Homework 2 Solutions

Communication Systems (EE 341), Autumn 2021

2) $c(t) = 50\cos(100\pi t)\,V \Rightarrow A_c = 50\,Volts$; $f_c = 50\,Hz$

$m(t) = 20\cos(2\pi t) \Rightarrow$ Single tone : $f_m = 1\,Hz$

$S(t) = 50[1 + \frac{20}{50}\cos(2\pi t)]\cos(100\pi t)$

$\therefore \mu = 0.4$

$S(t) = [50\cos(100\pi t) + 20\cos(2\pi t).\cos(100\pi t)]\,Volts$

$= \{50\cos(100\pi t) + 10[\cos(102\pi t) + \cos(98\pi t)]\}\,Volts$

Power developed across a $100\,\Omega$ load by this AM wave is,

$P = \frac{1}{2}\frac{50^2}{100} + \frac{1}{2}\frac{10^2}{100} + \frac{1}{2}\frac{10^2}{100}$

$= 12.5 + 0.5 + 0.5 = 12.5 + 1$

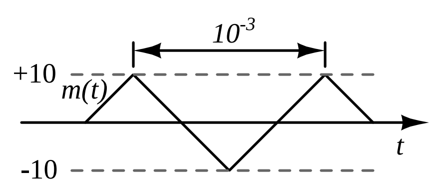
$= 13.5\,Watts$

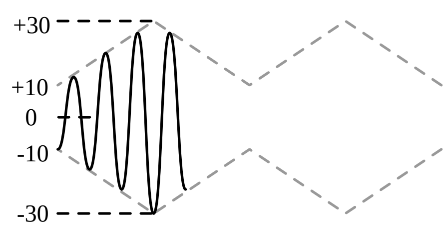
Another method to solve this problem:

$\mu = 0.4$

$P = \frac{A_c^2}{2}(1 + \frac{\mu^2}{2}) \times \frac{1}{100} = \frac{2500}{2\times 100}(1 + \frac{0.4^2}{2}) = 13.5\,Watts$

3)





$\frac{m_p}{A} = 0.5$

For $m_p = 10$

$\Rightarrow A = 20$

4) Let $m(t) = A_m \cos(2\pi f_m t)$. Then the bandwidth of $m(t)$ is f_m. Recall from the lectures that the following condition needs to be satisfied: $RC \ll \frac{1}{f_m}$.

5) The SSB wave $s_{USB}(t)$ is defined by

$$s_{USB}(t) = \frac{A_c}{2}\left[m(t)cos(2\pi f_c t) - m_h(t)sin(2\pi f_c t)\right] \tag{1}$$

Therefore, Hilbert transform of $s_{USB}(t)$ is

$$s^h_{USB}(t) = \frac{A_c}{2}\left[m(t)sin(2\pi f_c t) + m_h(t)cos(2\pi f_c t)\right] \tag{2}$$

From (1) and (2), we get:

$$s_{USB}(t)cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)cos^2(2\pi f_c t) - m_h(t)sin(2\pi f_c t)cos(2\pi f_c t)\right] \tag{3}$$

$$s^h_{USB}(t)sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)sin^2(2\pi f_c t) + m_h(t)cos(2\pi f_c t)sin(2\pi f_c t)\right] \tag{4}$$

Adding (3) and (4) and solving for $m(t)$, we get:

$$m(t) = \frac{2}{A_c}\left[s_{USB}(t)cos(2\pi f_c t) + s^h_{USB}(t)sin(2\pi f_c t)\right] \tag{5}$$

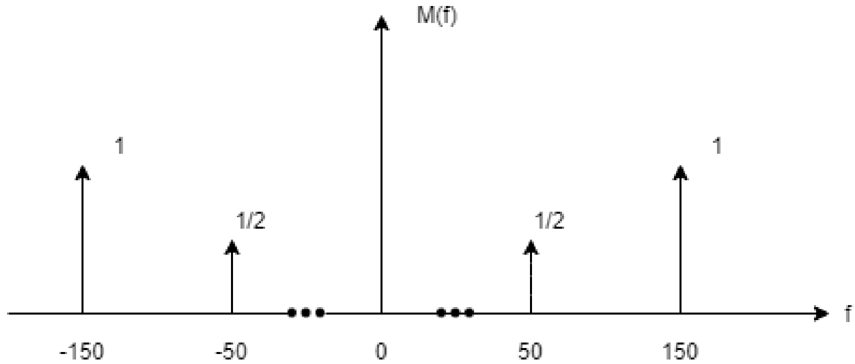
Similarly, we can show that,

$$m_h(t) = \frac{2}{A_c}\left[s^h_{USB}(t)cos(2\pi f_c t) - s_{USB}(t)sin(2\pi f_c t)\right] \tag{6}$$

