- **Bandwidth of Wide-band Frequency Modulation**

- **Generation of WBFM**

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- **Bandwidth of Wide-band Frequency Modulation**

WIDEBAND FM (WBFM)

- $m(t) = A_m \cos(2\pi f_m t)$
- $f_i(t) = f_c + k_f\, m(t)$
- $\theta_i(t) = 2\pi \int_0^t f_i(\tau)\, d\tau$
- $s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$

$\beta = \Delta f / f_m$; $\Delta f = k_f A_m$

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$$s_+(t) = A_c \exp\left\{ j2\pi f_c t + j\beta \sin(2\pi f_m t) \right\}$$

$\tilde{s}(t) \equiv$ complex envelope

$$= s_+(t)\, e^{-j2\pi f_c t}$$

$$s(t) = \mathrm{Re}\left\{ \tilde{s}(t) \exp(j2\pi f_c t) \right\}$$

$$\tilde{s}(t) = A_c \exp\left\{ j\beta \sin(2\pi f_m t) \right\} \leftarrow \text{periodic function}$$

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$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j 2\pi n f_m t)$$

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j 2\pi n f_m t)\, dt$$

$$= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp\left[j\beta \sin(2\pi f_m t) - j 2\pi n f_m t\right] dt$$

Let $x = 2\pi f_m t$

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$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{\pi} \exp\left[j\beta \sin x - nx\right] dx.$$

$$c_n = A_c J_n(\beta)$$

where $J_n(\beta) \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[j(\beta \sin x - nx)\right] dx$

n^{th} order Bessel function of the first kind

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j 2\pi n f_m t)$$

$$s(t) = A_c \cdot \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left[j2\pi(f_c + nf_m)t\right]\right]$$

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

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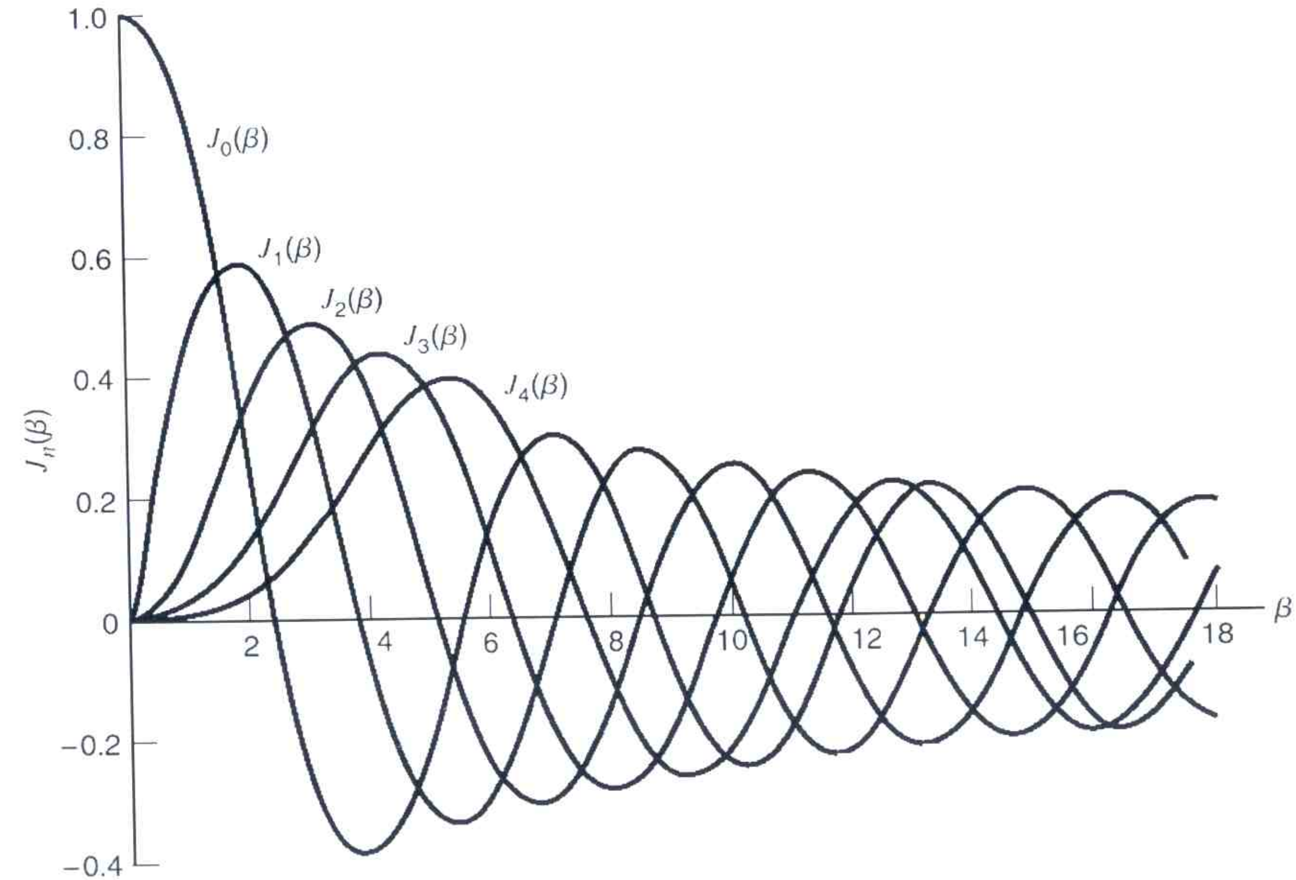
Plots of Bessel functions of the first kind for varying order



