- BAND-PASS SYSTEMS

Band-Pass Systems

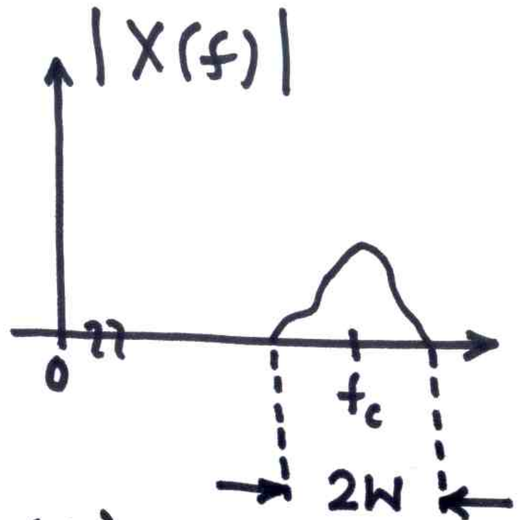
* Studied the complex low-pass representation of BP signals

* Logical to develop a corresponding procedure for handling the analysis of BP systems
* The analysis of BP systems can be greatly simplified by establishing an analogy between LP and BP systems

- $x(t)$: BP signal with $X(f)$ zero for $|f \pm f_c| > W$
- BP system : passband is the interval: $|f \pm f_c| < B$ where $B \leq W$
- Study the effect of a BP system on a BP input

BAND-PASS SIGNALS

BP: $x(t) \longleftrightarrow X(f)$



$W < f_c$

$$x_+(t) = x(t) + j\,x_h(t)$$

$$\tilde{x}(t) = x_+(t)\exp(-j2\pi f_c t)$$

$$x(t) = \mathrm{Re}\left[\tilde{x}(t)\exp(j2\pi f_c t)\right]$$

$$\tilde{x}(t) \equiv \text{complex envelope of } x(t)$$
$$\equiv x_{\ell p}(t)$$

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$$x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$$

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t) \equiv x_{lp}(t)$$

$$x(t) = \mathrm{Re}\left\{ \tilde{x}(t)\, e^{j2\pi f_c t} \right\}$$

$$X(f) = \frac{1}{2}\left[\tilde{X}(f-f_c) + \tilde{X}^*(-(f+f_c)) \right]$$

|||, let

$$\tilde{h}(t) = h_I(t) + j\, h_Q(t) \equiv h_{lp}(t)$$

$$h(t) = h_I(t)\cos 2\pi f_c t - h_Q(t)\sin 2\pi f_c t$$

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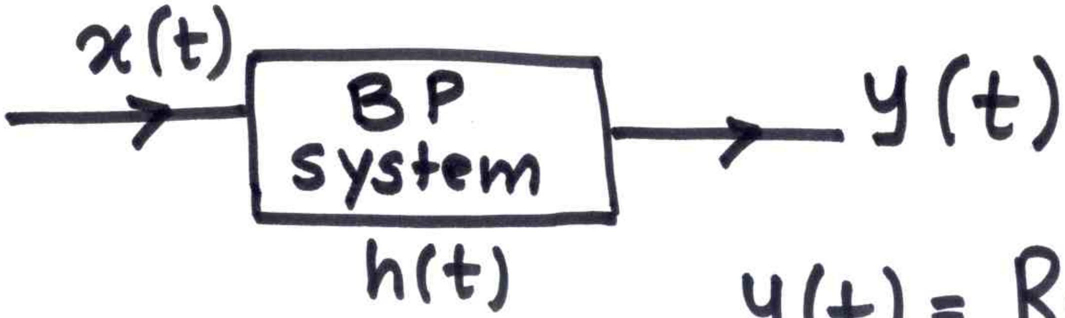
$$h(t) = \mathrm{Re}\left[\tilde{h}(t)\exp(j2\pi f_c t)\right]$$

