Superheterodyne Receiver, Sampling, PCM, Transmission of FM and AM Signals over Non-Linear Channels: Examples

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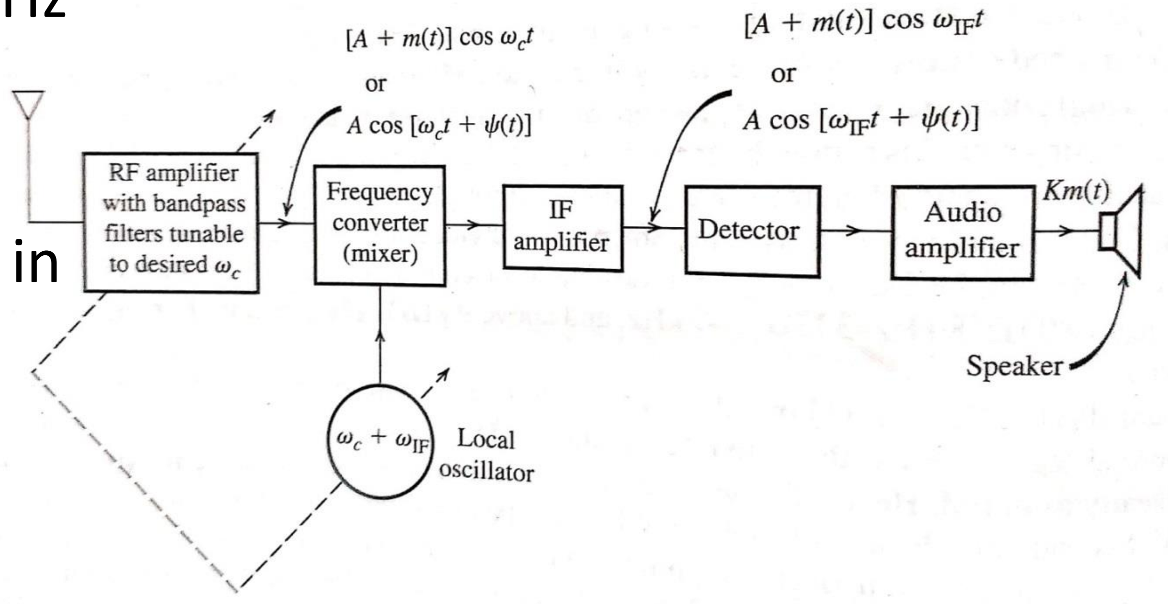
Example: Superheterodyne Receiver

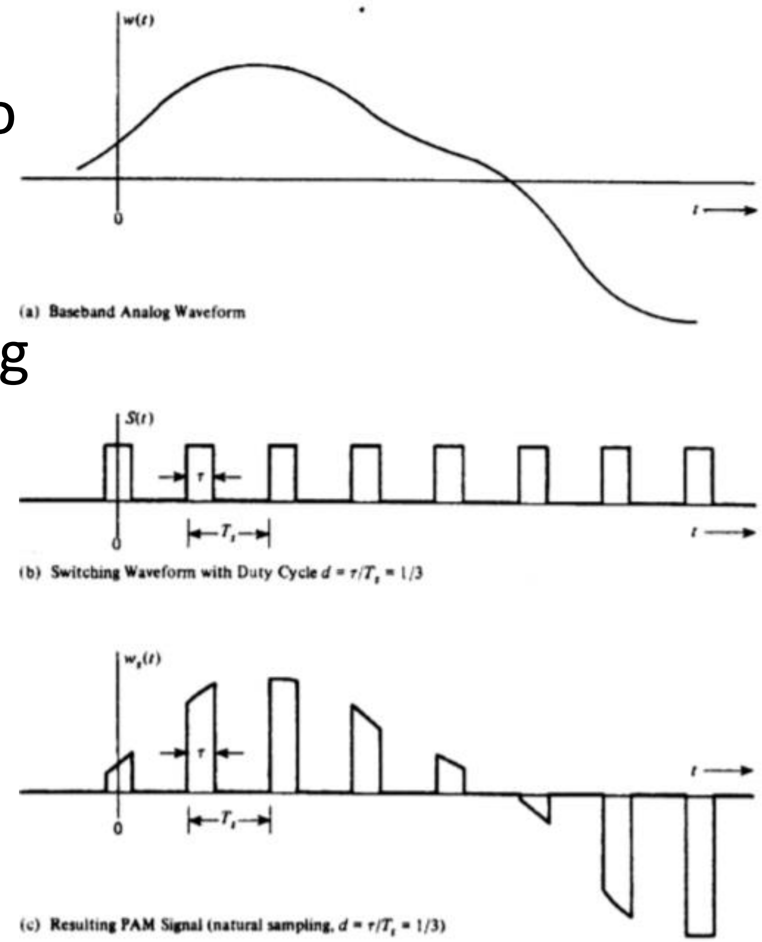
- A superheterodyne FM receiver operates in the frequency range: $f_c \in [88\text{ MHz}, 108\text{ MHz}]$
- We require that image frequency f_c' lie outside [88 MHz, 108 MHz] for every f_c
- Want minimum required f_{IF}
- Recall: $f_c' = f_c + 2f_{IF}$
- So $f_c + 2f_{IF} \geq 108$ MHz for every f_c
- So $f_{IF} \geq \frac{108-88}{2} = 10$ MHz
- Next, assume that $f_{IF} = 10$ MHz
- Want range of variations in f_{LO}
- Recall: $f_{LO} = f_c + f_{IF} = f_c + 10$
- So required range:
  - [98 MHz, 118 MHz]



