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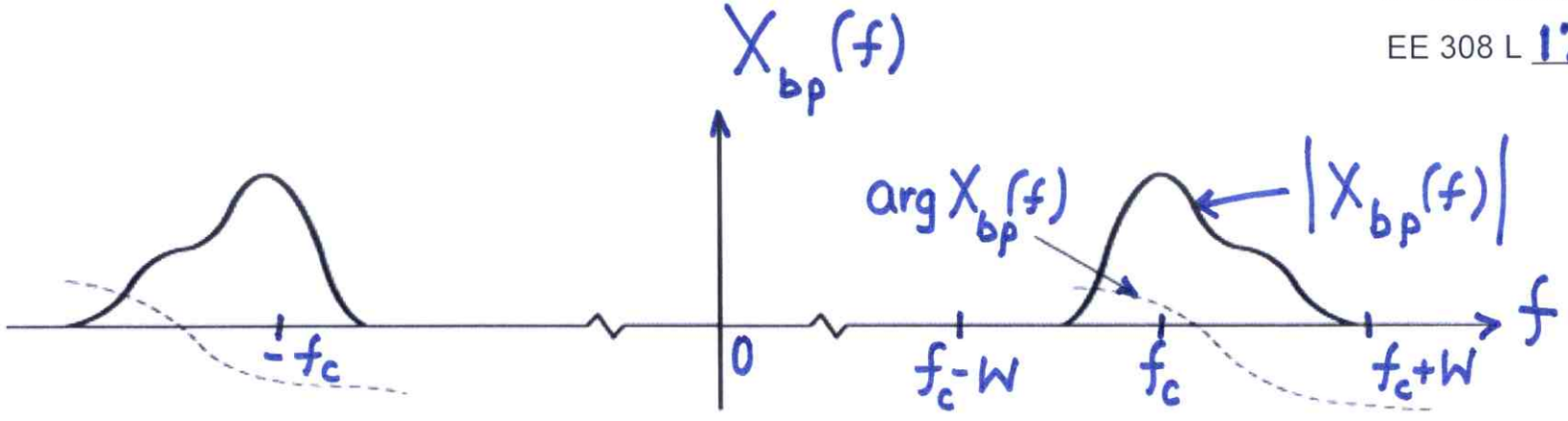
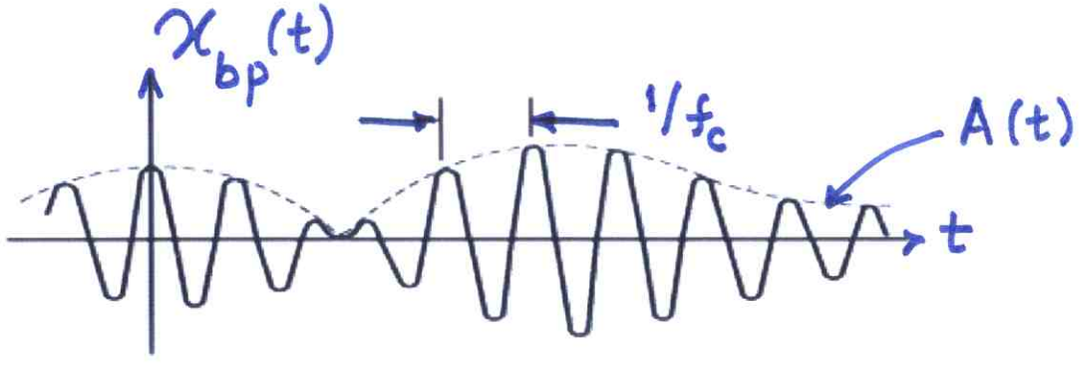
Analytic Signal or Pre-Envelope

Complex-Envelope Representation of Bandpass Signals


$X_{bp}(f)$
$\arg X_{bp}(f)$
$|X_{bp}(f)|$
$-f_c$
0
$f_c - W$
f_c
$f_c + W$
f


$x_{bp}(t)$
$1/f_c$
$A(t)$
t

Analytic Signal or Pre-Envelope

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$x(t)$ is a real-valued signal

