

High Level Synthesis

Binding

Virendra Singh

Computer Architecture and Dependable Systems Lab

Department of Electrical Engineering
Indian Institute of Technology Bombay

<http://www.ee.iitb.ac.in/~viren/>

E-mail: viren@ee.iitb.ac.in



EE-677: Foundations of VLSI CAD



Lecture 10 on 17 August 2021

CADSL

$$\frac{2M}{4A} \downarrow \downarrow \frac{2M+2A}{}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

ASAP

ASAP
ALAP

λ_{\min}
 λ_{\max}

slack
mobility

Cost

$$= \alpha \cdot \text{Area} + (1-\alpha) \text{ latency}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \leftarrow$$

$a_1 = \# \text{ multipl}$
 $a_2 = \# \text{ ALUs}$

$$\sum L \cdot \lambda_{ij}$$

$$\begin{pmatrix} \text{Maxi} \text{ latency} \\ \lambda \end{pmatrix} \leftarrow$$

Pareto optimal
solution



Concurrent Scheduling and Binding



Concurrent Scheduling and Binding

Concurrent Scheduling and Binding

- Define upper bound on resource usage and latency

Problems

- ❖ Intractable problem)

Scheduling
↓
Binding



Concurrent Scheduling and Binding

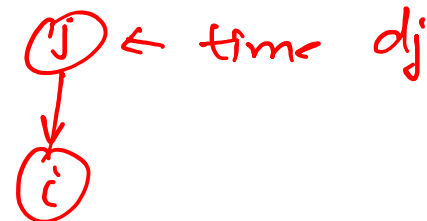
ILP Formulation

1. Start time of each operation is unique

$$\sum_i x_{ij} = 1 \quad \checkmark$$

2. Sequencing relations represented by $G_s(V,E)$ must be satisfied

$$\sum_i l_i x_{ij} \geq \sum_i l_i x_{ji} + d_j$$



3. Resource bound must be met at every schedule step

$$\sum_k \sum_m x_{im} \leq \underline{a_k}$$



Concurrent Scheduling and Binding

ϕ (a_1)

b_i

$$b_{11} + b_{12} + \dots + b_{1o_1} = 1$$

4. Operation has to bound one and only one resource

$$\sum_r (b_{ir}) = 1$$

5. Operation bound to same resource must not be concurrent

$$\sum_i b_{ir} \sum_{m=l-d_i+1}^l x_{im} \leq 1$$

$$\text{Latency: } \lambda = \sum_l l \cdot x_{nl} - \sum_l l \cdot x_{0l}$$

Minimize area and latency simultaneously



ILP Formulation

All operation must
start only once ✓

$$x_{0,1} = 1$$

$$x_{1,1} = 1$$

$$x_{2,1} = 1$$

$$x_{3,2} = 1$$

$$x_{4,3} = 1$$

$$x_{5,4} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

$$x_{7,2} + x_{7,3} = 1$$

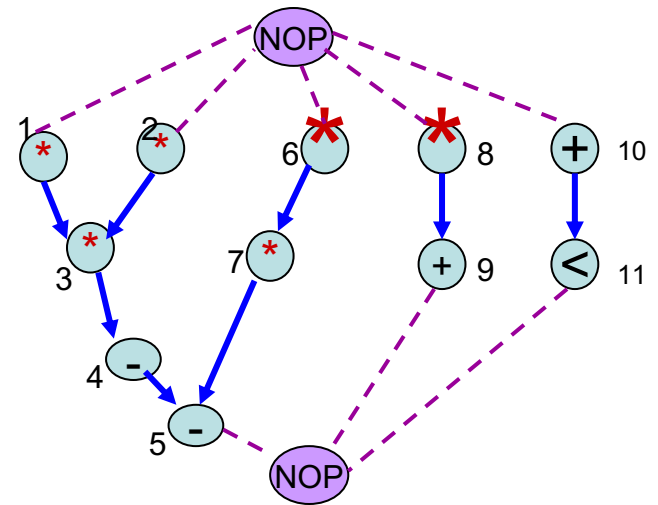
$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$

$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$

$$x_{10,1} + x_{10,2} + x_{10,3} = 1$$

$$x_{11,2} + x_{11,3} + x_{11,4} = 1$$

$$x_{n,5} = 1$$



ILP Formulation

Constraints – based on sequencing

$$\sum l_{ij} x_{ij} \Rightarrow \sum l_{ij} x_{ij} - d_i \geq 0$$

(more than one starting time for at least one operation)

$$2 x_{7,2} + \underline{3 x_{7,3}} - x_{6,1} - 2 x_{6,2} - 1 \geq 0$$

$$2 x_{9,2} + 3 x_{9,3} + 4 x_{9,4} - x_{8,1} - 2 x_{8,2} - 3 x_{8,3} - 1 \geq 0$$

$$2 x_{11,2} + 3 x_{11,3} + 4 x_{11,4} - x_{10,1} - 2 x_{10,2} - 3 x_{10,3} - 1 \geq 0$$