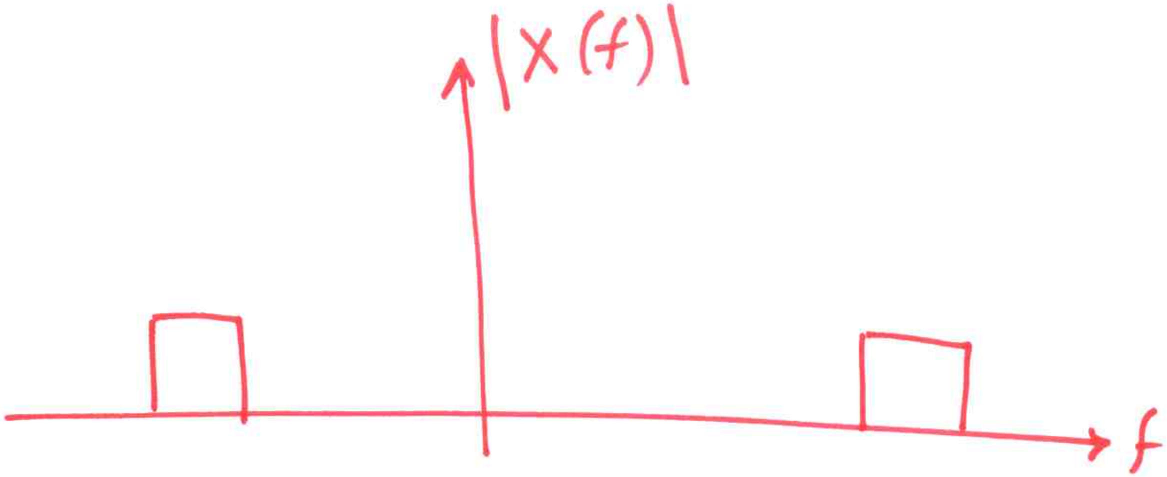
$$2h(t) = \tilde{h}(t)e^{j2\pi f_c t} + \tilde{h}^*(t)e^{-j2\pi f_c t}$$

$$H(f) = \cancel{\tilde{H}(f)}\ \frac{\tilde{H}(f-f_c) + \tilde{H}^*[-(f+f_c)]}{2}$$



$$y(t) = \mathrm{Re}\left[\tilde{y}(t)\exp(j2\pi f_c t)\right]$$

$$\rightarrow\ Y(f) = H(f)X(f)$$



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$$X(f)H(f) = \frac{1}{4}\left\{\left[\tilde{H}(f-f_c) + \tilde{H}^*[-(f+f_c)]\right] \times \left[\tilde{X}(f-f_c) + \tilde{X}^*[-(f+f_c)]\right]\right.$$

Consider the term

$$\tilde{H}(f-f_c)\,\tilde{X}^*[-(f+f_c)]$$

$\tilde{H}(f-f_c)$ has spectrum confined to the range

$$(f_c - B,\ f_c + B)$$

$\tilde{X}^*[-(f+f_c)]$ has non-zero spectral components in the range $\{-(f_c+W), -(f_c-W)\}$

III, $\tilde{H}^*[-(f+f_c)] \ast \tilde{X}(f-f_c) = 0$

$$X(f)H(f) = Y(f)$$

$$= \frac{\tilde{Y}(f-f_c) + \tilde{Y}^*[-(f+f_c)]}{2}$$

$$= \frac{1}{4}\tilde{H}(f-f_c)\tilde{X}(f-f_c)$$

$$+ \frac{1}{4}\tilde{H}^*[-(f+f_c)]\tilde{X}^*[-(f+f_c)]$$

$\tilde{Y}(f-f_c)$: non-zero spectral components in the range (f_c-B, f_c+B)

$$\frac{1}{2}\tilde{Y}(f-f_c) = \frac{1}{4}\left[\tilde{H}(f-f_c)\,\tilde{X}(f-f_c)\right]$$

and

$$\frac{1}{2}\tilde{Y}^*\left[-(f+f_c)\right] = \frac{1}{4}\left[\tilde{H}^*(-(f+f_c))\,\tilde{X}^*(-(f+f_c))\right]$$