Its analytic signal or pre-envelope is defined as

$$x_+(t) \triangleq x(t) + j x_h(t)$$

$x_h(t)$ is the Hilbert transform of $x(t)$

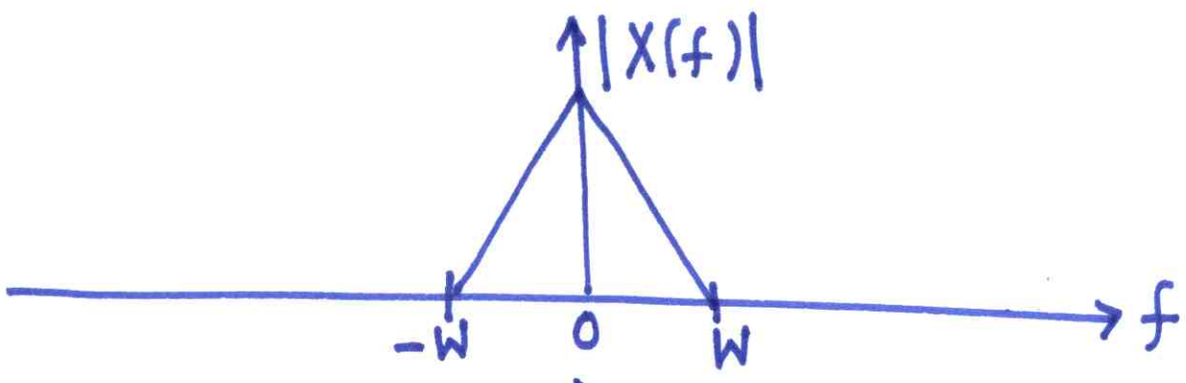
$$x_+(t) \longleftrightarrow X_+(f)$$

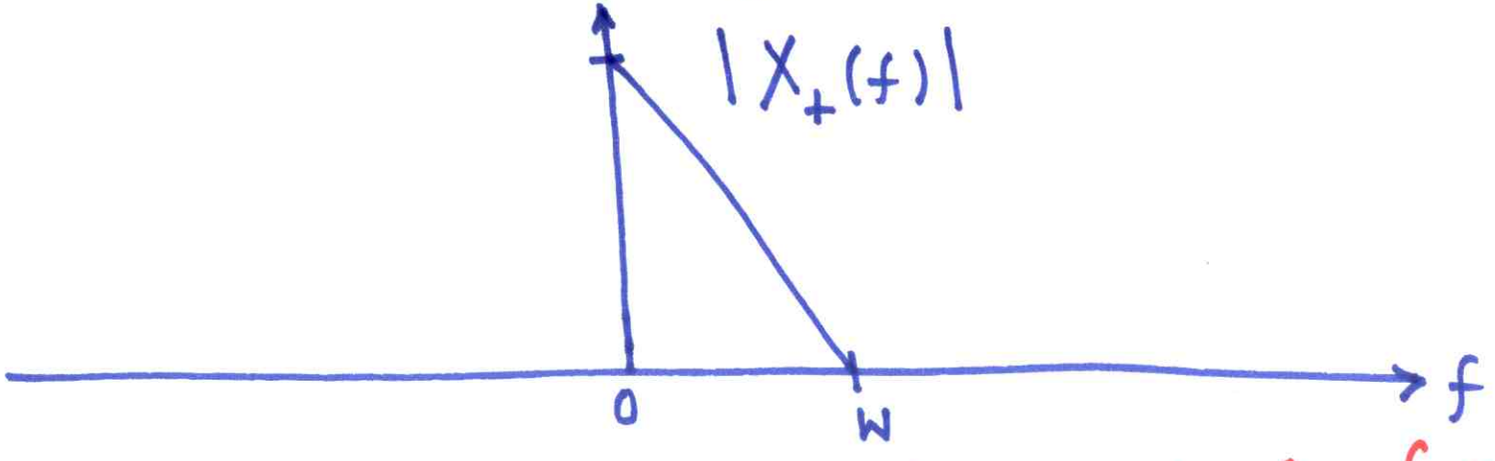
$$X_+(f) = X(f) + j\left[-j\,\mathrm{sgn}(f)\,X(f)\right]$$

$$= X(f) + \mathrm{sgn}(f)\,X(f)$$

$$= \begin{cases} X(f) + X(f) & \text{for } f > 0 \\ X(f) - X(f) & \text{for } f < 0 \end{cases}$$

