

Logic Optimization

Heuristic Based

✓
LOGIC
& 2 level
↓
Verification
↓
Testing
(TG.)

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EE-677: Foundations of VLSI CAD

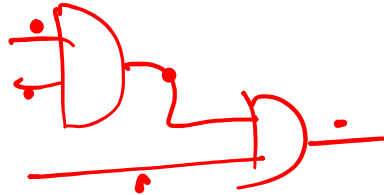


Lecture 25 on 07 Oct 2021

CADSL

Logic Testing Based Minimization

3 SAT
↑
NP Complete



$$\# \text{ faults} = 2 \times \# \text{ nets} \\ \geq 2 \times 5 = 10$$

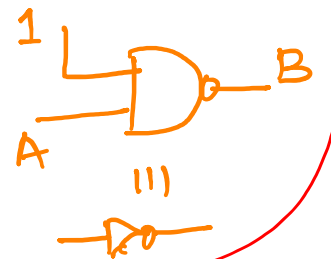
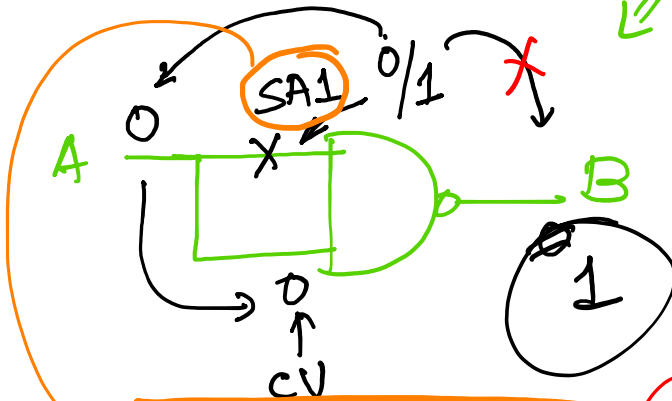
— if we can generate TV for all 10 faults ↑

→ One TV can detect multiple faults &

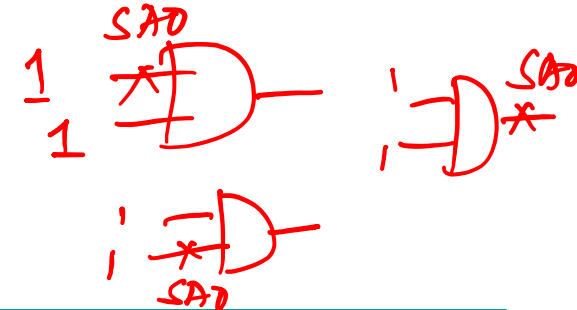
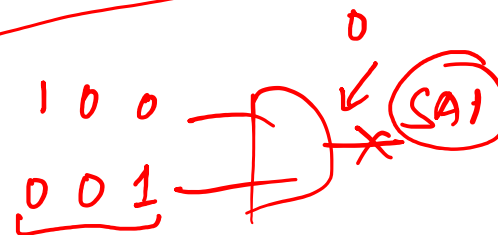
→ One fault can be tested by multiple TVs.

PODEM ⇒ { builds a decision tree at PI }

⇓



→ [Untestable fault]
Redundant fault



Logic Testing Based Minimization

Either we generate a TV for a fault or
it declare this fault as redundant

Identification of redundant fault.

1000
REDUNDANCY

950 ✓ TV
50 → Red.]

