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HILBERT TRANSFORM

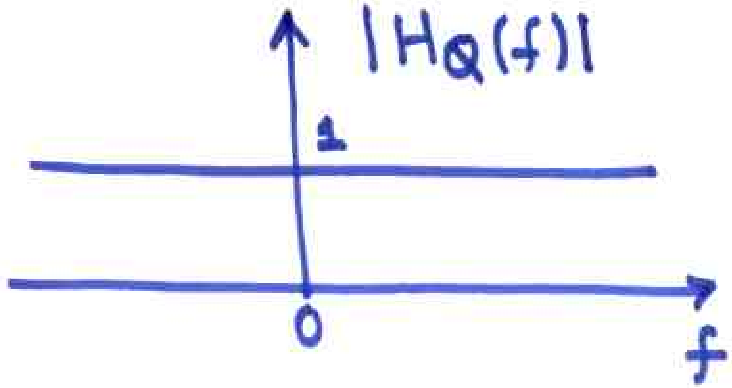
AND

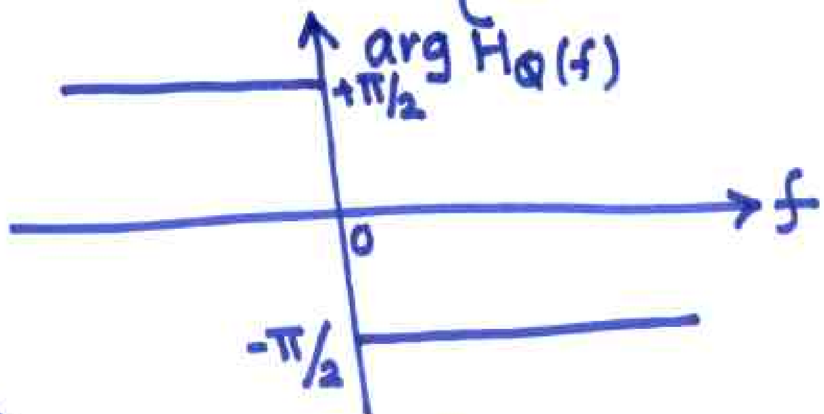
SINGLE SIDE BAND (SSB) MODULATION

QUADRATURE FILTER

All pass network

$$H_Q(f) = -j\,\mathrm{sgn}(f) = \begin{cases} -j & f>0 \\ +j & f<0 \end{cases}$$





$$\mathrm{sgn}(t) \longleftrightarrow \frac{2}{j2\pi f}$$

$$h_Q(t) = \frac{1}{\pi t}$$

$$\frac{1}{\pi t} \longleftrightarrow -j\,\mathrm{sgn}(f)$$

$$m(t) * h_Q(t) = m_h(t) \leftarrow \text{Hilbert Transform}$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\alpha)}{(t-\alpha)} \, d\alpha$$

