EE340: Communications Laboratory
Autumn 2021

Prelab Material

Lab 5: Non-linearity and its effects in communication systems

Non-linear Systems

- Linear Systems: Satisfy superposition principle
- However, any practical system is non-linear (amount of non-linearity may vary)
- Non-linearity results in generation of "new frequency components" – i.e. frequency components that are not there at the input of the system.
- Memoryless non-linearity can be modeled as:

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + a_4 x^4(t) \; \ldots$$

- Memoryless means present output depends only on the present output (also see Appendix – last slide)

Effects of Non-Linearity

Consider a simplified non-linear system described by

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t)$$

For $x(t) = A\cos(\omega t)$,

$$y(t) = \frac{1}{2}a_2 A^2 + \left(a_1 + \frac{3}{4}a_3 A^2\right)A\cos(\omega t) + \frac{1}{2}a_2 A^2 \cos(2\omega t) + \frac{1}{4}a_3 A^3 \cos(3\omega t)$$

Important observations:

- **Second order non-linearity (a_2 coefficient):**
  Adds DC + 2nd harmonic

$$\boxed{\frac{1}{2}a_2 A^2\left(1 + \cos(2\omega t)\right)}$$

- **Third order non-linearity (a_3 coefficient):**
  Adds 3rd harmonic and Gain becomes input amplitude (*A*) dependent
  Also, a_3 is generally negative
  => gain compression with increasing *A*

$$\left(a_1 + \frac{3}{4}a_3 A^2\right)A\cos(\omega t) + \frac{1}{4}a_3 A^3 \cos(3\omega t)$$

$$\text{Gain}: \quad \left(a_1 + \frac{3}{4}a_3 A^2\right)$$

Second Order Non-Linearity

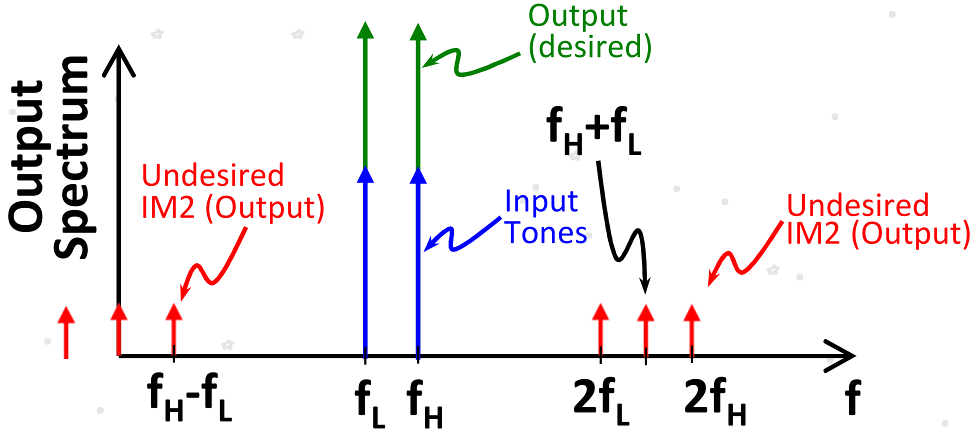
Consider a non-linear system described by

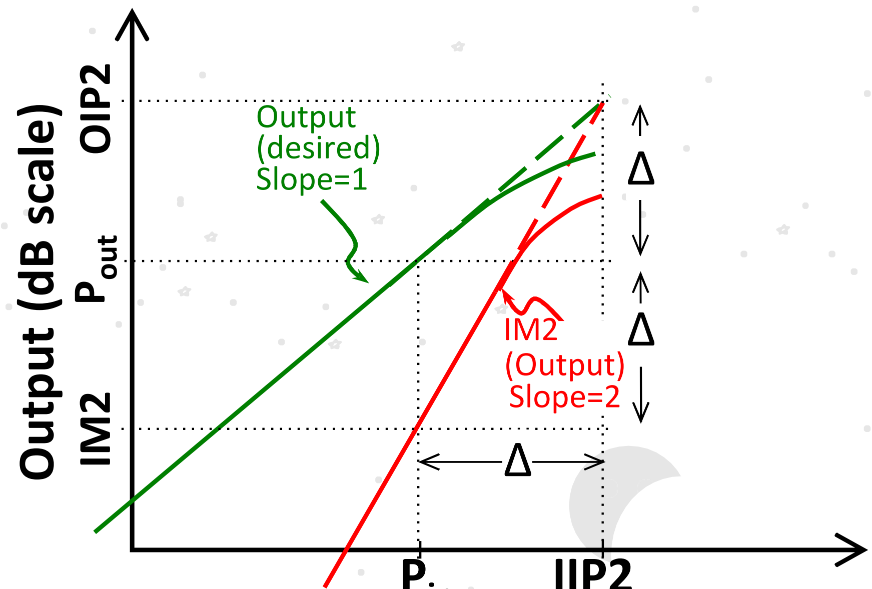
$$y(t) = a_1 x(t) + a_2 x^2(t); \qquad \Rightarrow \text{For } x = A(\cos\omega_1 t + \cos\omega_2 t):$$

