Midsem solutions.

Q2 P_1 : $\min \|x\|_1$ s.t.

$$\|y - Ax\|_2^2 \leq \varepsilon$$

$y = Ax +$ noise.

$$\|x^* - x\|_2 \leq C_0 \|x - x_s\|_1 + C_1 \varepsilon$$

C_0 and C_1 are increasing functions of δ_{2s} — theorem 3.

Theorem 6 — C_0 & C_1 are incr. fns of δ_s.

5

Worst case rec. error is guaranteed to be lower if δ_s is lower.

→ If A is orthonormal,

$$\|Ax\|^2 = \|x\|^2$$

$$(1-\delta_s)\|x\|^2 \leq \|Ax\|^2 \leq (1+\delta_s)\|x\|^2$$

$$\longrightarrow (1-\delta_s)\|x\|^2 \leq \|x\|^2 \leq (1+\delta_s)\|x\|^2$$

5 $1 - \delta_s \leq 1 \leq 1 + \delta_s \longrightarrow \underline{\delta_s = 0}$

$$\delta_s \leq \mu(S-1) \longrightarrow \delta_s \leq 0$$

μ → mutual coherence of A

$= 0$ for othonormal matrix

$\rightarrow \delta_s = 0$
as δ_s cannot be negative.

Q3 Single particle cryoelectron microscopy = tomography problem with micrographs.

In SPR, micrograph contains particles with unknown shifts & orientations.

Orientation bias in some macromolecules.
→ ∴ fourier space is incompletely filled.
→ Poor reconstruction.

→ this problem is partly resolved by tilting the slide out of plane.

Q5 Laplacian eigenmaps.

$x_1, x_2, \ldots, x_Q$ are the tomographic projection vectors.

$$\{x_i\}_{i=1}^{Q} \longrightarrow \{y_i\}_{i=1}^{Q}$$

$$\forall i,\ x_i \in R^N \quad (N \text{ bins})$$

$$y_i \in R^2$$

$$\theta_i = \tan^{-1}(y_{1i}/y_{2i})$$

$\longrightarrow$ sorting of projections.

smallest angle $\longrightarrow 0$.

2nd smallest $\longrightarrow \pi/Q$

7

3rd smallest $\longrightarrow 2\pi/Q$

largest $\longrightarrow \pi(Q-1)/Q$.

This assignment is based on order statistics of the uniform distribution.

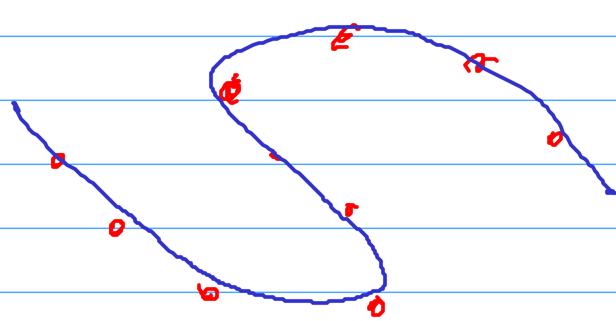
$Z_1, Z_2, \ldots Z_m \sim f_Z(\cdot)$

$E[\min\{Z_i\}]$ = 1st order statistic

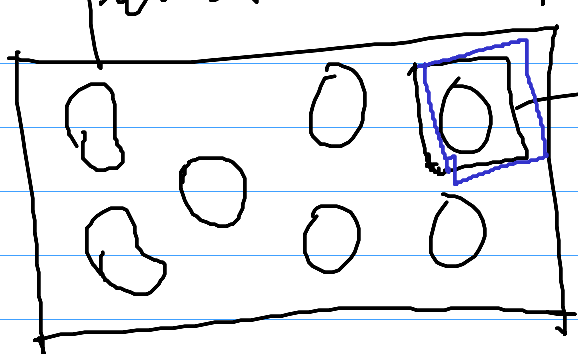
E[second smallest] = 2nd order statistic

Variance of each statistic $= O(1/a^2)$

3 If the distribution of angles is not uniform, but it is some other known distribution, then you use order stats of the other distribution.



