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CANONICAL REPRESENTATION OF BANDPASS SIGNALS

$$x(t) = \mathrm{Re}\{x_+(t)\}$$

$$= \mathrm{Re}\{\tilde{x}(t)\, e^{j2\pi f_c t}\} \longrightarrow ①$$

$\tilde{x}(t) \longrightarrow$ complex envelope of $x(t)$

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t) \longrightarrow ②$$

$$x(t) = \mathrm{Re}\{\tilde{x}(t)\, e^{j2\pi f_c t}\}$$

$$= \mathrm{Re}\{(x_I(t) + j x_Q(t))(\cos 2\pi f_c t + j \sin 2\pi f_c t)\}$$

$$= x_I(t) \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t$$

$$x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t \longrightarrow (3)$$

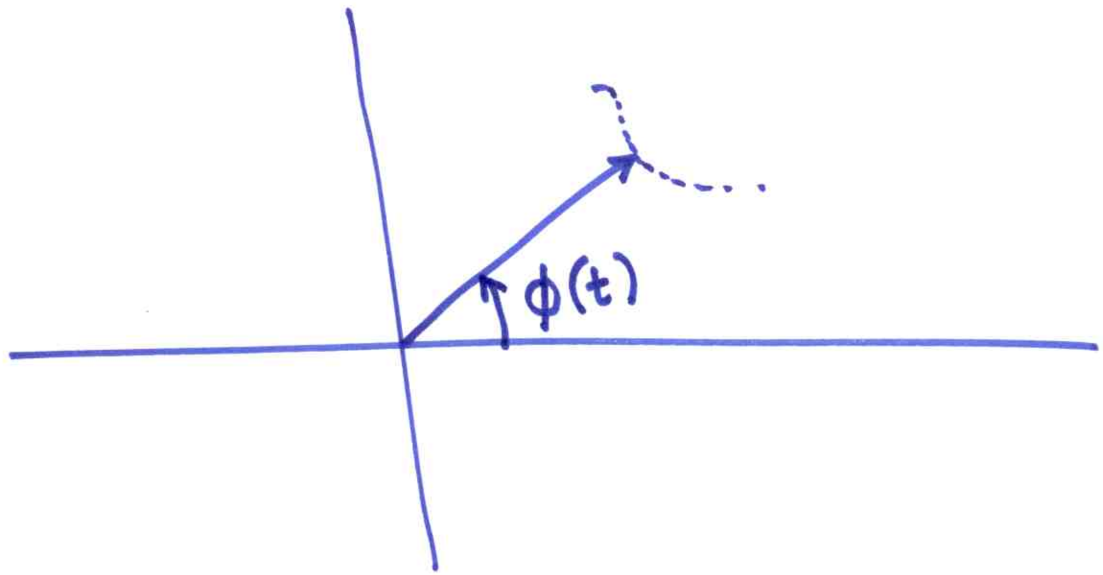
→ CANONICAL REPRESENTATION

→ IN-PHASE AND QUADRATURE COMPONENT REPRESENTATION

$$\tilde{x}(t) = x_I(t) + j\,x_Q(t)$$

$$a(t) = \sqrt{x_I^2(t) + x_Q^2(t)} \longrightarrow (4)$$

$$\phi(t) = \tan^{-1}\left\{\frac{x_Q(t)}{x_I(t)}\right\} \longrightarrow (5)$$


$\phi(t)$

$$\tilde{x}(t) = a(t)e^{j\phi(t)} \rightarrow (6)$$

POLAR FORM

$$x(t) = \mathrm{Re}\left\{ \tilde{x}(t) e^{j2\pi f_c t} \right\}$$

$$= \mathrm{Re}\left\{ a(t) e^{j\phi(t)} e^{j2\pi f_c t} \right\} \rightarrow (7)$$

$$x(t) = a(t)\cos(2\pi f_c t + \phi(t)) \rightarrow (8)$$

$$A\cos(2\pi f_c t + \phi) = \mathrm{Re}\left[A e^{j\phi} e^{j2\pi f_c t} \right]$$

sinusoid

Phasor representation of a sinusoid

$Ae^{j\phi} \rightarrow$ Phasor associated with the sinusoidal signal $A\cos(2\pi f_c t + \phi)$

3 Representations for the BP signal $x(t)$

(i) $$x(t) = \mathrm{Re}\left\{x_+(t)\right\} = \mathrm{Re}\left\{\tilde{x}(t)\, e^{j2\pi f_c t}\right\}$$

(ii) $$x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$$

(iii) $$x(t) = a(t)\cos\left(2\pi f_c t + \phi(t)\right) = \mathrm{Re}\left\{a(t)\, e^{j\phi(t)}\, e^{j2\pi f_c t}\right\}$$

