Computer Aided Design Optimization

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EE-677: Foundations of VLSI CAD

What is Mathematical Optimization?

- "Optimization" comes from the same root as "optimal", which means *best*. When you optimize something, you are "making it best".

Optimization Vocabulary

Your basic optimization problem consists of…

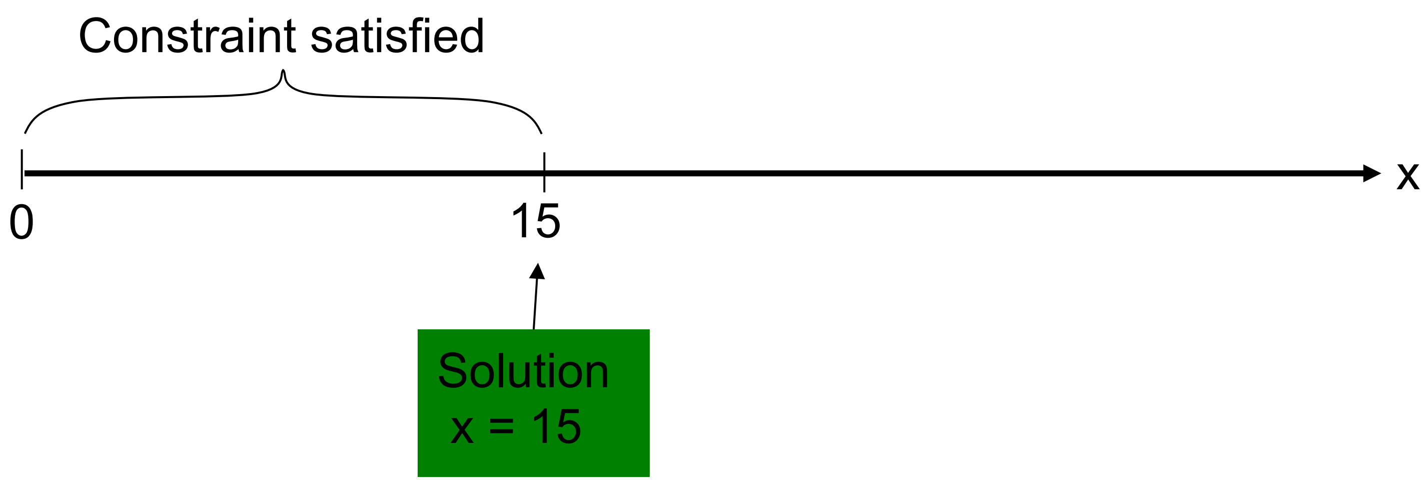
- The objective function, $f(x)$, which is the output you're trying to maximize or minimize.
- Variables, $x_1 x_2 x_3$ and so on, which are the inputs – things you can control. They are abbreviated x_n to refer to individuals or x to refer to them as a group.
- Constraints, which are equations that place limits on how big or small some variables can get. Equality constraints are usually noted $h_n(x)$ and inequality constraints are noted $g_n(x)$.

Types of Optimization Problems

- Some problems have constraints and some do not.
- There can be one variable or many.
- Variables can be discrete (for example, only have integer values) or continuous.
- Some problems are static (do not change over time) while some are dynamic (continual adjustments must be made as changes occur).
- Systems can be deterministic (specific causes produce specific effects) or stochastic (involve randomness/ probability).
- Equations can be linear (graph to lines) or nonlinear (graph to curves)

