Review of Signals and Systems: Part 2

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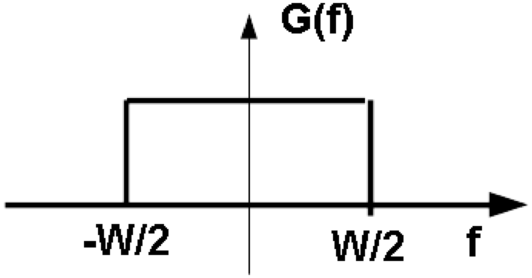
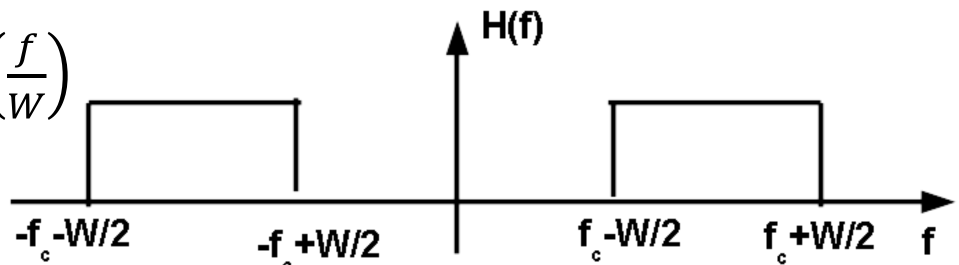
Bandwidth



- Intuitively, "bandwidth" of a signal:
  - ❑ provides a measure of extent of significant spectral content of the signal *for positive frequencies*
- Since the meaning of "significant spectral content" is mathematically imprecise, there is no universally accepted definition of bandwidth
  - ❑ several definitions have been proposed
- Following two examples illustrate how bandwidth is typically defined when signal is strictly band-limited
- E.g.:

1) Let $g(t)$ be such that $F[g(t)] = \text{rect}\left(\frac{f}{W}\right)$

- Bandwidth of $g(t)$:
  - ❑ $\frac{W}{2}$

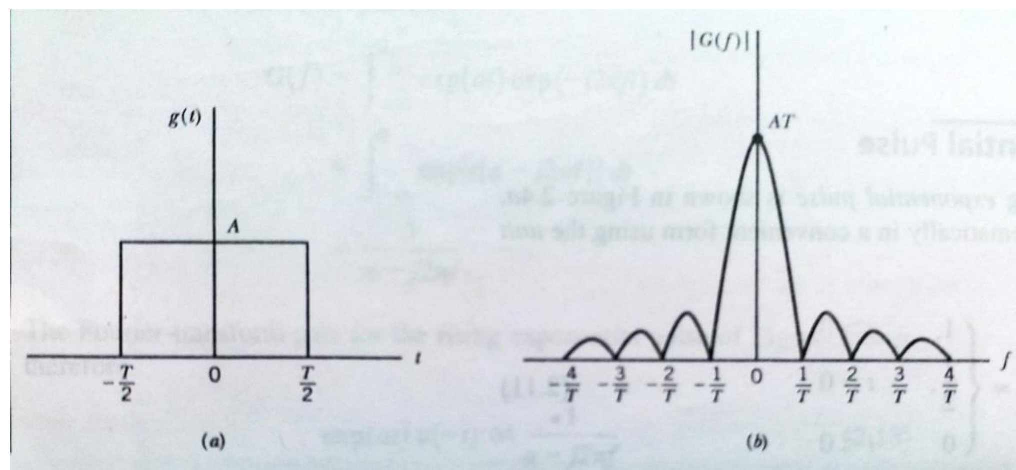


2) Let $h(t)$ be such that $F[h(t)] = \text{rect}\left(\frac{f-f_c}{W}\right) + \text{rect}\left(\frac{-f-f_c}{W}\right)$, where $f_c > \frac{W}{2}$

- Bandwidth of $h(t)$:
  - ❑ W
- Next, we study two commonly used definitions, which are often used for a signal that is not strictly band-limited

