Semiconductor Devices

Carrier Transport: Part 2

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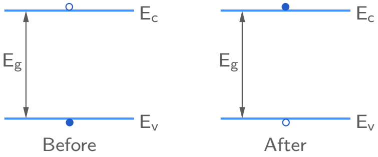
Generation and recombination



Generation

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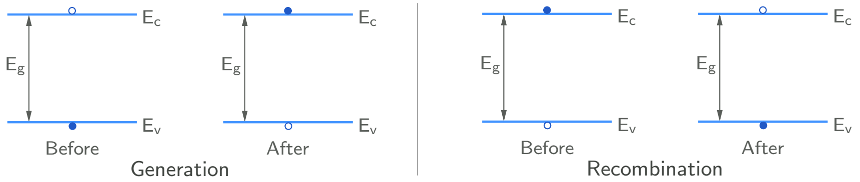

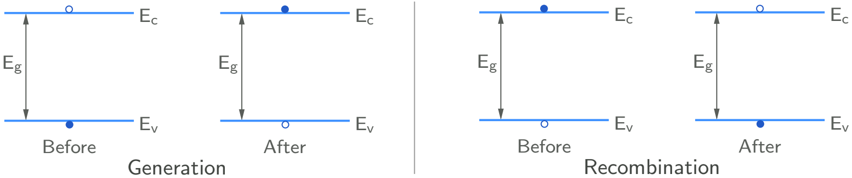

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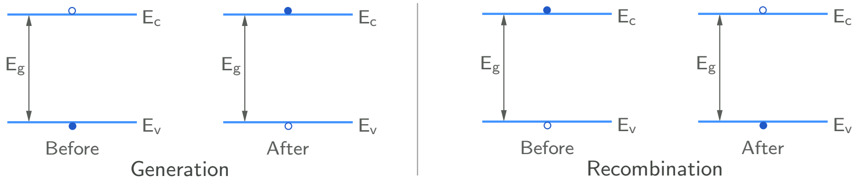
Generation and recombination



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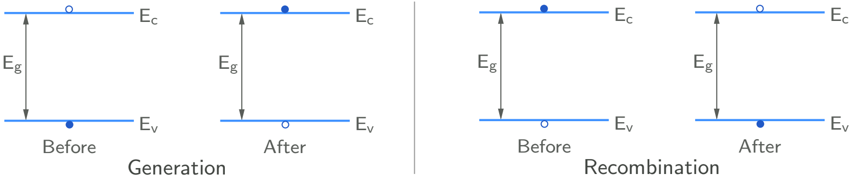

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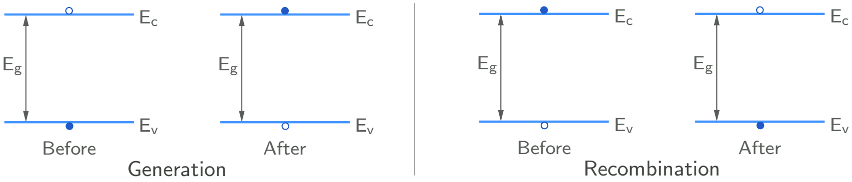

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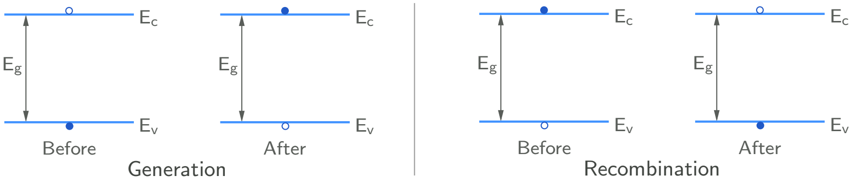

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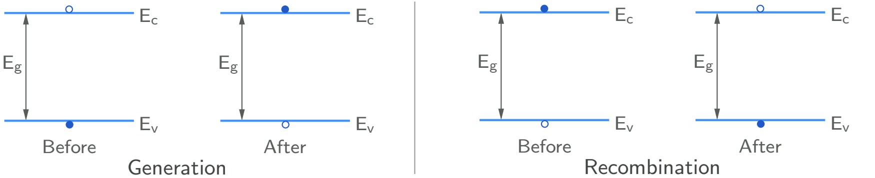

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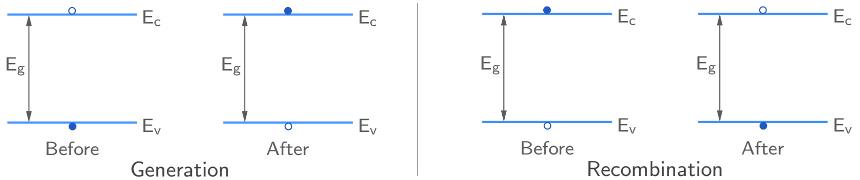

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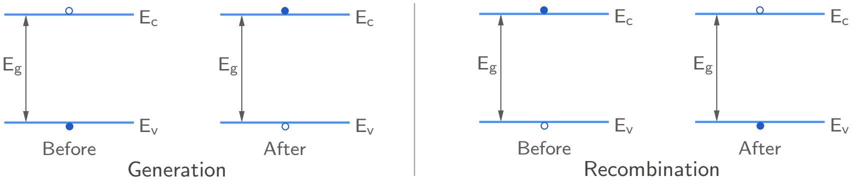

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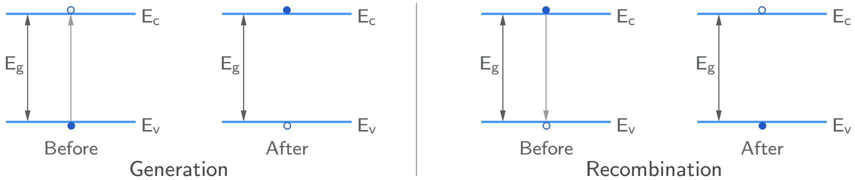
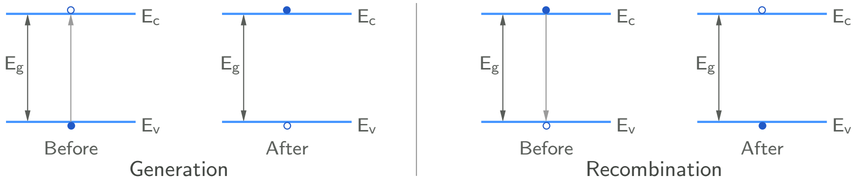
* Some of these processes, with very small rates, may be completely ineffective, while others may be dominant.



* Direct recombination

Direct G-R



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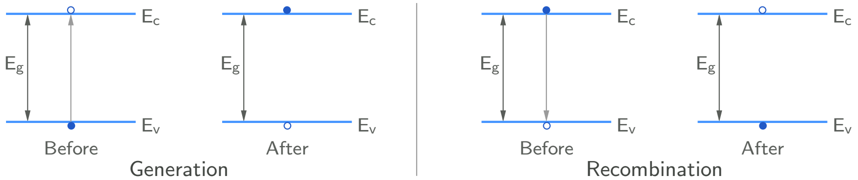

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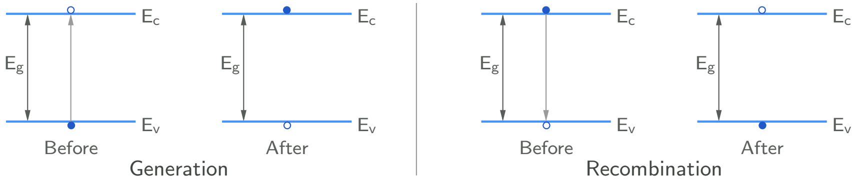

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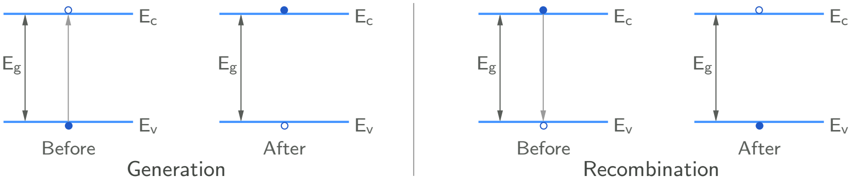

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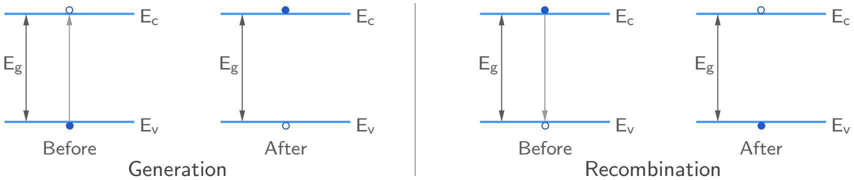

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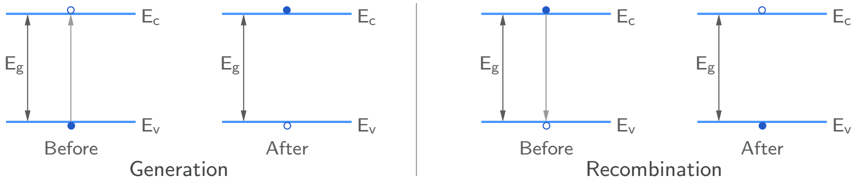

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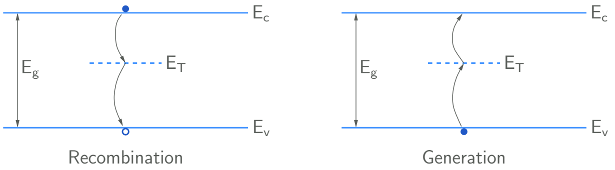
* Since particles from both condution and valence bands are simultaneously involved, the above processes are called "band-to-band" G-R.

Indirect G-R



* In indirect G-R, the transitions from the conduction band to the valence band (and *vice versa*) take place through a "G-R centre," with an energy level E_T located in the forbidden gap.

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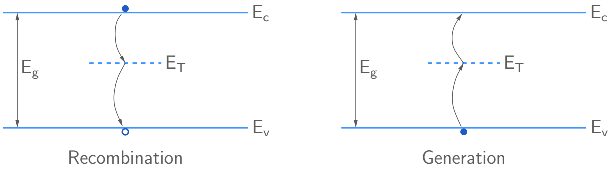

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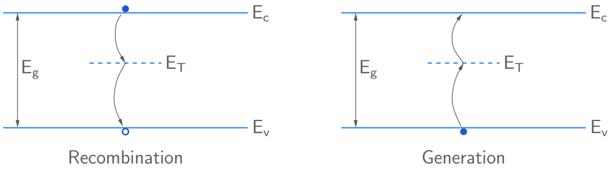

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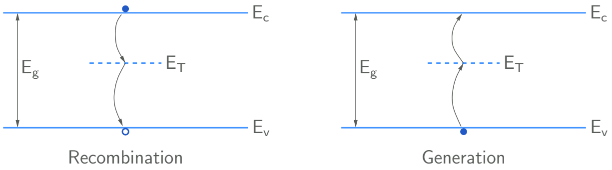

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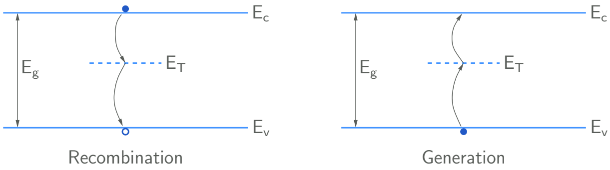

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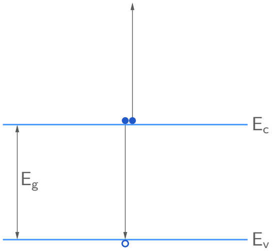
* Indirect G-R processes are particularly efficient when E_T lies close to the middle of the band gap.

