Q1:

1) a) $\frac{5(s+2)}{s\cdot(s^2+6s+9)} = \frac{a}{s} + \frac{b}{s+3} + \frac{c}{(s+3)^2}$

$\therefore\ 5s + 10 = (s+3)^2\cdot a + s\cdot(s+3)\cdot b + s\cdot c$

$s=0 \Rightarrow a = \frac{10}{9} \qquad \therefore\ b = \frac{-10}{9}$

$s=-3 \Rightarrow c = 5/3.$

$\therefore\ \mathcal{L}^{-1}(G) = \frac{10}{9} + \left(\frac{-10}{9}\right)\cdot e^{-3t} + \left(\frac{5}{3}\right)\cdot t\cdot e^{-3t}$

b) $\mathcal{L}(t\cdot f(t)) = F'(s)$

$G = \frac{s}{(s^2+1)^2} = \frac{1}{2} \times \frac{d}{ds}\ \frac{1}{s^2+1}$

$\therefore\ \mathcal{L}^{-1}\left(\frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right) = t\cdot \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) = t\sin t.$

$\therefore\ \text{Ans} = \frac{t\sin(t)}{2}$

c) $G = \frac{s^2+s+2}{s+1} = s + \frac{2}{s+1}$

$\mathcal{L}^{\circ}(\delta) = 1$

$\therefore\ \mathcal{L}^{\circ}(\delta') = s.$

$\therefore\ \mathcal{L}^{-1}(G) = \delta'(t) + 2e^{-t}$

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syms s
A = 5*(s+2)/(s*(s^2 + 6*s + 9));
ilaplace(A)

B = s/((s^2 + 1)^2);
ilaplace(B)

C = (s^2 + s + 2)/(s+1);
ilaplace(C)
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ans =

$$\frac{5\,t\,e^{-3t}}{3} - \frac{10\,e^{-3t}}{9} + \frac{10}{9}$$

ans =

$$\frac{t\sin(t)}{2}$$

ans = $2\,e^{-t} + \delta'(t)$

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2) $y'' + 6y' + 2y = 2r' + r$

$\therefore (s^2 + 6s + 2) \cdot Y(s) = (2s+1) R(s)$

Now $r(t) = \delta \Rightarrow R(s) = 1$

$$\therefore Y(s) = \frac{2s+1}{s^2+6s+9-7} = \frac{2s+1}{(s+3)^2 - 7} = \frac{2(s+3) - 5}{(s+3)^2 + (i\sqrt{7})^2}$$

$$\therefore y(t) = e^{-3t} \cdot 2 \cdot \cos(i\sqrt{7}t) - e^{-3t} \cdot \frac{5}{i\sqrt{7}} \sin(i\sqrt{7}t)$$

$$= e^{-3t}\left(2\cos(i\sqrt{7}t) + \frac{5i}{\sqrt{7}} \sin(i\sqrt{7}t)\right)$$

$$= e^{-3t}\left(2\cosh(\sqrt{7}t) - \frac{5}{\sqrt{7}} \sinh(\sqrt{7}t)\right)$$

3a) (i) $G = \dfrac{100}{s^2+1+s}$

(ii) $G = \dfrac{50}{s^2+1+2s}$

(iii) $G = \dfrac{25}{s^2+1+4s}$

(iv) $G = \dfrac{100}{s^2+1}$

b) $\omega_n = 1$ for all cases

$\zeta = 0.5, 1, 2, 0$ respectively

poles:

(i) $-0.5 \pm \dfrac{\sqrt{3}i}{2}$

(ii) -1 (both poles at same location)

(iii) $-2 \pm \sqrt{3}$

(iv) ± 1

c) (i) $t_{rise} = \dfrac{1.8}{\omega_n} = 1.8s$, $\%OS = e^{\frac{-0.5\pi}{\sqrt{3}/2}} = 16.3\%$, $t_{settle} = \dfrac{4}{\zeta\omega_n} = 8s$

(ii) $\%OS = 0\%$, $t_{settle} \approx \cancel{3.91}\ \dfrac{5.83}{p} = 5.83s$, $t_{rise} \approx 3.2s$

