

EE324 Control Systems Lab

Problem Sheet 1

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Question 1 and 2

We have:

$$G1 = 10/(s^2 + 2s + 10)$$

$$G2 = 5/(s + 5)$$

General Steps:

```
s = poly(0, 's')
G1 = 10 / (s^2 + 2*s + 10)
G2 = 5 / (s + 5)
// Calculate transfer function T
roots(T.num) // zeros
roots(T.den) // poles
// Manually check for pole-zero cancellation
```

Part A Cascade

For a Cascade system, the transfer function is $T = G1 * G2$

In Scilab: `T = G1 * G2`

$$T = \frac{50}{50 + 20s + 7s^2 + s^3}$$
$$Zeros = []$$
$$Poles = [-5, -1 + 3i, -1 - 3i]$$

Part B Parallel

For a Parallel system, the transfer function is $T = G1 + G2$

In Scilab: `T = G1 + G2`

$$T = \frac{100 + 20s + 5s^2}{50 + 20s + 7s^2 + s^3}$$
$$Zeros = [-2 + 4i, -2 - 4i]$$
$$Poles = [-5, -1 + 3i, -1 - 3i]$$

Part C Feedback

For a Closed loop feedback system, the transfer function is $T = G1/(1 + G1 * G2)$

In Scilab: `T = G1 / (1 + G1 * G2)`

$$T = \frac{50 + 10s}{100 + 20s + 7s^2 + s^3}$$
$$Zeros = [-5]$$
$$Poles = [-6.33, -0.33 + 3.96i, -0.33 - 3.96i]$$

Part D

In order to plot the unit step response for G1, we define a continuous time system, and use it to generate the plot

In Scilab:

```
T = syslin('c', G1)
time_steps = 0:0.05:5
plot2d([time_steps', time_steps'], [
    (csim('step', time_steps, G1))', 0*time_steps'])
```

$$T = \frac{10}{10 + 2s + s^2}$$

$$Zeros = []$$

$$Poles = [-1 + 3i, -1 - 3i]$$

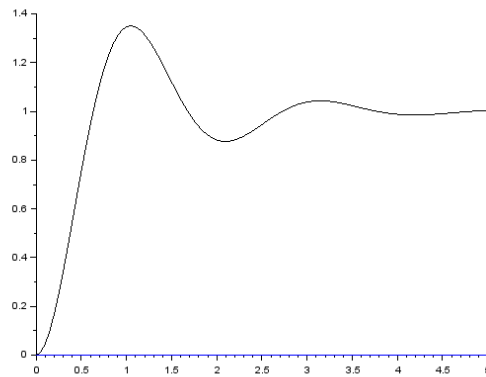


Figure 1: Unit step response of G1

Question 3

Practice

The following code was executed to understand how polynomial matrices work in Scilab.

```
s = poly(0, 's')
mat = [s, s^2; s+1, s^2+2*s+1]
mat * mat
det(mat)
mat^-1
mat^-1 * mat
```

Here are the results.

$$mat = \begin{bmatrix} s & s^2 \\ 1+s & 1+2s+s^2 \end{bmatrix}$$

$$mat * mat = \begin{bmatrix} 2s^2 + s^3 & s^2 + 3s^3 + s^4 \\ 1 + 4s + 4s^2 + s^3 & 1 + 4s + 7s^2 + 5s^3 + s^4 \end{bmatrix}$$

$$||mat|| = s + s^2$$

$$mat^{-1} = \begin{bmatrix} \frac{1+s}{s} & \frac{-s}{1+s} \\ \frac{-1}{s} & \frac{1}{1+s} \end{bmatrix}$$

$$mat^{-1} * mat = \begin{bmatrix} \frac{1}{1} & \frac{0}{1} \\ \frac{0}{1} & \frac{1}{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mesh Analysis

In taking the Laplace transform: $C \rightarrow 1/sC$, $L \rightarrow sL$

Analysing the circuit, we get:

$$\begin{bmatrix} 2 + 2s + \frac{1}{1+s} & \frac{-1}{1+s} & -(1+s) \\ \frac{-1}{1+s} & 3 + s + \frac{1}{1+s} & -2 \\ -(1+s) & -2 & 6 + s + \frac{1}{1+s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1(s) \end{bmatrix}$$

The following code is used:

```
Z = [2+2*s+1/(1+s), -1/(1+s), -(1+s);
-1/(1+s), 3+s+1/(1+s), -2;
-(1+s), -2, 6+s+1/(1+s)]
Z_inv = Z^-1 // find inverse
Z_inv(:, 3) // this is the answer
// scilab has 1-indexing, and we want the 3rd column of Z_inv
```

Answer:

$$\begin{bmatrix} \frac{I_1(s)}{V_1(s)} \\ \frac{I_2(s)}{V_1(s)} \\ \frac{I_3(s)}{V_1(s)} \end{bmatrix} = \begin{bmatrix} \frac{6 + 14s + 13s^2 + 6s^3 + s^4}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5} \\ \frac{7 + 16s + 13s^2 + 4s^3}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5} \\ \frac{11 + 28s + 27s^2 + 12s^3 + 2s^4}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5} \end{bmatrix}$$