

# High Level Synthesis

---

Virendra Singh

Computer Architecture and Dependable Systems Lab

Department of Electrical Engineering  
Indian Institute of Technology Bombay

<http://www.ee.iitb.ac.in/~viren/>

E-mail: [viren@ee.iitb.ac.in](mailto:viren@ee.iitb.ac.in)



*EE-677: Foundations of VLSI CAD*

---



Lecture 6 on 09 August 2021

**CADSL**

# Architectural Synthesis

---

## Architectural Level Abstraction

- Datapath
- Controller

## Architectural Synthesis

- Constructing the macroscopic structure of a digital circuit starting from behavioural models that can be captured from Data flow or Sequencing Graph



# Architectural Synthesis

---

Computation: Differential Equation Solver

$$x_l = x + dx$$

$$u_l = u - (3 * x * u * dx) - (3 * y * dx)$$

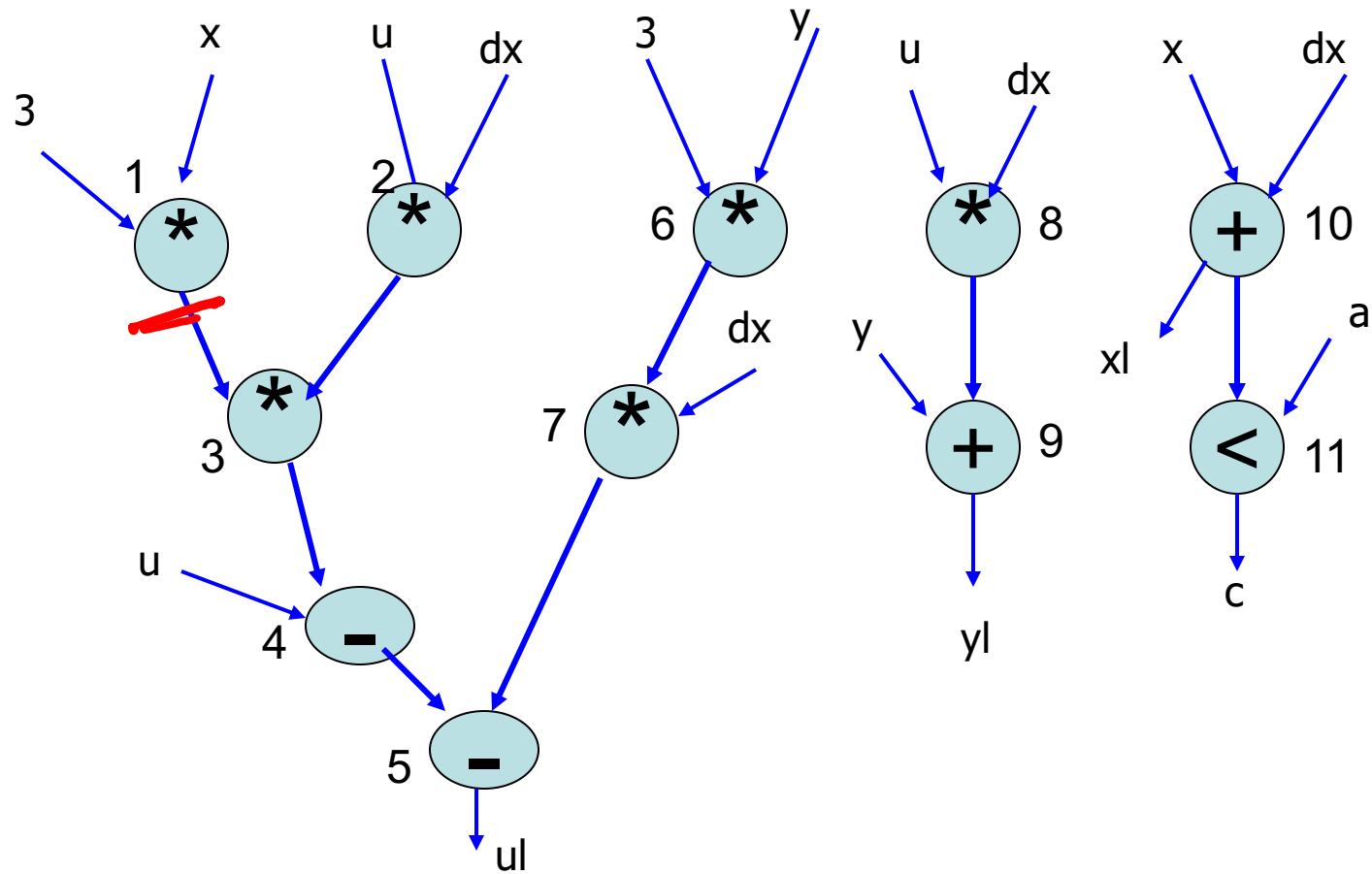
$$y_l = y + (u * dx);$$

$$c = x_l < a$$

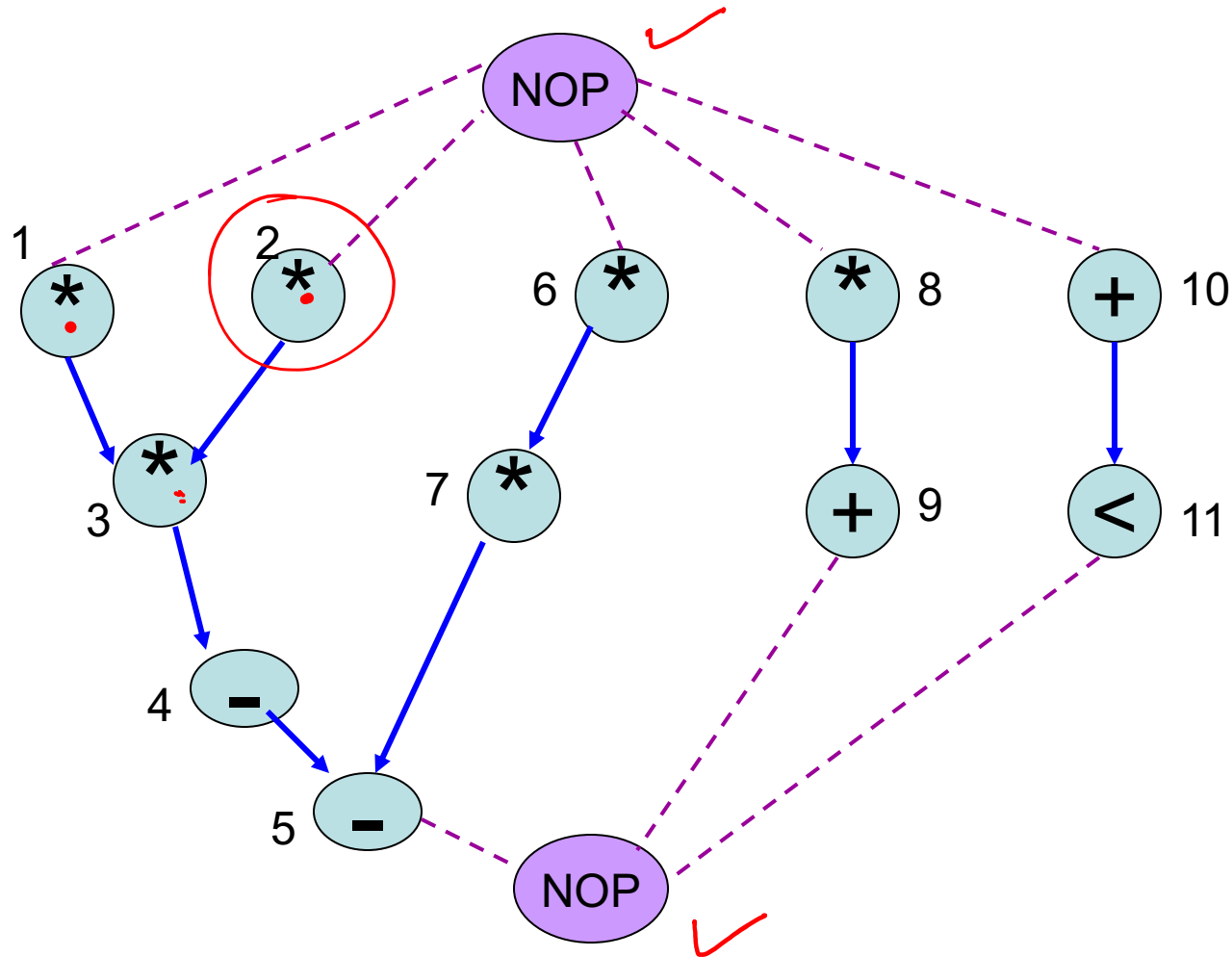
Data Flow Graph (**DFG**): represent operation and data dependencies