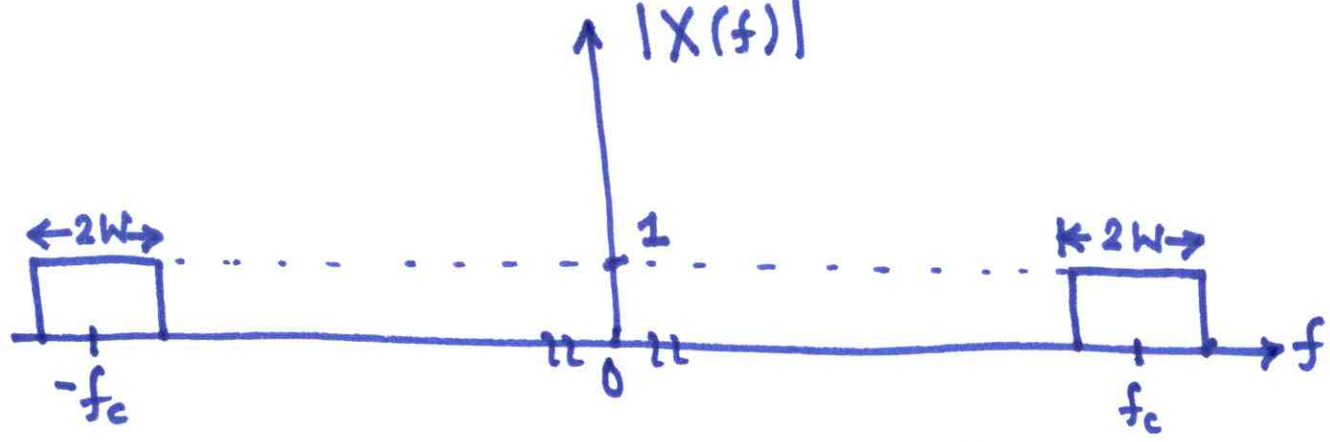
$$X_+(f) = \begin{cases} 2X(f) & f > 0 \\ 0 & f < 0 \end{cases}$$