G-R with three particles



Auger Recombination

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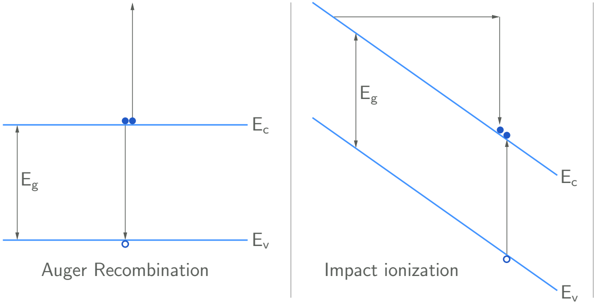

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* In the reverse process of impact ionisation, a high-energy electron loses its energy ($> E_g$) but remains in the conduction band, and the energy is used by a valence band electron to make a transition to the conduction band, thus generating an EHP.

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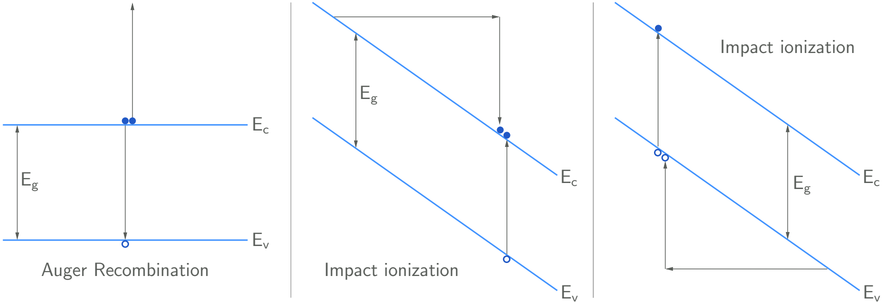

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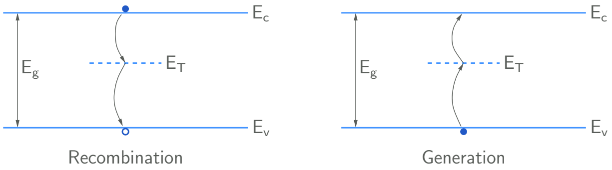
Non-radiative G-R in silicon



* In silicon, the dominant non-radiative G-R process is that involving a G-R centre.

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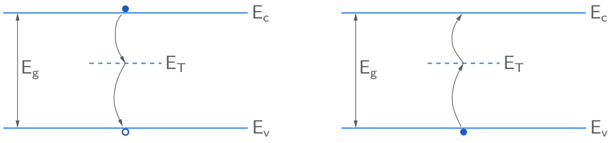

Recombination   Generation

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* The *net* rate of recombination per unit volume is given by the Shockley–Read–Hall (SRH) formula, $R - G = \dfrac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$.

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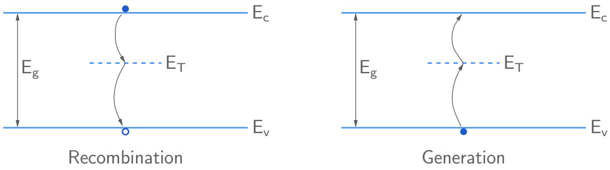

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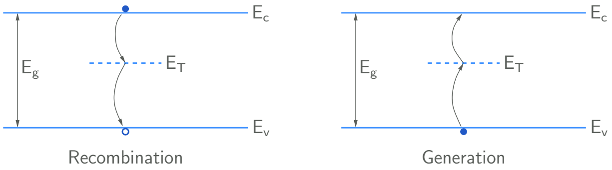

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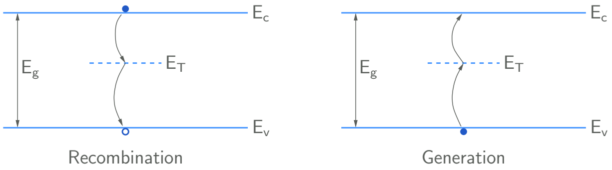

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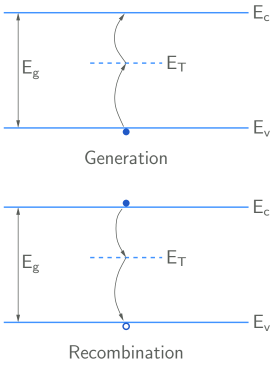
* Since the most effective G-R centres have $E_T \approx E_i$, n_1 and p_1 are generally much smaller than the majority carrier density in a doped semiconductor.

$$R - G = \frac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}.$$

Let us consider a semiconductor in which electrons are the majority carriers, with the equilibrium values of n and p given by $n_0 = 10^{16}\,\text{cm}^{-3}$, $p_0 = 10^4\,\text{cm}^{-3}$.



SRH formula: special case

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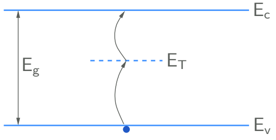
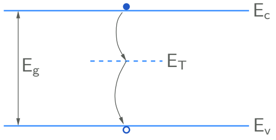


Generation



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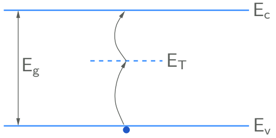

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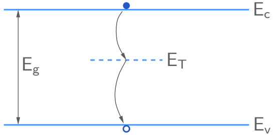

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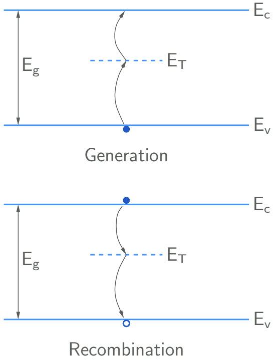

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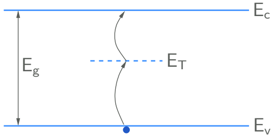

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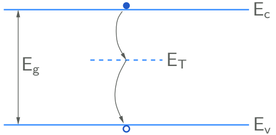

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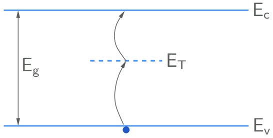
$$\approx \frac{p_0\Delta n + n_0\Delta p}{\tau_p(n_0 + \Delta n)} \qquad \because n_1 \ll n_0,\;\; p + p_1 \ll n_0$$



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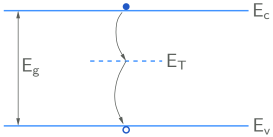


Recombination

SRH formula: special case

$$R - G = \frac{n\,p - n_i^2}{\tau_p(n+n_1) + \tau_n(p+p_1)}.$$

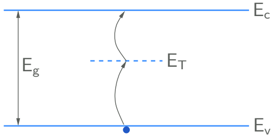
Let us consider a semiconductor in which electrons are the majority carriers, with the equilibrium values of n and p given by $n_0 = 10^{16}\,\text{cm}^{-3}$, $p_0 = 10^4\,\text{cm}^{-3}$.

Suppose that additional carriers are created in the semiconductor (e.g., by shining light on the sample), giving rise to "excess concentrations" $\Delta p = \Delta n = 10^{12}\,\text{cm}^{-3}$.

The total concentrations are $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$,

Applying the SRH formula, we get

$$\begin{aligned}
R - G &= \frac{(n_0+\Delta n)(p_0+\Delta p) - n_i^2}{\tau_p(n+n_1) + \tau_n(p+p_1)} \\
&= \frac{p_0\Delta n + n_0\Delta p + \Delta n\Delta p}{\tau_p(n+n_1) + \tau_n(p+p_1)} \qquad \because\ n_0 p_0 = n_i^2 \\
&\approx \frac{p_0\Delta n + n_0\Delta p}{\tau_p(n_0+\Delta n)} \qquad \because\ n_1 \ll n_0,\ \ p+p_1 \ll n_0 \\
&\approx \frac{n_0\Delta p}{\tau_p n_0} = \frac{\Delta p}{\tau_p}.
\end{aligned}$$



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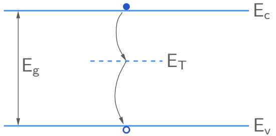

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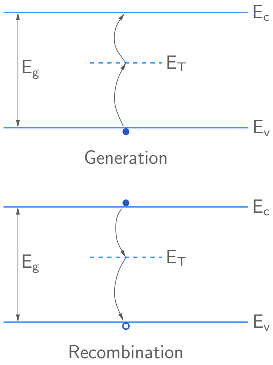

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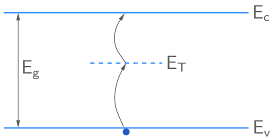

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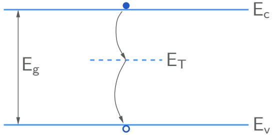

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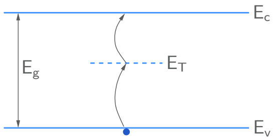
In conclusion,

* If the excess carrier densities Δn and Δp are small compared to the majority carrier density, the net recombination rate is governed by the minority carrier lifetime.



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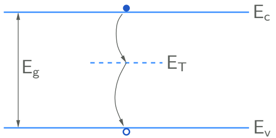

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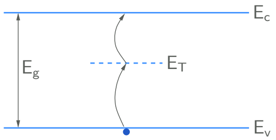
In conclusion,

- * If the excess carrier densities Δn and Δp are small compared to the majority carrier density, the net recombination rate is governed by the minority carrier lifetime.
- * We will find this result useful in our discussion of diodes and bipolar junction transistors.



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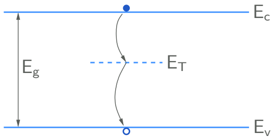

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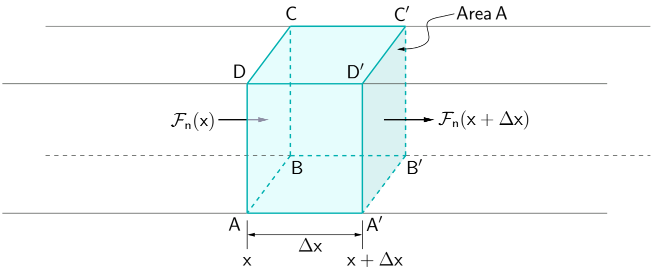
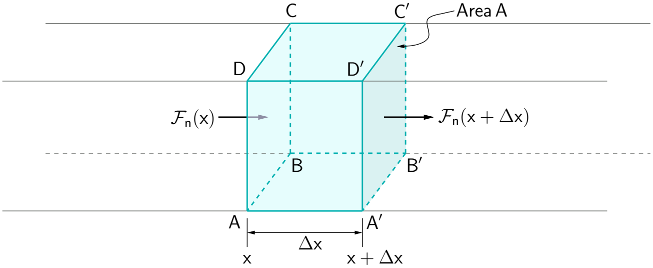


Two processes can change the number of electrons and holes in the box:



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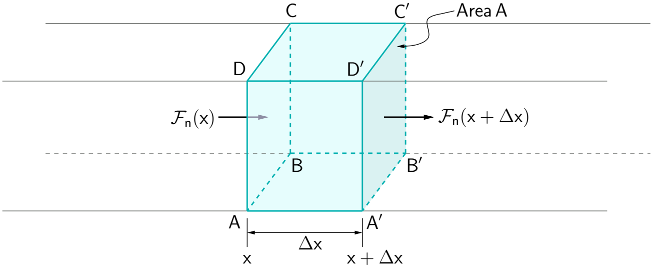

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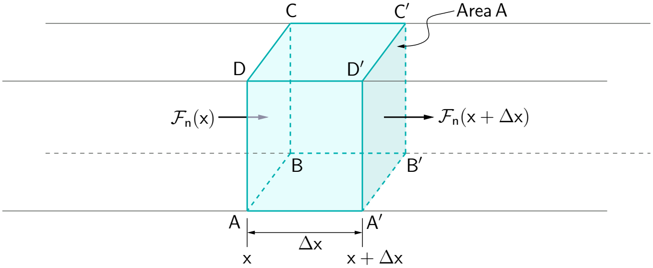
Continuity equations



Two processes can change the number of electrons and holes in the box:

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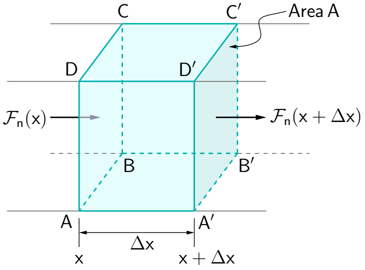
- generation and recombination of EHPs

The continuity equations serve to relate these phenomena.

Assume that there are no variations of n, p, ψ in the y and z directions.
$\rightarrow$ $\mathcal{F}_n$ and $\mathcal{F}_p$ in the y and z directions are zero.



