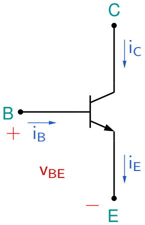
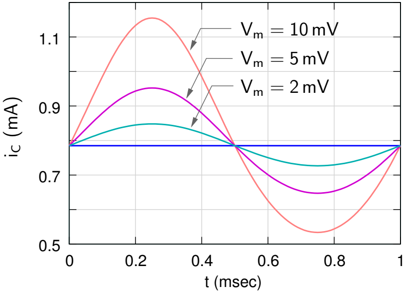
BJT: AC model



$v_{BE}(t) = V_0 + V_m \sin \omega t$

$V_0 = 0.65\,\text{V}, \quad f = 1\,\text{kHz}$



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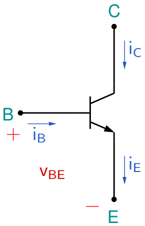

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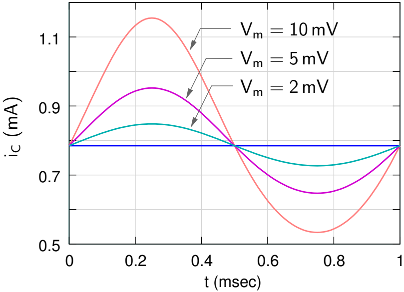
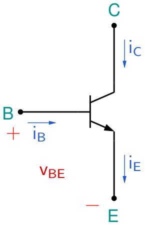


* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid $\rightarrow$ distortion.



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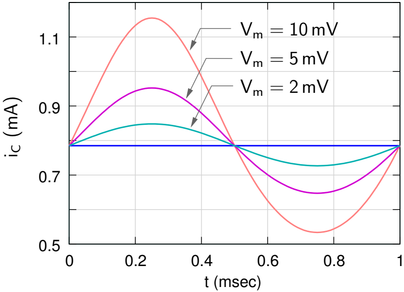
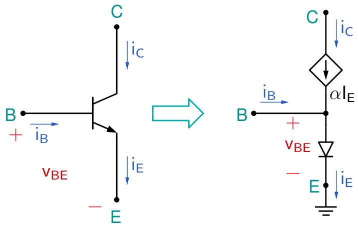
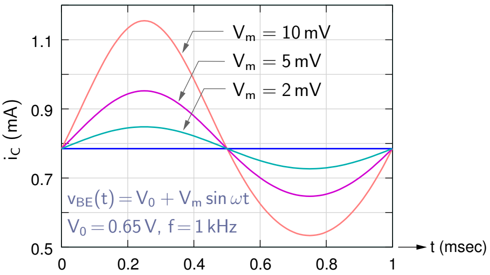


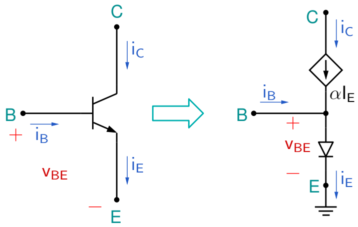
* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid $\rightarrow$ distortion.
* If $v_{be}(t)$, i.e., the time-varying part of v_{BE}, is kept small, i_C varies linearly with v_{BE}. How small? Let us look at this in more detail.

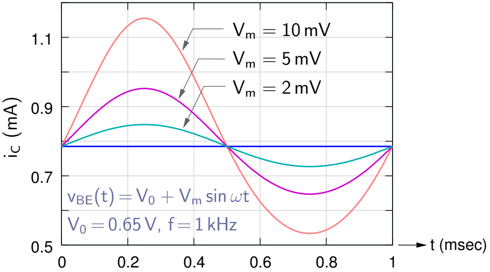
BJT: small-signal model





Let $v_{BE}(t) = V_{BE} + v_{be}(t)$ (bias+signal), and $i_C(t) = I_C + i_c(t)$.





