Review of Signals and Systems: Part 3

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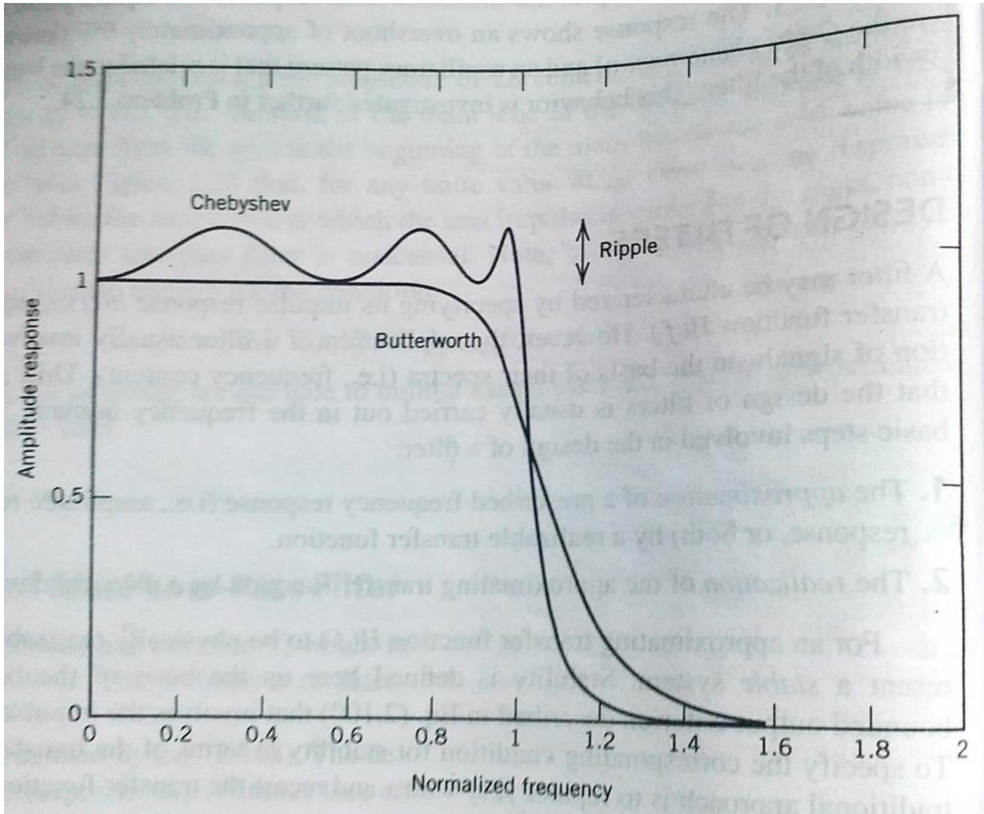
Transmission of Signals Through Linear Time-Invariant (LTI) Systems

- Filters and communication channels are often modeled as *LTI* systems
- Recall: if an input $x(t)$ is applied to an LTI system with impulse response $h(t)$, then its output $y(t)$ is given by:
  - ❑ $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$
- Fourier transform of output signal:
  - ❑ $Y(f) = X(f)H(f)$
  - ❑ $H(f)$ called "*frequency response*" or "*transfer function*" of the LTI system
- Recall: a system is *causal* if it does not respond before the excitation is applied
- Necessary and sufficient condition for an LTI system to be causal:
  - ❑ $h(t) = 0$, for $t < 0$
- Recall: a system is *bounded input-bounded output (BIBO) stable* if its output is bounded whenever input is bounded
- Necessary and sufficient condition for an LTI system to be BIBO stable:
  - ❑ $\int_{-\infty}^{\infty} |h(t)|dt < \infty$

