EE324, Control Systems Lab, Problem sheet 1
(Submission date: 2nd August 2021)

Analysis in Laplace domain:

In Scilab, type the following command in the command prompt:

```
s = poly(0,'s')
```

This defines the symbolic variable 's'. Once 's' is defined we can easily define polynomials in s and rational functions in s in the following manner:

```
f = s^3 + 6*s^2 + 11*s + 6
```

Like the polynomial 'f' above, define two more polynomials 'n' and 'd', and do the following:

```
G = n/d
```

```
sys = syslin('c',G)
```

This defines the continuous time system having transfer function G.

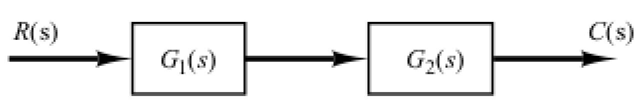
With this background, now solve the following problems:

Q1.Suppose there are two components with transfer functions G1(s) and G2(s) connected differently as
shown in Figure(a),(b) and (c), where

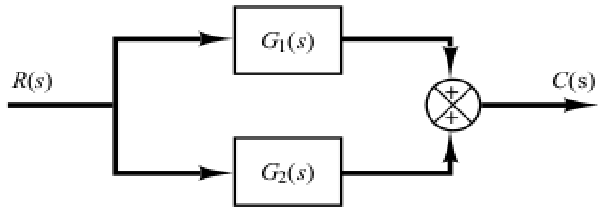
G1(s) = 10/(s^2+2s+10)   G2(s) = 5/(s+5)

Obtain the transfer function of the following systems using Scilab. In your report, write down every step that you followed for solving the problem for each of the cases.

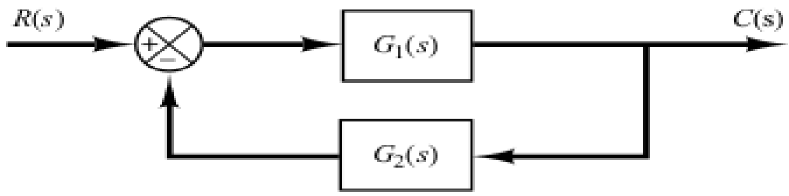
(a) Cascade system :



(b) Parallel system :



(c) Feedback(closed loop) system :



(d) Plot response to the unit step to the system with transfer function G1(s). (Use `csim` command.)

Q2. For each of the cases in Q1, find out the poles and zeros of the overall system (that is, having transfer function C(s)/R(s)). Write down all the steps you followed.
Hint: Find out using Scilab help or otherwise how to obtain roots of a given polynomial.

Scilab is quite powerful in handling polynomials and rational function in the variable s. Scilab can be used to deal with polynomial matrices as well. To start with, define a matrix in Scilab by executing the following command:

```
A = [1 2; 3 4]
```

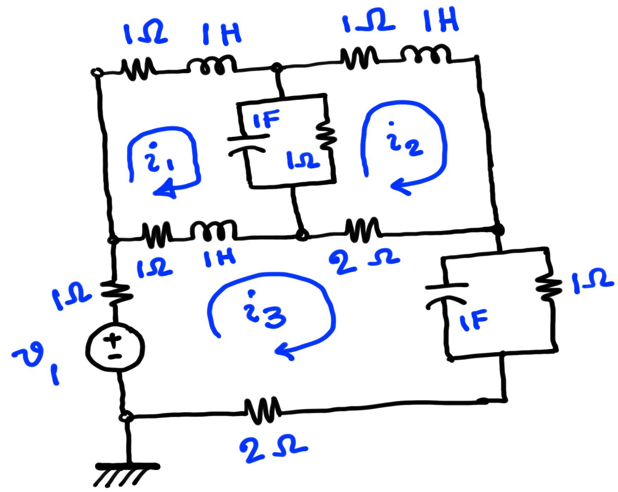
This defines the 2 x 2 matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

We can likewise define a matrix with polynomial or rational function entries. And then usual matrix operations, like addition, multiplication, taking determinants, inverses, etc. will work for polynomial matrices, too. Try these operations on polynomial matrices. Show your computations in your report.

Now solve the following problem based on these tools.