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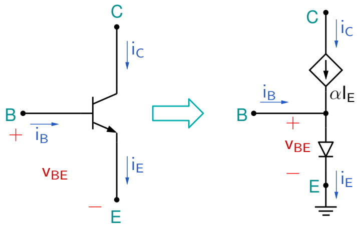
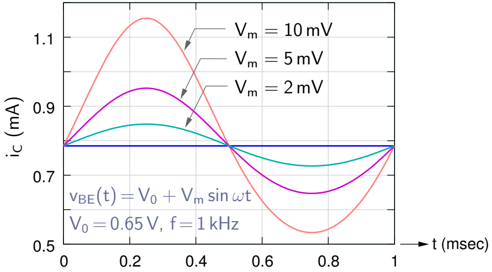
Assuming active mode, $i_C(t) = \alpha\, i_E(t) = \alpha\, I_{ES} \left[\exp\left(\dfrac{v_{BE}(t)}{V_T} \right) - 1 \right]$.





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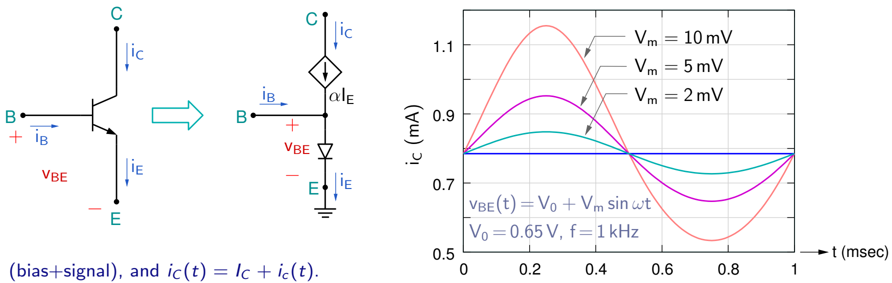
Since the B-E junction is forward-biased, $\exp\left(\dfrac{v_{BE}(t)}{V_T}\right) \gg 1$, and we get

$$i_C(t) = \alpha\, I_{ES}\, \exp\left(\frac{v_{BE}(t)}{V_T}\right) = \alpha\, I_{ES}\, \exp\left(\frac{V_{BE} + v_{be}(t)}{V_T}\right) = \alpha\, I_{ES}\, \exp\left(\frac{V_{BE}}{V_T}\right) \times \exp\left(\frac{v_{be}(t)}{V_T}\right).$$

BJT: small-signal model



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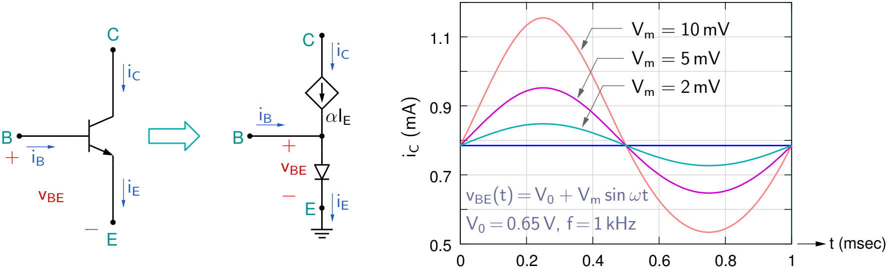
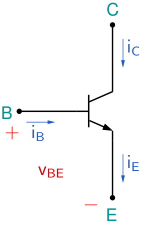
If $v_{be}(t) = 0$, $i_C(t) = I_C$ (the bias value of i_C), i.e., $I_C = \alpha\, I_{ES}\, \exp\left(\dfrac{V_{BE}}{V_T}\right)$

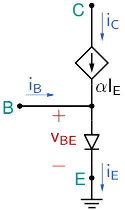
$$\Rightarrow\ i_C(t) = I_C\, \exp\left(\frac{v_{be}(t)}{V_T}\right).$$