$$4 x_{5,4} - 2 x_{7,2} - 3 x_{7,3} - 1 \geq 0$$

$$5 x_{n,5} - 2 x_{9,2} - 3 x_{9,3} - 4 x_{9,4} - 1 \geq 0$$

$$5 x_{n,5} - 2 x_{11,2} - 3 x_{11,3} - 4 x_{11,4} - 1 \geq 0$$



Concurrent Scheduling and Binding

4. Operation has to bound one and only one resource

$$\sum_r b_{ir} = 1$$

5. Operation bound to same resource must not be concurrent

$$\sum_i b_{ir} \sum_{m=l-d_i+1}^l x_{im} \leq 1$$

$$\text{Latency: } \lambda = \sum_l l \cdot x_{nl} - \sum_l l \cdot x_{0l}$$

Minimize area and latency simultaneously



ILP Formulation

Resource Constraints

$$x_{1,1} + x_{2,2} + x_{6,1} + x_{8,1} \leq a1$$

$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq a1$$

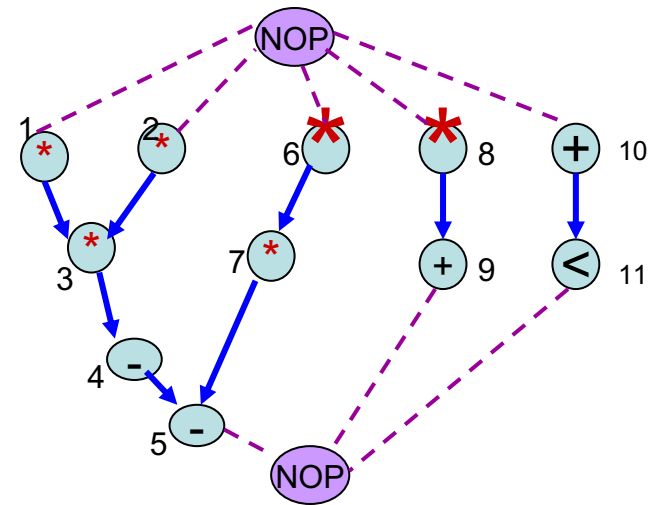
$$x_{7,3} + x_{8,3} \leq a1$$

$$x_{10,1} \leq a2$$

$$x_{9,2} + x_{10,2} + x_{11,2} \leq a2$$

$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq a2$$

$$x_{5,4} + x_{9,4} + x_{11,4} \leq a2$$



ILP Formulation

Resource Bind

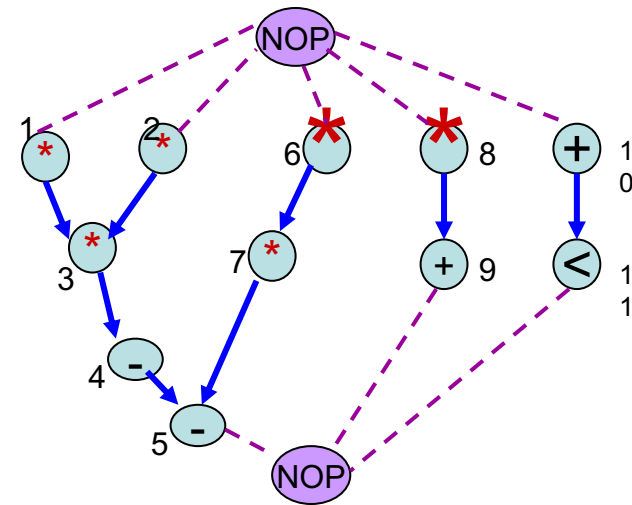
✓ $b_{1,1} + b_{1,2} + b_{1,3} + \dots + b_{1,a1} = 1$ ✓

$b_{2,1} + b_{2,2} + b_{3,3} + \dots + b_{4,a1} = 1$ ✓

$b_{10,1} + b_{10,2} + b_{10,3} + \dots + b_{10,a2} = 1$ ✓

Multi

ALU

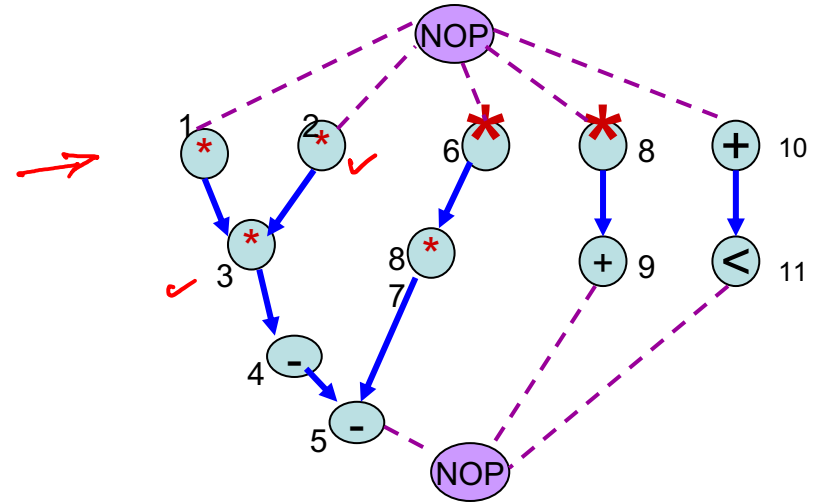


bir

ILP Formulation

~~χ~~
 $n \times \lambda$
 \underline{B}
 $n \times 1$

5



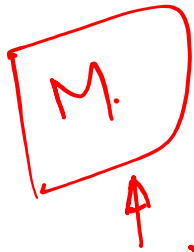
Resource Bind

$$\underline{b_{1,1} \cdot x_{1,1}} + \underline{b_{2,1} x_{2,1}} + \underline{b_{3,1} x_{3,1}} + \underline{b_{6,1} x_{6,1}} + \underline{b_{7,1} x_{7,1}} + \underline{b_{8,1} x_{8,1}} \leq \underline{1}$$

$$b_{1,2} \cdot x_{1,1} + b_{2,2} x_{2,1} + b_{3,2} x_{3,1} + b_{6,2} x_{6,1} + b_{7,2} x_{7,1} + b_{8,2} x_{8,1} \leq 1$$

$$b_{1,a1} \cdot x_{1,1} + b_{2,a1} x_{2,1} + b_{3,a1} x_{3,1} + b_{6,a1} x_{6,1} + b_{7,a1} x_{7,1} + b_{8,a1} x_{8,1} \leq 1$$





expensive.

Array

✓ $\underline{M1} \rightarrow \underline{A1}$, 1 cycle.
✓ $\underline{M2} \rightarrow \underline{A2}$, 2 cycles.
⇒ $\underline{A2 < A1}$.

⊗ $M1$

⊗ *

$M1$
or
 $M2$

Module Selection

Module Selection

- Resource type selection \longrightarrow Module selection
- Generalization of resource binding ✓
- We assume more than one resource type can match the functional requirement of an operation type
 - Resource types are compatible if they can perform the same operation
 - Resource types are characterized by the pair
 - $\{(\text{area_type}_k, \text{delay_type}_k); k = 1, 2, \dots, n_{\text{res}}\}$
$$\left. \begin{array}{lll} A_1 & d_1 & \\ A_2 & d_2 & \end{array} \right\} \begin{array}{lll} A_1 = 5a_1 & d_1 = 1 & \\ A_2 = 3a_1 & d_2 = 2 & \end{array}$$

