Transmission Line Impedance Calculation Using Detailed Line Geometry and HEM Soil Resistivity Measurements

Hamed Ahmadi and Mažana Armstrong
BC Hydro Transmission & Generation Engineering
6911 Southpoint Dr, Burnaby, BC V3N 4X8, Canada
Email: hamed.ahmadi@bchydro.com

***Abstract*—This paper describes a method for calculating overhead transmission line parameters using the available technology. The line geometry, i.e. conductor heights above ground and phase spacing, is collected using the LiDAR technology. The soil resistivity measurements are extracted from an airborne electromagnetic survey (HEM). Accurate line parameters are essential for real-time operation of interconnected power systems.**

***Index Terms*—Transmission line parameters, soil resistivity, LiDAR data, HEM measurements.**

I. Introduction

Transmission line parameters are important factors in engineering analyses conducted by utility engineers. The flow of power is mainly governed by the transmission system configuration and line impedances. The performance of protective relays, specifically distance protection [1], highly relies on the accuracy of the line parameters used in their settings. Despite the importance of having accurate line parameters, as also noted in [2], most utilities are not using the actual line geometry as collected, for example, by LiDAR, to obtain this information. A recent report from the Line Protection Subcommittee of the Power System Relay Committee elaborates on the importance of line parameters in protection performance of relays [3]. The performance of fault locating algorithms rely on the accuracy of the line impedance calculations [3].

Direct measurement of the line parameters is possible, as reported in [1] and [2]. As the new measurement technologies emerge for transmission systems, new techniques are developed to take advantage of the available data and estimate line parameters. For example, by using data measured by the phasor measurement units (PMU), the authors of [4] were able to estimate the line parameters. In a more rigorous approach, line parameters as well as conductor sag were estimated using the PMU data in [5]. The estimation results are highly sensitive to the “bad data”. In case of parallel lines, the mutual effects and measuring equipment limitations make it difficult to estimate the parameters of each line accurately.

In order to calculate the transmission line parameters, conductor positions with respect to one another as well as their height above ground is required. Also, soil resistivity within the right-of-way of the transmission line is needed. Important line parameters are its series impedance (resistance (R) and reactance (X)) and shunt capacitance (C). The impedance values can be stated in either phase coordinates (A, B, and C) or sequence coordinates (zero, positive, and negative sequence). A common method for calculating the line parameters is the Carson equations, derived by John R. Carson [6]. Those equations take into account the effect of ground conductivity (soil resistivity (ρ)) as well as the effects of frequency of the AC current/voltage in the line. A compact form of the equations were reproduced in Chapter 2 of the EPRI Red Book [7].

Transmission lines run along different terrain and the height of the conductors above ground varies over the length of the line. Traditionally, an approximate average value for the conductor height and spacing is used in calculations. Soil resistivity is a variable parameter which depends on many factors such as soil structure, moisture content, temperature, frost penetration, presence of human-made conductive objects (e.g., pipelines), etc. Despite the great impact of soil resistivity on the zero sequence impedance of the line, a reasonable value typically could not be estimated due to lack of measurement data. It is a time-consuming and expensive procedure to measure soil resistivity across the transmission right-of-way (ROW). A technique was developed in 1978 by Douglas Fraser [8] to measure soil resistivity for different penetration depth values using an airborne electromagnetic system. This technique has been adopted by several companies, used mostly for the mining and pipeline industries. Helicopters fly the ROW and collect soil resistivity measurements within the time-frame of hours compared to several weeks which would be needed for ground based spot measurements along the ROW. The Transmission Line Electrical Design Team at BC Hydro has employed such measurements for several new transmission lines. The results obtained by the helicopter survey of the ROW were aligned well with the measurements obtained using the conventional method of ground based soil resistivity measurements (Wenner Method).

The remaining sections of the paper are organized as follows. In Section II, the formulations for calculating line parameters are discussed and a sensitivity analysis is performed. A brief overview of the proposed data-driven parameter calculation method is given in Section III with a real example. The main findings of this paper are highlighted in Section IV.