(iii) $\%OS = 0\%$, $t_{settle} \approx \dfrac{3.91}{p_1} \approx \dfrac{4}{2-\sqrt{3}} = 14.9s$, $t_{rise} \approx 8.2s$

(iv) $\%OS = 100\%$, rise time $= \dfrac{2}{\pi \cdot \omega_n} = \dfrac{2}{\pi}$, $t_{settle} = \infty$
(0 → 100%)

4a) In steady state, $\dashv\vdash = --$

$\therefore V_c(0^-) = 1000$ for both capacitors

Also, $I_c(s) = sC\,V_c(s) - C\,v_c(0^-)$

$\therefore$

$$10^6 \underbrace{\begin{bmatrix} 1+\frac{1}{2s} & \frac{-1}{2s} \\ \frac{-1}{2s} & s+\frac{1}{2s}+\frac{1}{s} \end{bmatrix}}_{M} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -\frac{1000}{s} \\ 0 \end{bmatrix}$$

$$\therefore I_2(s) = \frac{\begin{vmatrix} 1+\frac{1}{2s} & -\frac{1000}{s} \\ \frac{-1}{2s} & 0 \end{vmatrix}}{|M|\cdot 10^6} = \frac{-\frac{1000}{2s^2} \times 10^{-6}}{\begin{vmatrix} 1+{}^1/_{2s} & -{}^1/_{2s} \\ -{}^1/_{2s} & s+{}^3/_{2s} \end{vmatrix}}$$

$$= \frac{-10^{-3}}{10s^2+8s+1}$$

$$\therefore V_{out}(s) = R\cdot I_2(s) = \frac{-5000}{10s^2+8s+1}$$

Inv. $\mathcal{L}$ was done using MATLAB

$$\therefore V_{out}(t) = \frac{-2500}{\sqrt{6}} \cdot \sinh\left(\frac{\sqrt{6}\,t}{10}\right)\cdot e^{-2t/5}$$

4b)

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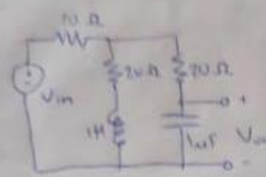


$$\begin{bmatrix} s+30 & -(s+20) \\ -(s+20) & s+40+\frac{10^6}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_{in}(s) \\ 0 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} s+30 & -(s+20) \\ -(s+20) & \end{bmatrix}} \quad \therefore I_2 = \frac{\begin{vmatrix} s+30 & V_{in} \\ -(s+20) & 0 \end{vmatrix}}{\begin{vmatrix} s+30 & -(s+20) \\ -(s+20) & s+40+\frac{10^6}{s} \end{vmatrix}}$$

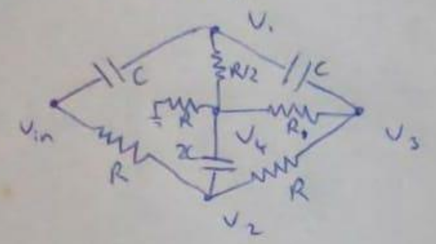
$$\therefore I_2 = \frac{V_{in}(s+20)}{s^2+70s+1001200}$$

$$V_{out}(s) = \frac{10^6}{s}\cdot I_2$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{10^6\cdot(s+20)}{s\cdot(s^2+70s+10^6+1200)}$$

4c) $V_3 = V_{out}$ (assuming ideal op amp)

Equivalent circuit:

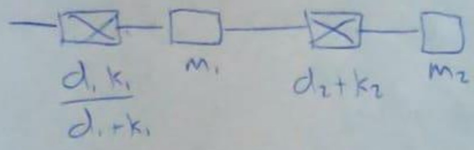


(assumed $R_1 = R_2 = R$)

$$\begin{bmatrix} 2sC + \frac{2}{R} & 0 & -sC & -2/R \\ 0 & 2sC + 2/R & -1/R & -2sC \\ -sC & -1/R & sC + 2/R & -1/R \\ -2/R & -2sC & -1/R & 2sC + \frac{4}{R} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} sC\,V_{in} \\ V_{in}/R \\ 0 \\ 0 \end{bmatrix}$$

$\frac{|A_3|}{|A|}$ gives us V_3.