Module Selection ✓

ILP Formulation

1. Start time of each operation is unique

$$\sum_i x_{ij} = 1 \quad \checkmark$$

2. Sequencing relations represented by $G_s(V,E)$ must be satisfied

$$\sum_i l_i x_{ij} \geq \sum_i l_i x_{ji} + d_j \quad \leftarrow$$



3. Operation has to bound one and only one resource

$$\sum_r b_{ir} = 1$$

Module Selection



4. Operation bound to same resource must not be concurrent

$$\sum_i b_{ir} \sum_{m=l-d_i+1}^l x_{im} \leq 1$$



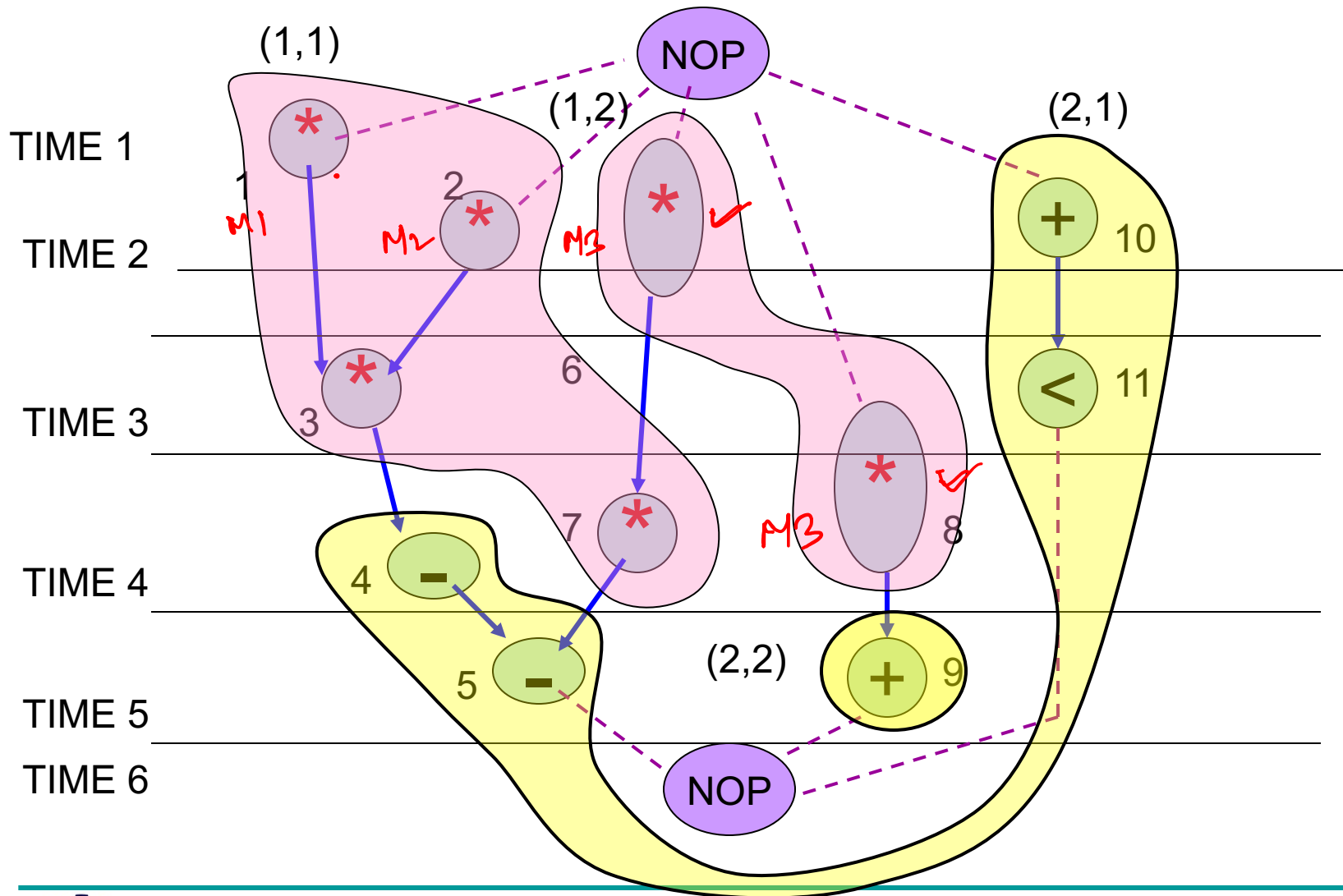
5. Execution delay for each bound operation

$$d_j = \sum_{r=1}^a b_{jr} \text{delay}_r ; j = 1, 2, \dots, n_{op}$$

Minimize area and latency simultaneously



Module Selection



Architectural Synthesis

Objective

- Area ✓
- Cycle time
- Latency
- Throughput

Resources

Schedule →
Binding → FU
Reg.
Mem port
BUs.