Q3. Consider the following electrical circuit. Write down the equation that one obtains via mesh analysis, in matrix-vector form $Z(s)I(s) = V(s)$.
Using Scilab, find out the transfer functions $I\_1(s)/V\_1(s)$, $I\_2(s)/V\_1(s)$, $I\_3(s)/V\_1(s)$. Show all the steps in your report.



1Ω
1H
1Ω
1H
1F
1Ω
i_1
i_2
1Ω
1H
2Ω
1Ω
i_3
1F
1Ω
v_1
2Ω

EE324, Control Systems Lab, Problem sheet 2
(Report submission date: 9th August 2021)

Analysis in time domain:

Q1a: In Scilab, build a continuous time LTI system with transfer function G(s) = a/(s+b), where a is equal to the last two digits of your roll number, and b is equal to the position in the alphabet of the first letter in your first name. Show all the steps in your report. **Hint:** use the command syslin.

b: Plot the response of this system to a unit step input. Save the plot and include it in your report (taking a screenshot is not allowed). Show the following points clearly in your plot: time constant, 2% settling time, and rise time.

c: Vary the parameter a from its original value to 100 times that in steps of a. Show by means of a plot how rise time varies with this variation in a.

d: Vary the parameter b from its original value to 100 times that in steps of b. Show by means of a plot how rise time varies with this variation in b.

Q2: Plot the step response of a standard, under-damped second order continuous time system with no zeros. Show the response in your report and, also, all the intermediate steps you followed to arrive at the plot. Write the damping ratio you used for this example.

Show by means of a series of plots (one single figure) the step responses of the above-mentioned standard second order system as its damping ratio varies from 0 to 2 in steps of 0.25. Observe how percentage-overshoot, rise-time, 2% settling time, and peak-time change with change in damping ratio.

Q3: Build two continuous time systems, one first order and the other second order, both without any finite zeros, such that both the systems have their step responses increase monotonically from initial value 0 to final value 1. Show these two step responses in a single plot, put it in your report. Write in your report the salient points of differences between these two responses. Check if step response is monotonic when we have repeated poles in the second-order system.

Q4a: Consider a continuous time single-integrator (G(s) = 1/s) . Plot response to the unit step of the system with transfer function G(s). (Use csim command.) Write commands in the report.

Q4b: Build a **discrete** time transfer function 1/z (again using syslin but not with the 'c' option) and now simulate using the **discrete** time step input. (For simulation, using 'dsimul'.) Comment in the report on how the response is different.

Q4c: Build just a ratio of two polynomials such as 1/z or 1/s and **try** giving them as input to csim. Compare the difference for Q4 a, b and c. Why are the conclusions different?

Q5: Compare three types of responses for the transfer function G(s)= (s+5)/[(s+4)(s+2)]. First: consider the output of G(s) for step input. Second: series (in this order): step input first through (s+5)/(s+4) and then 1/(s+2) and then vice-versa. Change the sampling period of the unit-step and check if some error creeps in the three plots superimposed. (In the command 'csim' (whose arguments are u, t, system), choose time series t (using syntax t=[0:tau:10]) as a vector of time-instants, in the time-step tau is each of 0.1 seconds, 0.5 seconds, and 2 seconds.)

EE324, Control Systems Lab, Problem sheet 3 (Report submission date: 16th August 2021)

Q1: On pole zero cancellation.
a) Consider the the transfer function (s+5+a)/(s^2+11*s+30), where ‘a’ is a real parameter to be varied from -1 to 1 in steps of 0.01. Plot the step response of the system for the various values of ‘a’. Check whether the response remains the same after pole-zero cancellation when a = 0. Hint: Use ‘simp’.

b) Plot the step response for the system 1/(s^2-s-6). State your observations on the system. Now generate a new transfer function by adding a zero to the system to cancel the pole on the right-half of the complex plane. Plot the step response of the new system. Comment on the stability (boundedness) of the response. Shift the zero slightly by adding a small variable parameter, ‘a’, and plot the response for different values of this parameter. Comment on the stability (boundedness) of these responses. From these observations try to justify the following statement: “an unstable plant cannot be rendered stable by canceling unstable poles by adding zeros attempting to cancel the unstable pole.