6)

$$m(t) = cos(100\pi t) + 2cos(300\pi t)$$

a)

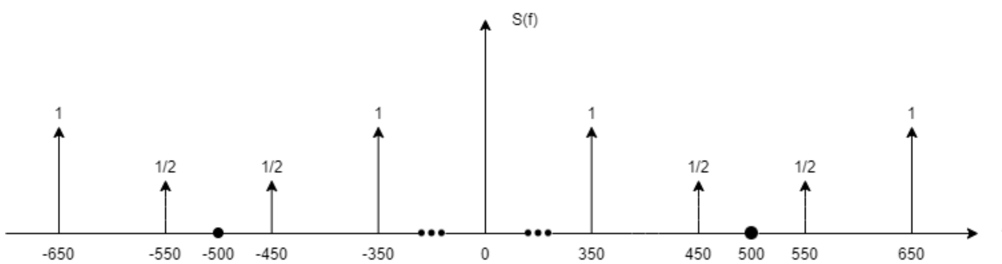


b)

$$2m(t)cost(100\pi t) = s(t)$$

i.e.,

$$f_c = 500Hz$$



c)

See Fig. 1.

d)

$$S_{usb}(t) = cos(1100\pi t) + 2cos(1300\pi t)$$

7) $s_{usb}(t) = m(t)cos(1000\pi t) - m_h(t)sin(1000\pi t)$

$= \{cos(100\pi t) + 2cos(300\pi t)\}cos(1000\pi t) - \{sin(100\pi t) + 2sin(300\pi t)\}sin(1000\pi t)$

$= cos(100\pi t)cos(1000\pi t) + 2cos(300\pi t)cos(1000\pi t) - sin(100\pi t)sin(1000\pi t) - 2sin(300\pi t)sin(1000\pi t)$

$= \frac{1}{2}cos(1100\pi t) + \frac{1}{2}cos(900\pi t) + cos(1300\pi t) + cos(700\pi t) - \frac{1}{2}cos(900\pi t) + \frac{1}{2}cos(1100\pi t)$

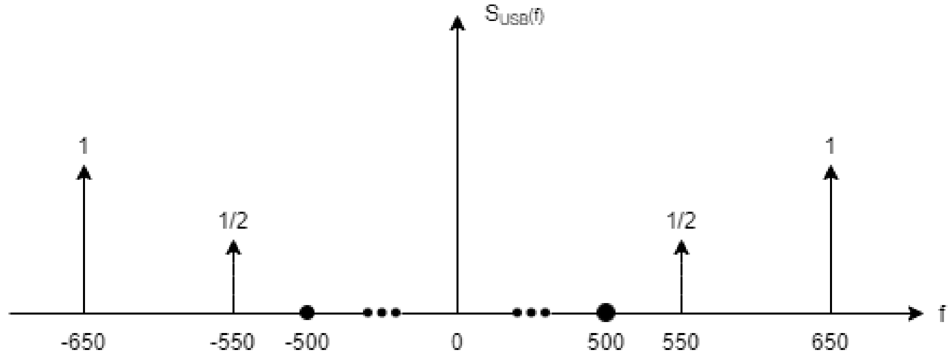


Fig. 1. The figure for Question 6 (c).

