Angle Modulation: Examples

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Example

- Suppose message signal $m(t)$ strictly band-limited with bandwidth B
- Also, suppose modulated signal:
  - $s(t) = A_c \cos(2\pi f_c t + k m^2(t))$,
  - where $A_c > 0$, $k > 0$ are constants and $f_c \gg B$
- Want to estimate bandwidth of $s(t)$
- Instantaneous frequency of $s(t)$:
  - $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$
  - $f_i(t) = f_c + \frac{k}{\pi} m(t) \frac{dm(t)}{dt}$
- So $s(t)$ is an FM signal with modulating signal $\widetilde{m}(t) = m(t) \frac{dm(t)}{dt}$ and frequency sensitivity $k_f = \frac{k}{\pi}$
- Bandwidth of signal $\widetilde{m}(t)$:
  - $2B$
- Assume that:
  - $-m_p \leq m(t) \frac{dm(t)}{dt} \leq m_p$, where m_p is a constant
- Frequency deviation:
  - $\Delta f = \frac{k m_p}{\pi}$
- So by Carson's rule, an estimate for bandwidth of $s(t)$:
  - $\frac{2k m_p}{\pi} + 4B$

Example

- Suppose carrier signal is $c(t) = 10\cos(2\pi f_c t)$ and message signal is $m(t) = \cos(20\pi t)$
- Above message signal used to frequency modulate carrier with frequency sensitivity $k_f = 50$
- Modulated signal $s(t)$:
  - $10\cos\left(2\pi f_c t + 2\pi k_f \int_0^t \cos(20\pi\tau)d\tau\right)$
  - $10\cos\big(2\pi f_c t + 5\sin(20\pi t)\big)$
- Suppose we define transmission bandwidth of above FM signal to be width of smallest band that contains at least $98\ \%$ of modulated signal power
- Want to find transmission bandwidth
- Modulation index β:
  - $\frac{k_f A_m}{f_m} = \frac{50\times 1}{10} = 5$
- $s(t)$ can be written as:
  - $A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$
  - $10\sum_{n=-\infty}^{\infty} J_n(5)\cos[2\pi(f_c + 10n)t]$
- Total power in f_c and first k side-frequencies to the right and left of f_c:
  - $\frac{10^2}{2}\sum_{n=-k}^{k} J_n^2(5)$
  - $50\big(J_0^2(5) + 2\sum_{n=1}^{k} J_n^2(5)\ \big)$
- So k must be such that:
  - $50\big(J_0^2(5) + 2\sum_{n=1}^{k} J_n^2(5)\ \big) \geq 0.98 \times \frac{10^2}{2}$
- Using a table of Bessel functions, we find that we need to choose $k = 6$
- So transmission bandwidth is:
  - $2 \times 10 \times 6 = 120$ Hz

Example

- Angle-modulated signal with carrier frequency $f_c = 10^5$ Hz given by:
  - ❑ $s(t) = 10\cos(2\pi f_c t + 5\sin(3000t) + 10\sin(2000\pi t))$
- Power of $s(t)$:
  - ❑ $\frac{10^2}{2} = 50$
- Want to find frequency deviation Δf
- Instantaneous frequency:
  - ❑ $\frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$
  - ❑ $\frac{1}{2\pi}[2\pi f_c + 15000\cos(3000t) + 20000\pi\cos(2000\pi t)]$
- So Δf:
  - ❑ $\frac{1}{2\pi}[15000 + 20000\pi]$
  - ❑ 12387.32 Hz
- Deviation ratio:
  - ❑ $\frac{\Delta f}{B}$
  - ❑ $\frac{12387.32}{1000} = 12.39$
- Bandwidth of $s(t)$:
  - ❑ $B_T \approx 2\Delta f + 2B$ (by Carson's rule)
  - ❑ $2 \times 12387.32 + 2 \times 1000 = 26.775$ kHz