Continuity equations

Assume that there are no variations of n, p, ψ in the y and z directions.
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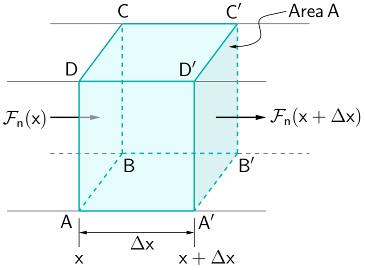
The number of electrons in the box can change because of the following factors.

* Flux $\mathcal{F}_n$ (if positive) brings electrons into the box at the rate $\mathcal{F}_n \times A$.



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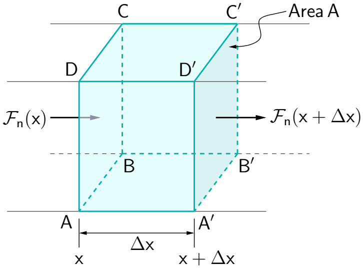

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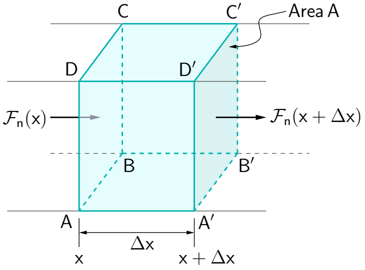

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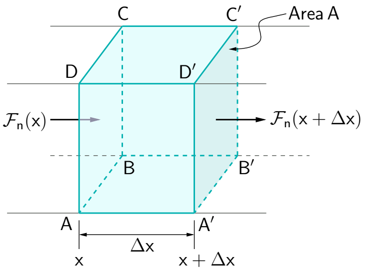
We can now relate the above factors to $\dfrac{\partial n}{\partial t}$:

$$(A\,\Delta x)\;\frac{\partial n}{\partial t} = \mathcal{F}_n(x)\,A - \mathcal{F}_n(x + \Delta x)\,A - (R - G)\,A\,\Delta x$$



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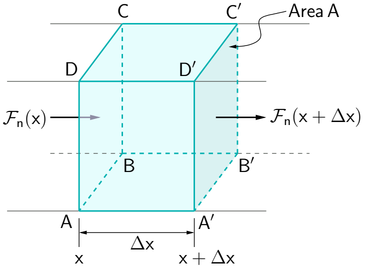

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Continuity equations

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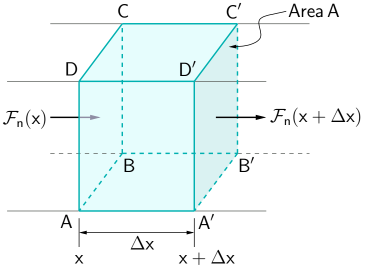

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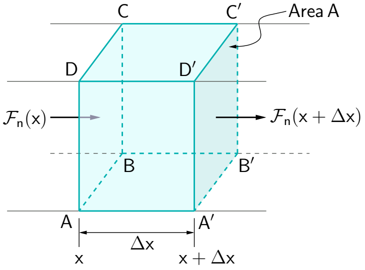

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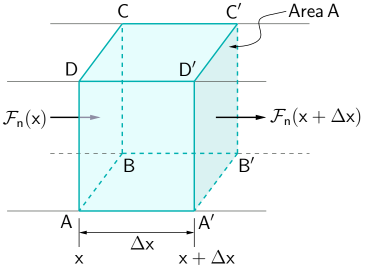

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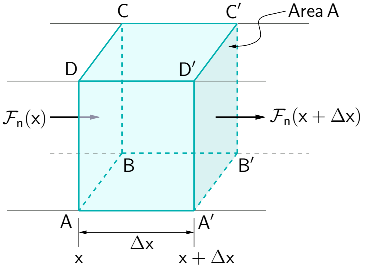

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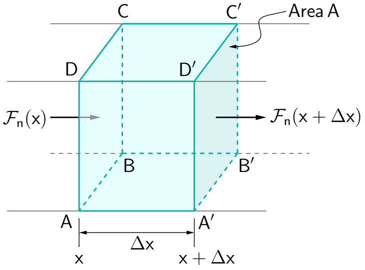

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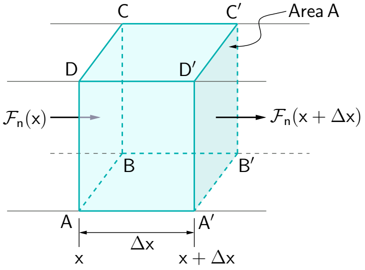

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A semiconductor device simulator solves the following coupled differential equations in a self-consistent manner:

Poisson's equation: $$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon} \quad \rightarrow \quad \frac{\partial^2 \psi}{\partial x^2} = -\frac{\rho}{\epsilon},$$

How does a semiconductor device simulator work?

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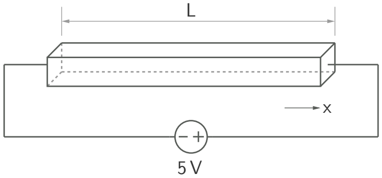
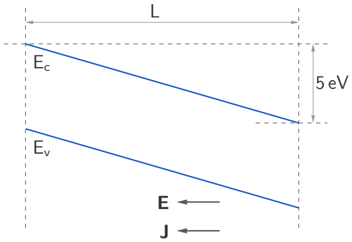
The general problem is very complex and needs to be solved numerically.

However, we can gain significant insight by considering examples which represent situations in real semiconductor devices.

Silicon bar example (revisited): consistency check





$L = 50\ \mu m$

$N_d = 5 \times 10^{17}\ cm^{-3}$

$T = 300\ K\ (n_i = 10^{10}\ cm^{-3})$

Solution:

(a) $\mathcal{E} = -\dfrac{d\psi}{dx} = -1\,\dfrac{kV}{cm}$, i.e., $\psi(x) = -\mathcal{E}x + \text{constant}$, say $\psi(x) = -\mathcal{E}x$.

(b) $n \approx N_d = 5 \times 10^{17}\ cm^{-3}$ for $0 < x < L$.

(c) $\mathcal{E} = 1\,kV/cm$, $J \approx J_n^{drift} = 3.2 \times 10^4\ A/cm^2$ ($J_p^{drift} \approx 0$, assuming p to be negligibly small).

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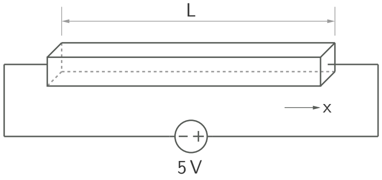

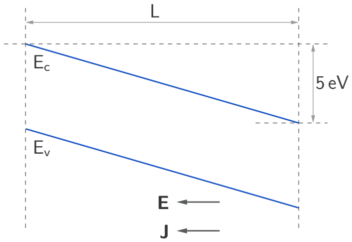

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* Poisson's equation: $\psi(x) = -\mathcal{E}x = (\text{constant})\,x \rightarrow \dfrac{d^2\psi}{dx^2} = 0$,

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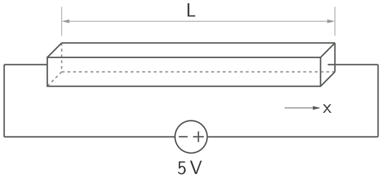

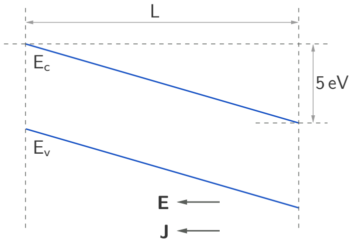

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$T = 300\,\text{K}$ ($n_i = 10^{10}\,\text{cm}^{-3}$)

Solution:

(a) $\mathcal{E} = -\dfrac{d\psi}{dx} = -1\,\dfrac{\text{kV}}{\text{cm}}$, i.e., $\psi(x) = -\mathcal{E}x + \text{constant}$, say $\psi(x) = -\mathcal{E}x$.

(b) $n \approx N_d = 5 \times 10^{17}\,\text{cm}^{-3}$ for $0 < x < L$.

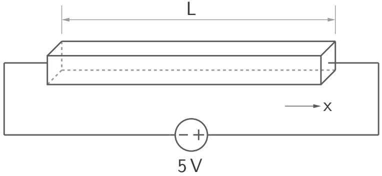
(c) $\mathcal{E} = 1\,\text{kV/cm}$, $J \approx J_n^{\text{drift}} = 3.2 \times 10^4\,\text{A/cm}^2$ ($J_p^{\text{drift}} \approx 0$, assuming p to be negligibly small).

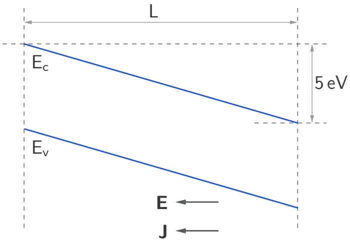
---

* Poisson's equation: $\psi(x) = -\mathcal{E}x = (\text{constant})\,x \rightarrow \dfrac{d^2\psi}{dx^2} = 0$,

  i.e., $\rho = q(N_d^+ - n + p)$ must be zero, which is satisfied by our solution.

Silicon bar example (revisited): consistency check





$L = 50\ \mu m$

$N_d = 5 \times 10^{17}\ cm^{-3}$

$T = 300\ K\ (n_i = 10^{10}\ cm^{-3})$

Solution:

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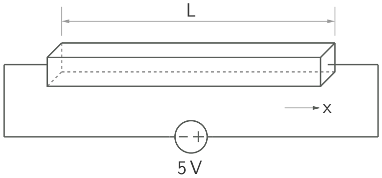

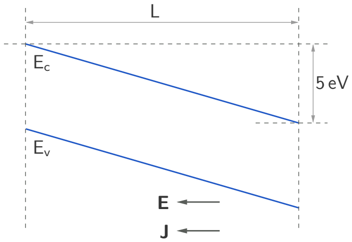

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* Continuity equation (electrons): Since this is a DC ("steady-state" situation), $\dfrac{\partial n}{\partial t} = 0 \rightarrow \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} = R - G$.

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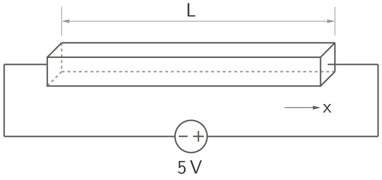

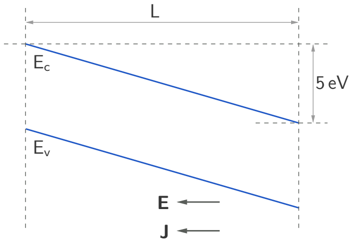

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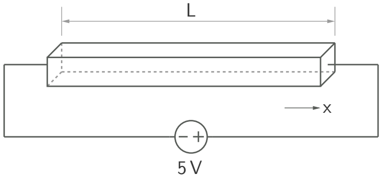

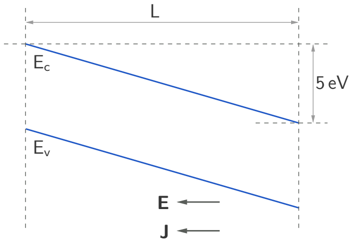

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  $\rightarrow \dfrac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} = 0 \rightarrow n\,p = n_i^2 \rightarrow p = \dfrac{n_i^2}{N_d} = p_0$ (equilibrium value).

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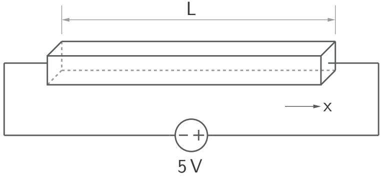

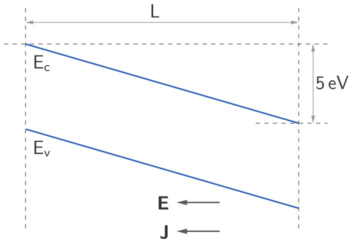

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  We see that p is indeed negligibly small.

