EE324 Control Systems Lab

Problem Sheet 7

Sheel Shah — 19D070052

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1 Question 1

Code

```
s = poly(0, 's')
g = 1 / (s * (s^2 + 4 * s + 8))
G = syslin('c', g)

// part A
K_A = 0
for k =0:0.1:100
    char_eqn = k*G
    gm = g_margin(char_eqn)
    pm = p_margin(char_eqn)
    if gm^2 + pm^2 <= 1e-10
        K_A = k
        break
```

```
    end
end
disp(K_A)


//part C
tf = K_A * G / (1 + K_A*G)
[z, p, g] = tf2zp(tf)
disp(p)
```

1.1 Part A

The code gives us K = 32

1.2 Part B

From looking at the bode plots, we see that the gain and phase margins are monotonic functions. Hence, there is no K for which only one of GM and PM is 0 and not the other.

1.3 Part C

The system is marginally unstable at K=32. The code above is used to find the pole and we find poles on the imaginary axis.

2 Question 2

2.1 Part A

2a)

$\% OS = 10\% = 0.1$

$\therefore 0.1 = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$$\therefore 2.303 = \frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$\therefore 1-\zeta^2 = \zeta^2 \cdot (1.86)$

$\therefore \zeta^2 = 0.349$

$\therefore \zeta = 0.5912$

$$\text{Now } TF = \frac{KG}{1+KG} = \frac{K}{s^2+3s+2+K}$$

$$\therefore \zeta = \frac{3}{2\sqrt{2+K}} = 0.5912$$

$\therefore \sqrt{2+K} = 2.537$

$\therefore \boxed{K = 4.437}$

2.2 Part B

2b) For SS, $s = 0$.

$\therefore$ SS $= \dfrac{K}{2+K} = 0.689$

lag controller:

pole at $s = -0.01$

zero at $s = -0.2$

$\therefore$ new G = old $G \cdot \dfrac{(s+0.2)}{(s+0.01)}$

$\therefore G'(0) = G(0) \cdot 20$

$= \frac{1}{2} \cdot 20 = 10$

$\therefore$ SS $= \dfrac{KG'(0)}{1+KG'(0)} = \dfrac{10K}{1+10K} = 0.978$

$\therefore$ $\boxed{\begin{array}{l} \text{old error} = 0.311 \\ \text{new error} = 0.022 \end{array}}$

2.3 Part C

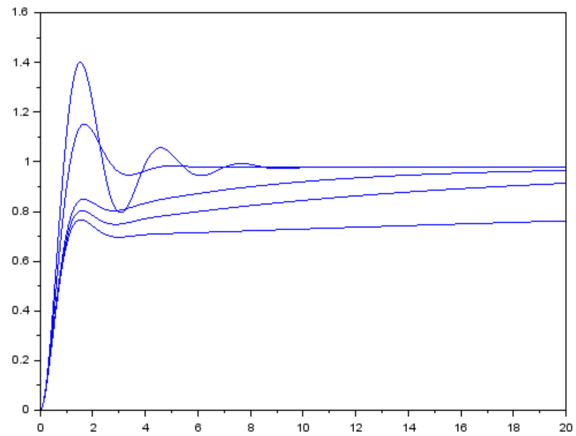

Figure 1: Plots for part C

Code

```
s = poly(0, 's')

g = 1 / (s^2 + 3 * s + 2)

G = syslin('c', g)

K = 4.437 // from part A

poles = [0.001, 0.005, 0.01, 0.05, 0.1]

t = 0:0.01:20

for pole = poles

    G_ = G * (s + 20*pole) / (s + pole)

    tf = (K*G_) / (1 + K*G_)

    resp = csim("step", t, tf)

    plot(t, resp)

end
```

3 Question 3

3.1 Working

3a) % OS is fixed

$\therefore \zeta$ is fixed at 0.5912

earlier $K = 4.437 \Rightarrow \omega_n = \sqrt{2+K} = 2.537$

$\therefore$ now for half settling time, $\omega_n = 5.074$

$$\therefore \frac{k(s+z)}{(s+1)(s+2)+k(s+z)} = \frac{ks+kz}{s^2+(3+k)s+(2+kz)}$$

is the new transfer function.

$$\therefore \sqrt{2+kz} = 5.074$$

$$\frac{3+k}{2\sqrt{2+kz}} = 0.5912$$

$$\therefore 3+k = 6$$

$$\therefore k = 3$$

$$\therefore \sqrt{2+3z} = 5.074$$

$$\therefore z = 7.915$$

$\therefore$ $z = 7.915$, $k = 3$ is the PD controller required

$\therefore$ $z = 9$, $p = 13$ (approx) could be the lead compensator.