Properties of $J_n(\beta)$

(1) $J_n(\beta)$ is always REAL (for all 'n' and 'β')

(2) $J_n(\beta) = (-1)^n J_{-n}(\beta)$

(3) For small values of β,

$$J_n(\beta) \cong \frac{(\beta/2)^n}{n!}$$

(for small values of β)

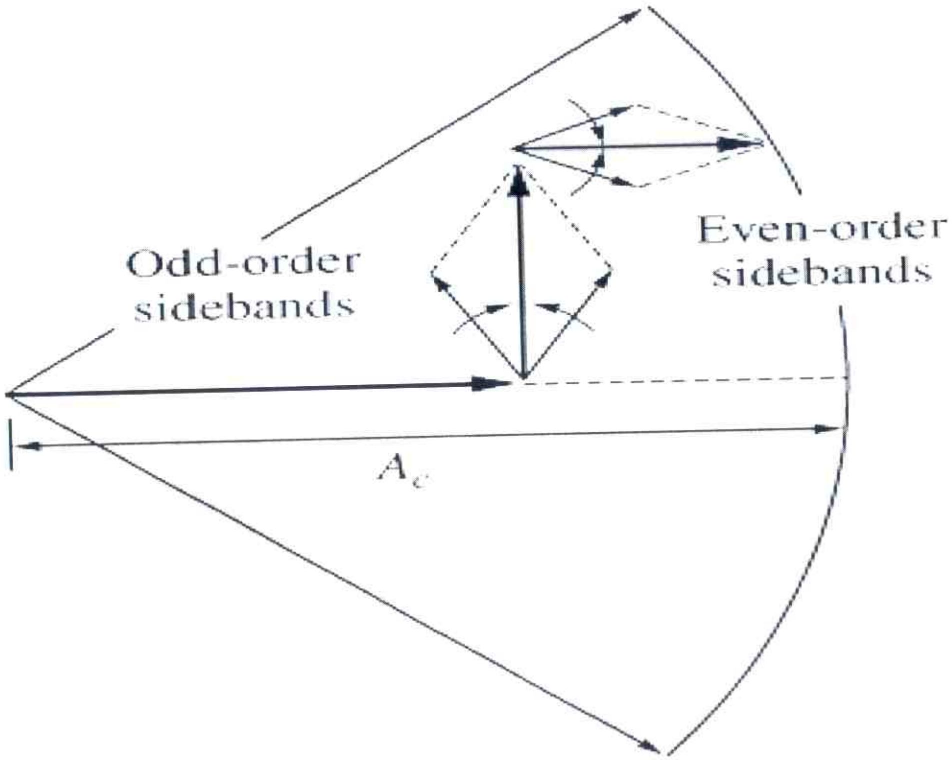
$$J_0(\beta) \cong 1$$

$$J_1(\beta) \cong \frac{\beta}{2}$$

$$J_n(\beta) \cong 0\,, \quad n > 2$$

(4) $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

FM phasor diagram for arbitrary β



Following observations regarding the spectrum:

