High Level Synthesis

Scheduling

Virendra Singh

Computer Architecture and Dependable Systems Lab

Department of Electrical Engineering

Indian Institute of Technology Bombay

http://www.ee.iitb.ac.in/~viren/

E-mail: viren@ee.iitb.ac.in

EE-677: Foundations of VLSI CAD

Architectural Synthesis

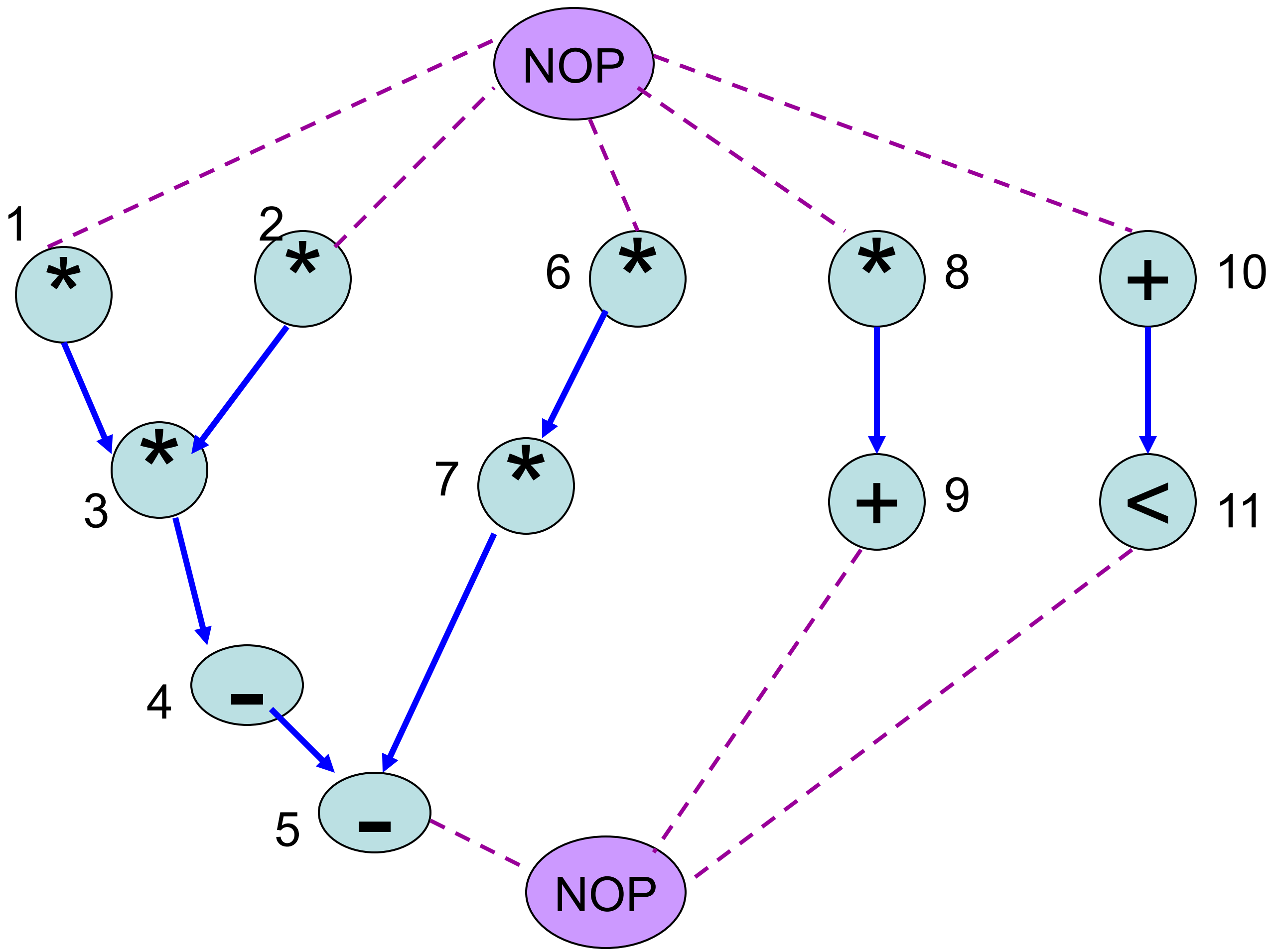
Architectural Level Abstraction

- Datapath
- Controller

Architectural Synthesis

- Constructing the macroscopic structure of a digital circuit starting from behavioural models that can be captured from Data flow or Sequencing Graph