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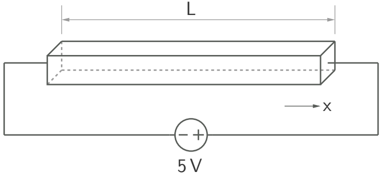

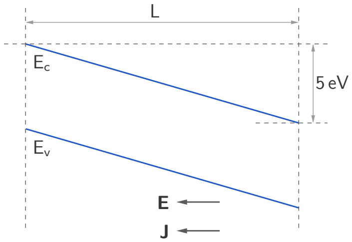

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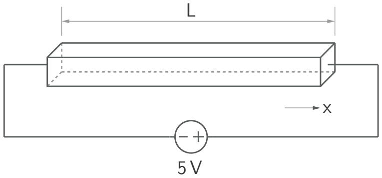

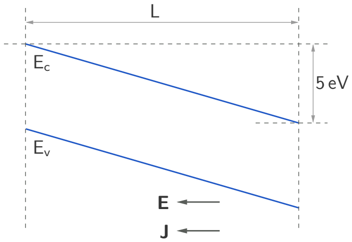

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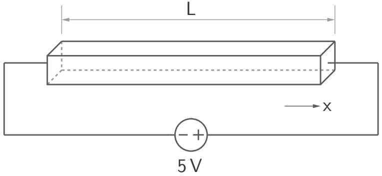
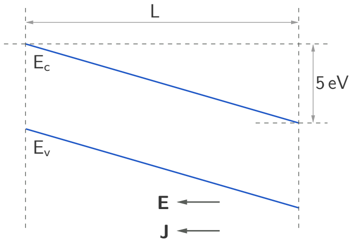
* Continuity equation (holes): $\dfrac{\partial p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - R - G.$





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  Since $R - G = 0$ and $\dfrac{\partial p}{\partial t} = 0$, $\dfrac{\partial J_p}{\partial x} = 0$, i.e., $J_p = \text{constant}$.

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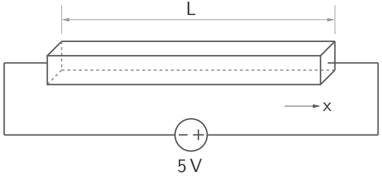

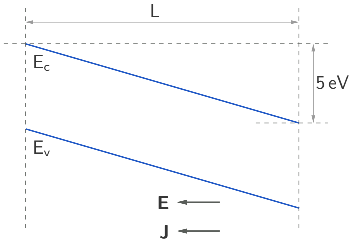

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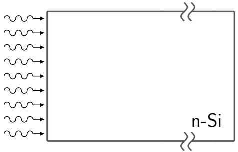
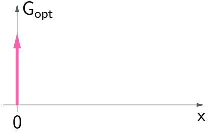
  $$\frac{J_p}{J_n} \approx \frac{J_p^{\text{drift}}}{J_n^{\text{drift}}} = \frac{\mu_p}{\mu_n}\frac{p_0}{n_0} = \frac{\mu_p}{\mu_n}\frac{n_i^2}{n_0^2} = \frac{\mu_p}{\mu_n}\left(\frac{10^{10}}{5 \times 10^{17}}\right)^2 = \frac{\mu_p}{\mu_n} \times 4 \times 10^{-16}, \text{ really small!}$$

Consider an n-type silicon sample with $N_d = 10^{17}\,\mathrm{cm}^{-3}$. Light is (continuously) incident on its surface, resulting in an optical generation rate shown in the figure.

(We are assuming here that the light is entirely absorbed in a very thin region near the semiconductor surface ($x = 0$) and does not penetrate deeper.)





Example

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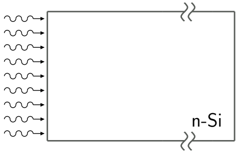
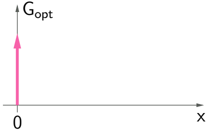
Assume that, as a result of the incident light, the excess minority carrier concentration (i.e., $p - p_0$) at $x = 0$ is maintained at $\Delta p_1 = 10^{10}\,\text{cm}^{-3}$.





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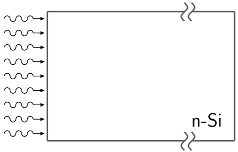
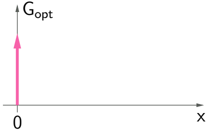
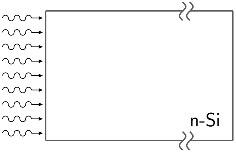
Solve the continuity equation for holes and obtain $\Delta p(x)$. ($T = 300\,\text{K}$)

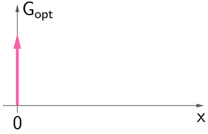


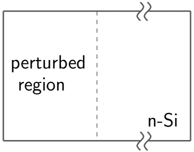


Example

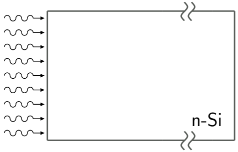
* Since only one end of the semiconductor is perturbed, we expect a region with a deviation from equilibrium conditions. We do not know at this point the extent of this region.

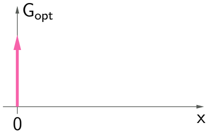


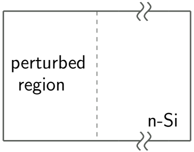




* Since only one end of the semiconductor is perturbed, we expect a region with a deviation from equilibrium conditions. We do not know at this point the extent of this region.
* We expect $p(x \rightarrow \infty) = p_0$, i.e., $\Delta p(x \rightarrow \infty) \equiv p(x \rightarrow \infty) - p_0 = 0$.







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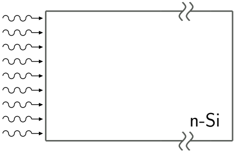

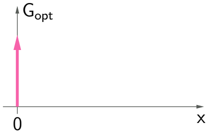

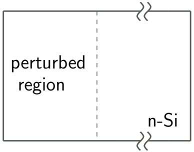

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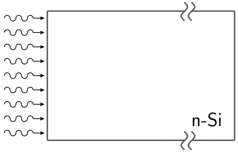
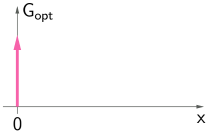
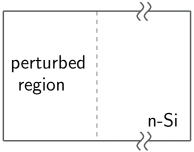
* At the surface $(x=0)$, EHPs are continuously generated; therefore, we expect some excess hole concentration there, i.e., $p(0) = p_0 + \Delta p_1$.
* We assume steady-state situation in which all quantities have settled to their steady-state forms, not varying with time.







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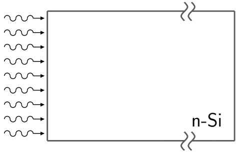
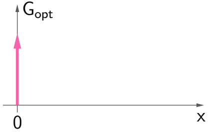
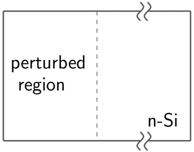
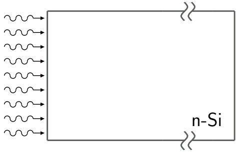
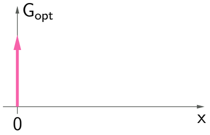


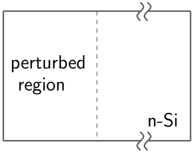




* Continuity equation for holes: $\dfrac{\partial p}{\partial t} = -\dfrac{\partial \mathcal{F}_p}{\partial x} - (R - G) = 0.$
* In steady state, the number of holes generated at $x = 0$ per second is equal to the total number of holes lost per second due to recombination.







Example

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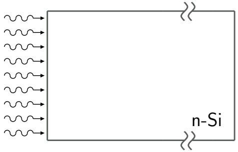
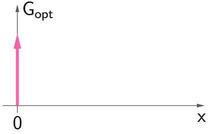
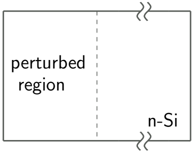
* Because of diffusion and recombination, the excess hole concentration decreases from Δp_1 at $x = 0$ to 0 at $x = \infty$.







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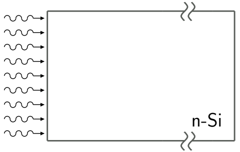
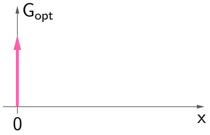
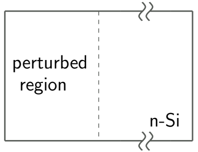
* Since $\Delta p_1 (= 10^{10}\,\text{cm}^{-3}) \ll n_0 (= 10^{17}\,\text{cm}^{-3})$, we can approximate $(R - G)$ with $\Delta p/\tau_p$.







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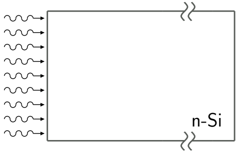
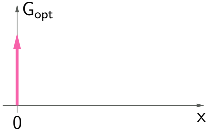
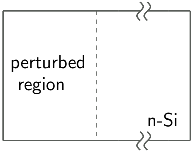
* Let us assume that $J_p^{\text{drift}} \ll J_p^{\text{diff}}$ (to be verified later)

$$\rightarrow \mathcal{F}_p \approx \mathcal{F}_p^{\text{diff}} = -D_p \frac{\partial p}{\partial x} = -D_p \frac{\partial (p_0 + \Delta p)}{\partial x} = -D_p \frac{\partial \Delta p}{\partial x}.$$







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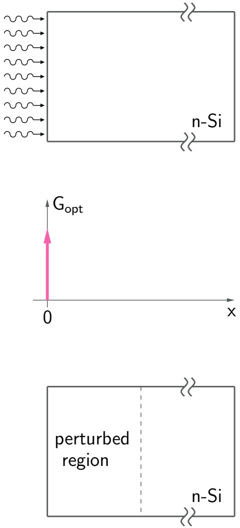

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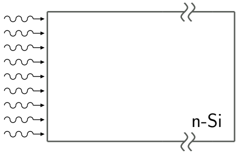

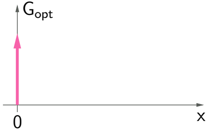

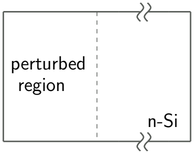

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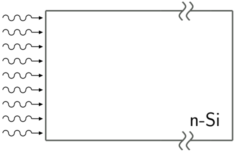
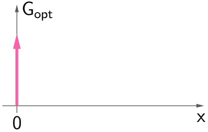
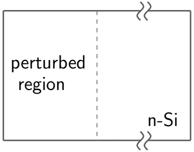
The quantity $\sqrt{D_p\tau_p}$ has units of $\sqrt{\frac{\text{cm}^2}{\text{s}} \times \text{s}} = \text{cm}$ and is called the "hole diffusion length" L_p — also, in this case, the "minority carrier diffusion length" since holes are the minority carriers.







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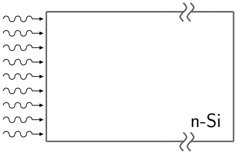

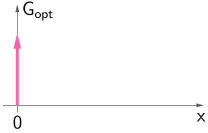

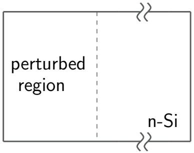

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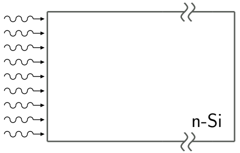

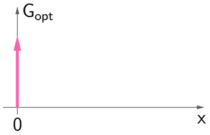

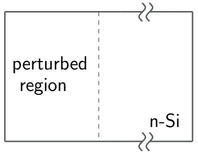

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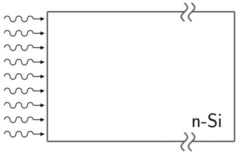
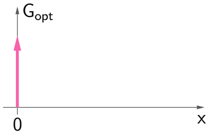
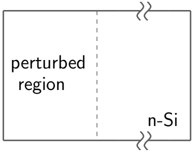
Note:

* Our solution is valid provided J_p^{drift} is small compared to J_p^{diff}.







