Online Learning

(i) Settings

(ii) Some solution without proof

— discussion intuition behind the ~~proof~~. solution.

$\{x_1, x_2 \dots x_n\} \longrightarrow$ Classifier / Regressor $\longrightarrow \hat{y}$

$y_1 \dots \dots y_n$

$(x, ?)$

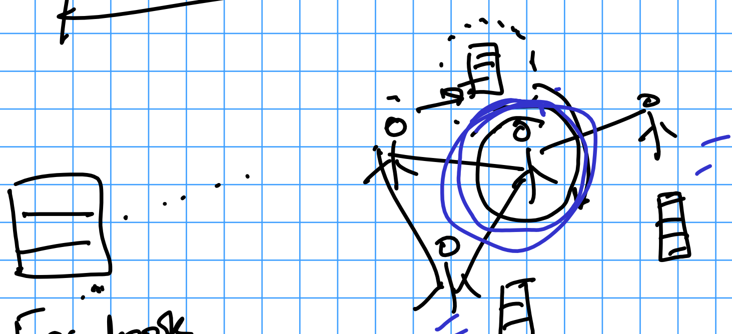
$\{x_t\} \longrightarrow$

→ Features are arriving across time

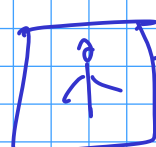
→ How to design ML algorithms that will be able to adapt across time.

Online algorithm → adapt across time

→ ~~a~~ small data at initial stage should not disturb the flow of the algorithm.



Facebook



Sports
Cinema.

Given that Facebook does not know the preference of the [person], what ~~post~~ topic it should float on the wall of [person]

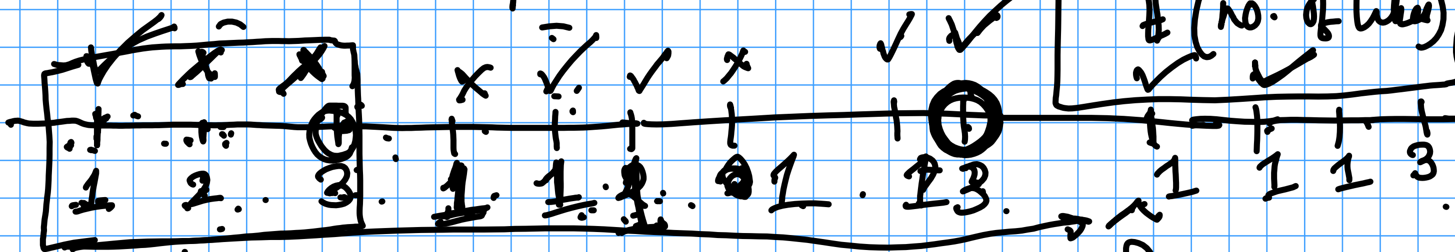
$P($ likes topic "topic"$) = P_{topic}$

$P($ does not like topic "topic"$) = 1 - P_{topic}$

	Topic	P_{Topic}
1	Sports	0.6
2	Cinema	0.3
3	Tech	0.7

Θ

Maximize E(no. of likes)



$\hat{P}_3 = 0.5$

$\hat{P}_1 = \frac{3}{5} = 0.6$

$\hat{P}_2 = 0.5$

Exploit

Explore

Venkata's algo

Until time t.

Topic "topic" is liked n_{topic} times

" " is posted N_{topic} times

at time $t+1$

topic [illegible] is posted with prob

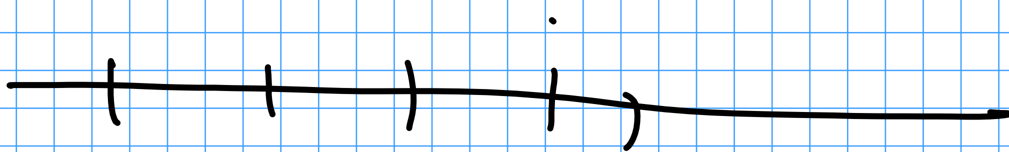
$$P = \frac{n_{topic}}{N_{topic}}$$

$\cong$ topic is posted if this topic has largest $\left(\frac{n_{topic}}{N_{topic}}\right)$