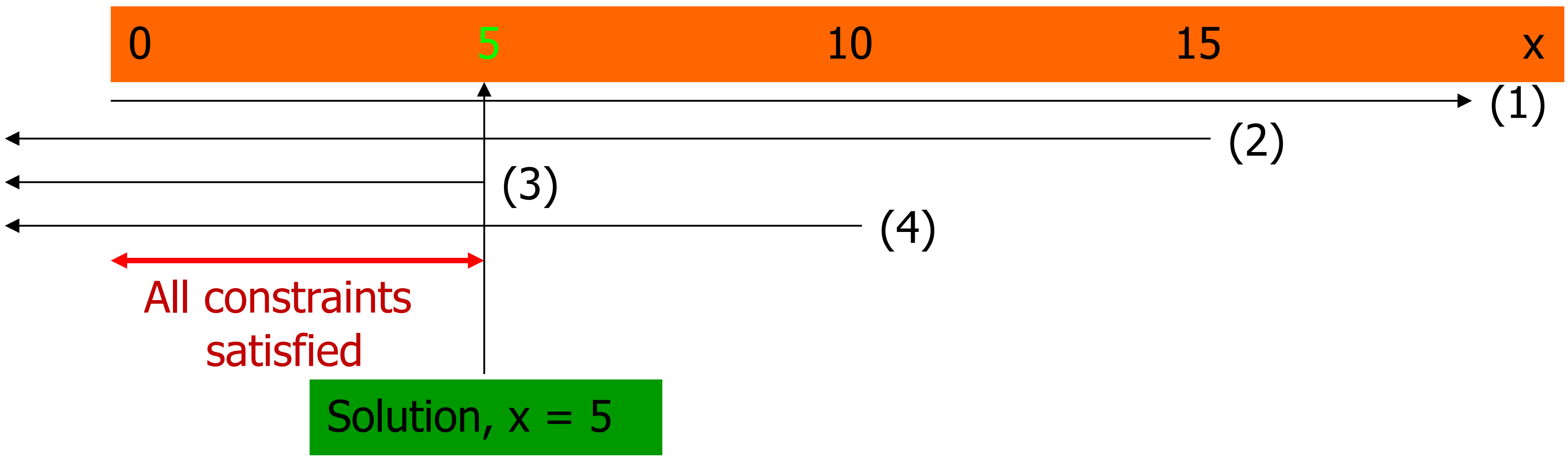
A Single-Variable Problem

- Consider variable x
- Problem: find the maximum value of x subject to constraint, $0 \leq x \leq 15$.
- Solution: $x = 15$.



Single Variable Problem (Cont.)

- Consider more complex constraints:
- Maximize x, subject to following constraints:
  - $x \geq 0$ (1)
  - $5x \leq 75$ (2)
  - $6x \leq 30$ (3)
  - $x \leq 10$ (4)



A Two-Variable Problem

- Manufacture of chairs and tables:
  - Resources available:
    - Material: 400 boards of wood
    - Labor: 450 man-hours
  - Profit:
    - Chair: $45
    - Table: $80
  - Resources needed:
    - Chair
      - 5 boards of wood
      - 10 man-hours
    - Table
      - 20 boards of wood
      - 15 man-hours
  - Problem: How many chairs and how many tables should be manufactured to maximize the total profit?