(1) The spectrum of an FM signal contains a carrier component ($n=0$) and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations of $f_m, 2f_m, 3f_m, \dots$

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(2) For the special case of β small compared with unity, only $J_0(\beta)$ and $J_1(\beta)$ have significant values, so that the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$

(NBFM)

(3) The amplitude of the carrier component of an FM signal is dependent on β.

- Envelope ~~of~~ an FM signal is CONSTANT

$$P_{av} = \frac{1}{2} A_c^2 \quad \left(\because P_{av} = \frac{1}{2} A_c^2 \underbrace{\sum_{n=-\infty}^{\infty} J_n^2(\beta)}_{=1} \right)$$

(4) $J_n(\beta)$ decays monotonically for $\frac{n}{\beta} > 1$ and that $|J_n(\beta)| << 1$ for $\left|\frac{n}{\beta}\right| >> 1$

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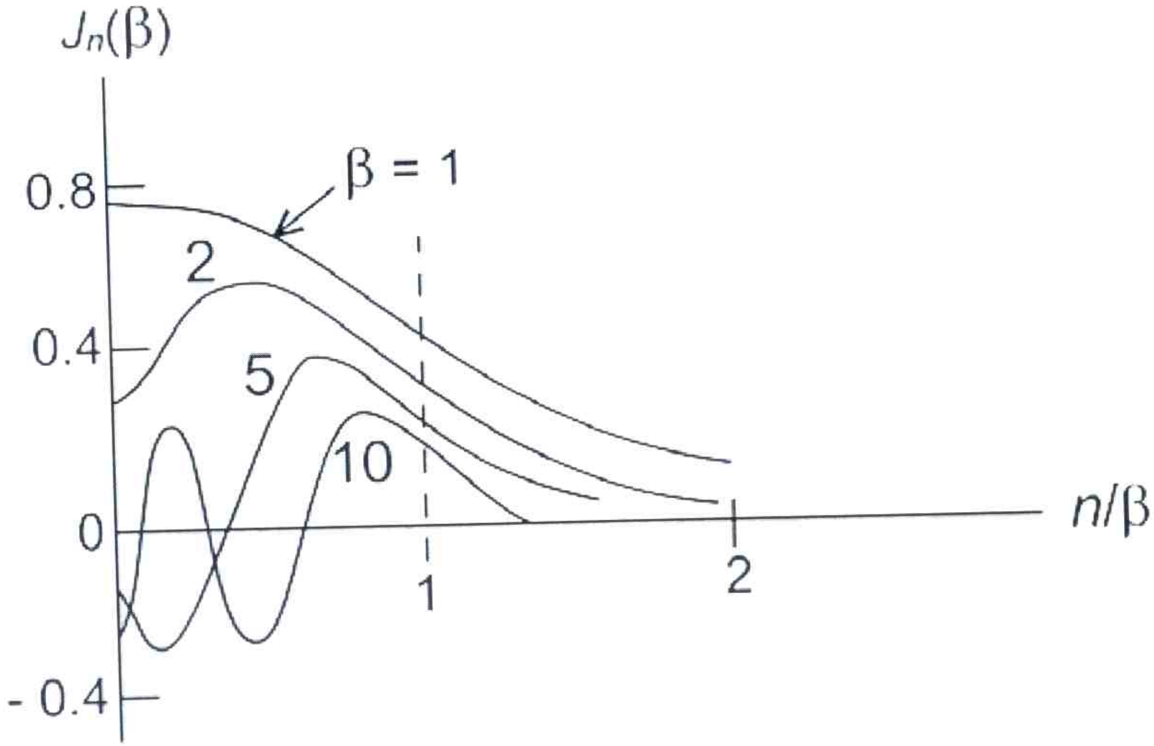
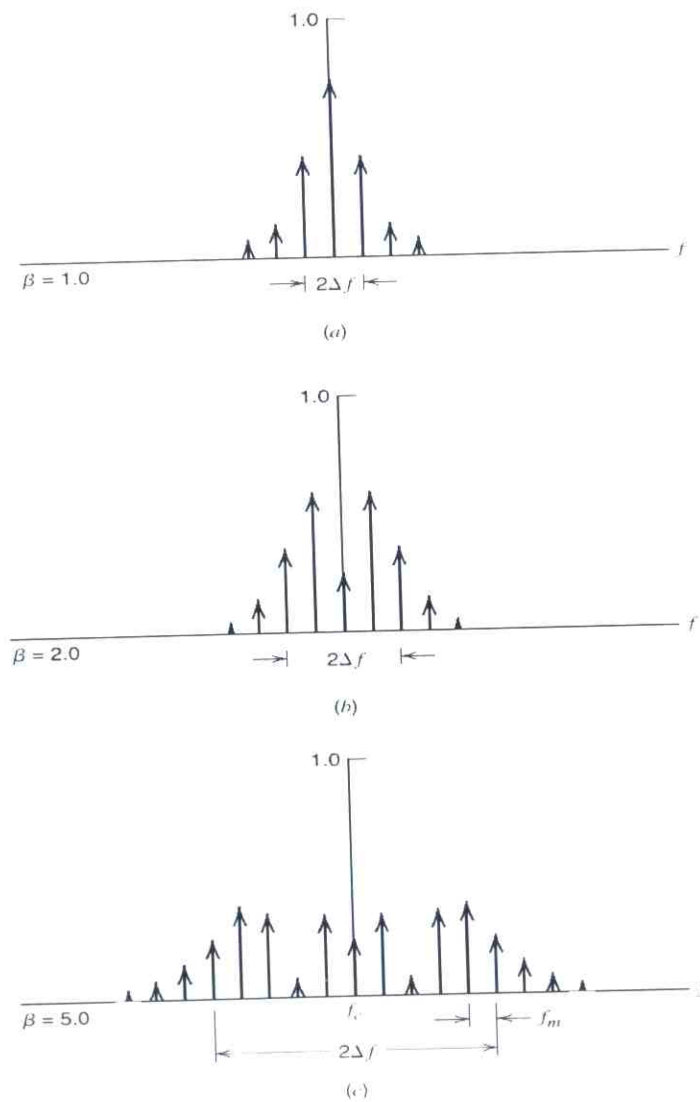
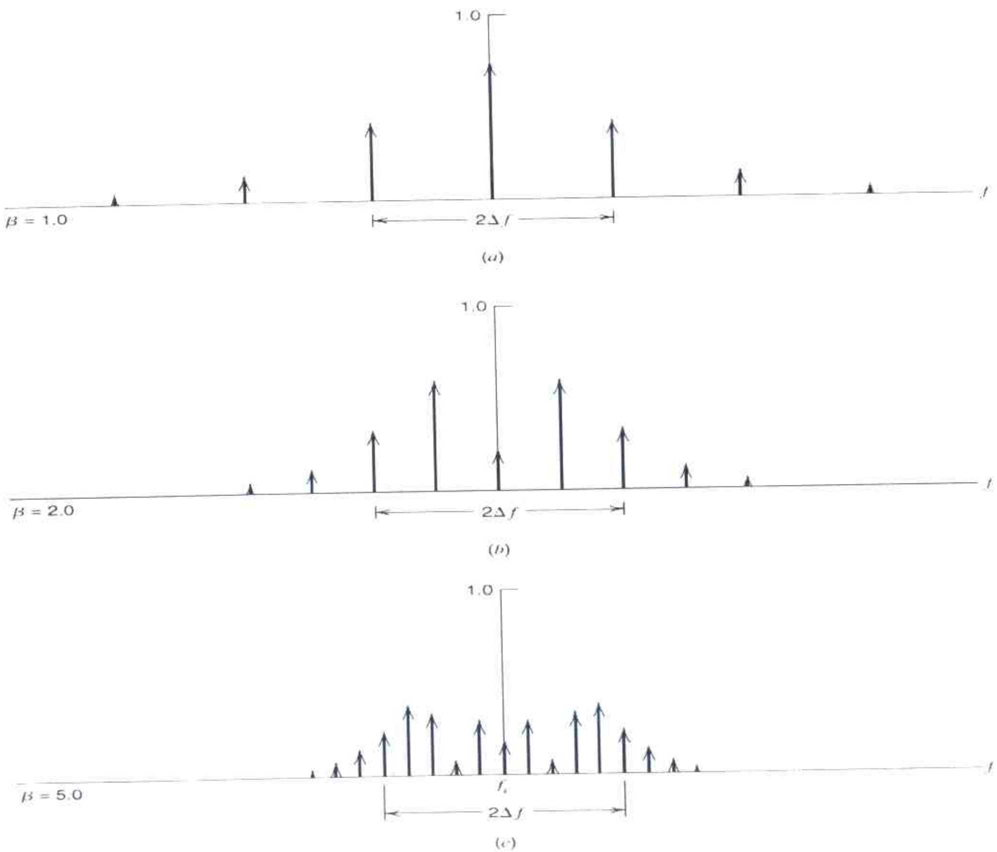




Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown



Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown

Transmission BW of FM signal

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- In theory, an FM wave contains an infinite number of side-frequencies, which implies infinite BW
- In practice, the FM wave is effectively limited to a finite number of significant side-frequencies compatible with a specified amount of distortion
- Consider tone-modulation:
  - In such an FM signal, the side frequencies that are separated from the carrier frequency f_c by an amount greater than the frequency deviation Δf decreases rapidly towards zero, so that the BW always exceeds the total frequency excursion, but nevertheless is limited

TRANSMISSION BW OF FM SIGNAL

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- Two limiting cases:
  - For large values of β , BW approaches, and is only slightly greater than the total frequency excursion
  - For small values of β , BW approaches $2f_m$

$$BW \cong 2\Delta f + 2f_m$$

CARSON'S FORMULA

UNIVERSAL CURVE FOR FM TRANSMISSION BANDWIDTH

- Carson's rule is simple to use, BUT, does not always provide a good estimate for WBFM

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- Use a definition based on retaining the maximum number of significant side frequencies whose amplitudes are all greater than some selected value
- BW of FM defined as separation between the two frequencies beyond which none of the side frequencies is greater than, say 1%, of the carrier amplitude obtained when the modulation is removed
- $BW = 2n_{max}f_m$ where f_m is the modulation frequency and n_{max} is the largest value of the integer n that satisfies the requirement $|J_n(\beta)| >> 0.01$