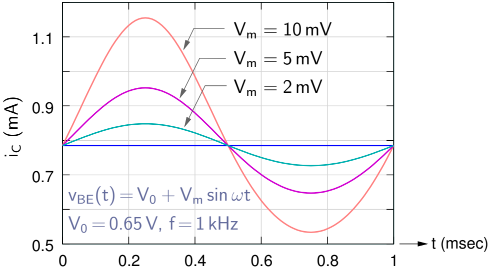
BJT: small-signal model



$$i_C(t) = I_C \exp\left(\frac{v_{be}(t)}{V_T}\right) = I_C \left[1 + x + \frac{x^2}{2} + \cdots\right], \quad x = v_{be}(t)/V_T\,.$$







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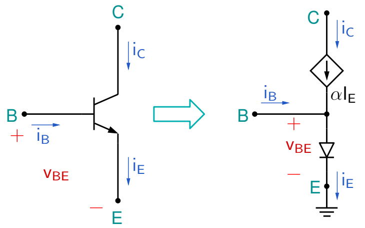
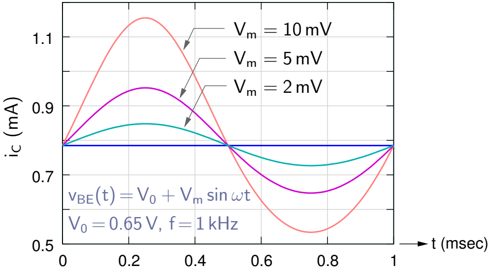
If x is small, i.e., if the amplitude of $v_{be}(t)$ is small compared to the thermal voltage V_T, we get

$$i_C(t) = I_C \left[1 + \frac{v_{be}(t)}{V_T}\right].$$





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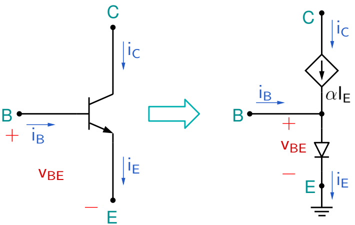
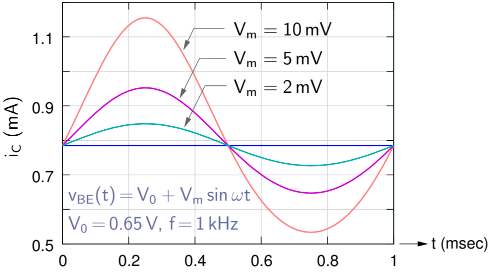
We can now see that, for $|v_{be}(t)| \ll V_T$, the relationship between $i_C(t)$ and $v_{be}(t)$ is linear, as we have observed previously.

BJT: small-signal model





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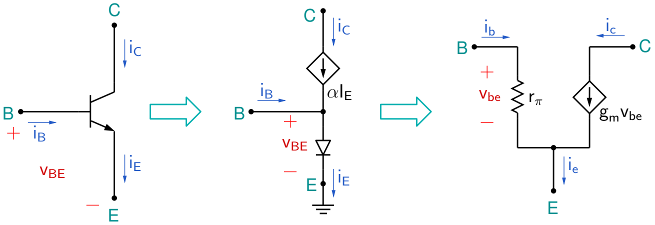
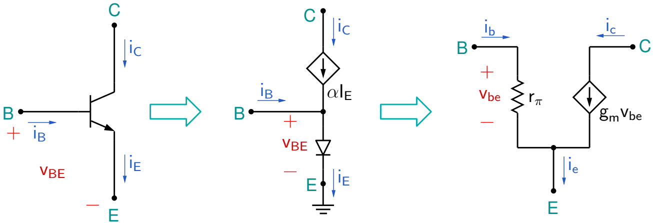
We can now see that, for $|v_{be}(t)| \ll V_T$, the relationship between $i_C(t)$ and $v_{be}(t)$ is linear, as we have observed previously.

$$i_C(t) = I_C + i_c(t) = I_C \left[1 + \frac{v_{be}(t)}{V_T}\right] \Rightarrow \boxed{i_c(t) = \frac{I_C}{V_T}\, v_{be}(t)}$$



The relationship, $i_c(t) = \dfrac{I_C}{V_T}\, v_{be}(t)$ can be represented by a VCCS, $i_c(t) = g_m\, v_{be}(t)$,

where $g_m = I_C/V_T$ is the "transconductance."