5a) (i)/(ii)



$$\therefore \left(\frac{d_1 s \cdot k_1}{d_1 s + k_1} + m_1 s^2\right) x_1 + (d_2 s + k_2)(x_1 - x_2) = F_1(s)$$

$$x_2 \cdot m_2 s^2 + (d_2 s + k_2)(x_2 - x_1) = F_2(s)$$

$$\therefore x_1(s) = \frac{3 \cdot F_2 \cdot (2s^2 + 9s + 4)}{5(2s^4 + 13s^3 + 72s^2 + 168s + 298)}$$

; setting $F_1 = 0$ and substituting given values

Method elaborated for x_2:
$F_2 = 0$.

$$\therefore x_1 = \frac{x_2(s^2 + 3s + 12)}{3s + 12}$$

$$\therefore x_2 \cdot \left(\left(\frac{s^2 + 3s + 12}{3s + 12}\right) \cdot \left(\frac{10s}{2s+1} + s^2 + 3s + 12\right) - (3s + 12)\right) = F_1(s)$$

This can now be simplified to get the answer.

5a) (iii) k_i : N/m

f_i : N

x_i : m

d_i : N-s/m

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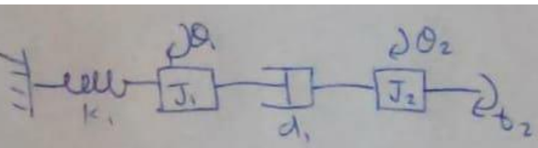
5b)



(i) k_1: newton - meter / radian

d_1: newton - meter · sec / radian

θ_1 / θ_2: radian

J_1 / J_2: kg · meter ^{2}

(ii) $T_2(s) = (J_2 s^2 + d_1 s)\theta_2 - d_1 s \theta_1$

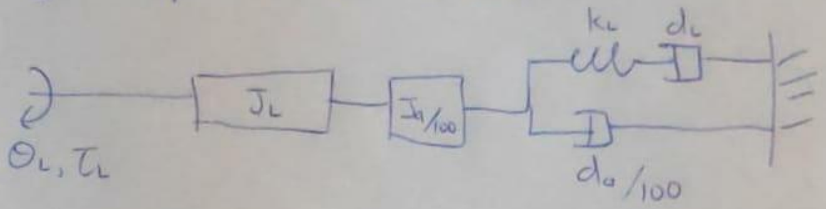
$0 = (J_1 s^2 + d_1 s + k_1)\theta_1 - d_1 s \theta_2$

$$\therefore \theta_2 = \frac{T_2(s) \cdot (J_1 s^2 + d_1 s + k_1)}{J_1 \cdot J_2 \cdot s^4 + d(J_1 + J_2)s^3 + d_1^2 s^2 + d_1 k_1 s}$$

$$\therefore \frac{\dot{\theta}_2}{T_2} = \frac{J_1 s^2 + d_1 s + k_1}{J_1 J_2 s^3 + d(J_1 + J_2)s^2 + d_1^2 s^2 + d_1 k_1}$$

5c) $E_a = I_a R_a - V_b$

Bring components to N_2 side:



$$\therefore T_L(s) = \Theta_L(s)\left(70.05\, s^2 + 0.02s + \frac{(200s)\cdot 20}{200s+20}\right)$$

But $\frac{T_L}{\Theta_L} = 100^{-1}\cdot\frac{T_m}{\Theta_m} = 100 \cdot k_t \cdot I_a$

$$= 100 \cdot k_t \frac{E_a + V_b}{R_a}$$

but $v_b = k_b \cdot \dot{\theta}$

$$\therefore \left(70.05s^2 + 0.02s + \frac{200s}{10s+1}\right)\cdot \Theta_m = \frac{k_t(E_a + s k_b \Theta_m)}{100\, R_a}$$

From the graph:

$$\frac{T_m}{5} + \omega_m \cdot 2 = 100$$

$\therefore R_a/k_t = 1/5, \; k_b = 2.$

$$\therefore 5E_a = \Theta_m\left(7005s^2 - 8s + \frac{20000s}{10s+1}\right)$$

$$\therefore \frac{\Theta_L}{E_a} = \frac{5 \times 10}{7005s^2 - 8s + \frac{2\times 10^4 \times s}{10s+1}}$$