Q2: On second order approximation. a) Plot the step response of the system 85/(s^3+7*s^2+27*s+85). Now determine a second order approximation of the system and plot the step response of the approximated system. Hint: Find out from the textbook (or any other reliable sources) how a second order approximation is done. (Plot the two step responses in the same figure.)

b) Plot the step response of the system (s+0.01)/(s^3+(101/50)*s^2+(126/25)*s+0.1). Now determine a second order approximation of the system and plot the step response of the approximated system (in the same figure).

Q3: Effect of additional poles, zeros.
a) Consider a system 9/(s^2+2s+9). Use ‘trfmod’ to determine the poles of the system. Now add a zero to the system and determine the time domain parameters - rise time, percentage overshoot of the step responses of the original system and the one with the additional zero.

b) To the system 9/(s^2+2s+9) now add a new pole closer to the origin from the existing poles and determine the time domain parameters. Repeat the same by adding a pole away from the origin from the existing poles and determine the time domain parameters.

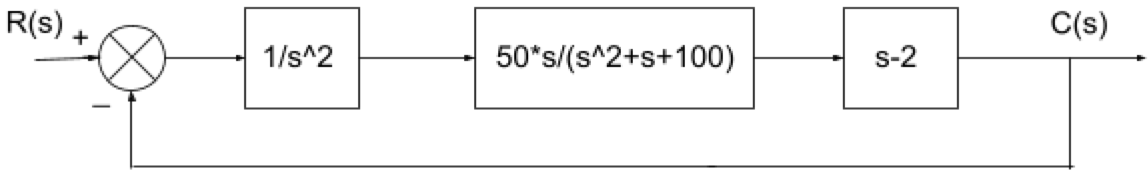
c) State your observations on the effect of additional poles and zeros on the system.

Q4: Plotting various time-domain parameters as functions of ζ and ω_n . a) Consider a standard closed loop second order transfer function with undamped natural frequency of 1 rad/sec. Plot the time-domain step response of the transfer function in three cases by choosing a damping ratio such that the system is undamped, underdamped, and overdamped. Print upto three decimal places the percentage peak overshoot, peak time, delay time, rise time, 2% settling time for the three cases. Observe how percentage-overshoot, rise-time, 2% settling time, and peak-time change with change in damping ratio.
Hint: Use ‘denom’ and ‘coeff’ to get the coefficients of the characteristic equation from the transfer function.

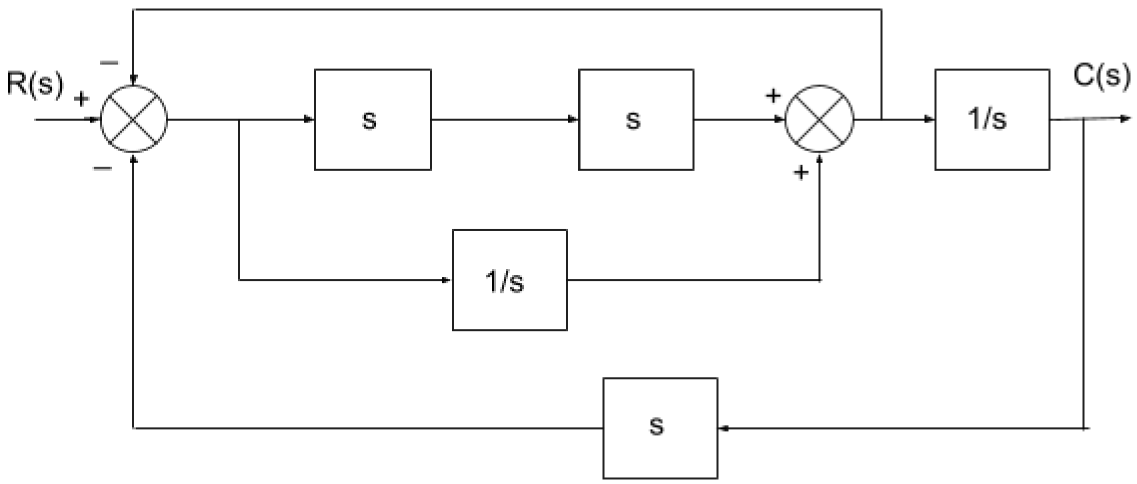
EE324, Control Systems Lab, Problem sheet 4
(Report submission date: 23rd August 2021)

Q1: Writing Scilab codes to obtain input-output transfer functions for complex interconnected Systems.