# Data Flow Graph



# Sequencing Graph



# Architectural Synthesis

---

Architectural Synthesis and optimization consists of two stages

1. **Placing the operation in time and in space**, i.e., determining their time interval of execution and binding to resources
2. **Determining detailed interconnection** of the datapath and the logic-level specifications of the control unit



# Temporal Domain: Scheduling

---

Delay  $\mathbf{D} = \{d_i; i = 0, 1, 2, \dots, n\}$

Start time  $T = \{t_i; i = 0, 1, \dots, n\}$

Scheduling: Task of determining the start timing,  
subject to preceding constraints specified by  
sequencing graph

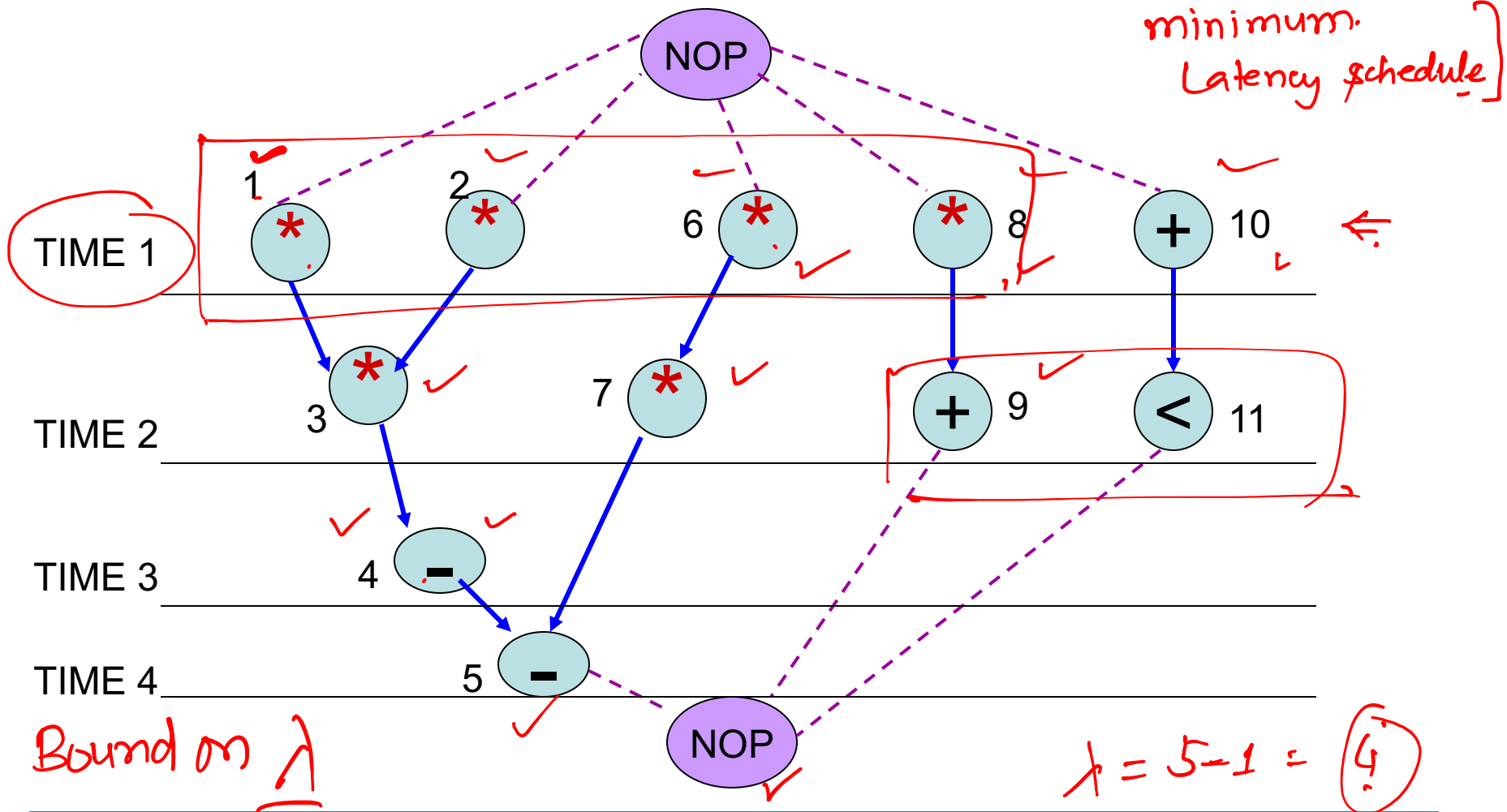
Latency  $\lambda = t_n - t_0$



4 mult + 2 ALU

# ASAP Scheduling

Best for performance





# ASAP Scheduling

---

ASAP( $G_s(V,E)$ ) {

?

Schedule  $v_0$  by setting  $t_0^s = 1$ ;

repeat {

    select vertex  $v_i$  whose predecessors are  
    all scheduled;

    schedule  $v_i$  by setting  $t_i^s = \max\{t_j^s + d_j\}$

    } untill ( $v_n$  is scheduled)

    return ( $t^s$ );

