Week 7 Monday:

One example (sketch) of ~~the~~ a partial calculation of monotone HF:

$T^n_{cl} \subseteq \mathbb{P}^n$ (monotone) Clifford torus:

Define $T^{n+1} = \left(S^1\left(\underbrace{\tfrac{1}{\sqrt{n+1}}}_{\text{radius}}\right)\right)^{n+1} \hookrightarrow S^{2n+1}(1) \subseteq \mathbb{C}^{n+1}$.

& $T^n := T^{n+1} / \text{diagonal } S^1 . \subseteq S^{2n+1}/S^1 \cong \mathbb{C}\mathbb{P}^n$.

Prop: [Oh]: $L := T^n_{cl}$ is monotone w.r.t. $(\mathbb{P}^n, \omega_{FS})$; w/ minimal Maslov # 2.

(follows from explicit analysis of ω & μ on $\pi_2(X, L)$:

via. SES

$$0 \to \pi_2(\mathbb{P}^n) \to \pi_2(\mathbb{P}^n, L) \to \pi_1(L) \to 0.)$$

Ex: T^2 in $\mathbb{C}\mathbb{P}^2$: presenting $\mathbb{C}^2 = \mathbb{C}\mathbb{P}^2 \setminus H_\infty$,

$T^2 \subseteq \mathbb{C}^2$ is one of the standard product tori $\left(S^1\left(\frac{1}{\sqrt{n+1}}\right) \times S^1\left(\frac{1}{\sqrt{n+1}}\right)\right)$

There is a canonical choice of spin str on T^n coming from the canonical trivialization $T^*T^n \cong T^n \times \mathbb{R}^n$.

$\rightsquigarrow$ ($\mathbb{Z}/2$-graded) brane structure.

Thm: [Cho]: Using the standard integrable J, there are $n+1$ families of Maslov index 2 disks:

T^2_{cl}

$\mathbb{P}^1$

$\mathbb{P}^2$ (toric picture

Can compute explicitly the homology classes they live in: basically the images of generators $\tilde{\alpha}_1, \dots, \tilde{\alpha}_{n+1} \in H_1(T^{n+1})$ under projection.

$\Rightarrow$ given a rank 1 local system ∇ on T^n w/ holonomies

$$x_1, \dots, x_n, \frac{1}{x_1 \cdots x_n} \in \mathbb{C}^*$$

$\text{im}(\tilde{\alpha}_1) - \text{im}(\tilde{\alpha}_n), \text{im}(\tilde{\alpha}_{n+1})$, get

$$m_0(L, \nabla) = x_1 + \cdots + x_n + \frac{1}{x_1 \cdots x_n}.$$

Can think of this as a hol. fxn.

$m_0(L) = W : (\mathbb{C}^*)^n \to \mathbb{C}$.

$\shortparallel$ space of rk. 1 local sys. on T^n.

Thm: [Cho]: There are exactly $n+1$ local systems for which $HF^*((L,\nabla),(L,\nabla)) \neq 0$

(otherwise, $HF^*((L,\nabla)) = 0$.)

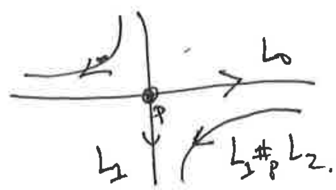
(in fact, these local systems are exactly $\longleftrightarrow$ "critical points of W" (won't explain this today).

<u>Exact triangles and idempotents</u> : → Goal: want to talk about LESs in Lag'n Floer cohomology
(will help us understand Floer cohomology of Lag'ns).

~~For A_∞ category~~

<u>ex</u>: If $L_0 \pitchfork L_1 = p$, w/ $\mathrm{ind}(p)=0$ (w.r.t. grading structures on L_i),

can define $\rightsquigarrow$ $L_1 \#_p L_2$ (Lag'n connect sum or "Polterovich surgery").



A result of [FOOO] implies for any K, $\exists$ a LES $HF^*(K, L_0) \to HF^*(K, L_1)$

$$\begin{tikzcd} HF^*(K,L_0) \arrow[r] & HF^*(K,L_1) \arrow[dl] \\ HF^*(K, L_1 \#_p L_2) \arrow[u] & \end{tikzcd}$$

This is a shadow of a relationship between the objects $L_0, L_1, L_1 \#_p L_2$:

<u>Def</u>: "$L_0 \#_p L_1$ is isomorphic to mapping cone of $L_0 \xrightarrow{p} L_1$, or there is an exact triangle
$$\begin{tikzcd} L_0 \arrow[r, "p"] & L_1 \arrow[dl] \\ L_0 \#_p L_1 \arrow[u] & \end{tikzcd}$$"

Let T_3 denote the category

$$\begin{tikzcd} X_0 \arrow[loop left, "e_{X_0}"] \arrow[rr, "\phi_0\ (\deg.0)"] & & X_1 \arrow[loop right, "e_{X_1}"] \arrow[dl, "\phi_1\ (\deg.0)"] \\ & X_2 \arrow[loop below, "e_{X_2}"] \arrow[ul, "\phi_2\ (\deg.1)"] & \end{tikzcd}$$

$\mu^2(\phi_{i+1 \bmod 2}, \phi_i) = 0$.

with $\mu^3(\phi_2, \phi_1, \phi_0) = e_{X_0}$ "abstract triangle".

$\mu^3(\phi_1, \phi_0, \phi_2) = e_{X_2}$

$\mu^3(\phi_0, \phi_2, \phi_1) = e_{X_1}$

meaning: $\hom(X_0, X_1) = \mathbb{K}\langle\phi_0\rangle$ ← deg. 0

$\hom(X_1, X_0) = 0$, etc.

$\hom(X_2, X_0) = \mathbb{K}\langle\phi_2\rangle$ ← deg. 1

<u>Def</u>: An <u>exact triangle</u> in an A_∞ category $\mathcal{C}$ is (the image of) an A_∞ functor $F: T_3 \to \mathcal{C}$.

(Nice aspect of this def'n: if $G: \mathcal{C} \to \mathcal{D}$ any functor, $F: T_3 \to \mathcal{C}$ triangle, then
$G \circ F: T_3 \to \mathcal{D}$ is a triangle. "exact triangles are functorial")

(what data is this: a triple of objects X, Y, Z, morphisms
$$\begin{tikzcd} X \arrow[r, "a\ (\deg.0)"] & Y \arrow[dl, "b\ (\deg.0)"] \\ Z \arrow[u, "c\ (\deg.1)"] & \end{tikzcd}$$
satisfying $\mu^1(a) = \mu^1(b) = \mu^1(c) = 0$, $\mu^2(b, a) =$ exact (need to fix the product) (if minimal, then μ^2(consecutive duples)$=0$))