Q1) $3s^7 - 9s^6 + 6s^5 - 4s^4 + 7s^3 - 8s^2 + 2s - 6$

s^7	3	6	7	2	
s^6	-9	-4	-8	-6	
s^5	14	13	0	0	(scaled by 3)
s^4	61	-112	-84	0	(scaled by 14)
s^3	38.7	19.3	0	0	
s^2	-142.4	-84	0		
s^1	-3.5	0			
s^0	-84	0			

no. of sign changes = 3.

no. of zeros rows = 0.

degree of polynomial = 7.

∴ 3 ORHP poles

0 jR poles

4 OLHP poles

Q2) $12s^7 - 4s^6 - 3s^5 + s^4 - 12s^3 + 4s^2 + 3s - 1$

(method 1)

s^7	12	-3	-12	3
s^6	-4	1	4	-1
s^5	0	0	0	0
s^4	1	8	-3	0
s^3	196	-64	0	0
s^2	8.33	-3		
s^1	6.59	0		
s^0	-3	0		

$\rightarrow$ -24 4 8 0 [dq/ds]

(scaled by 3)

$\therefore$ q has 2 ORHP, 2 OLHP, 2 jR

p/q has 1 ORHP

$\therefore$ p has 3ORHP, 2OLHP, 2jR roots.

Q2) $12s^7 - 4s^6 - 3s^5 + s^4 - 12s^3 + 4s^2 + 3s - 1$

(method 2) $= s^4(12s^3 - 4s^2 - 3s + 1) - (12s^3 - 4s^2 - 3s + 1)$

$= (s^4 - 1)(12s^3 - 4s^2 - 3s + 1)$

Now $(s^4 - 1)$ has roots $1, -1, j, -j$

We use routh-hurwitz table for roots of

$12s^3 - 4s^2 - 3s + 1$

$$\begin{array}{c|cc} s^3 & 12 & -3 \\ s^2 & -4 & 1 \\ s^1 & 0 & 0 \\ s^0 & & \end{array}$$

$\rightarrow \therefore\ 12s^3 - 4s^2 - 3s + 1$ is divisible by $(-4s^2 + 1)$.

$\therefore\ 12s^3 - 4s^2 - 3s + 1 = (-4s^2 + 1)(-3s + 1)$

$\therefore$ roots are $1, -1, j, -j, \frac{1}{2}, \frac{-1}{2}, \frac{1}{3}$ $\Rightarrow$ 3 ORHP, 2 $j\mathbb{R}$, 2 OLHP

Q3) $s^6 - s^4 - 7s^2 + 7s - 6$

s^6	1	-1	-7	-6
s^5	$\cancel{0}\,\epsilon$	0	7	0
s^4	-1	$-7/\epsilon$	-6	0
s^3	-7	7	0	
s^2	$-7/\epsilon$	-6	0	
s^1	7	0		
s^0	-6			

$\left[\frac{7+7\epsilon}{\epsilon} \approx 7/\epsilon\right]$

$\left[\frac{7+6\epsilon}{1} \approx 7\right]$

$\left[\frac{7+49/\epsilon}{7} \approx 7/\epsilon\right]$

$\left[\frac{49/\epsilon + 56}{7/\epsilon} \approx 7\right]$

$\epsilon > 0 \Rightarrow 3$ sign changes

$\epsilon < 0 \Rightarrow 3$ sign changes

$\therefore$ 3 ORHP, 0 $j\mathbb{R}$, 3 OLHP zeros

Q4) skipped

Q5) a) $\frac{1}{(s+1)(s+3)}$

b) $\frac{s-1}{s+1}$

c) $\frac{(s-2)}{(s-1)(s+1)}$

d) $\frac{(s+1)}{(s+2)(s+3)(s+4)}$

e) $\frac{(s+4)}{(s+1)(s+2)(s+3)}$

f) $\cancel{\frac{(s+1)(s+2)}{(s+3)(s+4)}}$ $\frac{(s+3)(s+4)}{(s+1)(s+2)}$

g) $\frac{1}{(s+1)(s+2)(s+3)}$

h) $\frac{(s-(-1+i))(s-(-1-i))}{(s+1)(s+2)}$

$= \frac{s^2+2s+2}{(s+1)(s+2)}$

i) $\frac{s^2+4s+5}{s^2+2s+2}$

j) $\frac{s+1}{s^2}$

Q6) $\frac{1}{(s+1)^3}$ $\rightarrow$ has 3 straight lines at $120°$ going out of $(-1,0)$.

