

CS 268P Introduction to Optimization
Homework 1 report

Implementation of Functioning one-dimensional black box unconstrained function optimizer [Golden Section Method]:

- Step size taken : 1.1
- Values taken for Stopping Criteria Parameters (Tolerance values):

$$\epsilon_r = 1e-16$$

$$\epsilon_{abs} = 1e-4$$

Function	Runs	Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	Dist_Rel_std	#Func_Evaluations
$4*x*x+5*x+2$	100	1.1	0.399231911	0.01192056	0.001215769	7.81E-05	15.26
e^x-7*x	100	1.1	0.382089615	0.012259614	0.000344731	2.26E-05	15.08
$\sin(x)$	100	1.1	0.131087303	0.002336913	10.94019787	0.849163085	10.28
$ x-20 $	100	1.1	0.449120998	0.011540997	1.28E-06	1.09E-07	23.32
$x*x-2*x+5$	100	1.1	0.400743484	0.014267315	0.001432619	0.000106387	13.98

- Values taken For Stopping Criteria Parameters (Tolerance values):

$$\epsilon_r = 1e-5$$

$$\epsilon_{abs} = 1e-1$$

Function	Runs	Step	RunTime_mean	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
$4*x*x+5*x+2$	100	1.1	0.238387585	0.005526118	0.032847297	0.002025265	8.22
e^x-7*x	100	1.1	0.215177536	0.004261199	0.012158927	0.000819219	7.94
$\sin(x)$	100	1.1	0.06862402	0.001505752	11.4795497	0.903507873	3
$ x-20 $	100	1.1	0.205621719	0.004928882	0.001814706	0.000118216	8.18

x^2-2x+5	100	1.1	0.21390676 5	0.005054 389	0.04677112 1	0.003147 059	7.06
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Comparison of Relative distance (error) values and Running time values, for different values of Stopping condition parameters and step sizes:

Function 1 : $4x^2+5x+2$

- **Values taken For Stopping Criteria Parameters (Tolerance values):**

$\epsilon_r = 1e-16$

$\epsilon_{abs} = 1e-4$

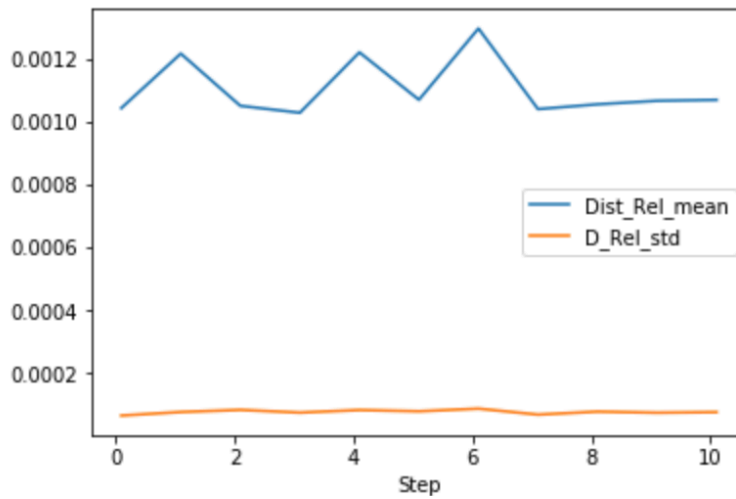
Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.440378189	0.012441517	0.00104279	6.69E-05	15.14
1.1	0.399231911	0.01192056	0.001215769	7.81E-05	15.26
2.1	0.49325943	0.018553811	0.001049984	8.49E-05	15.86
3.1	0.374517441	0.00646769	0.001027988	7.63E-05	16.08
4.1	0.39721489	0.020241272	0.001219817	8.44E-05	16.02
5.1	0.390012264	0.009090494	0.001069214	8.05E-05	16.42
6.1	0.37686348	0.008046272	0.001295565	8.88E-05	16.42
7.1	0.384879112	0.008468609	0.001039565	6.99E-05	16.64
8.1	0.396296978	0.009388202	0.001054837	7.93E-05	17
9.1	0.385909081	0.006134822	0.001065964	7.60E-05	17.2
10.1	0.392107964	0.007855385	0.001068877	7.77E-05	17.16