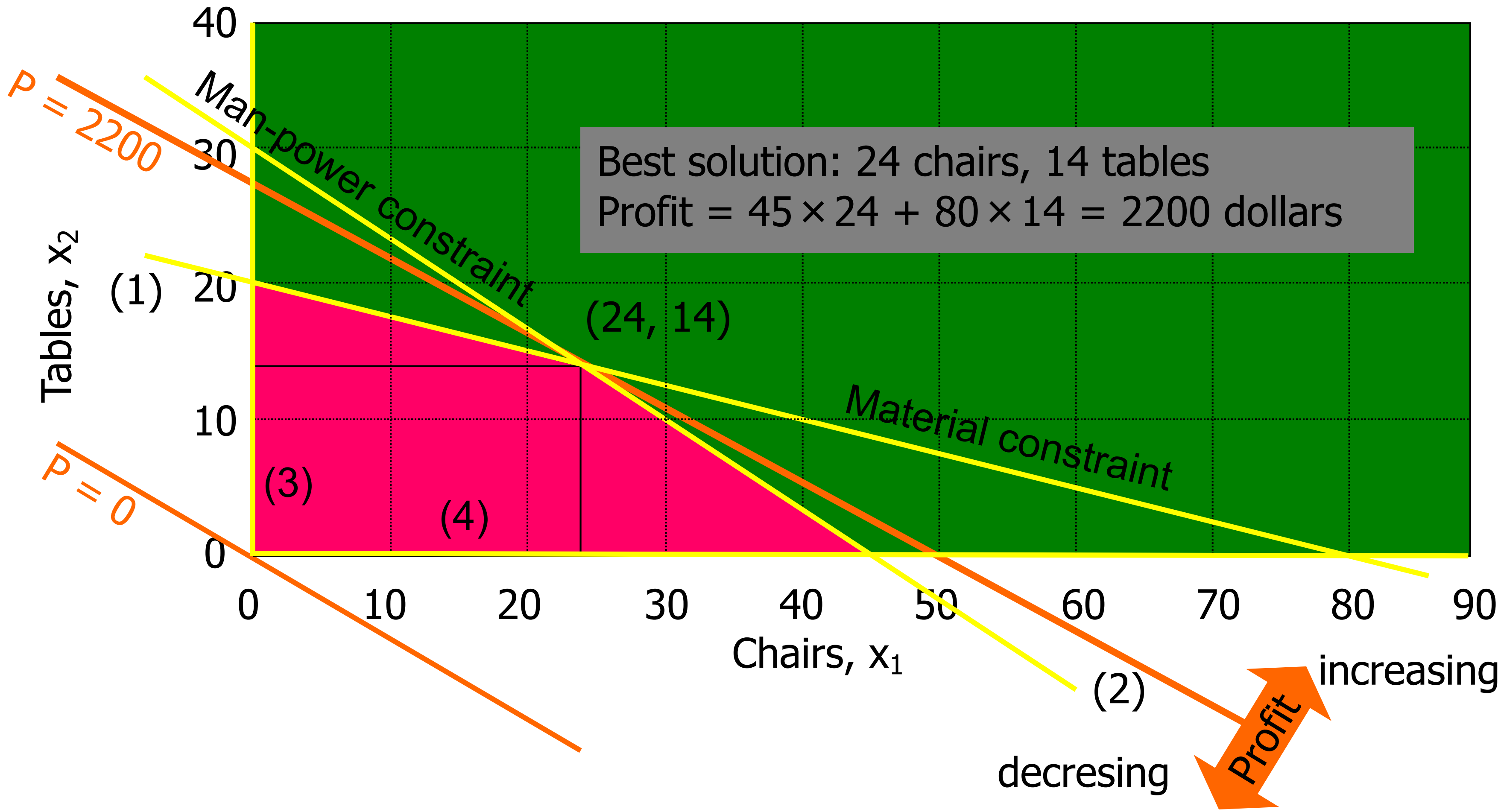
Formulating Two-Variable Problem

- Manufacture x_1 chairs and x_2 tables to maximize profit:

$$P = 45x_1 + 80x_2 \text{ dollars}$$

- Subject to given resource constraints:
  - 400 boards of wood, $5x_1 + 20x_2 \leq 400$ (1)
  - 450 man-hours of labor, $10x_1 + 15x_2 \leq 450$ (2)
  - $x_1 \geq 0$ (3)
  - $x_2 \geq 0$ (4)