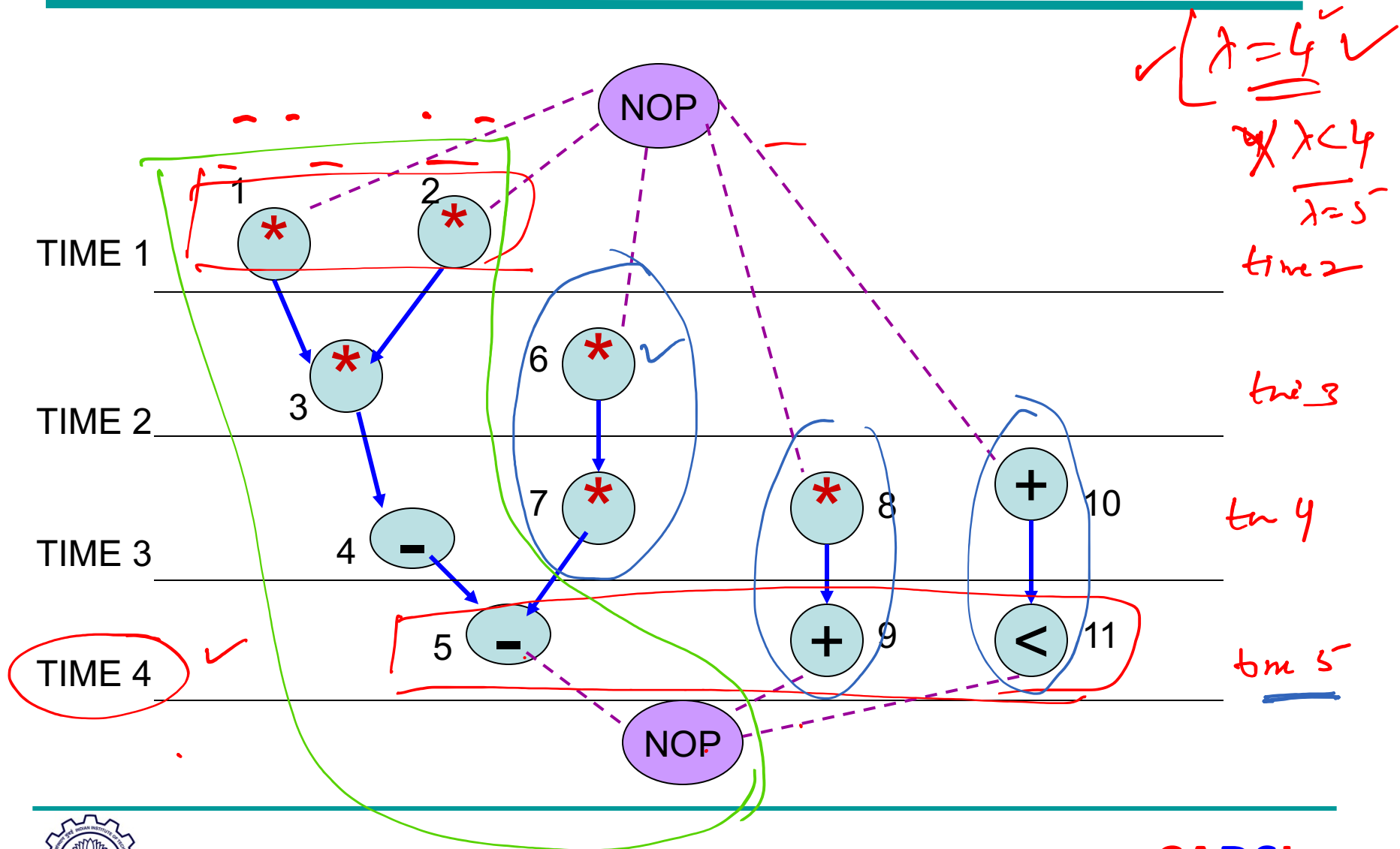
}



QMULT + 3ALU

# ALAP Scheduling

$\lambda = ?$



09 Aug 2021

EE-677@IITB

CADSL

# ALAP Scheduling

ALAP( $G_s(V,E), \lambda$ ) {

Schedule  $v_n$  by setting  $t_n^L = \lambda$ ;

repeat {

select vertex  $v_i$  whose successors are  
all scheduled;

schedule  $v_i$  by setting  $t_i^L = \min\{t_j^L - d_j\}$

} until ( $v_0$  is scheduled)

return ( $t^L$ );

}



# Scheduling with Resource Constraint

---

Scheduling under resource constraints

- computing area/latency trade-off points

Problems

- ❖ Intractable problem
- ❖ Area-performance trade-off points are affected by the other factors - non-resource dominated circuits



# Optimization

$$\min (\alpha \cdot \text{latency} + \beta \cdot \text{area})$$

$\alpha + \beta = 1$

## Area / Latency

Integer Linear

Programming (ILP)