Improve

{ chaining of operations } pipelining

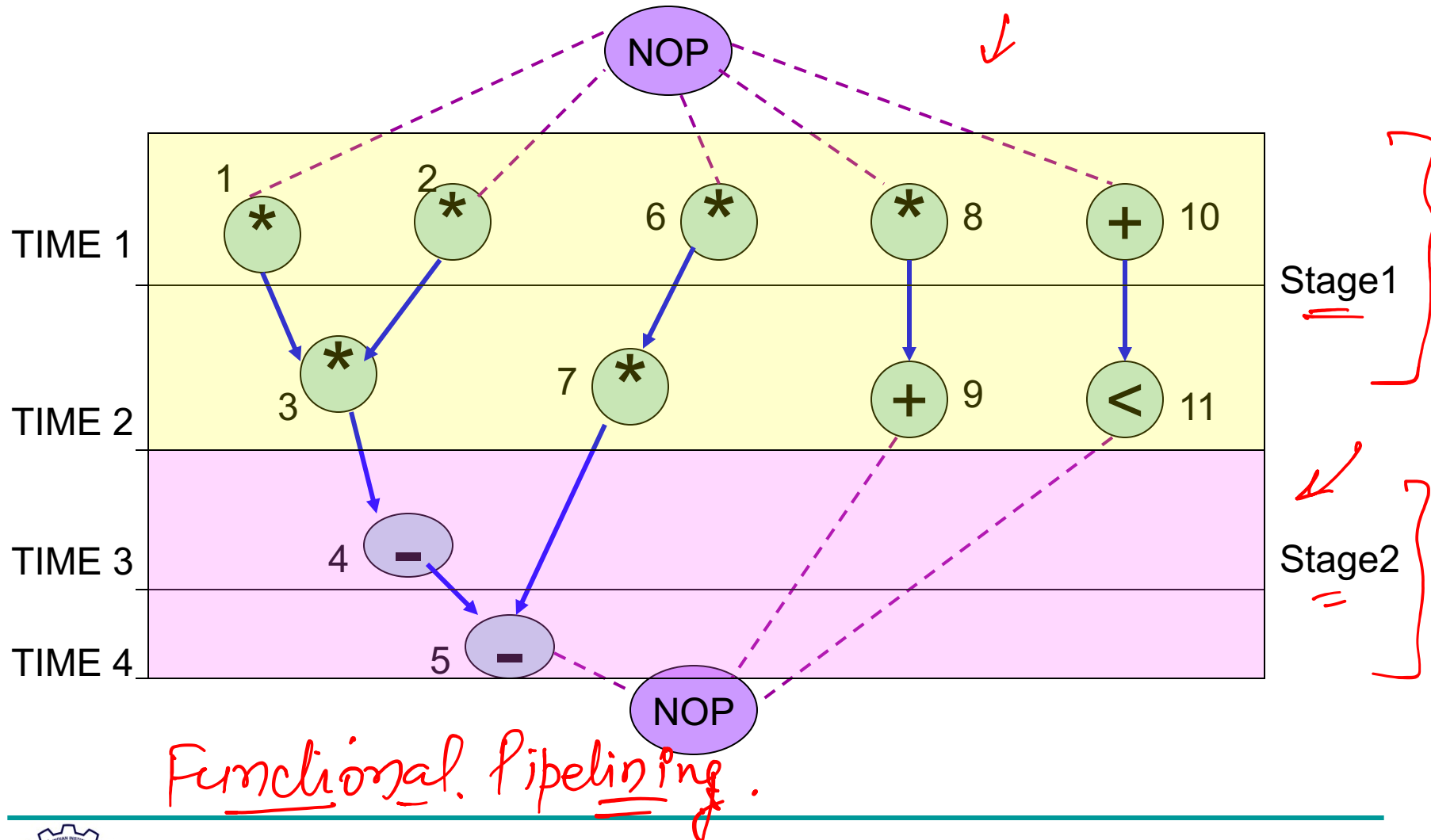
Pipelined Circuit Scheduling

Specification

- Sequencing Graph model ✓
 - Data Rate ✓]
1. Non-pipelined Sequencing graph ✓
 - Operations can be bound to pipelined model]
 2. Non hierarchical pipelined model with non-pipelined resources



Pipelined Sequencing Graph



Scheduling with Pipelined Resources

- Pipelined resources consume and produce data at time intervals that are smaller than execution delay

➤ Data introduction interval ✓

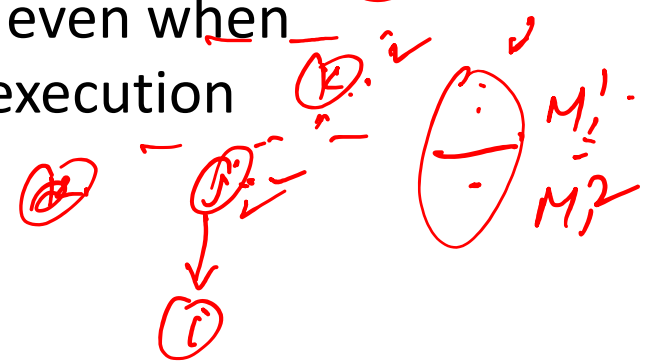
- Pipelined resources can be shared, even when corresponding operations overlap execution

❖ Necessary requirement

➤ no data dependency

➤ Operations do not start in the same time step

Mulh' 2 cycles
"Pipeline"