Average running time: 0.39319276809692383 +- 0.010270048451453835

Relative Distance: 0.0012415263987187739 +- 9.210778719319692e-05

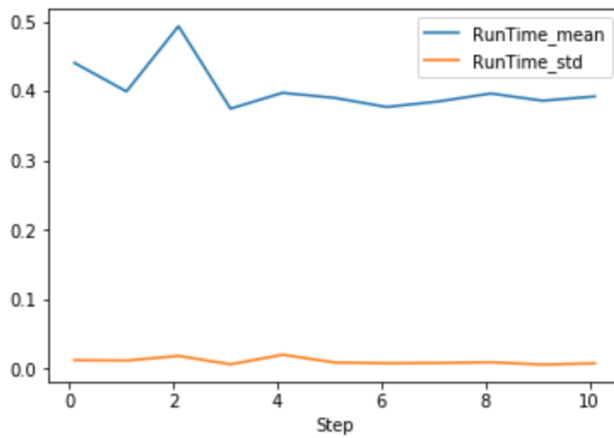
#Function Evaluations: 17.24 +- 0.06564879518897747

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean,in ms) , Runtime(standard deviation)

Out[53]: <matplotlib.axes._subplots.AxesSubplot at 0x10907f588>



- Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-5$

$\epsilon_{abs} = 1e-1$

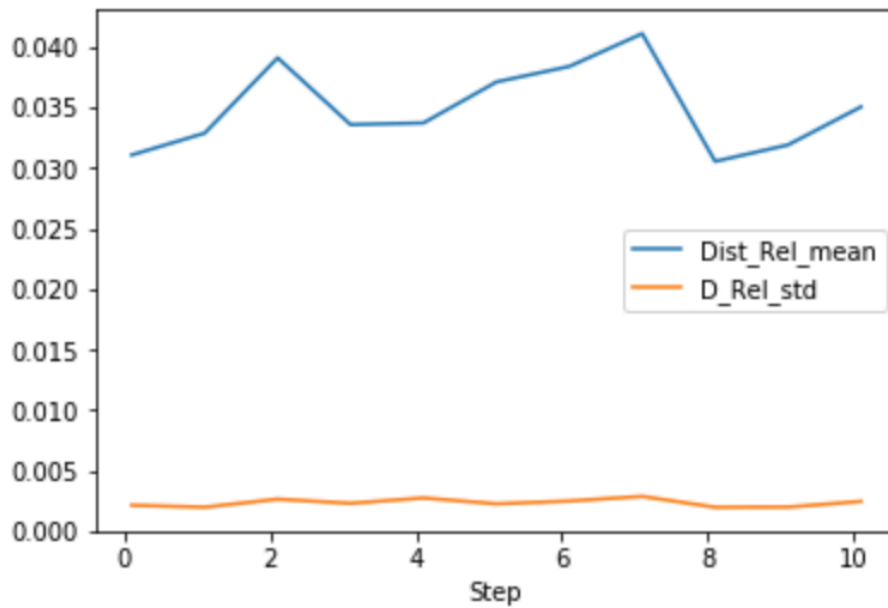
Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.29548645	0.011587947	0.031046712	0.002185475	7.66
1.1	0.238387585	0.005526118	0.032847297	0.002025265	8.22
2.1	0.253808498	0.005815683	0.0390539	0.002688434	8.6
3.1	0.257172585	0.006207276	0.033550326	0.002344438	9
4.1	0.255520344	0.007273759	0.033685835	0.002784568	8.76
5.1	0.243017673	0.004974417	0.037066431	0.002300678	9.16
6.1	0.24528265	0.004497022	0.038344455	0.00253332	9.5
7.1	0.236263275	0.004258987	0.041046101	0.002913746	9.4
8.1	0.24159193	0.004366431	0.030519831	0.002020138	9.72
9.1	0.240163803	0.003405046	0.031877423	0.002031891	9.78

Average running time: **0.24914264678955078 +- 0.00572908391069824**

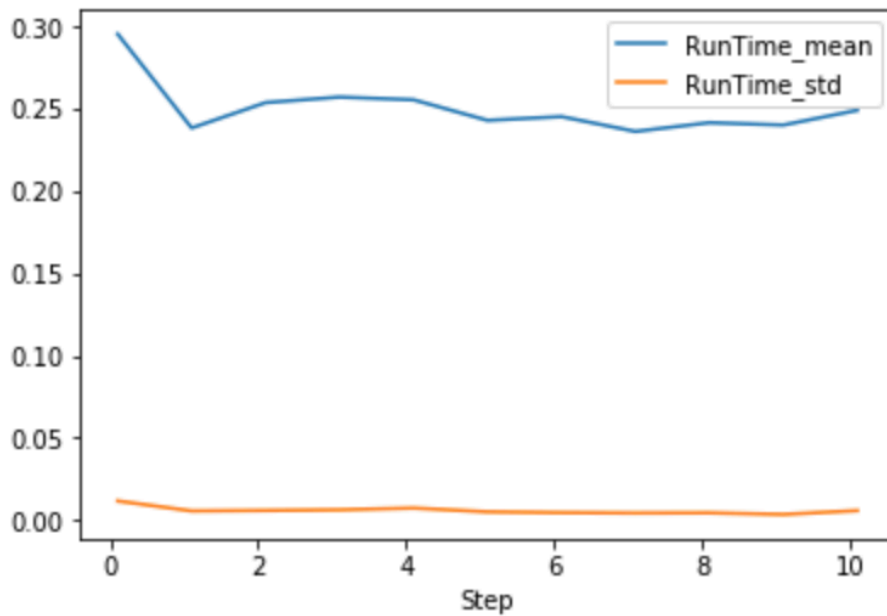
Relative Distance: **0.03501279928865557 +- 0.0025019806788933794**