Sequencing Graph



Temporal Domain: Scheduling

Delay **D** = $\{d_i;\ i = 0,1, 2, \ldots.. n\}$

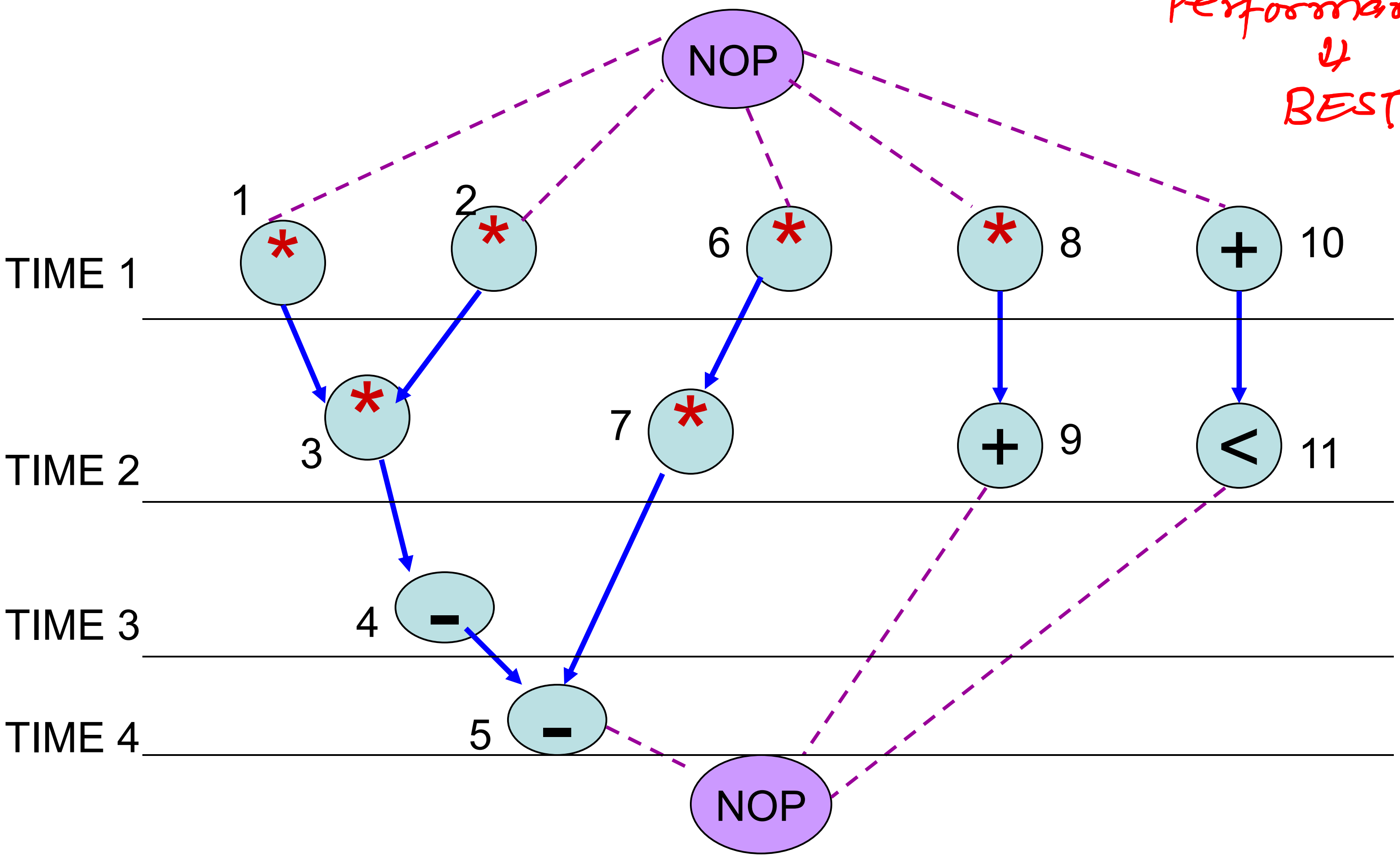
Start time T =$\{t_i;\ i= 0, 1, \ldots., n)$

Scheduling: Task of determining the start timing, subject to preceding constraints specified by sequencing graph

Latency $\lambda = t_n - t_0$

ASAP Scheduling

Performance 4 BEST



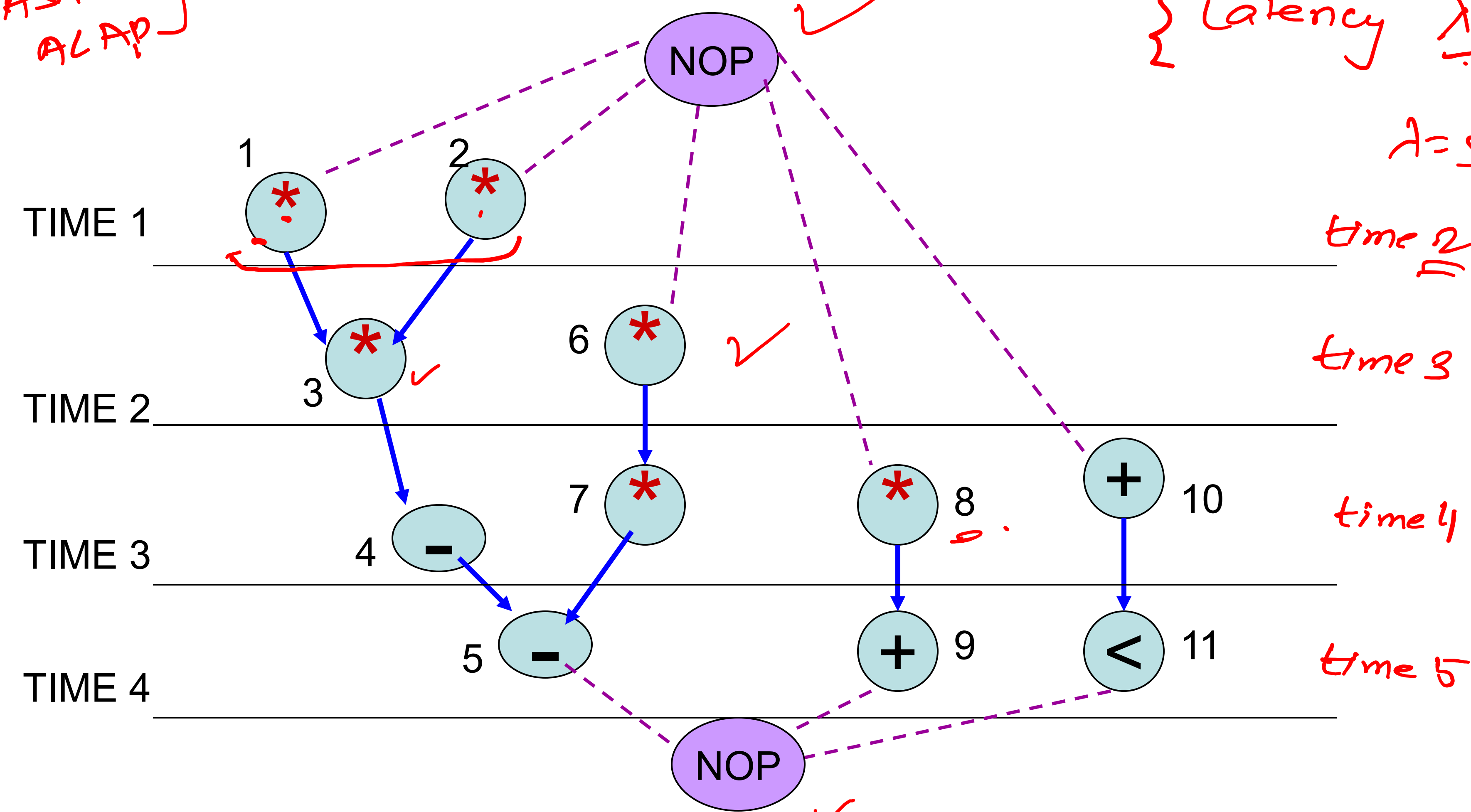
slack / flexibility

ASAP & ALAP

ALAP Scheduling

{Latency λ}

$\lambda = 5$



ILP

$X = \{x_1, x_2 \ldots x_n\}$

linear $\Rightarrow \min f(X)$

linear.
$$\left\{\begin{array}{l} h(X) = 0 \\ g(X) \leq 0 \end{array}\right]$$ constraints

Large problem.

