

SEMICONDUCTOR DEVICES

p - n Junctions: Part 3



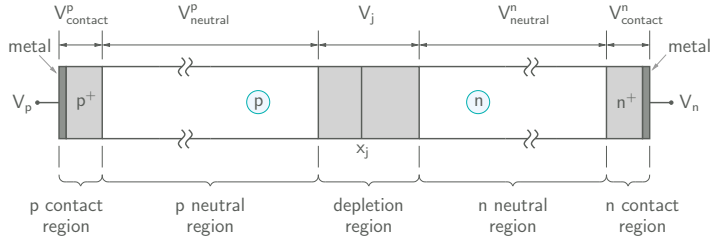
M. B. Patil

mbpatil@ee.iitb.ac.in

www.ee.iitb.ac.in/~sequel

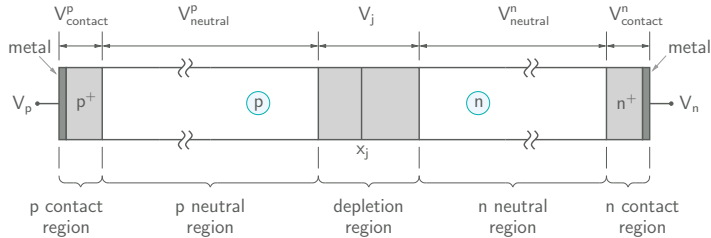
Department of Electrical Engineering
Indian Institute of Technology Bombay

pn junction: derivation of I - V equation



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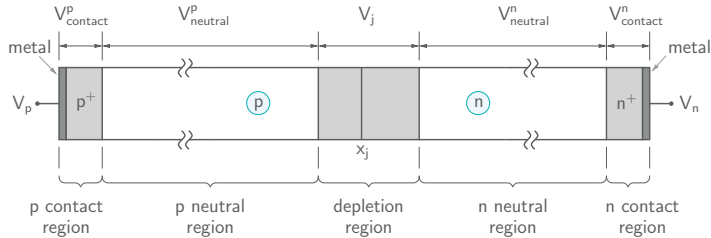
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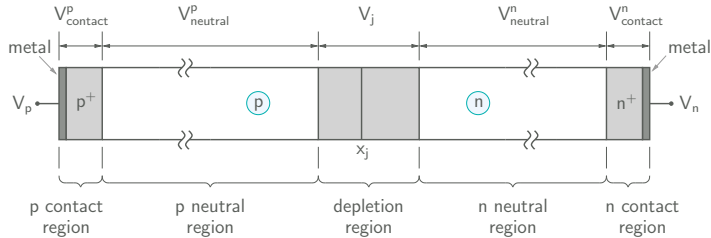


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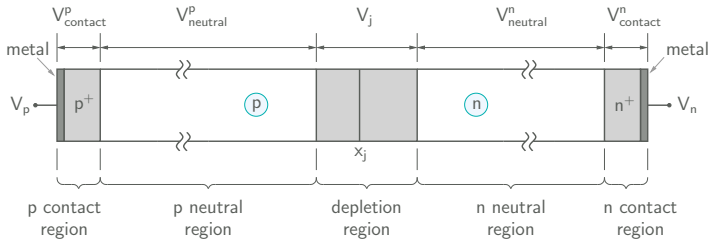


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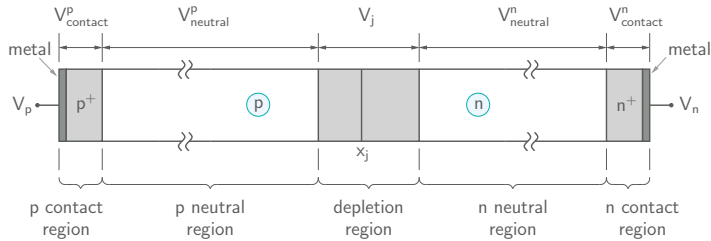
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$\rightarrow D_p \frac{d^2 p}{dx^2} - \frac{p - p_{n0}}{\tau_p} = 0$ or $\frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = 0$, where $L_p = \sqrt{D_p \tau_p}$ is the hole diffusion length.

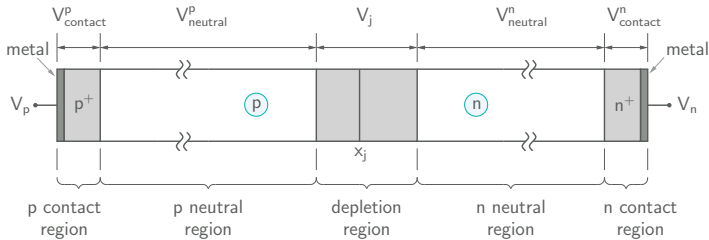


Example: For an abrupt, uniformly doped silicon pn junction at $T = 300$ K, with $N_a = 10^{16} \text{ cm}^{-3}$, $N_d = 10^{17} \text{ cm}^{-3}$, $\mu_p = 500 \text{ cm}^2/\text{V-s}$, calculate the diffusion length L_p for $\tau_p = 1 \text{ ns}$, 10 ns , 100 ns , $1 \mu\text{s}$, and $10 \mu\text{s}$, and compare it with the zero-bias value of W , the depletion width.



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Solution:
$$V_{\text{bi}} = V_T \log \left(\frac{N_a N_d}{n_i^2} \right) = (0.0259 \text{ V}) \log \left(\frac{10^{16} \times 10^{17}}{(1.5 \times 10^{10})^2} \right) = 0.75 \text{ V}.$$

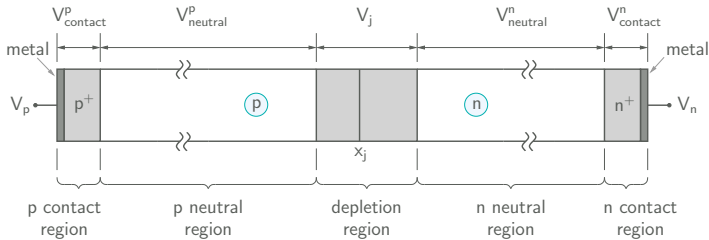


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The depletion width is $W = \sqrt{\frac{2\epsilon}{q} \frac{N_a + N_d}{N_a N_d} (V_{bi} - V_a)}$

$$= \sqrt{\frac{2 \times 11.8 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19}} \frac{1.1 \times 10^{17}}{10^{16} \times 10^{17}} \times 0.75} \text{ cm} = 0.33 \mu\text{m}.$$



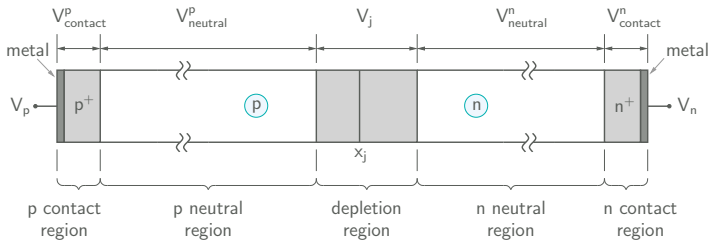
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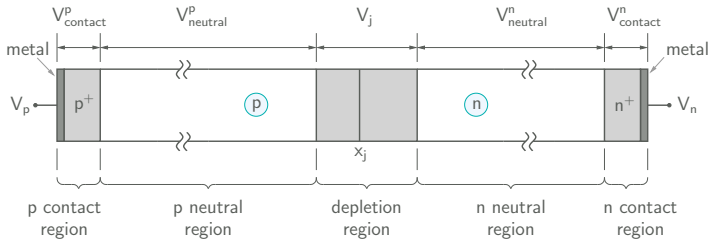
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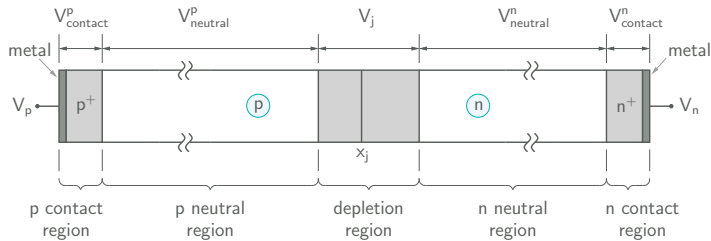
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For $\tau_p = 1 \text{ ns}$, $L_p = \sqrt{12.9 \frac{\text{cm}^2}{\text{s}} \times (1 \times 10^{-9} \text{ s})} = 1.14 \times 10^{-4} \text{ cm} = 1.14 \mu\text{m}.$

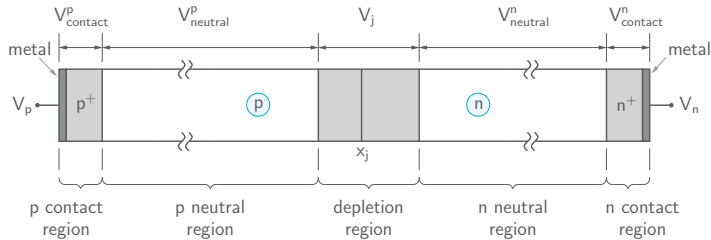


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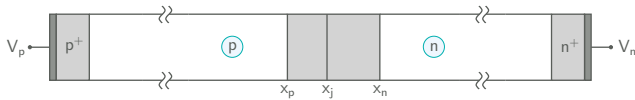
τ_p	$L_p (\mu\text{m})$
1 ns	1.14
10 ns	3.6
100 ns	11.4
1 μs	36.0
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Note that $L_p \gg W|_{0V}$ ($0.33 \mu\text{m}$), a typical situation.



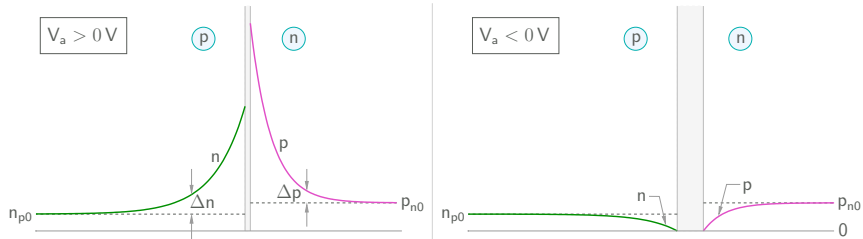
Hole continuity equation ($x > x_n$): $\frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = 0$.