$$\boxed{y(t) = a_2 A^2 + a_1 A\big(\cos(\omega_1 t) + \cos(\omega_2 t)\big) + a_2 A^2\left(\frac{\cos(2\omega_1 t) + \cos(2\omega_2 t)}{2} + \cos\big((\omega_1 - \omega_2)t\big) + \cos\big((\omega_1 + \omega_2)t\big)\right)}$$

The undesired spectral components generated due to the second order non-linearity coefficent a_2 at frequencies 0, $2\omega_1$, $2\omega_2$, $2(\omega_1 - \omega_2)$ and $2(\omega_1 + \omega_2)$ are called IM2 (Inter-Modulation products due to 2nd order non-linearity) components

Observations:



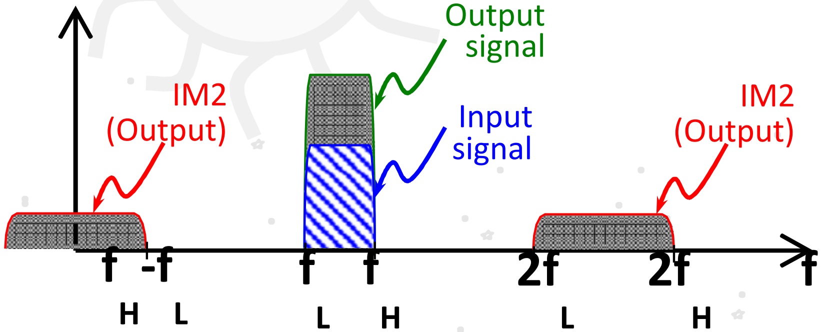


IIP2 = Pin + Δ (dB scale)

OIP2 = Pin + Δ + Gain (dB scale)

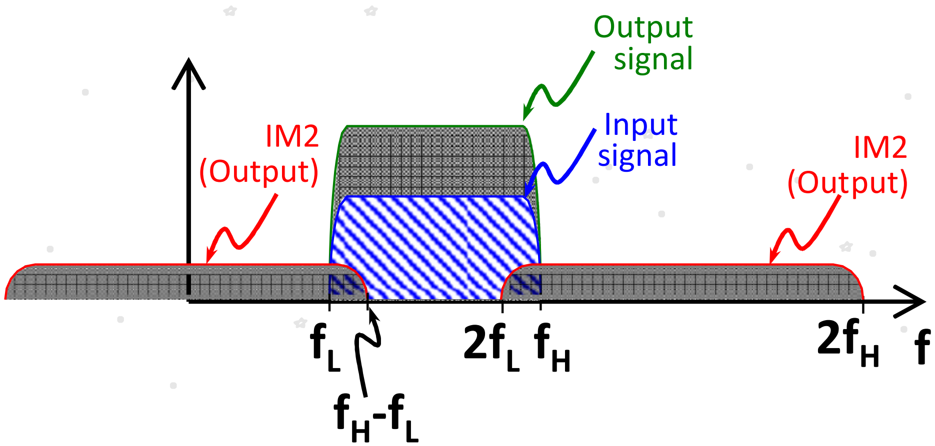
Second Order Non-Linearity

More observations:



Sub-octave: $f_H < 2f_L$ (i.e. BW < f_L)

- No in-band IM2 distortion –out-of-band IM2 components can easily be filtered out
- DC components can sometimes cause amplifier saturation



Multi-octave: $f_H > 2f_L$ (i.e. BW > f_L)

- In-band IM2 distortion present, can't be filtered out
- DC components may cause amplifier saturation

