Capacitance Calculation of Overhead Transmission Line Based on Moment Method

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***Abstract*—In this paper, the capacitance parameter of the overhead transmission lines is calculated by the moment method (MoM), which is based on the image method of electromagnetic theory. The effectiveness of the proposed method is verified by an overhead transmission line model, which has analytical solution. The comparison results show that the relative error between the MoM and analytical solution is small, which shows the accuracy of the program based on the MoM. Then the proposed MoM is applied to calculate the distributed capacitance parameters of three typical overhead transmission lines.**

Keywords—Transmission line; Image method; Moment Method; Capacitance;

I. Introduction

With the rapid development of UHV transmission technology, more and more attention has been paid to the electromagnetic transient phenomena in power systems. However, the parameters' accuracy of the overhead transmission line model directly affects the electromagnetic transient model and its analysis results. Therefore, the accurate calculation of the overhead transmission lines' distributed parameters is the basis for the study of the electromagnetic transient problems [1,2,3,4].

At present, research on the calculation method of the transmission line distributed parameters is constantly developing and updating. They usually used method to calculate the transmission line distribution parameters include the charge simulation method, finite difference method, finite element method and boundary element method [5]. The charge simulation method is based on the uniqueness theorem of the electrostatic field, which sets a set of discrete charges inside the conductor to simulate charge and replaces the free charge continuously distributed on the surface of the conductor electrode. Then, the superposition theorem is used to calculate the potential and electric field strength. Therefore, it is a simple method and has the advantages of being easy to deal with open-domain problems, however the calculation result is less accurate when the discrete charges are placed improperly [6,7,8,9]. The finite difference method divides the enclosed field into regular meshes and calculates the electromagnetic field. However, when the meshes are rough, the discretization accuracy is not enough and the error is large [10]. Based on the variational principle and interpolation theory, the finite element method can deal with the problems with complex boundary conditions, but the workload is large and the calculation process is complex [5,11,12,13]. The boundary element method is based on the finite element discretization technique, which converts the equivalent value of the boundary value problem into the boundary integral equation problem. It is only suitable for solving the linear and uniform problems of the regular region [14,15,16,17].

Based on the [18] and [19], this paper proposes the moment method to calculate the distributed capacitance parameters of the overhead transmission lines. The method is based on the principle of the image method, which convert the differential equation that is difficult to directly solve into a simple algebraic equation, and then use computer for calculation. Therefore, it has the advantages of high precision, reliability and simplicity [20,21,22]. In order to verify the effectiveness of the proposed method, the program is applied to analyze the distributed capacitance of two overhead transmission line models. The distribution parameters were calculated and compared with the analytical solutions, and the results were basically the same. Finally, the method is applied to calculate the distributed capacitance parameters of three typical overhead transmission lines.

II. Basic Principles

A. Principle of Moment Method

- Assume that there are N conductors (arbitrary shapes) above the earth, and the earth is at zero potential, as shown in Fig.1.

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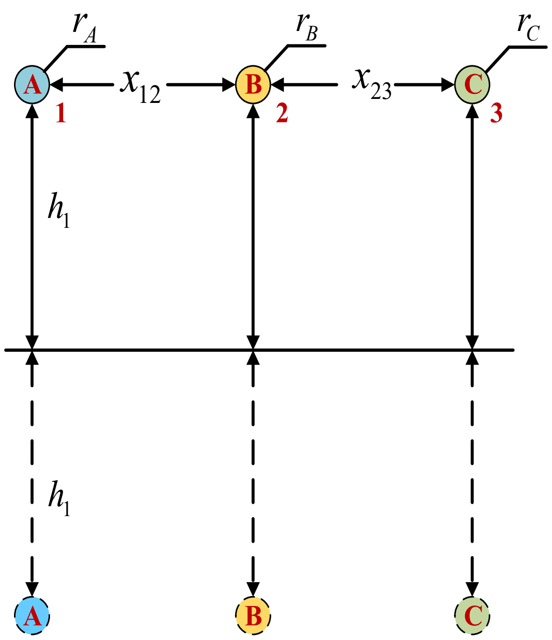

Fig. 1. Schematic of Image Method

- Based on the theory of moment method, the cross-sectional contours of the N conductor are evenly divided into discrete segments $\Delta l_{i1}, \Delta l_{i2}, \ldots, \Delta l_{iM}$ (i $=1,2,\cdots,N$; M is the number of discrete segments divided by each conductor).The central point of each discrete segment is taken as the matching point. Based on the image method, each conductor's voltage induced by the charge density of each discrete line segment can be obtained by the following equation.

$$V_i = \frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{N}\oint_{\Gamma_j}\sigma(x',y')\ln\frac{(x+x')^2+(y+y')^2}{(x-x')^2+(y-y')^2}dl' \quad (1)$$

Where V_i *is* the *ith* conductor's voltage, $\sigma(x',y')$ is the line charge density.