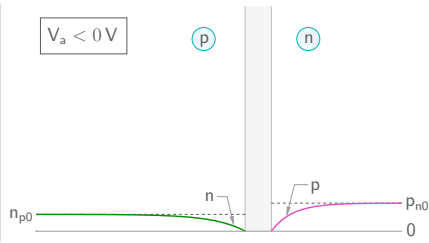
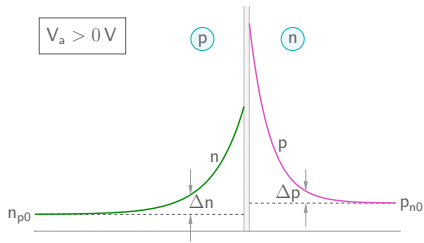
Boundary conditions: $\Delta p(x_n) = p_{n0} \exp\left(\frac{V_a}{V_T}\right) - p_{n0} = p_{n0} \left[\exp\left(\frac{V_a}{V_T}\right) - 1 \right]$

$$\Delta p(x \rightarrow \infty) = p(x \rightarrow \infty) - p_{n0} = 0$$

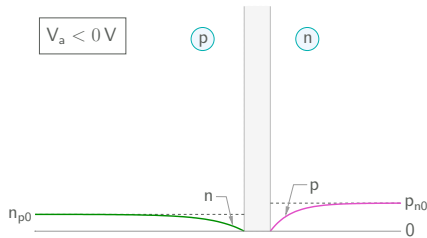
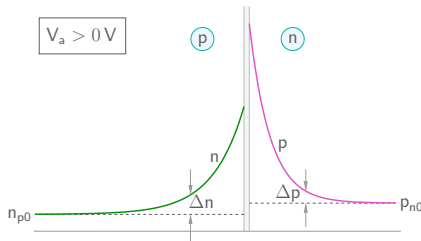
$$\rightarrow \Delta p(x) = p_{n0} \left[\exp\left(\frac{V_a}{V_T}\right) - 1 \right] \exp\left(-\frac{x - x_n}{L_p}\right), \quad x > x_n,$$

$$\Delta n(x) = n_{p0} \left[\exp\left(\frac{V_a}{V_T}\right) - 1 \right] \exp\left(-\frac{x_p - x}{L_n}\right), \quad x < x_p.$$



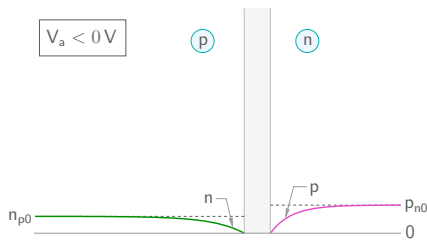
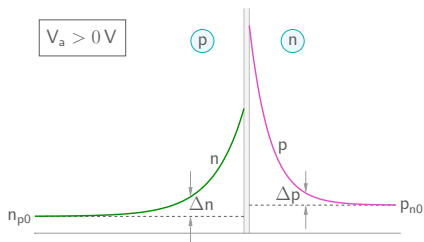


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- * When $x - x_n = 5L_p$, the exponential factor in $\Delta p(x)$ is $e^{-5} = 0.0067 \rightarrow$ In about five minority carrier diffusion lengths, the disturbance caused by the applied bias vanishes.

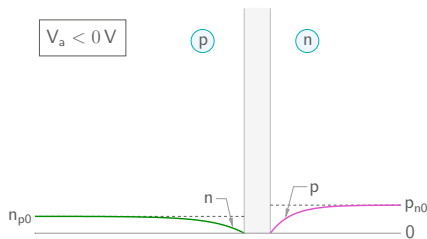
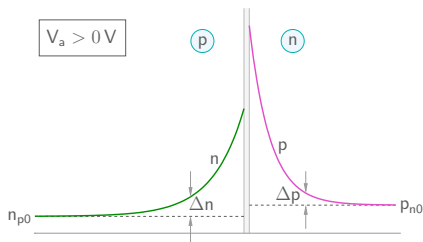


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- * When $x - x_n = 5L_p$, the exponential factor in $\Delta p(x)$ is $e^{-5} = 0.0067 \rightarrow$ In about five minority carrier diffusion lengths, the disturbance caused by the applied bias vanishes.
- * Consider the minority carrier concentrations at the depletion region edges.

$$\Delta p = p_{n0} \left[\exp \left(\frac{V_a}{V_T} \right) - 1 \right] \text{ at } x = x_n,$$

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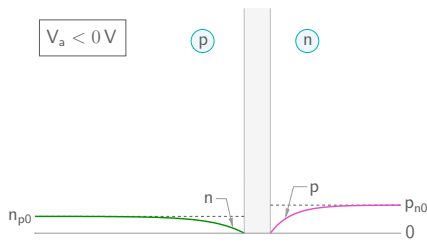
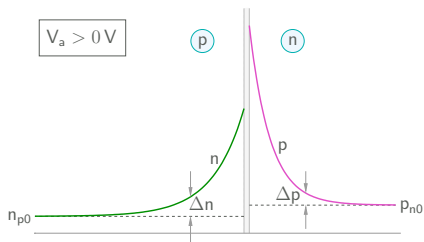
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For forward bias, $\Delta p(x_n)$ and $\Delta n(x_p)$ are positive.



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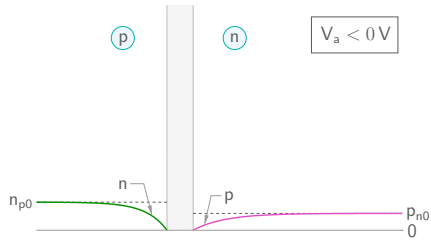
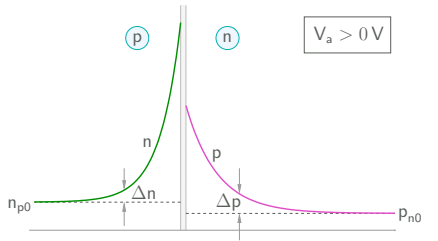
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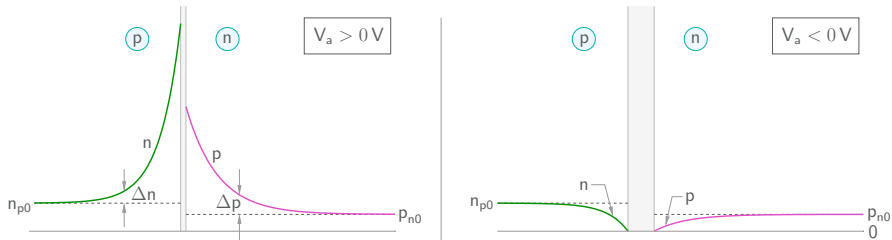
For forward bias, $\Delta p(x_n)$ and $\Delta n(x_p)$ are positive.

For reverse bias, $\Delta p(x_n)$ and $\Delta n(x_p)$ are negative.



Consider an abrupt, uniformly doped silicon pn junction at $T = 300\text{ K}$, with $N_a = 5 \times 10^{16}\text{ cm}^{-3}$ and $N_d = 10^{18}\text{ cm}^{-3}$. Compute $\Delta n(x_p)$ and $\Delta p(x_n)$ for $V_a = 0.1, 0.2, 0.3, 0.6, 0.7, -0.1, -0.2, -0.5, -1$, and -2 V . ($n_i = 1.5 \times 10^{10}\text{ cm}^{-3}$ for silicon at $T = 300\text{ K}$.)

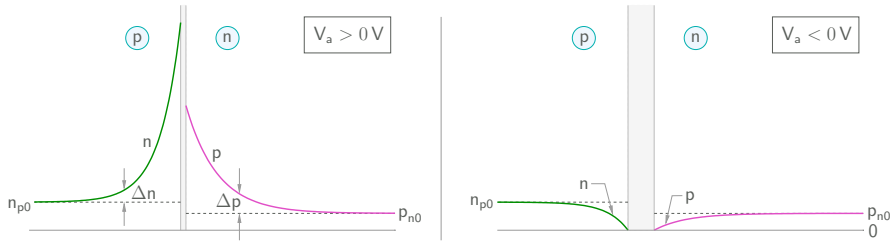
Solution: $p_{p0} \approx N_a = 5 \times 10^{16}\text{ cm}^{-3} \rightarrow n_{p0} = \frac{n_i^2}{p_{p0}} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3\text{ cm}^{-3}$.



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$$n_{n0} \approx N_d = 1 \times 10^{18}\text{ cm}^{-3} \rightarrow p_{n0} = \frac{n_i^2}{n_{n0}} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{18}} = 2.25 \times 10^2\text{ cm}^{-3}.$$

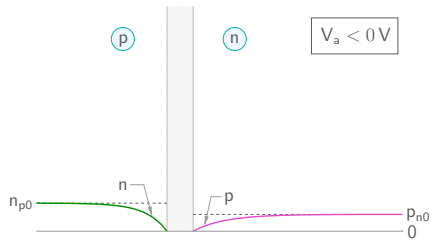
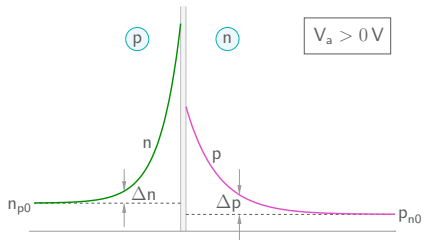


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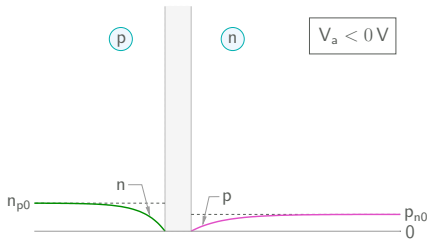
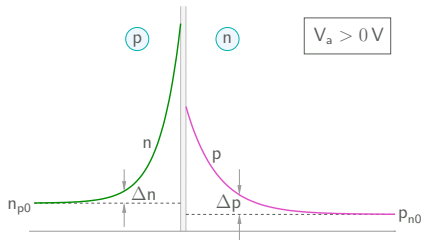
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$\Delta p(x_n) = p_{n0} \left[\exp\left(\frac{V_a}{V_T}\right) - 1 \right], \quad \Delta n(x_p) = n_{p0} \left[\exp\left(\frac{V_a}{V_T}\right) - 1 \right]$.

