Bandwidth and the Multipath Channel

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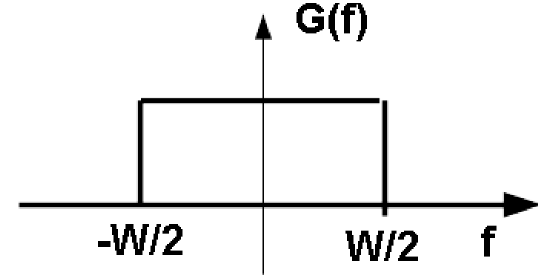
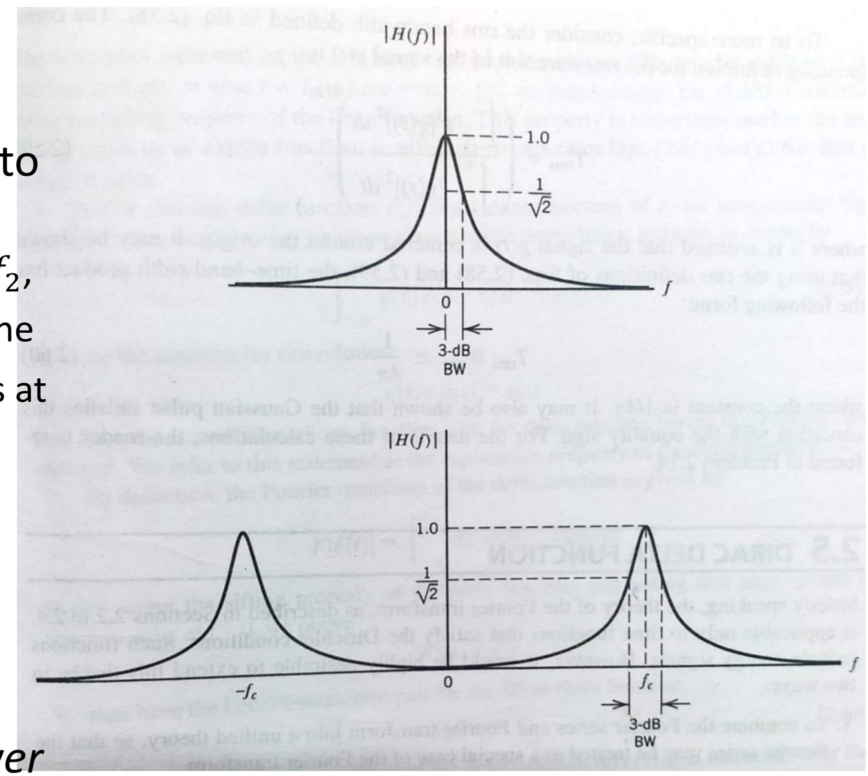
Bandwidth



- Recall: we discussed three definitions of bandwidth:
  1) bandwidth for the case where signal is strictly band-limited
  2) null-to-null bandwidth
  3) 3-dB bandwidth
- The definition 1) above is referred to as "*absolute bandwidth*":
  - ❑ $f_2 - f_1$, where the spectrum is zero outside the interval $f_1 < f < f_2$ along the positive f −axis
- E.g.: let $g(t)$ be such that $F[g(t)] = \mathrm{rect}\left(\frac{f}{W}\right)$
- Absolute bandwidth of $g(t)$:
  - ❑ $\frac{W}{2}$
- Recall: 3-dB bandwidth of a signal $g(t)$ is defined to be $f_2 - f_1$ if:
  - ❑ $f_2 > f_1 \geq 0$, for frequencies inside the band $f_1 < f < f_2$, the amplitude spectrum $|G(f)|$ falls no lower than $\frac{1}{\sqrt{2}}$ of the maximum value of $|G(f)|$, and the maximum value occurs at a frequency inside the band $[f_1, f_2]$
- *Note*:
  - ❑ The term "3-dB" used because $|G(f_a)| = \frac{1}{\sqrt{2}}|G(f_b)|$ iff $|G(f_a)|^2 = \frac{1}{2}|G(f_b)|^2$, i.e., $|G(f_a)|^2$ is lower than $|G(f_b)|^2$ by 3-dB
  - ❑ 3-dB bandwidth also referred to as "*half-power bandwidth*"
- Next, we discuss some more definitions of bandwidth