- The line charge density $\sigma(x',y')$ on each conductor can be solved by (1), then the charge (Q_1, Q_2, ..., Q_N) of each conductor per unit length can be calculated by:

$$Q_i = \int_{\Gamma_i}\sigma(x,y)dl, i = 1,2,\ldots,N \quad (2)$$

- The unit length coefficient of inductance matrix $\boldsymbol{K}$ can be obtained by:

$$\boldsymbol{K} = \boldsymbol{Q}\boldsymbol{U}^{-1} \quad (3)$$

- According to the obtained matrix of inductance coefficients $\boldsymbol{K}$, the capacitance matrix $\boldsymbol{C}$ can be obtained by the following formula:

$$C_{ij} = \begin{cases} -K_{ij}, i \neq j \\ \sum_{j=1}^{N} K_{ij}, i = j \end{cases} \quad (4)$$

B. Calculation steps for capacitors (and inductors)

For a multi-conductor system, if there are n wires, the surface charges of each wire are $Q_1, Q_2, Q_3, \cdots, Q_n$ respectively; The potential of each wire is $U_1, U_2, U_3, \cdots, U_n$, then we can get:

$$\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix} \quad (5)$$

Where Q_i is the surface charge per unit length on the wire i, $\boldsymbol{P}$ is called potential coefficient. The (5) can also be written as

$$\boldsymbol{U} = \boldsymbol{P}\boldsymbol{Q} \quad (6)$$

From (5) and (6) we can get:

$$\begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn} \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{pmatrix} \quad (7)$$

or

$$\boldsymbol{Q} = \boldsymbol{K}\boldsymbol{U} \quad (8)$$

Where, the $\boldsymbol{K}$ matrix is the inductance coefficient matrix, and obviously we can get the following equation,

$$\boldsymbol{K} = \boldsymbol{P}^{-1} \quad (9)$$

Where, K_{ii} represents the inductance of wire i to ground and the other wires; K_{ij} represents the inductance between the wires i and j.

According to the computed matrix of the inductance coefficients $\boldsymbol{K}$, the capacitance matrix $\boldsymbol{C}$ can be obtained by the following formula:

$$C_{ij} = \begin{cases} -K_{ij}, i \neq j \\ \sum_{j=1}^{N} K_{ij}, i = j \end{cases} \quad (10)$$

III. VALIDATION

In order to verify the moment method, taking a single wire system (Conductor Flat Tower) and a two-wire system (2 Conductor Flat Tower) above the earth as example, the capacitance parameters of the corresponding transmission lines are calculated separately.

A. Example 1-1 Conductor Flat Tower

The layout of 1 Conductor Flat Tower is shown in Figure 2, and the size parameters are given in Table I:

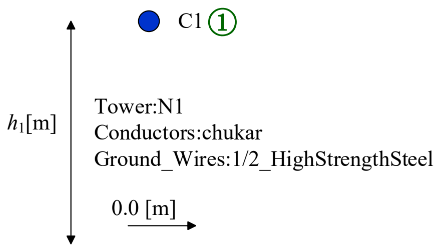

Fig. 2 Line parameter setting for 1 Conductor Flat Tower

TABLE I. SIZE PARAMETERS OF 1 CONDUCTOR FLAT TOWER

Parameter	Size (m)
Height of all conductors (h_1)	15
Conductor radius (r_{c1})	0.05
Total bundled sub－conductors (*N_FL*)	1

According to the size parameters of Table I, the calculated capacitance is:

$$C_{mom} = 7.322 \times 10^{-12} F$$

Where the calculated by analytical method:

$$C_{an} = 7.319 \times 10^{-12} F$$

The relative error between the numerical and analytical result is:

$$r_C = abs\left(\frac{C_{mom} - C_{an}}{C_{an}}\right) \times 100\% \approx 0.038\% \quad (11)$$

B. *Example 2-2 Conductor Flat Tower*

The layout of 2 Conductor Flat Tower is shown in Figure 3, and the size parameters are given in Table II:

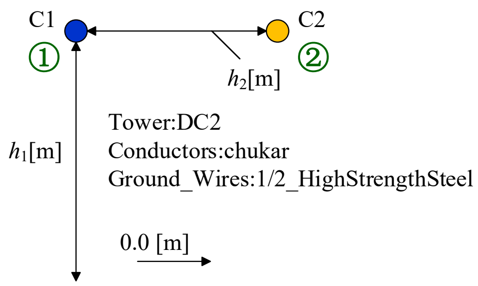

Fig.3. Parameter setting of 2 Conductor Flat Tower

TABLE II. SIZE PARAMETERS OF 2 CONDUCTOR FLAT TOWER

Parameter	Size (m)
Height of all conductors (h_1)	30
Horizontal spacing between conductors (h_2)	10
Conductor radius (r_{c1})	0.05
Total bundled sub-conductors (*N_FL*)	1

According to the size parameters of Table II, the results based on the moment method are obtained and given in Table III, the analytical solutions and the relative errors of the numerical results are also listed in Table III:

TABLE III. COMPARISON OF RESULTS BETWEEN THE MOM AND THE ANALYTICAL SOLUTION

Capacitance (pF)	MoM	Analytical method	Error
C_{10}	5.692	5.670	0.39%
C_{12}	1.651	1.651	0.02%
C_{20}	5.643	5.670	0.47%

According to (11) and Table III, we can see that the relative errors are small than 0.5%. The error of numerical results is within the allowable range of engineering, which proves the validity of the method of moments.

IV. APPLICATION

After the effectiveness of the MoM for calculating the capacitance parameters of the overhead transmission lines, the MoM is applied to analyze the distributed capacitance parameters of overhead transmission lines with more complex structures.

A. *Line Constants 3 Conductor Flat Tower*

The layout of the 3 Conductor Flat Tower is shown in Figure 4 and the size parameters are shown in Table IV.

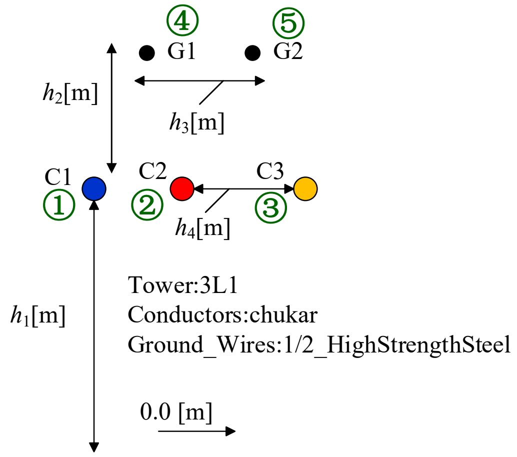

Fig. 4 Parameter setting of 3 Conductor Flat Tower

TABLE IV. SIZE PARAMETERS OF 3 CONDUCTOR FLAT TOWER

Parameter	Size (m)
Height of all conductors (h_1)	30
Height of ground wires over lowest conductors (h_2)	10
Spacing between ground wires (h_3)	10
Horizontal offset of center conductors (h_4)	10
Conductor radius (r_{c1})	0.0203454
Total bundled sub－conductors (*N_FL*)	1
Ground Wire radius (r_{c2})	0.0055245

According to the size parameters of Table IV, the calculated capacitance matrix $\boldsymbol{C}_{FLAT}$ is:

$$\boldsymbol{C}_{FLAT} = \begin{bmatrix} 4.284 & 1.251 & 0.518 & 1.036 & 0.537 \\ & 3.635 & 1.251 & 0.906 & 0.906 \\ & & 4.332 & 0.537 & 1.036 \\ & & & 3.004 & 0.984 \\ & & & & 3.007 \end{bmatrix} \times 10^{-12} F$$

B. *Line Constants 6 Conductor Concentric Tower*

The layout of the 6 conductors concentric tower is shown in Figure 5 and the size parameters are shown in Table V.