$$M_h(f) = -j\,\mathrm{sgn}(f)\, M(f)$$

<u>Example:</u> (i) $\cos 2\pi f_m t = m(t)$

$$m_h(t) = \cos(2\pi f_m t - \pi/2)$$

$$= \sin 2\pi f_m t$$

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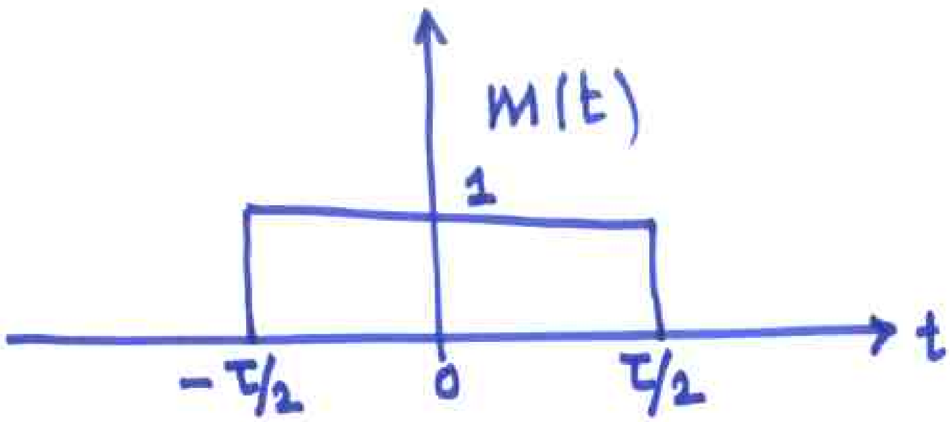
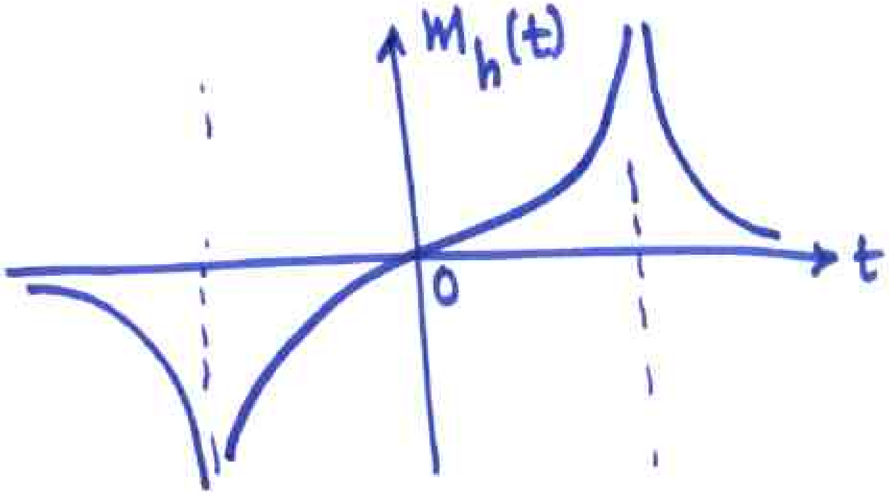

m(t)
1
t
−τ/2
0
τ/2


m_h(t)
t
0

PROPERTIES OF HT

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(1) $m(t)$ and $m_h(t) \rightarrow$ same amplitude spectrum

Energy or power in a signal & its HT are the same

(2) $m_h(t)$ is the HT of $m(t)$

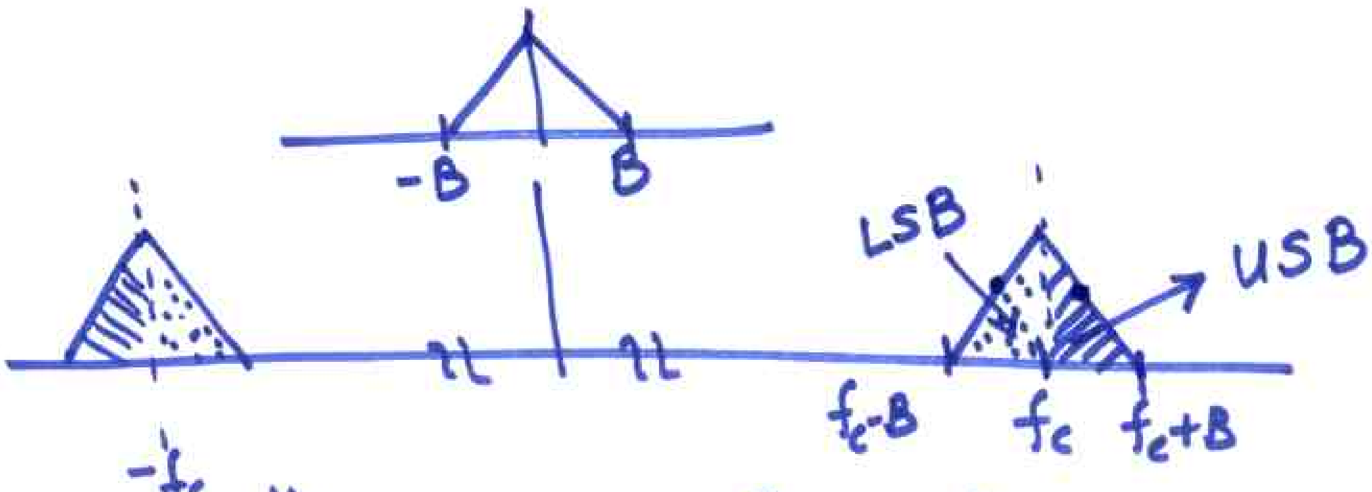
$-m(t)$ is the HT of $m_h(t)$

(3) $$\int_{-\infty}^{\infty} m(t)\, m_h(t)\, dt = 0$$ for energy signal

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} m(t)\, m_h(t)\, dt = 0$$ for power signal

$m(t)$ and $m_h(t)$ are orthogonal

SSB



"Selective fading"

