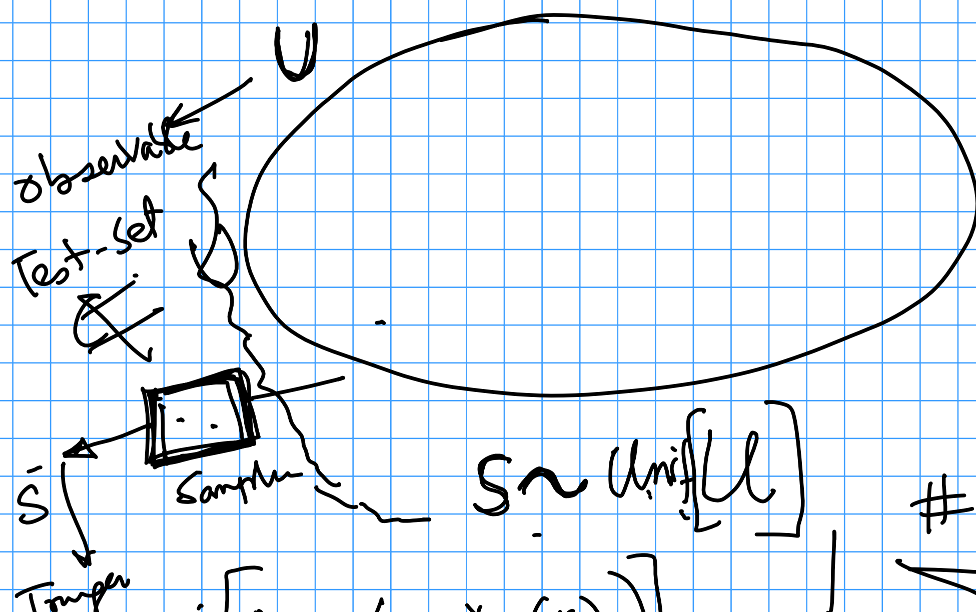
$$L_D(w^*(S)) = \underbrace{L_S(w^*(S))}_{\text{Avg - Traing - error}} + \boxed{L_D(w^*(S)) - L_S(w^*(S))}$$

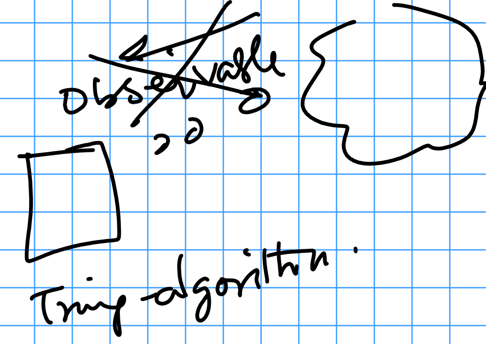
depends on S

Avg (Test - Traing) error.

$$\leq \frac{1}{|S|}\sum_{i\in S}\ell(0, z_i)$$

$$+ \underline{L_D(w^*(S)) - L_S(w^*(S))}$$





$S \sim \mathrm{Unif}[U]$

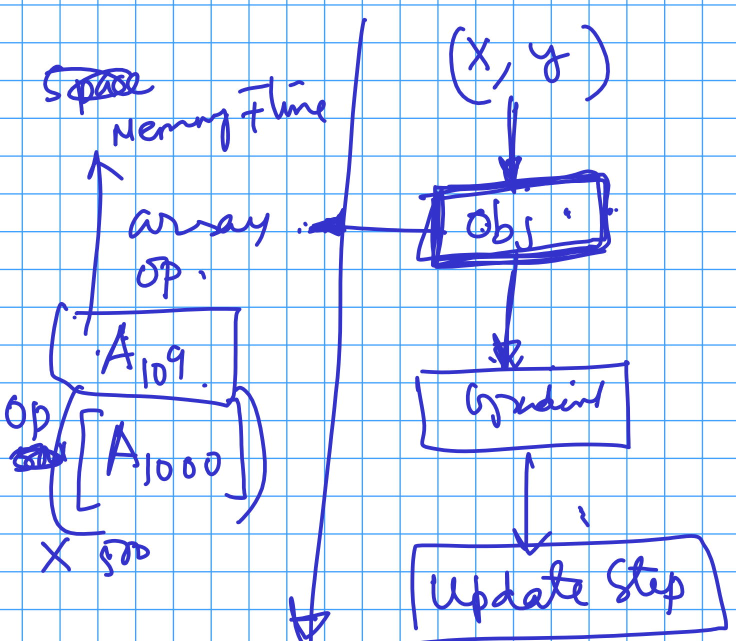
$$\mathbb{E}_S\left[L_D(w^*(S))\right] \approx \frac{1}{\#\text{ of sampled}} \sum_{k=1}^{\#\text{ of samples } S} L_D(w^*(S_k))$$

$$\mathbb{E}_S\left[L_D(w^*(S))\right] = \underbrace{\boxed{\mathbb{E}_S\left[L_S(w^*(S))\right]}}_{< \mathbb{E}_S L_S(0,S)} + \mathbb{E}_S\left(L_D(w^*(S)) - L_S(w^*(S))\right)$$

$U = 1 \cdots 10^9$

```
import random
for j = 1 : N
    S <- random.sample([1...10^9], 1000)
    w*(S) <- Learner(S)
      -> different w*(S) for different runs.
```

10. $N \ll 10^9$ 1000



$$\text{Train error} + \boxed{\text{Test error} - \text{Train error}}$$

$$E_{S}\left[L_D(w^*(S)) - L_S(w^*(S))\right]$$

$$= E_{S'}\left[L_D\left(w^*(S')\right)\right] - E_S\left(L_S(w^*(S))\right)$$

$$\overset{①}{=} E_{\substack{z \\ \sim U}}\left[\ell(w^*(S'),z) - \ell(w^*(S),z)\right]$$

① Assumes the distn of data in train and test are same.

$$S \setminus S' = \{z_i\}$$

$$S' \setminus S = \{z'_i\}$$

$$S = \{z_1 \,.\,.\, z_i \; z_n\}$$

$$S' = \{z_1 \,.\,.\, z'_i \; z_n\}$$

as z_i and z'_i are different

$$\text{Test error} \leq \text{Train error} + \frac{K}{\lambda|S|}$$