Multipath Propagation and Equalization

EE 340: Prelab Reading Material for Experiment 7

Autumn 2021

Multipath Propagation

Multipath propagation in wireless systems refers to the phenomenon in which the transmitted signal reaches the receiving antenna by taking two or more different paths as shown in Figure 1. It can be caused by various factors such as ionospheric reflection and refraction, reflection from water bodies, terrestrial objects, etc. Since the various paths are of different lengths, they arrive at the receiver with different delays. This can lead to Inter Symbol Interference (ISI). When ISI occurs, a part or, all of a given symbol which is transmitted is spread into the subsequent symbols, thereby resulting in errors at the receiver output.

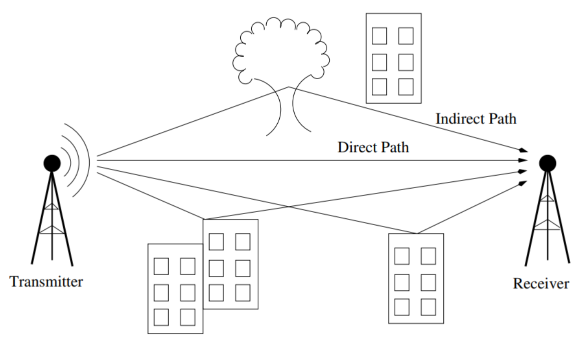

Figure 1: Multipath Propagation in a Wireless System

To model the multipath propagation at the receiver, consider the signal received through the direct path to be an impulse of unit amplitude. All other subsequent signals that arrive at the receiver after suffering reflections and refractions can be modeled as time delayed impulses with amplitude less than unity. Hence the addition of a signal and its time delayed versions obtained at the receiver input results in ISI.

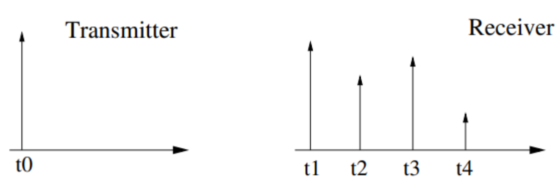

Figure 2: Multipath Model

We can use the z-transform to get a system transfer function for this multipath model (as shown in Figure 2, where the coefficients of the transfer function are called as taps.

Equalizer

An equalizer is a signal processing block that is used to reduce the effect of ISI on the transmitted symbols. Hence they are designed in such a way that they have the inverse transfer function of the multipath model. Let the transfer function H(z) of the multipath model be,

$$H(z) = 1 + \sum_{i=1}^{n} a_i z^{-i}$$

where a_i is the channel tap coefficient for i^{th} tap.
Then, the equalizer transfer function $E(z)$ is,

$$E(z) = \frac{1}{H(z)}$$

$$E(z) = 1 + \sum_{j=1}^{\infty} b_j z^{-j}$$

where b_j represent the equalizer tap coefficient for j^{th} tap and such an equalizer is called feed forward equalizer.

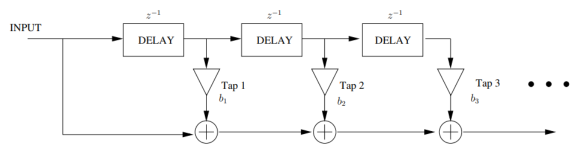

Figure 3: A Feed-Forward Equalizer

The channel tap coefficients (a_i s) can vary slowly with time due to changes in atmosphere or due to moving objects. In such cases, the equalizer tap coefficients (b_j s) also need to adapt to the time varying channel. In general adaptive equalizer algorithms are used as the channel coefficients are difficult to predict in advance (particularly for wireless channels). Most popular among them are Least Mean Square (LMS) algorithm and Constant Modulus Algorithm (CMA).

Least Mean Square Algorithm

LMS is a linear adaptive algorithm which minimises the mean square error (i.e. tap coeffiecients b_js are updated/adjusted by minimizing the $E[|error|^2])$. Let d[n] be the training sequence which is used for adjusting the taps/coefficients before random data/sequence arrives (can you think of the reason why adaptive mechanism is not performed when random data arrive ?), u[n] be the input to the receiver, y[n] be the output of the receiver as shown in Figure 4

Then, before the random data arrives, the taps/coefficients in LMS algorithm are updated by the equation

$$b_j[n+1] = b_j[n] + \mu \mathrm{u[n]} e^{*}\mathrm{[n]}$$

where $e[n] = d[n] - y[n]$, μ is the step size of LMS algorithm and determines the convergence rate. Too small a step size will make the algorithm take a lot of iterations while too large step size may diverge the weight taps (can you think of the reason?).

Constant Modulus Algorithm (CMA)

The CMA equalizer is an adaptive equalizer that works well when the signal has a constant modulus, i.e., when the signal constellation points lie on a circle (such as QPSK or 8-PSK). Multipath effects distort the received signal and its constellation diagram. The equalizer tries to ensure that the signal

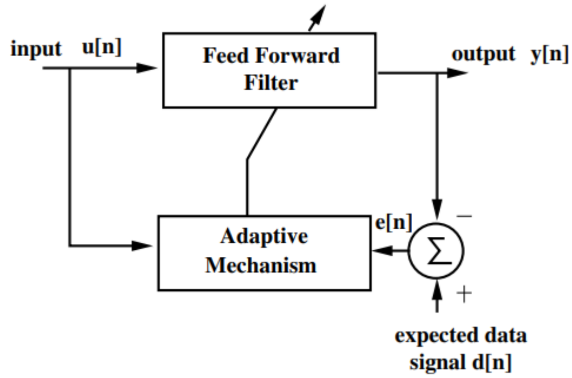

Figure 4: Least Mean Square Algorithm

samples from the equalizer output lie on a circle. This is done by minimizing the dispersion of the equalizer output $y[n]$ around a circular contour with a predefined radius R (for n^{th} symbol), which is termed as cost function and given by equation

$$J[n] = E\left[\left||y[n]|^2 - R^2\right|^2\right]$$

where $y[n] = y_I[n] + jy_Q[n]$ and $y_I[n], y_Q[n]$ are the in-phase and quadrature phase components of the signal obtained at the equalizer output for n^{th} symbol, and R is the radius of the circle. Let u[n] be the signal to be equalized and coefficient of k^{th} tap of the adaptive filter taps with L taps be $b_k[n]$, then the equalizer output and error are given by:

$$y[n] = \sum_{j=0}^{L-1} b_j^*[n]u[n-j] = B_n^H U_n$$

$$e[n] = |y[n]|^2 - R^2$$

Then the update equation of the filter coefficients for a CMA equalizer is given by:

$$b_j[n+1] = b_j[n] - \mu e[n]y^*[n]u[n]$$

where μ is the step factor which is to be carefully selected.

Usually, minimizing the difference ensures that the equalizer has compensated for the multipath effects added by the channel, and as a result, output samples lie on the desired constellation. It should be noted that if there are any phase and frequency offsets, the points may appear anywhere on a circle, and hence a carrier frequency and phase synchronization block (such as a costas loop) is required to remove these offsets to obtain the desired constellation plot.

Generally, some training sequences (symbol sequences know to the transmitter) are used for initial adjustment of tap coefficients. However, CMA algorithm can be used for "blind-adaptation," which means no training sequences are required (can you think of the reason?).

**Some of the project ideas include implementation of an adaptive equalizer (e.g., the equalizer used in lab can be made adaptive) or implementing other equalizers or developing GNU radio blocks to implement an equalizer. More information regarding equalizers can be found in Simon Haykin, Adaptive Filter Theory and GNU Radio wiki page.