2) The s-order restricted isometry constant (RIC) delta_s of a sensing matrix **A** is defined as the smallest non-negative real-valued scalar for which we have the following relationship for any s-sparse vector **x**: (1-delta_s) ||**x**||^2 <= ||**Ax**||^2 <= (1+\delta_s) ||**x**||^2 . From the point of view of worst-case upper bounds on the reconstruction error in compressed sensing using L1 norm optimization problems such as P1 (see definition of P1 in Q3 below), is it desirable to have a sensing matrix **A** with a larger value of the s-order RIC or a lower value of the s-order RIC? Justify. Also determine what is the s-order RIC of a *n* x *n* orthonormal matrix **A**. [5+5=10 points]

3) What is the advantage of cryo-electron tomography over single-particle cryo-electron microscopy reconstruction? [10 points]

4) Consider compressive measurements of the form **y** = **Ax** + **v** for sensing matrix **A**, signal vector **x**, noise vector **v** and measurement vector **y**. Consider the problem P1 done in class: Minimize ||**x**||_1 w.r.t. such that ||**y**-**Ax**||_2 <= e. Also consider the problem Q1: Minimize ||**Ax**-**y**||_2 w.r.t. **x** subject to the constraint ||**x**||_1 <= t. Prove that if **x** is a unique minimizer of P1 for some value e >= 0, then there exists some value t >= 0 for which **x** is also a unique minimizer of Q1. Note that ||**x**||_1 and ||**x**||_2 stand for the L1 and L2 norms of the vector **x** respectively.[10 points] (Hint: Consider t = ||**x**||_1 and now consider another vector **z** with L1 norm less than or equal to t).

5) Consider the tomography under unknown angles problem in 2D. How does the Laplacian eigenmaps based algorithm to solve this problem, make use of the fact that the distribution of the projection angles is uniform, in deciding the angle assignments? Recall that the angles themselves are unknown, but this algorithm assumes that the angle distribution is uniform. How would the angle assignment change if the known angle distribution weren't uniform, but some other known distribution? [7+3 = 10 points]

6) What are the physical constraints on the sensing matrix in (a) the CASSI camera for compressive hyperspectral image acquisition, and (b) pooled testing of COVID-19 samples? Explain briefly. [5+5=10 points]

7) Consider the problem of reconstruction of a 3D structure from N particle images in cryo-electron microscopy. We know that each particle image has an unknown orientation, represented by a 3D rotation matrix **Ri** for the i-th particle. What is the reason that the shifts of the particle images are also considered unknown? Are these shifts two-dimensional or three-dimensional? Why? Just like we have a global rotational ambiguity in cryo-em, do we also have a global translational ambiguity in cryo-em? Justify. [4+2+4=10 points]

8) Consider compressive measurements of the form **y** = **Ax** + **v** for sensing matrix **A**, signal vector **x**, noise vector **v** and measurement vector **y**. In addition, if you were told that exactly 3 elements in **x** were non-zero and the others were zero in value. You do not know the location of the non-zero elements of **x**. Describe an algorithm to determine **x** from **y**, **A**. Comment on how many measurements would be required for this algorithm to yield a good estimate of **x.** You may directly refer to algorithms done in class without describing them fully, but try to come up with an algorithm that requires as few samples in **y** as possible, given your knowledge of compressed sensing. [10 points]