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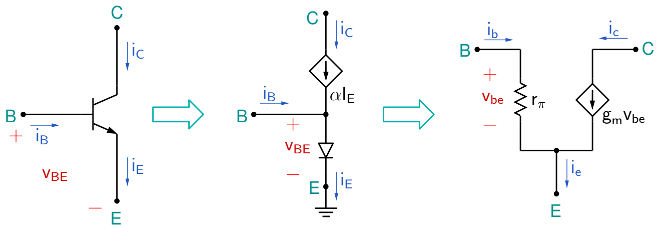
For the base current, we have,

$$i_B(t) = I_B + i_b(t) = \frac{1}{\beta}\,[I_C + i_c(t)]$$

$$\rightarrow i_b(t) = \frac{1}{\beta}\, i_c(t) = \frac{1}{\beta}\, g_m\, v_{be}(t) \rightarrow v_{be}(t) = (\beta/g_m)\, i_b(t).$$



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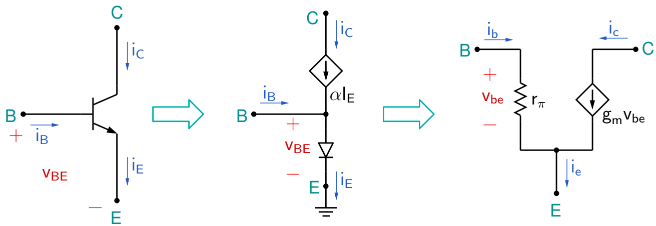
The above relationship is represented by a resistance, $r_\pi = \beta/g_m$, connected between B and E.

BJT: small-signal model



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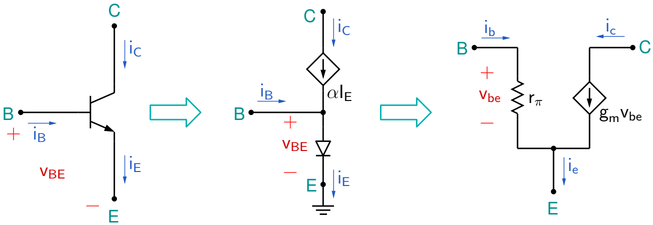
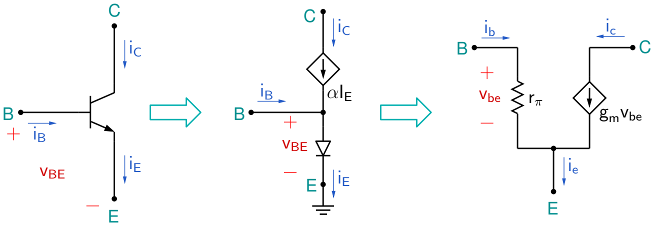
The resulting model is called the π-model for small-signal description of a BJT.

BJT: small-signal model



∗ The transconductance g_m depends on the biasing of the BJT, since $g_m = I_C / V_T$. For $I_C = 1\,mA$, $V_T \approx 25\,mV$ (room temperature), $g_m = 1\,mA/25\,mV = 40\,m\mho$ (milli-mho or milli-siemens).