linear

$$\min f(x)$$

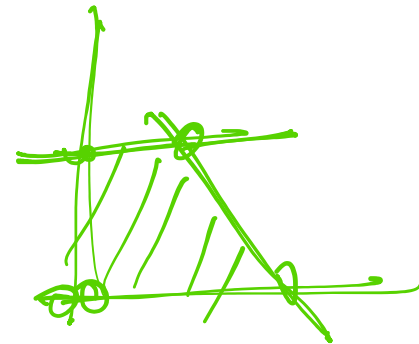
linear

$$\begin{cases} h(x) = 0 \\ g(x) \leq 0 \end{cases}$$

$$x = x_1, x_2, \dots, x_n$$

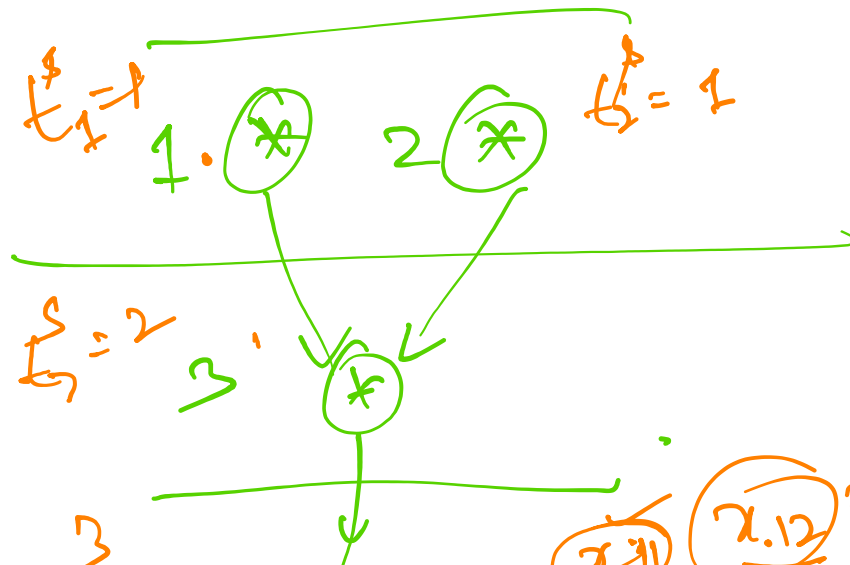
equality

in-equality



# Scheduling

decide start time of an operation



$\lambda$  - latency  
 $\{ n - \text{operations} \}$

$n \times \lambda$        $n = 11$   
 $\lambda = 4$

$11 \times 4 = 44$

2

$2 \times 3 = 6$

$x_{11}$   $x_{12}$   
 $x_{14}$   $x_{22}$   
 $x_{31}$   $x_{32}$

Boolean  
 variables

$\{0, 1\}$

$x_{11} = 1$



$$\checkmark$$

$$\underline{x_{11}} + \underline{x_{12}} = \underline{1}$$

$$\left. \begin{aligned} x_{21} + x_{22} &= 1 \\ x_{31} + x_{32} &= 1 \end{aligned} \right\}$$

---



# Scheduling with Resource Constraint <sup>of</sup>

## ILP Formulation

Binary decision variable  $X = \{x_{il}\}$

*$i^{th}$  operation is scheduled in  $l^{th}$  time step*