Filter

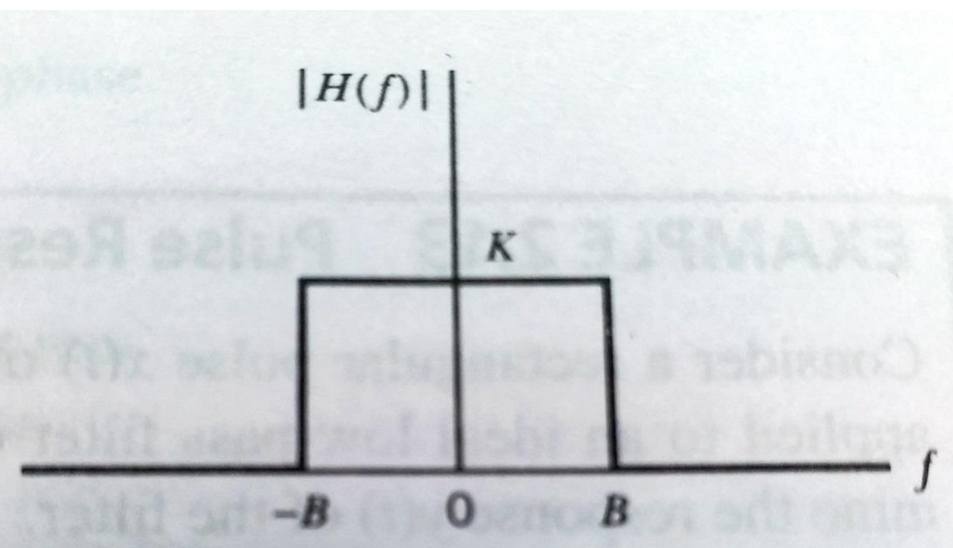
- A frequency-selective device
- Common use in communications:
  - ❑To limit the spectrum of a signal to some specified band of frequencies
- Frequency response of a filter has a "*passband*" and a "*stopband*"
  - ❑Frequencies of input inside passband are output with little or no distortion
  - ❑Frequencies of input inside stopband are blocked or significantly attenuated
- Common types of filters:
  - ❑low-pass, band-pass, high-pass, band-stop



Ref: "Communication Systems" by S. Haykin and M. Moher, 5th ed

Example

- Transfer function of an ideal low-pass filter:
  - $H(f) = \begin{cases} \exp(-j2\pi f t_0), & -B \le f \le B, \\ 0, & |f| > B. \end{cases}$
  - where $t_0 > 0$
- Is this a causal filter?
- Impulse response:
  - $h(t) = 2B\text{sinc}[2B(t - t_0)]$
- Impulse response shows that the filter is non-causal
- How can we design a causal filter that closely approximates an ideal low-pass filter?
  - consider filter similar to above, with t_0 large and part of impulse response for $t < 0$ truncated