$$FC = \frac{950}{1000} \times 100 = 95\%$$

✓ (FE) = $\frac{950}{1000 - 50} \times 100 = 100\%$

TC
Test coverage

$$FC = \frac{950}{1000} \times 100$$

$$FE = \frac{950}{1000 - 40} = \frac{950 \times 10}{960}$$

1000, 950 40 red
✓ = 10

Fault
Efficiency → FE

Fault coverage

Fault Efficiency

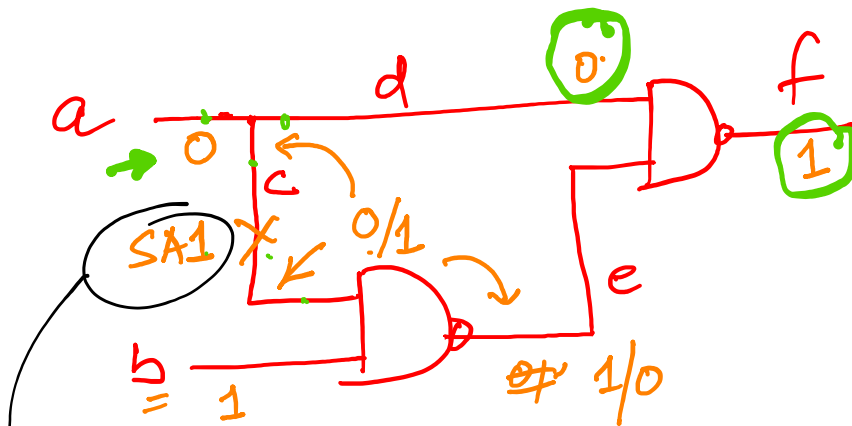
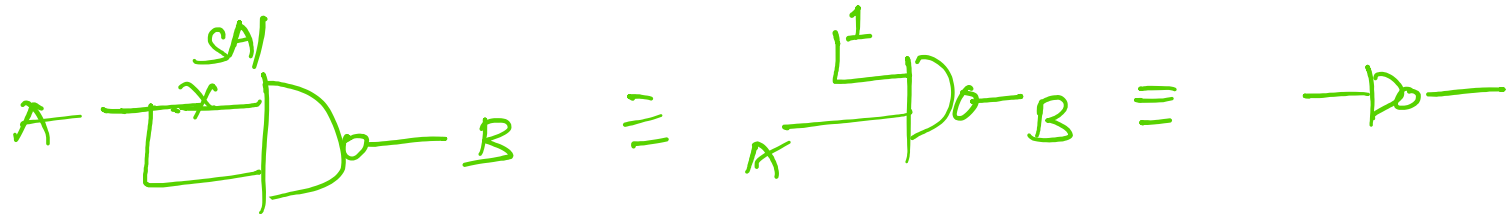
Boolean Diff.
SAT / PODEM

$$FC = \frac{\# \text{detectable fault}}{\# \text{Total fault}}$$

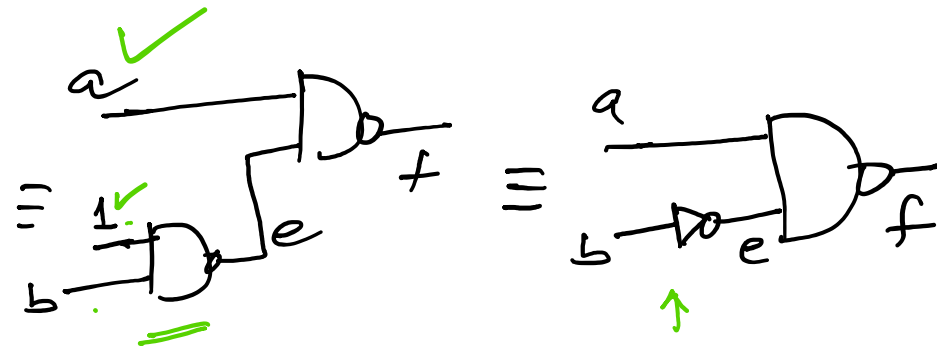
$$FE = \frac{\# \text{detectable}}{\# \text{Total faults} - \# \text{red. faults}}$$