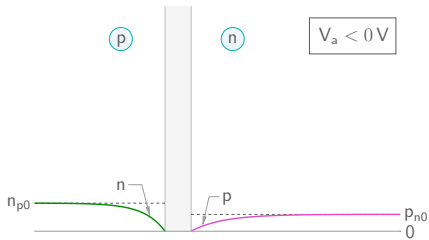
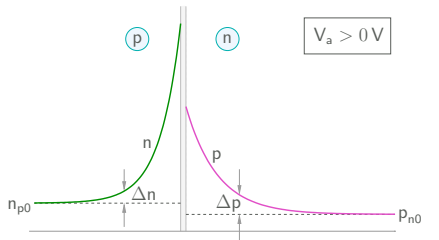


V_a (V)	$\Delta n(x_p)$ (cm^{-3})	$\Delta p(x_n)$ (cm^{-3})	V_a (V)	$\Delta n(x_p)$ (cm^{-3})	$\Delta p(x_n)$ (cm^{-3})
0	0	0	0	0	0
0.1	2.09×10^5	1.05×10^4	-0.1	-4.41×10^3	-2.20×10^2
0.2	1.02×10^7	5.08×10^5	-0.2	-4.50×10^3	-2.25×10^2
0.3	4.83×10^8	2.41×10^7	-0.5	-4.50×10^3	-2.25×10^2
0.6	5.18×10^{13}	2.59×10^{12}	-1	-4.50×10^3	-2.25×10^2
0.7	2.46×10^{15}	1.23×10^{14}	-2	-4.50×10^3	-2.25×10^2



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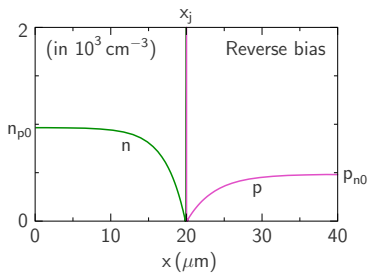
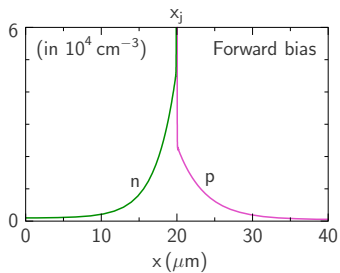
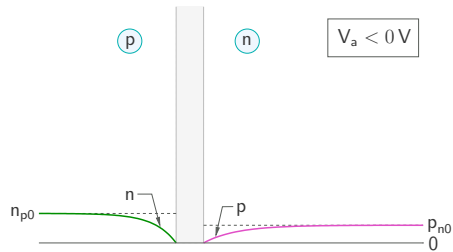
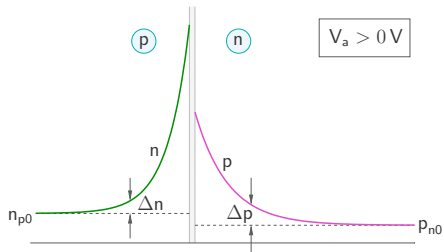
* Forward bias:
 $\Delta p(x_n)$ and $\Delta n(x_p)$ increase by several orders of magnitude as V_a is increased.



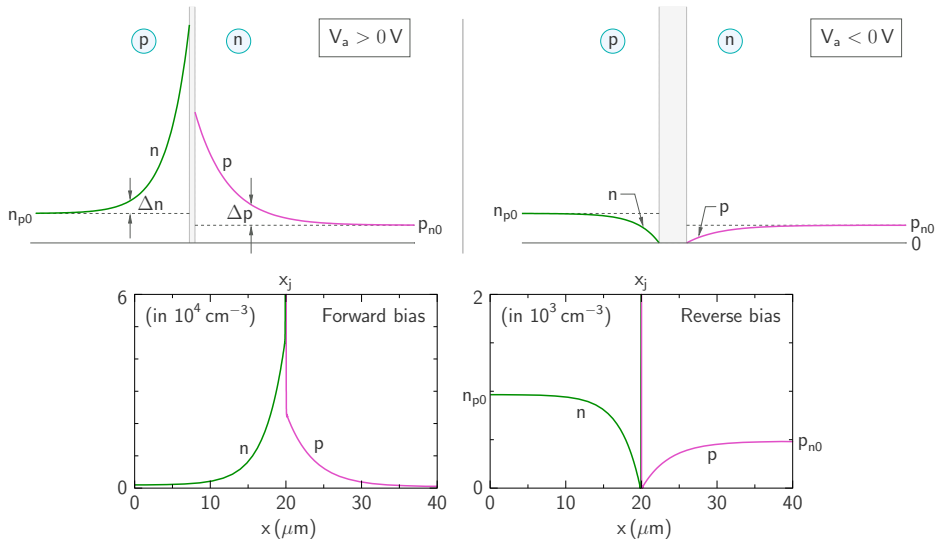
V_a (V)	$\Delta n(x_p)$ (cm ⁻³)	$\Delta p(x_n)$ (cm ⁻³)	V_a (V)	$\Delta n(x_p)$ (cm ⁻³)	$\Delta p(x_n)$ (cm ⁻³)
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- * Forward bias:
 $\Delta p(x_n)$ and $\Delta n(x_p)$ increase by several orders of magnitude as V_a is increased.
- * Reverse bias:
 $\Delta p(x_n) \approx -p_{n0}$, $\Delta n(x_p) \approx -n_{p0}$.

pn junction under forward bias: simulation results

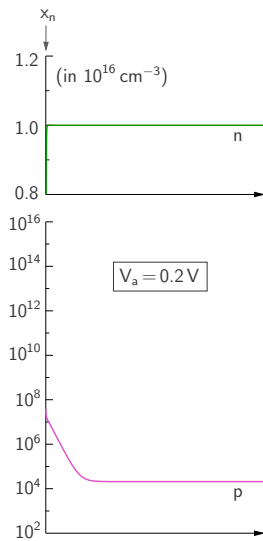


pn junction under forward bias: simulation results

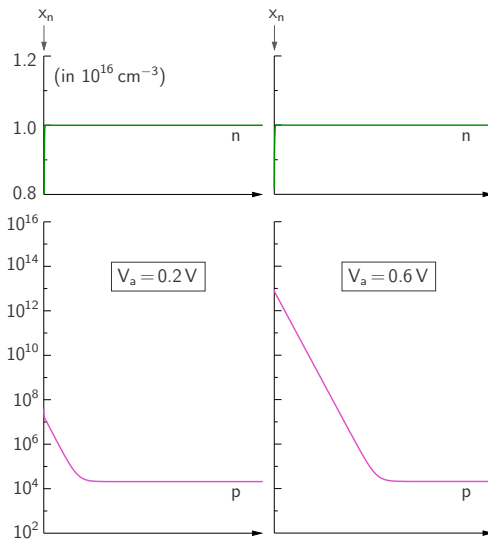


- * As we have seen earlier, the minority carrier diffusion lengths (i.e., L_n on the p -side, L_p on the n -side) are typically much larger than the depletion width.