$$\tilde{Y}(f) = \frac{1}{2}\tilde{X}(f)\,\tilde{H}(f)$$

$$\tilde{y}(t) = \frac{1}{2}\left[\tilde{x}(t) * \tilde{h}(t)\right]$$

$$\equiv \frac{1}{2}\left[x_{lp}(t) * h_{lp}(t)\right]$$

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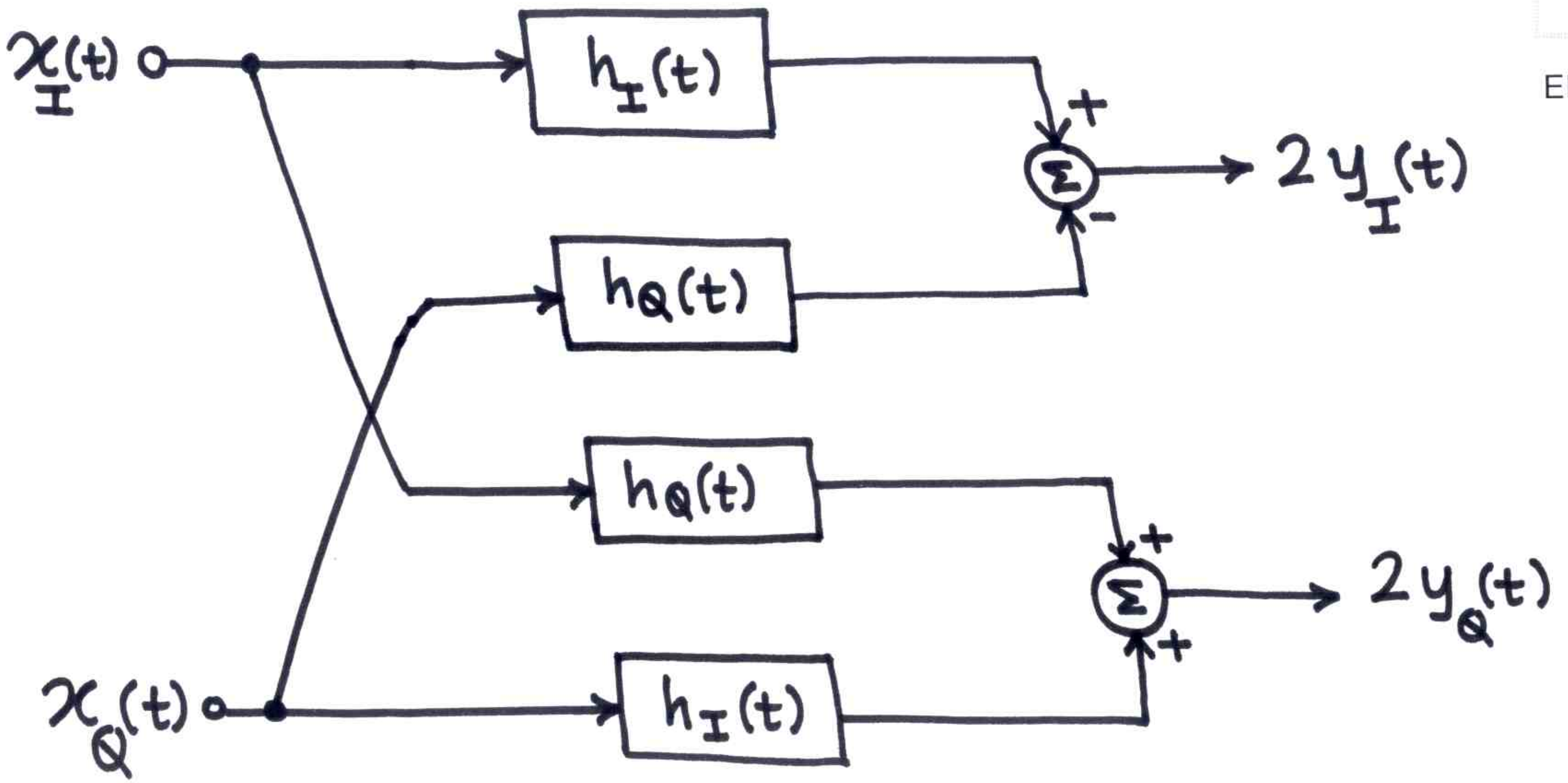
$$\tilde{y}(t) = \frac{1}{2}\left[\tilde{x}(t) * \tilde{h}(t)\right]$$

$$= \frac{1}{2}\left[x_I(t) + j\,x_Q(t)\right] * \left[h_I(t) + j\,h_Q(t)\right]$$

$$= y_I(t) + j\,y_Q(t)$$

$$y_I(t) = \frac{1}{2}\left\{x_I(t) * h_I(t) - x_Q(t) * h_Q(t)\right\}$$

$$y_Q(t) = \frac{1}{2}\left\{x_I(t) * h_Q(t) + x_Q(t) * h_I(t)\right\}$$



Block Diagram illustrating the relationships between the I-phase & Q-components of $y(t)$ & $x(t)$

Summary of the procedure for evaluating the response of a BP system (with mid-band f_c) to an i/p BP signal (of carrier freq. f_c):

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(1) BP: $x(t)$ is replaced by $\tilde{x}(t)\ (\equiv x_{lp}(t))$, which is related to $x(t)$ by

$$x(t) = \mathrm{Re}\left[\tilde{x}(t)\exp(j2\pi f_c t)\right]$$

(2) BP: $h(t) \longrightarrow \tilde{h}(t)\ (\equiv h_{lp}(t))$

$$h(t) = \mathrm{Re}\left[\tilde{h}(t)\exp(j2\pi f_c t)\right]$$

(3) $$\tilde{y}(t)\ (\equiv y_{lp}(t)) = \frac{1}{2}\left[\tilde{h}(t) * \tilde{x}(t)\right]$$

(4) $$y(t) = \mathrm{Re}\left[\tilde{y}(t)\exp(j2\pi f_c t)\right]$$

MODULE ENDS