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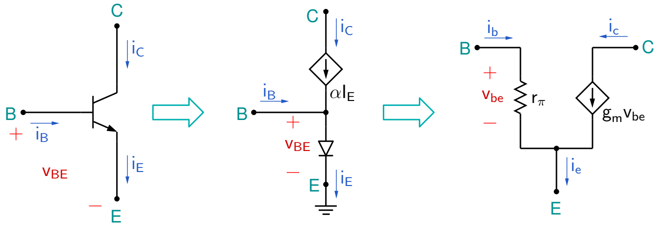

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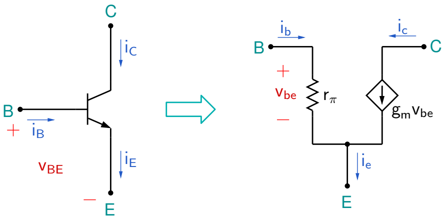
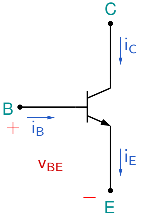
* Note that the small-signal model is valid only for small v_{be} (small compared to V_T).

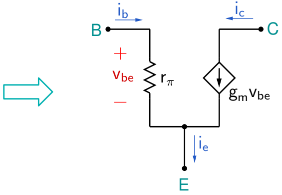
BJT: small-signal model

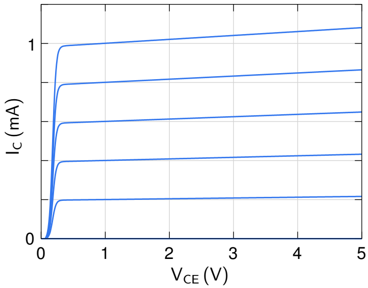


* In the above model, note that i_c is independent of v_{ce}.

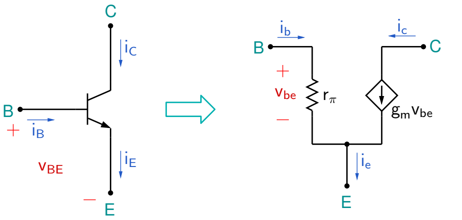
BJT: small-signal model

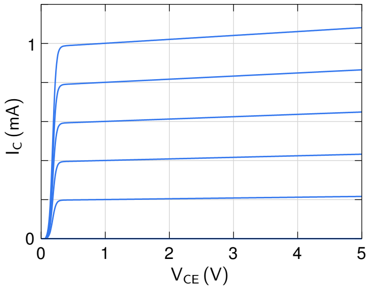






- ∗ In the above model, note that i_c is independent of v_{ce}.
- ∗ In practice, i_c does depend on v_{ce} because of the Early effect, and $\dfrac{dI_C}{dV_{CE}} \approx$ constant $= 1/r_o$, where r_o is called the output resistance.





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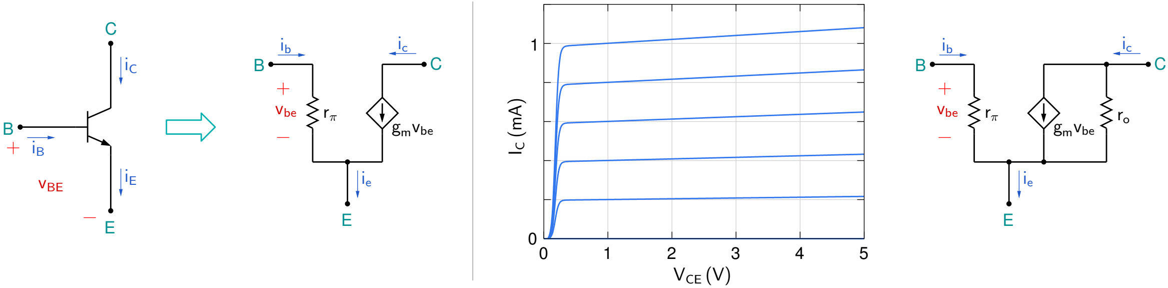
- A more accurate model includes r_o as well.