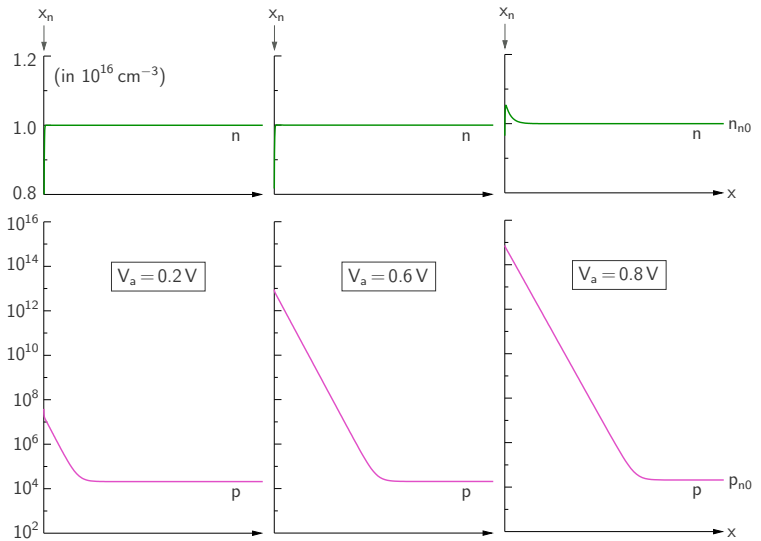
High-injection regime



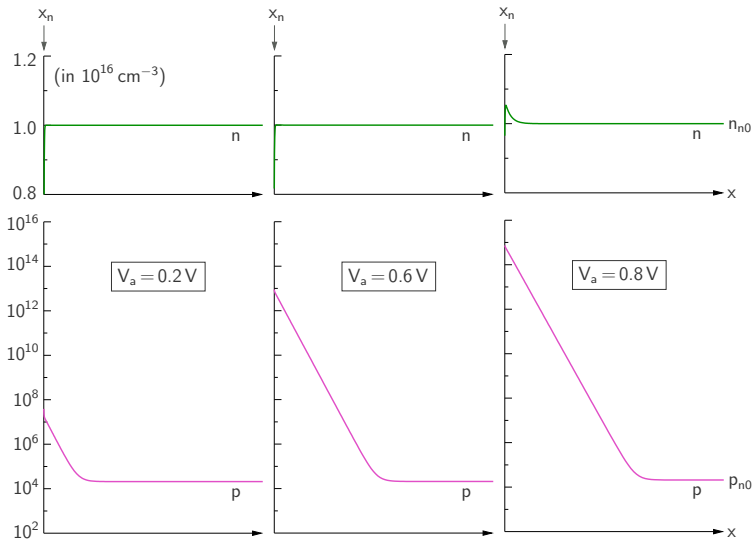
High-injection regime



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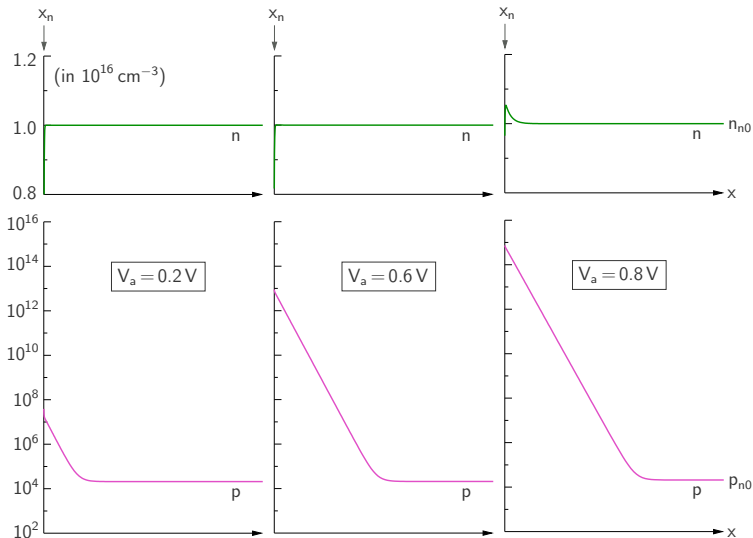


High-injection regime



- * As the forward bias is increased, the minority carrier concentration increases rapidly, and at some point becomes comparable to the majority carrier concentration. This regime is called the “high-injection” regime.

High-injection regime

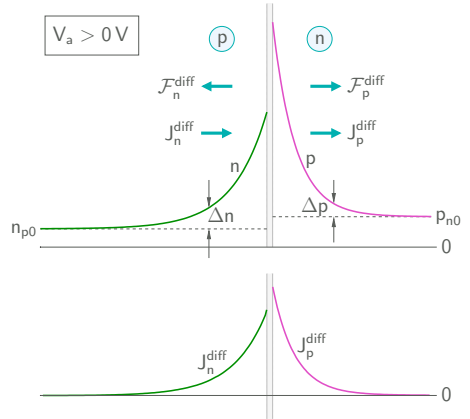


- * As the forward bias is increased, the minority carrier concentration increases rapidly, and at some point becomes comparable to the majority carrier concentration. This regime is called the “high-injection” regime.
- * In the high-injection regime, the majority carrier concentration also increases appreciably (e.g., $\Delta n \approx \Delta p$ on the n side), and the overall charge neutrality is maintained in the neutral regions.

pn junction: current flow under forward bias

$$\Delta p(x) = p_{n0} \left[\exp \left(\frac{V_a}{V_T} \right) - 1 \right] \exp \left(- \frac{x - x_n}{L_p} \right), \quad x > x_n.$$

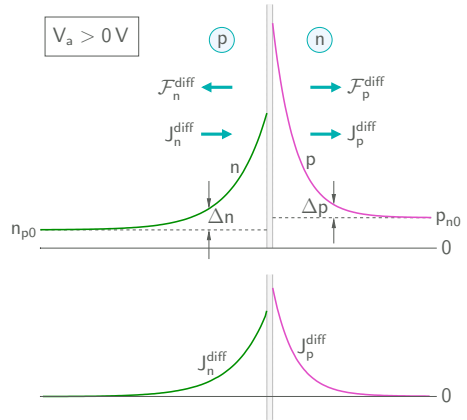
$$\Delta n(x) = n_{p0} \left[\exp \left(\frac{V_a}{V_T} \right) - 1 \right] \exp \left(- \frac{x_p - x}{L_n} \right), \quad x < x_p.$$



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Note that, although $\mathcal{F}_n^{\text{diff}}$ (for $x < x_p$) and $\mathcal{F}_p^{\text{diff}}$ (for $x > x_n$) are in opposite directions, J_n^{diff} (for $x < x_p$) and J_p^{diff} (for $x > x_n$) are in the same direction.



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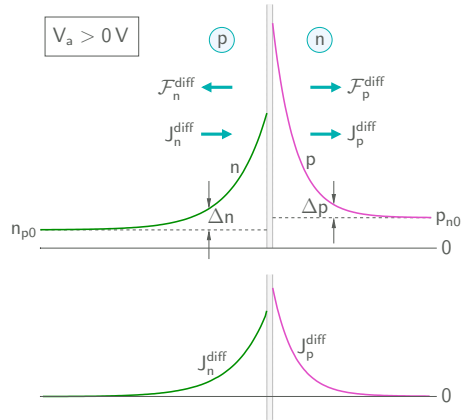
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In particular, we are interested in $J_n^{\text{diff}}(x_p)$ and $J_p^{\text{diff}}(x_n)$.

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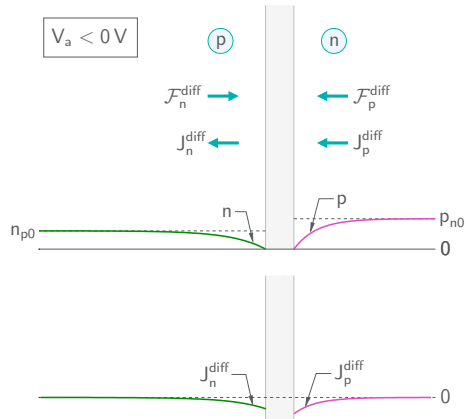
$$J_p^{\text{diff}}(x_n) = \frac{qD_p p_{n0}}{L_p} \left(e^{V_a/V_T} - 1 \right).$$



pn junction: current flow under reverse bias

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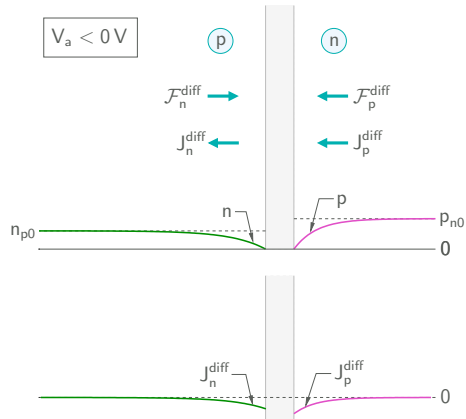


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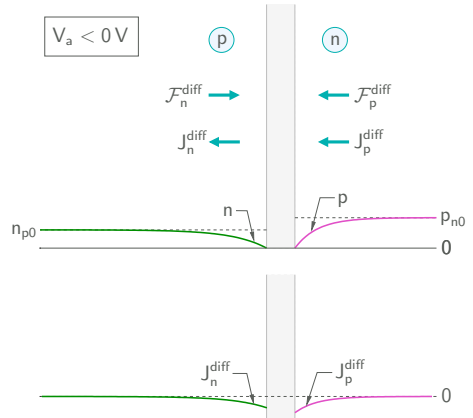
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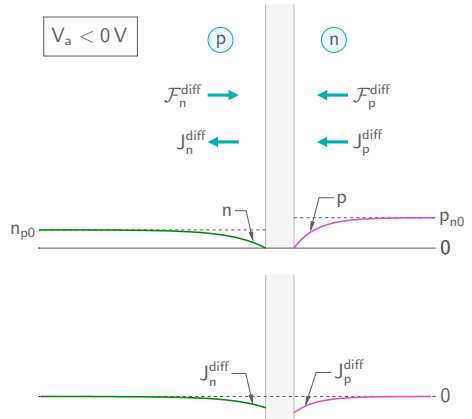
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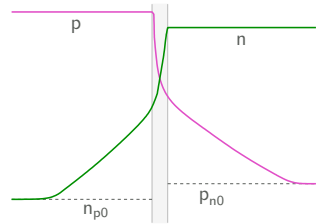
The currents are much smaller under reverse bias.



pn junction: What is happening inside the depletion region?

Consider x in the depletion region, i.e., $x_p < x < x_n$.

$$J_p^{\text{diff}} \approx -J_p^{\text{drift}} \rightarrow \int d\psi = -V_T \int \frac{1}{p} dp \rightarrow \frac{p(x)}{p(x_p)} = \exp \frac{\psi(x_p) - \psi(x)}{V_T}.$$

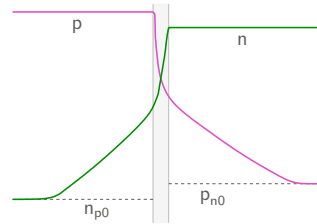


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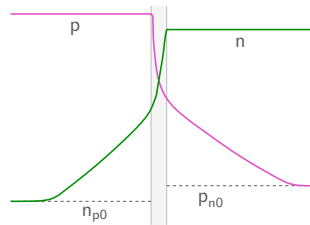
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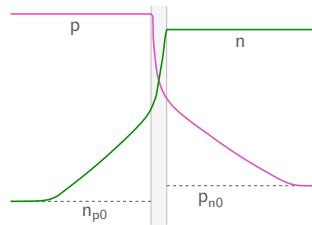
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If $V_a > 0$ V, $pn > n_i^2$ in the depletion region; else, $pn < n_i^2$.



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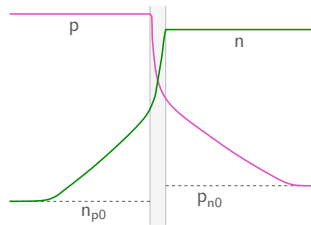
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If $V_a > 0\text{V}$, $pn > n_i^2$ in the depletion region; else, $pn < n_i^2$.

$$R - G = \frac{pn - n_i^2}{\tau_n(n + n_1) + \tau_p(p + p_1)}$$

\rightarrow we have a net recombination inside the depletion region if $V_a > 0\text{V}$, and a net generation if $V_a < 0\text{V}$.