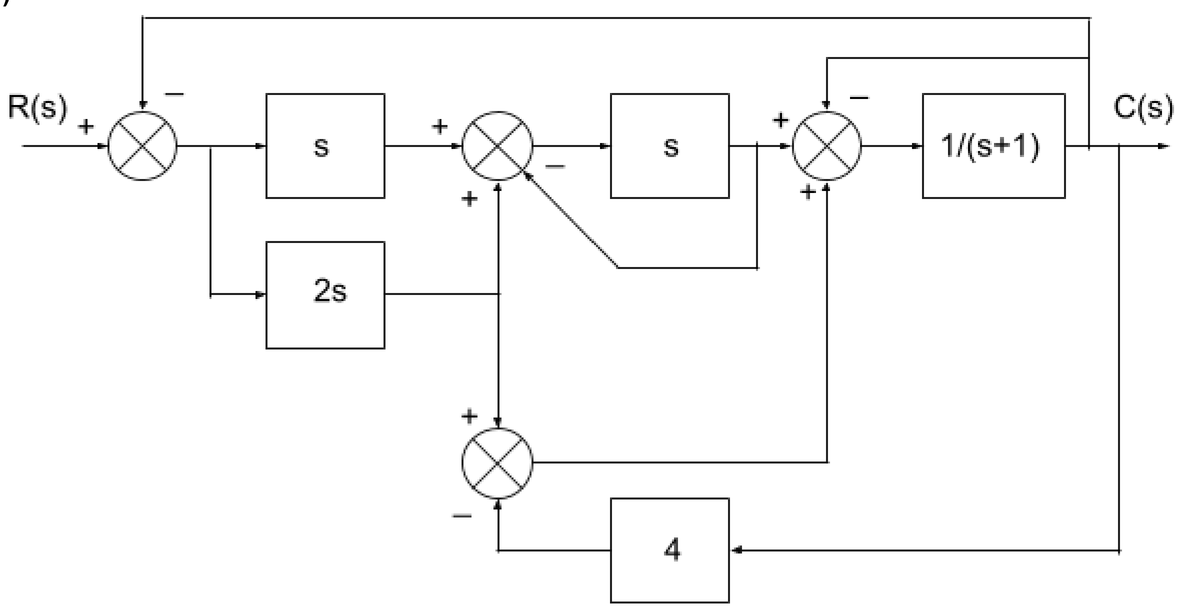
a)



b)



c)



Q2: Let G(s)=10/s(s+2)(s+4) be the transfer function of a plant. Suppose a proportionality gain K has been put in the forward path in series with the plant and then the feedback loop has been closed with unity negative feedback.

(a) Write a Scilab code that finds the closed-loop transfer function for a given value of K.
(b) Plot the loci of the closed-loop poles as K varies from 0 to 100 in steps of 0.1.
(c) From your plot, estimate the critical value of K that takes the closed-loop system to the verge of instability.
(d) Verify your estimation from Part (c) above with the R-H table.

Q3: Form the R-H table for the following polynomials. Use command routh_t

a) s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3
b) s^5 + 6*s^3 + 5*s^2 + 8*s + 20
c) s^5 - 2*s^4 + 3*s^3- 6*s^2 + 2*s - 4
d) s^6 + s^5 - 6*s^4 + s^2 + s - 6

Q4:

(a) Construct a degree 6 polynomial whose R-H table has its entire row corresponding to s^3 to be zero.
(b) Repeat Part (a) with a polynomial of degree 8 and having the entire row corresponding to s^3 to be zero.
(c) Construct a degree 6 polynomial whose R-H table has the first entry in its row corresponding to s^3 to be zero.

EE324, Control Systems Lab, Problem sheet 5
(Report submission date: 31st August 2021)

Q1) Plot the root locus of the following systems and observe the behavior of the closed-loop poles. **Hint**: use evans in Scilab.