978-1-4673-8721-7/16/$31.00 ©2016 IEEE

II. Calculation of Transmission Line Parameters

Consider a transmission line with N infinitely long horizontal conductors above ground. When a voltage a applied to the set of conductors, a potential to ground is established. The equivalent charges on each conductor can be calculated using the "coefficients of potential" matrix (P), as follows:

$$V = P\,Q \tag{1}$$

where $V \in \mathbb{R}^N$ is the vector of voltages, $P \in \mathbb{R}^{N\times N}$ is the coefficients of potential, and $Q \in \mathbb{R}^N$ is the vector of charges [7]. When the conductors configuration is known, elements of P are calculated as:

$$p_{ij} = \begin{cases} \frac{1}{2\pi\epsilon}\ln\left(\frac{D_{ii'}}{r_i}\right) & i = j \\ \frac{1}{2\pi\epsilon}\ln\left(\frac{D_{ij'}}{D_{ij}}\right) & i \neq j \end{cases} \tag{2}$$

where $D_{ii'}$ is the distance between conductor i and its image; $D_{ij'}$ is the distance between conductor i and the image of conductor j; r_i is the radius of conductor i; and ϵ is the permittivity of the medium $\epsilon = \epsilon_0\,\epsilon_r$ ($\epsilon_0 = 8.8442e{-}12$ F/m). Once P is calculated, conductor charges Q can be retrieved from (1) by taking the inverse of P. Current is the derivative of line charges with respect to time. Assuming a sinusoidal current, the following relation between voltages and currents can be established:

$$V = \frac{1}{j\omega}P\,I \;\rightarrow\; I = j\omega C\,V \tag{3}$$

where $I \in \mathbb{R}^N$ is the current in each conductor; $\omega = 2\pi f$; and $C \in \mathbb{R}^{N\times N}$ is the shunt capacitance matrix.

Convention ally, the line impedance has been calculated in BC Hydro based on Carson's equations.

$$Z_{ij} = \begin{cases} R_i + \frac{\mu_0\omega}{2\pi}(\hat{i}\ln\left(\frac{D_{ii'}}{\mathrm{GMR}_i}\right) + 2(\alpha_{ij} + \hat{i}\beta_{ij})) & i = j \\ \frac{\mu_0\omega}{2\pi}(\hat{i}\ln\left(\frac{D_{ij'}}{D_{ij}}\right) + 2(\alpha_{ij} + \hat{i}\beta_{ij})) & i \neq j \end{cases} \tag{4}$$

where R_i is the AC resistance of conductor i; GMR is the geometric mean radius; $\mu_0 = 4\pi \times 10^{-7}$H/m is the permeability constant of the vacuum; and $\hat{i}$ is the imaginary unit number ($\hat{i}^2 = -1$). The terms α and β are expressed as:

$$\alpha_{ij} = \frac{\pi}{8} - \frac{\xi_{ij}}{3\sqrt{2}}\cos(\theta_{ij}) + \frac{\xi_{ij}^2}{16}\cos(2\theta_{ij})(0.6728 + \ln(\frac{2}{\xi_{ij}})) + \frac{\xi_{ij}^2}{16}\theta_{ij}\sin(2\theta_{ij}) + \frac{\xi_{ij}^3}{45\sqrt{2}}\cos(3\theta_{ij}) - \frac{\pi\xi_{ij}^4}{1536}\cos(4\theta_{ij}) \tag{5a}$$

$$\beta_{ij} = -0.0386 + \frac{1}{2}\ln(\frac{2}{\xi_{ij}}) + \frac{\xi_{ij}}{3\sqrt{2}}\cos(\theta_{ij}) - \frac{\pi\xi_{ij}^2}{64}\cos(2\theta_{ij}) + \frac{\xi_{ij}^3}{45\sqrt{2}}\cos(3\theta_{ij}) - \frac{\xi_{ij}^4}{384}\theta_{ij}\sin(4\theta_{ij}) - \frac{\xi_{ij}^4}{384}\cos(4\theta_{ij})(\ln(\frac{2}{\xi_{ij}}) + 1.0895) \tag{5b}$$