ADVANTAGES OF SSB

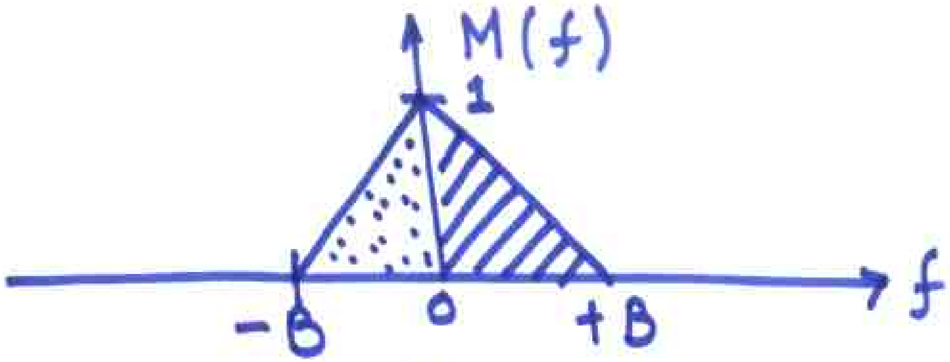
(1) Bandwidth saving

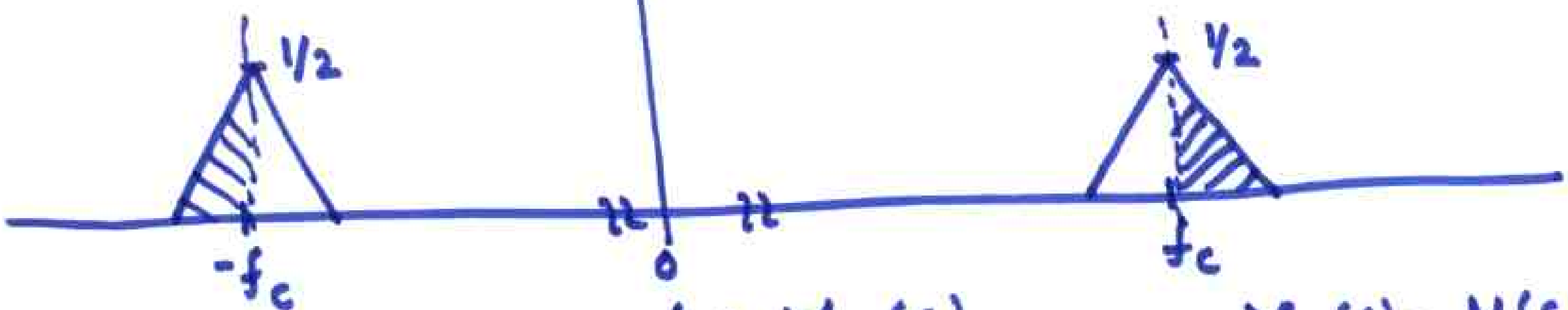
(2) Power saving

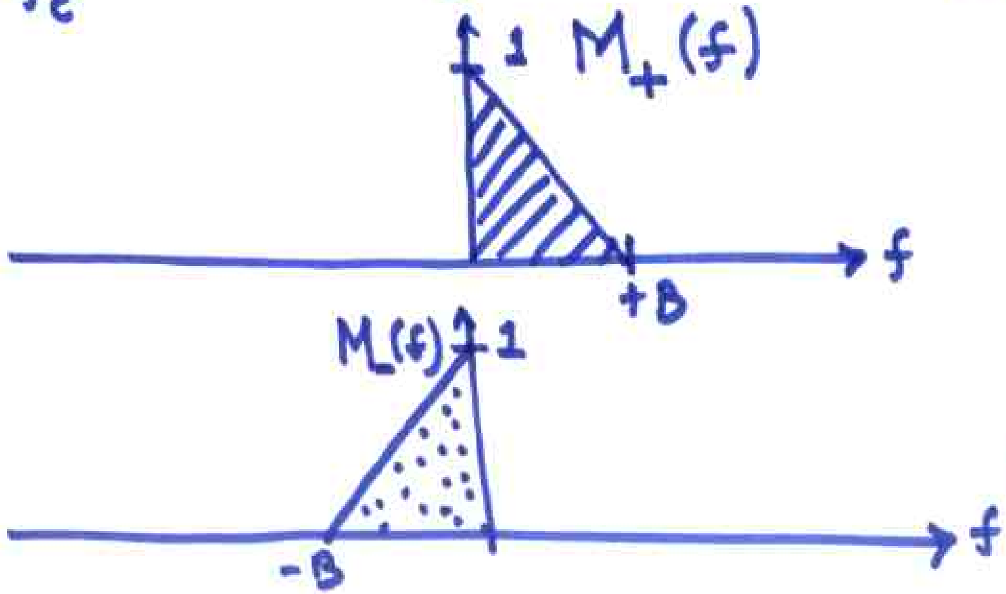
(3) Reduction of noise in the signal

(4) Selective fading of an SSB signal over long distances is REDUCED

↳ Different frequencies are affected in slighty different ways by the ionosphere and upper atmosphere, which have a great influence on radio signals of less than about 50 MHz