Null-to-null Bandwidth

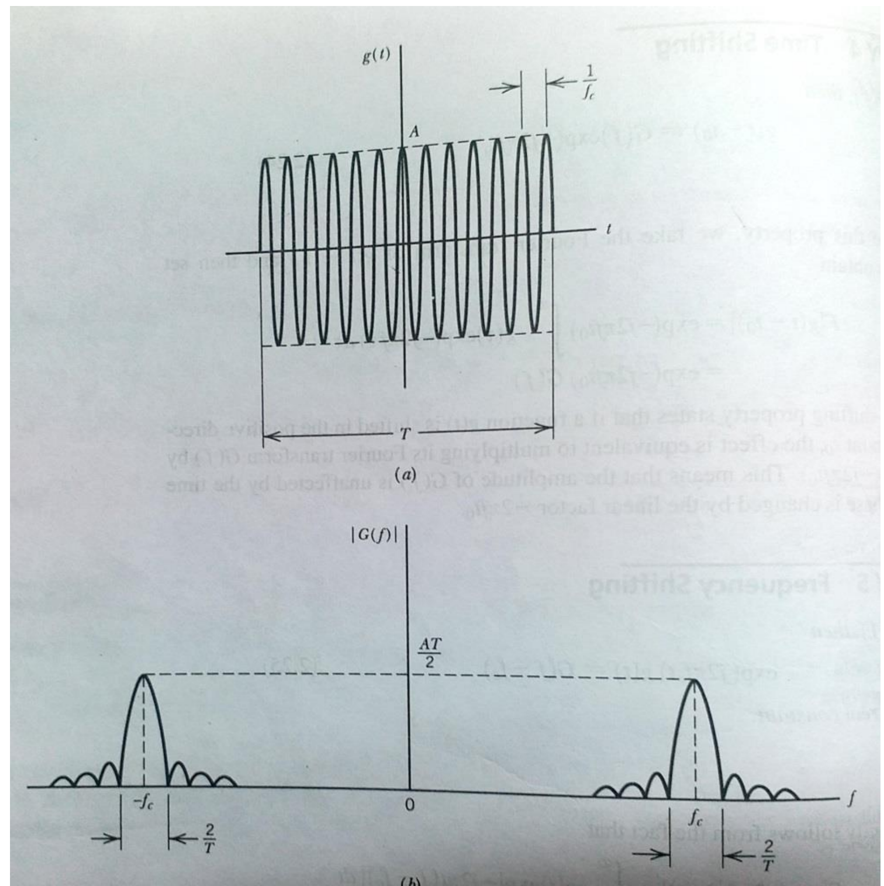
- Suppose spectrum of a signal is even function of frequency with "*main lobe(s)*" bounded by well-defined "*nulls*"
- If signal is low-pass, then bandwidth is defined as:
  - ❑half of total width of main lobe
- E.g.: Recall that if $g(t) = A\mathrm{rect}\left(\frac{t}{T}\right)$, where $A > 0$ and $T > 0$, then $G(f) = AT\mathrm{sinc}(fT)$
- Bandwidth of this signal:
  - ❑$\frac{1}{T}$



Ref: "Communication Systems" by S. Haykin and M. Moher, 5th ed

Null-to-null Bandwidth (contd.)

- If signal is band-pass with main spectral lobes centred around $\pm f_c$, where f_c is much larger than width of main-lobe, then bandwidth is defined as:
  - ❑ width of main lobe for positive frequencies
- E.g.: Consider the RF pulse $g(t) = \mathrm{A}\mathrm{rect}\left(\frac{t}{T}\right)\cos(2\pi f_c t)$, where $f_c T \gg 1$
- $G(f)$:
  - ❑ $\frac{AT}{2}\left\{\mathrm{sinc}\big((f - f_c)T\big) + \mathrm{sinc}\big((f + f_c)T\big)\right\}$
- Bandwidth of this signal:
  - ❑ $\frac{2}{T}$
- Above definition of bandwidth called "*null-to-null bandwidth*"
- Above example shows that shifting spectral content of a low-pass signal by a sufficiently large frequency has effect of doubling the bandwidth of the signal



