- NARROW-BAND FREQUENCY MODULATION
  (NBFM)

- WIDE-BAND FREQUENCY MODULATION
  (WBFM)

- Narrow-band Frequency Modulation

CDEEP
IIT Bombay

EE 308 L 21 / Slide 3

FM : $m(t) = A_m \cos 2\pi f_m t$

(Tone modulation)

- $f_i(t) = f_c + k_f\, m(t)$

$$= f_c + \underline{k_f A_m} \cos 2\pi f_m t$$

$$= f_c + \Delta f \cos 2\pi f_m t$$

$$\Delta f \triangleq k_f A_m \equiv \text{frequency deviation}$$

- $\theta_i(t) = 2\pi f_c t + \dfrac{2\pi \Delta f}{2\pi f_m} \sin 2\pi f_m t$

$$= 2\pi f_c t + \beta \sin 2\pi f_m t \qquad \beta \triangleq \frac{\Delta f}{f_m} \equiv \text{modulation index}$$

$\beta < 1$ radian Narrow-band FM

$\beta > 1$ radian Wide-band FM

- $s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$
- Expand the above relation, we get

$$s(t) = A_c \cos(2\pi f_c t) \cdot \cos[\beta \sin 2\pi f_m t] - A_c \sin(2\pi f_c t) \cdot \sin[\beta \sin 2\pi f_m t]$$

β is small compared to 1 radian

$\cos[\beta \sin 2\pi f_m t] \cong 1$, $\sin[\beta \sin 2\pi f_m t] \cong \beta \sin 2\pi f_m t$

$$\therefore s(t) \cong A_c \cos 2\pi f_c t - \beta A_c \sin(2\pi f_c t) \cdot \sin(2\pi f_m t)$$

Now, $\sin\alpha . \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$$\therefore s(t) \cong A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\left\{\cos\left[2\pi(f_c+f_m)t\right] - \cos\left[2\pi(f_c-f_m)t\right]\right\}$$

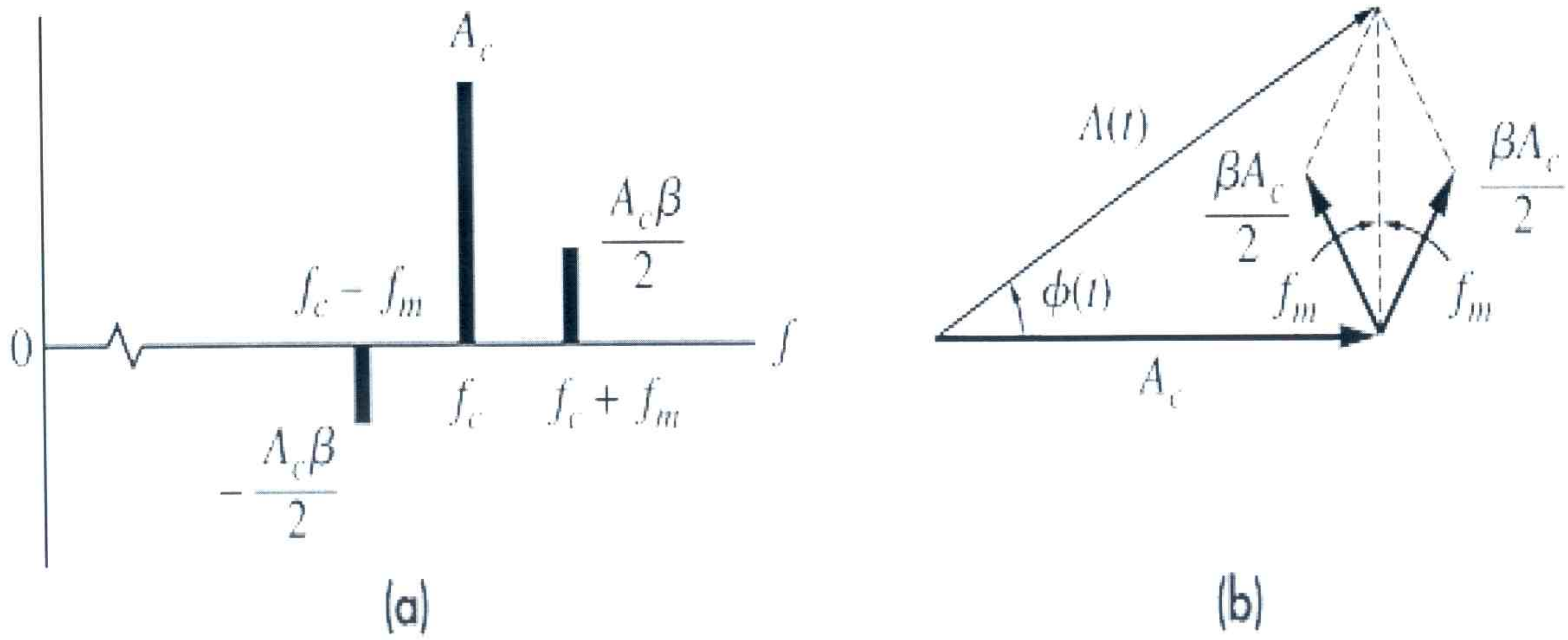
The above expression is somewhat similar to the corresponding one defining an AM signal

