Math 171 Midterm Examination

October 18, 2007

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Directions:

- 1. This is closed book/notes exam (only your single 8 1/2 by 11 inch information sheet allowed).
- 2. Your signature above indicates that you accept the University Honor Code.
- 3. Write your solutions on the exam sheet; you may use the back side of a page if you run out of space. Throughout the exam you should give complete and clear proofs of your statements, justifying your steps. If you are using a particular theorem, be sure to state clearly what you are using. If you have a question about what you may assume without proof, please be sure to ask.
- 4. This test is 2 hours and has 5 problems worth a total of 50 pts.
- 5. Good luck!

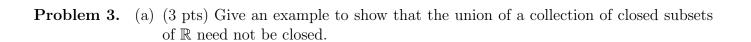
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Problem 1.	Let $\{x_n\}$	be a b	${ m ounded} \ { m s}$	equence of	of real	numbers.

(a) (5 pts) Define $\limsup x_n$ and $\liminf x_n$ and show that $\liminf x_n \leq \limsup x_n$.

(b) (5 pts) Show from the definitions that $\liminf x_n = \limsup x_n$ if and only if $\lim x_n$ exists.

- **Problem 2.** Given a norm $\|\cdot\|$ on \mathbb{R}^n , we let B denote its closed unit ball; that is, $B = \{x \in \mathbb{R}^n : \|x\| \le 1\}$.
 - (a) (4 pts) Determine B explicitly for the l^1 and l^∞ norms on \mathbb{R}^2 . Recall that $\|(x,y)\|_1 = |x| + |y|$ and $\|(x,y)\|_\infty = \max\{|x|,|y|\}.$

(b) (6 pts) Show that for any norm on \mathbb{R}^n , B is a convex set which contains a neighborhood of the origin. (Recall that a set is convex if it contains the line segment between any two of its points.)



(b) (7 pts) A collection of subsets of a metric space M is said to be *locally finite* if each point of M has a neighborhood which intersects only finitely many sets from the collection. Show that the union of a locally finite collection of closed subsets of a metric space is closed.

Problem 4. Let K be a compact subset of a metric space and O an open subset with $K \subseteq O$.

(a) (5 pts) Prove that there is a number $\epsilon>0$ so that for all $x\in K$ we have $D(x,\epsilon)\subseteq O.$

(b) (5 pts) Construct an open set U with $K \subseteq U$ and $cl(U) \subseteq O$.

Problem 5.	Given a metric space M , we define a relation \sim on M by defining $x \sim y$ if there is a
	continuous path in M from x to y .

(a) (5 pts) Prove that \sim is an equivalence relation.

(b) (5 pts) Show that each equivalence class is a path connected subset of M. Show that any path connected subset of M is contained in an equivalence class.