1) $T(s) = \dfrac{G(s)}{1+G(s)}$

a) $|T(j\omega)| = c$ is the locus for constant magnitude.

Let $G(j\omega) = a + jb$.

$$\therefore |a+jb| = c\,|1+a+jb|$$

$$\therefore a^2+b^2 = c^2\left((1+a)^2+b^2\right)$$

$$\therefore a^2+b^2 = c^2 + 2ac^2 + a^2c^2 + b^2c^2$$

$$\therefore a^2+b^2 + \frac{2c^2a}{c^2-1} + \frac{c^2}{c^2-1} = 0$$

$$\therefore \left(a^2 + 2\cdot a\cdot\frac{c^2}{c^2-1} + \frac{c^4}{(c^2-1)^2}\right) + b^2 = \frac{c^4}{(c^2-1)^2} - \frac{c^2}{c^2-1}$$

$$\therefore \left(a + \frac{c^2}{c^2-1}\right)^2 + b^2 = \frac{c^2}{(c^2-1)^2}$$

comparing to $(x+x_0)^2 + (y+y_0)^2 = r^2$, we have:

centre $= \left(\dfrac{-c^2}{c^2-1}, 0\right)$

radius $= \left|\dfrac{c}{c^2-1}\right|$

b) again, let $G(j\omega) = a+ib$

$$\therefore T(j\omega) = \frac{a+ib}{(1+a)+ib} = \frac{(a+ib)((1+a)-ib)}{(1+a)^2+b^2} = \frac{a^2+a+b^2+ib}{(1+a)^2+b^2}$$

Now, we have $\angle T(j\omega) = c$.

$$\therefore \tan^{-1}\left(\frac{Im(T)}{Re(T)}\right) = c$$

$$\therefore \frac{Im(T(j\omega))}{Re(T(j\omega))} = \tan c. \quad := d$$

$$\therefore \frac{b}{a^2+a+b^2} = d$$

$$\therefore a^2+a+b^2 - \frac{b}{d} = 0$$

$$\therefore (a^2+a+1/4) + \left(b^2 - \frac{b}{d} + \frac{1}{4d^2}\right) = \frac{1}{4d^2} + \frac{1}{4}$$

$$\therefore (a+1/2)^2 + \left(b - \frac{1}{2\tan c}\right)^2 = \frac{1}{4}\left(\frac{1+\tan^2 c}{\tan^2 c}\right)$$

$$= (4\sin^2 c)^{-1}$$

$$\therefore \text{centre} = \left(-0.5, \frac{1}{2\tan c}\right)$$

$$\text{radius} = (1/2\sin c)$$

2) A sinusoidal input creates a scaled and shifted
9 sinusoidal output through an LTI system.
Let transfer function be $G(s)$.

10

$$\therefore M(\omega) = |G(j\omega)| \quad , \quad \phi(\omega) = \angle G(j\omega)$$

11

Now $u(t) = \dfrac{A}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$

12

$$\therefore y(t) = \frac{A G(j\omega) e^{j\omega t}}{2j} + \frac{A G(-j\omega) e^{-j\omega t}}{-2j}$$

1

$$= 2\mathrm{Re}\left(\frac{A}{2j} \cdot G(j\omega) \cdot e^{j\omega t}\right)$$

$$\therefore |y(t)| = A \cdot |G(j\omega)| = A \cdot M(\omega)$$

$$\angle y(t) = -\frac{\pi}{2} + \angle G(j\omega) + \omega t$$

(from $\frac{1}{j}$)

$$\therefore y(t) = |y(t)| \cdot \cos(\angle y(t))$$
$$= A M(\omega) \cdot \sin(\omega t + \phi(\omega))$$

Sunday

$\therefore$ scale of magnitude gives $M(\omega)$
shift in phase gives $\phi(\omega)$

3) We plot nyquist plot for $k = 0.5$ to see that there are no encirclements of $(-1, 0)$.

$\therefore N = 0$, but $P = 1$

$\therefore Z = 1 \Rightarrow$ unstability.

