Midsem: CS 754, Advanced Image Processing, 29th Feb

Instructions: There are 120 minutes for this exam. This exam is worth 10% of your final grade. Attempt all questions. Write brief answers. *Wherever necessary, please write equations with the meaning of all terms clearly stated. You can quote results/theorems done in class directly without proving/justifying them.* Each question carries 10 points.

1. Consider video compressive sensing using a Rice single pixel camera versus using a snapshot camera (i.e. the Hitomi architecture). List any three differences between these two architectures and/or the reconstruction procedures (note: a total of three differences). [5+5 = 10 points]

2. Consider that you wish to minimize the cost function $J(\boldsymbol{\theta}) \triangleq \|\boldsymbol{y} - \boldsymbol{A\theta}\|_2^2$ using the majorization minimization technique used in ISTA. Consider a majorizer function $M_k(\boldsymbol{\theta})$ at the k^{th} iteration. What are the criteria that M_k must satisfy? Show that $J(\boldsymbol{\theta}_{k+1}) \leq J(\boldsymbol{\theta}_k)$ where the subscripts stand for the iteration index, when you iteratively minimize the majorizer. [5+5=10 points]

3. State whether true or false and justify: In orthogonal matching pursuit for estimating sparse $\boldsymbol{\theta} \in \mathbb{R}^n$ from measurements of the form $\boldsymbol{y} = \boldsymbol{A\theta}$ where $\boldsymbol{y} \in \mathbb{R}^m, \boldsymbol{A} \in \mathbb{R}^{m \times n}; m \ll n$, the same column of the matrix $\boldsymbol{A}$ never gets selected in more than one iteration.

4. Let $\boldsymbol{\theta}^\star$ be the result of the following minimization problem: (P1)$\min \|\boldsymbol{\theta}\|_1$ such that $\|\boldsymbol{y} - \boldsymbol{\Phi\Psi\theta}\|_2 \leq \varepsilon$, where $\boldsymbol{y}$ is an m-element measurement vector of the form $\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{\eta}$, $\boldsymbol{\Phi}$ is a $m \times n$ measurement matrix ($m < n$), $\boldsymbol{\Psi}$ is a $n \times n$ orthonormal basis in which n-element signal $\boldsymbol{x}$ has a sparse representation of the form $\boldsymbol{x} = \boldsymbol{\Psi\theta}$. Note that ε is an upper bound on the magnitude of the noise vector $\boldsymbol{\eta}$.
   Theorem 3 we studied in class states the following: If $\boldsymbol{\Phi}$ obeys the restricted isometry property with isometry constant $\delta_{2s} < \sqrt{2} - 1$, then we have $\|\boldsymbol{\theta} - \boldsymbol{\theta}^\star\|_2 \leq C_1 s^{-1/2} \|\boldsymbol{\theta} - \boldsymbol{\theta_s}\|_1 + C_2 \varepsilon$ where C_1 and C_2 are functions of only δ_{2s} and where $\forall i \in \mathcal{S}, [\boldsymbol{\theta_s}]_i = \theta_i; \forall i \notin \mathcal{S}, [\boldsymbol{\theta_s}]_i = 0$. Here $\mathcal{S}$ is a set containing the s largest magnitude elements of $\boldsymbol{\theta}$.
   Also consider the ℓ_0-norm minimization problem: P0 :$\min \|\boldsymbol{\theta}\|_0$ such that $\|\boldsymbol{y} - \boldsymbol{\Phi\Psi\theta}\|_2 \leq \varepsilon$.
   Answer the following questions in the case when $\boldsymbol{\theta}$ is s-sparse and there is no noise in the measurements. [5+5=10 points]
   (a) Under what condition on δ_{2s} are the solutions of the P_0 and P_1 problems the same? Give a *very brief* justification.
   (b) Under what condition on δ_{2s} is the solution of P_0 unique? Give a *very brief* justification.

5. Refer to theorem 3 in the question above. It appears that the upper bound on $\|\boldsymbol{\theta} - \boldsymbol{\theta}^\star\|_2$ becomes tighter as s increases. This seems to go against the spirit of compressed sensing which states that sparser vectors are reconstructed better. Explain how this contradiction is resolved. [10 points]

6. Consider that you are given the Radon projections of an image $f(x, y)$ (defined on domain Ω), in directions $\theta_1, \theta_2, ..., \theta_K; K > 1$. *Without* reconstructing the image, state how you will infer the following properties of the image *directly* from the projections? [3+3+4=10 points]
   (a) $\sum_{(x,y) \in \Omega} f(x, y)$
   (b) A slice of the Fourier transform of f in direction θ_1 in the frequency plane and passing through the origin of the frequency plane
   (c) The order-2 moments of the image. Recall that an image moment of order k is any moment $\nu_{p,k-p}$ of order $(p, k - p)$ (where $p \leq k$) defined as $\nu_{p,k-p} = \sum_{(x,y) \in \Omega} x^p y^{k-p} f(x, y)$

7. Write down the objective function (with the meaning of each term clearly stated) for compressed sensing based, coupled tomographic reconstruction of three structurally similar 2D image slices $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ (of equal size) from their respective tomographic measurements $\boldsymbol{y_1} = \boldsymbol{R_1}\boldsymbol{x_1}, \boldsymbol{y_2} = \boldsymbol{R_2}\boldsymbol{x_2}, \boldsymbol{y_3} = \boldsymbol{R_3}\boldsymbol{x_3}$ where $\boldsymbol{R_1}, \boldsymbol{R_2}, \boldsymbol{R_3}$ are the respective Radon transform matrices. State the advantages of the coupled reconstruction over independent reconstruction. What would happen to the coupled reconstruction if $\boldsymbol{R_1} = \boldsymbol{R_2} = \boldsymbol{R_3}$? What would happen to the coupled reconstruction if the three slices are not structurally similar? [2.5+2.5+2.5+2.5=10 points]