#Function Evaluations: **10.04 +- 0.10092926058017612**

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Function 2 : $e^x - 7 \cdot x$

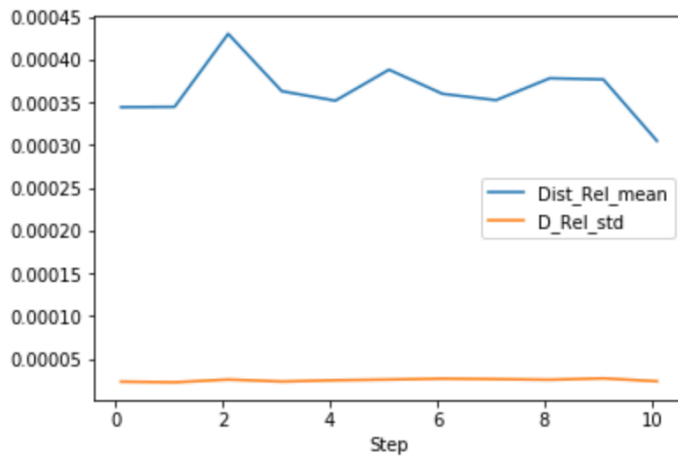
Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-16$

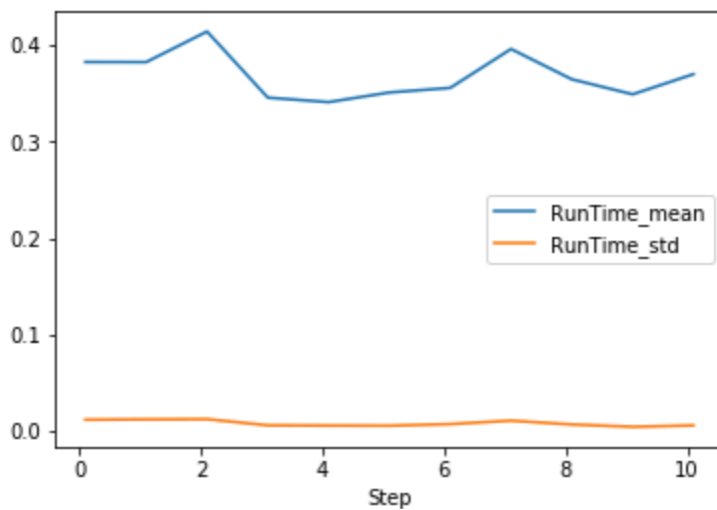
$\epsilon_{abs} = 1e-4$

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.38220644	0.011924882	0.000344375	2.34E-05	14.98
1.1	0.382089615	0.012259614	0.000344731	2.26E-05	15.08
2.1	0.413787365	0.01245883	0.000430136	2.58E-05	15.66
3.1	0.345299244	0.006061929	0.000363033	2.35E-05	15.7
4.1	0.340676308	0.005848922	0.000351909	2.50E-05	15.9
5.1	0.350666046	0.005685272	0.000388161	2.59E-05	16.22
6.1	0.355396271	0.007359773	0.000359909	2.69E-05	16.4
7.1	0.39567709	0.010906754	0.000352536	2.64E-05	16.34
8.1	0.364277363	0.006944662	0.000378141	2.56E-05	16.82
9.1	0.348753929	0.004526718	0.00037677	2.72E-05	16.94
10.1	0.369591713	0.005967279	0.000304933	2.38E-05	17.16

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Average running time: 0.36662817001342773 +- 0.009380663164791133
Relative Distance: 0.00032720313829729335 +- 2.709353284741512e-05
#Function Evaluations: 17.16 +- 0.0548067069343456