Recall: Narrow-band FM (NBFM) Generation

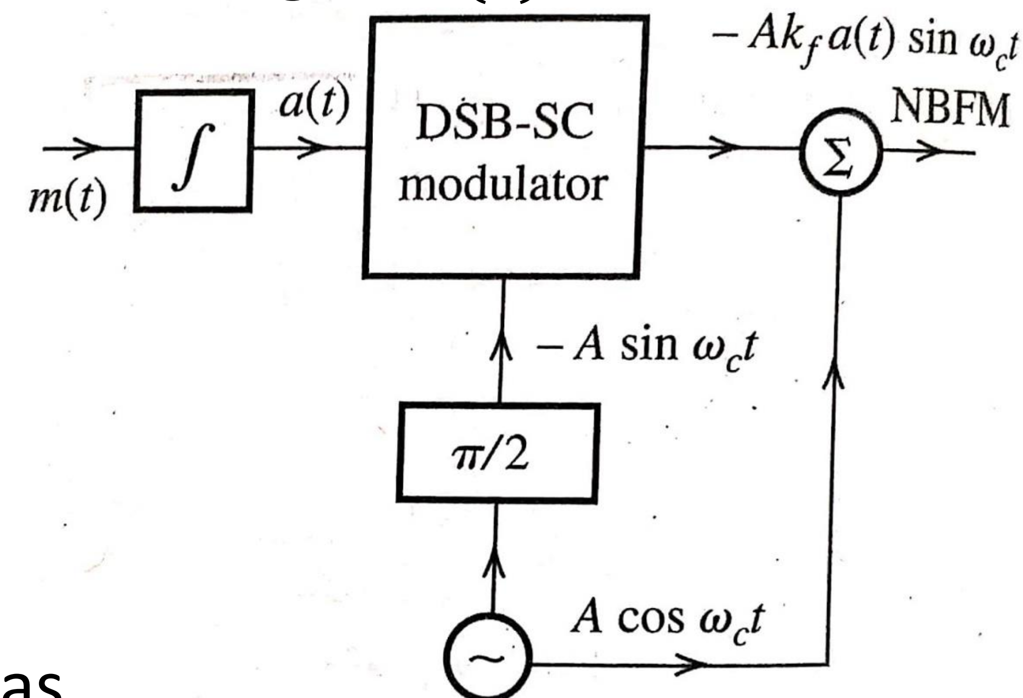
- FM signal given by:

  1) $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$

- Let $a(t) = \int_0^t m(\tau)d\tau$
- When $\left|k_f a(t)\right| \ll 1$ for all t, the signal $s(t)$ called narrow-band FM signal
- NBFM signal can be approximated by:

  2) $s(t) \approx A_c \cos[2\pi f_c t] - A_c 2\pi k_f a(t) \sin[2\pi f_c t]$

- Signal in 2) can be generated using:
  - a DSB-SC modulator as shown in fig.



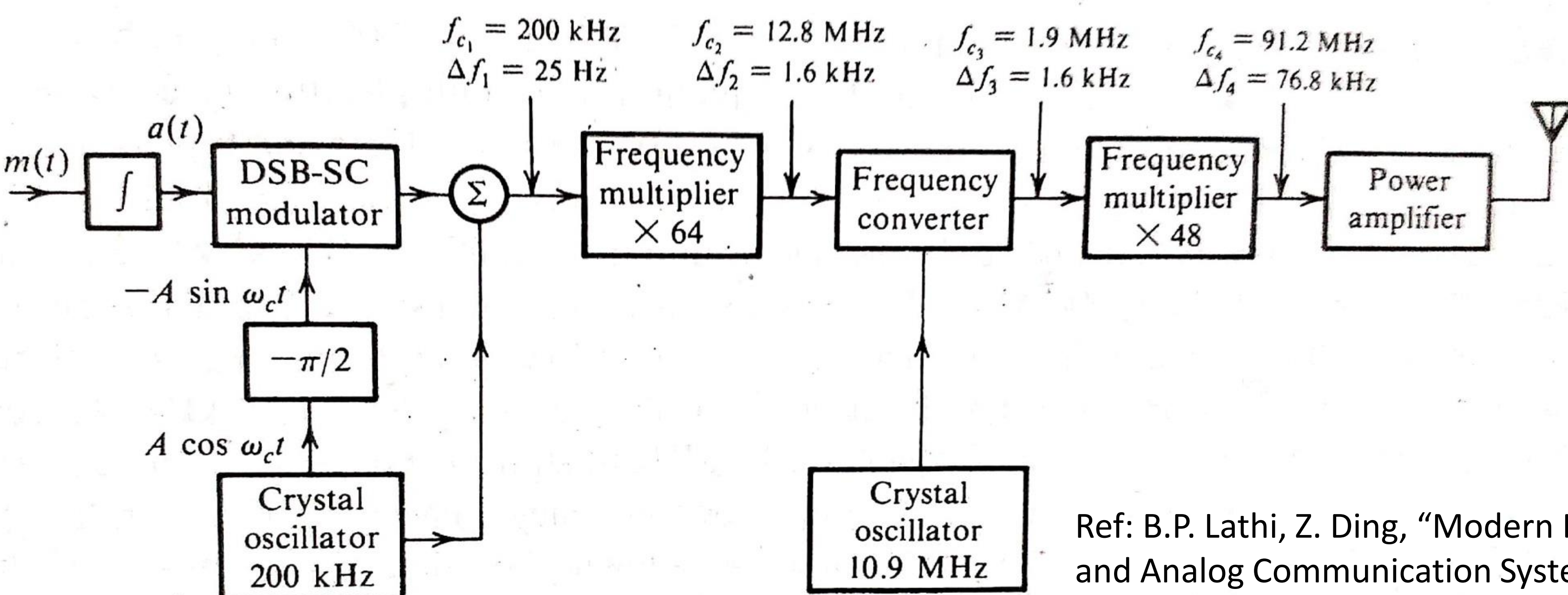
Ref: B.P. Lathi, Z. Ding, “Modern Digital and Analog Communication Systems”, 4th ed.