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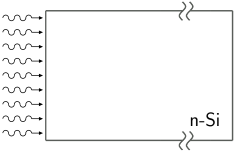
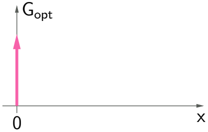
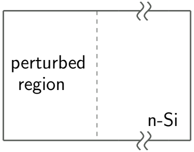
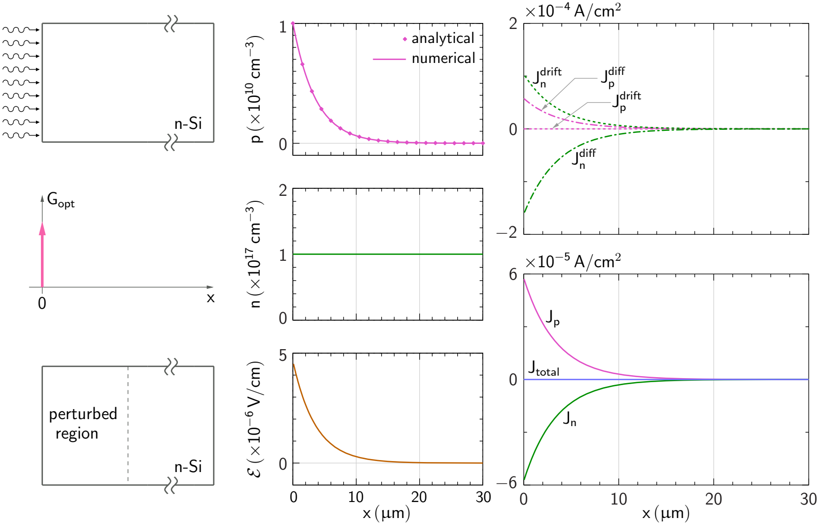
Note:

* Our solution is valid provided J_p^{drift} is small compared to J_p^{diff}.
* We could not have solved the continuity equations for electrons as easily since J_n^{drift} cannot be ignored; even a small electric field causes a significant J_n^{drift} because n is large.









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$\mu_n = 1400\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\tau_n = 10\ \mathrm{ns}$

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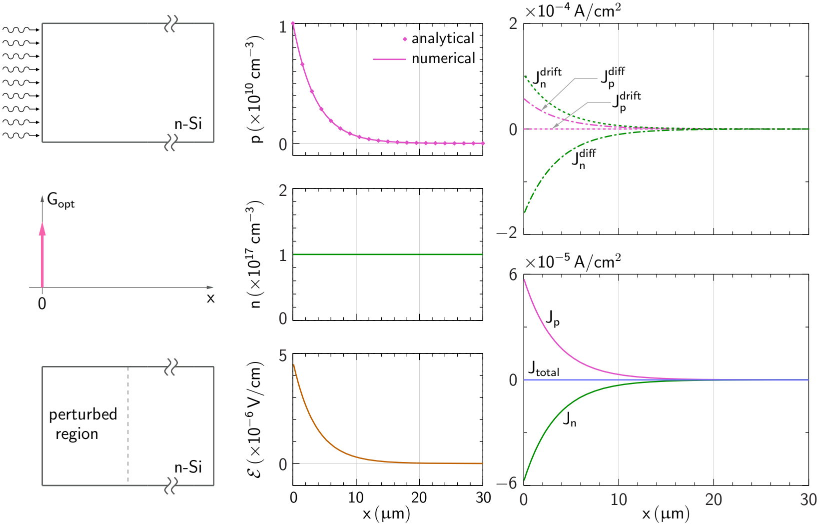
$E_T = E_i$

$T = 300\ \mathrm{K}$



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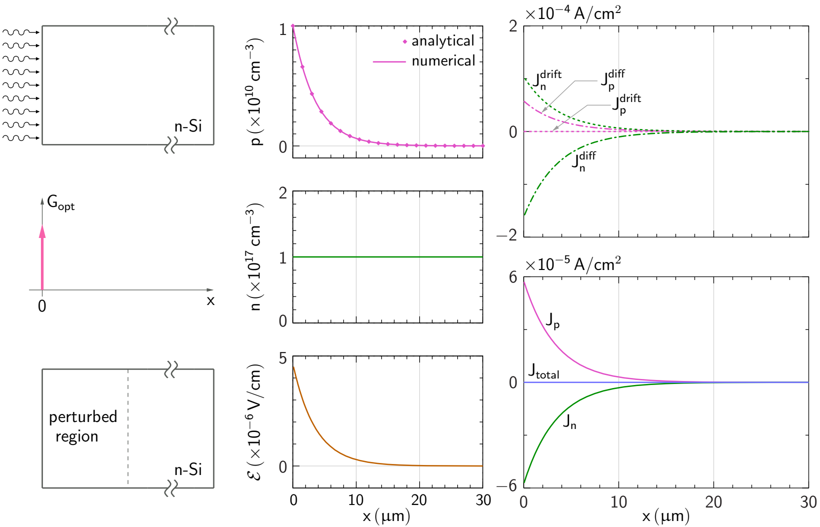
* The analytical solution $\Delta p = \Delta p_1 e^{-x/L_p}$ matches very well with the numerical solution.



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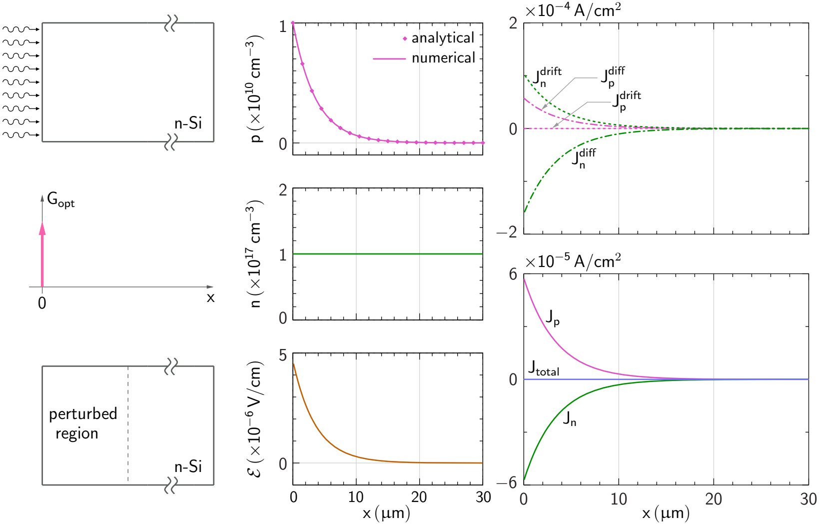
* J_p^{drift} is negligibly small, as we had assumed.



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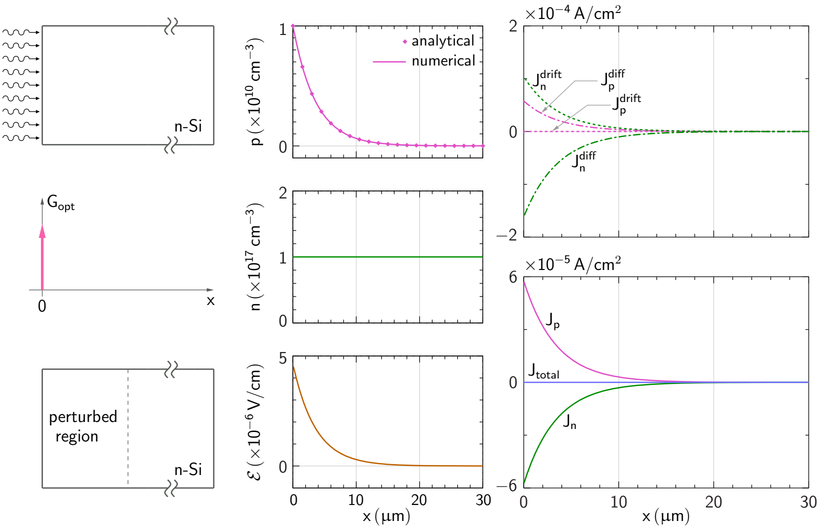

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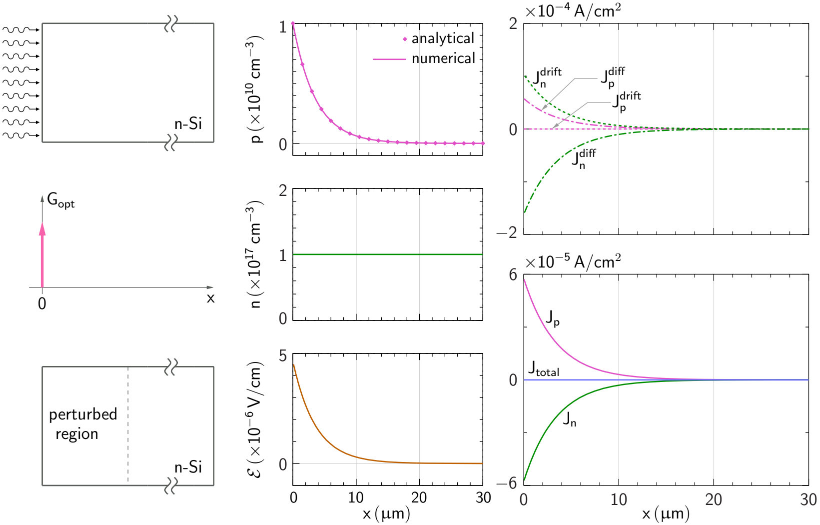
$$* \quad L_p = \sqrt{D_p \tau_p} = \sqrt{\mu_p\, V_T\, \tau_p} = \sqrt{500\ \frac{\text{cm}^2}{\text{V-s}} \times 0.0258\ \text{V} \times 10 \times 10^{-9}\ \text{s}} = 3.6 \times 10^{-4}\ \text{cm} = 3.6\ \mu\text{m}.$$



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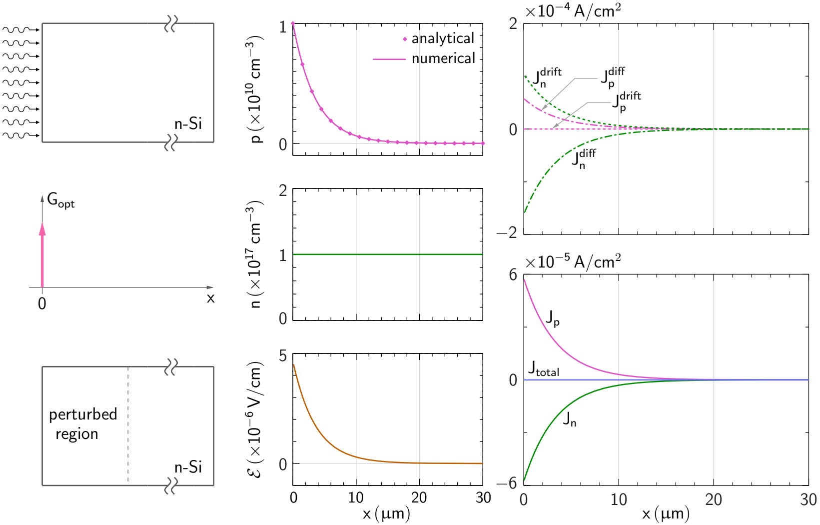
We expect the length of the perturbed region to be about $5\,L_p$ or $18\,\mu$m.