We find that system is marginally stable at $k = 1$ and stable for $k \geq 1$.

Note that open loop system is unstable.
And for $k = 1$, we have $\omega_{gcf} = \omega_{pcf}$.

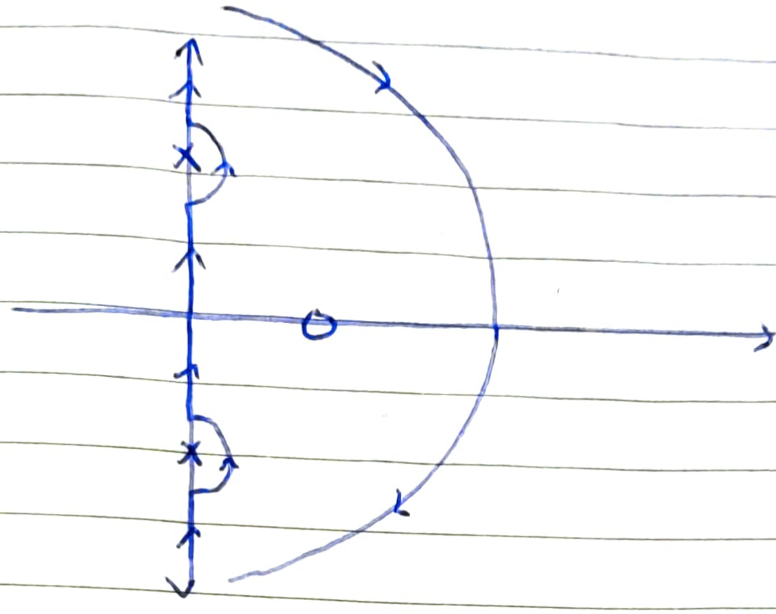
$\therefore k \geq 1 \Rightarrow$ stability.

$$\therefore k \in [1, \infty).$$

(marginal stability for $k = 1$)

(4)

a)



At origin, magnitude $= 1/9$, phase $= 180^\circ$

Till first pole detour, poles have no phase contribution.

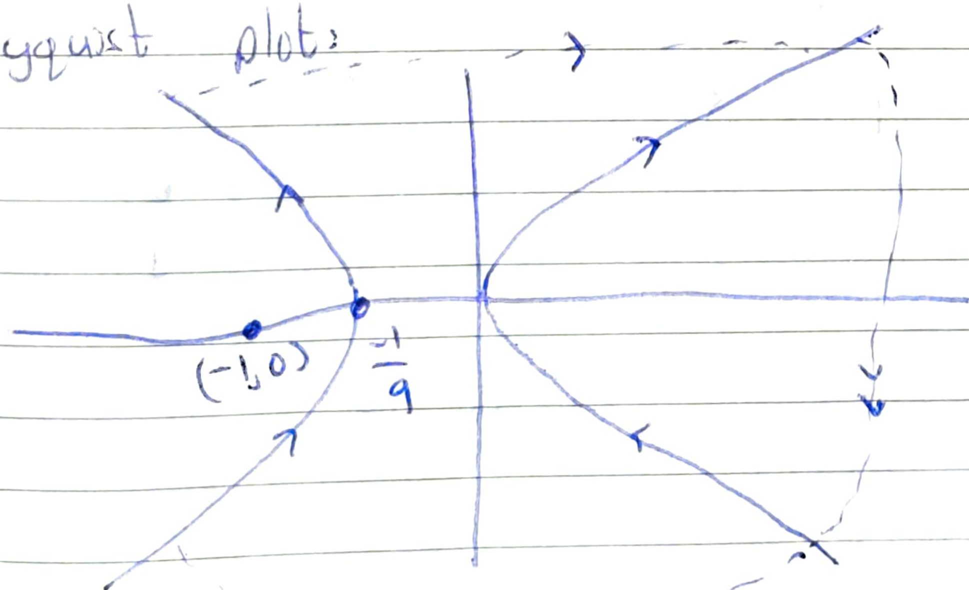
Final phase $= 180^\circ - \tan^{-1}(3) = 108^\circ$

Around pole, phase contribution goes from -90° to $+90^\circ$.