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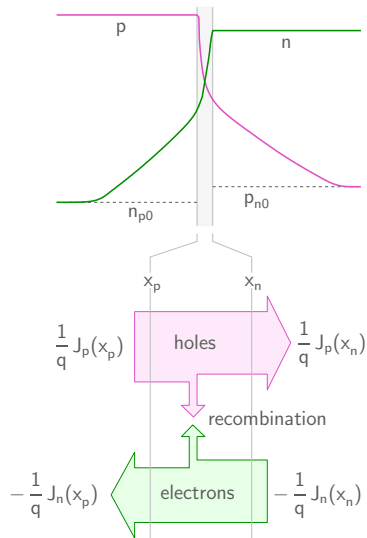
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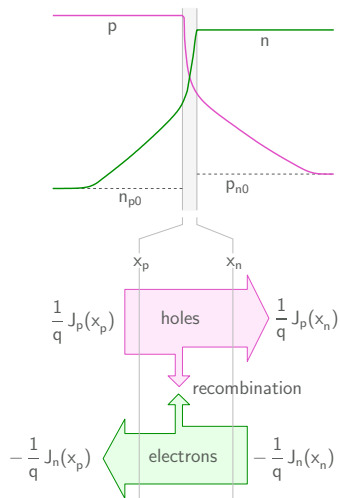
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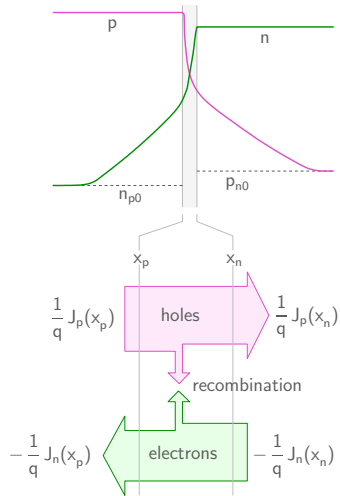
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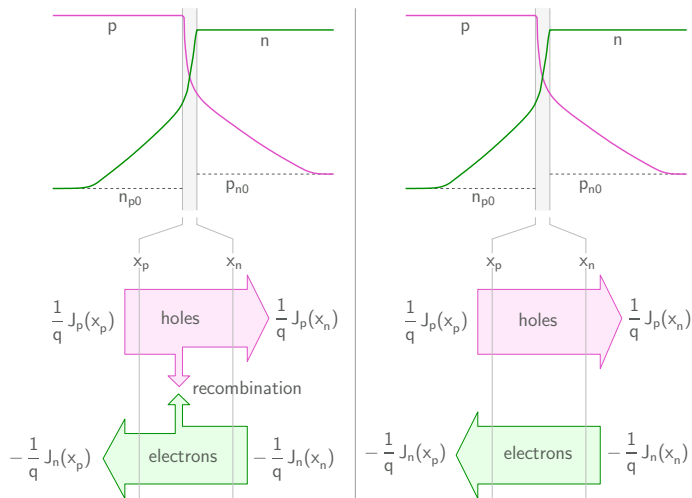


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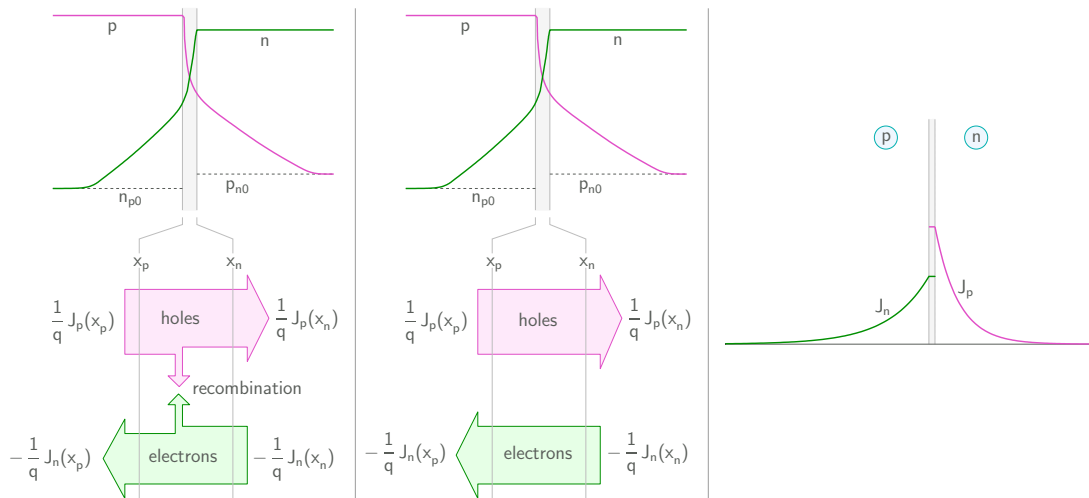
* To obtain a first-order *I-V* model, we ignore G-R in the depletion region.

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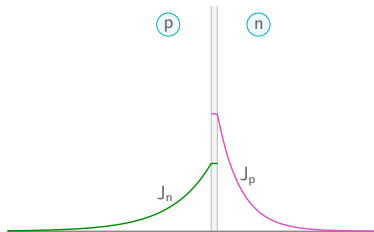
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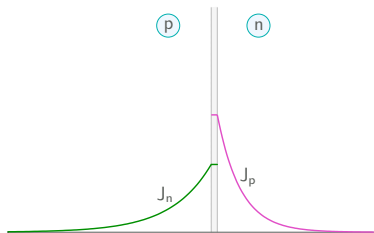
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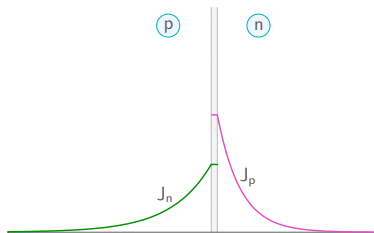
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pn junction: total current density

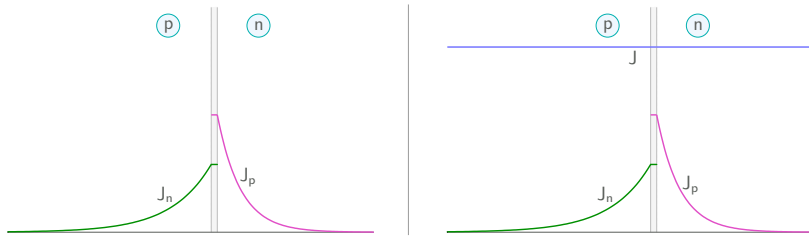




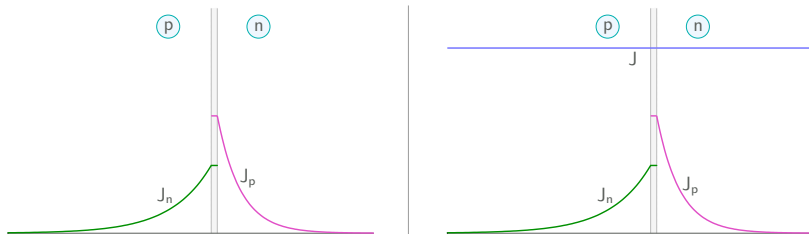
- * The total current density is the same throughout the device.



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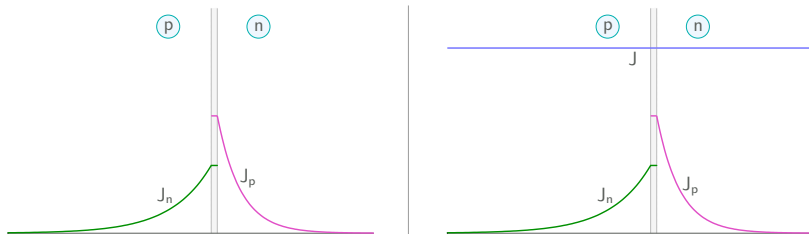


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- * The total current density is the same throughout the device.
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- * Using our earlier results for $J_p(x_n)$ and $J_n(x_p)$, we get

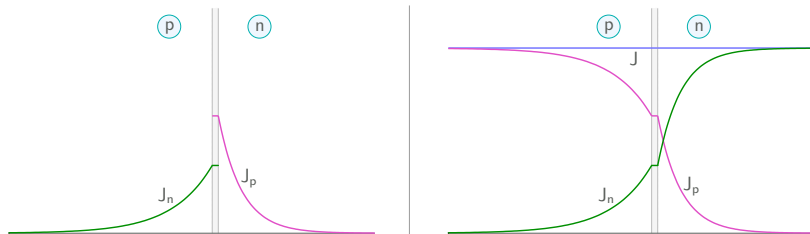
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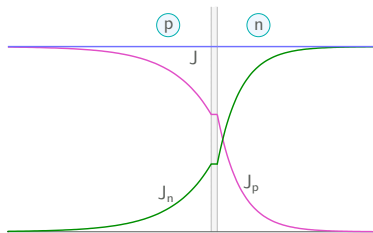
- * We can now obtain J_n ($x > x_n$) and J_p ($x < x_p$) using $J_n(x) + J_p(x) = J$.



