Homework 1

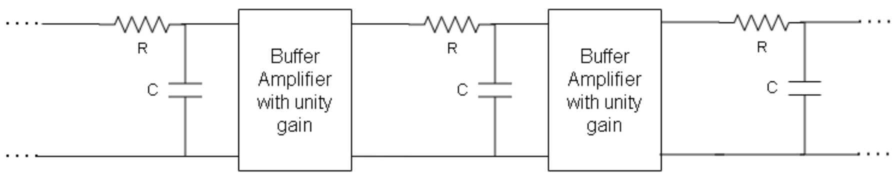
Communication Systems I (EE 341), Autumn'21

1) The following problems from the book "Communication Systems" by S. Haykin and M. Moher, 5th edition, Chapter 2: 2.1 to 2.13 and 2.15 on pp. 70-72.
2) Evaluate the Fourier transform $G(f)$ of the functions below:
   a) $g(t) = \cos(2\pi f_c t)\,\mathrm{rect}\left(\frac{t}{T}\right)$, where $\mathrm{rect}(t) = \begin{cases} 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$
   b) $g(t) = \cos(2\pi f_c t)\,u(t)$, where $u(t) = \begin{cases} 1 & t > 0 \\ 0, & \text{otherwise} \end{cases}$
   c) $g(t) = \exp(-\alpha t)\cos(2\pi f_c t)\,u(t)$, where $u(t)$ is a unit step function and $\alpha > 1$
   d) $g(t) = \mathrm{rect}\left(\frac{t}{T}\right) * \mathrm{rect}\left(\frac{t}{T}\right)$
   e) $g(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right)$
   f) $g(t) = \mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right) - \mathrm{rect}\left(\frac{t+\frac{T}{2}}{T}\right)$
3) Determine the inverse Fourier transform of the following frequency domain functions:
   a) $G(f) = \exp\left(-\frac{f^2}{2\sigma^2}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$
   b) $G(f) = -\cos\left(\pi\frac{f}{2B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$
   c) $G(f) = \begin{cases} 1e^{-j2\pi f t_o} & |f| \leq B \\ 0, & \text{otherwise} \end{cases}$
   d) $G(f) = \frac{1}{(2\pi f)^2}\left(e^{j2\pi f} - j2\pi f e^{j2\pi f} - 1\right)$
4) Consider the function $g(t) = \delta(t + \frac{1}{2}) - \delta(t - \frac{1}{2})$,
   a) What does the integration of $g(t)$ w.r.t time yield ?
   b) Using the below relationship
      $\int_{-\infty}^{t} g(\tau)d\tau \longleftrightarrow \frac{1}{j2\pi f}G(f) + \frac{1}{2}G(0)\delta(f)$
      Obtain the Fourier transform of the output of the integrator in a) above.

5) Find the Fourier transform of a general periodic signal $g_{T_0}(t)$ of period T_0 (Your answer should include the Fourier transform of $g(t)$ which is just equal to one period of the signal $g_{T_0}(t)$).

6) a) Determine the overall amplitude response of the cascade connection shown in the figure below consisting of 'N' identical stages, each with a time constant RC equal to τ_0.

b) Show that as N $\rightarrow \infty$, the amplitude response of the cascade connection $\rightarrow$ exp($\frac{-1}{2} f^2 T^2$), where for each value of N, $\tau_0^2 = \frac{T^2}{4\pi^2 N}$ is satisfied.



7) Let $y(t) = \int\limits_{t-T}^{t} g(\tau) d\tau$ denote the output of an LTI system with $g(t)$ as its input. Determine the frequency response of this LTI system.

8) Obtain the spectral density, autocorrelation, and signal energy
   a) $g(t) = A\text{rect}\left(\frac{t-t_0}{T}\right)$
   b) $g(t) = Ae^{-\alpha t}u(t)$
   c) $g(t) = A_0 + A_1 \sin(2\pi f_0 t + \varphi)$
9) Obtain the autocorrelation of $g(t) = Au(t)$. Use your result to find the signal power and spectral density.
10) Estimate the essential bandwidth B (in Hz) of the signal $e^{-\alpha t}u(t)$ if the essential band is required to contain 95% of the signal energy.
11) The power spectral density (PSD) of a signal is given by:

$$H(f) = \begin{cases} 1, & 0 \leq |f| \leq a, \\ \frac{b-|f|}{b-a}, & a < |f| \leq b, \\ 0, & |f| > b, \end{cases}$$

where $0 < a < b$. Find the:

(a) absolute bandwidth,
(b) 3-dB bandwidth,
(c) equivalent noise bandwidth,
(d) root-mean-square (RMS) bandwidth,
(e) null-to-null bandwidth,
(f) 50 dB bounded spectrum bandwidth, and
(g) power bandwidth

of the signal.