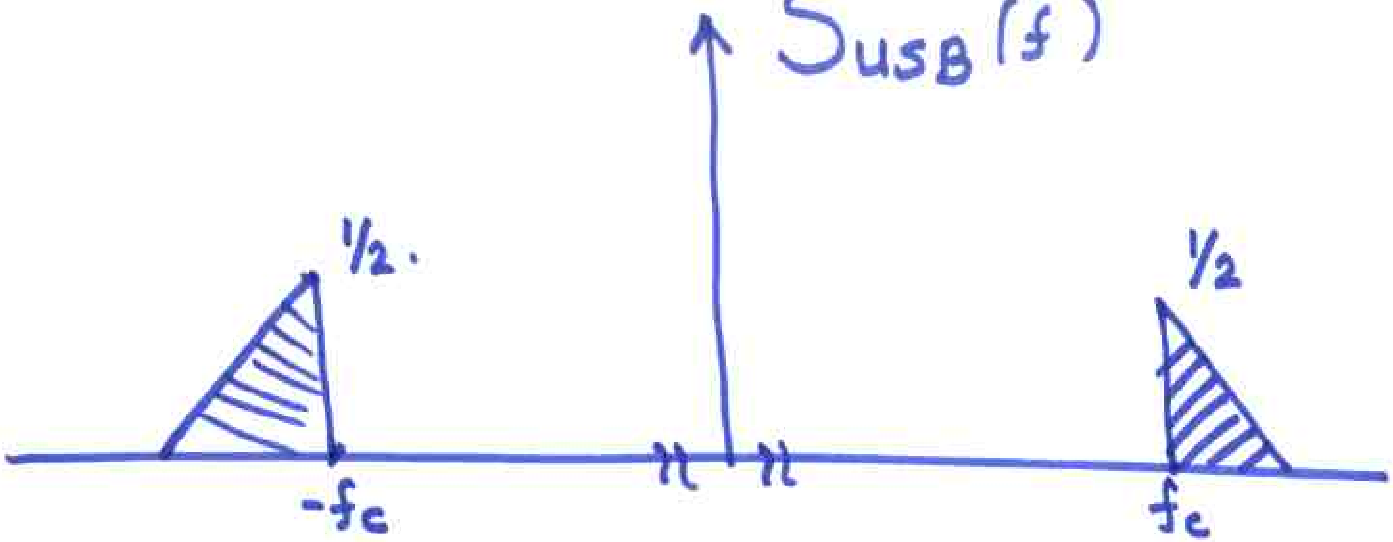






$$M_+(f) = M(f)\,u(f)$$
$$= M(f)\frac{\left[1+\mathrm{sgn}(f)\right]}{2}$$
$$= \frac{1}{2}\left[M(f) + jM_h(f)\right]$$

$$M_-(f) = \frac{1}{2}\left[M(f) - jM_h(f)\right]$$



$$S_{USB}(f) = \frac{1}{2}\left[M_+(f-f_c) + M_-(f+f_c) \right]$$

$$= \frac{1}{4}\left[M(f-f_c) + jM_h(f-f_c) + M(f+f_c) - jM_h(f+f_c) \right]$$

$$= \frac{1}{4}\left[M(f-f_c) + M(f+f_c) \right] - \frac{1}{4j}\left[M_h(f-f_c) - M_h(f+f_c) \right]$$

$$S_{USB}(t) = \frac{1}{2} m(t)\cos 2\pi f_c t - \frac{1}{2} m_h(t) \sin 2\pi f_c t$$

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$$s_{LSB}(t) = \frac{1}{2} m(t)\cos 2\pi f_c t + \frac{1}{2} m_h(t)\sin 2\pi f_c t$$

$$s_{SSB}(t) = \frac{1}{2}\left[m(t)\cos 2\pi f_c t \mp m_h(t)\sin 2\pi f_c t\right]$$

$- \rightarrow$ USB

$+ \rightarrow$ LSB