Ref: "Communication Systems" by S. Haykin and M. Moher, 5th ed

Equivalent Noise Bandwidth

- Consider a power signal, $h(t)$, whose power spectral density (PSD) is $S_h(f)$
- Equivalent noise bandwidth, B_{eq}, is the width of a fictitious rectangular spectrum such that the power in that rectangular band is equal to the power associated with the actual spectrum over positive frequencies
- Let $f_0 > 0$ be a frequency at which $S_h(f)$ attains its maximum value
- Then B_{eq} can be found using:

  1) $B_{eq}S_h(f_0) = \int_0^\infty S_h(f)df$
- RHS of 1) is actual power of signal $h(t)$ for positive frequencies
- LHS of 1) is power of a rectangular spectrum with height $S_h(f_0)$ and bandwidth B_{eq}
- Hence,

  ❑ $B_{eq} = \frac{1}{S_h(f_0)}\int_0^\infty S_h(f)df$

Root Mean Square (RMS) Bandwidth

- RMS bandwidth is the square root of the second moment of a properly normalized form of the squared amplitude spectrum of the signal about a suitably chosen point
- Consider a low-pass signal, $g(t)$, whose Fourier transform is $G(f)$
- RMS bandwidth of $g(t)$:
  - $W_{rms} = \left(\frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2 df}{\int_{-\infty}^{\infty} |G(f)|^2 df} \right)^{1/2}$
- Mathematical evaluation easier than measurement in the lab

Bounded Spectrum Bandwidth and Power Bandwidth

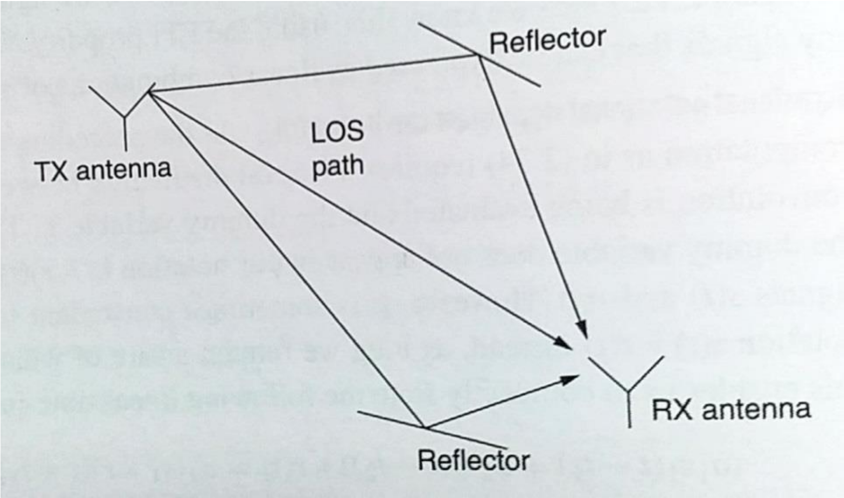
- *Bounded spectrum bandwidth* is $f_2 - f_1$ such that outside the band $f_1 < f < f_2$, the PSD, $S_h(f)$, must be down by at least a certain amount, say 50 dB, below the maximum value of the PSD
- *Power bandwidth* is $f_2 - f_1$, where $f_1 < f < f_2$ defines the frequency band in which 99 % of the total power resides
  - ❑ Similar to the Federal Communications Commission (FCC) definition of *occupied bandwidth*, which states that the power above the upper band edge f_2 is $\frac{1}{2}\%$ and the power below the lower band edge f_1 is $\frac{1}{2}\%$, leaving 99% of the total power within the occupied band

Multipath Channel

- Recall: in wireless communication, receiver often receives:
  - ❑ transmitted signal directly from transmitter
  - ❑ and also several delayed versions of it reflected from objects in environment
  - ❑ such a channel called "*multipath channel*" (see figure)
- Impulse response of a multipath channel:
  - ❑ $h(t) = \alpha_1 \delta(t - \tau_1) + \cdots + \alpha_m \delta(t - \tau_m)$
- Frequency response:
  - ❑ $H(f) = \alpha_1 e^{-j2\pi f \tau_1} + \cdots + \alpha_m e^{-j2\pi f \tau_m}$
- The different complex exponentials (in the frequency domain) can interference with each other constructively or destructively
  - ❑ leads to significant fluctuation in $H(f)$ as f varies
  - ❑ called "*frequency-selective fading*"
- WLOG, assume that $\tau_1 < \tau_2 < \cdots < \tau_m$