in which the terms ξ_{ij} and θ_{ij} are defined as follows:

$$\xi_{ij} = 2.81 \times 10^{-3} D_{ij'}\sqrt{\frac{f}{\rho}} \tag{6a}$$

$$\theta_{ij} = \sin^{-1}\left(\frac{H_{ij}}{D_{ij'}}\right),\ (\theta_{ii} = 0) \tag{6b}$$

where ρ is the soil resistivity in $\Omega.m$ and H_{ij} is the horizontal distance between conductors i and j.

It is important to understand the sensitivity of the line parameters to conductor configuration. A simple configuration, common in BC Hydro 500 kV transmission system, is considered here to perform the analysis. This flat configuration along with the notations is shown in Fig. 1, with the initial values of $D_{12} = D_{23} = 12$ m, $h = 20$ m, $R_{AC} = 0.10396\ \Omega$/km, $r = 14.06$ mm, $f = 60$ Hz, and $\rho = 100\ \Omega.m$. In the first scenario, the conductor height (h) is increased from 5 m to 50 m and the line parameters are calculated, as shown in Fig. 2. A large variation has been observed in C^0, a 45% reduction in its magnitude when h is increased from 5 m to 50 m. This parameter is not shown in Fig. 2 due to large variations. Other parameters not shown in Fig. 2 had negligible variations with height.

The spacing between phases, denoted by $D_{12} = D_{23}$ in Fig. 1, affects the line parameters. The line parameters versus D_{12} are shown in Fig. 3. As can be seen, both the positive and zero sequence reactances are somewhat sensitive to the change in phase spacing. The line capacitance, on the other hand, is more sensitive to the phase spacing. Despite the variations shown in Fig. 3, in reality, the variance of phase spacing is not significant. For instance, in a 500 kV line, the phase spacing is usually around 12 m ±20% in BC Hydro system.

Soil resistivity, denoted by ρ, has significant impact on the zero sequence reactance of the line, as shown in Fig. 4. The zero sequence resistance is also influenced by ρ, although to a less degree. Other line parameters, i.e. positive sequence series impedance and shunt capacitance are not significantly sensitive to ρ.

The short background on sensitivity analysis of line parameters with respect to line configuration and soil resistivity motivated the authors to use the available more advanced techniques for accurately determining the conductor configuration and soil resistivity. This data is used to calculate more reliable values for transmission line impedance and admittance matrices.

III. Detailed Line Geometry and Soil Resistivity Data

A remote sensing technique is used in BC Hydro to measure distance by targeting an object with a laser and analyzing the reflected light. This technology, known as LiDAR, has been adopted to create a high-resolution 3D model of overhead conductors and the ground underneath within the right-of-way (ROW). For example, Fig. 5 shows three spans of a 500 kV transmission line. The elevation data for the ground level is used to calculate the height of the conductors above ground.

An airborne electromagnetic system was used to measure the soil resistivity. This technique provides information about

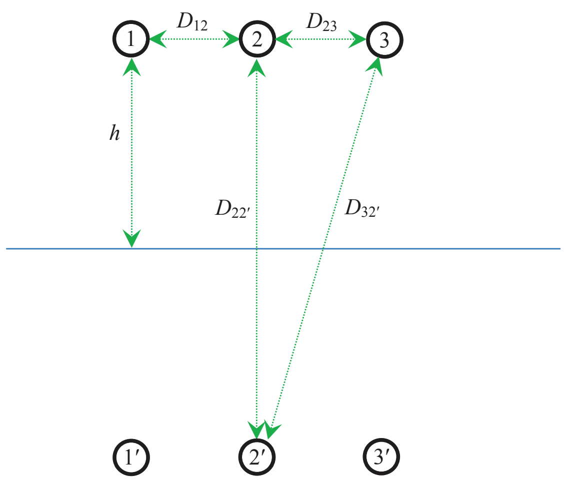

Fig. 1. Conductor configuration for a common 500 kV transmission line.