- $S_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c\left\{\cos\left[2\pi(f_c+f_m)t\right] + \cos\left[2\pi(f_c-f_m)t\right]\right\}$

  where μ is the modulation factor of the AM signal

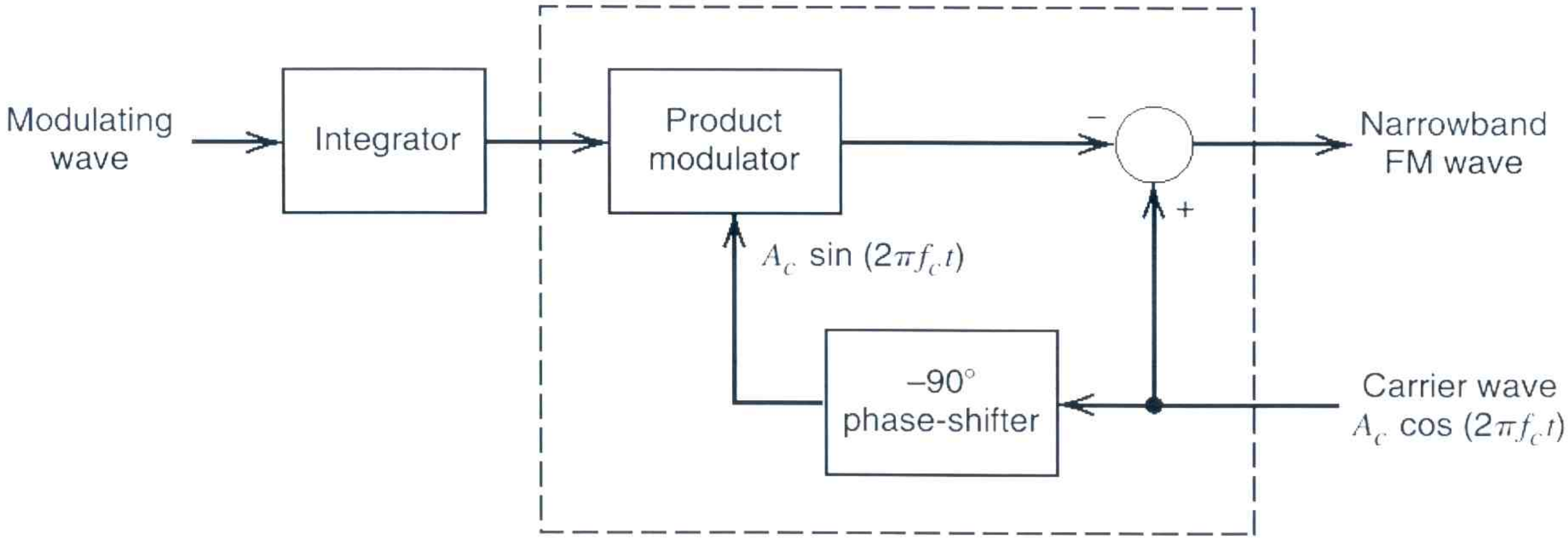
- Basic difference: NOTE THE ALGEBRAIC SIGN of the lower side frequency in the narrow-band FM is REVERSED