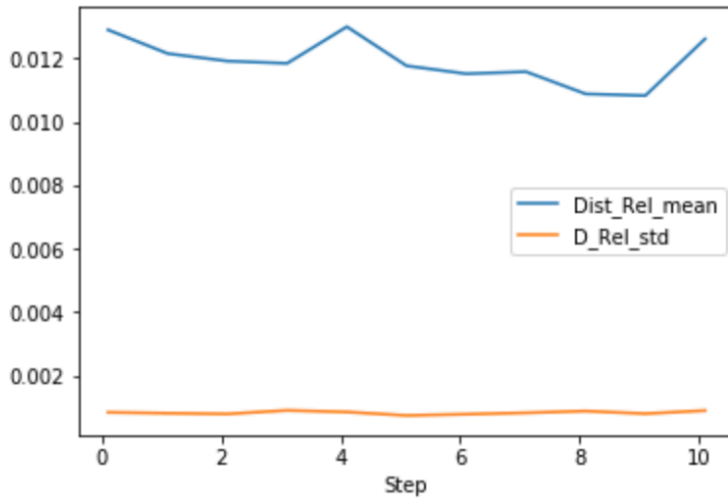
- Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-5$

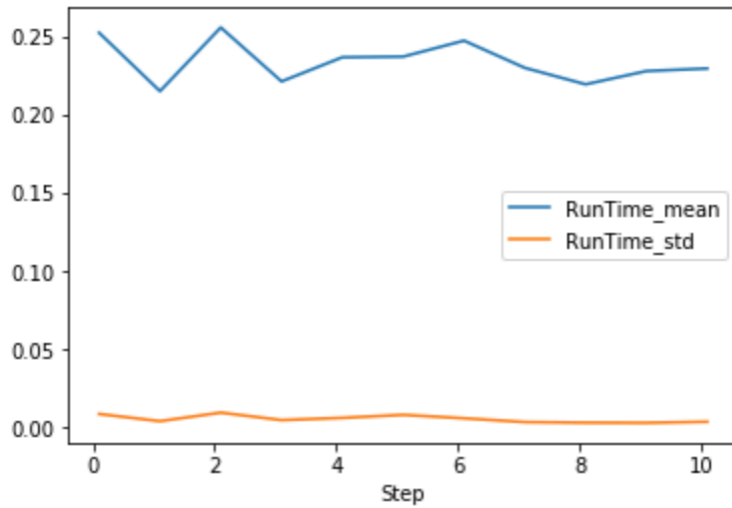
$\epsilon_{abs} = 1e-1$

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.252480507	0.008849302	0.012906762	0.000847977	7.74
1.1	0.215177536	0.004261199	0.012158927	0.000819219	7.94
2.1	0.255877972	0.009721859	0.011922786	0.000797775	8.3
3.1	0.221400261	0.00495101	0.011851536	0.000909461	8.48
4.1	0.236856937	0.006380725	0.013005537	0.000858465	8.88
5.1	0.237238407	0.00830882	0.011773188	0.000748786	8.98
6.1	0.247471333	0.006105521	0.011522199	0.000789106	9.44
7.1	0.230078697	0.00369293	0.011586621	0.000832189	9.44
8.1	0.219545364	0.003292937	0.010886455	0.000887844	9.34
9.1	0.228028297	0.003205763	0.010834314	0.000806453	9.76
10.1	0.229666233	0.003893888	0.012621745	0.000903838	9.8

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Average running time: 0.22966623306274414 +- 0.0038938883238880855
Relative Distance: 0.012621744984598327 +- 0.000903838274644192
#Function Evaluations: 9.8 +- 0.09896928253669407

Function 3 : sin(x)

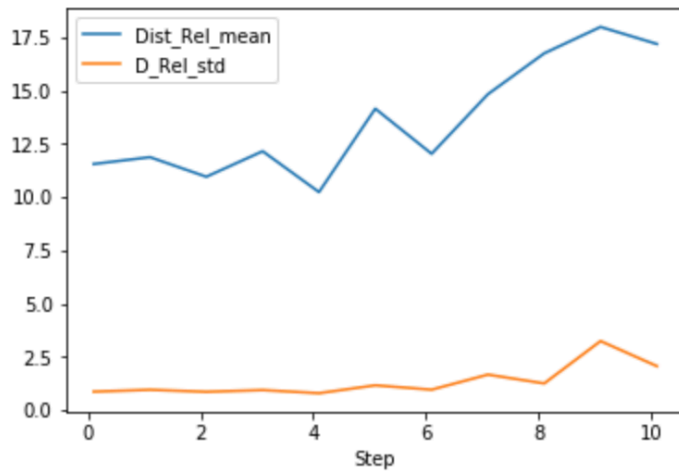
- Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-16$