1. Start time of each operation is unique ✓

$$\sum_l x_{il} = 1$$

2. Sequencing relations represented by  $G_s(V,E)$  must be satisfied ✓

$$\sum_l l x_{il} \geq \sum_l l x_{jl} + d_j$$

3. Resource bound must be met at every schedule step

$$\sum_k \sum_m x_{im} \leq a_k$$





# ILP Formulation

$\lambda \leq \lambda_{max}$

All operation must start only once

$$x_{0,1} = 1$$

$$x_{1,1} = 1$$

$$x_{2,1} = 1$$

$$x_{3,2} = 1$$

$$x_{4,3} = 1$$

$$x_{5,4} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

$$x_{7,2} + x_{7,3} = 1$$

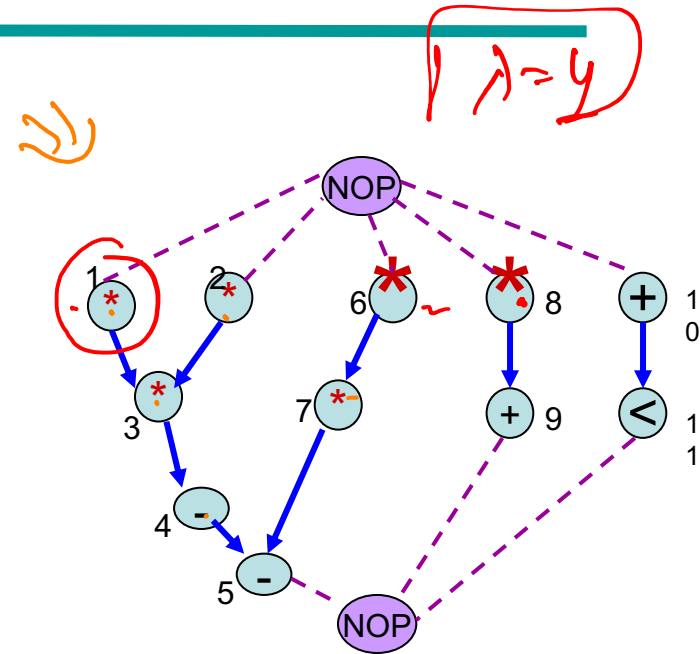
$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$

$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$

$$x_{10,1} + x_{10,2} + x_{10,3} = 1$$

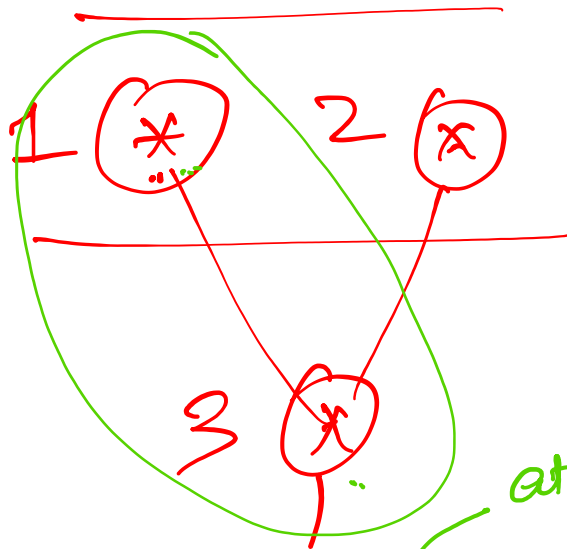
$$x_{11,2} + x_{11,3} + x_{11,4} = 1$$

$$x_{n,5} = 1$$



$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_{11} + x_{12} = 1 \quad \checkmark$$



