

Logic Optimization

Heuristic Based

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EE-677: Foundations of VLSI CAD



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CADSL

Logic Minimization

(f) Minimum \Rightarrow (\bar{F}) \leftarrow Cover
Min terms.
 \checkmark ON set / DC set \checkmark
should not cover OFF-set

(PI)
 F
 \uparrow
check

Heuristic logic minimization

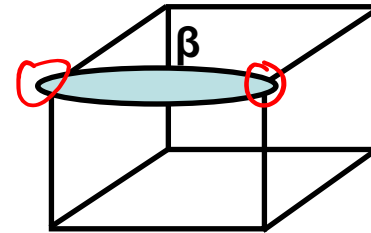
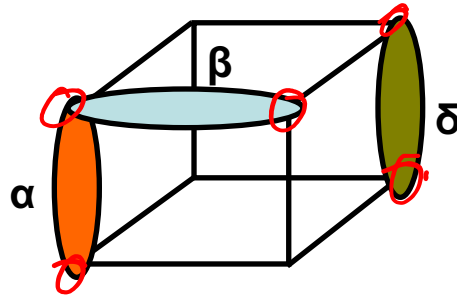
- Provide irredundant covers with “reasonably small” sizes
- Fast and applicable to many functions → *EXPRESSO - EXACT*
– Much faster than exact minimization
{ QM - set of PI
• Logic, ILP
- Avoid bottlenecks of exact minimization
– Prime generation and storage
– Covering ✓
- Motivation
– Use as internal engine within multi-level synthesis tools



Minimal or irredundant cover

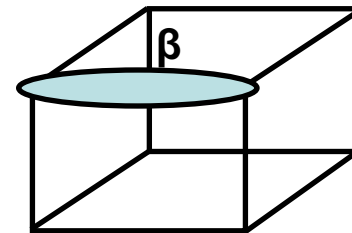
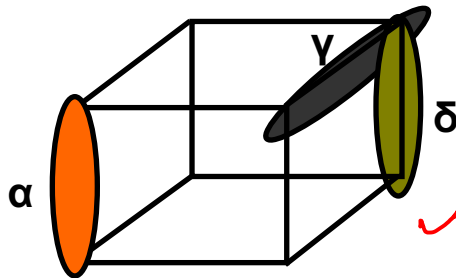
- $f_1 = a'b'c' + a'b'c + ab'c + abc + abc'$; $f_2 = a'b'c + ab'c$

Minimum cover



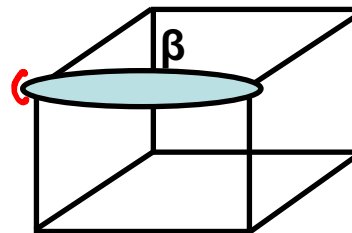
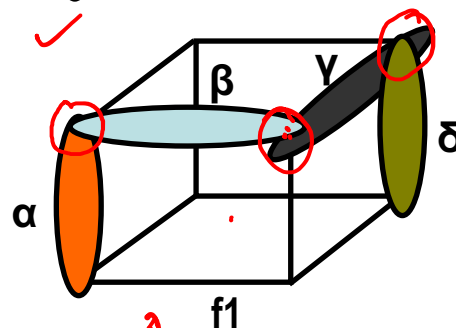
α, β, γ

Irredundant cover



$\alpha, \beta, \gamma, \delta$

Minimal cover w.r. to single implicant containment



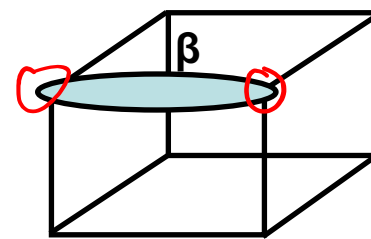
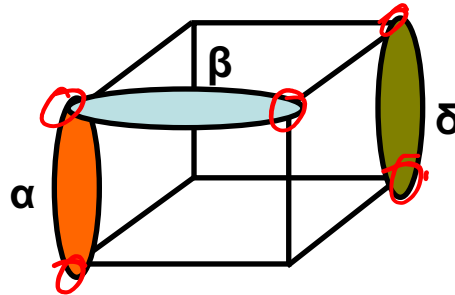
$\alpha, \beta, \gamma, \delta$