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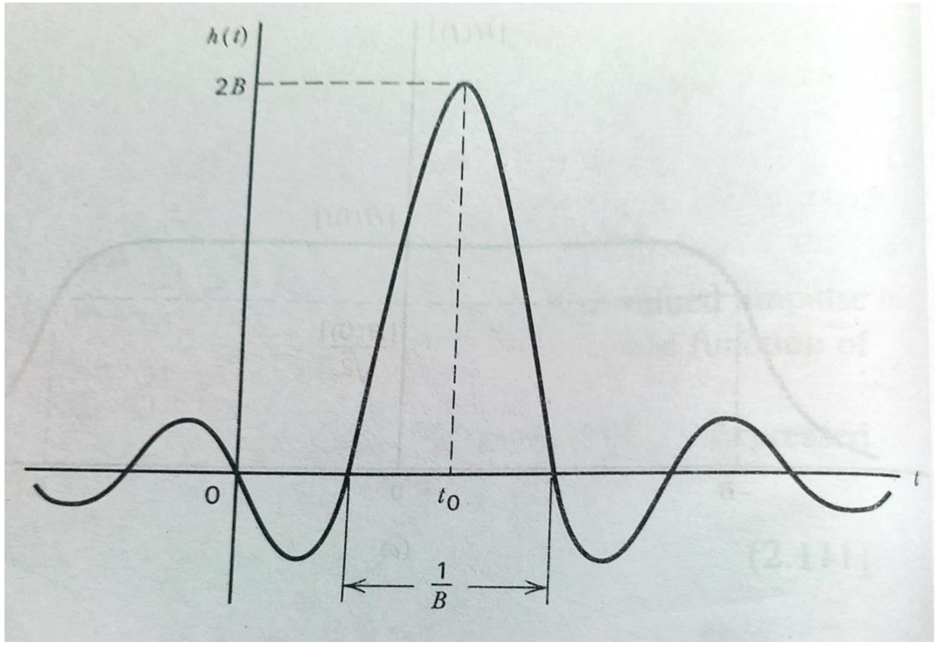
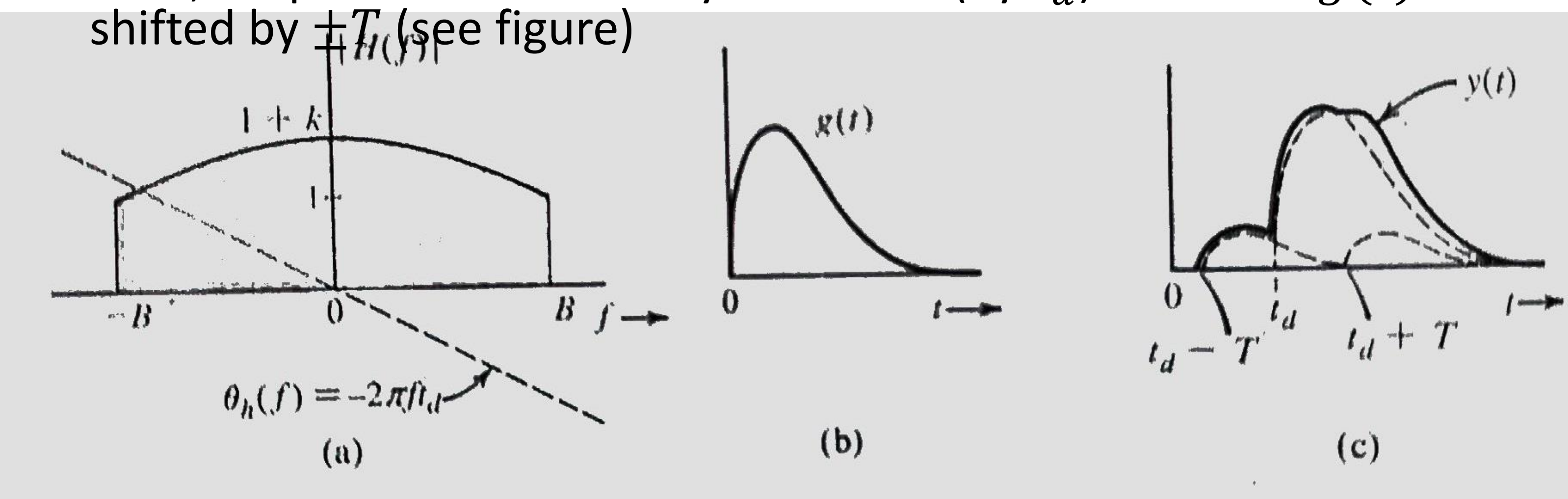


Communication Link Viewed as a Filter: LTI Channel

- Communication channel usually acts as a filter
- First, consider a LTI channel with frequency response $H(f)$
- What properties must $H(f)$ satisfy so that the input, say $g(t)$, is passed undistorted?
  - ❑ $H(f)$ must be of the form $ke^{-2\pi f t_d}$ for some constants k and t_d over the frequency band on which $g(t)$ has non-zero spectral content
- If $H(f)$ does not satisfy above properties, then $g(t)$ may get distorted

