Term Paper: Theory of Small Reflections for Conjugately Characteristic-Impedance Transmission Lines

Sheel F. Shah[1,*]

[1] *Student, B. Tech. Electrical Engineering Department, IIT Bombay, Mumbai 400076, India*
[*] *19D070052@iitb.ac.in*

1. Introduction

The paper [1] being reviewed is centered around building the theory of small reflections (TSR) for Conjugately Characteristic-Impedance Transmission Lines (CCITLs). CCITLs is a deeply researched class of transmission lines that are essentially lossless transmission lines with conjugate characteristic impedances Z_0^- and Z_0^+ along the reverse and forward directions, respectively [2].In TSR, only first order reflections are used to determine the voltage reflection coefficient Γ.In this paper, TSR has been expanded to include the case of a cascade of CCITLs with small discontinuities in a transformer made of several sections.

2. Motivation

Impedance matching is an extremely useful application of transmission lines. It is necessary in order to avoid reflections of the signal from the load and interfere with the originally transmitted signal [3]. It is also important to ensure that the power delivered to the load is as high as possible. Impedance matching techniques such as single stub matching and quarter wave transformers do not have sufficient bandwidth to support modern applications. Hence, multi-section matching transformers that rely on TSR are used. The use of CCITLs in multi-section impedance matching transformers motivates the extension of TSR to CCITLs.

3. Body

We only consider CCITls with passive characteristic impedances, that are also terminated by passive loads.Hence, we have:

$$Z_0^{\pm} = |Z_0| e^{\mp j\phi_0} \quad \text{where} \quad -90^o \leq \phi_0 \leq 90^o \tag{1}$$

where $|Z_o|$ is the absolute value of Z_0^-, and ϕ_0 is its phase.
Consider a CCITL of length l terminated by a passive impedance Z_L, with propagation constants β_0^+ and β_0^- and conjugate characteristic impedances Z_0^- and Z_0^+. For this CCITL, we can write:

$$\tilde{\beta_0} = \frac{\beta_0^+ + \beta_0^-}{2} \quad \text{and} \quad \Gamma_L = \frac{Z_L Z_0^- - Z_0^+ Z_0^-}{Z_L Z_0^+ + Z_0^+ Z_0^-} \tag{2}$$

$$Z_{in} = Z_0^+ Z_0^- \frac{1 + \Gamma_L e^{-j2\tilde{\beta}_0 l}}{Z_0^- - Z_0^+ \Gamma_L e^{-j2\tilde{\beta}_0 l}} \tag{3}$$

Now consider the CCITL transformer shown in Fig. 1.
Based on (2), and (3), with Z_1 instead of Z_0, we can write:

$$Z_{in} = Z_1^+ Z_1^- \frac{1 + \Gamma_1 e^{-j2\tilde{\theta}}}{Z_1^- - Z_1^+ \Gamma_1 e^{-j2\tilde{\theta}}} \tag{4}$$

$$\tilde{\beta_1} = \frac{\beta_1^+ + \beta_1^-}{2} \quad \text{and} \quad \Gamma_1 = \frac{Z_L Z_1^- - Z_1^+ Z_1^-}{Z_L Z_1^+ + Z_1^+ Z_1^-} \quad \text{and} \quad \tilde{\theta} = \tilde{\beta_1} l \tag{5}$$

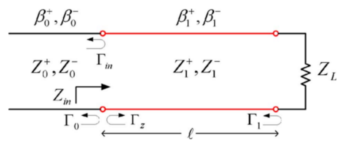

Fig. 1. Single section CCITL based transformer

Furthermore, using (2) at the $Z_0 - Z_1$ junction we also have:

$$\Gamma_{in} = \frac{Z_{in}Z_0^- - Z_0^+ Z_0^-}{Z_{in}Z_0^+ + Z_0^+ Z_0^-} \tag{6}$$

Substituting Z_{in} given by (4) into (6), and using the partial reflection coefficients Γ_0, Γ_z gives:

$$\Gamma_{in} = \Gamma_0 \frac{1 + \Gamma_x^{-1}\Gamma_1 e^{-j2\tilde{\theta}}}{1 - \Gamma_z \Gamma_1 e^{-j2\tilde{\theta}}} \quad \text{where} \quad \Gamma_x = \frac{Z_1^+ Z_1^- - Z_0^+ Z_1^-}{Z_1^+ Z_1 - + + Z_0^+ Z_1^+} \tag{7}$$

All of this now gives the TSR for Single Section CCITL Transformer:

$$\text{If} \quad Z_1^+ \approx Z_L \; \& \; Z_1^- \approx Z_0^- \implies |\Gamma_1 \Gamma_z| << 1, \quad \text{then} \quad \Gamma_{in} \approx \Gamma_0 + \frac{\Gamma_0}{\Gamma_x}\Gamma_1 e^{-j2\tilde{\theta}} \tag{8}$$

Hence, the above TSR statement for single section says that if the discontinuities in the transformer are small, then TSR can be applied to CCITL based transformer as in the equation shown.

4. Novelty of the work

The TSR statement derived can be very easily extended in a single step to obtain the TSR statement for a multi-section transformer. The paper [1] also discusses a mathematical approximation for TSR called the Approximate theory of small reflections(ATSR), but we not discuss these in detail, since they can be easily derived from (8). Numerical results show that TSR matches extremely well with the exact reflection coefficient for a multi-section CCITL transformer. Hence, this study can be used in the implementation of multi-section for band pass responses (e.g., Binomial or Chebyshev) for higher frequencies using CCITLs [4].

5. Future directions of this work

The work in this paper [1] has been used to develop a method for designing multi-section transformers for multiband applications [5], and for the design of T-matching networks for Reciprocal Conjugately Characteristic-Impedance Transmission Lines [6] (CCITLs for which signal transmission doesn't depend on direction). The theory of small reflections could also possibly be extended to bi-characteristic-impedance transmission lines(BCITLs).

References

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