Recall: Indirect Method of Armstrong

- FM signal given by:
  - $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$
- Let $a(t) = \int_0^t m(\tau) d\tau$
- In indirect method of Armstrong:
  - first NBFM signal is generated (possibly using DSB-SC modulator method we discussed)
  - then NBFM signal converted to WBFM signal of desired center frequency and frequency deviation by using frequency multipliers and frequency converters

Example

- Want to generate WBFM signal with:
  - ❑ carrier frequency $f_c = 91.2$ MHz and
  - ❑ frequency deviation $\Delta f = 76.8$ kHz
- We initially generate NBFM signal with $\Delta f = 25$ Hz and $f_c = 200$ kHz and then convert it to required WBFM signal as shown in fig.
- Note that we need to multiply 25 Hz by 3072 to get 76.8 kHz
- However, if carrier frequency of 200 kHz multiplied by 3072, then we get 614.4 MHz
- Hence, we need a frequency converter stage



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Example

- Want to design an Armstrong indirect FM modulator to generate FM signal with:
  - carrier frequency 97.3 MHz and $\Delta f = 10.24$ kHz
- An NBFM generator is available with:
  - $f_{c_1} = 20$ kHz and $\Delta f_1 = 5$ Hz
- Only frequency doublers can be used as multipliers
- Also, a local oscillator (LO) with adjustable frequency between 400 and 500 kHz is available for frequency mixing
- Solution:
  - NBFM output is input to frequency multiplier with factor 16
  - Frequency multiplier output is input to mixer with frequency 440.15 kHz followed by band-pass filter that retains band around 760.15 kHz
  - Band-pass filter output is input to frequency multiplier with factor 128

Direct Generation of Wideband FM Signals

- In a voltage-controlled oscillator (VCO), frequency of output sinusoid controlled by input voltage
- FM signal can be generated using a VCO with input being:
  - message signal $m(t)$
- Instantaneous frequency:
  - $f_i(t) = f_c + k_f m(t)$
- VCO can be implemented:
  - by varying the capacitance value according to $m(t)$ in an oscillator containing inductor and capacitor
  - e.g., Hartley oscillator, Colpitts oscillator
- Implementation of capacitor with variable capacitance:
  - reverse-biased diode acts as capacitor whose capacitance varies with bias voltage
  - variable capacitor also known as “varicap”, “varactor” or “voltacap”

Direct Generation of Wideband FM Signals (contd.)

- In Hartley or Colpitts oscillator, frequency of oscillation given by:
  - ❑ $f_0 = \frac{1}{2\pi\sqrt{LC}}$
- Suppose capacitance C varied using $m(t)$:
  - ❑ $C = C_0 - km(t)$
- Then f_0:
  - ❑ $\frac{1}{2\pi\sqrt{L(C_0 - km(t))}}$
- $f_0 \approx$:
  - ❑ $\frac{1}{2\pi\sqrt{LC_0}}\left[1 + \frac{km(t)}{2C_0}\right]$ when $\frac{km(t)}{C_0} \ll 1$
- So $f_0 = f_c + k_f m(t)$,
  - ❑ where $f_c = \frac{1}{2\pi\sqrt{LC_0}}$
  - ❑ $k_f = \frac{kf_c}{2C_0}$
- Max. capacitance deviation:
  - ❑ $\Delta C = km_p$, where $-m_p \leq m(t) \leq m_p$
- $\frac{\Delta C}{C_0}$:
  - ❑ $\frac{2\Delta f}{f_c}$,
  - ❑ where $\Delta f = k_f m_p$ (frequency deviation)
- In practice, $\frac{\Delta f}{f_c}$ is small even in WBFM; hence distortion due to above approximation is small
- So direct FM generation can be used to produce sufficient frequency deviation and does not require much frequency multiplication