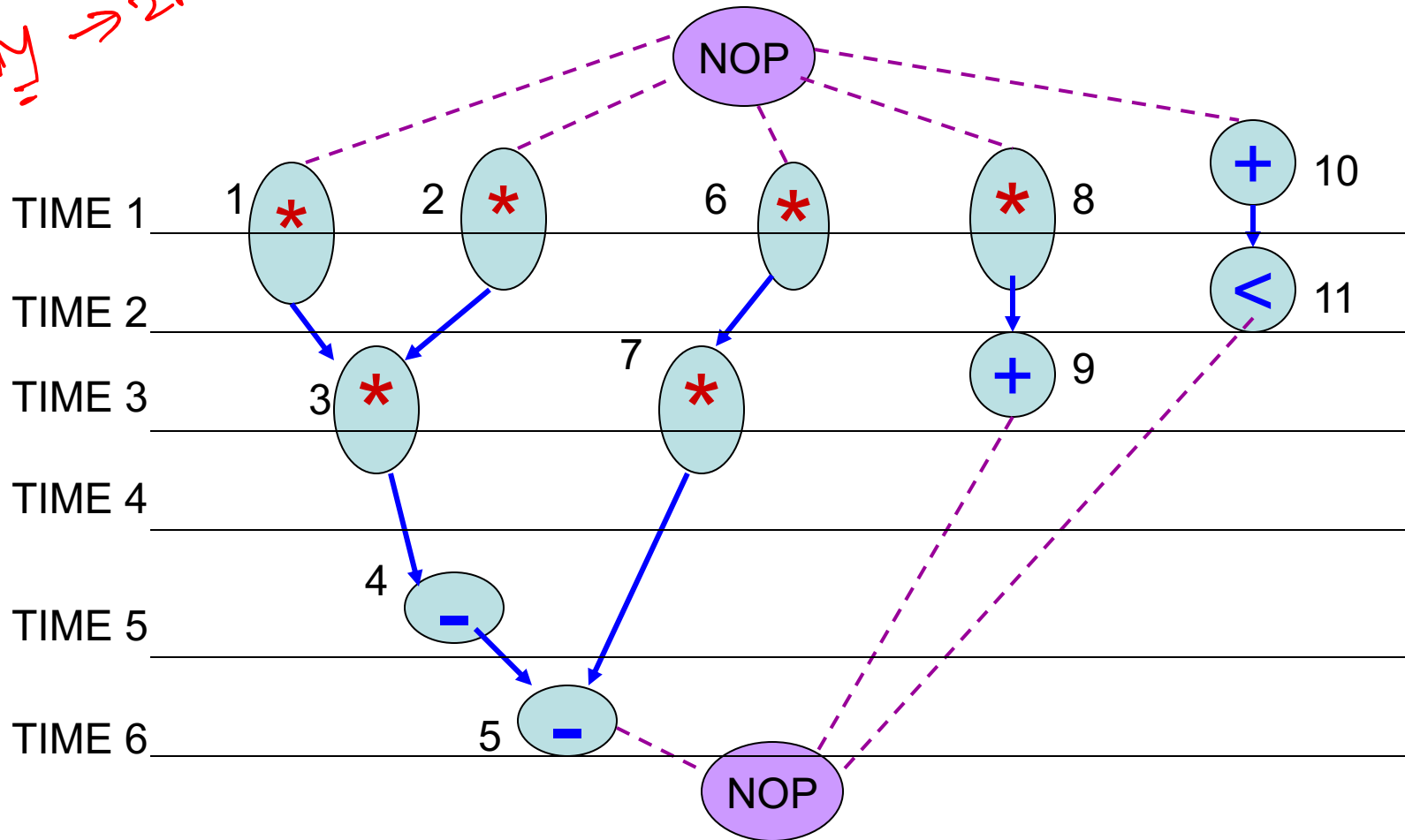
Scheduling with Pipelined Resources

- List scheduling algorithm can be extended to handle pipelined resources by allowing
 - Scheduling of overlapping operations with different start time, and
 - No data dependencies ✓

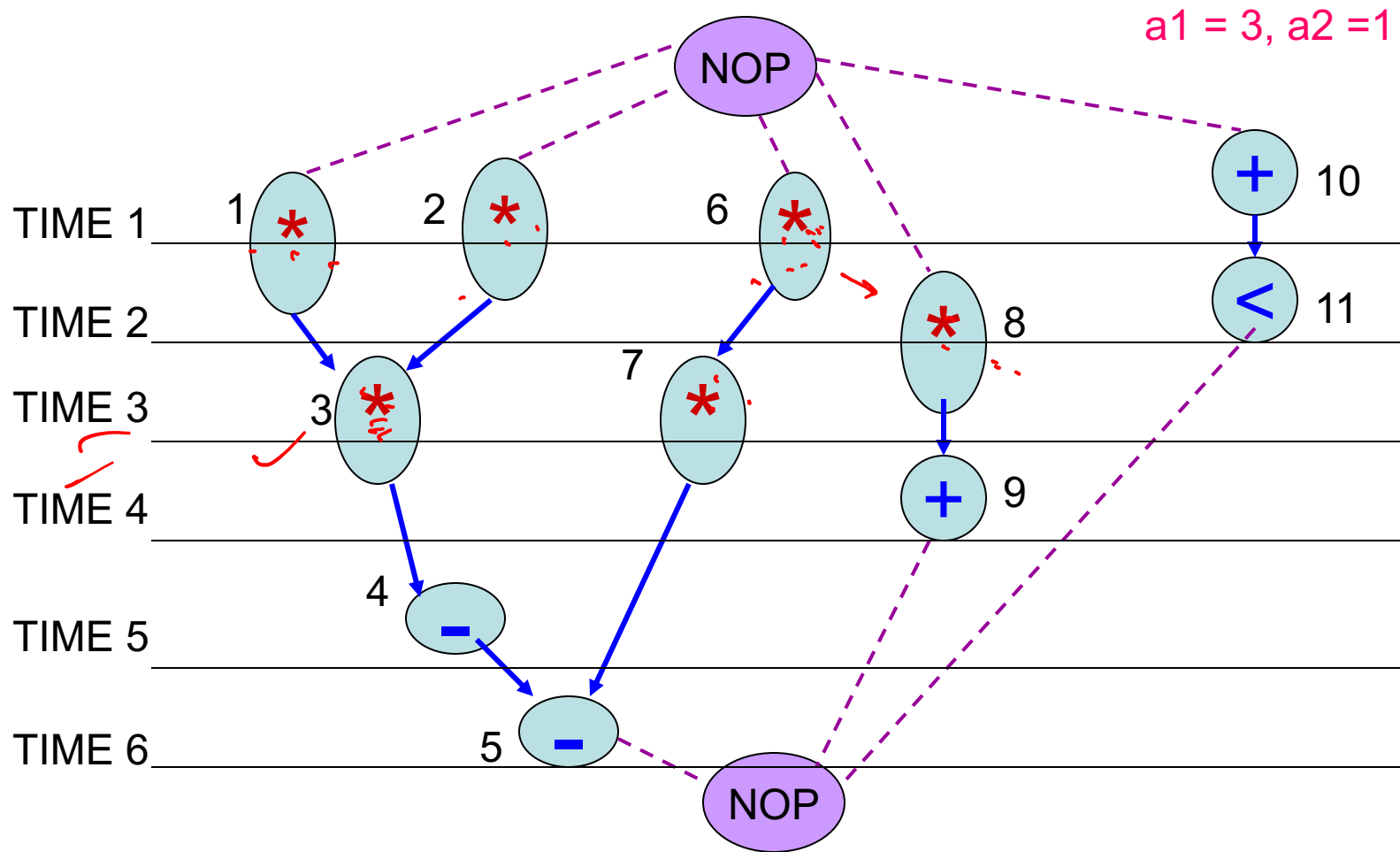


Scheduled Sequencing Graph

M → 2M.



Scheduled Seq. Graph with Pipelined Resources



Functional Pipelining

Assumption

- ❖ Resources are not pipelined
- ❖ Graph model is not hierarchical

data introduction interval.

- The number of pipelined resources depend on δ_0
- The higher δ_0 , larger the operations executing concurrently
- Upper bound on resource uses implies lower bound on δ_0