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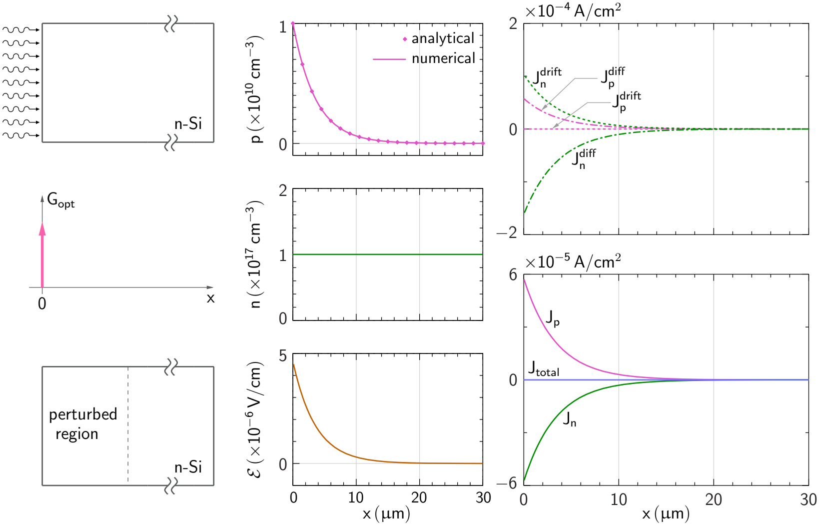

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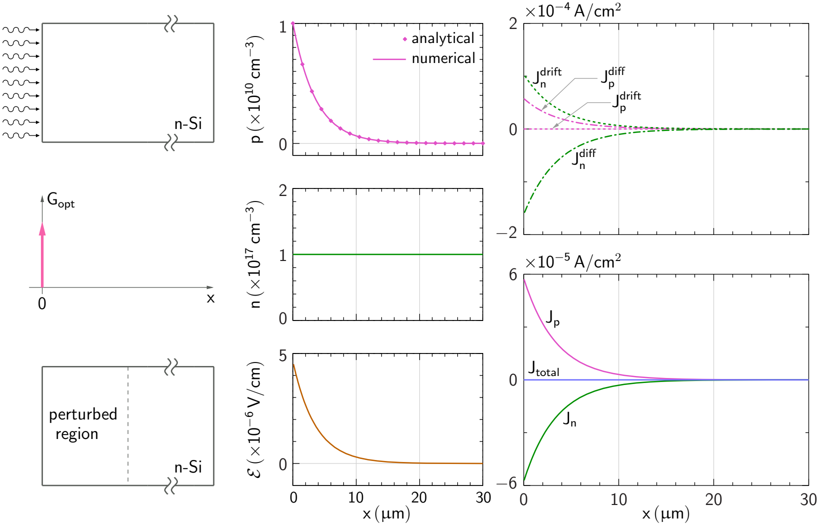
$T = 300\ \mathrm{K}$

* The condition $\Delta p \ll n$ (in general, the excess minority carrier concentration being much smaller than the majority carrier concentration) is called "low-level injection," i.e., injection of a small number of minority carriers in a sea of majority carriers.



$N_d = 10^{17}\ \mathrm{cm}^{-3}$

$\mu_n = 1400\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

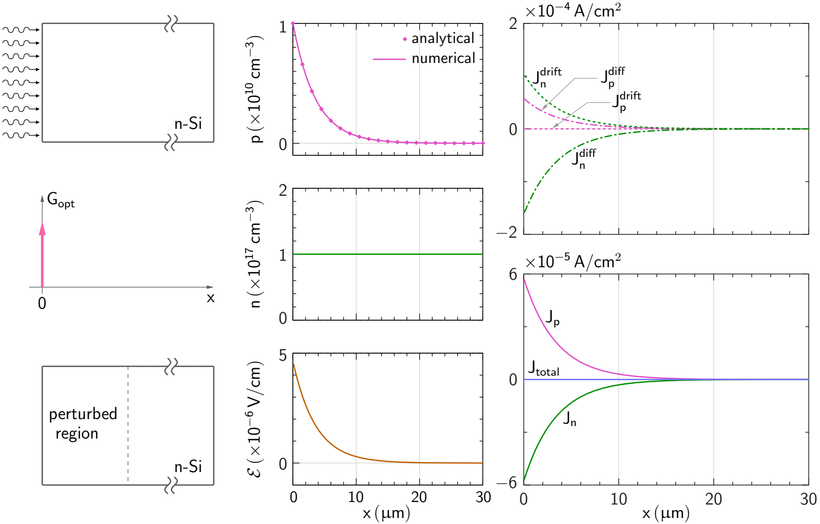
$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\tau_n = 10\ \mathrm{ns}$

$\tau_p = 10\ \mathrm{ns}$

$E_T = E_i$

$T = 300\ \mathrm{K}$



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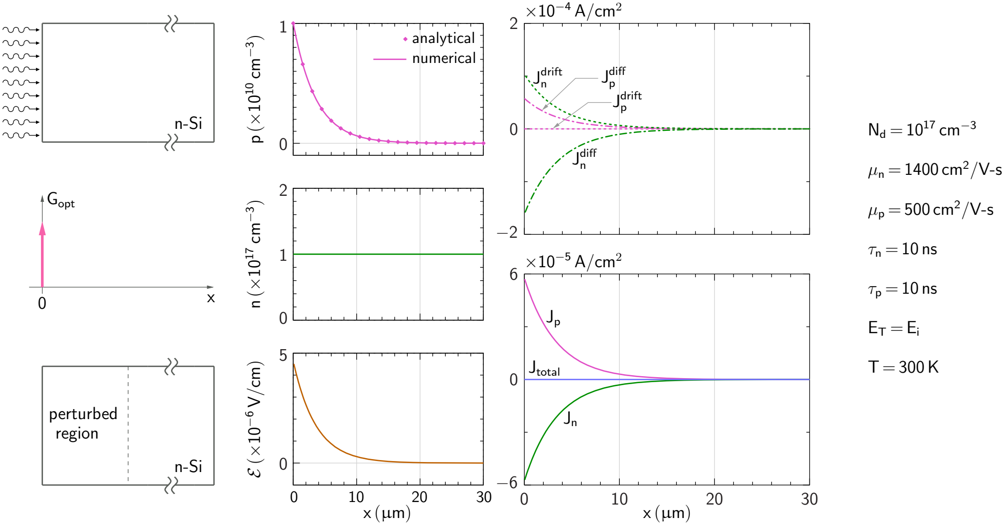
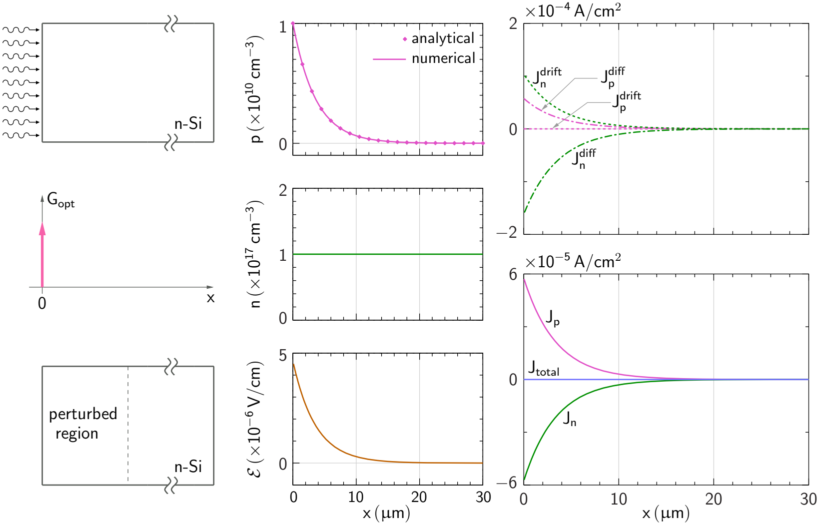
* The electron density is essentially constant ($= n_0$), but there is a small change in n with respect to x which gives rise to a non-zero J_n^{diff}. $\Delta n(x) \approx \Delta p(x)$ (not shown).



* The electron density is essentially constant ($= n_0$), but there is a small change in n with respect to x which gives rise to a non-zero J_n^{diff}. $\Delta n(x) \approx \Delta p(x)$ (not shown).
* The electric field is very small, but it is sufficient to cause a significant J_n^{drift} because n is large.



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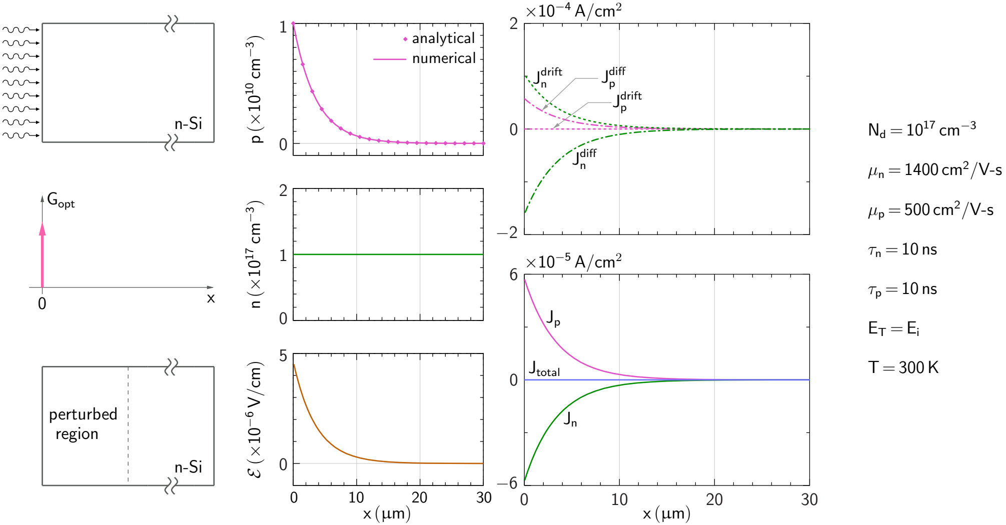
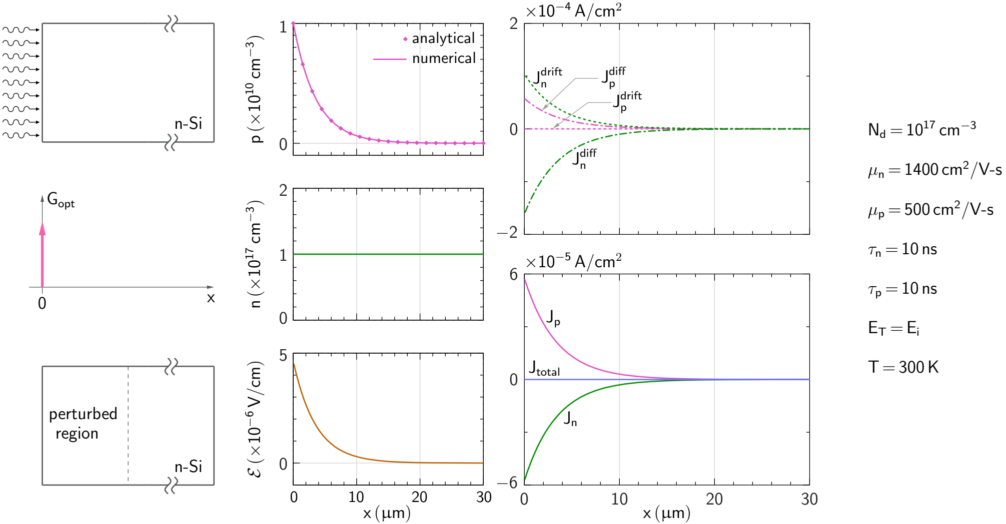


* The total current $J = J_n + J_p$ is zero throughout since we have an open-circuit condition.