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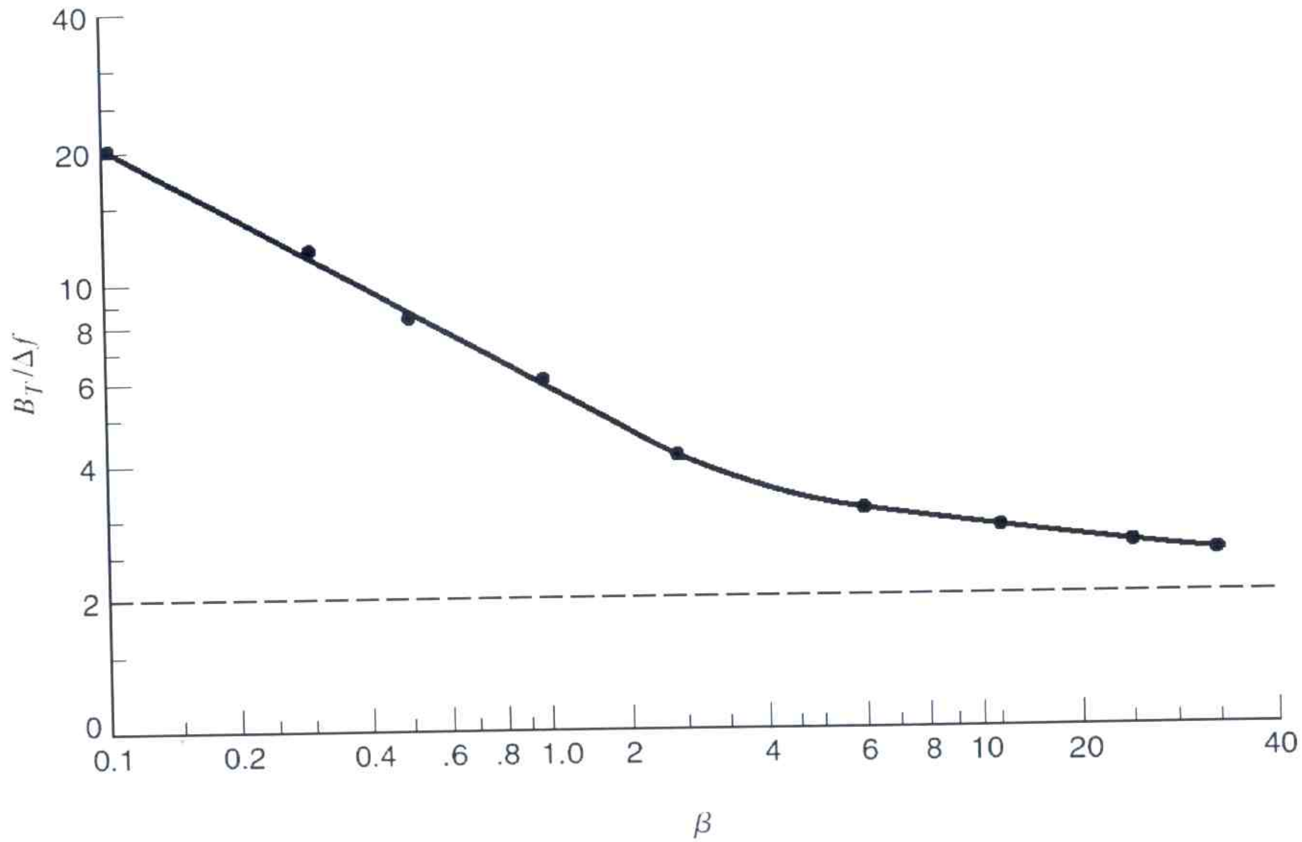
Universal curve for evaluating the 1 percent bandwidth of an FM wave