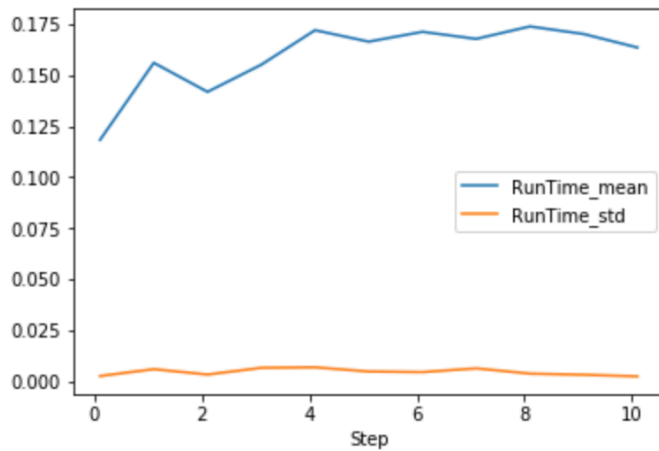
$\epsilon_{abs} = 1e-4$

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.118403435	0.002504606	11.56001244	0.864470628	8.2
1.1	0.15619278	0.005873393	11.87999298	0.951775849	10.42
2.1	0.141947269	0.003238808	10.96002157	0.857321518	11.12
3.1	0.155260563	0.006532098	12.16015049	0.937920531	12.28
4.1	0.172200203	0.006783209	10.2400153	0.791405591	13
5.1	0.16658783	0.004737883	14.15985568	1.161116073	13.42
6.1	0.171413422	0.004437446	12.03983392	0.956964419	14.04
7.1	0.167937279	0.006270299	14.84012652	1.664258905	13.28
8.1	0.174040794	0.003696373	16.7599625	1.25018363	14.76
9.1	0.170331001	0.003110184	18.00033068	3.240908888	15.04
10.1	0.163710117	0.002355984	17.19995845	2.062587023	15.46

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Average running time: 0.1637101173400879 +- 0.002355984260529712

Relative Distance: 17.199958453429673 +- 2.062587023355564

#Function Evaluations: 15.46 +- 0.098369525850019

- **Values taken For Stopping Criteria Parameters (Tolerance values):**

€_r= 1e-5

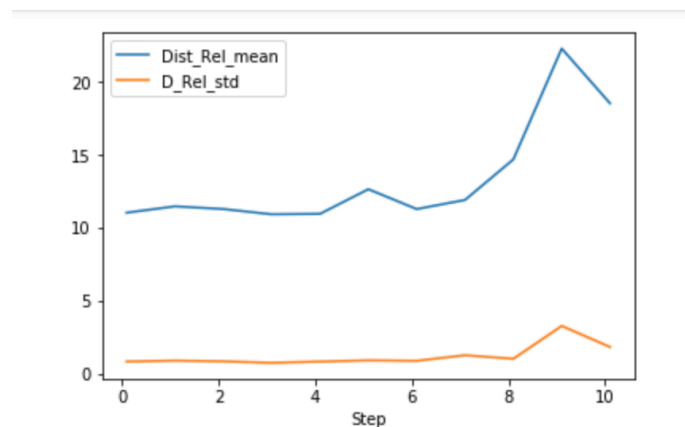
€_{abs}=1e-1

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.059480667	0.001768786	11.0443413	0.842933354	1.7
1.1	0.06862402	0.001505752	11.4795497	0.903507873	3

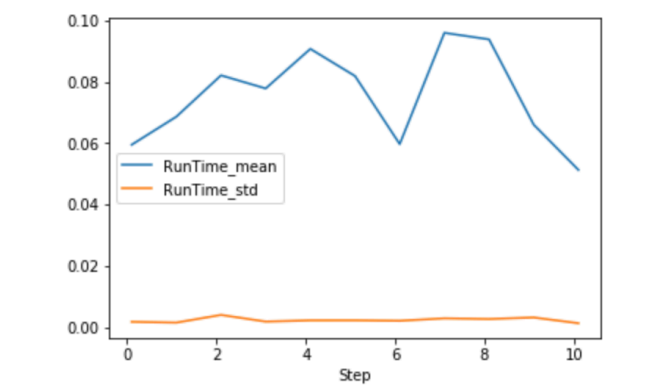
2.1	0.082097054	0.00400762	11.29571758	0.855663072	3.54
3.1	0.077824593	0.001840041	10.9340235	0.753144706	4.48
4.1	0.090727806	0.00223167	10.96453349	0.838883942	5.58
5.1	0.081880093	0.002241381	12.65420869	0.923437054	4.84
6.1	0.059688091	0.002101679	11.29241534	0.889437006	2.4
7.1	0.095956326	0.002891835	11.91731411	1.273685697	6.04
8.1	0.093827248	0.002674435	14.70013369	1.03038066	6.28
9.1	0.066027641	0.003170437	22.29660095	3.282613121	3.24
10.1	0.051279068	0.001309539	18.53705851	1.836161375	1.54