And mag $\to \infty$ ($\therefore$ offset $= -180^\circ$, as it is from pole)

Now $\omega \to \infty \Rightarrow$ mag $\to 0$ and phase falls by 180° (curved part)

Nyquist plot:



∴ at $k=1$, 0 encirclements of $(-1,0)$

∴ $N=0$, $P=0$, $Z=0$

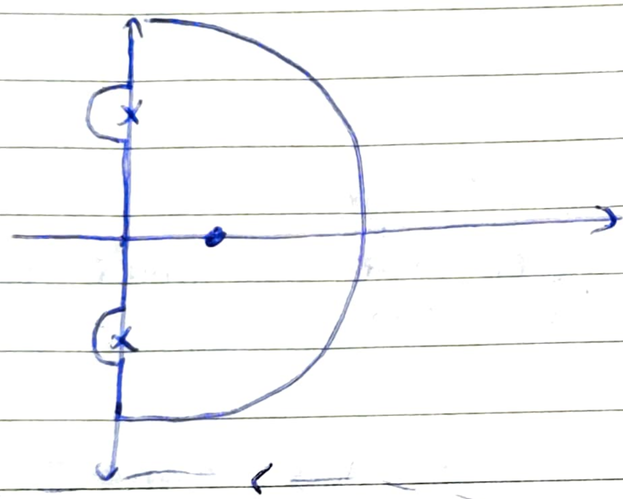
for $k \geq 9$, 1 encirclement (clockwise)

∴ $N=-1$.

∴ $Z=1$ ⇒ unstable.

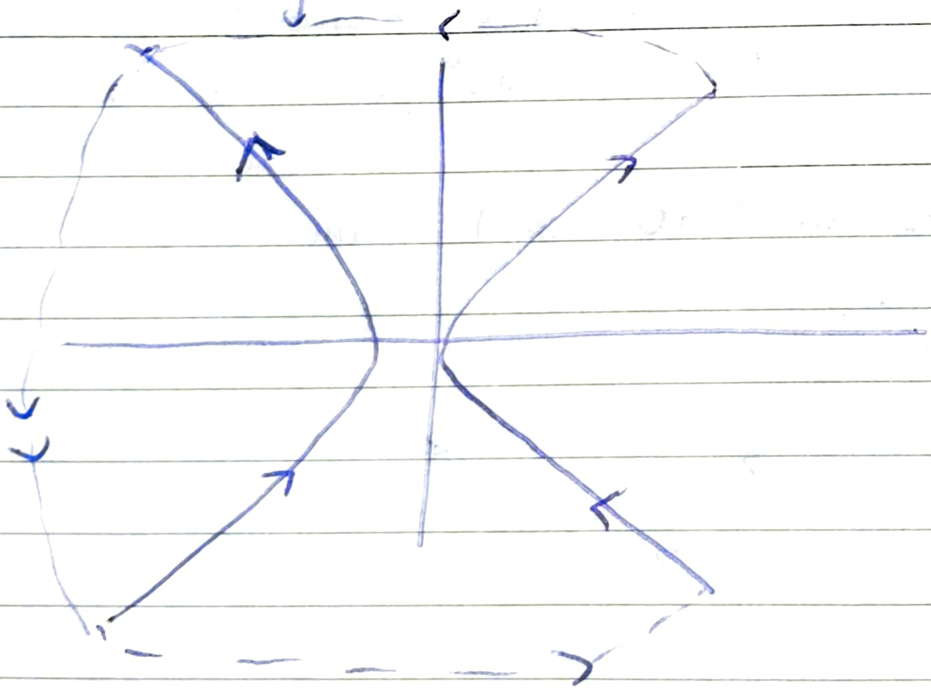
∴ stability for $k \in [0, 9)$

b)



very similar to previous case.