$$- cos(700\pi t) + cos(1300\pi t)$$
$$= cos(1100\pi t) + 2cos(1300\pi t)$$

8)

$$s_{usb+c}(t) = A_c \cos(2\pi f_c t) + [m(t)\cos(2\pi f_c t) - m_h(t)\sin(2\pi f_c t)]$$
$$= [A_c + m(t)]\cos(2\pi f_c t) - m_h(t)\sin(2\pi f_c t)$$
$$= E(t)\cos(2\pi f_c t + \phi)$$
$$\text{where } E(t) = \left[(A_c + m(t))^2 + m_h^2(t)\right]^{\frac{1}{2}}$$
$$= A_c\left[1 + \frac{2m(t)}{A_c} + \frac{m^2(t)}{A_c^2} + \frac{m_h^2(t)}{A_c^2}\right]^{\frac{1}{2}}$$

if $A_c >> m(t)$, then in general $A_c >> |m_h(t)|$ (may not be true for all t, but it's true for most t), then

$$E(t) \simeq A_c\left[1 + \frac{2m(t)}{A_c}\right]^{\frac{1}{2}}$$
$$\simeq A_c\left[1 + \frac{m(t)}{A_c}\right] (Using\ Taylor\ Series\ Expansion)$$
$$= A_c + m(t)$$

9)

$$V_{lp}(f) \equiv \tilde{V(f)} = 2rect(\frac{f+100}{400})$$

$$v_{lp}(t) = 800(sinc400t)e^{j2\pi 100t}$$

$$= 800(sinc400t)(cos2\pi 100t + jsin2\pi 100t)$$

$$v_p(t) = 800(sinc400t)cos(2\pi 100t)$$

$$v_q(t) = 800(sinc400t)sin(2\pi 100t)$$

10)

$$v_{bp}(t) = 2z(t)[cos(\pm\omega_0 t + \alpha)cos\omega_c t - sin(\pm\omega_0 t + \alpha)sin\omega_c t]$$

so,

$$v_i(t) = 2z(t)cos(\pm\omega_0 t + \alpha)$$

$$v_q(t) = 2z(t)sin(\pm\omega_0 t + \alpha)$$

$$v_{lp}(t) = 2z(t)[cos(\pm\omega_0 t + \alpha) + jsin(\pm\omega_0 t + \alpha)]$$

$$= 2z(t)e^{(\pm\omega_0 t + \alpha)}$$

11)

$$H_{lp}(f) = \frac{2}{1 + j\frac{2f}{B}} = \frac{2\pi B}{\pi B + j2\pi f}$$

$$\Rightarrow h_{lp}(t) = 2\pi B e^{-\pi B t}u(t)$$

$$x_{bp}(t) = Re[Ae^{j2\pi f_c t}u(t)]$$

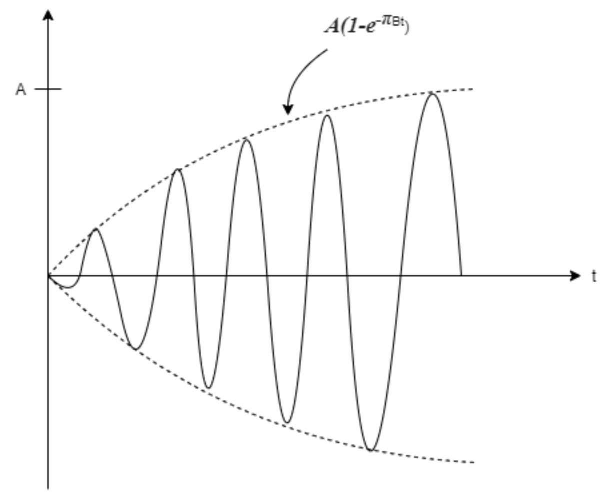
$$\Rightarrow x_{lp}(t) = Au(t)$$

$$y_{lp}(t) = \frac{1}{2}h_{lp}(t) * x_{lp}(t)$$

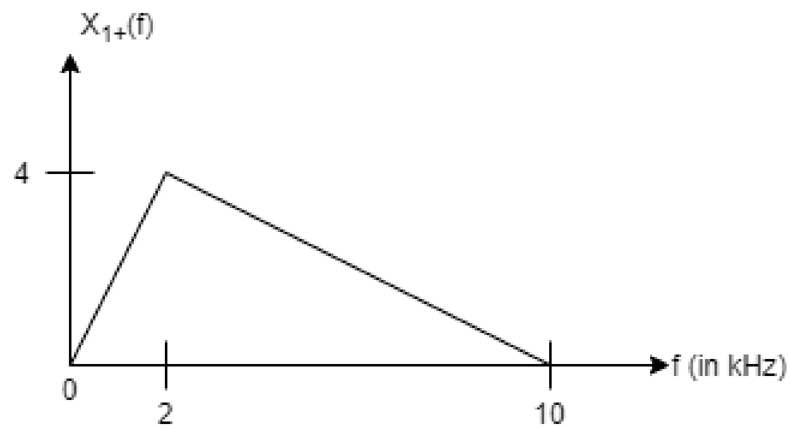
$$= \frac{1}{2}2\pi BA\int_0^t e^{-\pi B(t-\tau)}\,d\tau$$