3.2 Plots

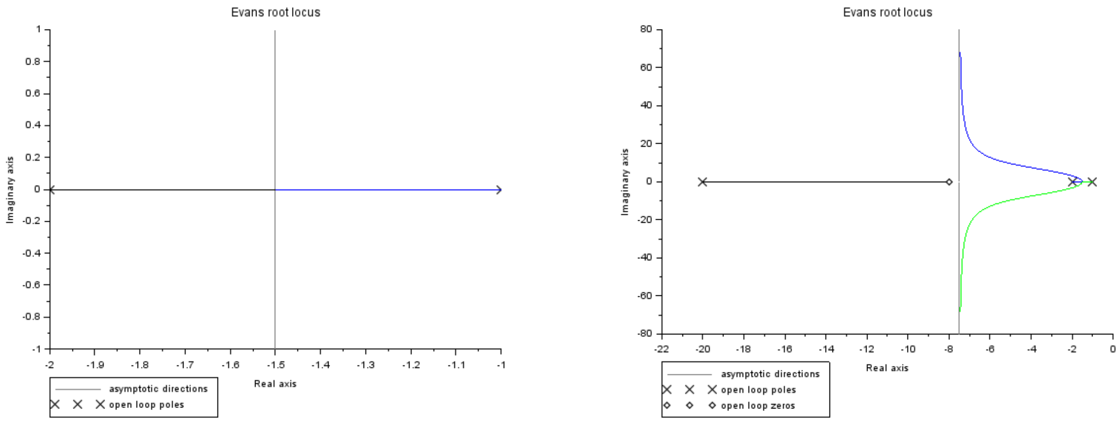

Figure 2: Old and new root loci (part A)

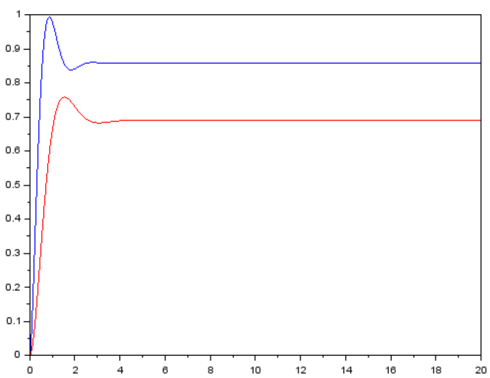

Figure 3: Old and new step responses (part A)

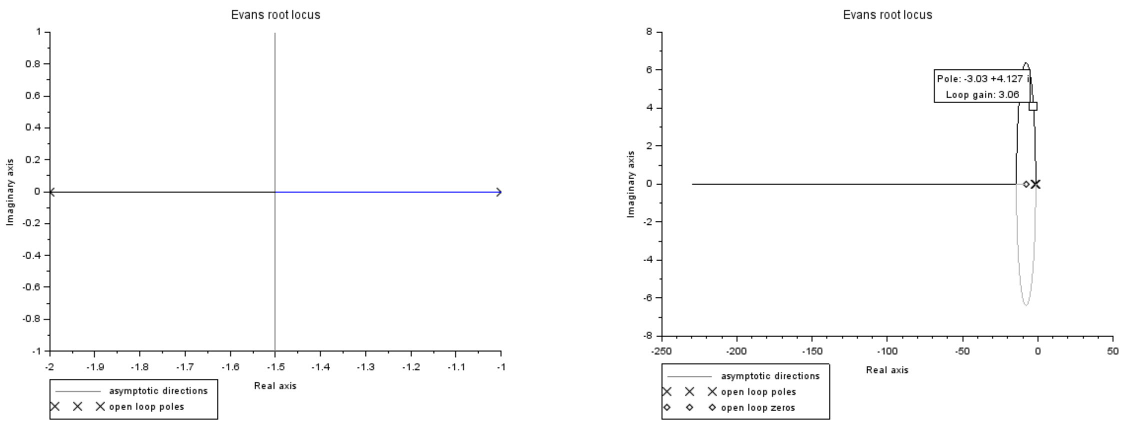

Figure 4: Old and new root loci (part B)

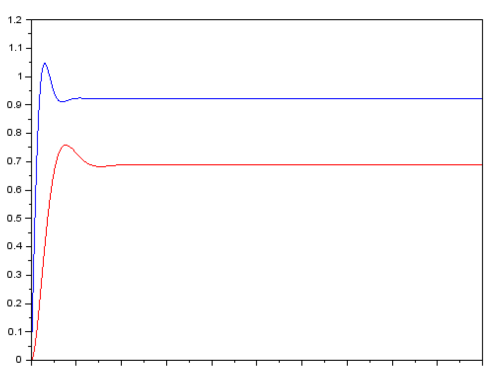

Figure 5: Old and new step responses (part B)