2) Consider the Hitomi video compressed sensing camera architecture. Such a video camera acquires T snapshots where the t-th snapshot ($1 <= t <= T$) acquires the form

$$y_t = \sum_{a=1}^{F} x_{at} C_{at}$$

where F is the number of sub-frames that are modulated and add up to create a single snapshot, x_{at} is the a-th subframe ($1 <= a <= F$) that contributes to y_t and C_{at} is its corresponding binary modulation code. Explain, with an appropriate cost function with clearly defined symbol meanings, how dictionary learning can be used in the recovery of the underlying unknown subframes x_{at} from the snapshots y_t and the corresponding modulation codes C_{at}. Assume that the dictionary has been learned offline on representative data. Furthermore, using a suitable cost function or algorithm sketch with clearly defined symbol meanings, explain how the dictionary can also be inferred on the fly from y_t and C_{at}. [10 points]

3) Let **A** be a linear forward model that converts a m x n matrix to a vector of p elements. Let $\delta_r(\mathbf{A})$ be the r-restricted isometry constant of **A**. Let **Z** be a matrix of rank r such that $\mathbf{A}(\mathbf{Z}) = \mathbf{b}$ where **b** is a vector with p elements. If $\delta_{2r}(\mathbf{A}) < 1$, then show that **Z** is the only matrix of rank at the most r which satisfies $\mathbf{A}(\mathbf{Z}) = \mathbf{b}$. Recall that if r is an integer less than or equal to $\min(m,n)$, then the r-restricted isometry constant of matrix **A** is the smallest number $\delta_r(\mathbf{A})$ such that $(1-\delta_r(\mathbf{A})) \|\mathbf{X}\|_F <= \|\mathbf{A}(\mathbf{X})\|_2 <= (1+\delta_r(\mathbf{A}))\|\mathbf{X}\|_F$ for any matrix **X** of rank at the most **r**. [10 points]

4) Explain any one application of robust principal components analysis in image processing. Clearly state the physical meaning of the matrix and its low rank and sparse components. [10 points]

5) What are the advantages and disadvantages of an overcomplete dictionary over an orthonormal dictionary? [10 points]

6) Let $\boldsymbol{g}$ be the 1D Radon projection of a 2D image $\boldsymbol{f}$ in the direction **d**. Write an expression for the 1D Radon projection of a transformed image $a\boldsymbol{f} + b$ in the same direction **d**, in terms of a, b, $\boldsymbol{g}$ and z, where z denotes the Radon projections of a 2D image containing all ones. Here a and b are known scalars and the operation $a\boldsymbol{f} + b$ is performed pixelwise.
Let **D** be a dictionary used to sparsely represent a class of images. Then write an expression for the equivalent dictionary used to sparsely represent the Radon transforms of these images in the direction **d**. [5+5=10 points]

7) Explain briefly what is meant by "common lines" in cryo-electron microscopy. Given projections of a 3D structure in directions **d1** and **d2**, explain how the common line between them can be found out. [10 points]

8) At its core, we know that compressive sensing exploits the sparsity or compressibility of signal coefficients in some transform domain such as wavelet or DCT, for the purpose of signal reconstruction from undersampled measurements. However we also know that the signal coefficients exhibit a much stronger property than just sparsity – namely the power law, which says that on an average, the signal coefficient magnitude decreases with frequency. Write a cost function that in some form uses this power law for the sake of compressive reconstruction, with meanings of all symbols clearly explained. (Note: There is more than one "correct" answer to this question, and you need to write just one). [10 points]