But around pole, offset is $+180°$



$P = 2$

At $k=1$,

$N=2$

∴ $Z=0$

At $k \geq 9$,

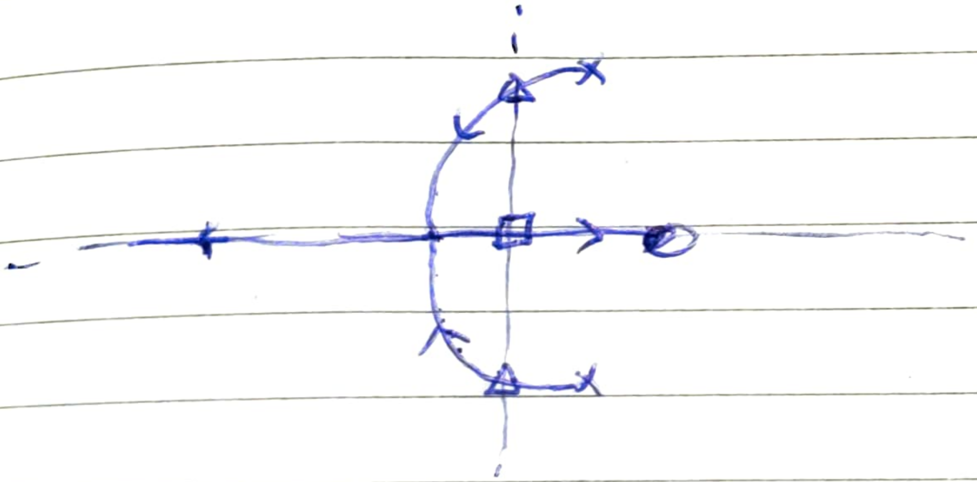
$N = 1$

∴ $Z = 1$ ⇒ unstable.

∴ stability in $k \in [0, 9)$

5) Root locus method:

$$G = \frac{(s-2)}{(s-(1+3i))(s-(1-3i))}$$



There are two crossovers.

case 1:

both poles cut jw axis and enter LHP (see Δ)

case 2:

1 poles re enters ORHP (see $\square$)

~~from matlab, range of~~

Let values be k_1 (Δ) and k_2 ($\square$)

At k_1, poles are roots of:

$$1 + \frac{k_1(s-2)}{s^2-2s+10} = 0$$

$\therefore$ $s^2 + (k_1-2)s + (10-2k_1)$ has 2 imaginary roots.

$\therefore$ sum of roots $= 0$.

$\therefore$ $k_1 = 2$.

Similarly at k_2, one root is $s=0$.

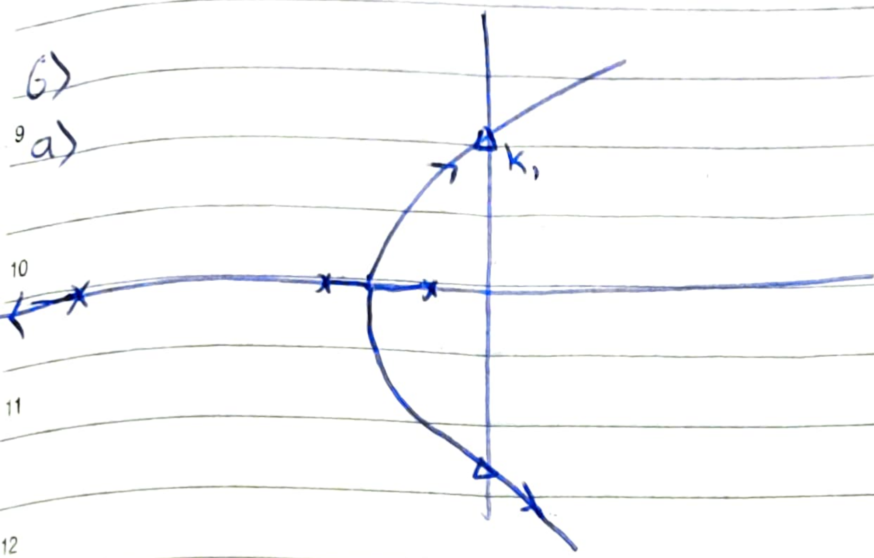
$\therefore$ product of roots $= 0$.