Functional Pipelining

ILP Formulation

- Uniqueness constraints ✓
- Sequencing constraints ✓
- **Resource constraints**

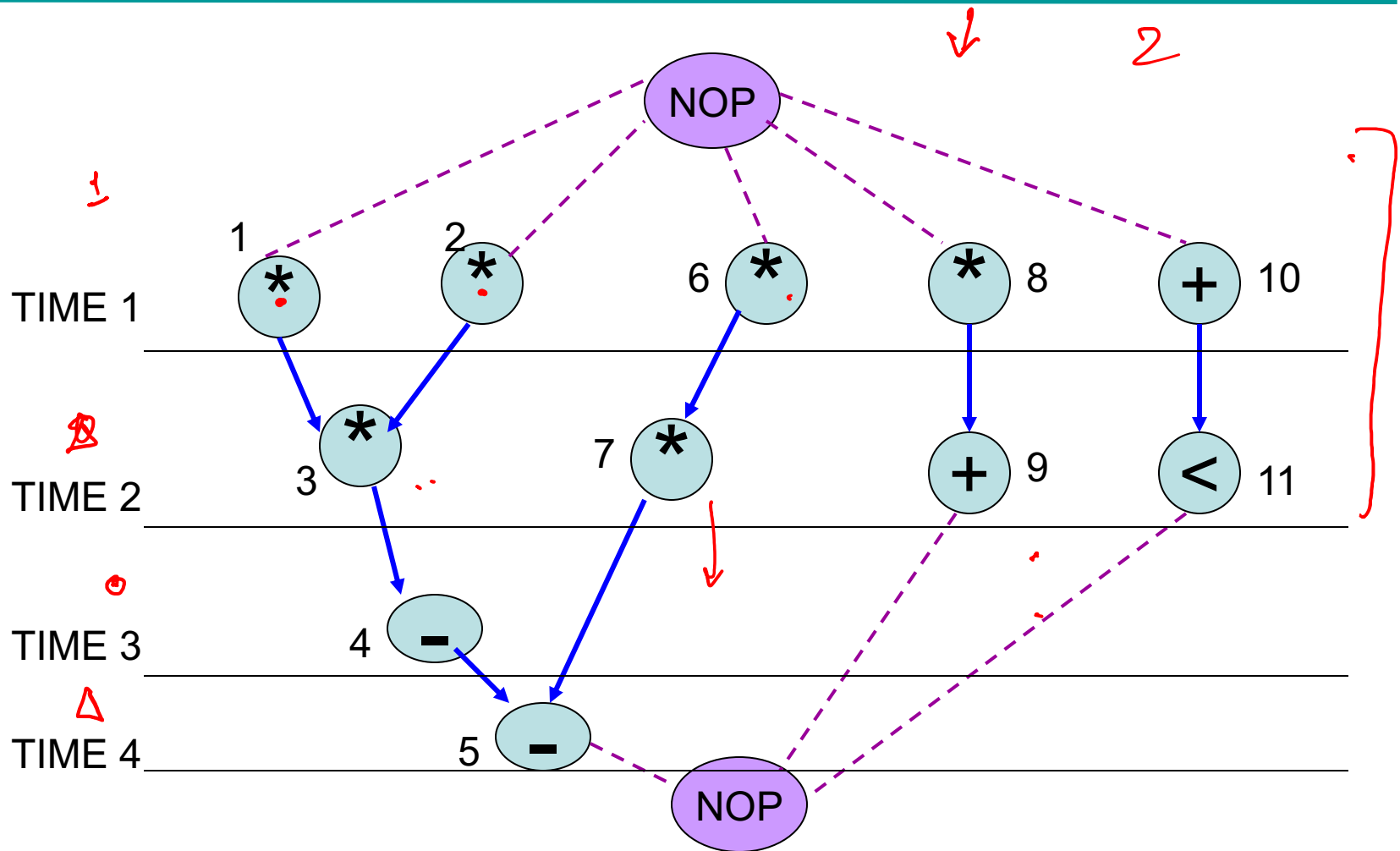
$$\sum_p \sum_k x_{i,l+p\delta_0} \leq a_k$$

$$\delta_0 = 2$$

$$x_{1,1} \quad x_{1,\underline{3}}$$



Sequencing Graph



Functional Pipelining

All operation must
start only once



$$x_{0,1} = 1$$

$$x_{1,1} = 1$$

$$x_{2,1} = 1$$

$$x_{3,2} = 1$$

$$x_{4,3} = 1$$

$$x_{5,4} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

$$x_{7,2} + x_{7,3} = 1$$

$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$

$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$

$$x_{10,1} + x_{10,2} + x_{10,3} = 1$$

$$x_{11,2} + x_{11,3} + x_{11,4} = 1$$

$$x_{n,5} = 1$$



Functional Pipelining

Constraints – based on sequencing

(more than one starting time for at least one operation)

$$2 x_{7,2} + 3 x_{7,3} - x_{6,1} - 2 x_{6,2} - 1 \geq 0$$

$$2 x_{9,2} + 3 x_{9,3} + 4 x_{9,4} - x_{8,1} - 2 x_{8,2} - 3 x_{8,3} - 1 \geq 0$$

$$2 x_{11,2} + 3 x_{11,3} + 4 x_{11,4} - x_{10,1} - 2 x_{10,2} - 3 x_{10,3} - 1 \geq 0$$

$$4 x_{5,4} - 2 x_{7,2} - 3 x_{7,3} - 1 \geq 0$$

$$5 x_{n,5} - 2 x_{9,2} - 3 x_{9,3} - 4 x_{9,4} - 1 \geq 0$$

$$5 x_{n,5} - 2 x_{11,2} - 3 x_{11,3} - 4 x_{11,4} - 1 \geq 0$$



Functional Pipelining

Resource constraints

$$\sum_p \sum_k x_{i,l+p\delta 0} \leq a_k$$

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} + x_{7,3} + x_{8,3} \leq 3$$