Solution: Two-Variable Problem



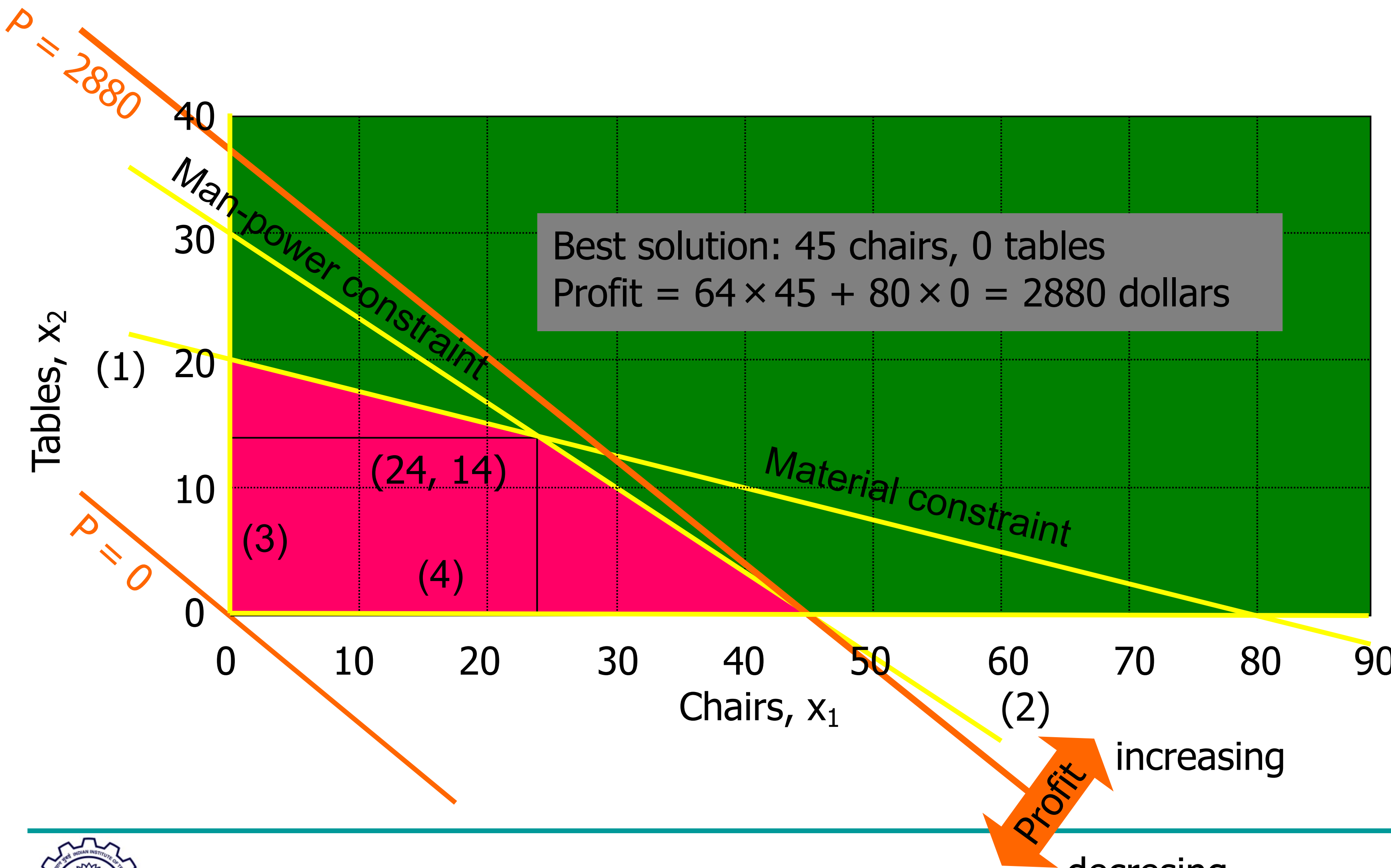
Change Profit of Chair to \$64/Unit

- Manufacture x_1 chairs and x_2 tables to maximize profit:

$$P = 64x_1 + 80x_2 \text{ dollars}$$

- Subject to given resource constraints:
  - 400 boards of wood, $5x_1 + 20x_2 \leq 400$ (1)
  - 450 man-hours of labor, $10x_1 + 15x_2 \leq 450$ (2)
  - $x_1 \geq 0$ (3)
  - $x_2 \geq 0$ (4)

Solution: \$64 Profit/Chair



A Dual Problem

- Explore an alternative.
- Questions:
  - Should we make tables and chairs?
  - Or, auction off the available resources?
- To answer this question we need to know:
  - What is the minimum price for the resources that will provide us with same amount of revenue as the profits from tables and chairs?
  - This is the dual of the original problem.

Formulating the Dual Problem

- Revenue received by selling off resources:
  - For each board, w_1
  - For each man-hour, w_2
- Minimize $400w_1 + 450w_2$
- Subject to constraints:
    - $5w_1 + 10w_2 \geq 45$
    - $20w_1 + 15w_2 \geq 80$
    - $w_1 \geq 0$
    - $w_2 \geq 0$

The Duality Theorem

- If the primal has a finite optimal solution, so does the dual, and the optimum values of the objective functions are equal.

Primal-Dual Problems

- **Primal problem**
  - **Fixed resources**
  - **Maximize profit**
- **Variables:**
  - **x_1 (number of chairs)**
  - **x_2 (number of tables)**
- **Maximize profit $45x_1+80x_2$**
- **Subject to:**
  - **$5x_1 + 20x_2 \leq 400$**
  - **$10x_1 + 15x_2 \leq 450$**
  - **$x_1 \geq 0$**
  - **$x_2 \geq 0$**
- **Solution:**
  - **$x_1 = 24$ chairs, $x_2 = 14$ tables**
  - **Profit = \$2200**

- **Dual Problem**
  - **Fixed profit**
  - **Minimize value**
- **Variables:**
  - **w_1 (\$ value/board of wood)**
  - **w_2 (\$ value/man-hour)**
- **Minimize value $400w_1+450w_2$**
- **Subject to:**
  - **$5w_1 + 10w_2 \geq 45$**
  - **$20w_1 + 15w_2 \geq 80$**
  - **$w_1 \geq 0$**
  - **$w_2 \geq 0$**
- **Solution:**
  - **$w_1 = \$1$, $w_2 = \$4$**
  - **value = \$2200**

Thank You