Ankan's algorithm

	Topic	P_{Topic}		n_{like}	$\bar{n}_{like}$
1	Sports	0.6	⋮	.	.
2	Cinema	0.3	→ 0	.	.
3.	Tech	0.7	1		

$$\cancel{\frac{n_{like}}{N_{like}}}$$



at each time t

Toss coin with prob. p = head

if head

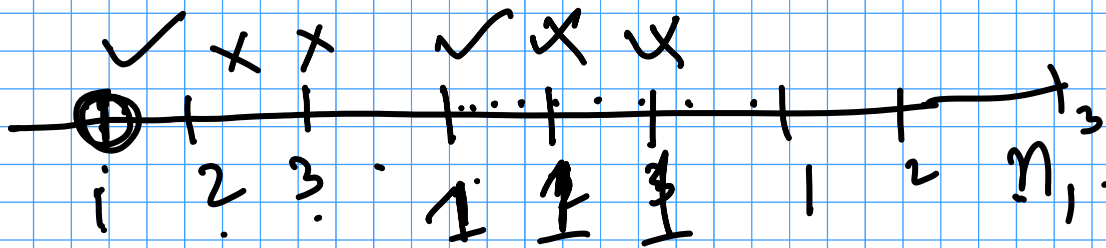
post a topic with prob.

$$\frac{n_{topic}}{\sum\limits_{\text{Topic in sanitized list}} n_{Topic}}$$

where sanitized list contains topics with [illegible] $n_{topic} > \bar{n}_{Topic}$.

if tail

post uniformly random.



$n_1 \to 10+$

$n_2 \to 10+$

$n_3 \to 10+$

prior

$\bar{p}_1$

$\bar{p}_2$

$\bar{p}_3$

$$\hat{p}_1 = \frac{\#\text{ likes of } 1}{\#\text{ posts of } 1}$$

$\hat{p}_2, \hat{p}_3$ are defined similarly

$t \to \hat{p}_1(t), \hat{p}_2(t), \hat{p}_3(t)$

t: $n_i(t)$ likes for topic "i"

$N_i(t)$ posts " " "i"

$$p_i \sim \text{Beta}\left(n_i(t) + n_0,\ N_i(t) + N_0\right)$$

$\hat{p}_1(t)$ + noise

$\hat{p}_2(t)$ + noise

$\hat{p}_3(t)$ + noise

$\cancel{\dfrac{n_i(t)}{N_i(t)}}$

$\max_i \left(\hat{p}_i(t) + \text{noise}\right)$

$$\boxed{\text{Beta}(\alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{*}}$$

$$p_i \sim \text{Beta}\left(n_i(t)+n_0,\ N_i(t)+N_0\right)$$

$$E(p_i) = \frac{n_i(t)+n_0}{N_i(t)+N_0}$$

$$\boxed{i = \arg\max_i p_i}$$

Topic $\{1, 2, \ldots, n\}$

$$\begin{array}{ccc} 1 & 2 & \cdots & n \\ \downarrow & \downarrow & & \downarrow \\ p_1 & p_2 & \cdots & p_n \end{array}$$

$$i = \arg\max_i p_i$$

$$E(\#\text{ of likes})$$

$$= E\left(\sum_{t=1}^{T} \mathbb{1}(\text{post at time } t \text{ is liked})\right)$$

$$= \sum_{t=1}^{T} P(\text{post at time } t \text{ is liked})$$

$$= \sum_{t=1}^{T} p_i \, \underline{\mathbb{1}(\text{topic } i \text{ is posted at time } t)}$$

$\downarrow$

is maximized

when Topic i is the one

with highest probability.

$$E(\# \text{ of likes}) = T\, P_{max}$$

max possible value of

\# likes ≈ 0

\# dislikes ≈ 0 for topic i, $i = 1 \ldots N$

for each $t: 1 \ldots T$

for each ~~pos~~ topic $i: 1 \ldots N$

$\theta_i \sim Beta(S_i + 1, f_i + 1)$

\# of likes of topic i

\# of no-likes of topic i

$i^* = \arg\max_i \theta_i$

post the topic with topic-id $\underline{i^*}$.

$S_{i^*} = S_{i^*} + 1$ if you observe a "like"

$F_{i^*} = f_{i^*} + 1$ otherwise

Mean of the beta distn $= \hat{P}_i = \dfrac{S_i + 1}{S_i + 1 + F_i + 1} = \dfrac{n_i + 1}{N_i + 2}$

total no of posts of topic i

total no of posts of topic i that has been liked.

$\mathbb{E}$(# likes upto time T) =

Thompson sampling

~~$= \mathbb{E}$(# max~~ max value of E(# likes upto time T)

$- O(\log T)$

max value of E(# likes upto T)

$= T P_{max}$

E(# likes upto time T)

$= T P_{max} - c \log T.$

$\lim_{t \to \infty} \frac{1}{T}$ E(# likes upto time T) $= P_{max} - \frac{c \log T}{T}$

$\downarrow$

0

$\frac{1}{T}$ E(# likes upto time T) $= P_{max}$