M - ~~no~~ variables

M_1 - constraints

Scheduling with Resource Constraint

ILP Formulation

$i \quad l$ $\quad n \times \lambda$ variable

Binary decision variable $X = \{x_{il}\}$

$x_{ij} = 1 \qquad x_{ij} = 0 \quad l \neq j$

1. Start time of each operation is unique

   $\Sigma_l \; x_{il} = 1$

2. Sequencing relations represented by $G_s(V,E)$ must be satisfied

   $\Sigma_l \; l.x_{il} \geq \Sigma_l \; l.x_{jl} + d_j$

3. Resource bound must be met at every schedule step

   $\Sigma_k \; \Sigma_m \; x_{im} \leq a_k$

Minimum Resource Scheduling under Latency Constraint

Binary decision variable $X = \{x_{il}\}$ $\{0, 1\}$

1. Start time of each operation is unique

   $\Sigma_l \, x_{il} = 1$

2. Sequencing relations represented by $G_s(V,E)$ must be satisfied

   $\Sigma_l \, x_{il} \geq \Sigma_l \, xjl + d_j$

3. Resource bound must be met at every schedule step

   $\Sigma_k \, \Sigma_m \, x_{im} - a_k \leq 0$ variable

4. Latency Constraints

   $\boxed{\Sigma_l \, l.x_{nl} \leq \lambda + 1}$ 8

4 7

Minimum Resource Scheduling under Latency Constraint

$\lambda = 4$

All operation must start only once

$\boxed{\sum_{l} x_{il} = 1}$

$x_{0,1} = 1$

$x_{1,1} = 1$

$x_{2,1} = 1$

$x_{3,2} = 1$

$x_{4,3} = 1$

$x_{5,4} = 1$

$x_{1,2} = 0$
$x_{1,3} = 0$
$x_{1,4} = 0$

$x_{6,1} + x_{6,2} = 1$

$x_{7,2} + x_{7,3} = 1$

$x_{8,1} + x_{8,2} + x_{8,3} = 1$

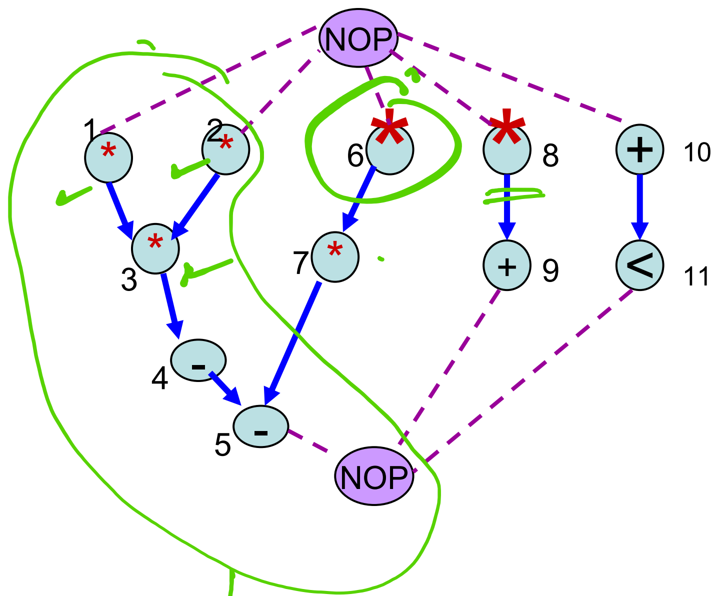
$x_{9,2} + x_{9,3} + x_{9,4} = 1$

$x_{10,1} + x_{10,2} + x_{10,3} = 1$

$x_{11,2} + x_{11,3} + x_{11,4} = 1$

$x_{n,5} = 1$

$x_{63} = 0 \quad x_{64} = 0$

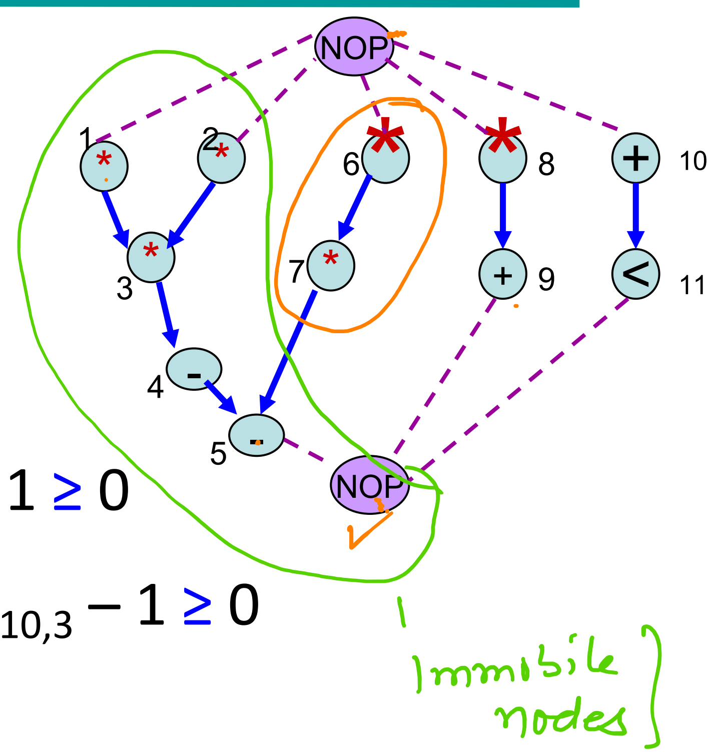


$\lambda = 4$ no flexibility

Minimum Resource Scheduling under Latency Constraint

Constraints – based on sequencing

(more than one starting time for at. least one operation)



$$2\,x_{7,2} + 3\,x_{7,3} - x_{6,1} - 2\,x_{6,2} - 1 \geq 0$$

$$2\,x_{9,2} + 3\,x_{9,3} + 4\,x_{9,4} - x_{8,1} - 2\,x_{8,2} - 3\,x_{8,3} - 1 \geq 0$$

