

# Homework 4 Solutions

Communication Systems - I (EE 341), Autumn 2021

[2] Let  $f_c$  be the required frequency and  $f'_c = 1500$  kHz.  $f'_c$  is the image frequency corresponding to the frequency  $f_c$ . Hence,  $f'_c = f_c + 2f_{IF} = f_c + 2 \times 455$ . Hence,  $f_c = 1500 - 2 \times 455 = 590$  kHz.

[3]

The spectrum of the flat-top pulses is given by

$$\begin{aligned} H(f) &= T \operatorname{sinc}(fT) \exp(-j\pi fT) \\ &= 10^{-4} \operatorname{sinc}(10^{-4}f) \exp(-j\pi f 10^{-4}) \end{aligned}$$

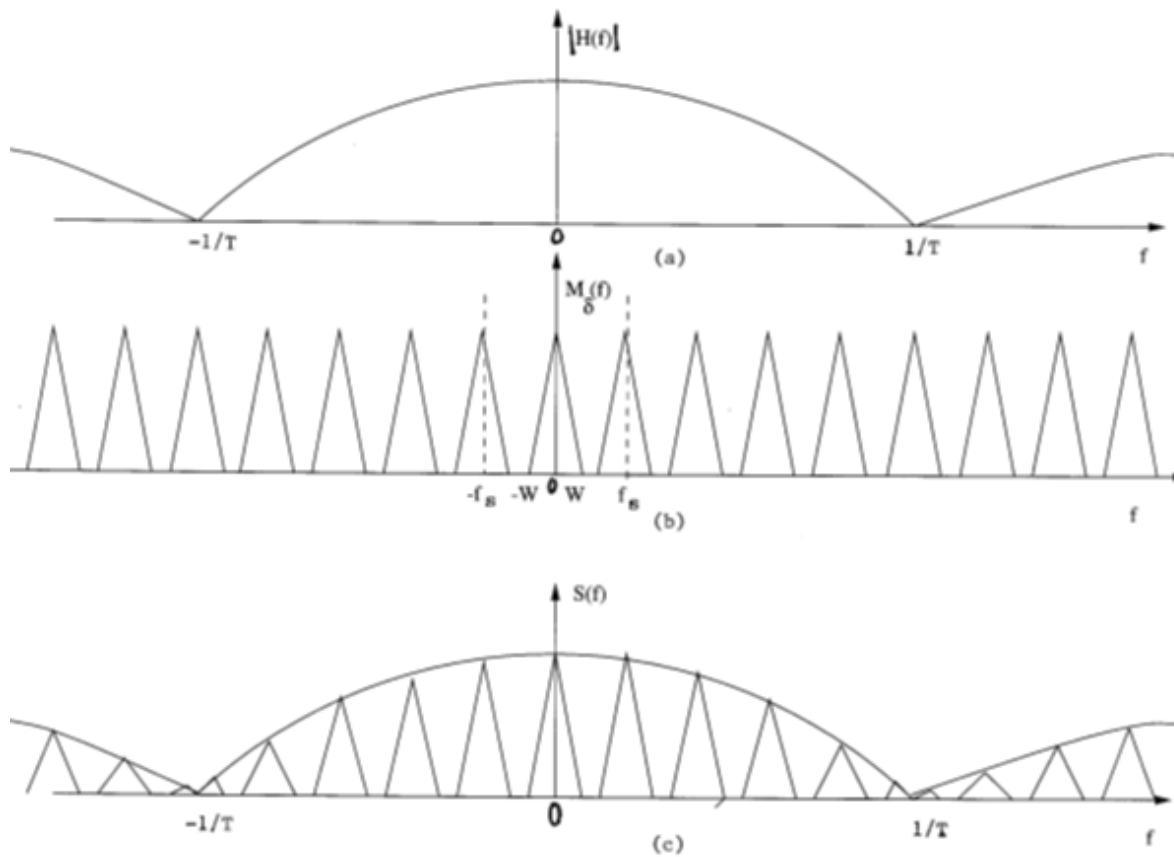
Let  $s(t)$  denote the sequence of flat-top pulses:

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

The spectrum  $S(f) = F[s(t)]$  is as follows:

$$\begin{aligned} S(f) &= f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f) \\ &= f_s H(f) \sum_{k=-\infty}^{\infty} M(f - kf_s) \end{aligned}$$

The magnitude spectrum  $|S(f)|$  is thus as shown in Fig. 1c.



$$\begin{aligned} 1/T &= 20,000 \text{ Hz} \\ f_s &= 1,000 \text{ Hz} \\ W &= 450 \text{ Hz} \end{aligned}$$

[4]

The minimum number of bits per sample is 7 for a signal-to-quantization noise ratio of 40 dB. Hence,

$$\begin{aligned} \left( \begin{array}{l} \text{The number of samples} \\ \text{in a duration of } 10s \end{array} \right) &= 8000 \times 10 \\ &= 8 \times 10^4 \text{ samples} \end{aligned}$$

The minimum storage is therefore

$$\begin{aligned} &= 7 \times 8 \times 10^4 \\ &= 5.6 \times 10^5 \\ &= 560 \text{ kbits} \end{aligned}$$

[5]

Suppose that baseband signal  $m(t)$  is modeled as the sample function of a Gaussian random process of zero mean, and that the amplitude range of  $m(t)$  at the quantizer input extends from  $-4A_{rms}$  to  $4A_{rms}$ . We find that samples of the signal  $m(t)$  will fall outside the amplitude range  $8A_{rms}$  with a probability of overload that is less than 1 in  $10^4$ . If we further assume the use of a binary code with each code word having a length  $n$ , so that the number of quantizing levels is  $2^n$ , we find that the resulting quantizer step size is

$$\delta = \frac{8A_{rms}}{2^R} \quad (1)$$

Substituting Eq. (1) to the formula for the output signal-to-quantization noise ratio, we get

$$(SNR)_o = \frac{3}{16}(2^{2R}) \quad (2)$$

Expressing the signal-to-noise ratio in decibels:

$$10\log_{10}(SNR)_o = 6R - 7.2 \quad (3)$$

This formula states that each bit in the code word of a PCM system contributes 6dB to the signal-to-noise ratio. It gives a good description of the noise performance of a PCM system, provided that the following conditions are satisfied:

1. The system operates with an average signal power above the error threshold, so that the effect of transmission noise is made negligible, and performance is thereby limited essentially by quantizing noise alone.
2. The quantizing error is uniformly distributed.
3. The quantization is fine enough (say  $R > 6$ ) to prevent signal-correlated patterns in the quantizing error waveform.
4. The quantizer is aligned with the amplitude range from  $-4A_{rms}$  to  $4A_{rms}$ .

In general, conditions (1) through (3) are true of toll quality voice signals. However, when demands on voice quality are not severe, we may use a coarse quantizer corresponding to  $R \leq 6$ . In such a case, degradation in system performance is reflected not only by a lower signal-to-noise ratio, but also by an undesirable presence of signal-dependent patterns in the waveform of quantizing error.

[6]

$f_s = 60 \text{ kHz}$  Recover using LPF  $25 \leq B \leq 35 \text{ kHz}$

$f_s = 45 \text{ kHz}$  Can't recover by filtering

$f_s = 25 \text{ kHz}$  Recover using BPF over  $f_l \leq |f| \leq 25 \text{ kHz}$  with  $10 < f_l < 15 \text{ kHz}$