$\therefore$ $k_2 = 5$.

$\therefore$ $k \in (2, 5)$

6)

a)



At $k = k_1$

$$1 + \frac{k_1}{(s+1)(s+10)(s+100)} = 0$$ has 2 purely imaginary roots.

$\therefore\ s^3 + s^2(111) + s(1110) + 1000 + k_1 = 0$

$\therefore\ -j\omega^3 + j\omega \cdot 1110 = 0$ (Im part $= 0$)

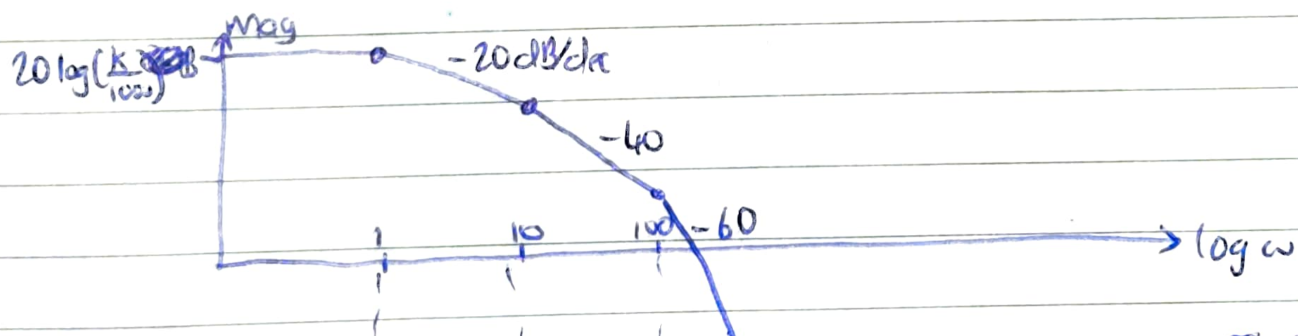
$\therefore\ \omega = \sqrt{1110}$

$-\omega^2 \cdot 111 + 1000 + k_1 = 0.$ (Re part $= 0$)

$\therefore\ k_1 = 122210$

$\therefore\ k \in [0, 122{,}210)$

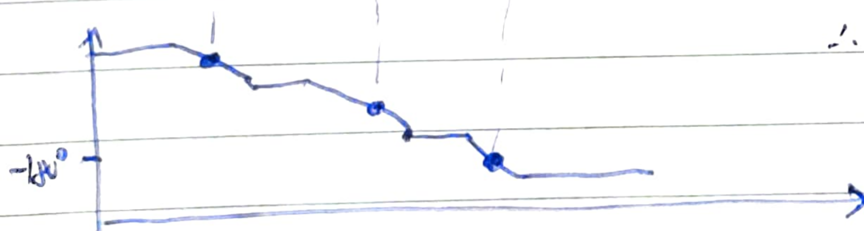
b)