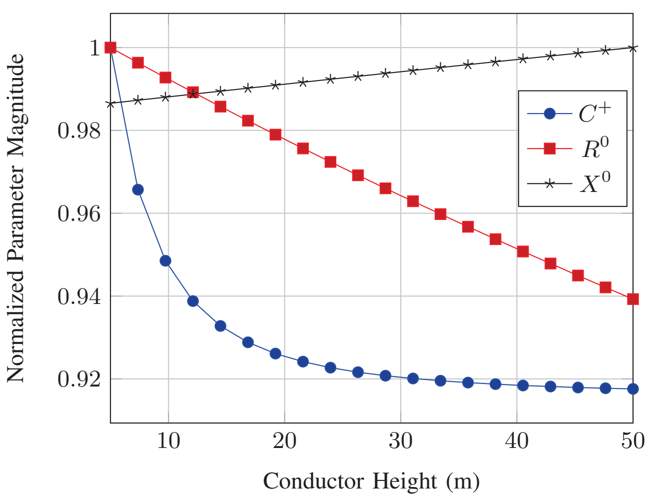

Fig. 2. Impact of conductors' height (h) on line parameters.

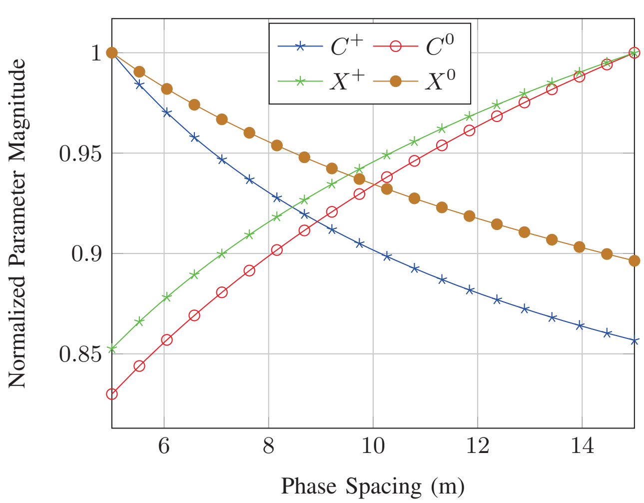

Fig. 3. Impact of conductor spacing ($D_{12} = D_{23}$) on line parameters.

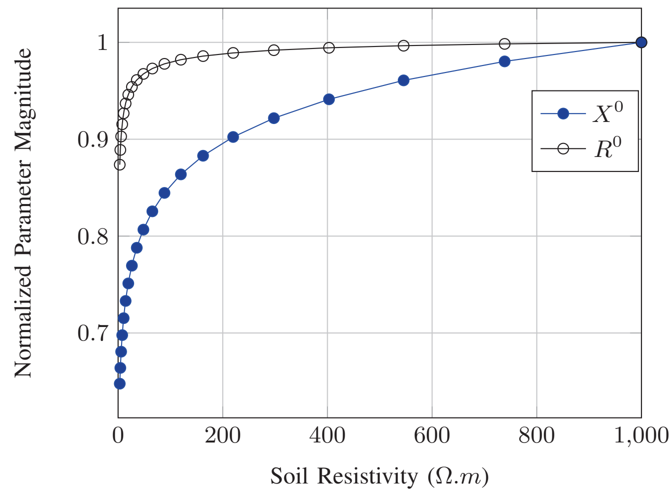

Fig. 4. Impact of soil resistivity (ρ) on line parameters.

soil resistivity at different depth levels in soil. For example, the soil resistivity of a small section of the ROW for a transmission line is shown in Fig. 6. As can be seen, this data is tagged with GPS coordinates and can be readily used for any parts of the transmission line. These measurements are usually conducted at a particular time of the year, with known weather conditions. The results should be carefully adjusted for different conditions. For example, moisture content and temperature largely affect the soil resistivity [9] and [10].

