Home-work/Quiz Boolean Formulas

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We will be considering Boolean formulas on n variables $x_1, x_2, ..., x_n$ and the corresponding Boolean functions $f : B_2^n \rightarrow B_2$.

1. Corresponding to every Boolean formula, there is a well defined Boolean function. Explain.

2. Consider the Boolean formula ($n = 3$):

   $$((x_1.(x_2 + x.3)) + ((\neg x_1).((\neg x_2) + (\neg x_3))))$$

   What is the Boolean function (truth-table) represented by this formula?

3. Corresponding to every Boolean formula, there is a well defined subset of B_2^n. Explain.

4. Prove the following equivalences between Boolean formulas.

   (a) $(x_1.(x_2 + x_3)) \equiv ((x_1.x_2) + (x_1.x3))$

   (b) $(x_1 + (x_2.x_3)) \equiv ((x_1 + x_2) + (x_1 + x3))$

   (c) $(\ (x_1 + x_2)) \equiv ((\ x_1).(\ x_2))$

   (d) $((x_1.x_2) + (x_1.(\ x_2))) \equiv x_1$

5. Let $n = 4$. Describe the subsets of B_2^4 defined by the following Boolean Formulas.

   (a) x_1

   (b) $x_1.x_2$

   (c) $x_1.x_2.x_3$

(d) $x_1.x_2.x_3.x_4$

(e) $x_1 + x_2$

(f) $x_1 + x_2 + x_3 + x_4$

6. For each of the formulas in the previous question illustrate the Shannon decomposition using variable x_1.

7. Let $f(x_1, x_2, \ldots, x_n)$ be a Boolean formula. Remember that we defined:

$$\exists_{x_1} f \;=\; f|_{x_1} + f|_{\overline{x_1}} \qquad \forall_{x_1} f \;=\; f|_{x_1}.f|_{\overline{x_1}}$$

Show that

$$\exists_{x_1} f = (\neg \forall_{x_1} (\neg f))$$

and

$$\forall x_1 f = (\neg \exists_{x_1} (\neg f))$$