19D070052 Sheel Shah Week 1

a) Let $s_1 = s_2 = \cdots = s_n = \alpha$

Probability of this happening is:

$$P(s_1=\alpha) \cdot P(s_2=\alpha) \cdots P(s_n=\alpha)$$

$$= \prod_i {}^m C_\alpha \cdot p_i^\alpha \cdot (1-p_i)^{m-\alpha}$$

$$= \left({}^m C_\alpha\right)^n \cdot (p_1 \cdot p_2 \cdots p_n)^\alpha \cdot \left[(1-p_1)(1-p_2)\cdots(1-p_n)\right]^{m-\alpha}$$

$$\therefore \text{Ans} = \sum_{\alpha=0}^{\alpha=m} \left({}^m C_\alpha\right)^n \cdot (p_1 p_2 \cdots p_n)^\alpha \cdot \left[(1-p_1)(1-p_2)\cdots(1-p_n)\right]^{m-\alpha}$$

b) Each s_i is a sum of m Bernoulli, iid RVs with $P(1) = p_i$

$\therefore$ s_i is Binomial(p_i, m) and $s_i \perp s_j \ \forall \ i \neq j$

$\therefore$ $E[s_i] = p_i \cdot m$, $Var(s_i) = p_i(1-p_i)m$

$\therefore$ $E[s_1 + s_2 \cdots s_n] = E[s_1] + E[s_2] \cdots E[s_n]$

$\therefore$ $E[S] = (p_1 + p_2 \cdots p_n) \cdot m$

Since s_is are independent, $Var(\Sigma s_i) = \Sigma Var(s_i)$

$\therefore$ $Var(S) = Var(s_1) + Var(s_2) \cdots Var(s_n)$

$= p_1(1-p_1)m + p_2(1-p_2)m \cdots p_n(1-p_n)m$

$\therefore$ $Var(S) = m\left((p_1 + p_2 + \cdots p_n) - (p_1^2 + p_2^2 \cdots p_n^2)\right)$