Average running time: 0.05127906799316406 +- 0.001309538717313371
 Relative Distance: 18.537058509164908 +- 1.8361613752945176
 #Function Evaluations: 1.54 +- 0.13356270445305532

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Function 4 : $|x-20|$

- Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-16$

$\epsilon_{abs} = 1e-4$

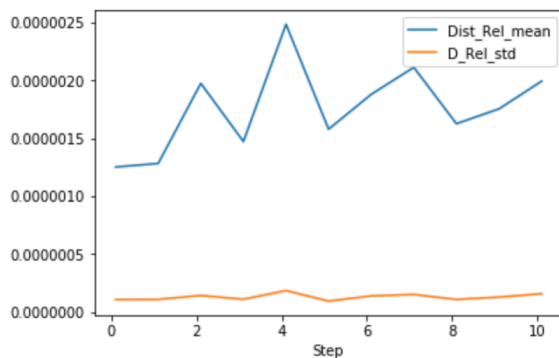
Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.459909439	0.009595711	1.25E-06	1.08E-07	22.9
1.1	0.449120998	0.011540997	1.28E-06	1.09E-07	23.32
2.1	0.453023911	0.010212041	1.98E-06	1.43E-07	23.02
3.1	0.42927742	0.00764475	1.47E-06	1.11E-07	23.5
4.1	0.404376984	0.005979424	2.49E-06	1.85E-07	23.08
5.1	0.420002937	0.007526573	1.58E-06	9.48E-08	23.56
6.1	0.420260429	0.007573363	1.88E-06	1.39E-07	23.52
7.1	0.421407223	0.007531935	2.11E-06	1.52E-07	23.58
8.1	0.482068062	0.034547039	1.63E-06	1.09E-07	24.12
9.1	0.420894623	0.006265379	1.76E-06	1.29E-07	24.12
10.1	0.407328606	0.005618993	1.99E-06	1.58E-07	23.76

Average running time: 0.4276418685913086 +- 0.010852961868967349

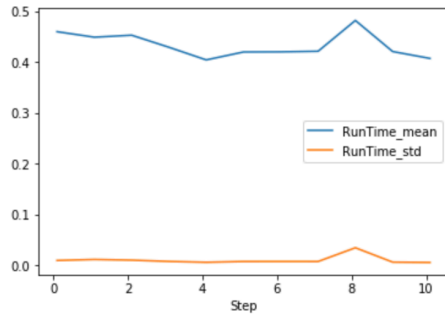
Relative Distance: 2.0298509196621237e-06 +- 1.464527276412593e-07

#Function Evaluations: 23.74 +- 0.10557367782184986

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



- Values taken For Stopping Criteria Parameters (Tolerance values):