NBFM with tone modulation (a) Line spectrum; (b) Phasor diagram



Block diagram of a method for generating a narrowband FM signal

CDEEP
IIT Bombay

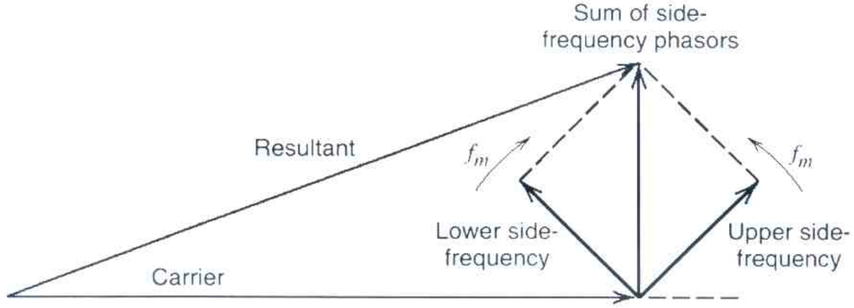


Narrowband phase modulator

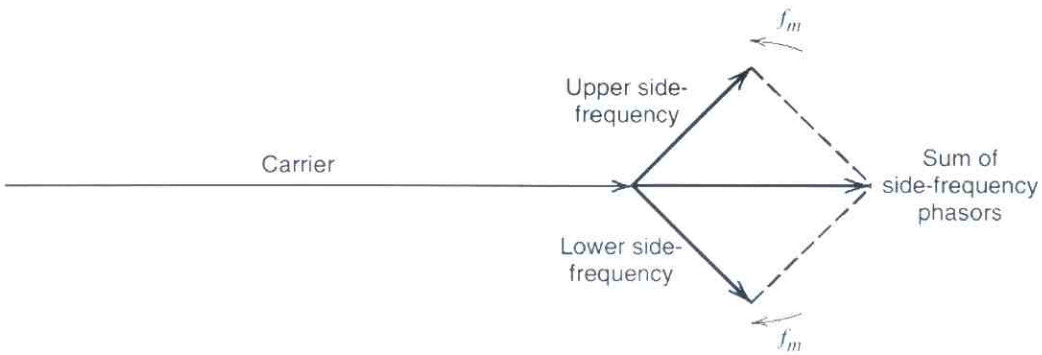
CDEEP
IIT Bombay

EE 308 L 21 / Slide 8

A phasor comparison of narrowband FM and AM waves for sinusoidal modulation. (*a*) Narrowband FM wave. (*b*) AM wave



(*a*)



(*b*)

- If $m(t)$ is not a sinusoidal signal

- $$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

Let $m_{in}(t) = \int_{-\infty}^{t} m(\tau)\,d\tau$ & $2\pi k_f = \alpha_f$

Then,

$$s(t) = A_c \cos\left[2\pi f_c t + \alpha_f m_{in}(t)\right]$$

$$s_+(t) = A_c \exp\left[j\,(2\pi f_c t + \alpha_f m_{in}(t)\right]$$

$$s_+(t) = A_c\left[1 + j\alpha_f m_{in}(t) - \frac{\alpha_f^2}{2!}\left(m_{in}(t)\right)^2 + \cdots + j^n\frac{\alpha_f^n}{n!}\left(m_{in}(t)\right)^n + \cdots\right] \times e^{j2\pi f_c t}$$

$$s(t) = \mathrm{Re}\left\{ s_+(t) \right\}$$

$$\therefore s(t) = A_c\left[\cos(2\pi f_c t) - \alpha_f m_{in}(t)\sin(2\pi f_c t) - \frac{\alpha_f^2}{2!}\left(m_{in}(t)\right)^2\cos(2\pi f_c t) + \cdots\right]$$

$m(t) \longleftrightarrow M(f) \rightarrow BW: W \Rightarrow m_{in}(t) \longleftrightarrow M_{in}(f) \rightarrow$ BL to W

$\left(\because M_{in}(f) = \dfrac{M(f)}{j2\pi f}\right)$

$\left(m_{in}(t)\right)^2 \longrightarrow BW: 2W$

$\left(m_{in}(t)\right)^3 \longrightarrow BW: 3W$

$\left(m_{in}(t)\right)^n \longrightarrow BW: nW$

Narrow-band FM (NBFM)

if $|\alpha_f m_{in}(t)|_{max} << 1$ then

$$s(t) \cong A_c[\cos(2\pi f_c t) - \alpha_f m_{in}(t)\sin(2\pi f_c t)]$$

$$\left\{ s(t) \cong A_c[\cos(2\pi f_c t) - k_p m(t)\sin(2\pi f_c t)] \right\}$$

Narrow-band PM

* If $m(t) = A_m \cos(2\pi f_m t)$

then NBFM $s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \cdot \sin(2\pi f_m t)$

where $\beta = \Delta f / f_m$ and $\Delta f = k_f A_m$

- **Wide-band Frequency Modulation**

WIDE-BAND FM (WBFM)