Example

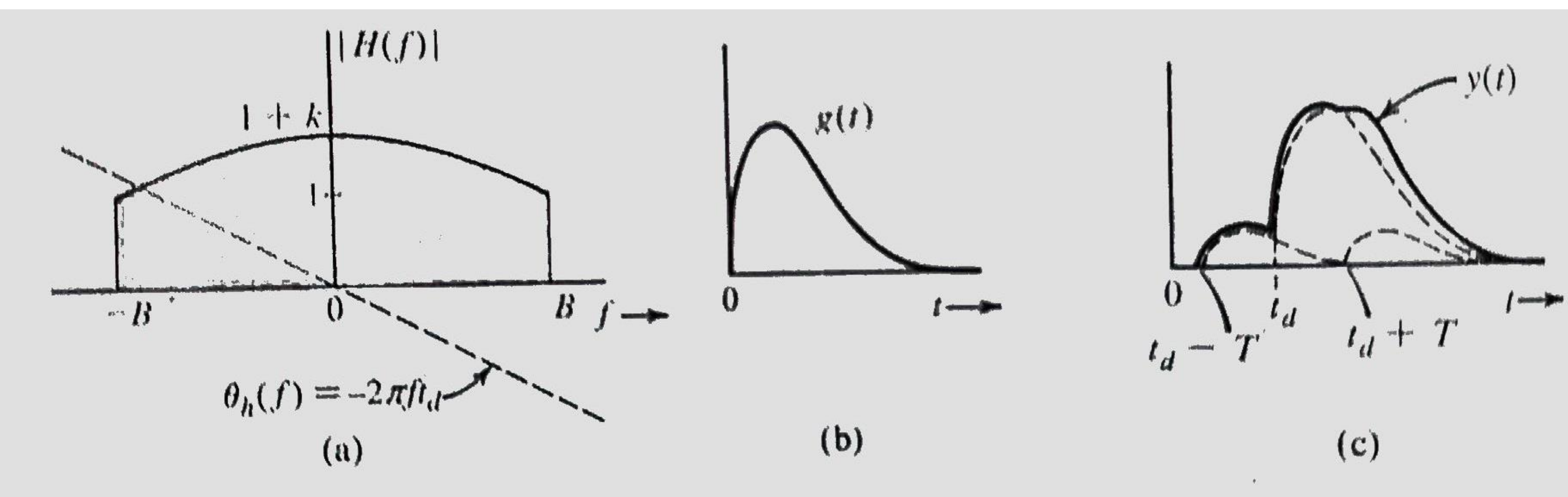
- Consider a channel with frequency response:
  - $H(f) = \begin{cases} (1 + k\cos(2\pi f T))e^{-j2\pi t_d f}, & |f| < B, \\ 0, & |f| \geq B, \end{cases}$
  - where k, t_d and T are positive constants
- A pulse, $g(t)$, which is low-pass and band-limited to B Hz, is applied at the input of this channel
- Want to find output signal
- $H(f)$ can be written as:
  - $H(f) = \begin{cases} e^{-j2\pi t_d f} + \frac{k}{2}e^{-j2\pi f(t_d - T)} + \frac{k}{2}e^{-j2\pi f(t_d + T)}, & |f| < B, \\ 0, & |f| \geq B, \end{cases}$
- Output signal:
  - $y(t) = g(t - t_d) + \frac{k}{2}[g(t - t_d - T) + g(t - t_d + T)]$
- Thus, output consists of delayed version (by t_d) of sum of $g(t)$ and its echoes shifted by $\pm T$ (see figure)



Ref: Lathi, Ding, "Modern Digital and Analog Communication Systems", 4th ed.

Example (contd.)

- Recall: output signal:
  - $y(t) = g(t - t_d) + \frac{k}{2}[g(t - t_d - T) + g(t - t_d + T)]$
- Note that channel causes input signal $g(t)$ to spread out in time
- In digital communications, this causes "*intersymbol interference*" (ISI)
  - digital symbol, when passed through channel like in the above example, spreads more widely than its allotted time
  - so adjacent symbols interfere with each other, thus increasing the probability of detection error at receiver



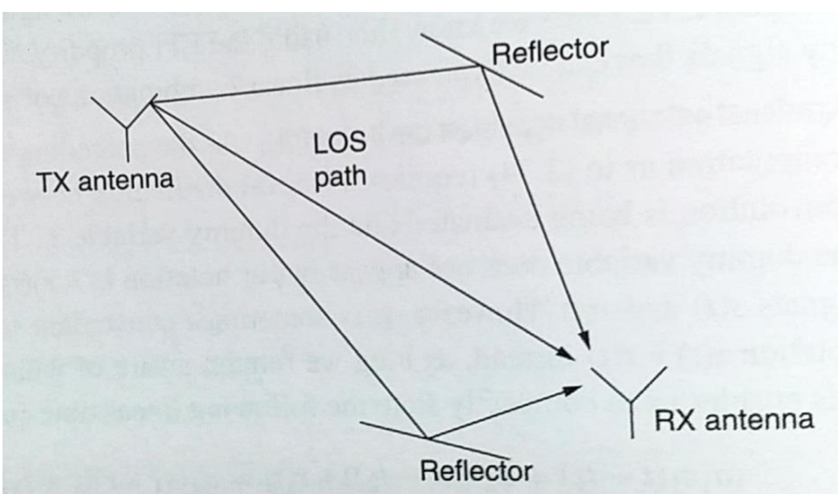
Ref: Lathi, Ding, "Modern Digital and Analog Communication Systems", 4th ed.