$\omega_{pcf} \approx 5 \cdot 10 = 50$

$\therefore\ 20\log(\frac{K}{1000}) - 20$

$-40 \cdot 0.699 = 0$



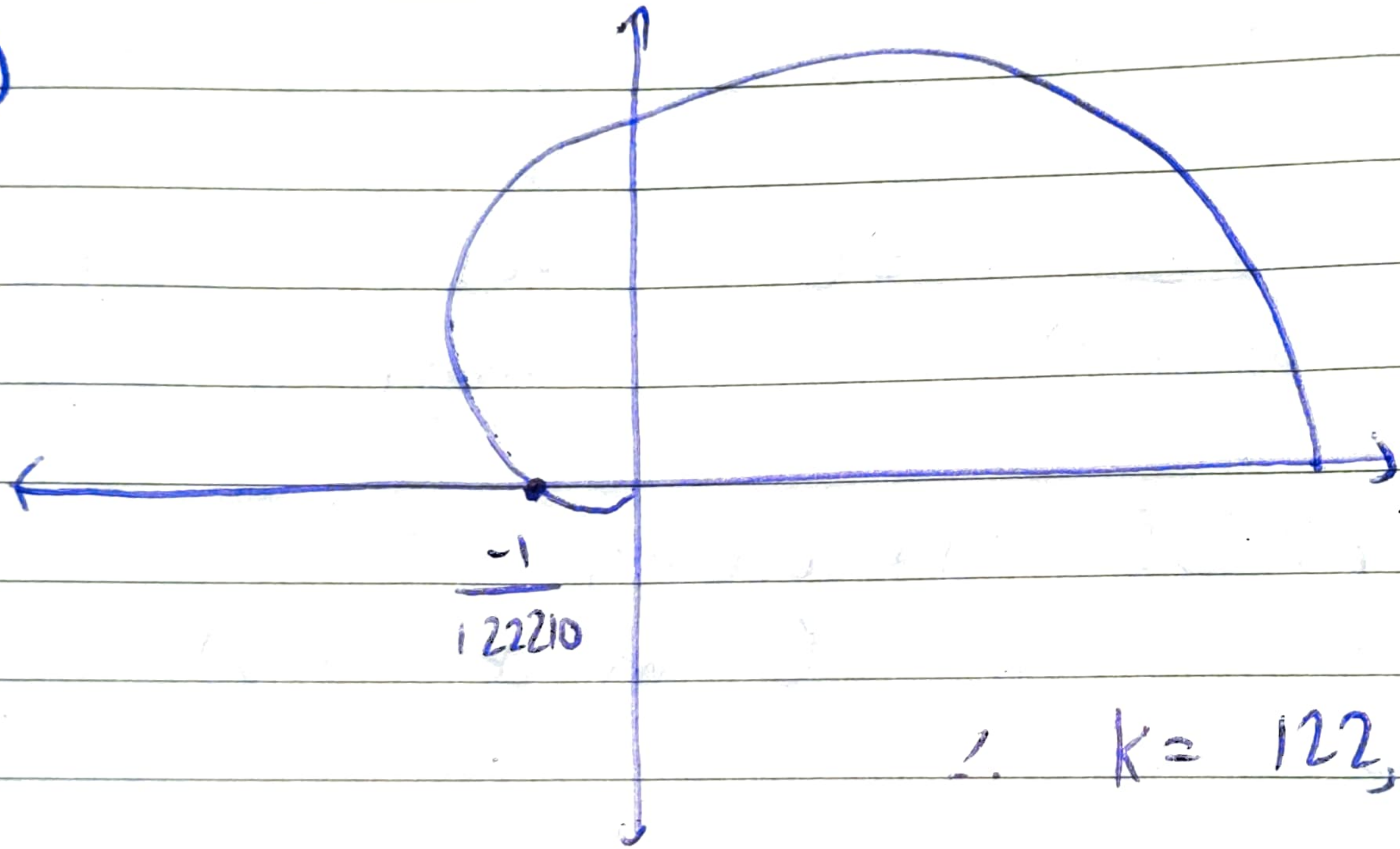
We want $\omega_{pcf} = \omega_{gcf}$.

$\therefore 20\log(k/1000) - 20\times 1 - 40\cdot\log(5) = 0$ dB

$\therefore k = 250{,}000$.

