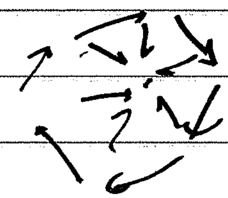
Kontsevich: Deformation-quantization

Iso from $HH^\bullet$ to ~~[illegible]~~ $H^\bullet(X, \Omega^\bullet)$, iso corrected by something



qt-equal Todd class,
bernoulli numbers

S^1-equiv. Todd genus of $(D^2 \longrightarrow \cdot)$

A Construction of Fuk that's automatically Karoubi-closed:
(& triangulated)

(X, ω) cpct. sympl, $c_1 = 0$.

Assume $[\omega] \in H^2(X; \mathbb{Z})$

$\rightsquigarrow$ A_∞ category $/ \mathbb{Q}((q))$ $\underline{\mathcal{C}}$

triangulated, Karoubi-closed $\qquad 2n = \dim X$

suppose X has a CY metric (choose J, $\Omega^{n,0}$).

Then $\mathcal{C}$ has Bridgeland stability.

Guess: $q \to 0$

(semi-)Stable objects:

$L \subset X$ spin, sLag, oriented; $(\mathcal{E}, \nabla)$ flat conn.

$\pi_1(L) \xrightarrow{\rho} GL(N, \mathbb{C})$

cplx. monodromy, semi-simple,

$\underline{\det(\rho) \subset U(1)}$.

Interesting geometry: Moduli space has cplx. structure:

SL part of rep'n.

det : $U(1)$ local systems, (cplx. part)

& deform L in transversal dimension (real part)

so complex manifolds

Claim: this is compact. ~~cpx~~ mfld.

(similar to spectral curve for Hitchin system :)

(L, g) riem. mfld.

$\rho_\epsilon : \pi_1(L) \longrightarrow GL(N, \mathbb{C})$ when $\epsilon \to 0$ blows up.

Claim: Limiting object real lag'n submanifold $\Sigma \subset T^*L$ $\quad \Sigma \xrightarrow{N:1} L$ ramified.

graph of harmonic multi-valued one-form. ↑ immersed (horribly) but compact.

(~~Cath~~ Pantev: ~~Cath~~ Corlat's thm? (sp?)).

geometrically: consider N copies of L, get ramified covering.

very bad limit of copies of L.

ex: complex curve, $\frac{d}{dt} + \epsilon^{-1} A$ ← hol. 1-form w/ values in matrices.

Have a phase for each stable object.

$\mathrm{Arg}(\Omega^{n,0}|_{L_i})$ strictly decreasing phase

$L_1, L_2,$ upper triangular, filtered by

stability condition

~~Vasserot~~

Category w/ stability conditions is automatically Karoubi closed.

Looks like one can get arbitrary upper bound on what all objects look like.

Ab categories are Reps of fundamental groups.
(has higher exts, but not negative exts)

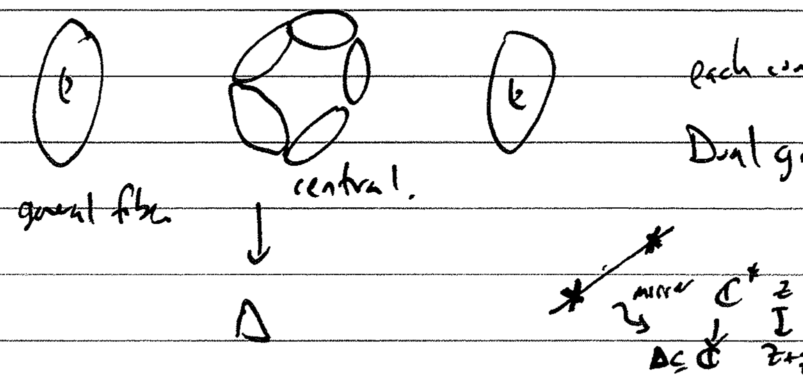
Simple test for local C-Y: spectral curves,
count geodesics.

can compactify $\det(\rho) \in U(1)$ direction,
need to compactify ~~Q~~ in L direction.

unclear picture: any Lag'n maslov 0, can define grade flow.
fixed points choices of special Lag'ns.

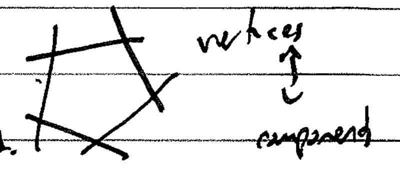
why cpct. w/ boundary?
if go to ∞, get first order deformations glues cloud multiple one

Tony Pantev: simplest example: degenerating ell. curve to a wheel.