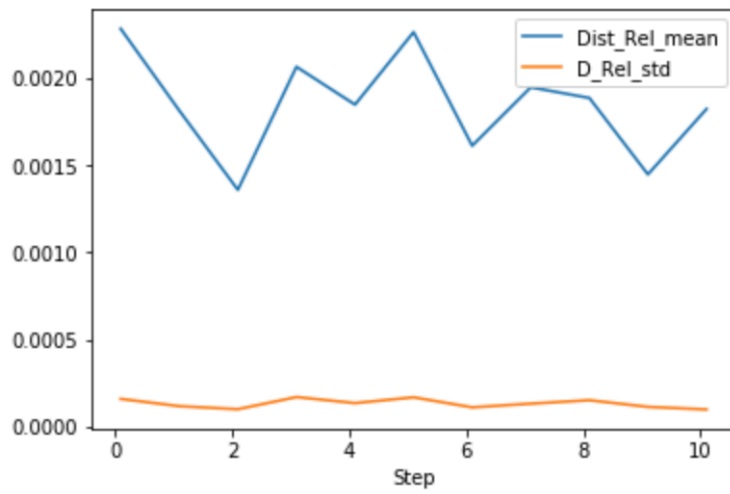
$\epsilon_r = 1e-5$

$\epsilon_{abs} = 1e-1$

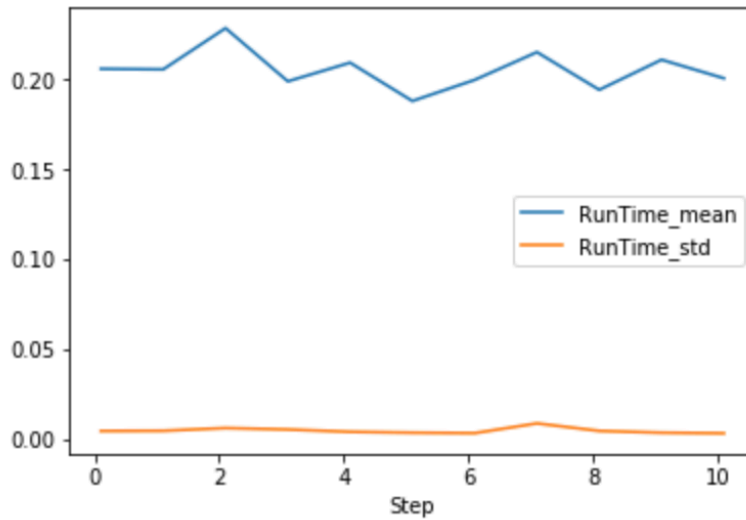
Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.205917358	0.004823341	0.002284058	0.000160042	7.56
1.1	0.205621719	0.004928882	0.001814706	0.000118216	8.18
2.1	0.228507519	0.006419783	0.001359384	0.000100855	9.18
3.1	0.198974609	0.005640634	0.002065585	0.000170481	8.38
4.1	0.20932436	0.004355169	0.001848523	0.000135859	9.04
5.1	0.188040733	0.003863476	0.002264628	0.000168951	8.6
6.1	0.199773312	0.003624011	0.001612443	0.000111493	9.32
7.1	0.215172768	0.009028329	0.001947857	0.000132738	9.22
8.1	0.19425869	0.004859521	0.001886983	0.000152442	8.9
9.1	0.210959911	0.00385766	0.001448202	0.00011416	9.94
10.1	0.200684071	0.003536753	0.00182337	9.95E-05	9.66

Average running time: 0.2006840705871582 +- 0.003536753361302632
Relative Distance: 0.001823369630542128 +- 9.953257046295489e-05
#Function Evaluations: 9.66 +- 0.09499246824653533

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Function 5 : $x^2 - 2x + 5$

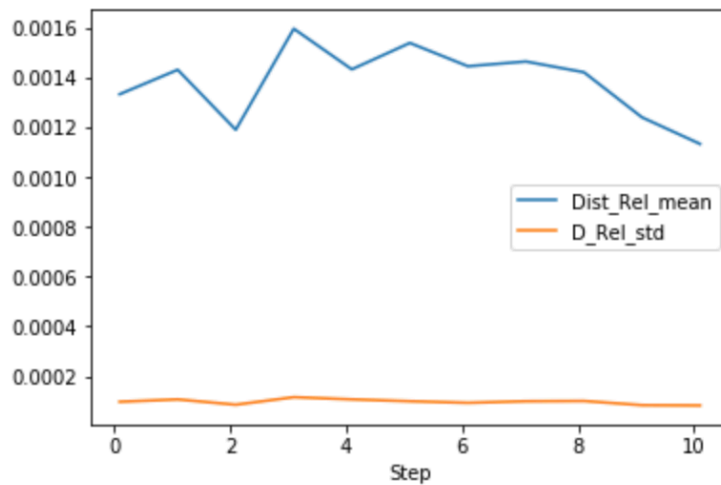
- Values taken For Stopping Criteria Parameters (Tolerance values):

$\epsilon_r = 1e-16$

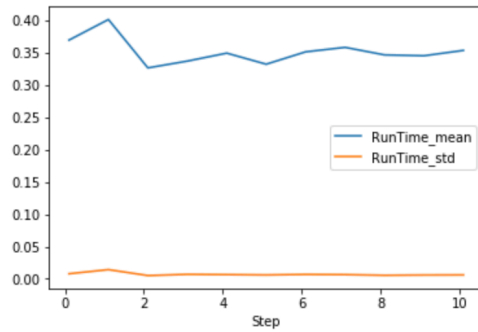
$\epsilon_{abs} = 1e-4$

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.368945599	0.007942614	0.001334328	9.69E-05	13.6
1.1	0.400743484	0.014267315	0.001432619	0.000106387	13.98
2.1	0.325987339	0.005148947	0.001191337	8.51E-05	14.28
3.1	0.336492062	0.006958699	0.001597739	0.000115364	14.36
4.1	0.348789692	0.006555057	0.001434373	0.000106542	15.08
5.1	0.331687927	0.006061	0.001540504	9.90E-05	14.62
6.1	0.350797176	0.006844459	0.001446837	9.28E-05	15.24
7.1	0.357704163	0.00653081	0.0014659	9.90E-05	15.5
8.1	0.346064568	0.005561915	0.001422676	0.000100029	15.32
9.1	0.344724655	0.005964308	0.00124129	8.33E-05	15.64
10.1	0.353007317	0.006098576	0.001134306	8.21E-05	15.94