$$2\,x_{11,2} + 3\,x_{11,3} + 4\,x_{11,4} - x_{10,1} - 2\,x_{10,2} - 3\,x_{10,3} - 1 \geq 0$$

$$4\,x_{5,4} - 2\,x_{7,2} - 3\,x_{7,3} - 1 \geq 0$$

$$5\,x_{n,5} - 2\,x_{9,2} - 3\,x_{9,3} - 4\,x_{9,4} - 1 \geq 0$$

$$5\,x_{n,5} - 2\,x_{11,2} - 3\,x_{11,3} - 4\,x_{11,4} - 1 \geq 0$$

Minimum Resource Scheduling under Latency Constraint

Resource Constraints

step1 $x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} - a_1 \leq 0$

step2 $x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} - a_1 \leq 2$

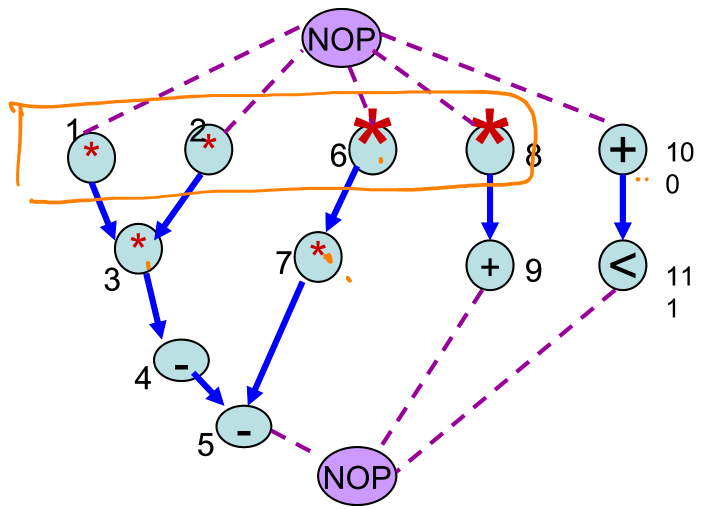
$x_{7,3} + x_{8,3} - a_1 \leq 0$

$x_{10,1} - a_2 \leq 0$

$x_{9,2} + x_{10,2} + x_{11,2} - a_2 \leq 0$

$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} - a_2 \leq 0$

$x_{5,4} + x_{9,4} + x_{11,4} - a_2 \leq 0$



$\boxed{x_{il} \,/\, a_i}$

Minimum Resource Scheduling under Latency Constraint

Mult ALU

5 1

a_1 a_2

Optimize $c^T a$

$c = [5, 1]$

Objective function to minimize = $c^T a$ = $5.\, a_1 + 1.\, a_2$

(a_1, a_2)

