Exercise 1

1.

Factorized probability is non-negative.

因為 $P(x_k|Pa_k)$ 是 legal 的機率分佈

$\Rightarrow P(x_k|Pa_k) \geq 0$

$\Rightarrow P(x_1, \dots, x_n) = \prod_{k=1}^{n} P(x_k|Pa_k) \geq 0.$

Factorized probabilities 全加起來是 1.

因為 $P(x_k|Pa_k)$ 是 legal 的機率分佈

$$\Rightarrow \sum_{k=1}^{n} P(x_k|Pa_k) = 1$$

$$\sum_{x_1,\dots,x_n} P(x_1, \dots, x_n) = \sum_{x_1,\dots x_n} P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_n|x_1, \dots, x_{n-1})$$

$= \sum_{x_1,\dots x_{n-1}} P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_{n-1}|x_1, \dots, x_{n-2}) \sum_{x_n} P(x_n|x_1, \dots, x_{n-1}),$

where $\sum_{x_n} P(x_n|x_1, \dots, x_{n-1}) = 1$

$= \sum_{x_1,\dots x_{n-2}} P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_{n-2}|x_1, \dots, x_{n-3}) \sum_{x_{n-1}} P(x_{n-1}|x_1, \dots, x_{n-2}),$

where $\sum_{x_{n-1}} P(x_{n-1}|x_1, \dots, x_{n-2}) = 1$

$= \dots = \sum_{x_1} P(x_1) \sum_{x_2} P(x_2|x_1) = \sum_{x_1} P(x_1) = 1.$

2.

(1) $X_7 \perp X_9$
True. 藉 v-structure 定理: X_3 or X_6 可以是 v-structure within the path between X_7 and X_9, 然後 所有的 descendants of X_3 or X_6 都不屬於 ϕ.

(2) $X_{10} \perp X_7 \mid X_4$
True. 藉 cascade 定理: X_4 可以是 cascade within the path between X_{10} and X_7, and $X_4 \in \{X_4\}$.

(3) $X_4 \perp X_8 \mid X_1$
False. 藉 v-structure 定義, fixing X_1 couples X_4 and X_8(4,8,1). Thus, X_4 and X_8 are not necessarily independent.

(4) $X_4 \perp X_3 \mid X_1$
True. 藉 cascade 定理: X_1 可以是 cascade within all paths between X_4 and X_3, and $X_1 \in \{X_1\}$.

(5) $X_1 \perp X_5 \mid X_3$

False. 藉 v-structure 定義, fixing X_3 couples X_1 and X_5(1,3,5). Thus, X_1 and X_5 are not necessarily independent.

(6) $X_1 \perp X_5 \mid X_6$
False. (1,3,5) as X_6 is the descendant of X_3 or (1,2,6,5) as fixing X_6 couples X_2 and X_5. Thus, X_1 and X_5 are not necessarily independent.

(7) $X_9 \perp X_7 \mid X_6$
False. X_6 is the descendant of all the other vertices, and thus the condition of rule 1 and 2 could never be met, which means X_9 and X_7 are not necessarily independent.

3.

(1)

multinomial_nb - Training acc: 0.9585714285714286
multinomial_nb - Test acc: 0.77

(2)

bernoulli_nb - Training acc: 0.9414285714285714
bernoulli_nb - Test acc: 0.7866666666666666