Minimal or irredundant cover

- $f_1 = a'b'c' + a'b'c + ab'c + abc + abc'$; $f_2 = a'b'c + ab'c$

Minimum cover



α, β, γ

Minimal cover or irredundant cover

Cover of the function that is not a proper superset of another cover

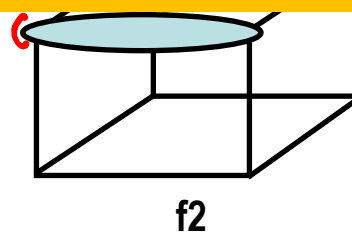
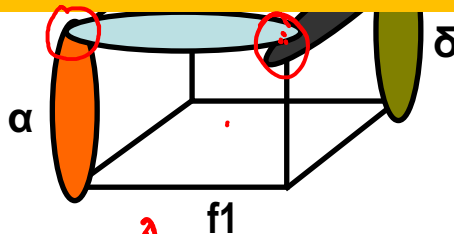
~~No implicant can be dropped~~ ✓

Local optimum ✓

$\alpha, \beta, \gamma, \delta$

$\alpha, \beta, \gamma, \delta$

Minimal cover w.r. to single implicant containment



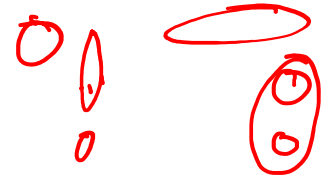
Heuristic minimization -- principles

- Start from initial cover F_{init}
 - Provided by designer or extracted from hardware language model
- Modify cover under consideration
 - Make it prime and irredundant \Leftarrow
 - Perturb cover and re-iterate until a small irredundant cover is obtained
- Typically the size of the cover decreases
 - Operations on limited-size covers are fast



Heuristic minimization - operators

$abcd' \rightarrow abcd' \downarrow$
 $abc*$ expansion



- Expand

- Make implicants prime
- Removed covered implicants

- Reduce

- Reduce size of each implicant while preserving cover

- Reshape

- Modify implicant pairs: enlarge one and reduce the other

- Irredundant

- Make cover irredundant



Example

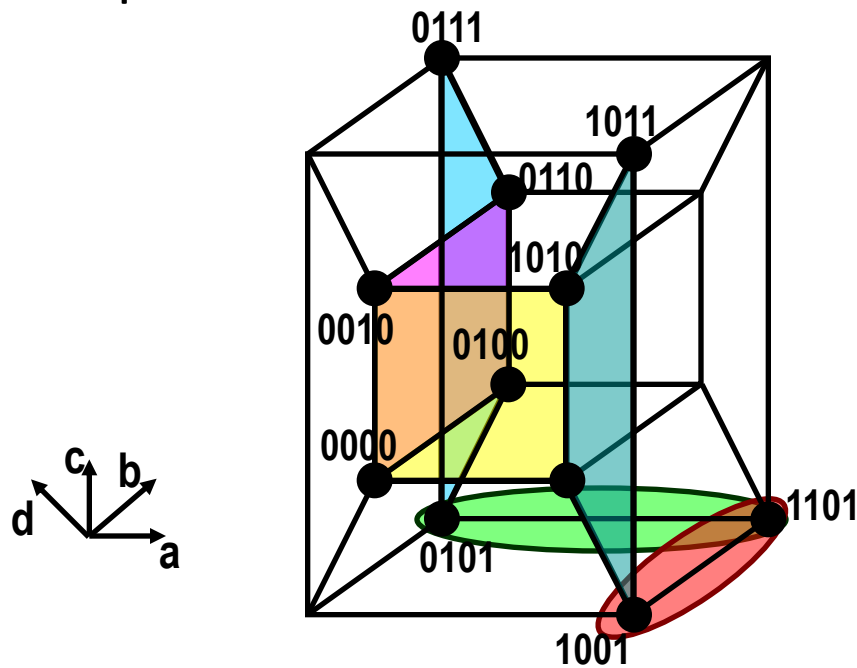
- Initial cover
 - (without positional cube notation)

	a	b	c	d	
0	0	0	0	0	1
2	0	0	1	0	1
4	0	1	0	0	1
6	0	1	1	0	1
8	1	0	0	0	1
	1	0	1	0	1
	0	1	0	1	1
	0	1	1	1	1
	1	0	0	1	1
	1	0	1	1	1
	1	1	0	1	1