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t)$$

$x(t) \rightarrow$ Narrow Band Bandpass

spectrum $X(f)$: $|f \pm f_c| \leq W$

$x_I(t)$ and $x_Q(t)$ are lowpass signals with spectrum confined to $|f| \leq W$

$\tilde{x}(t) \longleftrightarrow \tilde{X}(f)$

$\tilde{X}(f) = X_+(f+f_c)$ is nonzero only for $|f| \leq W$

$x_I(t)$ is the real part of $\tilde{x}(t)$

$\therefore$ $x_I(t) = \left[\dfrac{\tilde{x}(t) + \tilde{x}^*(t)}{2} \right]$

$$X_I(f) = \frac{\tilde{X}(f) + \tilde{X}^*(-f)}{2}$$

Both $\tilde{X}(f)$ and $\tilde{X}^*(-f)$ are nonzero only for $|f| \leq W$

$\Rightarrow X_I(f)$ is also nonzero for $|f| \leq W$

$|||$ $x_Q(t)$ is also a lowpass signal

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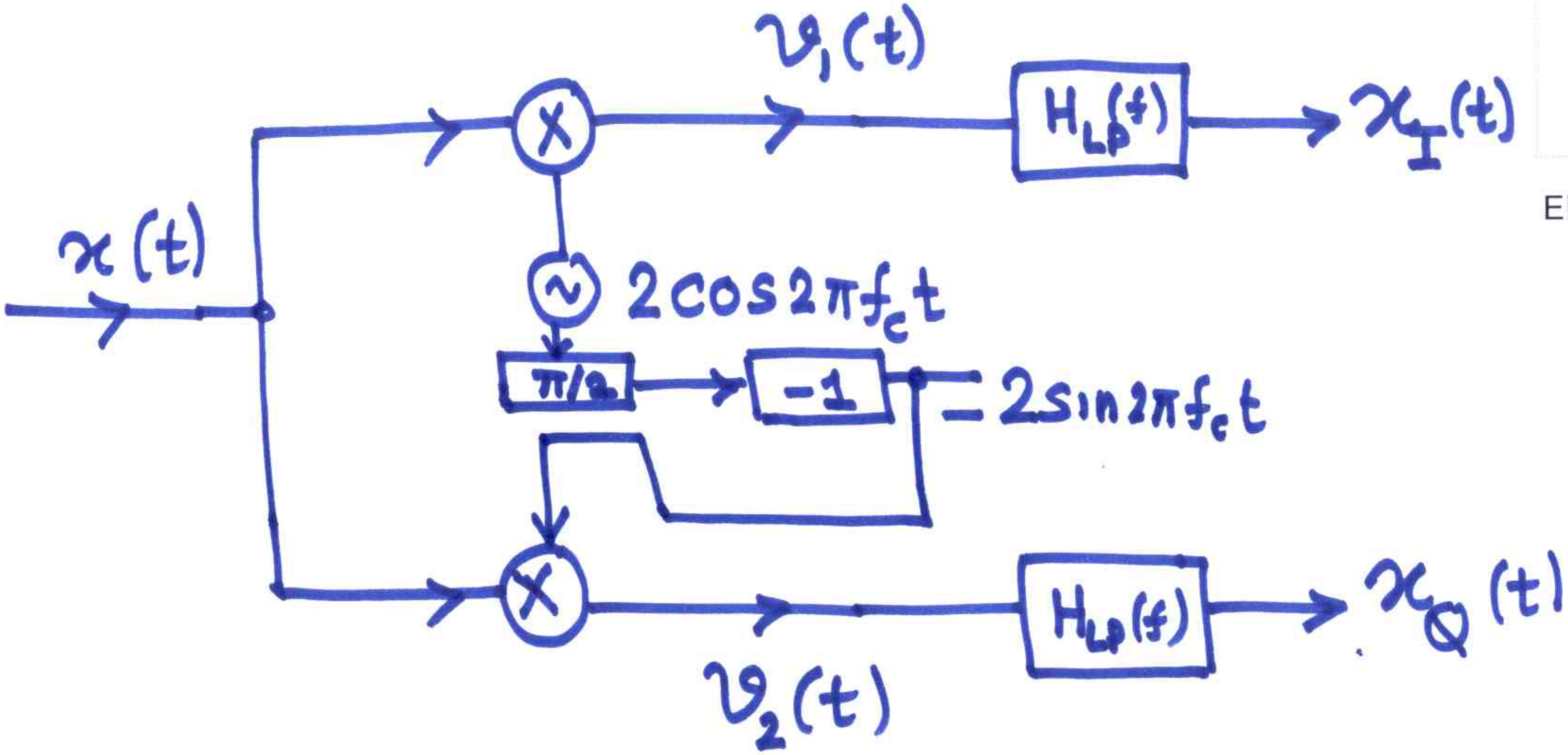

$$v_1(t) = 2\,x(t)\cos 2\pi f_c t$$
$$= 2\left(x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t\right)\cos 2\pi f_c t$$

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$$v_1(t) = 2\left(x_I(t)\cos^2 2\pi f_c t - x_Q(t)\sin 2\pi f_c t \cdot \cos 2\pi f_c t\right)$$

$$= 2x_I(t)\cos^2 2\pi f_c t - x_Q(t)\sin 4\pi f_c t$$

$$= 2x_I(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right] - x_Q(t)\sin 4\pi f_c t$$

$$= x_I(t) + x_I(t)\cos 4\pi f_c t - x_Q(t)\sin 4\pi f_c t.$$

$$\rightarrow x(t) = \mathrm{Re}\left\{ \tilde{x}(t)\, e^{j2\pi f_c t} \right\}$$

$$\rightarrow x(t) = \underbrace{a(t)}_{\text{natural envelope (envelope)}} \cos\left(2\pi f_c t + \underbrace{\phi(t)}_{\text{(PHASE)}}\right)$$

$$a(t) = |\tilde{x}(t)| = |x_+(t)|$$

$x_+(t) \equiv$ pre envelope

$\tilde{x}(t) \equiv$ complex envelope

Example: Let $x(t) = \cos 2\pi f_c t$

Let us find $x_+(t)$, $\tilde{x}(t)$, $a(t)$, $\phi(t)$

$$X(f) = \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$X_+(f) = \delta(f-f_c) \Rightarrow x_+(t) = e^{j2\pi f_c t}$$

$$\tilde{X}(f) = \delta(f) \implies \tilde{x}(t) = 1$$

$\therefore$ $\tilde{x}(t) \rightarrow$ is real & positive

$$\implies \phi(t) = 0$$

$$a(t) = |\tilde{x}(t)| = 1$$

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MODULE END