It is important to understand the use of zero-sequence impedance for line protection purposes. When a single line-to-ground fault occurs, assuming that there is a transformer with a neutral connection to ground at the source side, a fault current flows through the loop created by the earth return path from the source to the faulted location. If the fault happens at the tower, then tower footing resistance is the main factor in determining the zero-sequence impedance. In such cases, with a known structure of the tower foundation and grounding (rods, counterpoises, etc.), the ground resistivity is crucial for calculating the tower grounding resistance at that particular location. This case is different from a study performed during normal operation to, for instance, calculate the unbalance in the system. In such cases, the zero-sequence impedance is calculated only based on the soil resistivity in the surroundings of the overhead line and the tower footing resistances do not play a role.

Based on the available data, accurate values for conductor height, phase spacing, and soil resistivity is calculated for each span of the transmission line. The results are added up to find the total impedance and shunt admittance of the line. An example is provided here to illustrate the application of the developed method. Figure 7 shows a 277 km-long 500 kV line in BC Hydro system. The coordinates are shifted for data privacy reasons. A comparison is made for the parameters obtained using the proposed method and those obtained using the conventional method adopted in BC Hydro in Table I. The conventional method is based on using the average values of conductor height and phase spacing for long subsections of the line (three subsections in this case: 81 km, 56 km, and 139 km)

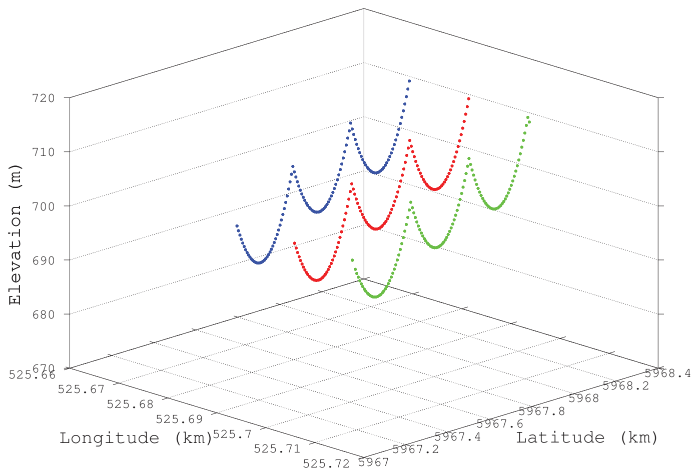

Fig. 5. Conductor positions obtained using LiDAR technology for three spans of a 500 kV transmission line.

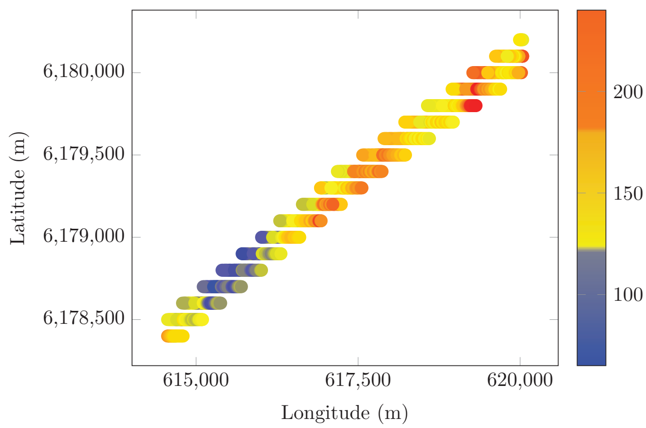

Fig. 6. Soil resistivity measurements data ($\Omega.m$) at 5m below ground for a section of a transmission line.

and considering a constant soil resistivity of 150 $\Omega.m$. As can be seen, the differences are considerable for the zero-sequence resistance and reactance values.