Example

- Set of primes

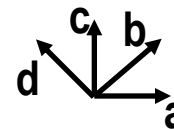
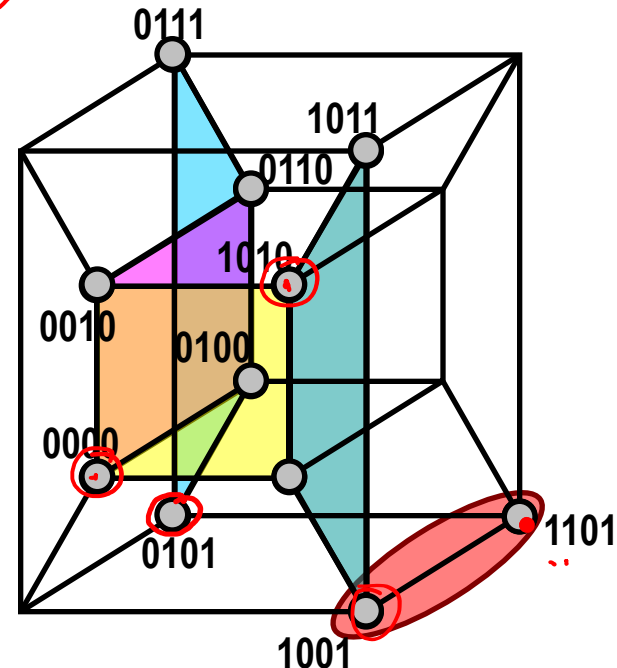


α	0 * * 0	1
β	* 0 * 0	1
γ	0 1 * *	1
δ	1 0 * *	1
ϵ	1 * 0 1	1
ζ	* 1 0 1	1

$\rightarrow \bar{a}\bar{d}, \bar{b}\bar{d}$

Example of expansion

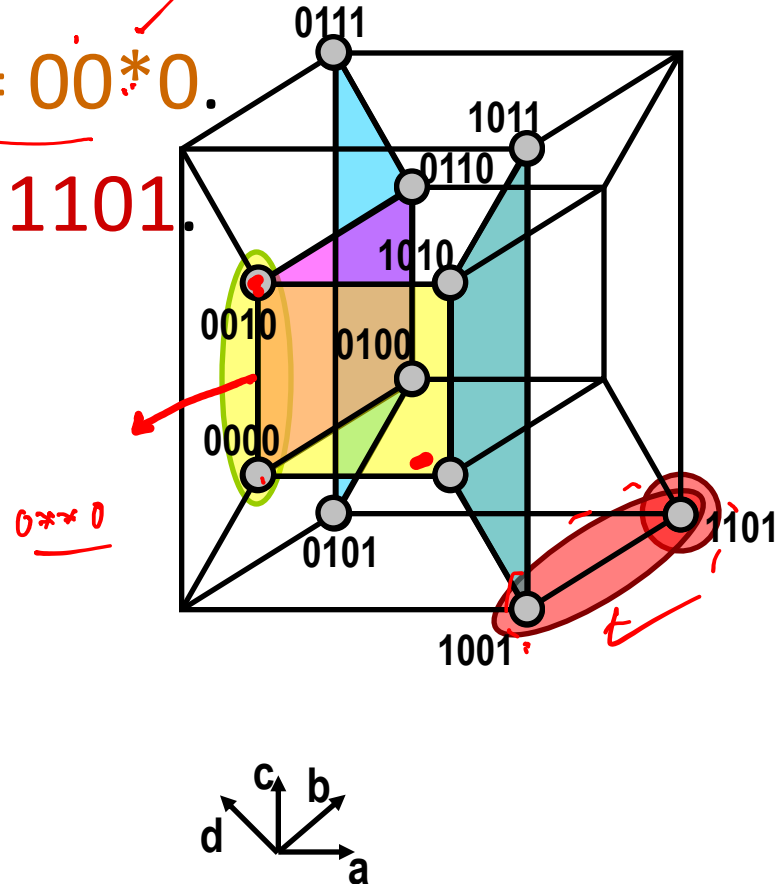
- Expand 0000 to $\alpha = 0^{**}0$.
– Drop 0100, 0010, 0110 from the cover. ✓
- Expand 1000 to $\beta = *0^{**}$. $\bar{b}\bar{d}$
– Drop 1010 from the cover.
- Expand 0101 to $\gamma = 01^{**}$.
– Drop 0111 from the cover.
- Expand 1001 to $\delta = 10^{**}$.
– Drop 1011 from the cover.
- Expand 1101 to $\epsilon = 1^{*}01$.
- Cover is: $\{\alpha, \beta, \gamma, \delta, \epsilon\}$. ✓