$|\alpha_f m_{in}(t)|_{max} \not\ll 1$ NOT SATISFIED

$f_i(t) = f_c + k_f m(t)$

Let $m_p = m(t)_{max} = |m(t)_{min}|$

Then, $f_i(t)$ varies in the range

$$(f_c - k_f m_p) \text{ to } (f_c + k_f m_p)$$

$\therefore$ frequency deviation (with centre at f_c) is

$$2k_f m_p$$

$$(B_T)_{FM} \overset{?}{=} 2k_f m_p$$

Let Δf denote the maximum frequency deviation

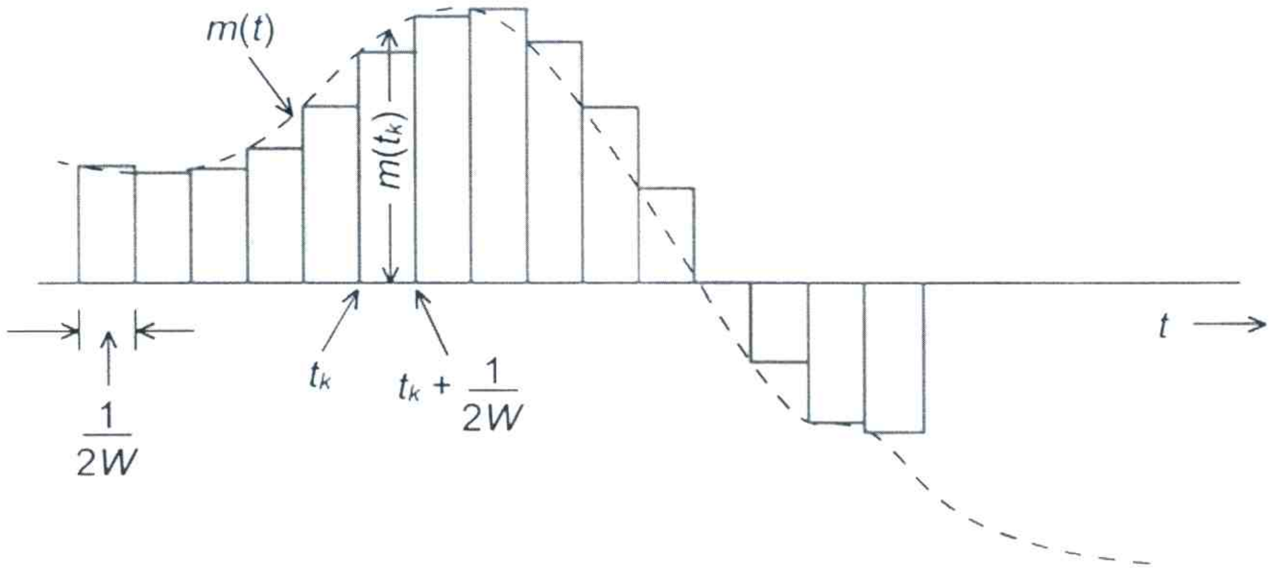
i.e., $\Delta f = k_f m_p$

Then, $(B_T)_{FM} \overset{?}{=} 2\Delta f$

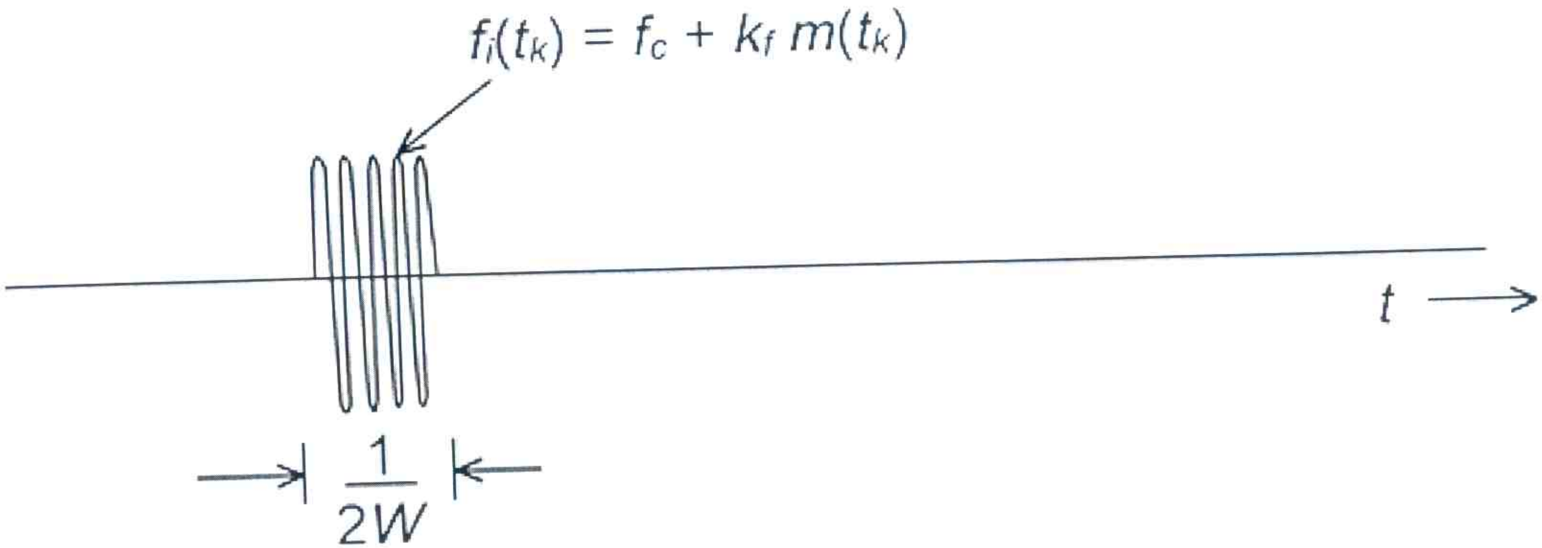
- It is valid only for $\Delta f \gg W$
- for $\Delta f \ll W$, $(B_T)_{FM} \neq 2\Delta f$ but $2W$
- fallacy: equating $\underline{f_i(t)}$ (time dependent) to the spectral frequency (independent variable)