Third Order Non-Linearity

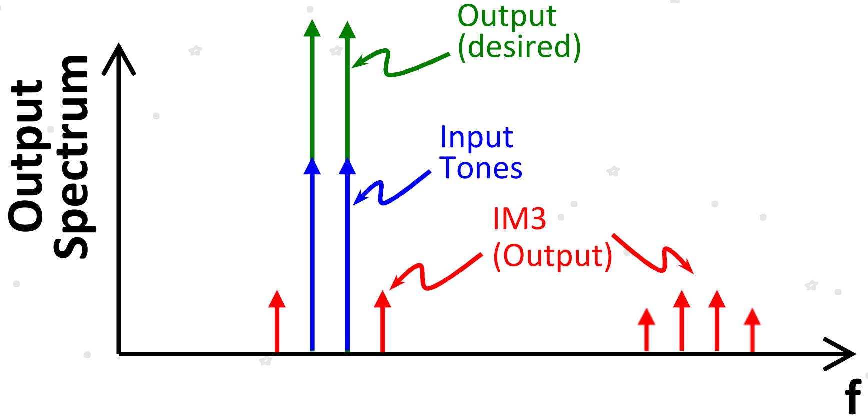
Consider a non-linear system described by

$$y(t) = a_1 x(t) + a_3 x^3(t); \qquad \Rightarrow \text{For} \quad x(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t):$$

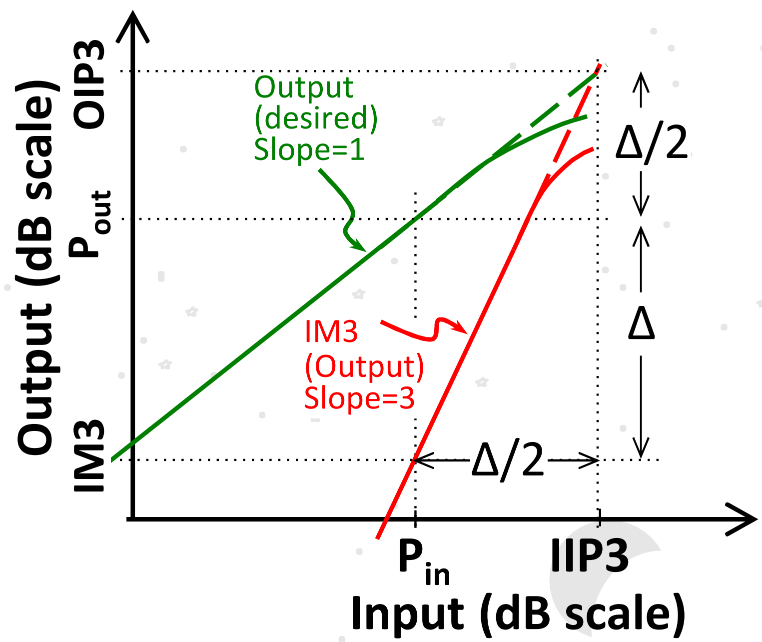
$$y(t) = A\left(a_1 + \frac{9a_3A^2}{4}\right)\left(\cos(\omega_1 t) + \cos(\omega_2 t)\right) + \frac{1}{4}a_3A^3\left(\cos(3\omega_1 t) + \cos(3\omega_2 t)\right)$$

$$+\frac{3}{4}a_3A^3\left[\cos\left((2\omega_1 - \omega_2)t\right) + \cos\left((2\omega_1 + \omega_2)t\right) + \cos\left((2\omega_2 - \omega_1)t\right) + \cos\left((2\omega_2 + \omega_1)t\right)\right]$$

Observations:



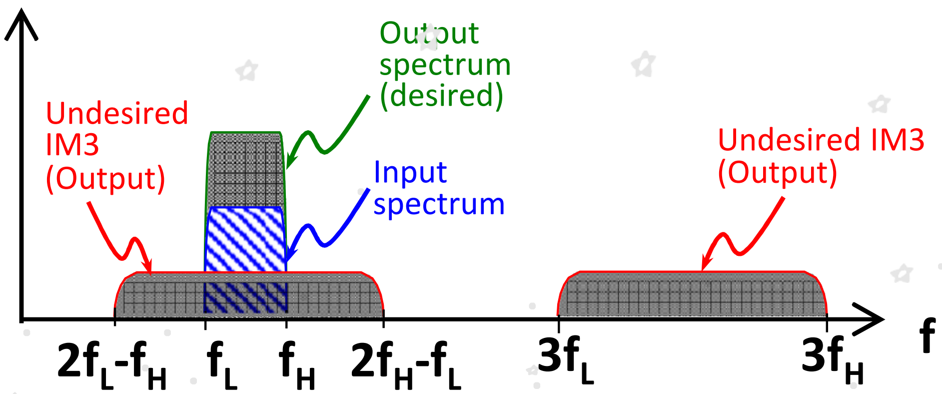
Generates in-band/adjacent band, out-of-band components, but no DC



IIP3 = Pin + Δ/2 (dB scale)