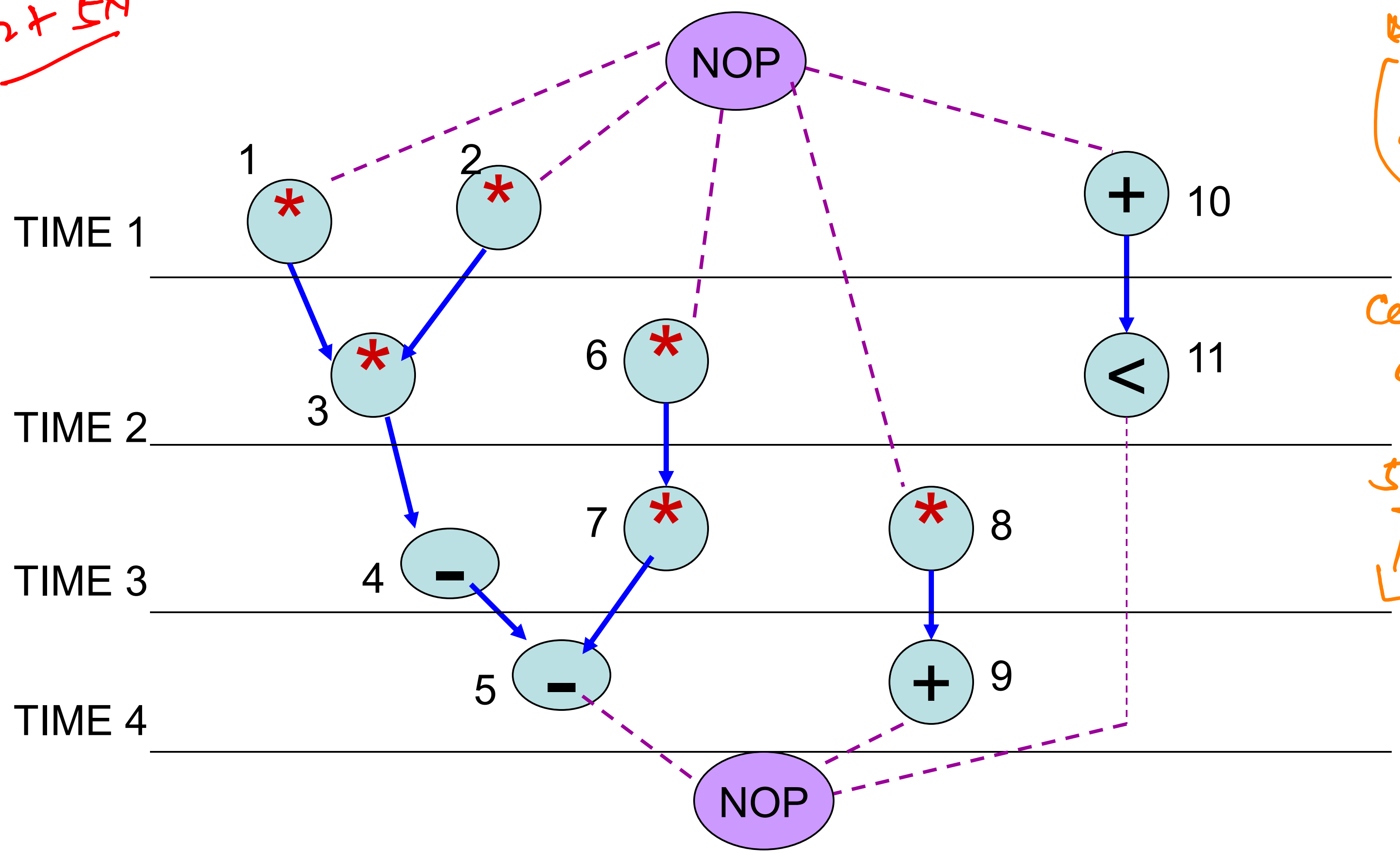
$\min f(x)$ ✓

Optimum Scheduling under Latency Constraint

max {dj} @ 2

6×2 + 5×1

λ = 4



~~M=2~~
$a_1 = 2$
$a_2 = 2$

cost $c^T a$

$5a_1 + a_2$

$10 + 2 = 12$

Cost

70%

30%

$\alpha = .7$

$\beta = .3$

min $(\alpha \cdot \text{latency} + \beta \, \text{area} \cdot)$

$\boxed{\alpha + \beta = 1}$

Co-optimization

ILP

Long time

Approximation Scheduling Algorithms

optimal ✓

- Hardware scheduling is modeled by sequencing graph with possibly multiple-cycle operations with different types
- Minimum latency resource constraint scheduling is known to be intractable
- Heuristic Solution
  - ✓ List Scheduling ✓
  - ✓ Force-directed Scheduling
  - ✓ Trace scheduling

List Scheduling

- Minimize latency under resource constraint a
- Candidate operation $U_{l,k}$ i
  - Operation of type k whose predecessors have already been scheduled early enough so that the corresponding operations are completed at step l
- Unfinished operations $T_{l,k}$
- Priority list

10 2

List Scheduling

LIST_SCHED ($G_s(v,E)$){

l = 1;

Repeat {

for each resource type $k = 1,2,.....,n_{res}$ {

- Determine candidate operation $U_{l,k}$;
- Determine unfinished operations $T_{l,k}$; — on going
- Select $S_k \equiv U_{l,k}$ vertices, s.t. $|S_k| + |T_{l,k}| \leq a_k$;
- Schedule the S_k operation at step l by setting $t_i = l$ for all i: $v_k \in S_k$;

}

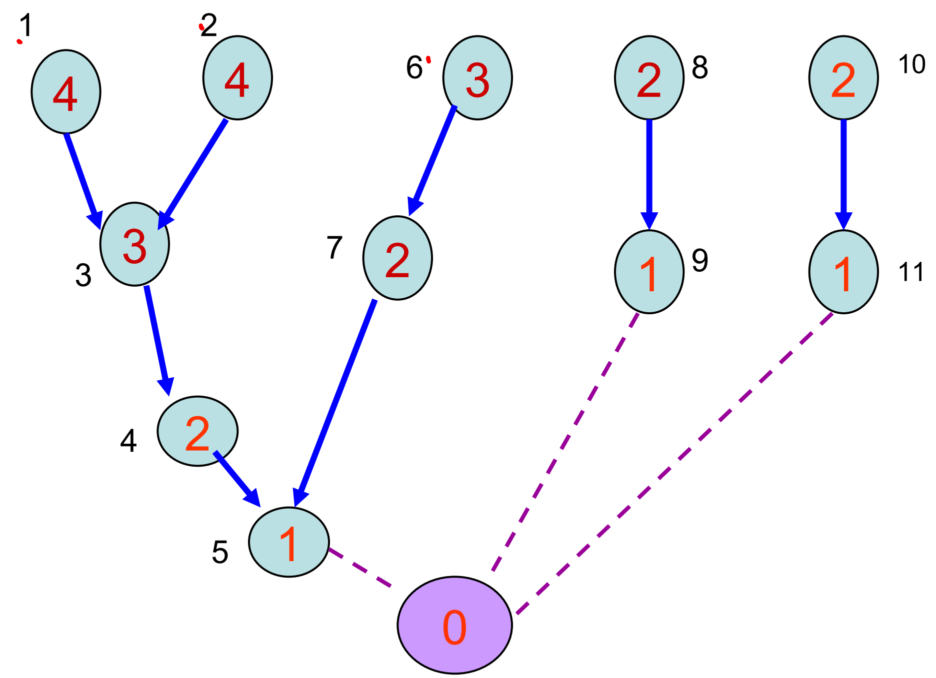
- $l = l+1$;

}

- Until (v_n is scheduled);
- Return (t); }

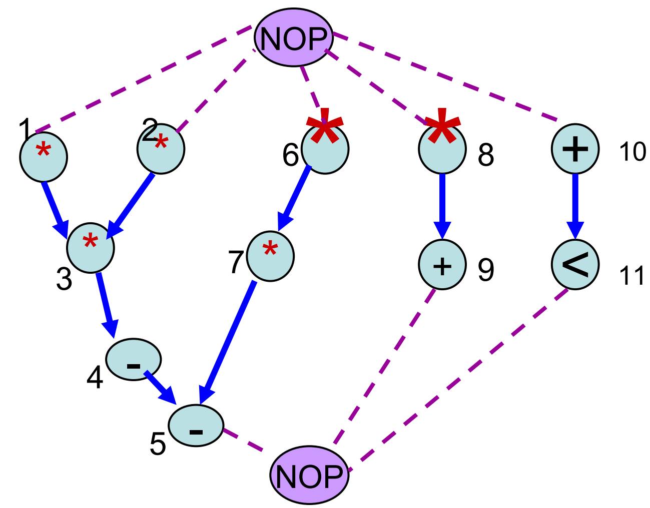
List Scheduling

- Assume a_1 = 2 multipliers and a_2 = 2 ALUs
- Assume operations have unit delay
- Priority function



List Scheduling

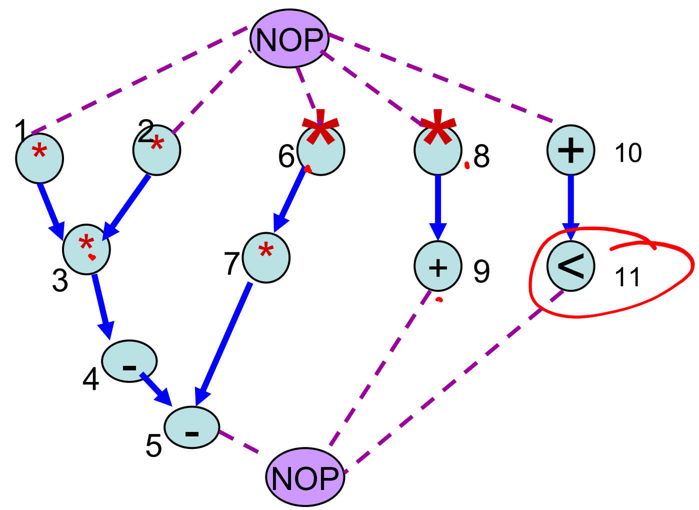
- First Step
  - $K = 1, U_{1,1} = \{v_1, v_2, v_6, v_8\}$
  - Selected Operations are
    - $\{v_1, v_2\}$
    - Label is maximal
  - $K = 2, U_{1,2} = \{v_{10}\}$
  - Selected Operation
    - $\{v_{10}\}$