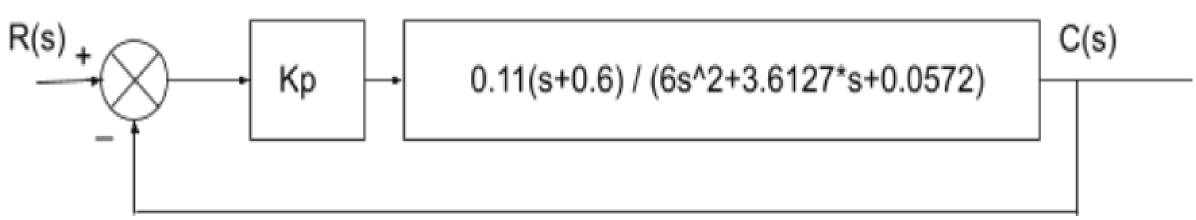
- **a)** 10 / (s^3 + 4s^2 + 5s + 10) is the **closed** loop transfer function of a system with unity negative feedback.
- **b)** (s + 1) / (s^2*(s + 3.6)) is the open-loop transfer function of a system. (In practical implementation, k has to be real, even though break-away/break-in points are allowed to be complex).
- **c)** (s + 0.4) / (s^2*(s + 3.6)) is the open loop transfer function of a system.
- **d)** (s+p) / (s*(s+1)(s+2)) vary the parameter p and comment on the stability of the system as p changes.

Q2) Design a transfer function (individual for each sub-part) such that:

- **a)** The breakaway and breakin points coincide. Hint: Symmetric poles about the origin.
- **b)** The number of branches at the breakaway or breakin point is more than 4.
- **c)** The branches of the root locus coincide with their asymptotes.
- **d)** The breakaway or break in points are complex numbers by following the steps given below.
  - **i)** Consider a transfer function with no zeros and with poles as real and symmetric about the jw axis.
  - **ii)** Now substitute s^2 with -s^2 (write the higher powers such as s^4, s^6 in terms of s^2) in the transfer function you have designed in part (i), and plot the root locus.
  - **iii)** Now substitute s with s-k where k being a positive integer of your choice, in the transfer function you have designed in part (ii) and plot the root locus.

Q3) Design a Proportional controller (with gain Kp) in Scilab to cascade the given third-order system 1 / (s*(s^2 + 3*s +5)) to attain the required closed-loop time domain specification of 1.5 seconds as the rise time, on giving the step input. Also, find the minimum possible rise time for the given system (maintaining stability).

Q4) Design a Proportional controller in Scilab for the given second-order system to attain a steady-state error of 1 percent for step response. Plot the step response and root locus of the system and identify the Proportional gain (Kp) on the root locus for which the closed-loop system is marginally stable.



Q5) Consider a 3rd order system, with no zeros and 2 dominant poles, and the 3rd pole very left on the complex plane. Plot the root locus of the system, and then compare it to the root locus of the system when the leftmost pole is ignored. Till what value of K is the step response of the closed loops of both the systems are similar.

EE324, Control Systems Lab, Problem sheet 6
(Report submission date: 28th Sept 2021)

Q1) Design a Proportional (P) controller with gain *K* using the root-locus method so that the closed-loop system with unity negative feedback attains the following specifications. The open loop system has the transfer function G(s)=1/((s+3)(s+4)(s+12)).

a) To obtain a steady state error of 0.489 on applying step input.
b) To attain a damping ratio of 0.35.
c) What is the gain value at the break away point?
d) Now for the given open loop system increase the controller gain *K* in a small range of 0 to 1 in appropriate steps and compare the step responses on a graph. What can you conclude about the locus of these closed loop poles on the root locus of the open loop system? And what can you conclude about the steady state errors?
e) Now do the same for a larger range of proportional gain such as 1 to 1000 in appropriate steps and compare the step responses on a graph. What can you conclude about the locus of these closed loop poles on the root locus of the open loop system? and What can you conclude about the settling times (5%) and steady state errors? What can conclude on the stability of the system?

Q2) For the same open loop system as above, design a Proportional-Integral (PI) controller with transfer function (K(s+z)/s) using the root-locus method to attain the following specifications for the closed-loop.

a) To reach a damping ratio of 0.2 for an initial value of z =0.01.
b) To obtain undamped natural frequencies of 8 and 9 rad/s.
c) Vary the value of ‘z’ and observe its effect on the root locus of the system.
d) Is it possible to alter the pole locations of a system using a PI controller without changing the damping ratio?