Ref: B.P. Lathi, Z. Ding, “Modern Digital and Analog Communication Systems”, 4th ed.

Example: Natural Sampling

- In *natural sampling*, an analog signal $g(t)$ is multiplied by a periodic train of rectangular pulses $c(t)$
  - ❑ pulse repetition frequency of periodic train: f_s
  - ❑ duration (respectively, amplitude) of each rectangular pulse: T (respectively, $1/T$)



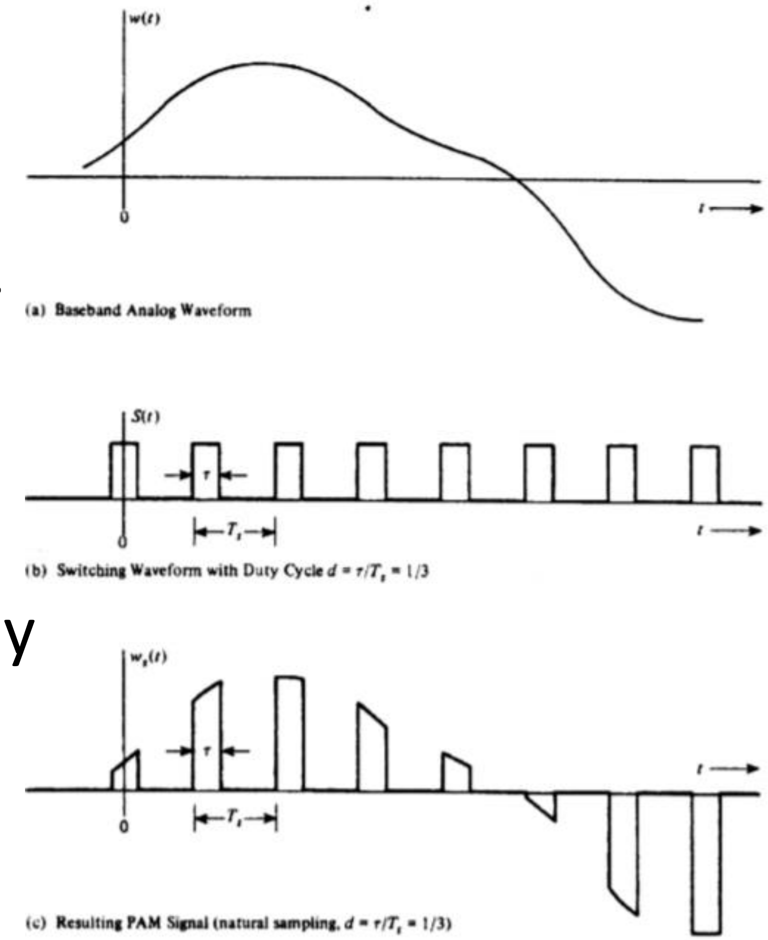
- Assume that time $t = 0$ corresponds to midpoint of a rectangular pulse in $c(t)$
- Want spectrum of signal $s(t)$ that results from the use of natural sampling
- Fourier series expansion of $c(t)$:
  - ❑ $c(t) = \sum_{n=-\infty}^{\infty} f_s \mathrm{sinc}(nf_sT)\, e^{j2\pi n f_s t}$
- $s(t)$:
  - ❑ $c(t)g(t) = \sum_{n=-\infty}^{\infty} f_s \mathrm{sinc}(nf_sT)\, g(t)e^{j2\pi n f_s t}$
- $S(f)$:
  - ❑ $\sum_{n=-\infty}^{\infty} f_s \mathrm{sinc}(nf_sT)\, G(f - nf_s)$

Ref: https://www.ques10.com/p/37214/explain-natural-sampling-and-flat-top-sampling-w-1/

Example: Natural Sampling (contd.)