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- **Generation of WBFM**

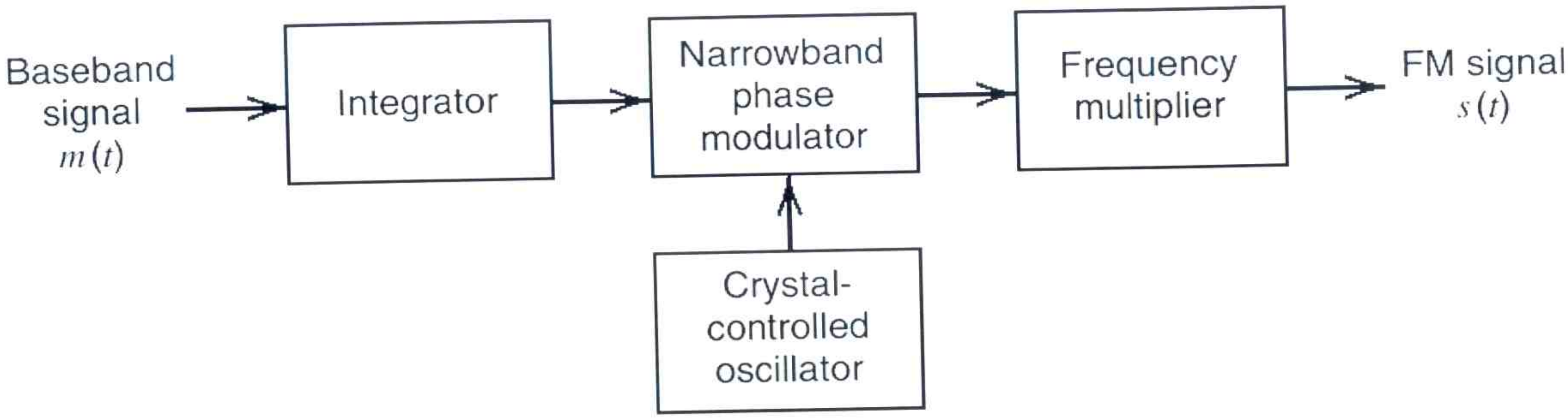
DIRECT METHOD

- Uses a sinusoidal oscillator, with one of the Reactive elements (e.g., capacitive element) in the tank circuit of the oscillator being directly controllable by the message signal
- Conceptually, it is straightforward to implement
- Moreover, it is capable of providing large frequency deviation

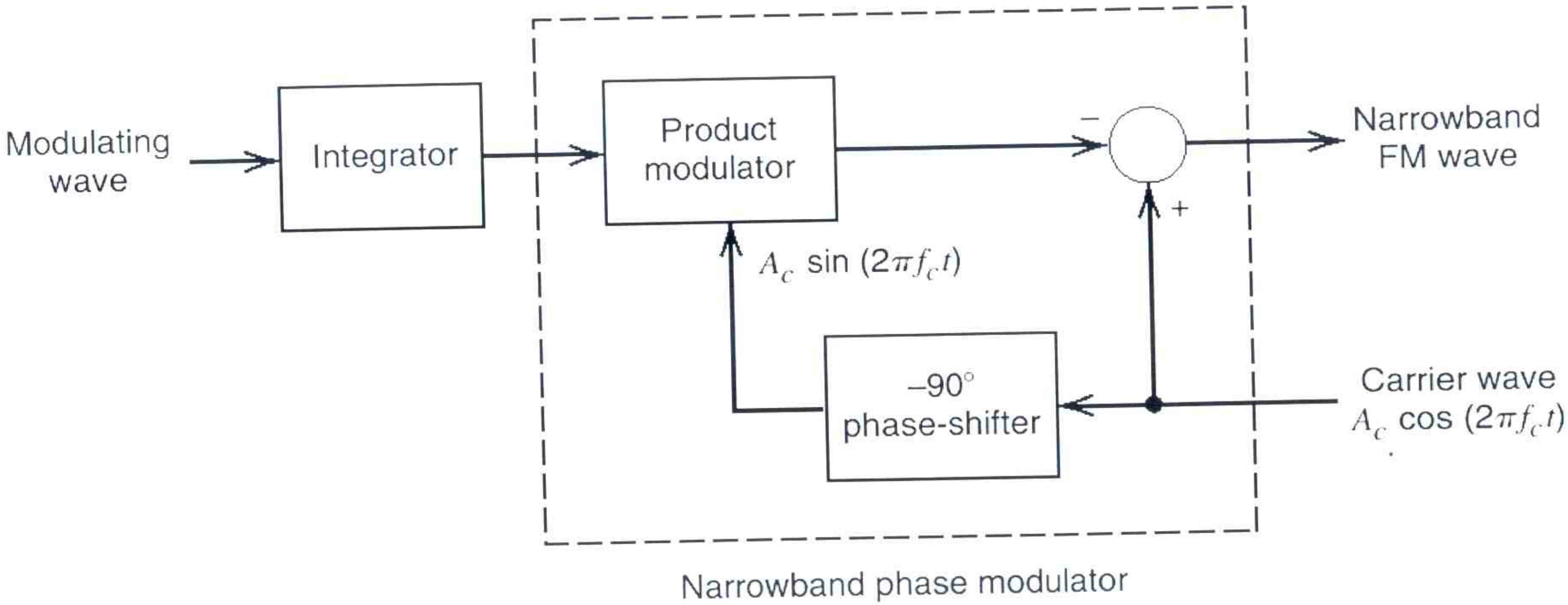
Serious Limitation:

- tendency for the carrier frequency to shift, which is unacceptable for commercial radio applications
- frequency stabilization of the FM generator is required, which adds system complexity to the design of the frequency modulator