$$\begin{array}{cc} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{array}$$

at least  
 { 3 can start one time step  
 after 1 starts

$$2x_{32} + 3x_{33} - x_{11} - 2x_{12} - 1 \geq 0$$

$$2x_{32} \geq 1x_{11} + 1$$

$$\left. \begin{array}{l} 2x_{32} + 3x_{33} \geq 1x_{11} + 2x_{12} + 1 \\ 2 \geq 1x_{11} + 2x_{12} + 1 \end{array} \right\}$$

$$x_{32} = 1$$

$$x_{12} = 0$$

$$x_{11} = 1$$



# ILP Formulation

Constraints – based on sequencing

(more than one starting time for at least one operation)

$$2x_{7,2} + 3x_{7,3} - x_{6,1} - 2x_{6,2} - 1 \geq 0$$

$$2x_{9,2} + 3x_{9,3} + 4x_{9,4} - x_{8,1} - 2x_{8,2} - 3x_{8,3} - 1 \geq 0$$

$$2x_{11,2} + 3x_{11,3} + 4x_{11,4} - x_{10,1} - 2x_{10,2} - 3x_{10,3} - 1 \geq 0$$

$$4x_{5,4} - 2x_{7,2} - 3x_{7,3} - 1 \geq 0$$

$$5x_{n,5} - 2x_{9,2} - 3x_{9,3} - 4x_{9,4} - 1 \geq 0$$

$$5x_{n,5} - 2x_{11,2} - 3x_{11,3} - 4x_{11,4} - 1 \geq 0$$