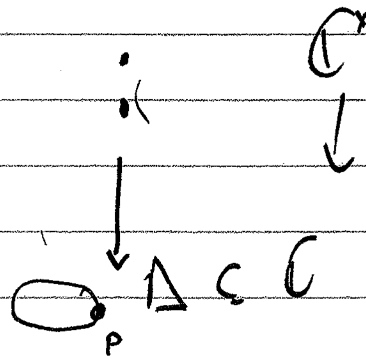


each component is Fano.

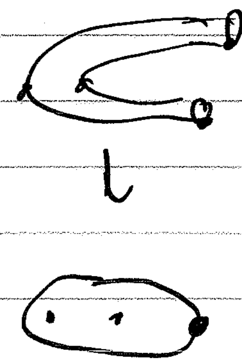
Dual graph is



at point on $\partial\Delta$, have two points.



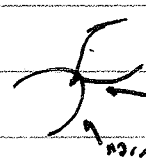
have



branch points.

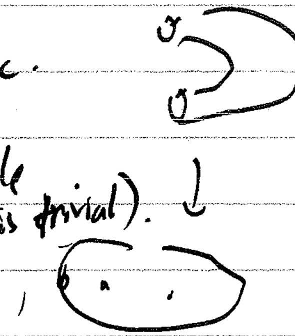
n copies of this.

identification of



⟿ global monodromy.

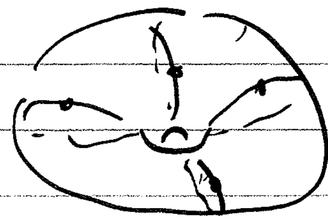
another disc.



(monodromy trivial b/c normal bundle above is trivial).

this allows identifying two boundaries, according to dual complex:

given by automorphism ⟿ get ell. curve.



n-cut circles

marked point in each cut.

Interesting: higher dimensional dual intersection complexes
$\Rightarrow$ get more than one potential.

Take E w/ punctures, B-model, try to construct mirror of that & see what it is.

Chow motives of general type gen. by pure chow motives

Need to construct mirror for Fano w/ CY divisor
gen. type embedded in CY.

need these families, pairs of objects here, CY anchors
unnatural from motivic picture.

Q: Not clear one can have a theory of motivic invts, i-sympl.-picture w/o additional data

Any idea why convergence has anything to do w/ saturation of Fukaya category.
(large volume.)

locally convergent near 0, rather
misleading question:
two conditions: - locally convergent > both follow from saturation
- well-posedness / axiom
oppositeness.

maybe saturation $\Rightarrow$ convergence

Good candidate for $\mathcal{IG}$ data for non-proper varieties

LG model of disc, mfold w/ boundary, pseudo-convex local condition on bdry.

Can see nc-mixed HS here, esp. with mirrors.

should guarantee that obvious connection from GW / D & weight filtration is a mixed H.S.

Kontsevich: Affine str. & reconstruction of mfolds over local fields

B loc. cpct top. space

$B \supset B_0$ dense open subset w/ $\mathbb{Z}$-aff structure

$\pi_1(B_0) \to SL(n, \mathbb{Z}) \ltimes \mathbb{R}^n$.

$\operatorname{codim}(B - B_0) \geq 2$.

codim $= 2$, assume $\circlearrowright$ is conjugate to $\begin{pmatrix} 1 & 1 & & \\ 0 & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

i.e. focus-focus singularity.

lift local sys. to $SL(n, \mathbb{Z}) \ltimes (K^\times)^n$

K non-arch. field w/ valuation $\mathrm{val}: K^\times \to \mathbb{R}$.

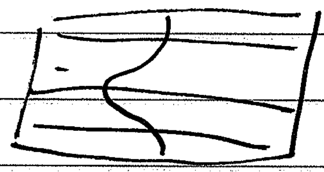
$\rightsquigarrow$ should give a canonical C-Y over K.

(only need 2, 3 b/c of Hartogs' phenomenon??).

Hartogs coherent sheaf up to codim 3, just extend.

get sheaf of functions. ([illegible] of f.f. singularity).

$n=3$



monodromy standard.

arbitrary contour curve, generalized graph of fcn

cod. 3 condition guess:

should make a notion of moving focus-focus singularities correspond, non-arch. guys will be the same

some bizarre things can't be seen from alg. geom.

Q: $\# S^3 \times S^3$ non-alg. C-Y.

what is variation of Hodge st. coming from these guys?

$\exists$ v. of HS is alg.-geom.

mirror to rigid C-Y, we have variation of H-S