EE709 HW CNF 19D070052

1) We know that product of maxterms is a valid cnf, so we simply use that. Since all expressions have around 3 variables, the POM won't be too large $\Rightarrow$ no intermediate variables are needed.

a) $x_1 \cdot x_2 + \bar{x}_1 \cdot x_2$:

x_1	x_2	y	
0	0	0	✓
0	1	1	
1	0	0	✓
1	1	1	

$$y = (x_1 + x_2) \cdot (\bar{x}_1 + x_2)$$

b) $x_1 \oplus x_2 \oplus x_3 \oplus x_3$:

x_1	x_2	x_3	y	
0	0	0	0	✓
0	0	1	0	✓
0	1	0	1	
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	0	✓
1	1	1	0	✓

$$y = (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \bar{x}_3) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

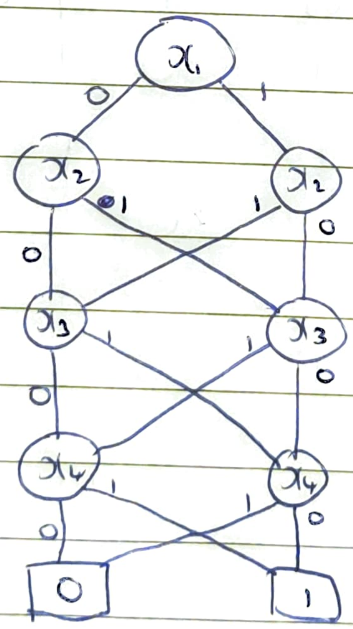
c) $(x_1 + x_2 \cdot x_3) \cdot (\bar{x}_1 + x_2 \cdot x_3)$:

x_1	x_2	x_3	y	
0	0	0	0	✓
0	0	1	0	✓
0	1	0	0	✓
0	1	1	1	
1	0	0	0	✓
1	0	1	0	✓
1	1	0	0	✓
1	1	1	1	

$$y = (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + x_3) \cdot (\bar{x}_1 + x_2 + x_3)$$
$$\cdot (\bar{x}_1 + x_2 + \bar{x}_3) \cdot (\bar{x}_1 + \bar{x}_2 + x_3)$$

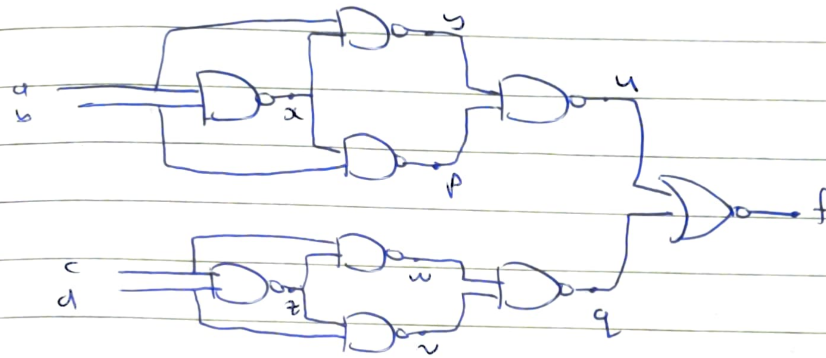
2) To convert ROBDD to CNF, we can look at all the paths that lead to $\boxed{0}$, and use their maxterm form (like in q1) to come up with the CNF.

$x_1 \oplus x_2 \oplus x_3 \oplus x_4$:



$$y = (x_1 + x_2 + x_3 + x_4) \cdot (x_1 + x_2 + \bar{x}_3 + \bar{x}_4) \cdot (x_1 + \bar{x}_2 + \bar{x}_3 + x_4) \cdot (x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$
$$\cdot (\bar{x}_1 + x_2 + x_3 + \bar{x}_4) \cdot (\bar{x}_1 + x_2 + \bar{x}_3 + x_4) \cdot (\bar{x}_1 + \bar{x}_2 + x_3 + x_4)$$
$$\cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$

3)