Q3) a) Plot input and output sinusoid of varying frequency (choose 5 different frequencies) and check for a stable transfer function G(s) = 1/(s^2 + 5s + 6) and check that the ratio of the amplitude of output to input is |G(jω)| and the phase difference is the angle of G(jω), with omega equal to these 5 frequencies.

b) The desired relation (between phase difference and angle of G(jω) is for frequency measured in Hz or in rad/s ?

c) Consider G(s) = 60/(s^3+6s^2+11s+6). Find answers to Q4a for this case. Find a frequency when the phase angle difference is 180 degrees. (Find the frequency by trial and error or by any other method.) Did numerator 60 play a role in this argument (of finding the frequency for which we have 180 degrees phase difference between input and output)?

EE324, Control Systems Lab, Problem sheet 7
(Report submission date: 5th October 2021)

Q1) For the open-loop transfer function $1/s(s^2+4s+8)$.
a) Find the value of K (gain) for which the closed-loop characteristic equation has gain margin and phase margin equal to zero.
b) Can you have a K for which the gain margin is non-zero but phase margin is 0? And vice versa?
c) Can you comment on the stability of the system for the K in part a?

Q2) Consider having a **lag-compensator** that has a ratio of zero-magnitude to pole-magnitude of say 20. This assignment aims to change the absolute pole-zero pair location of this lag-compensator (maintaining the ratio of 20) and see the effect on the transients.
a) Consider G(s)=1/(s^2+3s+2) and first choose a constant gain K to achieve 10% OS in the closed-loop.
b) Find the steady-state error and now add the above lag-compensator and (with the above ratio of 20), find the new steady-state error.
c) Change the location of the pole-zero pair (with say 5 different pole-zero locations) to see the degrading effect on the planned %OS and the trade-off with how late the lag-compensator effect comes into action.

Q3) Design the following:
a) Design a lead compensator for G(s) of Q2 to have 2% settling time made half of the case for Q2-a, and %OS still the same.
b) Design a PD controller to achieve the specification in Q3a.

Report Format:

Q1)
- **a)** Show the calculation of the K value and add a picture of the Bode Plot(Magnitude and Phase response).
- **b)** State your answer along with the reason/explanation.
- **c)** State your answer along with the reason/explanation.

Q2)
- **a)** Show the calculation of the K value.
- **b)** Show the calculation of steady-state error before and after adding the lag-compensator. Add the picture of step response before and after the addition of the lag-compensator.
- **c)** Show the plot of the step-response for different pole-zero locations.

Q3)
- **a)** Show the calculation of the lead compensator. Show the plot of root locus and step-response before and after the addition of the lead compensator.
- **b)** Show the calculation of the PD controller. Show the plot of root locus and step-response before and after the addition of the PD controller.

Note:
- **At least these are things to be added to the report and if anyone wants to add extra required plots/values can be also added.**
- **Add the Scilab code for all questions.**

EE324, Control Systems Lab, Problem sheet 8
(Report submission date: 12th October 2021)

Q1) For a lag compensator with transfer function $\frac{s+K_1}{s+K_2}$

a) Keeping the ratio of K1 and K2 constant (say = 5) move both the pole and zero away from the origin and towards the origin, and comment on the transient behavior of the system.
b) What can you comment on the impulse response of the system as you move the pole and zero in the same manner as in the above (1a)?

Q2) Find a transfer function that is open-loop stable and has two intersections of the root locus on the imaginary axis. (Hence, you will get two phase-crossover frequencies), using the following steps:
a) Consider 4 non-repeating poles on the imaginary axis and one real pole. (For example, -1, $\pm 2i$, $\pm i$ and plot the root locus.
b) Now shift the origin of the root locus such that all the poles lie in the left half-plane (refer to question 2d of problem sheet 5 for a hint). Plot the corresponding bode plot of this system
c) Now using the above bode plot, design the location and number of zeros to achieve two phase-crossover frequencies.
d) Plot the root locus of the new system and verify that it satisfies the problem statement.

Q3) For the magnitude plot shown (an actual magnitude plot and its asymptotic approximation is given) in the figure below, figure out the corresponding transfer function. Afterward, plot the phase plot for the same.