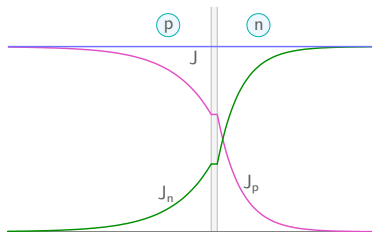
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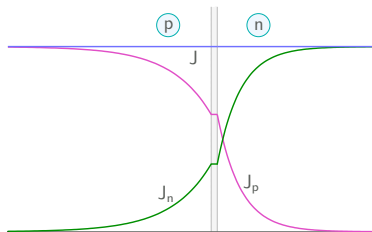


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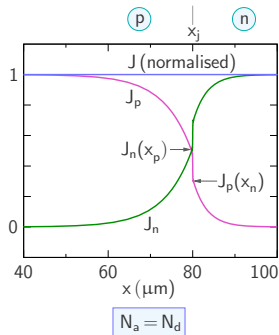
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- * The current density is due to majority carriers (drift component).
- * Since the majority carrier concentration is large, a very small electric field suffices to produce the required current density ($J_n^{\text{drift}} = qn\mu_n\mathcal{E}$, $J_p^{\text{drift}} = qp\mu_p\mathcal{E}$).

pn junction under forward bias: numerical results

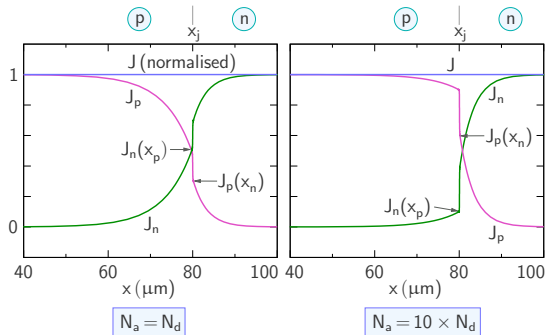


Doping densities:

(1) $N_a = N_d = 10^{16} \text{ cm}^{-3}$

(Parameters: $V_a = 0.5 \text{ V}$, $\mu_n = 1400 \text{ cm}^2/\text{V-s}$, $\mu_p = 500 \text{ cm}^2/\text{V-s}$, $\tau_n = 10 \text{ ns}$, $\tau_p = 10 \text{ ns}$, $T = 300 \text{ K}$)

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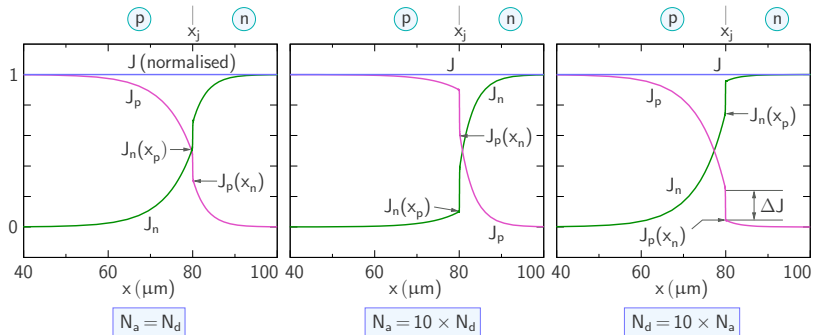
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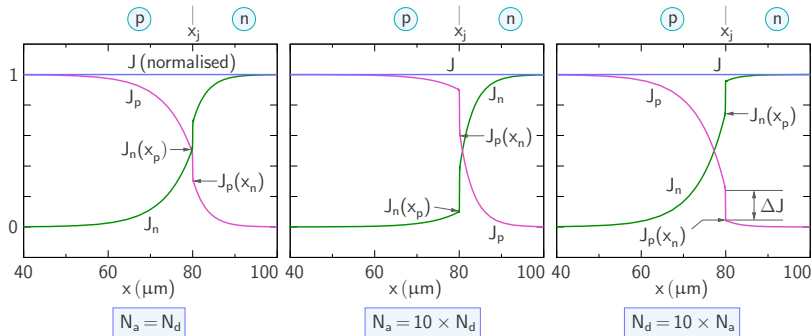
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pn junction under forward bias: numerical results



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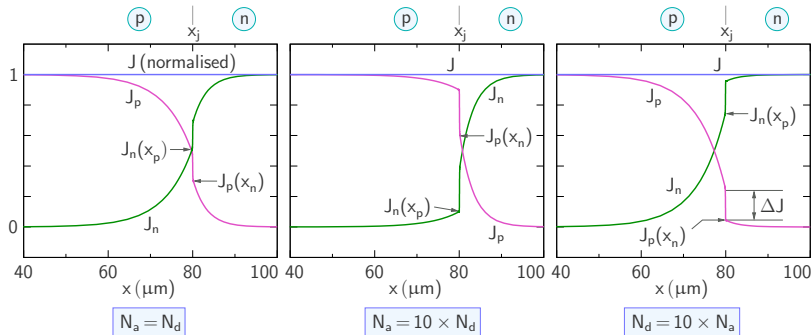
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pn junction under forward bias: numerical results



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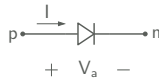
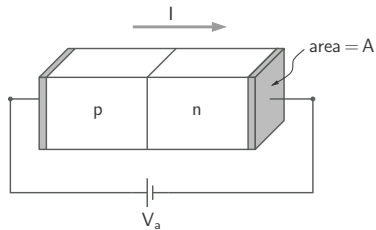
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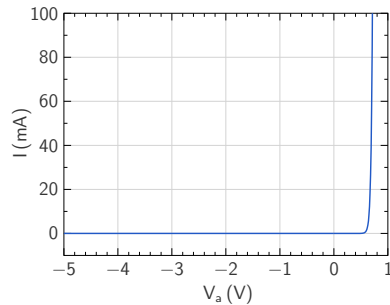
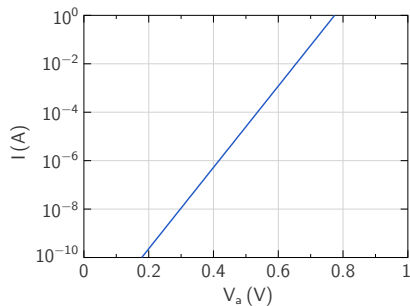
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- * Because of recombination, there is a change in J_p and J_n across the depletion region (which has been ignored in our analysis). This change is seen as vertical lines in the figure since the depletion width is much smaller than the diffusion lengths.

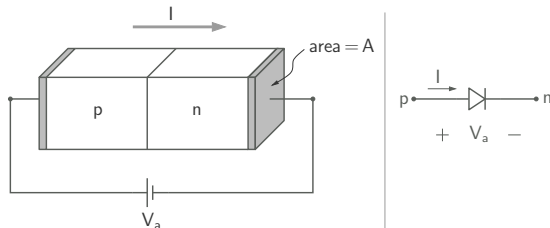
Diode I - V equation



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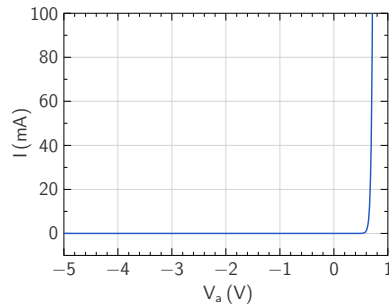
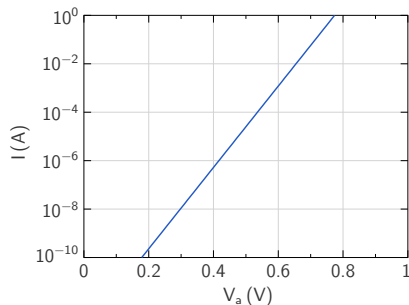


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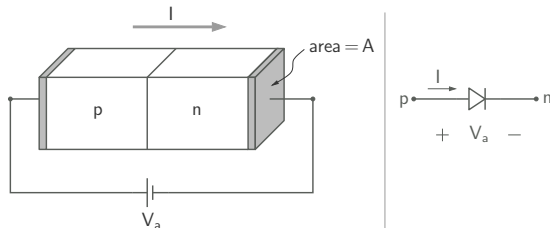


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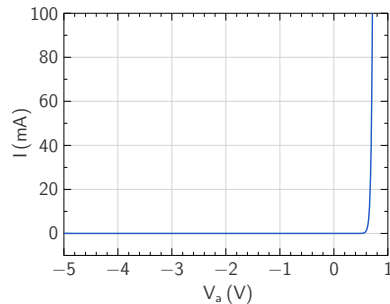
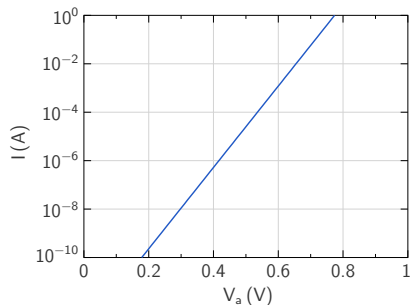
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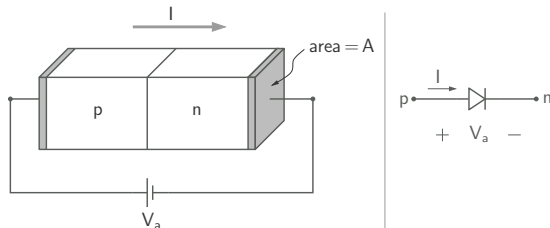
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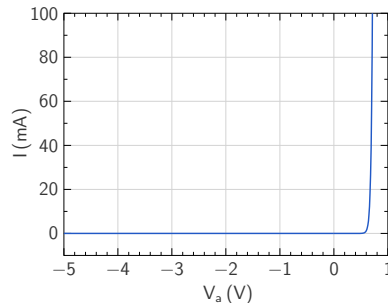
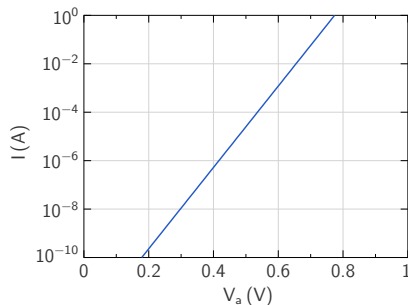
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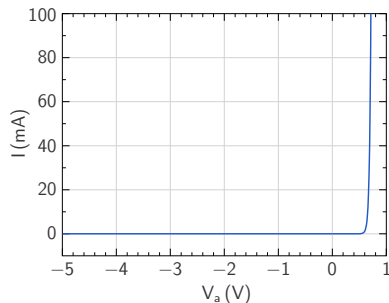
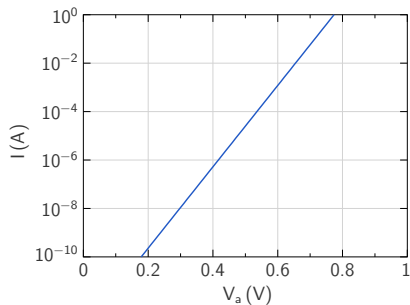
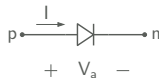
- * This equation is known as the “Shockley diode equation.”
- * Under reverse bias, with V_R equal to a few V_T or larger, $e^{V_a/V_T} = e^{-V_R/V_T} \approx 0$, and $I \approx -I_s$, i.e., the diode current “saturates” (at $-I_s$). I_s is therefore called the “reverse saturation current.”