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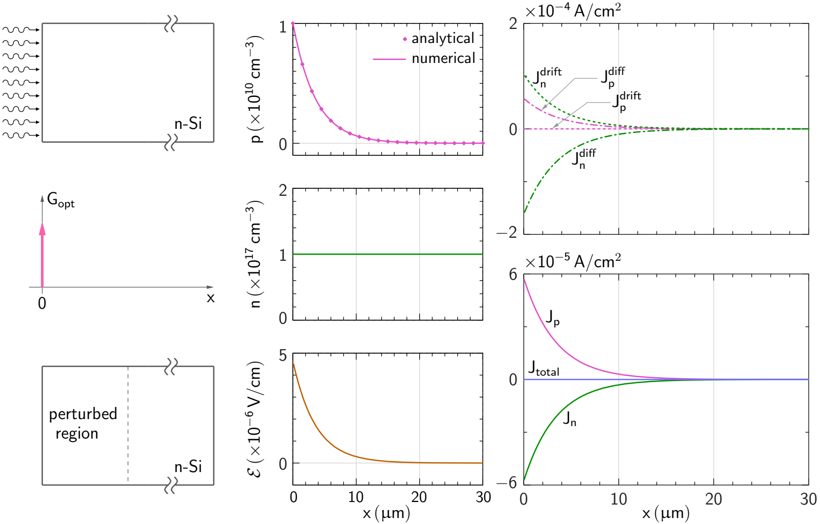

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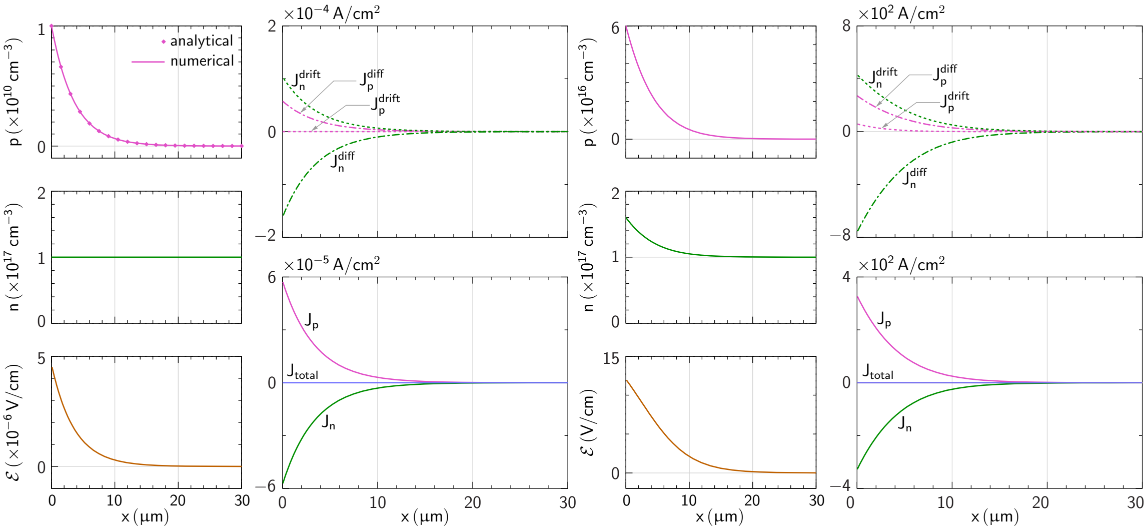
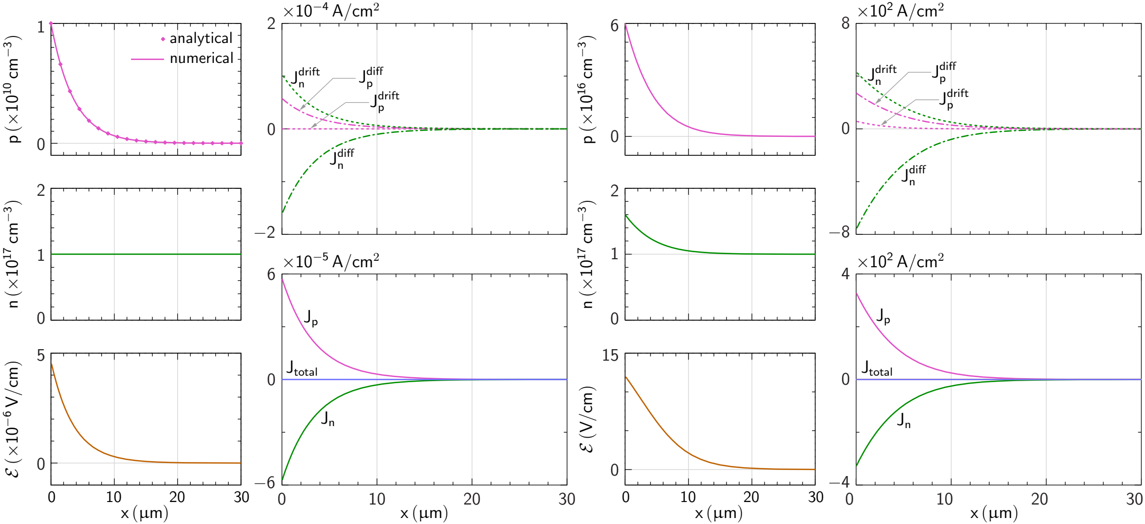
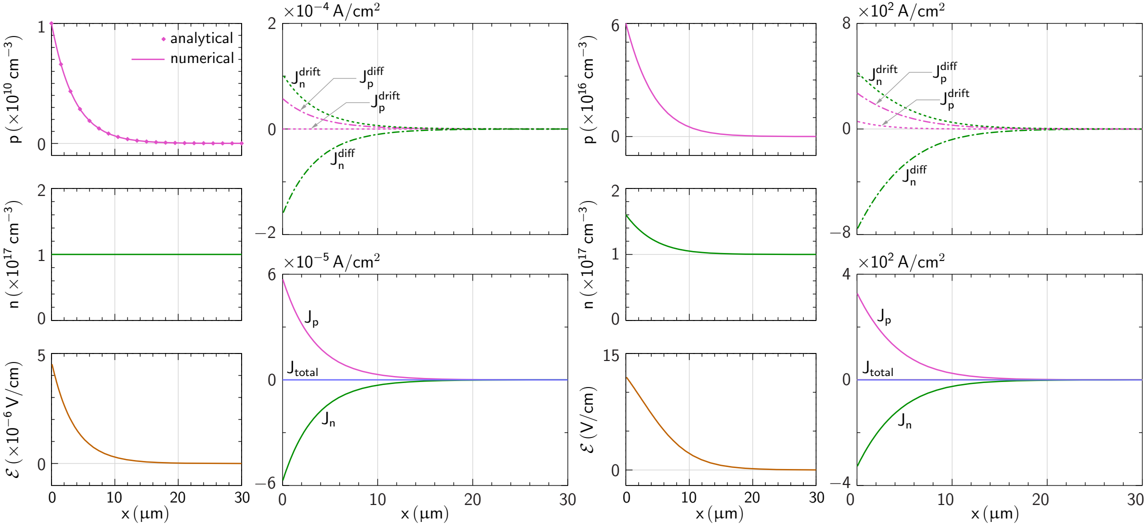
* The total current $J = J_n + J_p$ is zero throughout since we have an open-circuit condition.
* The voltage drop ($\int \mathcal{E} dx$ in magnitude) is very small.
* To summarise, $\Delta p \ll n_0$, $J_p \approx J_p^{\text{diff}}$, $\Delta n(x) \approx \Delta p(x)$ $\rightarrow$ charge neutrality $\rightarrow$ small $\mathcal{E}$, and $J^{\text{total}} = 0$.



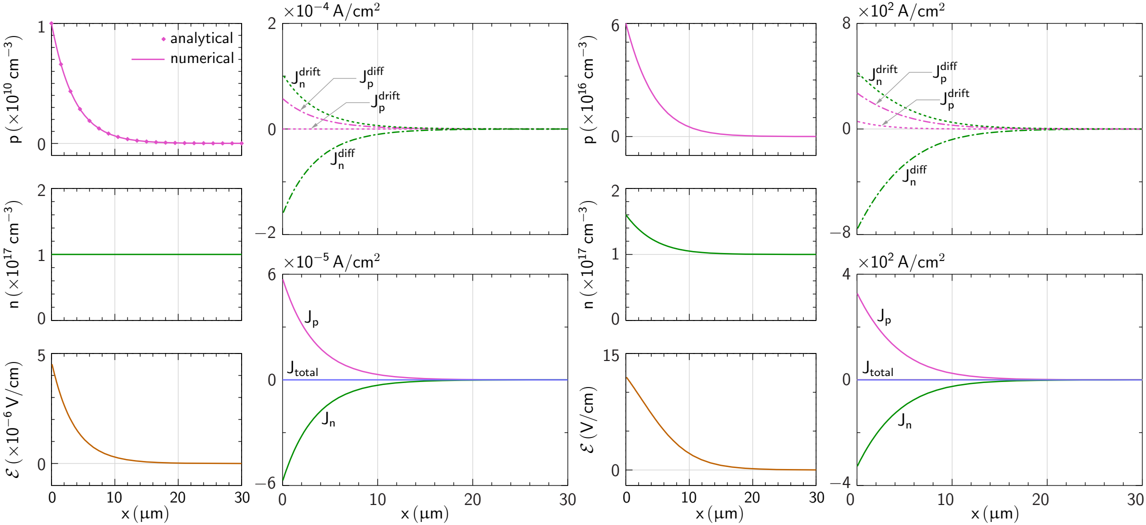
p (×10^10 cm^−3)
analytical
numerical
n (×10^17 cm^−3)
ℰ (×10^−6 V/cm)
x (μm)
×10^−4 A/cm^2
J_n^drift
J_p^diff
J_p^drift
J_n^diff
×10^−5 A/cm^2
J_p
J_total
J_n
p (×10^16 cm^−3)
n (×10^17 cm^−3)
ℰ (V/cm)
×10^2 A/cm^2



* Suppose G_{opt} is increased such that $\Delta p_1 = 6 \times 10^{16}\ \mathrm{cm}^{-3}$. $\Delta p(0)$ is now comparable to the majority carrier density, and we have a high-level injection situation.



* Suppose G_{opt} is increased such that $\Delta p_1 = 6 \times 10^{16}\,\text{cm}^{-3}$. $\Delta p(0)$ is now comparable to the majority carrier density, and we have a high-level injection situation.
* J_p^{drift} is comparable to J_p^{diff} with high-level injection $\rightarrow$ We would not be able to solve the continuity equation for holes analytically.



analytical
numerical
$p\,(\times 10^{10}\,\mathrm{cm}^{-3})$
$n\,(\times 10^{17}\,\mathrm{cm}^{-3})$
$\mathcal{E}\,(\times 10^{-6}\,\mathrm{V/cm})$
$\times 10^{-4}\,\mathrm{A/cm^2}$
J_n^{drift}
J_p^{diff}
J_p^{drift}
J_n^{diff}
$\times 10^{-5}\,\mathrm{A/cm^2}$
J_p
J_{total}
J_n
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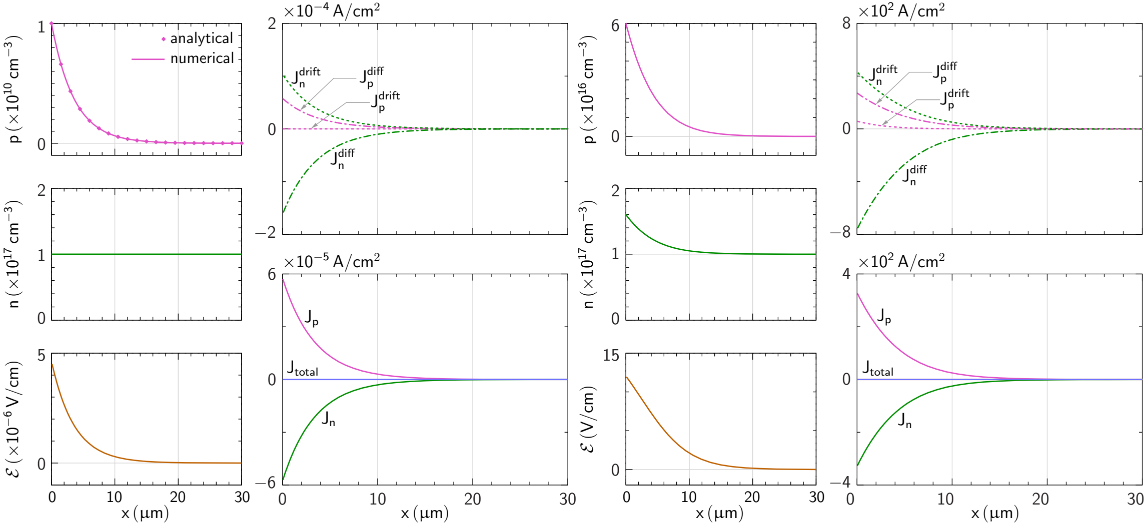
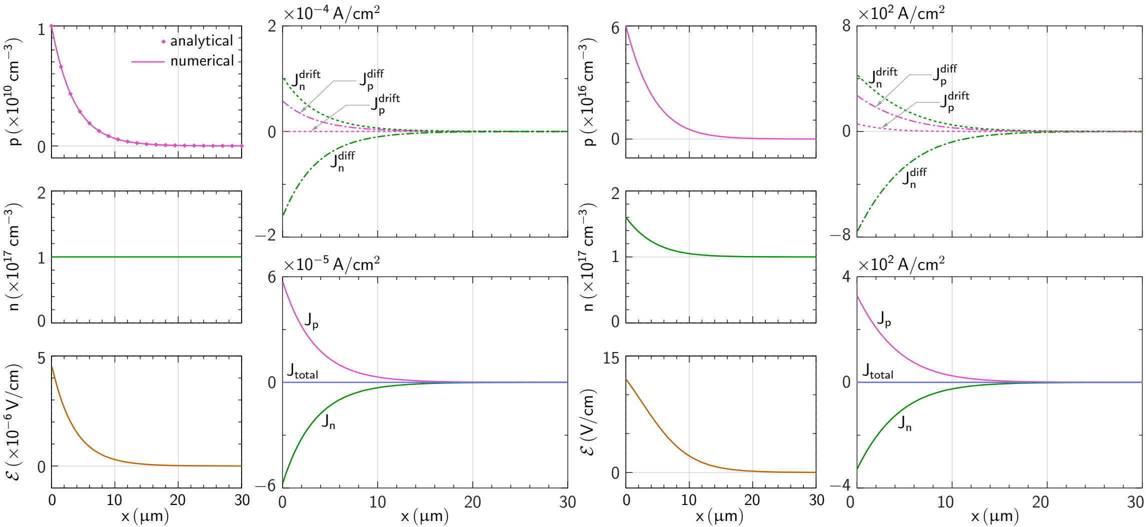
$n\,(\times 10^{17}\,\mathrm{cm}^{-3})$
$\mathcal{E}\,(\mathrm{V/cm})$
$\times 10^{2}\,\mathrm{A/cm^2}$
$x\,(\mu\mathrm{m})$