$S_1 = \{v_1, v_2, v_{10}\}$

List Scheduling

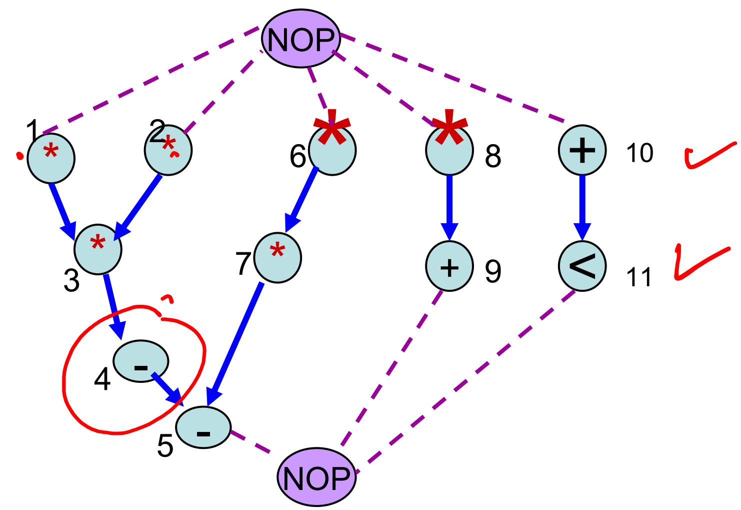
- Second Step
  - $K = 1, U_{2,1} = \{v_3, v_6, v_8\}$
  - Selected Operations are
    - $\{v_3, v_6\}$
    - Label is maximal
  - $K = 2, U_{1,2} = \{v_{11}\}$
  - Selected Operation
    - $\{v_{11}\}$



$S_2 = \{v_3, v_6, v_{11}\}$

List Scheduling

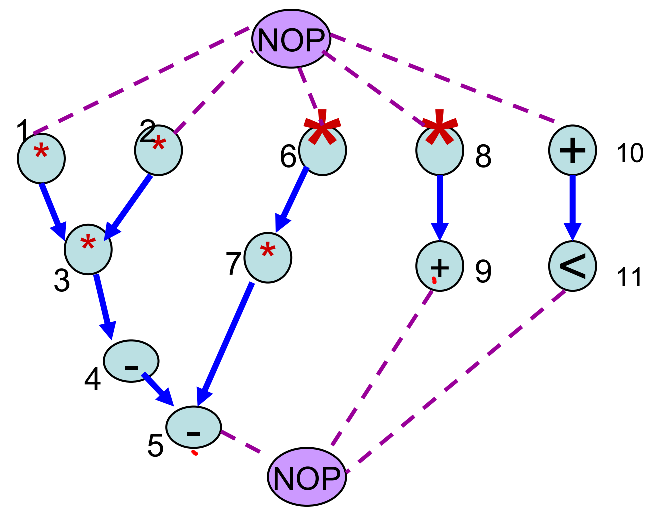
- Third Step
  - K = 1, $U_{3,1} = \{v_7, v_8\}$
  - Selected Operations are
    - $\{v_7, v_8\}$
  - K = 2, $U_{1,2} = \{v_4\}$
  - Selected Operation
    - $\{v_4\}$