$\therefore$ the breakaway angle can't be $90°$

$\frac{(s+1)^3}{s(s^2+1)}$: at point very close to $(-1,0)$,

angle due to complex pole pair $= 0 - 360 \approx 0$

angle due to $s=0$ pole $= -180°$

$\therefore$ angle due to zeros $= 3\theta$ $\Rightarrow 3\theta - 180 = (2k+1)180$

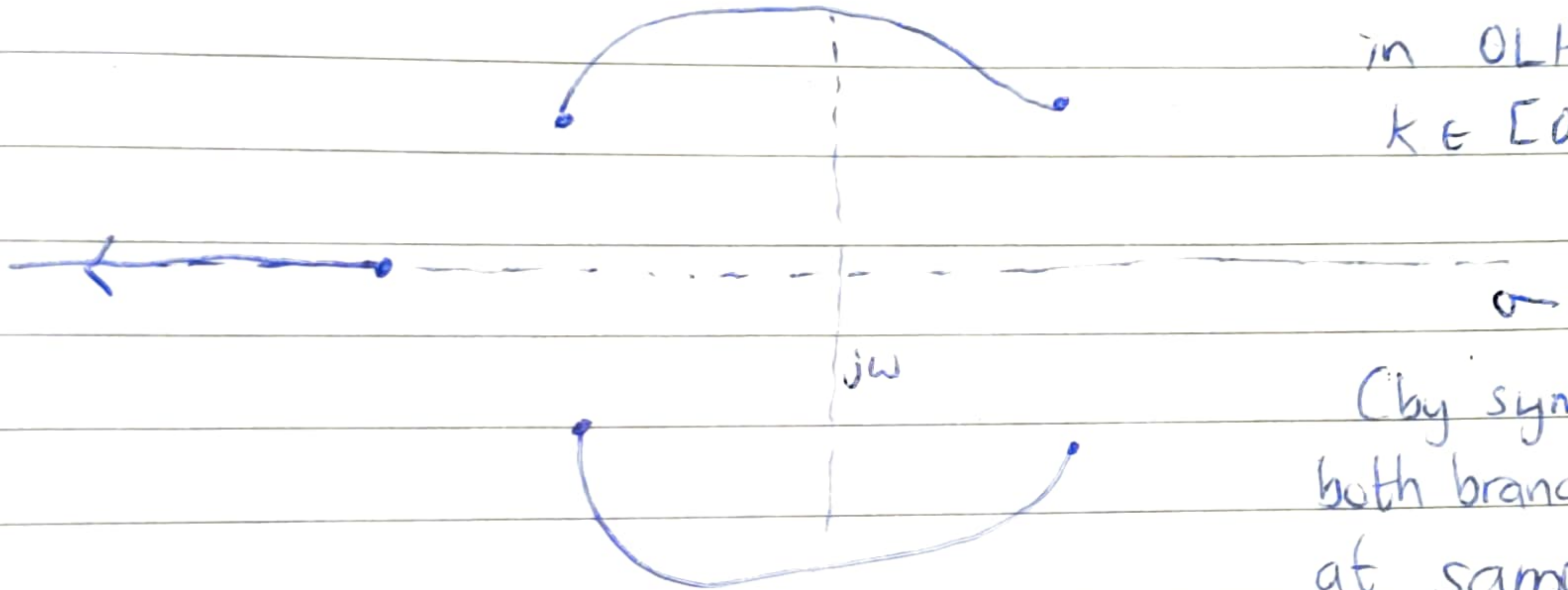
$\therefore 3\theta$ is even multiple of 180 (we take 360)

$\therefore \theta = 120° \neq 90°$

Q7) poles: $-1+i$, $-1-i$, -2

zeros: $1+i$, $1-i$

$\therefore$ root locus looks like:



$\therefore$ all three branches in OLHP for $k \in [0, 2.72)$

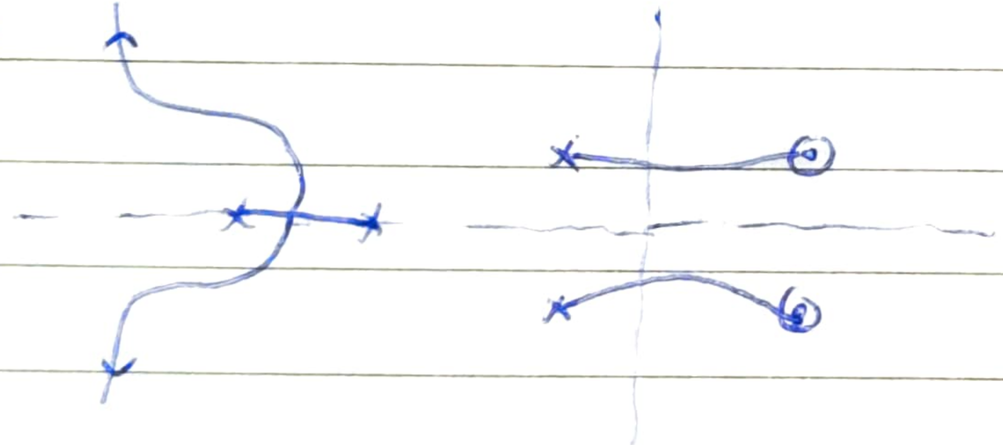
(by symmetry) both branches hit $j\mathbb{R}$ at same k

Q8) $$\frac{\frac{K}{(s+3)(s+4)}}{1+\frac{K}{(s+3)(s+4)}\cdot\frac{s^2-4s+8}{s^2+2s+5}}$$

$\therefore$ $d(s) = (s+3)(s+4)(s^2+2s+5)$

$n(s) = (s^2-4s+8)$

a)



b) $\omega_0 = 1.84$, $K = 7.16$

c) $(-3.52, 0)$

d) $183.4°$

e) $2 \pm 2i$