Ref: U. Madhow, "Introduction to Communication Systems"

- Then frequency response can be written as:
  - ❑ $H(f) = e^{-j2\pi f \tau_1} \sum_{k=1}^{m} \alpha_k e^{-j2\pi f (\tau_k - \tau_1)}$
- The term $e^{-j2\pi f \tau_1}$ corresponds to a delay τ_1 seen by all frequencies and can be dropped by taking τ_1 as the time origin
- So:
  - ❑ $H(f) = \alpha_1 + \sum_{k=2}^{m} \alpha_k e^{-j2\pi f (\tau_k - \tau_1)}$



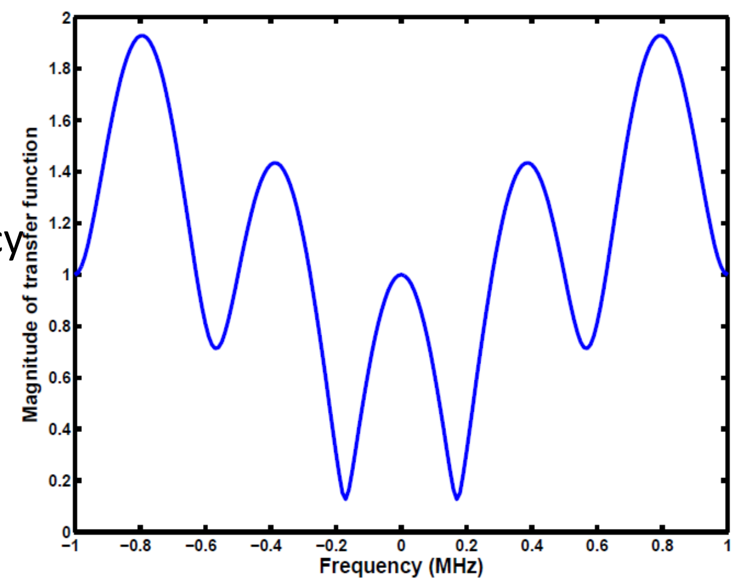
Multipath Channel (contd.)

- Recall:
  - $\tau_1 < \tau_2 < \cdots < \tau_m$
  - $H(f) = \alpha_1 + \sum_{k=2}^{m} \alpha_k e^{-j2\pi f(\tau_k - \tau_1)}$
- For $k \geq 2$, period of k'th sinusoid above (in frequency domain) is:
  - $\frac{1}{(\tau_k - \tau_1)}$
- Hence, fastest fluctuations as a function of f occur due to the sinusoid with period $\frac{1}{(\tau_m - \tau_1)}$
- Define the "*delay spread*" of the channel as:
  - $\tau_d = \tau_m - \tau_1$
- Then for a frequency interval that is significantly smaller than $\frac{1}{\tau_d} = \frac{1}{(\tau_m - \tau_1)}$:
  - *variation of $H(f)$ over the interval is small*
- Hence, we define the "*coherence bandwidth*" of the channel as:
  - $B_c = \frac{1}{\tau_d} = \frac{1}{(\tau_m - \tau_1)}$
  - *$H(f)$ can be modeled as approximately constant over intervals significantly smaller than coherence bandwidth*