$S_3 = \{v_7, v_8, v_4\}$

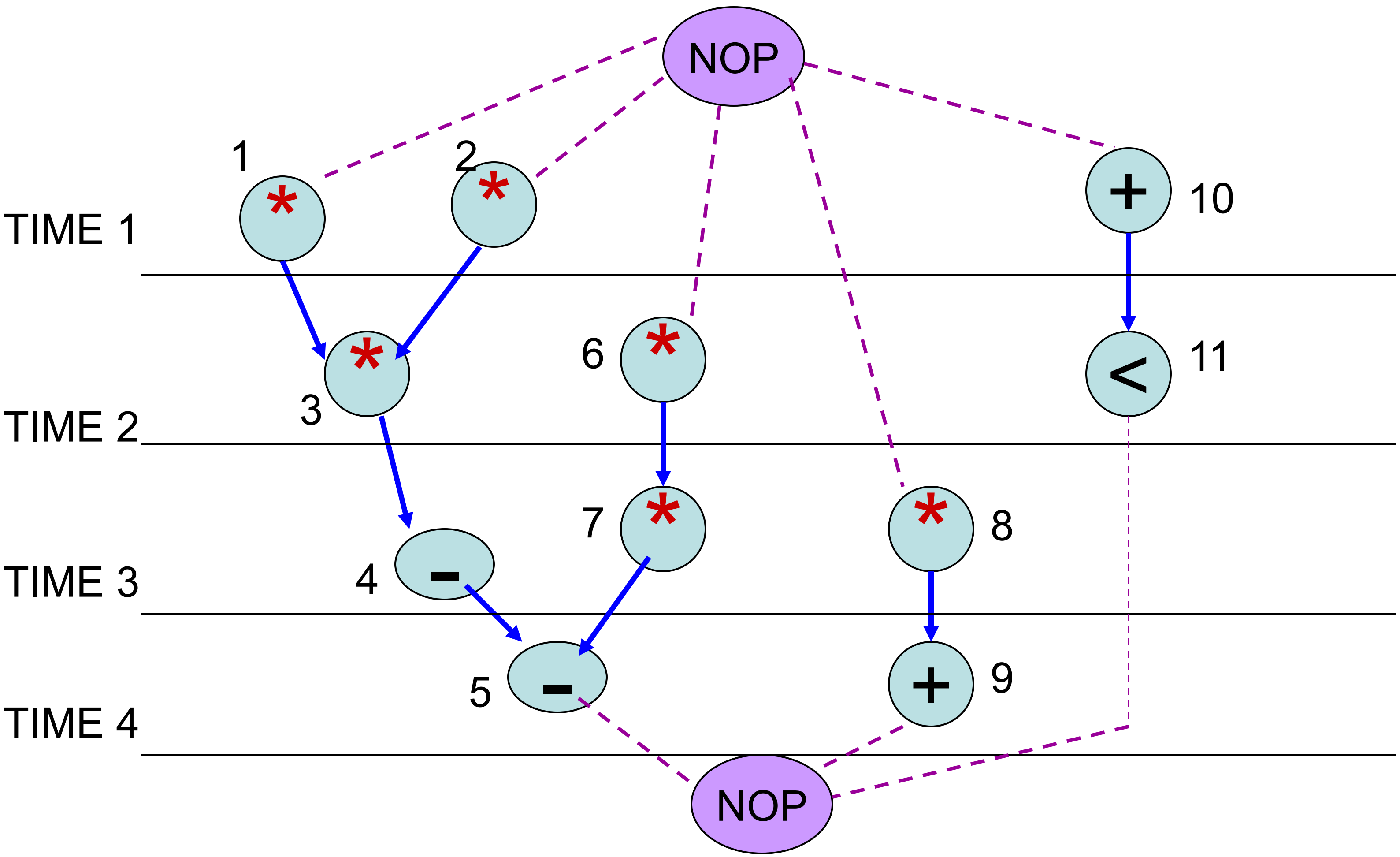
List Scheduling

- Fourth Step
  - K = 1, $U_{3,1} = \Phi$
  - K = 2, $U_{1,2} = \{v_{5,} v_9\}$
  - Selected Operation
    - $\{v_{5,} v_9\}$



$S_4 = \{v_5, v_9\}$

Optimum Scheduling under Resource Constraint



List Scheduling

- Assume $a_1 = 3$ multipliers and $a_2 = 1$ ALU
- Multiplier Delay = 2 and ALU delay = 1
- Priority function (weight of longest path to sink node)