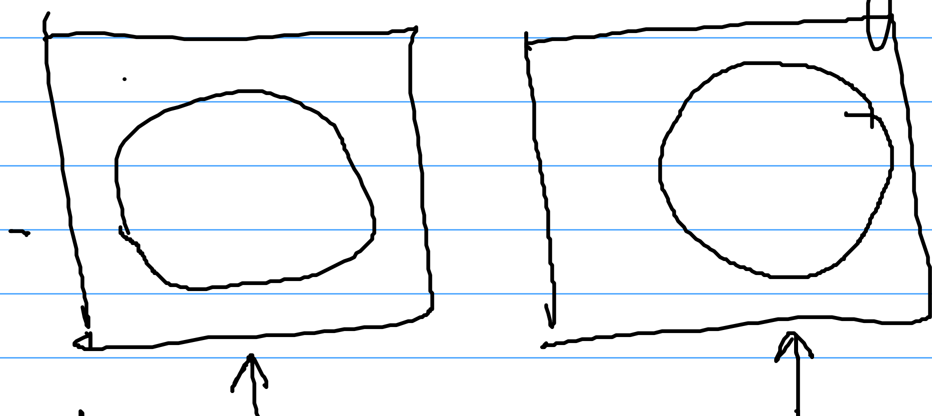
Q7 Why are the shifts of the particle images unknown?



particle picking algorithm.

Shift = difference between centroid of bounding box and centroid of actual particle.
→ 2D vector.

Translational ambiguity?



Yes it exists. Shift of macromolecule & sensor (camera) together does not change image. Hence this ambiguity exists.

Q4 $P1$: $\min \|x\|_1$ s.t. $\|y - Ax\|_2 \leq e$

$Q1$: $\min \|y - Ax\|_2$ s.t. $\|x\|_1 \leq t$

If $\bar{x}$ is a unique minimizer of P_1 for some e, then it is also a unique minimizer of Q_1 for some t.

Proof: Let $t = \|x\|_1$.

Consider $z \neq x$ s.t. $\|z\|_1 \leq t$.

Then $\|y - Az\|_2 > \|y - Ax\|_2$

otherwise the solution of P_1 would not be unique.

By definition of Q_1, x must be the unique minimizer of Q_1.

Q6 a CASSI matrix

$y = \Phi x$

$$\Phi = \underbrace{\left(C_1 \,|\, C_2 \,|\, \ldots \,|\, C_{N_\lambda} \right)}_{N_x N_y \times N_x N_y N_\lambda}$$

where each C_i has size

$N_x N_y \times N_x N_y$ (image size)

Each C_i is diagonal & non-negative.

The diagonals of each of the C_is are shifts of each other — due to the prism.

The binary pattern on the diagonals of each C_i is due to the aperture code.

Q6b Pooling matrix

- Sparse & binary → ease of pipetting

$A_{ij} = 1$ if j^{th} sample contributes of i^{th} pool and 0 otherwise

$A_{ij} = 1$ or 0

# of 1s in any column - should be small for ease of pipetting & to prevent dilution effects.

Q8 ISTA/OMP need $O(s \log n)$ measurements for successful recovery.

Brute force search over all possible triples requires only 2s measurements. Time complexity is $O(n^3)$ which is feasible.

Which triple is the best?

The one with smallest value of $\|y - A\hat{x}\|_2^2$

$$\underset{m\times 1}{y} = Ax = \underset{m\times 3}{A_S}\,\underset{3\times 1}{x_S}$$

S = support of x

for every S, obtain $\hat{x}_S$ using $A_S^{+} y$.

Choose S with least value of $\|y - A_S^{\dagger} \hat{x}_S\|$

Note $|S| = 3$.

In order to get accurate results with brute force search, you need $m \geq 6$.