Communication Link Viewed as a Filter: Nonlinear Channel

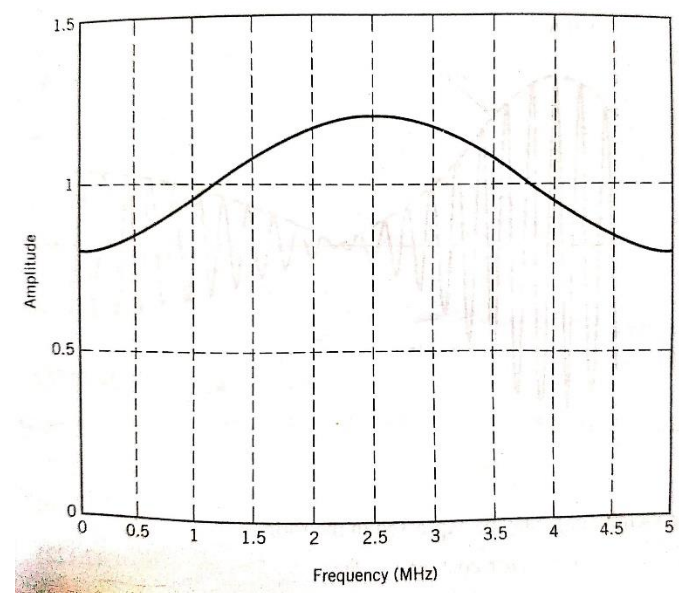
- Several communication channels (e.g., some fiber optic channels) are nonlinear
- E.g.: consider a channel whose output, $y(t)$, for a given input $g(t)$ is of the form:
  - ❑ $y(t) = a_0 + a_1 g(t) + a_2 g(t)^2 + \cdots + a_k g(t)^k$
  - ❑ where k is a positive integer
- If $g(t)$ is band-limited to bandwidth B, what is bandwidth of $y(t)$?
  - ❑ kB
- Thus, a nonlinear channel can cause bandwidth of transmitted signal to increase
- This can cause interference among signals using different frequency channels

Communication Link Viewed as a Filter: Multipath Channel

- In wireless communication, receiver often receives:
  - transmitted signal directly from transmitter
  - and also several delayed versions of it reflected from objects in environment
- Such a channel called "*multipath channel*"
- E.g.: Consider a multipath channel with:
  - a direct path and
  - a second (reflected) path that is delayed by duration τ w.r.t. direct path, undergoes attenuation α and phase change of ϕ during reflection
- Impulse response of this channel may be modeled as:
  - $h(t) = \delta(t) + \alpha e^{j\phi}\delta(t-\tau)$
- Amplitude spectrum of this channel for $\alpha = 0.2$, $\phi = \pi$ and $\tau = 0.2$ shown in fig.
- Clearly, channel causes distortion of transmitted signal



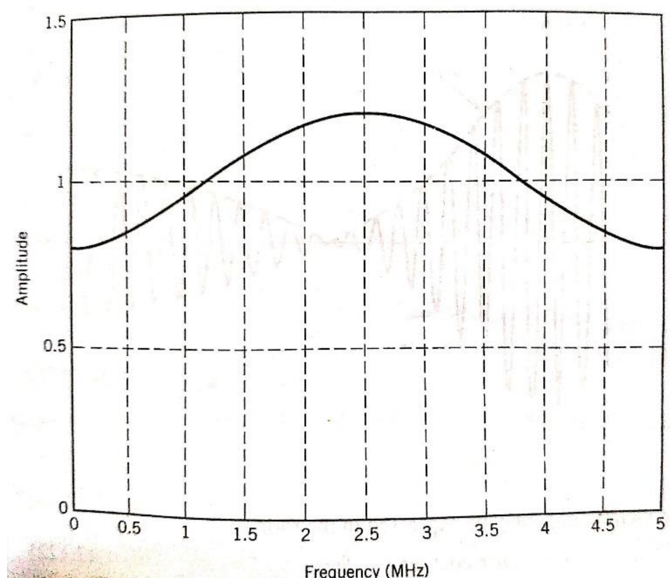
Ref: U. Madhow, "Introduction to Communication Systems"



Ref: "Communication Systems" by S. Haykin and M. Moher, 5th ed

Communication Link Viewed as a Filter: Multipath Channel (contd.)