List Scheduling

- Step 1

$U_{1,1} = \{v_1, v_2, v_6, v_8\}$ $\quad U_{1,2} = \{v_{10}\}$

$T_{1,1} = \Phi$ $\quad T_{1,2} = \Phi$

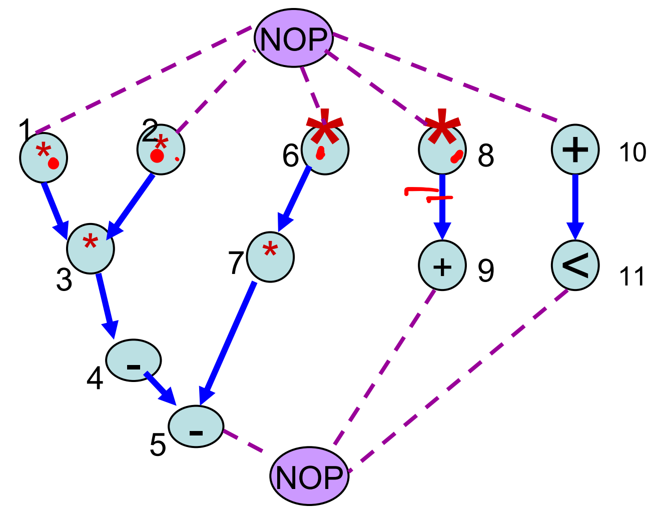
$S_1 = \{v_1, v_2, v_6, v_{10}\}$

- Step 2 $\quad |V_{21}| + |T_{21}| \le 3$

$U_{2,1} = \{v_8\}$ $\quad U_{2,2} = \{v_{11}\}$

$T_{2,1} = \{v_1, v_2, v_6\}$ $\quad T_{2,2} = \Phi$

$S_2 = \{ v_{11}\}$



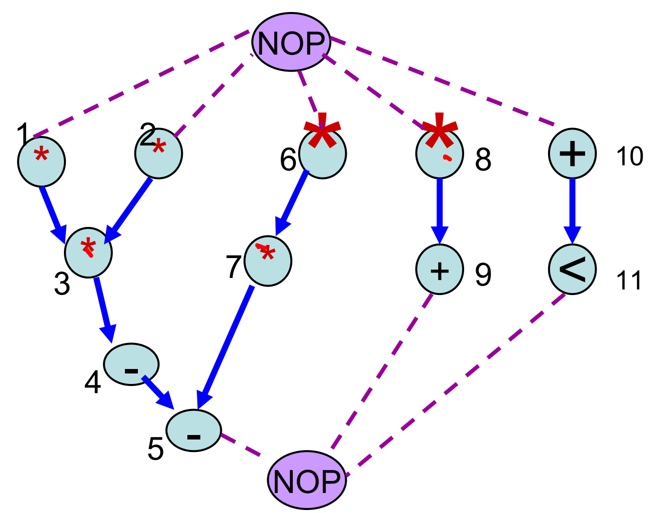
List Scheduling

- Step 3

$U_{3,1} = \{v_8, v_3, v_7\}$ $U_{3,2} = \Phi$

$T_{3,1} = \Phi$ $T_{3,2} = \Phi$

$S_3 = \{v_8, v_3, v_7\}$



- Step 4

$U_{4,1} = \Phi$ $U_{4,2} = \Phi$

$T_{4,1} = \{v_8, v_3, v_7\}$ $T_{4,2} = \Phi$

$S_4 = \Phi$

List Scheduling

- Step 5

$U_{5,1} = \Phi$ $U_{5,2} = \{v_4, v_9\}$

$T_{5,1} = \Phi$ $T_{5,2} = \Phi$

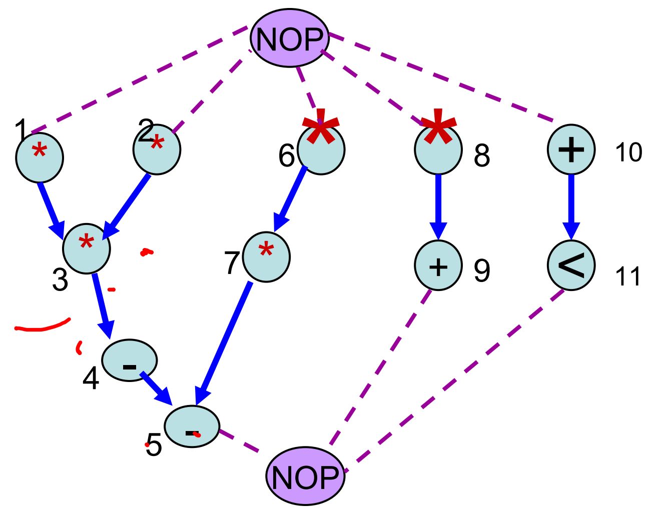
$S_5 = \{v_4\}$

- Step 6

$U_{6,1} = \Phi$ $U_{6,2} = \{v_5, v_9\}$

$T_{6,1} = \Phi$ $T_{6,2} = \Phi$

$S_6 = \{v_5\}$



List Scheduling

- Step 7

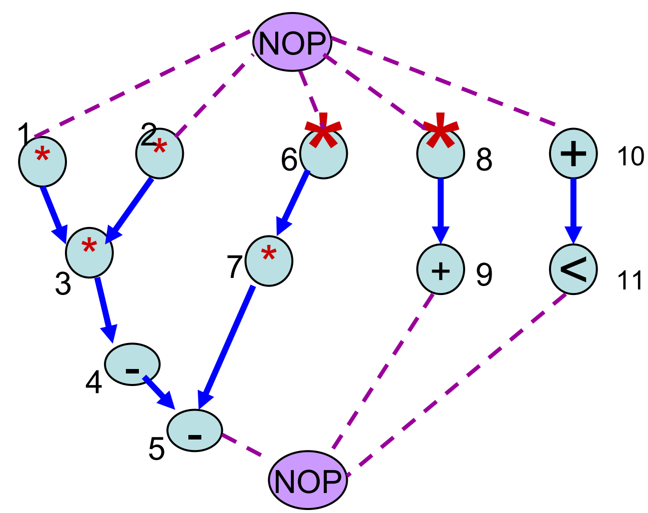
$U_{7,1} = \Phi$

$U_{7,2} = \{v_9\}$

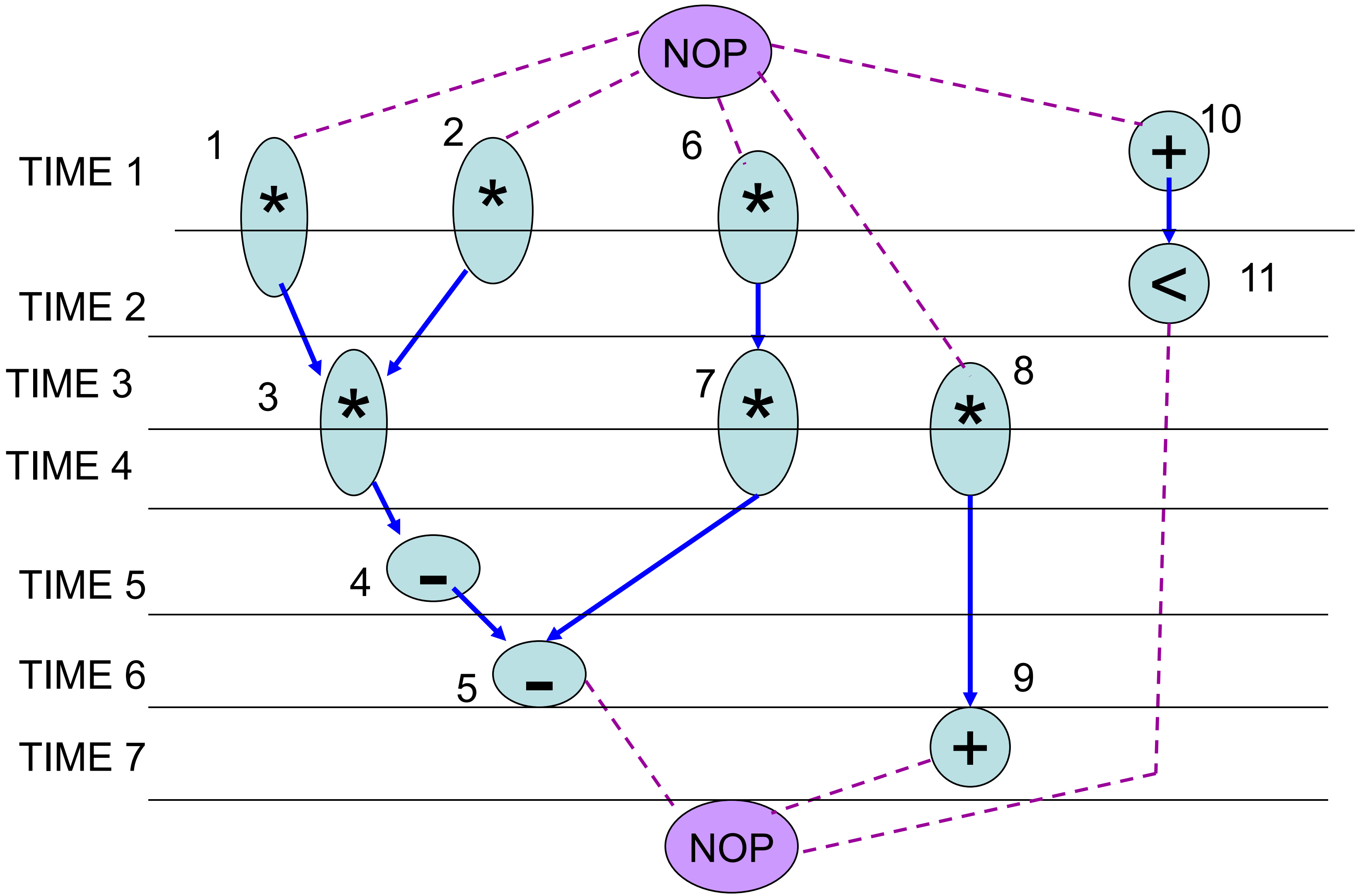
$T_{7,1} = \Phi$

$T_{7,2} = \Phi$

$S_7 = \{v_9\}$



List Scheduling



List Scheduling

$\Downarrow$

Variable

$$\sum_l l \cdot x_{il} \geq \sum_l l \cdot x_{jl} + d_j$$

Binding

Automation

Thank You