IV. Conclusion

Based on the available technology, an accurate method for calculating the overhead transmission line parameters was discussed. It was shown that the line shunt capacitance is sensitive to the height of the conductors above ground, while line impedance is relatively less sensitive. Soil resistivity has been shown to have a significant impact on the zero sequence components of the line impedance. For areas with variable soil resistivity, it is important to conduct extensive measurements to determine more accurate values for soil resistivity. Whether-related factors such as moisture content, temperature, frost, etc., need to be considered when applying soil resistivity measurements for a different time of the year.

TABLE I

Comparison between line parameters obtained using the proposed method and the conventional method used in BC Hydro. (R and X in Ω and C in μSi).

Method	R^+	R^0	X^+	X^0	C^+	C^0
Conventional	7.26	56.98	92.47	342.26	1357.24	780.38
Proposed	7.09	50.66	91.59	301.30	1377.40	758.00
Difference (%)	2.3	11.1	0.9	11.9	1.5	2.9

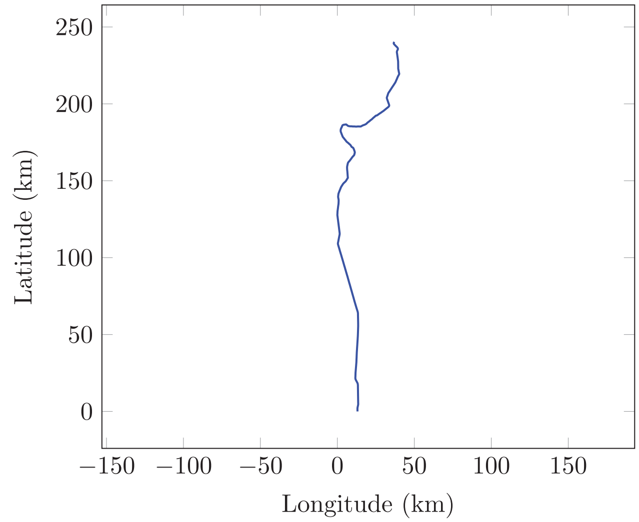

Fig. 7. Overview of the corridor of a 500 kV line in BC Hydro system. Coordinates are shifted for privacy protection.

References

[1] V. Sellwood, U. Klapper, M. Kruger, and S. Kaiser, “A new technique for setting distance protection and fault location by measurement of transmission line system impedance characteristics,” in *8th IEE Inter. Conf. AC and DC Power Transmission*, Mar. 2006, pp. 197–199.

[2] A. Dierks, H. Troskie, and M. Kruger, “Accurate calculation and physical measurement of transmission line parameters to improve impedance relay performance,” in *IEEE PES Inaugural Conference and Exposition in Africa*, Jul. 2005, pp. 455–461.

[3] Working Group D6, “AC transmission line model parameter validation,” Power System Relay Committee of the IEEE Power & Energy Society, Tech. Rep., Sep. 2014.

[4] C. S. Indulkar and K. Ramalingam, “Estimation of transmission line parameters from measurements,” *Int. J. Elec. Power & Energy Syst.*, vol. 30, no. 5, pp. 337–342, 2008.

[5] Y. Du and Y. Liao, “On-line estimation of transmission line parameters, temperature and sag using PMU measurements,” *Electric Power Syst. Res.*, vol. 93, pp. 39–45, 2012.

[6] J. R. Carson, “Wave propagation in overhead wires with ground return,” *Bell system technical journal*, vol. 5, no. 4, pp. 539–554, 1926.

[7] EPRI, *EPRI AC Transmission line reference book-200 kV and above*, 3rd ed. Palo Alto, CA: Electric Power Research Institute, 2005.

[8] D. C. Fraser, “Resistivity mapping with an airborne multicoil electromagnetic system,” *Geophysics*, vol. 43, no. 1, pp. 144–172, 1978.

[9] G. F. Tagg, *Earth Resistances*. London, England: George Newnes, 1964.

[10] G. V. Keller and F. C. Frischknecht, *Electrical methods in geophysical prospecting*, ser. International series of monographs on electromagnetic waves. New York: Pergamon Press, 1966.