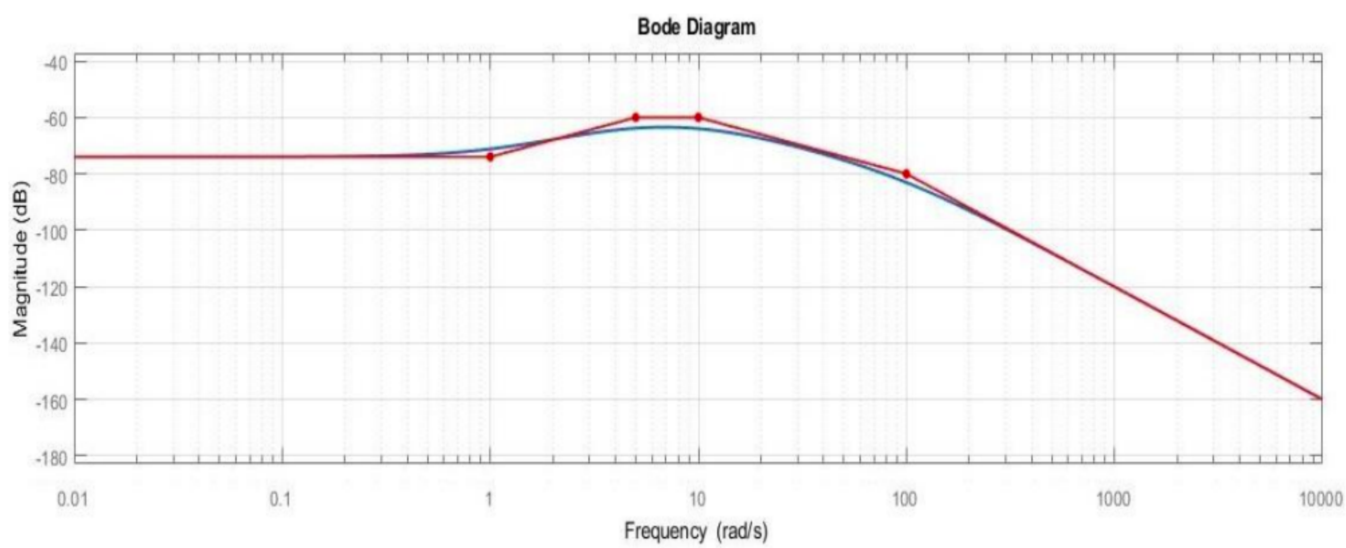


Fig: Magnitude plot of the transfer function

Report Format:

Q1)

a) Show the plot of step response for various values of pole-zero location and write your observations from the plot.

b) Show the plot of the impulse response for various values of pole-zero locations (maintaining K1/K2=Constant) and write your observations from the plot.

Q2)

a) Show the plot of the root locus.

b) Show the Bode plots.

c) Show the calculation of zeros location.

d) Show root locus for the new system.

Q3)

a) Show the calculation of the Transfer Function.

b) Show the phase plot.

Note:

- **At least these are things to be added to the report and if anyone wants to add extra required plots/values can be also added.**
- **Add the Scilab code for all questions.**

EE324, Control Systems Lab, Problem sheet 9
(Report submission date: 19th Oct 2021)

Q1) Compare Nyquist plots of a transfer function G(s) = $\dfrac{10}{s\,(\frac{s}{5}+1)\,(\frac{s}{20}+1)}$ and C(s)G(s), where C(s) is:

i) A lag compensator with transfer function $\dfrac{s+3}{s+1}$

ii) A lead compensator with transfer function $\dfrac{s+1}{s+3}$

Comment on the variation in the gain margin and phase margin for C(s)G(s) in comparison to the gain margin and phase margin of G(s) for the cases (a) and (b).

Q2) A "Notch filter" is a band stop filter or band-reject filter that has a very low gain at a particular frequency. Determine a transfer function of a notch filter that rejects (or attenuates) a 50 Hz signal. Comment on a method to modify (or adjust) the steepness of the magnitude plot for the notch filter and prove the same by comparing the corresponding bode plots.