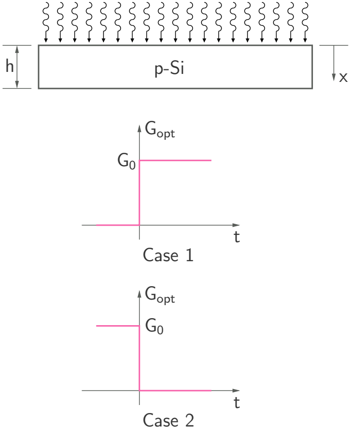
* Charge neutrality is still maintained almost perfectly because
  $\Delta n \approx \Delta p \rightarrow \Delta n + n_0 \approx \Delta p + N_d^+ \rightarrow n \approx p + N_d^+$ (since p_0 is small.)



* Charge neutrality is still maintained almost perfectly because $\Delta n \approx \Delta p \rightarrow \Delta n + n_0 \approx \Delta p + N_d^+ \rightarrow n \approx p + N_d^+$ (since p_0 is small.)
* $J^{\mathrm{total}} = J_n + J_p$ remains equal to zero even with high-level injection.

Example

Consider a p-type silicon sample with $N_a = 5 \times 10^{17}\,\mathrm{cm}^{-3}$ at $T = 300\,\mathrm{K}$. When it is illuminated uniformly with light of a certain wavelength, there is uniform generation throughout the sample at the rate of $G_0\,/\mathrm{cm}^3$-s, where G_0 depends on the intensity of the incident light.



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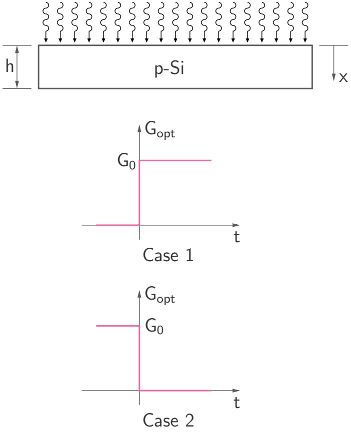
For the excitations shown as Case 1 and Case 2, how does the excess electron concentration $\Delta n = n - n_0$ vary with time?



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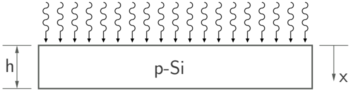
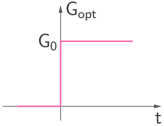
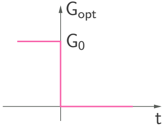
Note: The condition of uniform generation holds if the thickness of the sample h is much smaller than $1/\alpha$, where α is the absorption coefficient of silicon at the wavelength of the incident light.





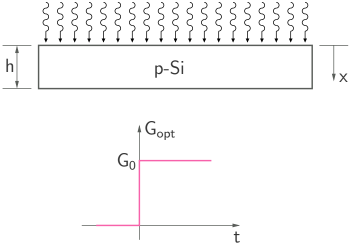
Case 1



Case 2

Continuity equation for the minority carriers (electrons) for $t > 0$ is,

$$\frac{\partial n}{\partial t} = -\frac{\partial \mathcal{F}_n}{\partial x} - (R - G)_{\text{SRH}} + G_{\text{opt}}$$



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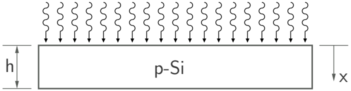
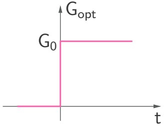
$$= -(R - G)_{\mathrm{SRH}} + G_0 \quad \because \text{there is no variation in space}$$





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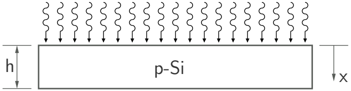

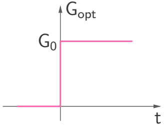

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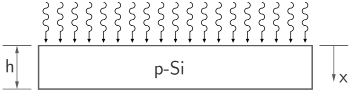
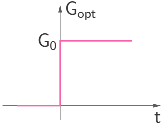
$$\therefore \quad \frac{\partial (n_0 + \Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G_0.$$





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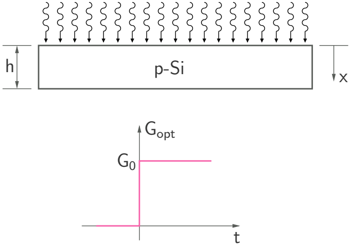

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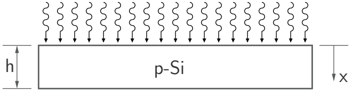

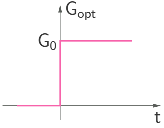

Homogeneous solution: $\Delta n^{(h)} = A\, e^{-t/\tau_n}$.

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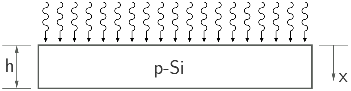

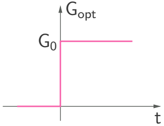

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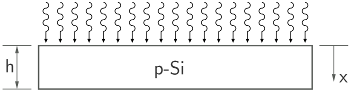

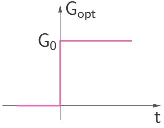

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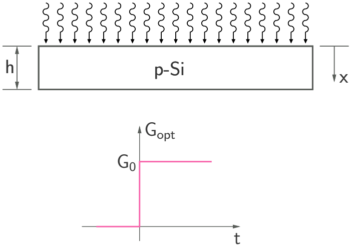
$\rightarrow \ \Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

Example: Case 1

$\Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

At $t = 0^-$, $G_{\text{opt}} = 0 \to \Delta n = 0.$



Example: Case 1

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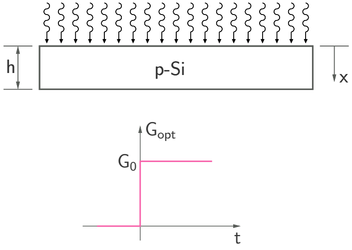
$\Delta n(0^+) = \Delta n(0^-)$ since Δn cannot change suddenly (that would require an infinite electron flux or an infinite G-R rate).



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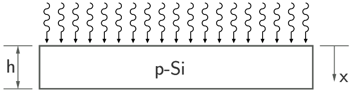

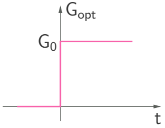

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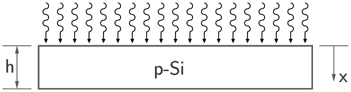

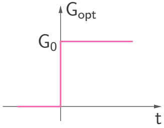

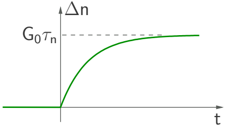

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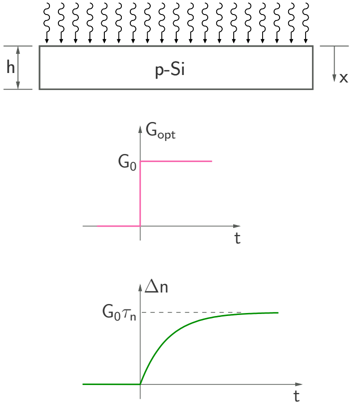
In steady state, as $t \rightarrow \infty$, the rate of optical generation must be equal to the rate of net thermal recombination.



Example: Case 1

$\Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

At $t = 0^-$, $G_{\mathrm{opt}} = 0 \rightarrow \Delta n = 0.$

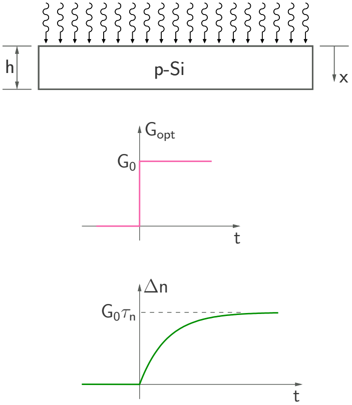
$\Delta n(0^+) = \Delta n(0^-)$ since Δn cannot change suddenly (that would require an infinite electron flux or an infinite G-R rate).

Using $\Delta n(0^+) = 0$, we get $A = -G_0 \tau_n$, and

$\Delta n(t) = G_0 \tau_n \left(1 - e^{-t/\tau_n}\right)$

In steady state, as $t \rightarrow \infty$, the rate of optical generation must be equal to the rate of net thermal recombination.

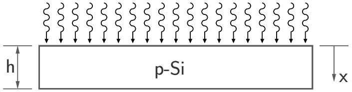
$\rightarrow G_0 = \dfrac{\Delta n}{\tau_n} \rightarrow \Delta n = G_0 \tau_n$, as predicted by the above equation.

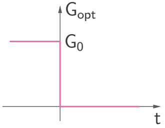


Example: Case 2

Continuity equation for the minority carriers (electrons) for $t > 0$ is,

$$\frac{\partial n}{\partial t} = -\frac{\partial \mathcal{F}_n}{\partial x} - (R - G)_{\mathrm{SRH}} \quad \text{(Note: no } G_{\mathrm{opt}} \text{ term here)}$$





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&= -(R-G)_{\mathrm{SRH}} \quad \because \text{ there is no variation in space} \\
&= -\frac{\Delta n}{\tau_n}, \quad \text{assuming low-level injection}
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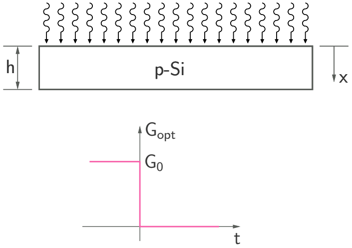

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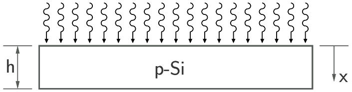

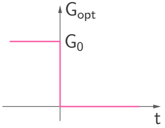

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Homogeneous solution: $\Delta n^{(h)} = A'\, e^{-t/\tau_n}$.

Particular solution: $\Delta n^{(p)} = 0$.

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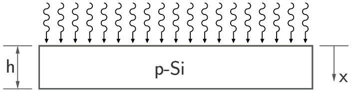

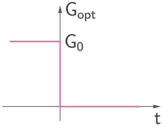

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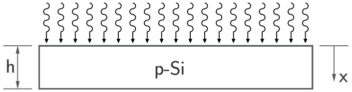

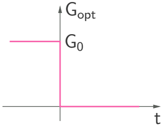

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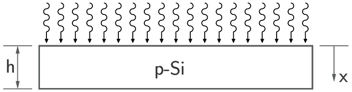

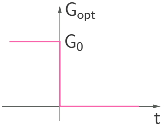

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