$x = \overline{a\cdot b}$

$y = \overline{x\cdot a} = \overline{\overline{a\cdot b}\cdot a} = a\cdot b + \bar{a} = b + \bar{a}$

$p = \overline{x\cdot b} = a\cdot b + \bar{b} = a + \bar{b}$

$u = \overline{y\cdot p} = \overline{(a+\bar{b})\cdot(b+\bar{a})} = \bar{a}\cdot b + \bar{b}\cdot a = a\oplus b$

or $= \overline{(\bar{a}+b)\cdot p} = a\cdot\bar{b} + \bar{p}$

$z = \overline{c\cdot d}$

$w = d + \bar{c}$

$v = \bar{d} + c$

$q = c\oplus d = (c+d)(\bar{c}+\bar{d})$

$f = \overline{a\oplus b + c\oplus d}$

or $\overline{(a\cdot\bar{b} + \bar{p}) + c\oplus d} = (\overline{a\cdot\bar{b}})\cdot p\cdot \overline{c\oplus d}$

$= (\bar{a}+b)\cdot p\cdot(c\cdot d + \bar{c}\cdot\bar{d})$

$= (\bar{a}+b)\cdot p\cdot(c+\bar{d})\cdot(\bar{c}+d)$

or $\overline{(a\oplus b) + q} = (a\cdot b + \bar{a}\cdot\bar{b})\cdot\bar{q}$

$= (a+\bar{b})(\bar{a}+b)\cdot\bar{q}$

a) p-sa-0:

① a, b, c, d st. $p = 1$

② a, b, c, d st $\frac{\partial f}{\partial p} = f|_{p=0} \oplus f|_{p=1} = 1$

①: $p = 1 \iff (a + \bar{b})$

②: $f = (\bar{a} + \bar{b}) \cdot p \cdot (c + \bar{d}) \cdot (\bar{c} + d)$

$\therefore f|_{p=0} = 0$

$f|_{p=1} = (\bar{a} + \bar{b})(c + \bar{d})(\bar{c} + d)$

$\therefore \frac{df}{dp} = 0 \oplus f|_{p=1} = f|_{p=1}$

$\therefore$ ①·②: $(a + \bar{b})(\bar{a} + \bar{b})(c + \bar{d})(\bar{c} + d)$

From minisat:

$a = b = c = d = 0$ is a valid test

b) p-sa-1:

again, $\frac{df}{dp} = (\bar{a} + \bar{b})(c + \bar{d})(\bar{c} + d)$

but we want a, b, c, d st. $p = 0$

$\therefore \overline{(a + \bar{b})} = \bar{a} \cdot b$

$\therefore$ ①·② $= (\bar{a}) \cdot (b) \cdot (\bar{a} + \bar{b}) \cdot (c + \bar{d})(\bar{c} + d)$

From minisat:

~~unsatisfiable $\Rightarrow$ p-sa-1 isn't testable~~

~~$\Rightarrow$ p-sa-1 doesn't affect the circuit.~~

$a = c = d = 0$, $b = 1$ is a valid test.

c) q - sa-0:

① $q = 1 \iff c \oplus d = (c+d)(\bar{c}+\bar{d})$

② $\frac{df}{dq} = 1$

$f = (a+\bar{b})(\bar{a}+b) \cdot \bar{q}$

$\therefore f|_{q=1} = 0$

$f|_{q=0} = (a+\bar{b})(\bar{a}+b)$

$\therefore \frac{df}{dq} = (a+\bar{b})(\bar{a}+b)$

① · ② : $(a+\bar{b})(\bar{a}+b)(c+d)(\bar{c}+\bar{d})$

From minisat:

$a=b=d=0, c=1$ is a valid test.

d) q - sa-1:

② $\frac{df}{dq} = (a+\bar{b})(\bar{a}+b)$

① $q = 0 \iff \overline{c \oplus d} = (c+\bar{d})(\bar{c}+d)$

① · ② : $(a+\bar{b})(\bar{a}+b)(c+\bar{d})(\bar{c}+d)$

From minisat:

$a=b=c=d=0$ is a valid test.