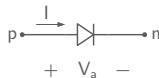
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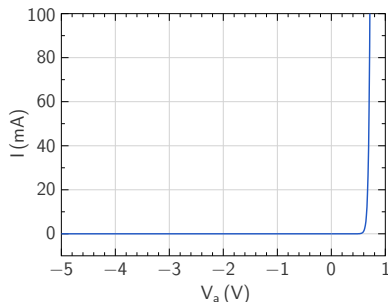
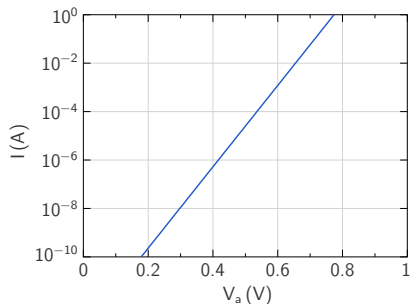
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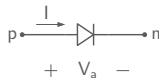
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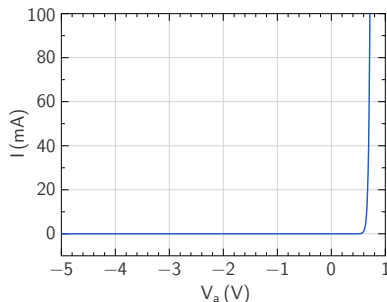
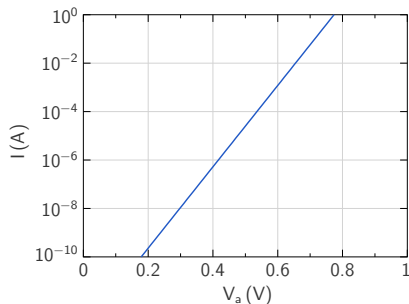


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- * Recombination in the depletion region under forward bias can be incorporated in the Shockley equation with an “ideality factor” η ($1 < \eta < 2$):

$$\begin{aligned} I &= I_{s1} \exp\left(\frac{V_a}{\eta_1 V_T}\right) + I_{s2} \exp\left(\frac{V_a}{\eta_2 V_T}\right) \\ &\approx I_s^{\text{eff}} \exp\left(\frac{V_a}{\eta V_T}\right) \end{aligned}$$



Example

For an abrupt, uniformly doped silicon pn junction diode, $N_a = 10^{17} \text{ cm}^{-3}$, $N_d = 2 \times 10^{16} \text{ cm}^{-3}$, $\mu_n = 1500 \text{ cm}^2/\text{V-s}$, $\mu_p = 500 \text{ cm}^2/\text{V-s}$, $\tau_n = 2 \mu\text{s}$, $\tau_p = 5 \mu\text{s}$, $A = 10^{-3} \text{ cm}^2$. Compute the following for a forward bias of 0.65 V at $T = 300 \text{ K}$:

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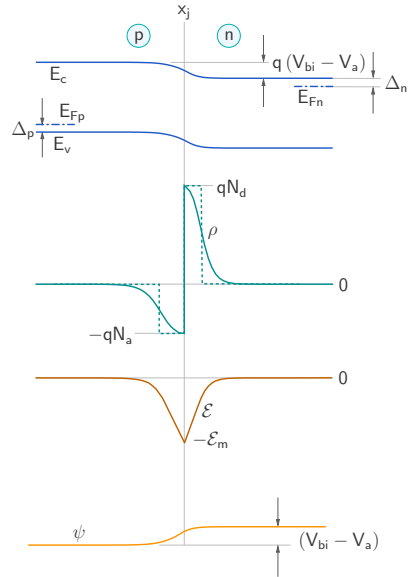
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Example (continued)

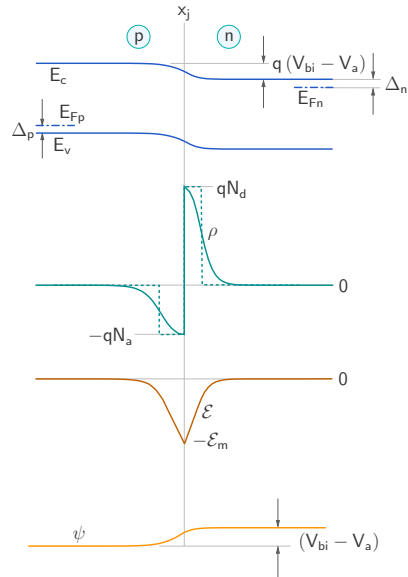
$$V_{bi} = V_T \log \left(\frac{N_a N_d}{n_i^2} \right) = (0.0259 \text{ V}) \log \frac{(10^{17})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} = 0.77 \text{ V}.$$



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$$V_{bi} = V_T \log \left(\frac{N_a N_d}{n_i^2} \right) = (0.0259 \text{ V}) \log \frac{(10^{17})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} = 0.77 \text{ V}.$$

$$W = \sqrt{\frac{2\epsilon}{q} \frac{N_a + N_d}{N_a N_d} (V_{bi} - V_a)}$$

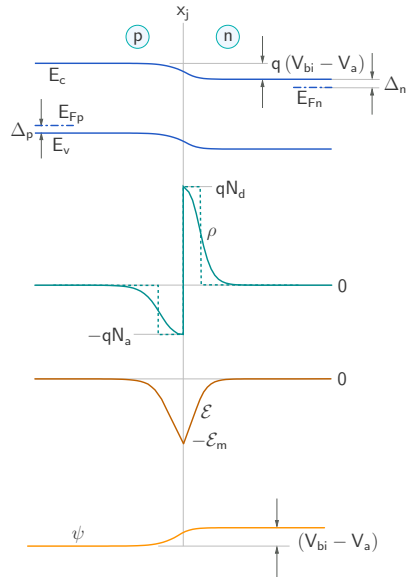


Example (continued)

$$V_{bi} = V_T \log \left(\frac{N_a N_d}{n_i^2} \right) = (0.0259 \text{ V}) \log \frac{(10^{17})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} = 0.77 \text{ V}.$$

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$$= \sqrt{\frac{2 \times 11.8 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19}} \frac{1.2 \times 10^{17}}{2 \times 10^{33}} \times 0.12} \text{ cm} = 0.097 \mu\text{m}.$$



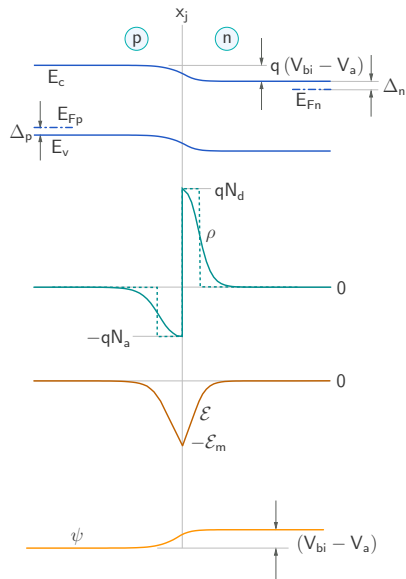
Example (continued)

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$$(V_{bi} - V_a) = \frac{1}{2} \mathcal{E}_m W$$



Example (continued)

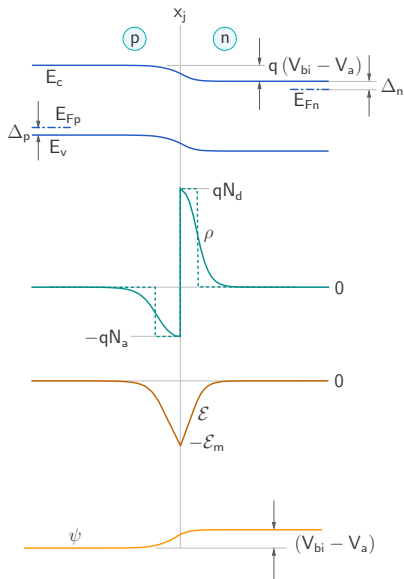
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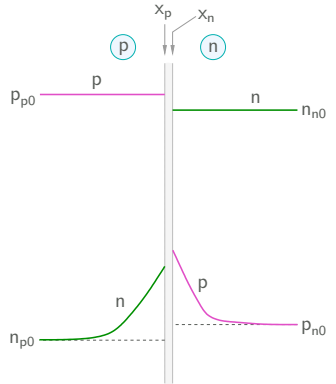
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$$(V_{bi} - V_a) = \frac{1}{2} \mathcal{E}_m W$$

$$\rightarrow \mathcal{E}_m = \frac{2(V_{bi} - V_a)}{W} = \frac{2 \times 0.12 \text{ V}}{0.097 \times 10^{-4} \text{ cm}} = 25 \text{ kV/cm}.$$

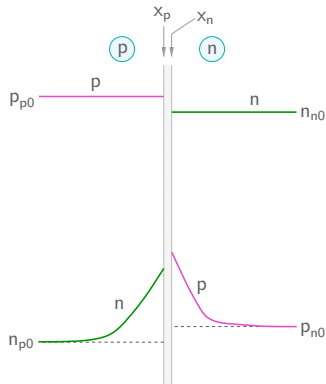


Example (continued)