Logic Testing Based Minimization

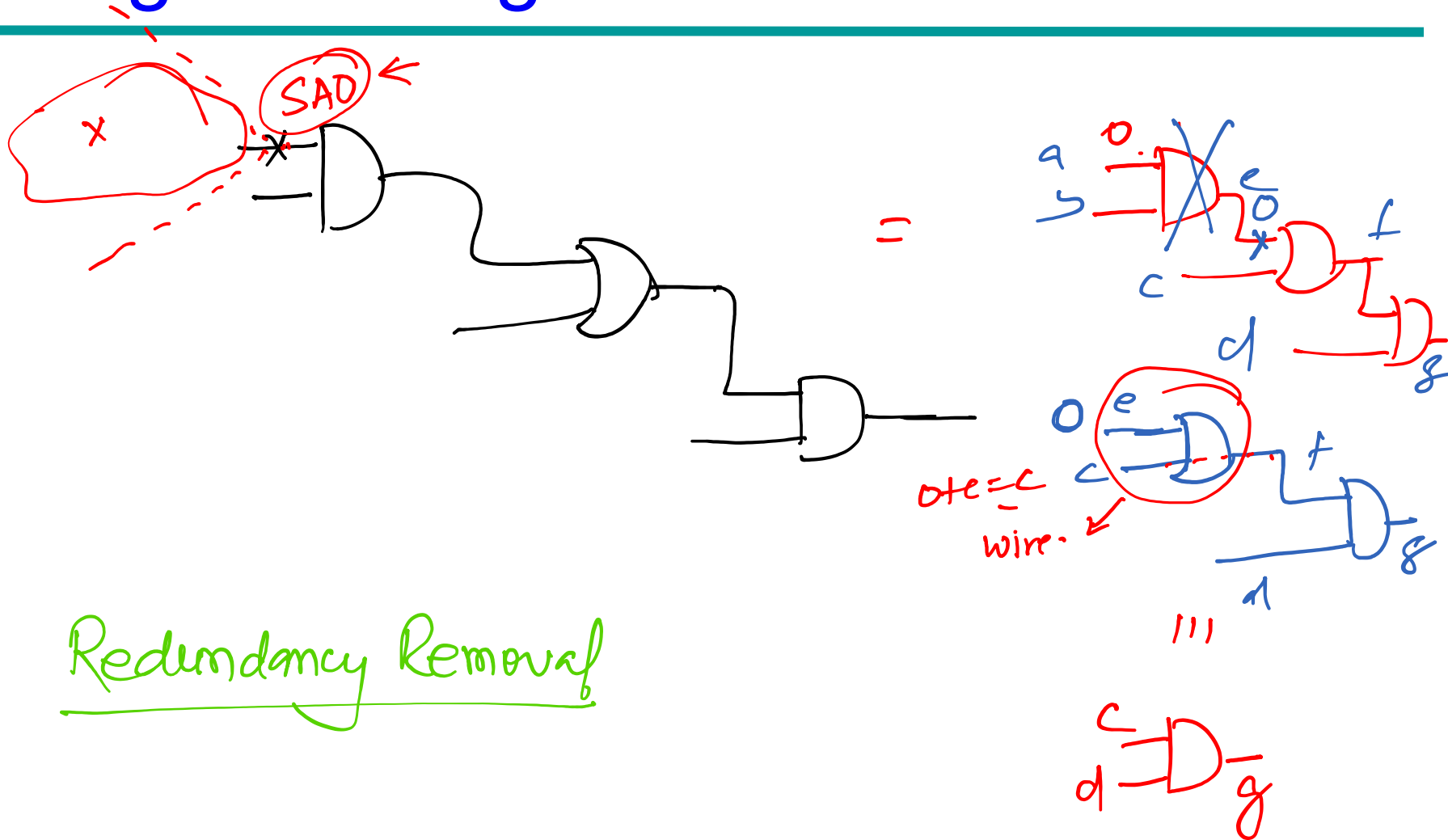


No test vector
Redundant fault



FC & FE

Logic Testing Based Minimization



Heuristic based 2 Level Logic Minimization

Exact Method \rightarrow QM algo. \rightarrow all PI_2 \checkmark
(\overline{EPI}) \checkmark
- Pebrick's method.
 \rightarrow ILP \checkmark



Some more background

- Function $f(x_1, x_2, \dots, x_i, \dots, x_n)$
- Cofactor of f with respect to variable x_i
 - $f_{x_i} = f(x_1, x_2, \dots, 1, \dots, x_n)$
- Cofactor of f with respect to variable x_i'
 - $f_{x_i'} = f(x_1, x_2, \dots, 0, \dots, x_n)$
- Boole's expansion theorem:
 - $f(x_1, x_2, \dots, x_i, \dots, x_n) = \underline{x_i f_{x_i} + x_i' f_{x_i'}}$
 - Also credited to Claude Shannon



Example

- Function: $f = ab + bc + ac$
- Cofactors:
 - $f_a = b + c$
 - $f_{a'} = bc$
- Expansion:
 - $f = a f_a + a' f_{a'} = a(b + c) + a'bc$



RM $\rightarrow \frac{n+4}{2}$

Unateness \rightarrow

• Function $f(x_1, x_2, \dots, x_i, \dots, x_n)$

• Positive unate in x_i when:

$$- f_{x_i} \geq f_{x_i'}$$

• Negative unate in x_i when:

$$- f_{x_i} \leq f_{x_i'}$$

• A function is positive/negative unate when positive/negative unate in all its variables

UNATE

$x_1x_2 + x_2x_3 + x_1x_4$
positive

negative
 $\bar{x}_1\bar{x}_2 + \bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_4$

$x_i \rightarrow$
 $x_1 \rightarrow$
 $x_1x_2 + x_1\bar{x}_3 + \bar{x}_2x_3$

$$f_{x_1} = x_2 + \bar{x}_3 + \bar{x}_2x_3$$

$$f_{\bar{x}_1} = \bar{x}_2\bar{x}_3$$

Operators

- Function $f(x_1, x_2, \dots, x_i, \dots, x_n)$
- *Boolean difference* of \underline{f} w.r.t. variable x_i :
 - $\partial f / \partial x_i \equiv f_{x_i} \oplus \underline{f_{x_i'}}$
- *Consensus* of \underline{f} w.r.t. variable x_i :
 - $C_{x_i} \equiv \underline{f_{x_i}} \cdot \underline{f_{x_i'}}$
- *Smoothing* of \underline{f} w.r.t. variable x_i :
 - $S_{x_i} \equiv \underline{f_{x_i}} + \underline{f_{x_i'}}$



$$f = ab + bc + ac$$

-

BEBAUT



Thank You