Q3) For the transfer function C(s) = $\dfrac{100}{s+30}$ how much **minimum delay** (in seconds) would be needed to destabilize the **closed-loop system**. Compare the Bode plots (magnitude and phase) of C(s) and C(s)G(s) where G(s) is the delay calculated to achieve the destabilization. What can you comment on the gain and phase margins, with and without the delay?

Q4) For the given open-loop system G(s) = $\dfrac{1}{(s^3+3s^2+2s)}$, observe the difference in the gain margin magnitudes on applying the following four techniques:

i) Root-locus
ii) Nyquist plot
iii) Bode-plots: using the asymptotic plot for calculating the gain margin
iv) Bode-plots: using the actual plot for calculating the gain margin

Q5) For the given open-loop system G(s) = $\dfrac{10s+2000}{(s^3+202s^2+490s+18001)}$, perform the following operations:

i) Plot the bode plot (magnitude and phase) of the system and comment on the gain margin and phase margin of the system.
ii) Add a proportional gain 'K' to improve the steady-state error to 10% for the step response of the closed-loop system.
iii) Observe the new phase and gain crossover frequencies and the gain and phase margins for the system obtained in question 5 (ii).

iv) To improve the phase margin of the system obtained in question 5 (ii), cascade the open-loop system of question 5 (ii) with a zero such that the phase margin is greater than or equal to 90 degrees, but **without altering the dc gain** of the closed-loop system.
v) Comment on the closed-loop system stability for the transfer function obtained in question 5 (iv).

Report Format:
Q1) Shows the nyquist plots of all the three required systems and comments as per question.

Q2) Show the Bode-plot of the obtained transfer function and write your answer along with the reason/explanation.

Q3) Show the Bode-plot of the transfer function and delayed transfer function and write your answer along with the reason/explanation.

Q4) a) Show the plot of the root locus.
b) Show the Nyquist plot of the transfer function
c) Show the Asymptotic bode-plot of the transfer function
d) Show the Bode-plot of the transfer function.

Q5) a) Show the Bode-plot of the transfer function
b) Show the Bode-plot of the closed-loop transfer function
c) Write your observation.
d) Show the Bode-plot
e) State your answer along with the reason/explanation.

Note:

- **At least these are things to be added to the report and if anyone wants to add extra required plots/values can be also added.**
  Add the Scilab code for all questions.

EE324, Control Systems Lab, Problem sheet 10
(Report submission date: 26 Oct 2021, viva on 27th Oct)

All examples below have only real numbers within A, and preferably (i.e. as far as possible) only integer values for A, B, C and D.

Notation: Matrix P is n X m means n-rows and m-columns. (i,j)-th entry of A means entry in the i-th row and j-th column.

Q1) Consider state space system $\frac{dX}{dt} = AX + BU$, and $y = CX + DU$

- Take any nonsingular 3 X 3 matrix T, and any 3 X 3 matrix A, B of size 3 X 1 and C of size 1 X 3 (and D : scalar: 1 X 1). Check that $G(s) = D + C(sI - A)^{-1}B$ is the same even if A, B, C are changed using $T: A \rightarrow T^{-1}AT,\ B \rightarrow T^{-1}B,\ C \rightarrow CT$
- Check that eigenvalues of A are the poles of G(s) (for the above example).
- Take G(s) : two examples: denominator of degree two: one which is proper, and one which is strictly proper. Obtain state space realizations for both. Comment about value of D in each of the two cases.

Q2) Obtain state space realization (any of the canonical forms) for $G(s) = \frac{(s+3)}{(s^2+5s+4)}$

- Then get state space realization for G(s), but with zero at -1 instead of -3: Get state space realization of size 2 X 2.

Q3) Choose a 3 X 3 diagonal matrix A, and B and C of appropriate size (with one column and one row respectively). Check that if corresponding entry of B is zero, then that diagonal entry of A is no longer a pole of G(s) (due to pole/zero cancellation). Same happens with C also.

Q4) Choose A to be upper-triangular, 3 X 3 matrix, and entry in (1,3) is zero. Check that if entries along the diagonal are repeated, then pole/zero cancellation can happen even if entries in B or C are nonzero (in continuation of question Q3).

Note:

1) Show your calculation in the report either by hand-written equations(Images) or typed in.
2) If there are any plots in question inculde them in your report
3) Add the Scilab code for all questions.