The equilibrium minority carrier densities are

$$p_{n0} = \frac{n_i^2}{n_{n0}} \approx \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{16}} = 1.125 \times 10^4 \text{ cm}^{-3},$$

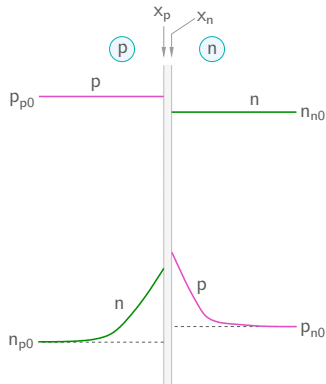


Example (continued)

The equilibrium minority carrier densities are

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$$n_{p0} = \frac{n_i^2}{p_{p0}} \approx \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}.$$



Example (continued)

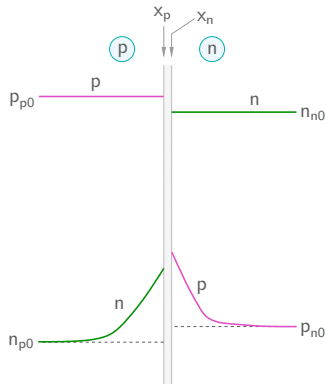
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The minority carrier densities at x_p and x_n are

$$n(x_p) = n_{p0} (e^{V_a/V_T} - 1) = 2.25 \times 10^3 \times e^{0.65/0.0259} = 1.8 \times 10^{14} \text{ cm}^{-3},$$



Example (continued)

The equilibrium minority carrier densities are

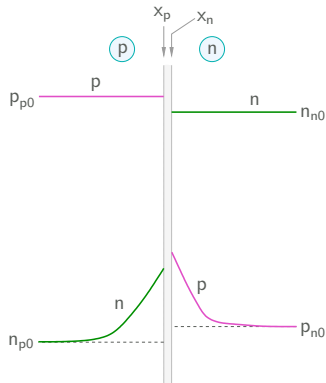
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Example (continued)

The diffusion coefficients are

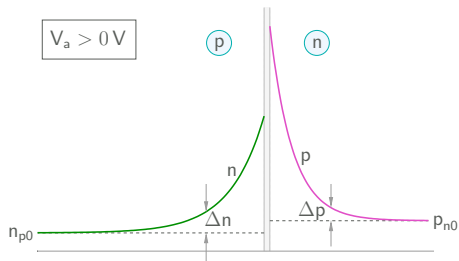
$$D_p = V_T \mu_p = 0.0259 \times 500 = 12.9 \text{ cm}^2/\text{s},$$

$$D_n = V_T \mu_n = 0.0259 \times 1500 = 38.7 \text{ cm}^2/\text{s}.$$

The minority carrier diffusion lengths in the neutral regions are

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.9 \times 5 \times 10^{-6}} \text{ cm} = 80.3 \text{ } \mu\text{m},$$

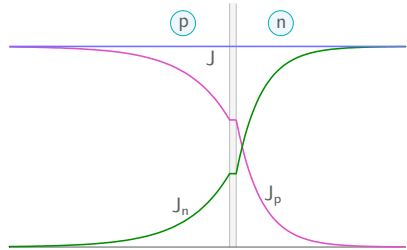
$$L_n = \sqrt{D_n \tau_n} = \sqrt{38.7 \times 2 \times 10^{-6}} \text{ cm} = 88 \text{ } \mu\text{m}.$$



Example (continued)

The minority carrier current densities at x_n and x_p are

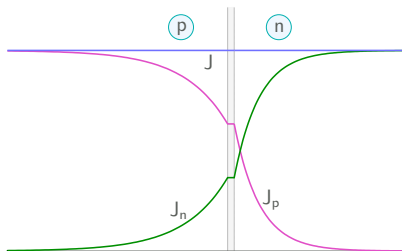
$$J_p(x_n) = \frac{qD_p p_{n0}}{L_p} (e^{V_a/V_T} - 1)$$



Example (continued)

The minority carrier current densities at x_n and x_p are

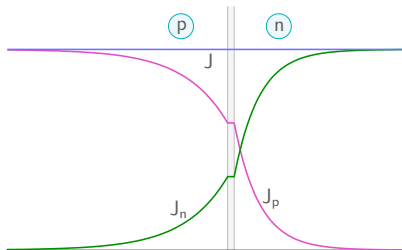
$$\begin{aligned} J_p(x_n) &= \frac{qD_p p_{n0}}{L_p} (e^{V_a/V_T} - 1) \\ &= \frac{1.6 \times 10^{-19} \times 12.9 \times 1.125 \times 10^4}{80.3 \times 10^{-4}} \times 8.12 \times 10^{10} \end{aligned}$$



Example (continued)

The minority carrier current densities at x_n and x_p are

$$\begin{aligned} J_p(x_n) &= \frac{qD_p p_{n0}}{L_p} (e^{V_a/V_T} - 1) \\ &= \frac{1.6 \times 10^{-19} \times 12.9 \times 1.125 \times 10^4}{80.3 \times 10^{-4}} \times 8.12 \times 10^{10} \\ &= 0.235 \text{ A/cm}^2. \end{aligned}$$

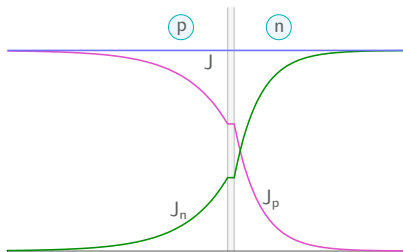


Example (continued)

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Example (continued)

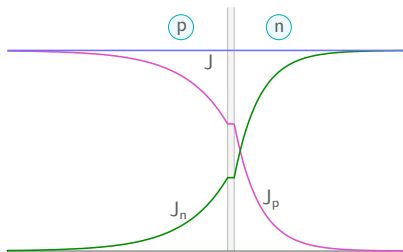
The minority carrier current densities at x_n and x_p are

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The diode current I is

$$I = A(J_p(x_n) + J_n(x_p))$$



Example (continued)

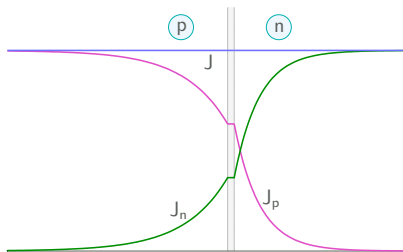
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The diode current I is

$$\begin{aligned}I &= A(J_p(x_n) + J_n(x_p)) \\&= 10^{-3} \text{ cm}^2 \times (0.235 + 0.13) \text{ A/cm}^2\end{aligned}$$



Example (continued)

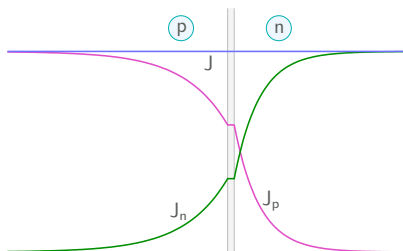
The minority carrier current densities at x_n and x_p are

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$$\begin{aligned}I &= A(J_p(x_n) + J_n(x_p)) \\&= 10^{-3} \text{ cm}^2 \times (0.235 + 0.13) \text{ A/cm}^2 \\&= 0.365 \text{ mA}.\end{aligned}$$

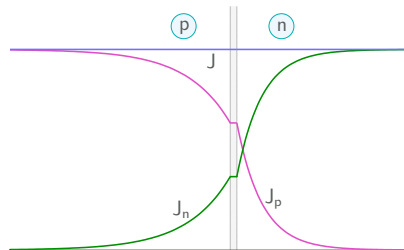


Example (continued)

In the neutral n region more than $5L_p$ away from the depletion region,

$J \approx J_n = q\mu_n \mathcal{E}_{\text{neutral}}^n n_{n0}$, leading to

$$\mathcal{E}_{\text{neutral}}^n = \frac{J}{q\mu_n n_{n0}} = \frac{0.365 \left[\frac{\text{A}}{\text{cm}^2} \right]}{1.6 \times 10^{-19} [\text{C}] \times 1500 \left[\frac{\text{cm}^2}{\text{V-s}} \right] \times 2 \times 10^{16} \left[\frac{1}{\text{cm}^3} \right]}$$

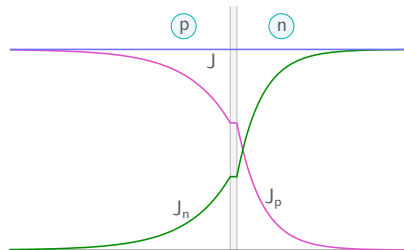


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Example (continued)

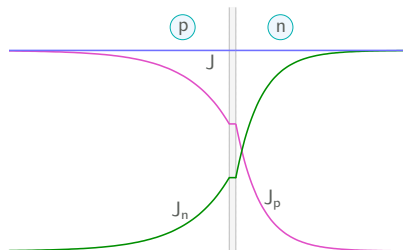
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Similarly,

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Example (continued)

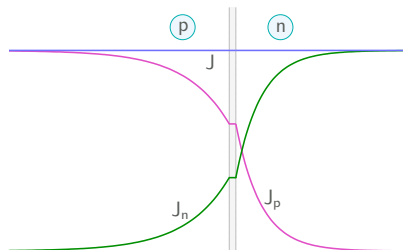
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Example (continued)

In the neutral n region more than $5L_p$ away from the depletion region,

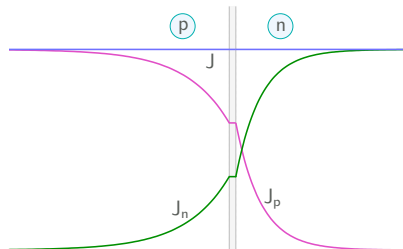
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Note that these values are much smaller than \mathcal{E}_m in the depletion region (25 kV/cm).



The reverse saturation current I_s is given by

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Note how small I_s is. The only reason we can get significant currents (\sim mA) in forward bias is the *huge* exponential factor (e^{V_a/V_T}) in the Shockley equation.