$$y_{bp}(t) = Re[y_{lp}(t)e^{j2\pi f_c t}]$$

$$= A(1 - e^{-\pi Bt})cos(2\pi f_c t)u(t)$$



12) a)



b)

See Fig. 2.

13)

$$g(t) \longleftrightarrow G(f)$$

$$\frac{1}{1+t^2} \longleftrightarrow \pi e^{-|2\pi f|}$$

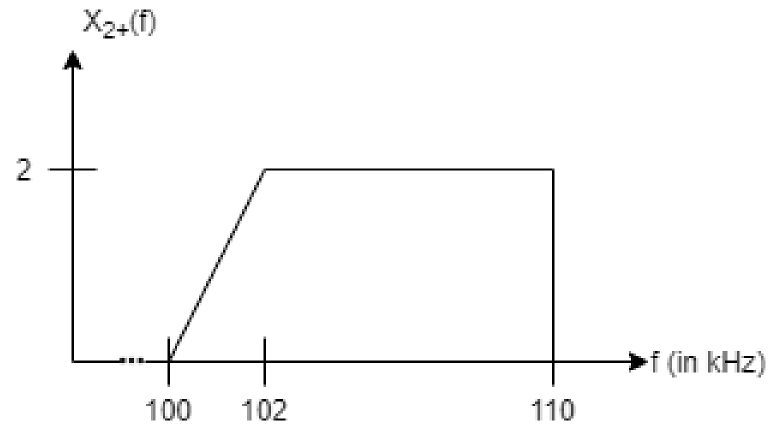


Fig. 2. The figure for Question 12 (b).

Hence,

$$G_+(f) = 2\pi e^{-2\pi f} u(f)$$

We require

$$\mathcal{F}^{-1}[G_+(f)]$$

Now,

$$e^{-t} \longleftrightarrow \frac{1}{1 + j2\pi f}$$

$$e^{-2\pi t} u(t) \longleftrightarrow \frac{1}{2\pi} \frac{1}{1 + jf}$$

From duality theorem,

$$2\pi e^{-2\pi f} u(f) \longleftrightarrow \frac{1}{1 - jt}$$

Pre-envelope of g(t)

$$g_+(t) = \frac{1}{1 - jt} = \frac{1 + jt}{1 + t^2} = \frac{1}{1 + t^2} + j\frac{t}{1 + t^2}$$

14) x(t) can be taken as a BP signal.

$x_{lp}(t) \equiv \tilde{x}(t) = 2, \qquad 0 \le t \le 1msec$

$h(t) = 2cos[2\pi 10^6 (T - t)]$

$= 2\{cos(2\pi 10^6 T) cos(2\pi 10^6 t) + sin(2\pi 10^6 T) sin(2\pi 10^6 t)\}$

$= 2\{cos(2\pi 10^3) cos(2\pi 10^6 t) + sin(2\pi 10^3) sin(2\pi 10^6 t)\}$

$= 2cos(2\pi 10^6 t), \qquad 0 \le t \le 1msec$

Thus, h(t) is also a BP signal

$h_{lp}(t) \equiv \tilde{h}(t) = 2, \qquad 0 \leq t \leq 1msec$

. $\qquad\qquad\qquad = 0, \qquad otherwise$

$x_{lp}(t) = h_{lp}(t) = 2rect(\frac{t-(T/2)}{T})$

$y_{lp}(t) = \frac{1}{2}x_{lp}(t) * h_{lp}(t)$

$= 2 \times 10^{-3} triangle(\frac{t-T}{T})$

$y(t) = y_{lp}(t) cos(2\pi f_c t)$