# ILP Formulation

2M ✓  
2AL

## Resource Constraints

$$X_{1,1} + X_{2,1} + X_{6,1} + X_{8,1} \leq 2$$

$$X_{3,2} + X_{6,2} + X_{7,2} + X_{8,2} \leq 2$$

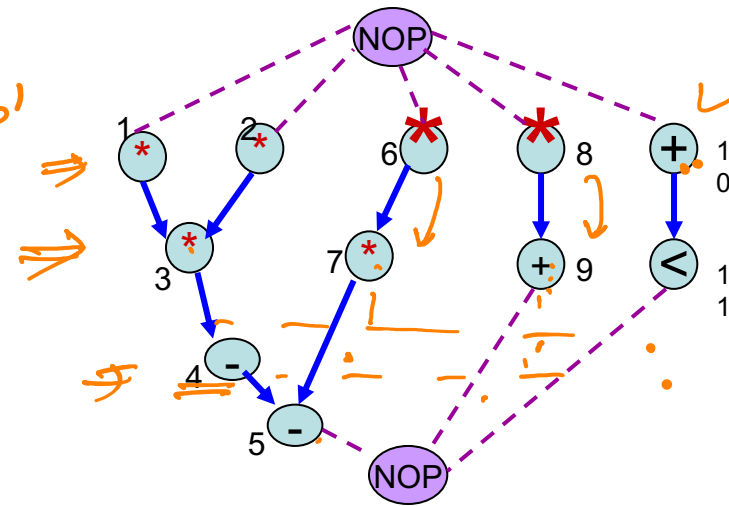
$$X_{7,3} + X_{8,3} \leq 2$$

$$X_{10,1} \leq 2$$

$$X_{9,2} + X_{10,2} + X_{11,2} \leq 2$$

$$X_{4,3} + X_{9,3} + X_{10,3} + X_{11,3} \leq 2$$

$$X_{5,4} + X_{9,4} + X_{11,4} \leq 2$$



# ILP Formulation

---

Optimize  $\sum_i \sum_j c_{ij} x_{ij}$

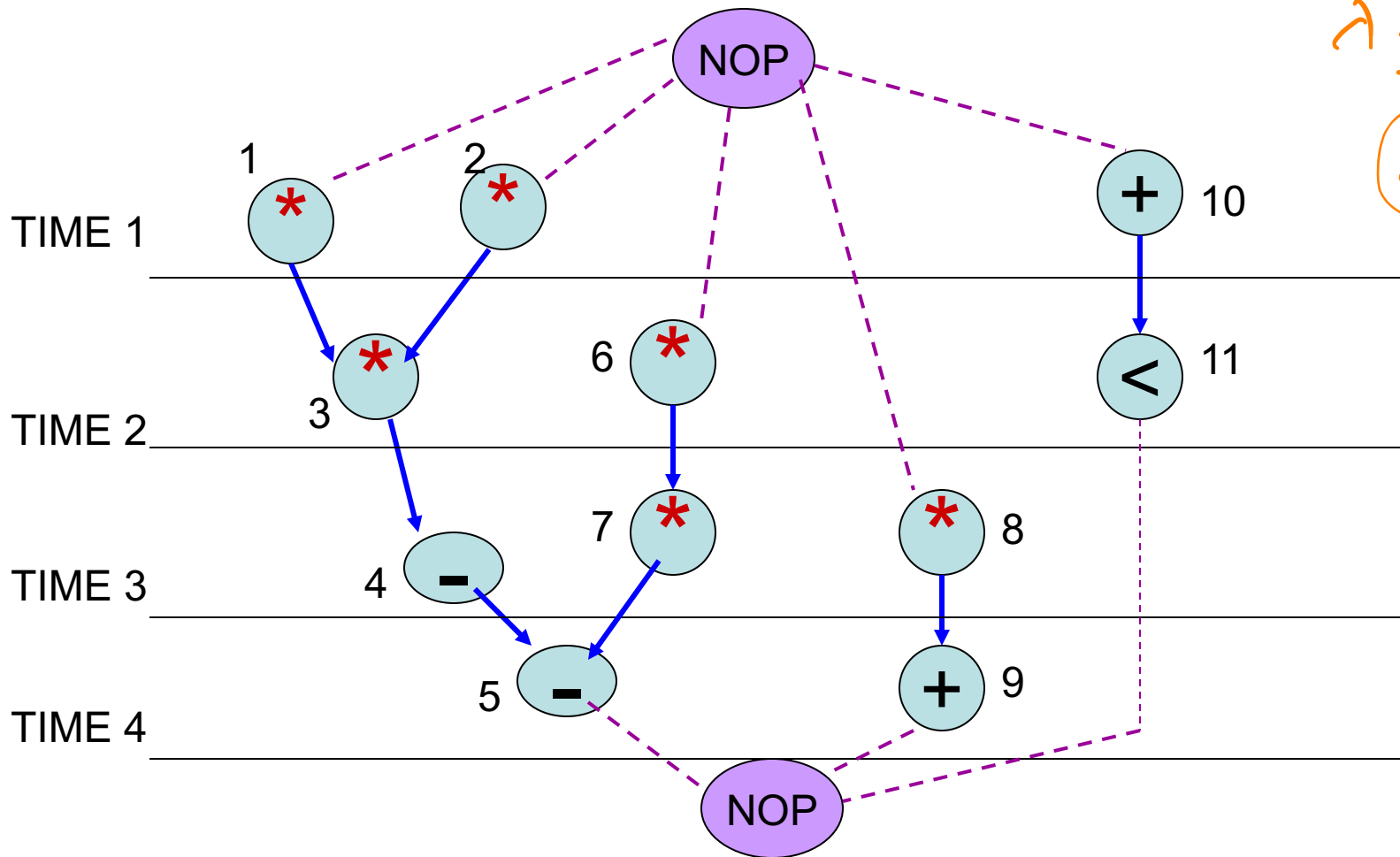
El. die

$$\begin{aligned} & x_{6,1} + 2 x_{6,2} + 3 x_{7,2} + 3 x_{7,3} + x_{8,1} + 2 x_{8,2} + 3 x_{8,3} \\ & + 2 x_{9,2} + 3 x_{9,3} + 4 x_{9,4} + x_{10,1} + 2 x_{10,2} + 3 x_{10,3} \\ & + 2 x_{11,2} + 3 x_{11,3} + 4 x_{11,4} \end{aligned}$$



# Optimum Scheduling under Resource Constraint

2M + 2A



$\lambda = 4$   
 $\lambda = 5$



# Thank You



09 Aug 2021

EE-677@IITB

**CADSL**