$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq 3$$

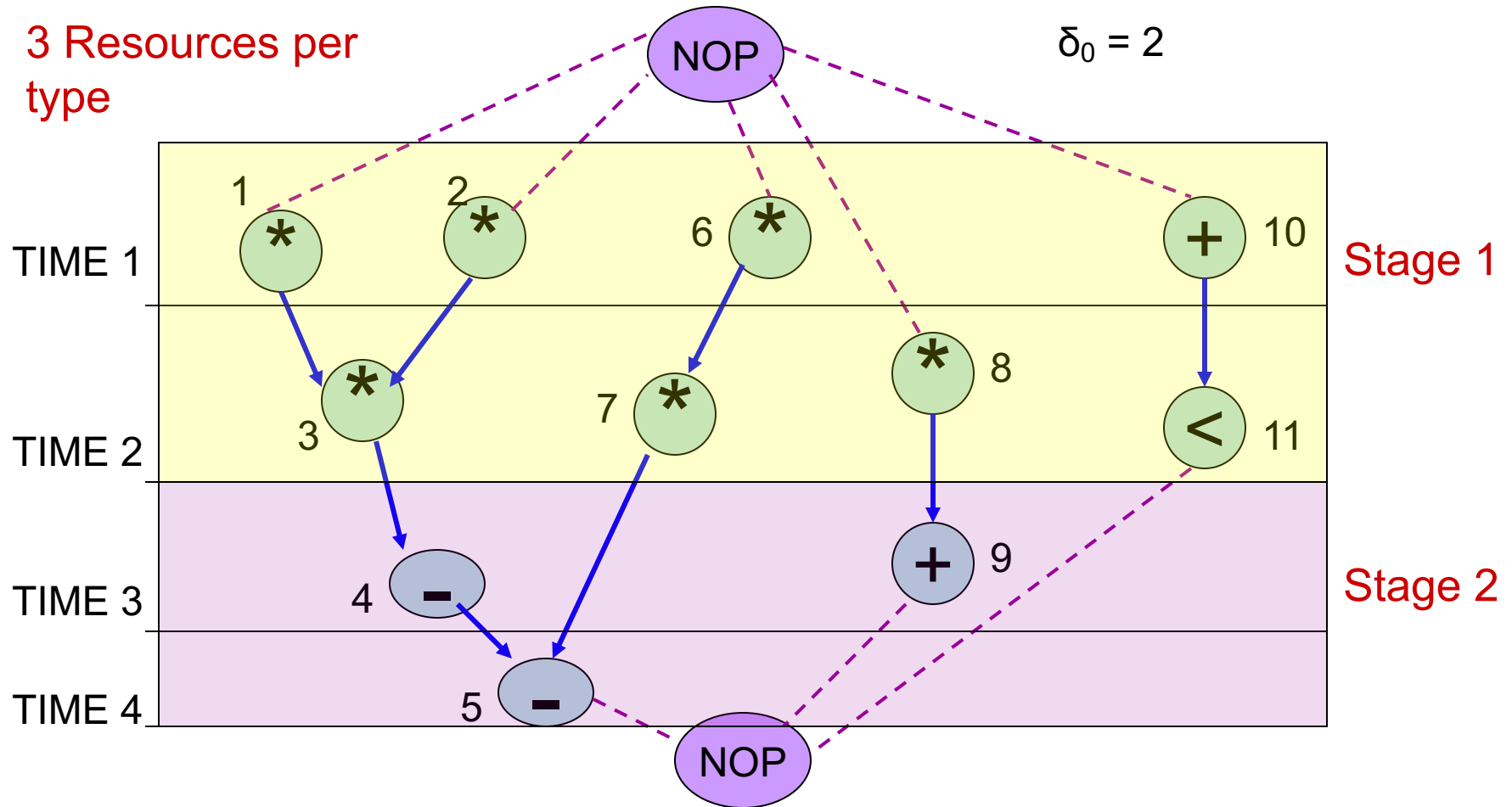
$$x_{10,1} + x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq 3$$