Staircase Approximation of m(t)

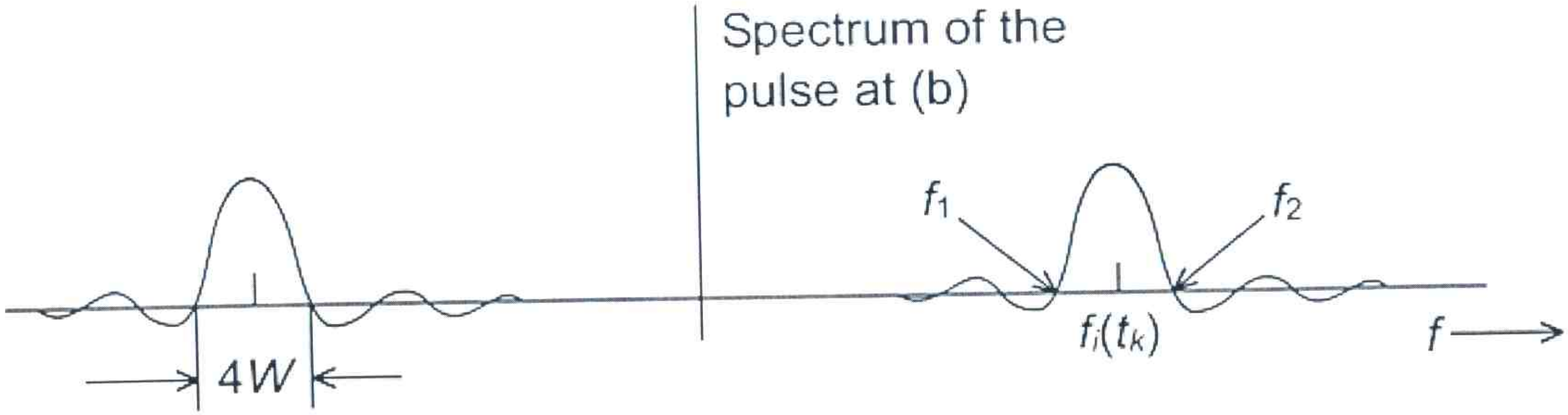
$M(f)$: $BW \equiv W$



The frequency of the RF pulse in the interval $\left(t_k, t_k + \frac{1}{2W}\right)$



Spectral range of the RF pulse in the interval $\left(t_k, t_k + \frac{1}{2W}\right)$



$$f_1 = f_i(t_k) - 2W$$
$$f_2 = f_i(t_k) + 2W$$

One possible value of the transmission BW of an FM signal,

$$(B_T)_{FM} = 2k_f m_p + 4W$$
$$= 2\Delta f + 4W$$
$$= 2(\Delta f + 2W)$$

- For the WBFM case, where $\Delta f \gg W$,

$$(B_T)_{FM} \cong 2\Delta f$$

- Other rules of thumb for $(B_T)_{FM}$ are found in the literature
- CARSON'S RULE: $(B_T)_{FM} = 2(\Delta f + W)$
  (better estimate for NBFM

CDEEP
IIT Bombay

EE 308 L 21 / Slide 18

Define "deviation ratio" $\equiv D$

$$D \triangleq \frac{\Delta f}{W}$$

$$(B_T)_{FM} = 2W(D+k)$$

$$1 < k < 2$$