Example of reduction

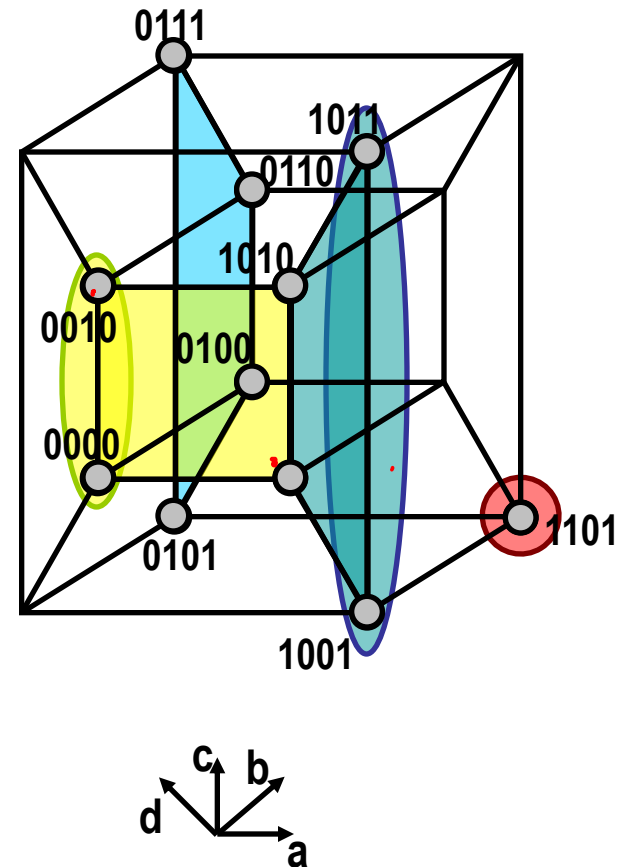
Irredundant

- Reduce $0^{**}0$ to nothing. ✓
- Reduce $\beta = *0*0$ to $\beta' = 00^{*}0$. ✓
- Reduce $\varepsilon = 1*01$ to $\varepsilon' = 1101$. ✓
- Cover is: $\{\beta', \gamma, \delta, \varepsilon'\}$.
↑



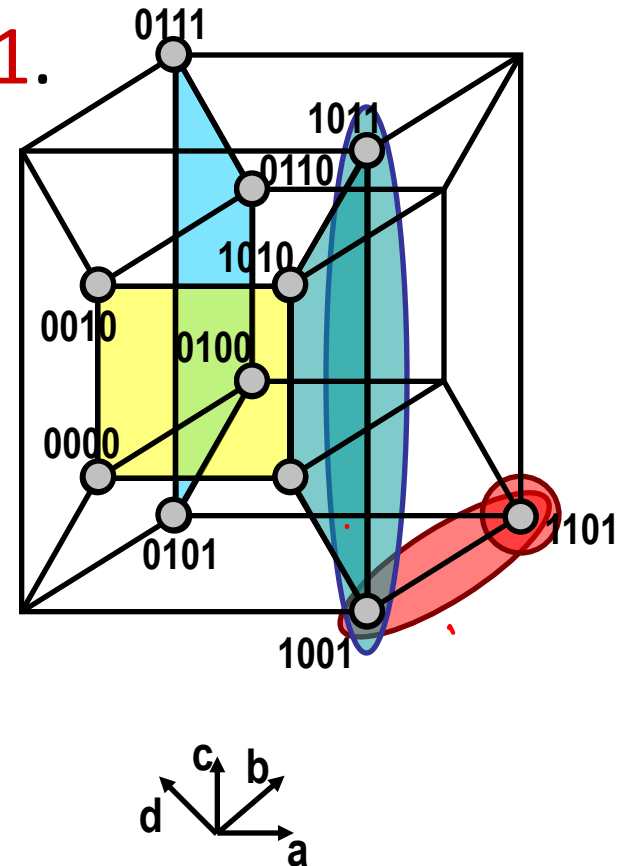
Example of reshape

- Reshape $\{\beta', \delta\}$ to: $\{\beta, \delta'\}$.
 - Where $\delta' = 10*1$.
- Cover is: $\{\beta, \gamma, \delta', \varepsilon'\}$.



Example of second expansion

- Expand $\delta' = 10^*1$ to $\delta = 10^{**}$.
- Expand $\varepsilon' = 1101$ to $\varepsilon = 1^*01$.