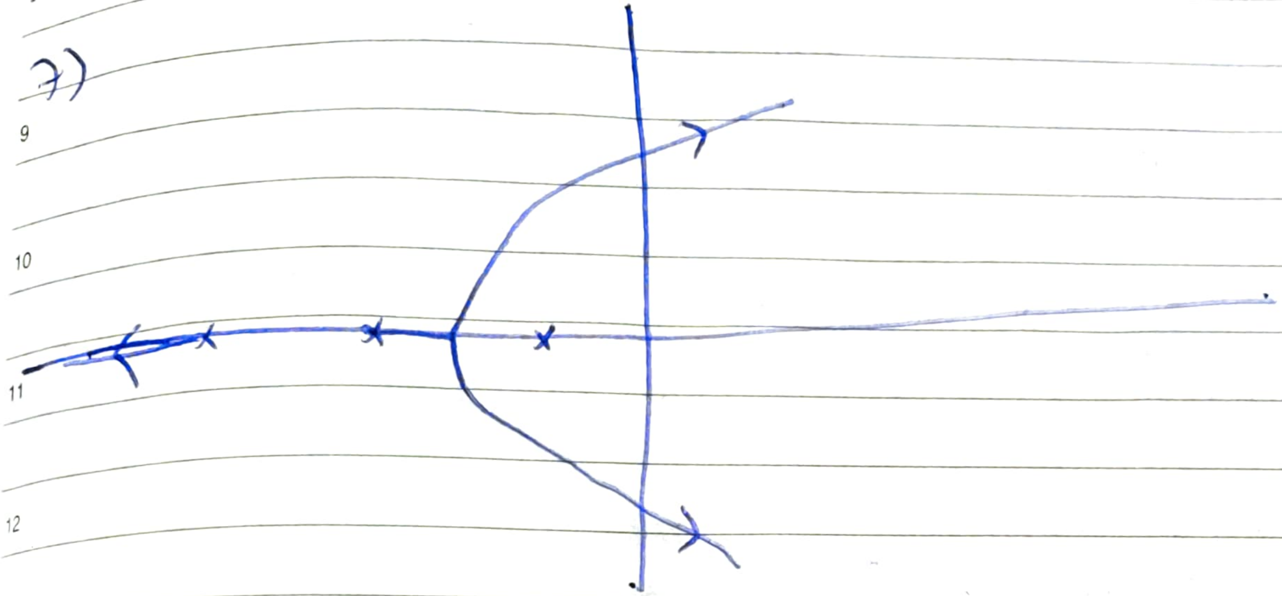
(this is because we used saturation at 5x)

c)



$\therefore k = 122{,}210$

7)



we ignore 3rd pole.

$$t_{settle} = \frac{4}{|Re(poles)|} \leq 2$$

$$\therefore |Re(poles)| \geq 2$$

$$\% OS = 0.05 \geq e^{-\pi \cdot |Re(poles)| / |Im(poles)|}$$

$\therefore$ ~~0.954~~ $0.953 \cdot |Im| \leq |Re|$

$\therefore y = mx$, $m \geq -1.05$

Now $Re = -2$.

$\therefore Im = 2.1$

$\therefore (-2, 2)$ is a desired pole location

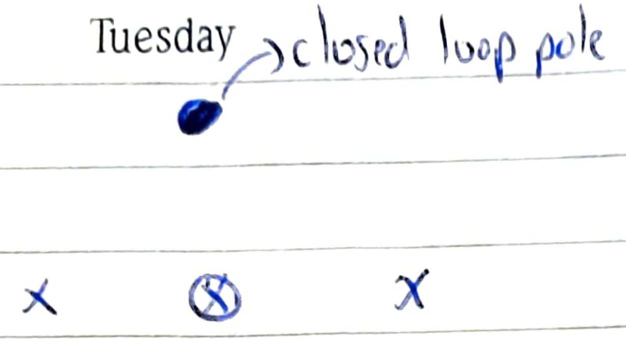
angle contributions:

$$-(90° + 26.6° + 90° + 63.43) = -270°$$

$\therefore$ zero should give $+90°$.

$\therefore$ zero at $s = 2$

Tuesday



is the diagram post PD controller.

~~For~~ $\dfrac{k}{(s+1)(s+3)} + 1$ has pole $(-2, 2)$

$\therefore (-1+2j)(1+2j) + k = 0$

$\therefore -1-4+k=0$

$\therefore k=5$

$\therefore sse = \dfrac{1}{1+\dfrac{5}{1\cdot 3}} = \dfrac{3}{8} = 0.375$

consider pole at -0.005
zero at -0.1

$\therefore sse = \dfrac{1}{1+\dfrac{5\times 0.1}{1\cdot 3\cdot 0.005}} = \dfrac{1}{1+\dfrac{100}{3}} = \dfrac{3}{103}$

$\therefore sse = \cancel{0.00}\ 0.03.$

k stays almost same, ∴ taken as 5.