Homework 3 Solutions

Communication Systems - I (EE 341), Autumn 2021

2) For FM:

$f_i = f_c + k_f m(t)$

$f_i = 10^8 + 10^5 m(t)$

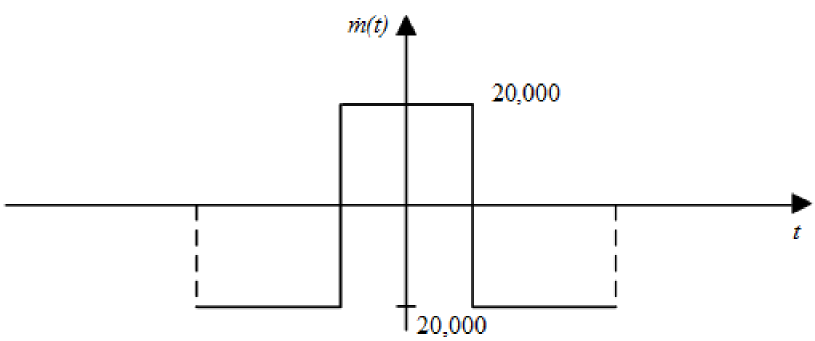
$(f_i)_{min} = 10^8 + 10^5[m(t)]_{min} = 99.9\ MHz$

$(f_i)_{max} = 10^8 + 10^5[m(t)]_{max} = 100.1\ MHz$

$\therefore m(t)$ increases and decreases linearly with time, $f_i(t)$ increases linearly from $99.9\ MHz$ over a half-cylce and decreases linearly from $100.1\ MHz to 99.9\ MHz$ over the remaining half-cycle of the $m(t)$.

For PM:

$\dot{m}(t) = f_i(t) = f_c + \frac{k_p}{2\pi}\dot{m}(t)$



$= 10^8 + \frac{10\pi}{2\pi}\dot{m}(t) = 10^8 + 5\dot{m}(t)$

$(f_i)_{min} = 10^8 + 5[\dot{m}(t)]_{min} = 10^8 - 10^5 = 99.9\ MHz$

$(f_i)_{max} = 10^8 + 5[\dot{m}(t)]_{max} = 10^8 + 10^5 = 100.1\ MHz$

The carrier switches between $99.9\ MHz$ and $100.1\ MHz$

3) a) The envelope of the NBFM wave is

$A(t) = A_c\sqrt{1 + \beta^2 \sin^2(2\pi f_m t)}$

$[A(t)]_{maxm} = A_{max} = A_c\sqrt{1 + \beta^2}$

$[A(t)]_{minm} = A_{min} = A_c$

$\therefore \frac{A_{max}}{A_{min}} = \sqrt{1+\beta^2}$

b) $S(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c \cos[2\pi(f_c + f_m)t] - \frac{1}{2}\beta A_c \cos[2\pi(f_c - f_m)t]$

$\therefore$ Average power $= P_{av} = \frac{1}{2}A_c^2 + \frac{\beta^2 A_c^2}{8} + \frac{\beta^2 A_c^2}{8}$

$= \frac{A_c^2}{2}(1 + \frac{\beta^2}{2})$

Average power of the unmodulated carrier is

$P_c = \frac{A_c^2}{2}$

$\therefore \frac{P_{av}}{P_c} = 1 + \frac{\beta^2}{2}$

c) $\tan^{-1}(x) \cong x - \frac{x^3}{3}$ (Given) for $x << 1$

$\theta_i(t) = 2\pi f_c t + tan^{-1}[\frac{S_I(t)}{S_Q(t)}] = 2\pi f_c t + tan^{-1}[\beta \sin(2\pi f_m t)]$

$\therefore \theta_i(t) \cong 2\pi f_c t + \beta \sin(2\pi f_m t) - \frac{\beta^3}{3}\sin^3(2\pi f_m t)$

$\therefore$ Power ratio of the third and first harmonics is given by

$D_h = \frac{(\frac{1}{3}\beta^3)^2}{\beta^2} = \frac{\beta^4}{9}$

For $\beta = 0.3, D_h = 0.09\%$

4) a) Δf = $k_f A_m$ = 25 x 10^3 x 20 = 500kHz = 5 x 10^5 Hz

β = Δf/f_m = 5 x 10^5/10^5 = 5

BW using Carson's formula = B = 2f_m(1+ β) = 2 x 100 (1+5) = 1200kHz = 1.2MHz

b) If the amplitude of the modulating wave is doubled, we find that

Δf = 1 MHz and β = 10

$\therefore$ B = 2f_m (1+β) = 2 x 100 x 11 = 2200kHz = 2.2MHz

c) If f_m is doubled Δf remains unchanged

β = 2.5

$\therefore$ B = 2f_m(1+ β) = 2 x 200 x 3.5 = 1.4MHz

5)

$$s(t) = A_c cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \tag{1}$$

Assuming that f_c is large compared to the BW of $s(t)$, $\implies$ $s(t)$ is a bandpass signal.

$$\tilde{s}(t) \equiv s_{lp}(t) = A_c \exp\left(j2\pi k_f \int_0^t m(\tau)d\tau\right) \tag{2}$$

$$s_\dagger(t) \equiv \text{Pre-envelope of } s(t) \tag{3}$$

$$= \tilde{s}(t)\exp(j2\pi f_c t)$$

$$= s(t) + js_h(t)$$

where $s_h(t)$ is the Hilbert transform of $s(t)$.

$$\therefore s(t) + js_h(t) = A_c \exp\left(j2\pi k_f \int_0^t m(\tau)d\tau\right)\exp(j2\pi f_c t) \tag{4}$$

$$= A_c cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$
$$+ jA_c sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \tag{5}$$

Equating real and imaginary parts, we get

$$s_h(t) = A_c sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \tag{6}$$

6)