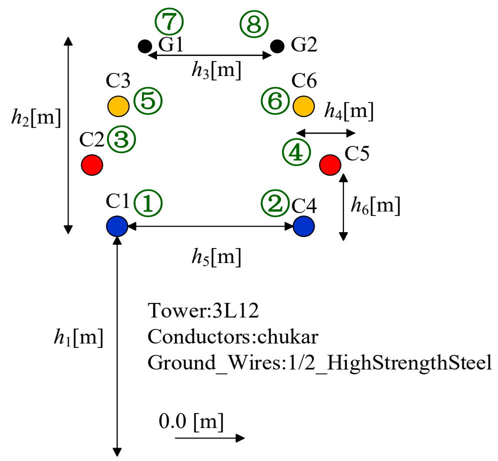

Fig. 5. Parameter setting of 6 Conductor Concentric Tower

TABLE V. SIZE PARAMETERS OF 6 CONDUCTOR CONCENTRIC TOWER

Parameter	Size (m)
Height of all conductors (h_1)	30
Height of ground wires over lowest conductors (h_2)	15
Spacing between ground wires (h_3)	10
Horizontal offset of center conductors (h_4)	2
Horizontal spacing between lowest conductors (h_5)	10
Vertical spacing between phase conductors (h_6)	5
Conductor radius (r_{c1})	0.0203454
Total bundled sub－conductors (N_FL)	1
Ground Wire radius (r_{c2})	0.0055245

According to the size parameters of Table V, the capacitance matrix $\boldsymbol{C}_{CON}$ based on the Moment Method is obtained:

$$\boldsymbol{C}_{CON}=\begin{bmatrix} 3.131 & 0.949 & 1.718 & 0.549 & 0.774 & 0.454 & 0.385 & 0.269 \\ & 3.127 & 0.549 & 1.718 & 0.454 & 0.774 & 0.269 & 0.385 \\ & & 2.564 & 0.434 & 1.634 & 0.497 & 0.691 & 0.348 \\ & & & 2.563 & 0.497 & 1.634 & 0.348 & 0.691 \\ & & & & 2.178 & 0.783 & 1.549 & 0.631 \\ & & & & & 2.182 & 0.631 & 1.549 \\ & & & & & & 2.186 & 0.743 \\ & & & & & & & 2.186 \end{bmatrix}\times 10^{-12}F$$

C. *Line Constants 6 Conductor Offset Tower*

The layout of the 6 Conductor Offset Tower is shown in Figure 6, and the size parameters are shown in Table Ⅵ.

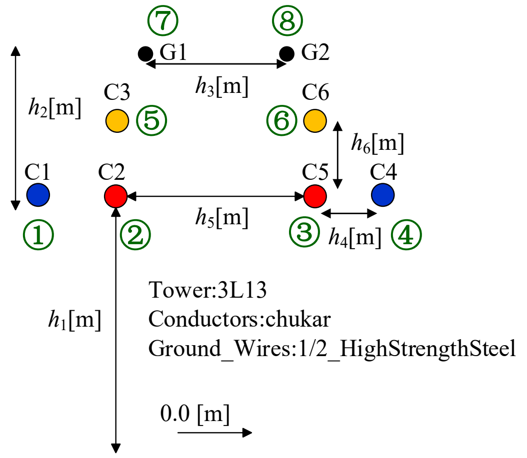

Fig. 6 Parameter setting of 6 Conductor Offset Tower

TABLE VI. SIZE PARAMETERS OF 6 CONDUCTOR OFFSET TOWER

Parameter	Size (m)
Height of all conductors (h_1)	30
Height of ground wires over lowest conductors (h_2)	10
Spacing between ground wires (h_3)	10
Horizontal offset of center conductors (h_4)	2
Horizontal spacing between lowest conductors (h_5)	15
Vertical spacing between phase conductors (h_6)	5
Conductor radius (r_{c1})	0.0203454
Total bundled sub－conductors (N_FL)	1
Ground Wire radius (r_{c2})	0.0055245

According to the size parameters of Table Ⅵ, the capacitance matrix $\boldsymbol{C}_{OFF}$ is calculated, and the calculation result is:

$$\boldsymbol{C}_{OFF}=\begin{bmatrix} 2.805 & 3.056 & 0.307 & 0.236 & 1.410 & 0.288 & 0.558 & 0.281 \\ & 2.550 & 0.415 & 0.307 & 1.491 & 0.374 & 0.581 & 0.335 \\ & & 2.641 & 3.056 & 0.374 & 1.491 & 0.335 & 0.581 \\ & & & 2.866 & 0.288 & 1.410 & 0.281 & 0.558 \\ & & & & 2.520 & 0.457 & 1.435 & 0.531 \\ & & & & & 2.518 & 0.531 & 1.435 \\ & & & & & & 2.281 & 0.780 \\ & & & & & & & 2.276 \end{bmatrix}\times 10^{-12}F$$

V. CONCLUSION

In this paper, based on the theory of moment method, the capacitance per unit length of overhead transmission lines is studied. The applicability and accuracy of the method are proved by comparison with analytical methods. The method is applied to three common overhead transmission line layout types, and the capacitance parameters per unit length of transmission line are calculated, which provides a foundation for further study of the transmission line's transient problems.

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