Example

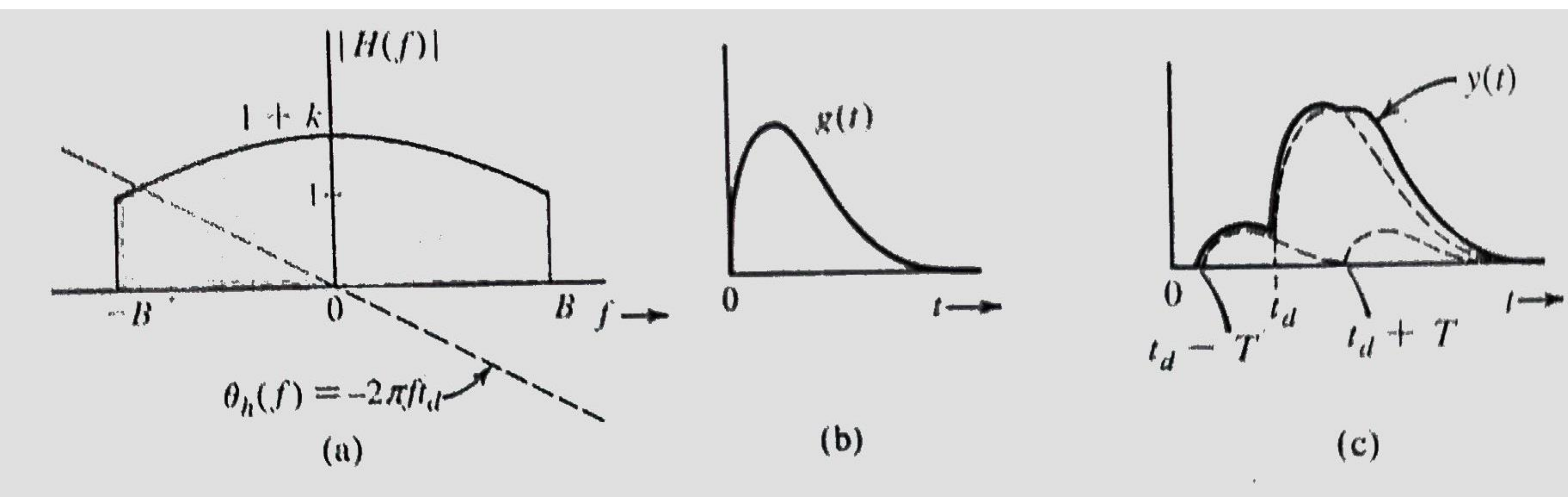
- Consider a multipath channel with impulse response:
  - $h(t) = \delta(t-1) - 0.5\delta(t-1.5) + 0.5\delta(t-3.5)$
- Dropping the first delay, as above, frequency response is:
  - $H(f) = 1 - 0.5e^{-j\pi f} + 0.5e^{-j5\pi f}$
- Suppose time is measured in μs and frequency in MHz
- Delay spread:
  - $2.5\ \mu$s
- Coherence bandwidth:
  - 400 kHz
- So we can assume that $H(f)$ is approximately constant over a frequency interval of 40 kHz (10% of coherence bandwidth)
  - Note: the above choice is somewhat arbitrary
- Fig. shows plot of $|H(f)|$ vs f on a linear scale
  - significant variations in $|H(f)|$ with f
  - however, if we zoom in to a window of width 40 kHz, then there are fewer fluctuations
- Suppose we send a "narrowband" signal on this channel, whose bandwidth is of order of 40 kHz
  - *magnitudes at all frequencies get scaled by roughly same amount*
- If it is affected by a "severe fade" (e.g., if its center frequency is around 0.2 MHz), then quality of received signal will be poor
- Solution to this problem:
  - use "*diversity*", e.g., send multiple narrowband signals with different center frequencies
  - likely that for some of them, received signal will be of good quality

Ref: U. Madhow, "Introduction to Communication Systems"



Time-Domain Interpretation of Delay Spread and Coherence Bandwidth

- Recall:
  - ❑ delay spread: $\tau_d = \tau_m - \tau_1$
  - ❑ coherence bandwidth: $B_c = \frac{1}{\tau_d} = \frac{1}{(\tau_m - \tau_1)}$
- Suppose we send a pulse $g(t)$ of duration T_w over a multipath channel as shown in figure
- Note that $\tau_d = 2T$
- If $T_w \gg \tau_d$, then intuitively, amount of distortion suffered by $g(t)$ will be low (see figure)
- This is consistent with fact that if $T_w \gg \tau_d$, then bandwidth of pulse will be $\ll B_c$ and hence $|H(f)|$ will be approximately constant over the frequency band on which $g(t)$ has non-zero spectral content



Ref: Lathi, Ding, "Modern Digital and Analog Communication Systems", 4th ed.