4 terms'

Exp



Example

Summary of the steps taken by MINI

- Expansion:
 - Cover: $\{\alpha, \beta, \gamma, \delta, \epsilon\}$.
 - Prime, redundant, minimal w.r. to scc.
- Reduction:
 - α eliminated.
 - $\beta = *0*0$ reduced to $\beta' = 00*0$.
 - $\epsilon = 1*01$ reduced to $\epsilon' = 1101$.
 - Cover: $\{\beta', \gamma, \delta, \epsilon'\}$. ✓
- Reshape:
 - $\{\beta', \delta\}$ reshaped to: $\{\beta, \delta'\}$ where $\delta' = 10*1$.
- Second expansion:
 - Cover: $\{\beta, \gamma, \delta, \epsilon\}$. ✓
 - Prime, irredundant.

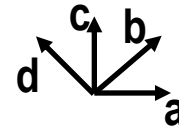
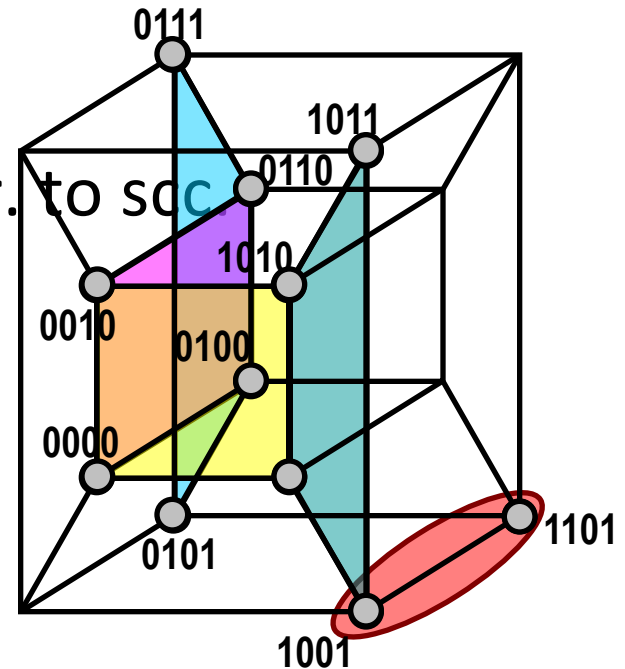
irredundancy



Example

Summary of the steps taken by ESPRESSO

- Expansion:
 - Cover: $\{\alpha, \beta, \gamma, \delta, \epsilon\}$.
 - Prime, redundant, minimal w.r. to soc
- Irredundant:
 - Cover: $\{\beta, \gamma, \delta, \epsilon\}$.
 - Prime, irredundant.



$f = \beta \gamma \delta \epsilon$
 $f = \alpha \beta \gamma \delta \epsilon$
 $f = \alpha \beta \gamma \delta \epsilon$

$\alpha, \beta, \gamma, \delta, \epsilon$



Rough comparison of minimizers

- MINI
 - Iterate EXPAND, REDUCE, RESHAPE ✓
- Espresso
 - Iterate EXPAND, IRREDUNDANT, REDUCE ✓✓
- Espresso guarantees an irredundant cover
 - Because of the irredundant operator
- MINI may return irredundant covers, but can guarantee only minimality w.r.to single implicant containment



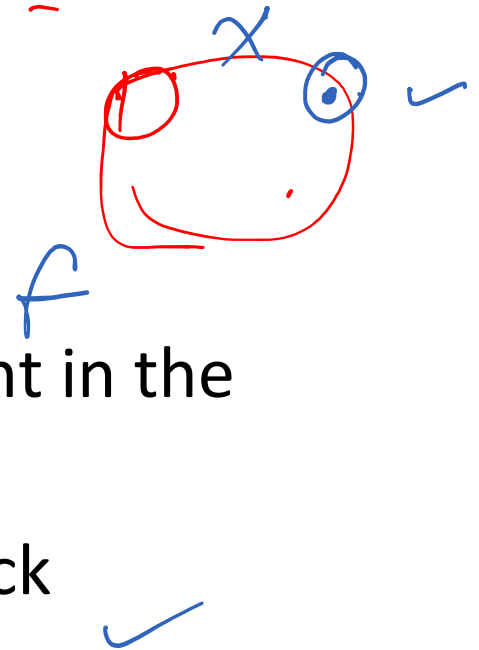
Expand: Naïve implementation

- For each implicant
 - For each care literal ✓
 - Raise it to don't care if possible
 - Remove all implicants covered by expanded implicant
- Issues
 - Validity check of expansion
 - Order of expansion



Validity check

- Espresso, MINI
 - Check intersection of expanded implicant with OFF-set
 - Requires complementation
- Presto
 - Check inclusion of expanded implicant in the union of the ON-set and DC-set
 - Reducible to recursive tautology check



FFFLZQ.

Thank You