$$x_{9,2} + x_{10,2} + x_{11,2} + x_{5,4} + x_{9,4} + x_{11,4} \leq 3$$

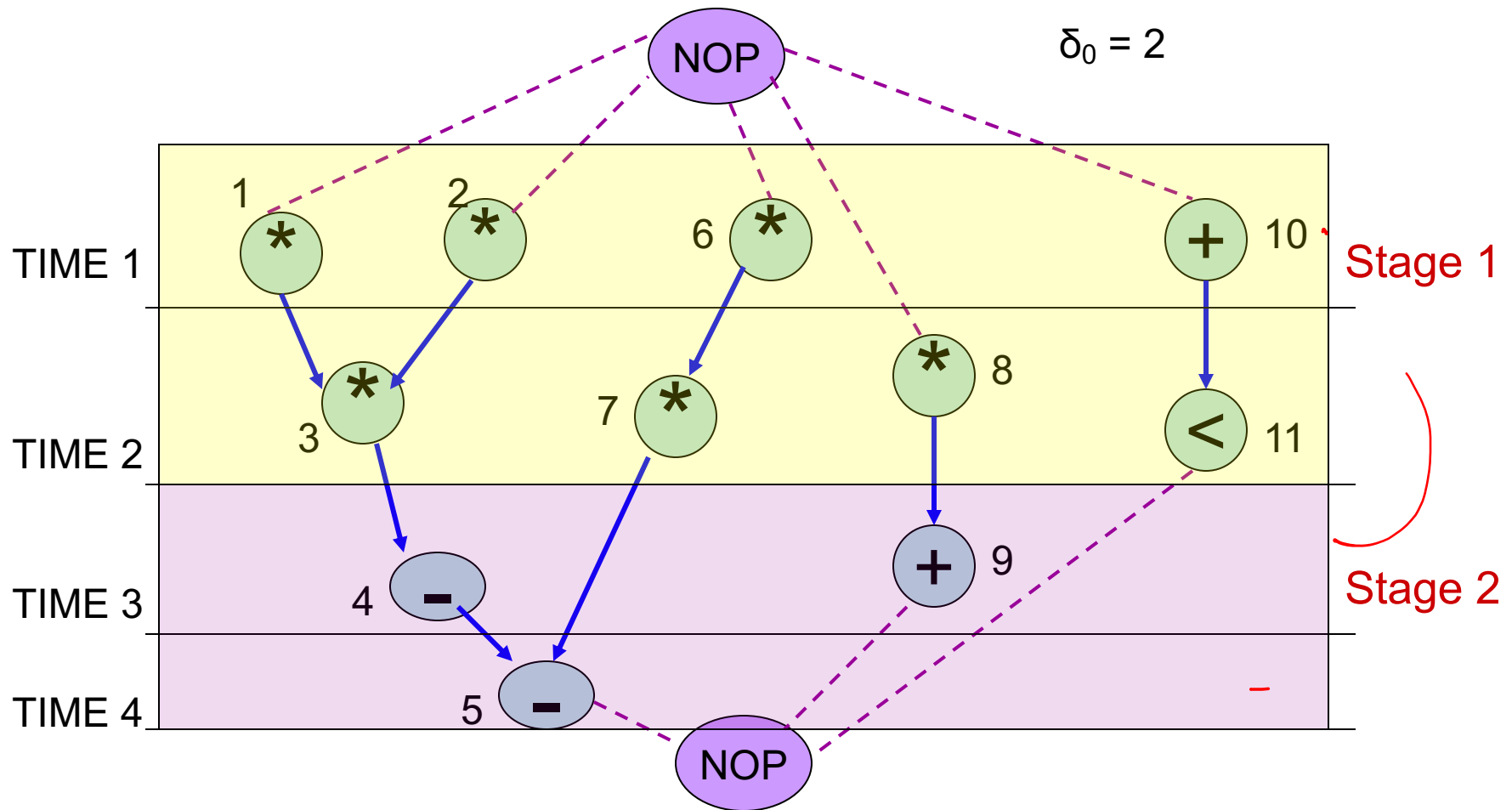


Scheduled Sequencing Graph

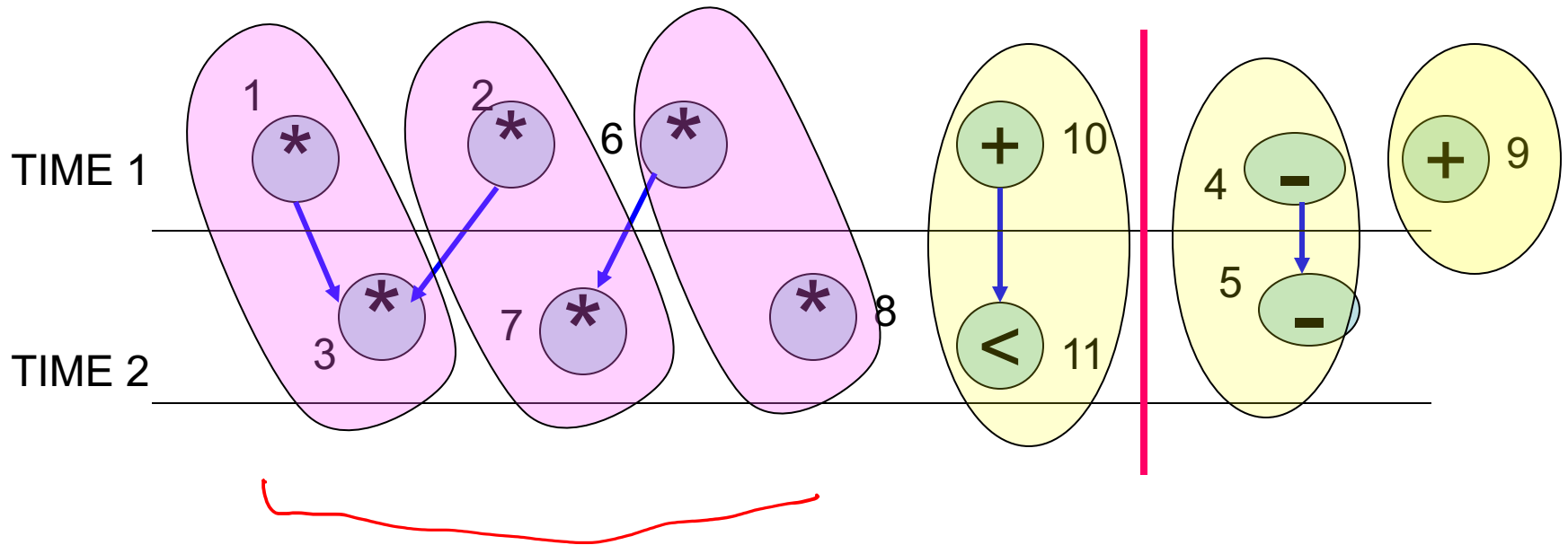
3 Resources per type



Scheduled Sequencing Graph

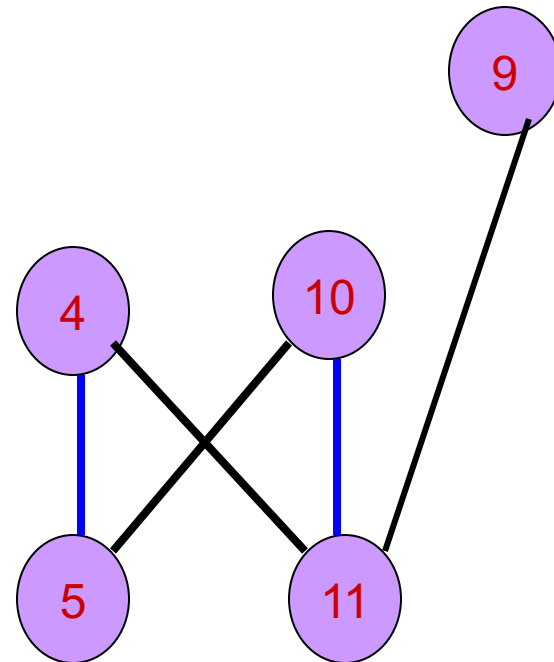
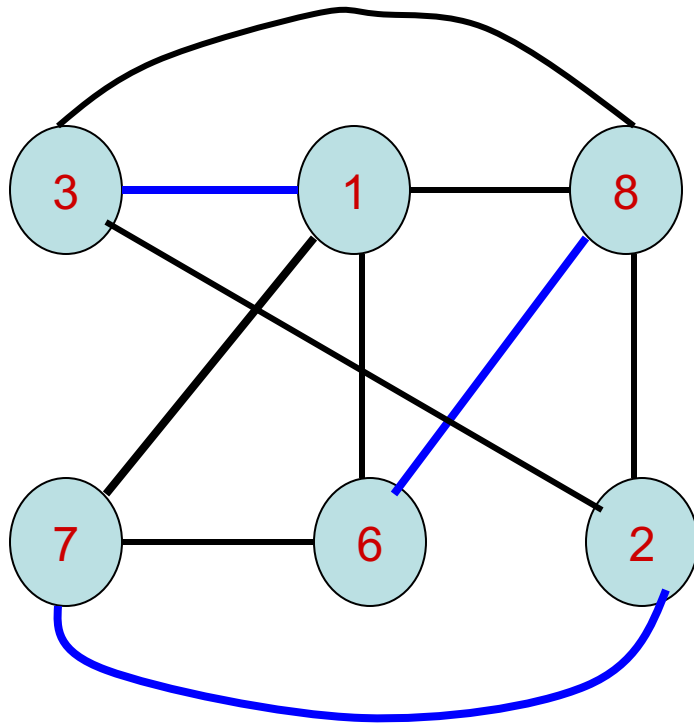


Sequencing Graph



Compatibility Graph

ZOLP.



Thank You



17 Aug 2021

CAD@IITB

CADSL