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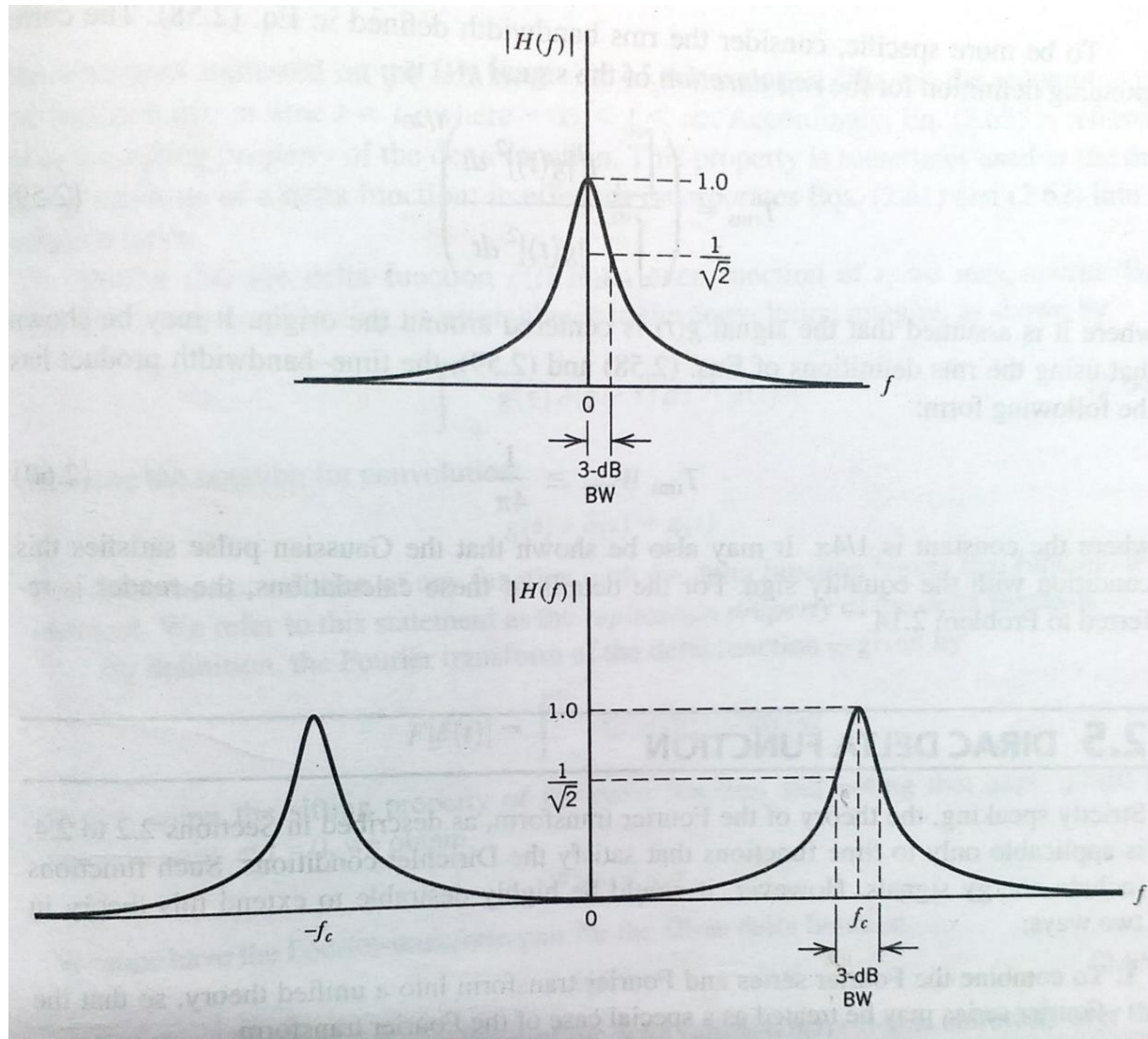
3-dB Bandwidth

- 3-dB bandwidth of a signal $g(t)$ is defined to be $f_2 - f_1$ if:
  - $f_2 > f_1 \geq 0$, for frequencies inside the band $f_1 < f < f_2$, the amplitude spectrum $|G(f)|$ falls no lower than $\frac{1}{\sqrt{2}}$ of the maximum value of $|G(f)|$, and the maximum value occurs at a frequency inside the band $[f_1, f_2]$



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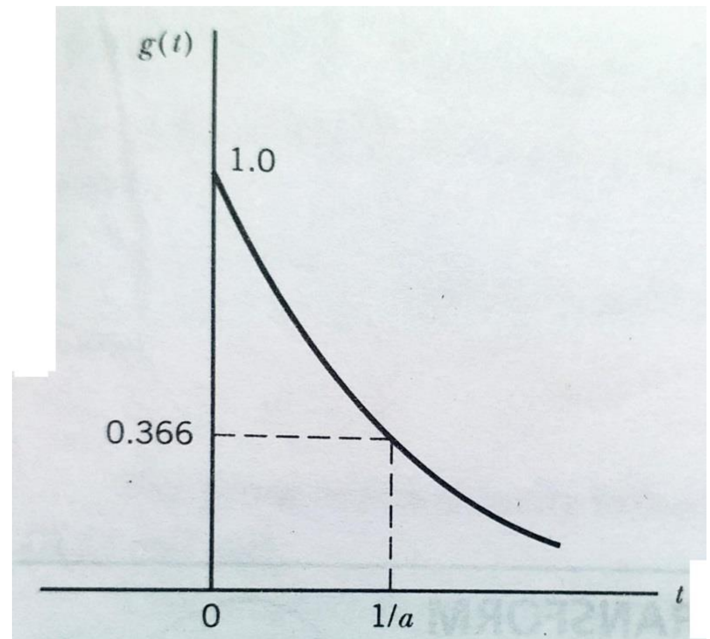
3-dB Bandwidth (contd.)

- E.g.: recall that Fourier transform of $g(t) = e^{-at}u(t)$, where $a > 0$, is:
  - ❑ $G(f) = \frac{1}{a+j2\pi f}$
- 3-dB bandwidth of $g(t)$:
  - ❑ obtained by solving the equation $\frac{1}{\sqrt{a^2+4\pi^2 f^2}} = \frac{1}{\sqrt{2}}\frac{1}{a}$
  - ❑ $\frac{a}{2\pi}$ Hz
- If signal is band-pass, centred at $\pm f_c$, as in lower fig. on previous slide, then 3-dB bandwidth is separation, along positive frequency axis, between:
  - ❑ the two frequencies at which amplitude spectrum of signal drops to $\frac{1}{\sqrt{2}}$ of its peak value at f_c



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