EE 621 **Spring 2021**

HW 2: Exponential distribution & Poisson process

Assigned: 22/03/21 Due: 29/03/21 (11.59 pm)

1 Memorylessness

The random variable X taking values in $\mathbb{R}_+$ satisfies the following property.

$$P(X > t + s \mid X > s) = P(X > t) \qquad \forall\, s, t \geq 0.$$

Prove that X is exponentially distributed.

Hint: Use the fact that $\bar{F}_X(t+s) = \bar{F}_X(t)\bar{F}_X(s)$ for all $s, t \geq 0$.

2 Memorylessness continued

Suppose $X \sim \mathrm{Exp}(\lambda)$, $t > 0$.

1. Compute $\mathbb{E}[X \mid X > t]$
2. Compute $\mathbb{E}\left[X^2 \mid X > t\right]$

3 Insurance planning

Suppose that the damage cost D of an automobile accident is exponentially distributed with mean $1/\lambda$. Your insurance company will cover the part of the cost exceeding d, i.e., your insurance company will pay the amount X, given by

$$X = \max(D - d, 0).$$

Note that d is a fixed number, that is pre-determined by your insurance policy. What is the mean value of the amount X that the insurance company pays per accident?

4 SBI woes

You go to the SBI branch on campus to get your passbook updated. You can be served at either of two counters. Both counters are busy (i.e., someone is being served at each counter), but there is no one else waiting. You will get served once either counter becomes free. Suppose that the service times of the two counters are exponentially distributed, with means $1/\lambda_1$ and $1/\lambda_2$ respectively. Find $\mathbb{E}[T]$, where T is the amount of time you spend in the bank to get your passbook updated.

5 Poisson warm up

Starting at time $t = 0$, suppose that people arrive at a bus-stop according to a Poisson process with rate λ. The bus departs at time t. Let X denote the sum of the waiting times of all those who get on the bus at time t. (You may take X to be 0 if there are no arrivals until time t.)

Compute the variance of X.

Hint: Condition on the number of arrivals until time t.

6 Poisson arrivals in a random period

Consider a Poisson process with rate λ. Let A_X denote the number of arrivals in this process in a random interval of length X. Here, X is a non-negative random variable independent of the Poisson process.

1. Compute $\mathbb{E}[A_X]$
2. Compute $\mathrm{Var}(A_X)$