- Recall: impulse response of above channel:
  - ❑ $h(t) = \delta(t) + \alpha e^{j\phi}\delta(t-\tau)$
  - ❑ amplitude spectrum of this channel for $\alpha = 0.2$, $\phi = \pi$ and $\tau = 0.2$ shown in fig.
- If bandwidth of transmitted signal is narrow (e.g., 100 kHz wide), then:
  - ❑ there is not much distortion, but only an attenuation
  - ❑ called "*flat fading*" channel
- However, if bandwidth of transmitted signal is wide (e.g., 3 MHz wide), then:
  - ❑ it experiences significant distortion
  - ❑ called "*frequency-selective*" channel



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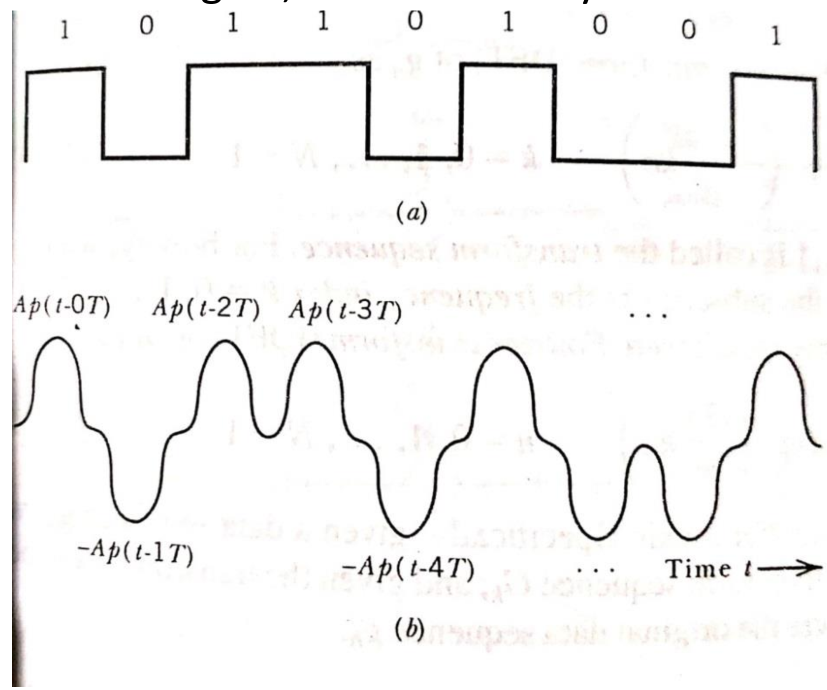
Sources of Information

- E.g. of sources of information:
  - ❑ speech, music, video, text and pdf files
- Some sources (e.g., speech) generate an *analog* signal $m(t)$
  - ❑ either directly used for modulating a carrier signal or
  - ❑ sampled, quantized and converted into a sequence of bits
- Some sources (e.g., a text file) generate a sequence of bits, say $b_1, \dots, b_T$
- In each of the above cases, sequence of bits can be represented by a sequence of pulses:
  - ❑ $g(t) = \sum_{k=1}^{K} b_k\, p(t - kT)$
  - ❑ where $p(t)$ is a pulse, which may be rectangular or a different shape
  - ❑ pulse shape chosen to limit bandwidth of transmitted signal, reduce inter-symbol interference, etc.
- The function $g(t)$ is an *analog* waveform, although it represents a bit sequence
- Hence, any modulation technique that can be used to modulate an analog message signal $m(t)$ can also be used to modulate a signal $g(t)$ that represents a bit sequence
  - ❑ however, note that typically different modulation techniques are used for analog and digital message signals in practice since their statistical properties are different



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