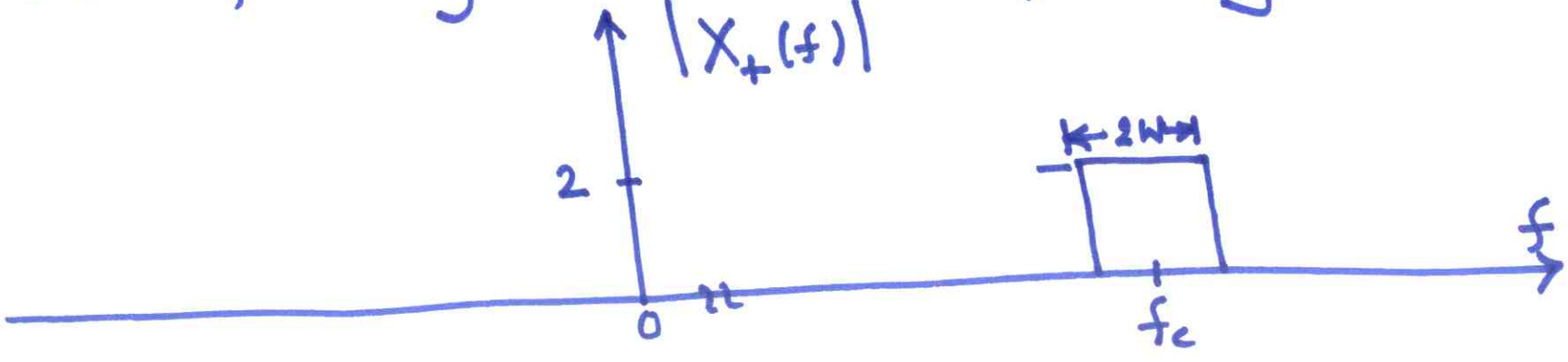


i.e., pre-envelope has spectrum only for $f \geq 0$

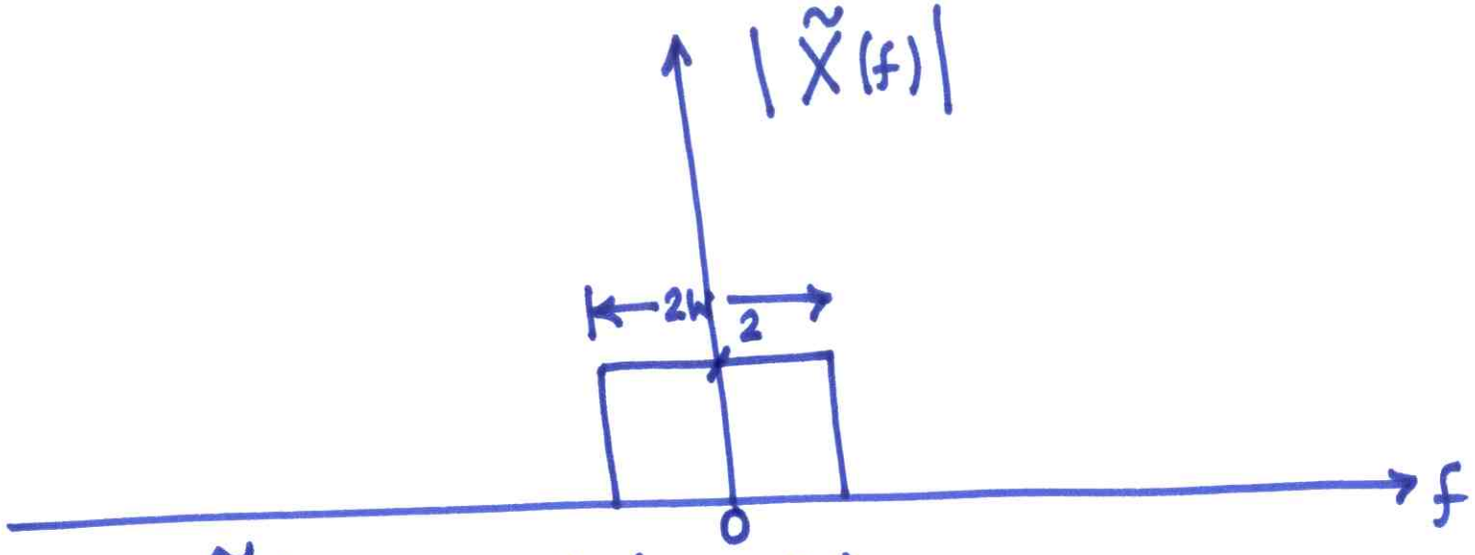
Complex-Envelope Representation of Bandpass Signals



The amplitude spectrum of the analytic signal corresponding to the bandpass signal $x(t)$:



$$x_+(t) \times e^{-j2\pi f_c t} \longleftrightarrow X_+(f+f_c) \equiv \tilde{X}(f)$$



$$\tilde{X}(f) = X_+(f+f_c)$$

$$\tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t}$$

$$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t} \quad \leftarrow \text{COMPLEX}$$

$$\rightarrow \quad x_+(t) = x(t) + j\, x_h(t)$$

$$x(t) = \mathrm{Re}\{x_+(t)\} = \mathrm{Re}\left\{\tilde{x}(t)\, e^{j2\pi f_c t}\right\}$$

$\tilde{x}(t) \rightarrow$ COMPLEX-ENVELOPE OF $x(t)$

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$x(t)$ is Bandpass Signal

$\tilde{x}(t)$ is a COMPLEX-valued LP signal

$\tilde{x}(t) \rightarrow$ COMPLEX ENVELOPE

??

Suppose:

$$x(t) = a(t)\cos(2\pi f_c t + \theta(t))$$

where $a(t)$ and $\theta(t)$ are lowpass signals (real-valued)

then $\rightarrow$

$$x(t) = \mathrm{Re}\left[\left\{a(t)e^{j\theta(t)}\right\}e^{j2\pi f_c t}\right]$$

$\left\{a(t)e^{j\theta(t)}\right\}$: COMPLEX envelope

$e^{j2\pi f_c t}$: complex CARRIER

$$\Rightarrow \tilde{x}(t) = a(t)e^{j\theta(t)}$$

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MODULE END