BJT: small-signal model



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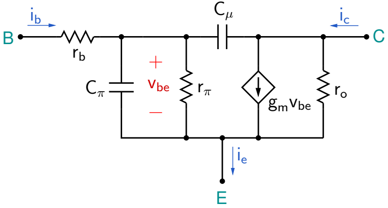
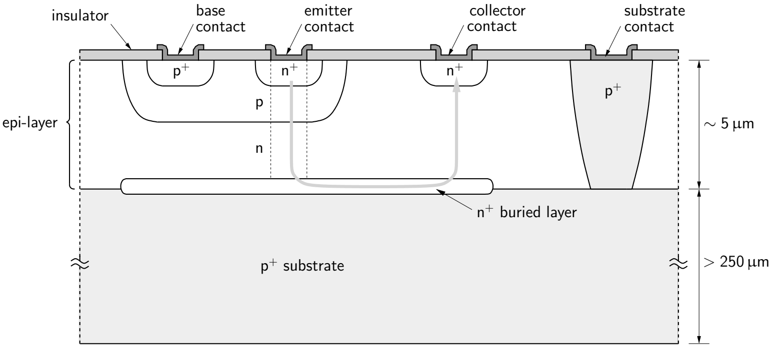
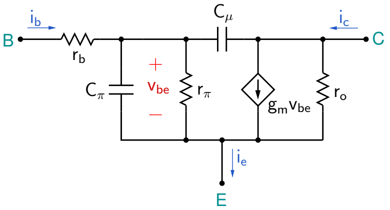
BJT: small-signal model

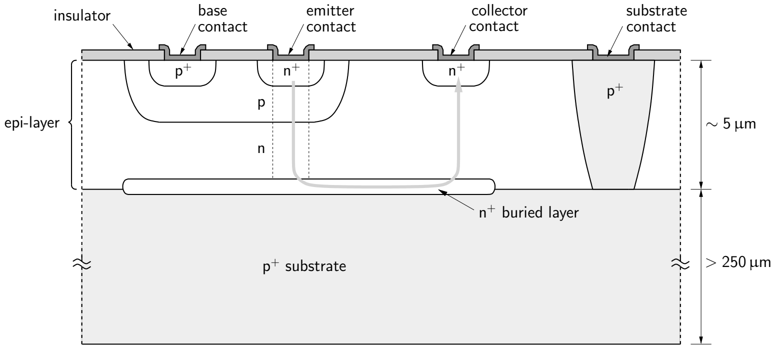




* A few other components are required to make the small-signal model complete:
  r_b: base spreading resistance
  C_π: base charging capacitance + B-E junction capacitance
  C_μ: B-C junction capacitance

BJT: small-signal model





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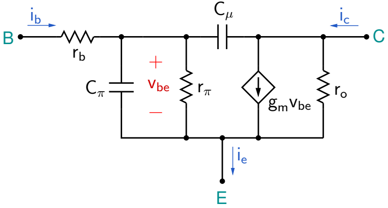
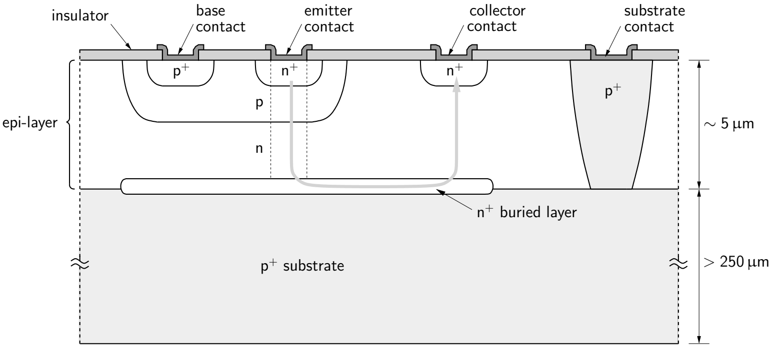
* The capacitances are typically in the pF range. At low frequencies, $1/\omega C$ is large, and the capacitances can be replaced by open circuits.

BJT: small-signal model





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* Note that the small-signal models we have described are valid in the active region only.