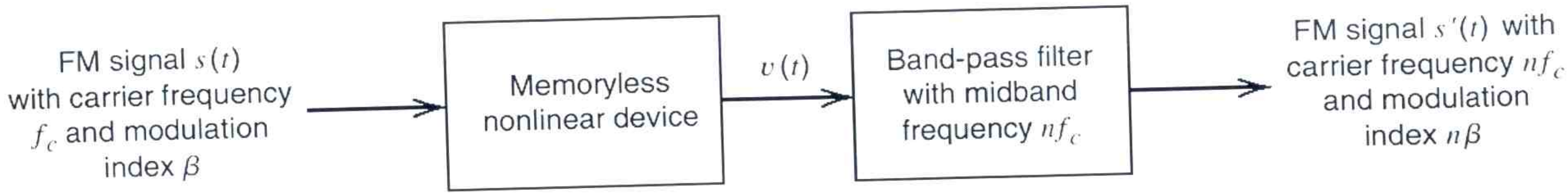
Block diagram of the indirect method of generating a wideband FM signal



Block diagram of a method for generating a narrowband FM signal



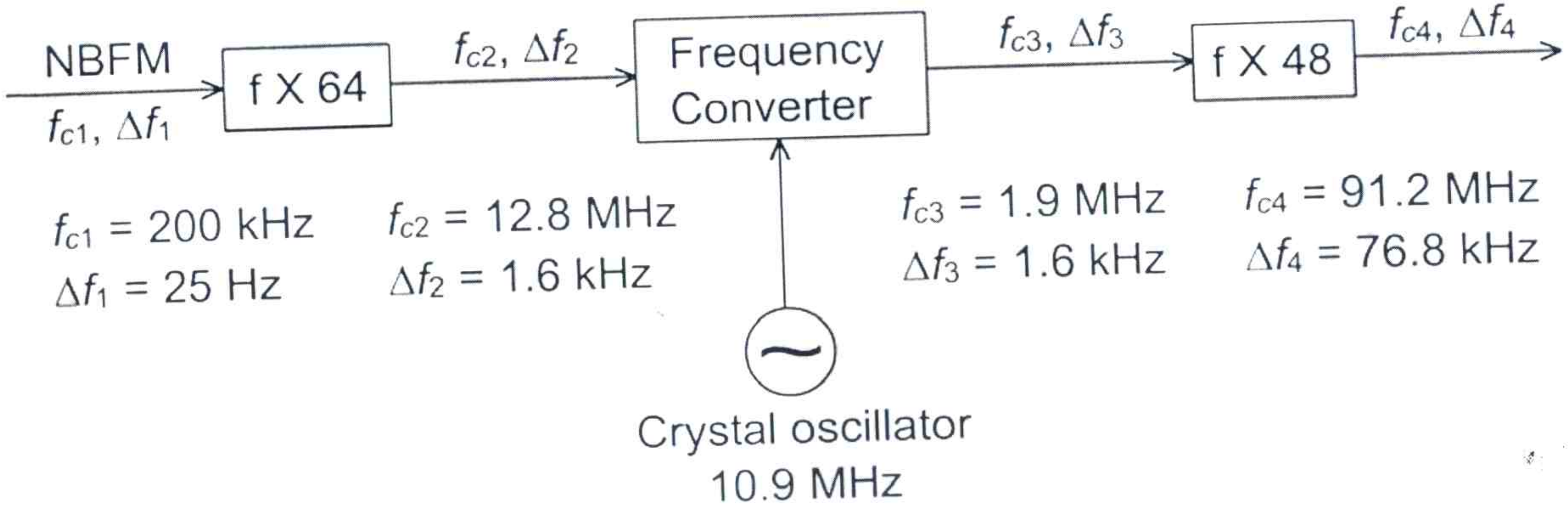
Block diagram of frequency multiplier



$$v(t) = \alpha_1 s(t) + \alpha_2 s^2(t) + \cdots + \alpha_n s^n(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$

$$s'(t) = A_c \cos\left[2\pi n f_c t + 2\pi n k_f \int_0^t m(\tau)\,d\tau\right]$$



COMMERCIAL FM RADIO MODULATOR IMPLEMENTATION