OIP3 = Pin + Δ/2 + Gain (dB scale)

Third Order Non-linearity

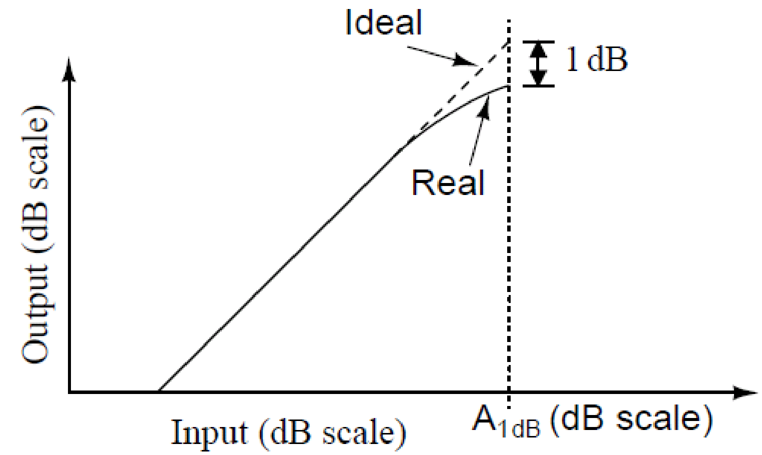


- The figure above shows the undesired spectrum generated by 3rd order non-linearity (i.e. due to non-zero a_3 co-efficient)
- The undesired spectrum generated is called IM3 component, i.e. Inter-Modulation products due to 3rd order non-linearity component.
- Due to 3rd or odd order non-linearities (unlike 2nd or even order non-linearities), part of the spectrum is in-band and hence CANNOT be removed by filtering even for narrow-band inputs.
- Therefore, effects of 3rd (or odd) order non-linearities are more difficult to remove in general (then of even order non-linearities).

Compression Point and Jamming

1-dB compression point: Amplitude (A_{-1dB}) at which gain decreases by 1-dB (without interferer) – because a_3 is "almost always" negative.

$$20\log\left[\frac{\left(a_1+\frac{3}{4}a_3A_{-1dB}^2\right)A_{-1dB}}{a_1A_{-1dB}}\right]=-1\,\text{dB}\quad\Rightarrow A_{\text{-1dB}}\approx 0.40\sqrt{\left|\frac{a_1}{a_3}\right|}$$



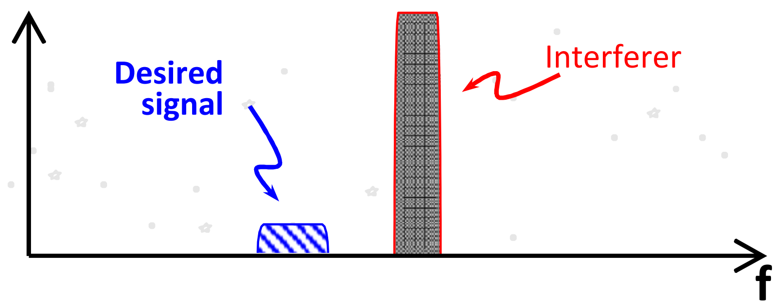
Jamming / Blocking / Desensitization

Desired signal

Interferer

$$\text{For}\quad x(t)=A\cos(\omega t)+B\cos(\omega_I t),$$

$$y(t)=\left(a_1+\frac{3}{4}a_3A^2+\frac{3}{2}a_3B^2\right)A\cos(\omega t)+\text{other terms}$$



- **Therefore, if interferer amplitude B>>A, the receiver is jammed**
- **The transmitter can jam the receiver if they are operating concurrently, for example in full duplex systems (and isolation is poor)**

APPENDIX: Real Systems are not memory-less or linear: Non-linear dynamical behaviour

- **Transfer function of a dynamic non-linear system**
  - Very complex, commonly expressed as the Volterra series

$$y(t) = a_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_0^{\infty} \ldots\ldots \int_0^{\infty} a_n(\tau_1, \tau_2, \cdots, \tau_n) x(t-\tau_1) x(t-\tau_2) \cdots x(t-\tau_n)\, d\tau_1 d\tau_1 \ldots d\tau_n$$

a_n is called the n^th order Volterra kernel

Therefore, a 2^nd order dynamic non-linear system can be modeled as

$$y(t) = a_0 + \int_0^{\infty} a_1(\tau_1) x(t-\tau_1) d\tau_1 + \frac{1}{2} \int_0^{\infty} \int_0^{\infty} a_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2)\, d\tau_1 d\tau_2$$