Average running time: **0.36192893981933594 +- 0.010555986556422578**
Relative Distance: **0.0013298370212317257 +- 8.373863601749386e-05**
#Function Evaluations: **15.84 +- 0.09970878932729751**



Step size vs Runtime(mean) , Runtime(standard deviation)



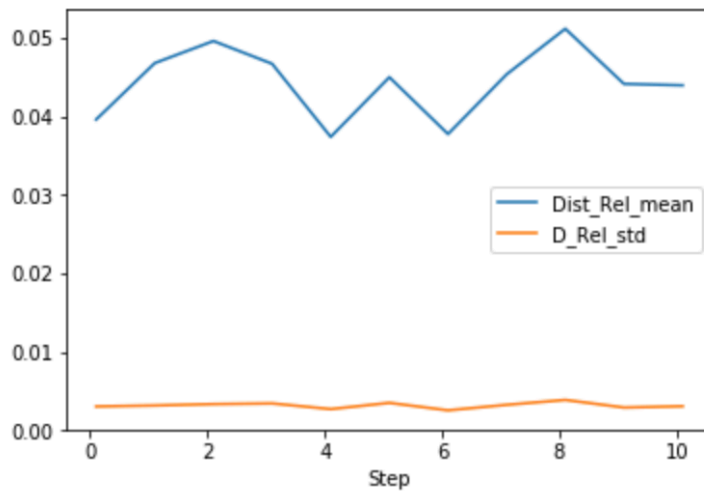
- **Values taken For Stopping Criteria Parameters (Tolerance values):**

$\epsilon_r = 1e-5$

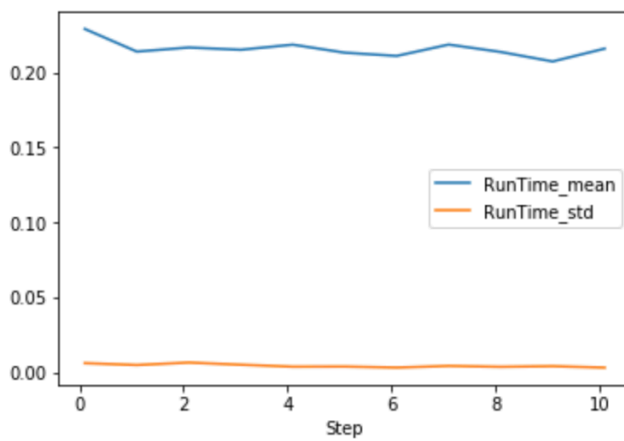
$\epsilon_{abs} = 1e-1$

Step	RunTime_mean(ms)	RunTime_std	Dist_Rel_mean	D_Rel_std	Func_Evaluations
0.1	0.228989124	0.006282645	0.039581661	0.003017588	6.36
1.1	0.213906765	0.005054389	0.046771121	0.003147059	7.06
2.1	0.216610432	0.006679517	0.049582395	0.003319289	7.1
3.1	0.215110779	0.005245732	0.046682131	0.003406893	7.22
4.1	0.218527317	0.003939274	0.037367916	0.002684031	7.74
5.1	0.21317482	0.004037424	0.044971867	0.003476738	7.8
6.1	0.211014748	0.003334513	0.037727021	0.002526581	7.96
7.1	0.218584538	0.004444019	0.045337792	0.003227614	8.26
8.1	0.213654041	0.003857592	0.051144269	0.003850047	8.16
9.1	0.207316875	0.004275201	0.04411921	0.002883785	8.1
10.1	0.215878487	0.003310194	0.043936566	0.003046054	8.54

Step size vs Relative distance(error) mean, Relative distance(error) standard deviation



Step size vs Runtime(mean) , Runtime(standard deviation)



Average running time: 0.21587848663330078 +- 0.0033101935703801484
Relative Distance: 0.04393656576529107 +- 0.0030460543798242227
#Function Evaluations: 8.54 +- 0.1024344026996947