1) Recall: $S(f) = \sum_{n=-\infty}^{\infty} f_s \mathrm{sinc}(n f_s T)\, G(f - n f_s)$

- Under what conditions can $g(t)$ be recovered exactly from its naturally sampled version?
- Assume that $g(t)$ is bandlimited with $G(f) = 0$ for $f \notin (-W, W)$; also, $f_s > 2W$
- Then by 1), the different frequency-shifted replicas of $G(f)$ in the spectrum $S(f)$ will not overlap
- Hence, $G(f)$, and therefore the signal $g(t)$, can be recovered exactly by passing $s(t)$ through a low-pass filter of bandwidth W



(a) Baseband Analog Waveform

(b) Switching Waveform with Duty Cycle $d = \tau/T_s = 1/3$

(c) Resulting PAM Signal (natural sampling, $d = \tau/T_s = 1/3$)

Ref: https://www.ques10.com/p/37214/explain-natural-sampling-and-flat-top-sampling-w-1/

Example: PCM

- A PCM system uses a uniform quantizer and represents each quantized value using 7 bits
- Bit rate of system equals 50×10^6 bps
- Want maximum message bandwidth for which system operates satisfactorily
- Nyquist rate:
  - $2W$, where W is message bandwidth
- So $2W \times 7 \leq 50 \times 10^6$
- Hence, $W \leq 3.57$ Mbps
- Suppose a sinusoidal modulating wave of amplitude A_m and frequency 1 MHz is applied to input
- Assume that quantizer divides the range $[-A_m, A_m]$ into intervals of equal sizes
- Output signal-to-quantization noise ratio:
  - $\left(\frac{3P}{m_{max}^2}\right) 2^{2R}$
- P:
  - $\frac{A_m^2}{2}$
- m_{max}:
  - A_m
- Substituting above values of P and m_{max}, and $R = 7$, we get the above ratio to be:
  - $1.5 \times 2^{14} = 43.91$ dB

Effect of Sending FM Signal over Non-linear Channel

- Consider FM signal:
  - $s(t) = A_c \cos[2\pi f_c t + \phi(t)]$
  - where $\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau$
- Sent over a channel with following input-output characteristic:
  - $v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t)$
  - where a_1, a_2 and a_3 are constants
- Recall the trigonometric identity: $\cos^3\theta = \frac{1}{4}(3\cos(\theta) + \cos(3\theta))$
- Output of channel:
  - $v_o(t) = \frac{1}{2}a_2 A_c^2 + \left(a_1 A_c + \frac{3}{4}a_3 A_c^3\right)\cos[2\pi f_c t + \phi(t)] + \frac{1}{2}a_2 A_c^2 \cos[4\pi f_c t + 2\phi(t)] + \frac{1}{4}a_3 A_c^3 \cos[6\pi f_c t + 3\phi(t)]$
- Can message signal $m(t)$ be recovered from $v_o(t)$?
  - Yes, assuming that f_c is much larger than bandwidth of FM signal
- Recovery:
  - Input $v_o(t)$ to band-pass filter with mid-band frequency f_c
- Output:
  - $v_o'(t) = \left(a_1 A_c + \frac{3}{4}a_3 A_c^3\right)\cos[2\pi f_c t + \phi(t)]$
- $v_o'(t)$ is scaled version of $s(t)$
- Thus, FM signals robust to transmission over non-linear channels of above type
- So FM widely used in satellite communication systems
  - permits the use of highly non-linear amplifiers, which are required for producing a high-power output at radio frequencies

Effect of Sending AM Signal over Non-linear Channel

- Consider AM signal:
  - $s(t) = A_c[1 + k_a m(t)]\cos[2\pi f_c t]$
- Sent over a channel with following input-output characteristic:
  - $v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t)$
  - where a_1, a_2 and a_3 are constants
- Output of channel:
  - $v_o(t) = \frac{1}{2}a_2 A_c^2[1 + k_a m(t)]^2 + \left(a_1 A_c[1 + k_a m(t)] + \frac{3}{4}a_3 A_c^3[1 + k_a m(t)]^3\right)\cos[2\pi f_c t] + \frac{1}{2}a_2 A_c^2[1 + k_a m(t)]^2\cos[4\pi f_c t] + \frac{1}{4}a_3 A_c^3[1 + k_a m(t)]^3\cos[6\pi f_c t]$
- Can message signal $m(t)$ be recovered from $v_o(t)$?
  - Difficult to recover it; cannot be recovered using envelope detection
- Thus, AM signal gets distorted when transmitted over non-linear channels of above type