Shefali E.

HW2: 2.3(1, 2, 4, 5, 11, 12) 2.4 (1, 2, 3, 8, 12)

2.4

① ⓔ $x(n) = x(\frac{n}{3}) + 1$ for $n > 1$, $x(1) = 1$ (solve for $\underbrace{n = 3^k}$)

$$x(3^k) = x(\tfrac{3^k}{3}) + 1$$
$$= x(3^{k-1}) + 1$$
$$= (x(3^{k-2}) + 1) + 1 \quad = x(3^{k-2}) + 2$$
$$= (x(3^{k-3}) + 1 + 1 + 1 \quad = x(3^{k-3}) + 3$$
$$= x(3^{k-i}) + i$$
if $i = k$ $\quad = x(3^{k-k}) + k = x(1) + k = 1 + k$

$\log_3 n$

$k = \log_3 n$

$\log n = k \log_3$

or

$\frac{\log n}{\log 3} = k = \log_3 n$

ⓐ $x(n) = x(n-1) + 5$ for $n > 1$; $x(1) = 0$

$x(1) = x(1-1) + 5$
$= x(0) + 5$

$x(2) = x(2-1) + 5$
$= x(1) + 5$

$x(3) = x(3-1) + 5$
$= x(2) + 5$
$= x(0) + 15$

n	x(n)
1	x(0)+5
2	x(1)+10
3	x(2)+15

+5
+5

$\rightarrow x(n) = x(n-1) + 5$

$x(n-1) = x(n-2) + 5 + 5$
$= x(n-3) + 15$
$= x(n-4) + 20$
$= x(n-5) + 25$
$= x(n-w) + w \cdot 5$

$x(1) = 0$
$x(2) = x(1) + 5$
$x(3) = x(2) + 10$
$x(4) = x(3) + 15$

$f(n) = 5(n-1)$
$5(1-1) = 0$

ⓑ $x(n) = 3x(n-1)$ for $n > 1$; $x(1) = 4$

$$= 3x(3x(n-2)) = 9x(n-2) = 3^2 x(n-2)$$
$$= 3(3(3x(n-3))) = 27x(n-3) = 3^3 x(n-3)$$
$$= 3(3(3(3x(n-4)))) = 81x(n-4) = 3^4 x(n-4)$$
$$= 3^i x(n-i)$$

$x(1) = 4$

$x(2) = 3x(2-1) = 3x(1) = 3 \cdot 4 = 12$

$x(3) = 3x(3-1) = 3x(2) = 3 \cdot 12 = 36$

n	x(n)
1	4
2	12
3	36

$\cdot 3$
$\cdot 3$

$x(n) = 3^{n-1} x(1)$

$x(0) = 0$
$x(1) = x(1-1) + 1 = x(0) + 1 = 1$
$x(2) = x(2-1) + 2 = x(1) + 2 = 3$
$x(3) = x(3-1) + 3 = x(2) + 3 = 6$
$x(4) = x(4-1) + 4 = x(3) + 4 = 10$

c) $x(n) = x(n-1) + n$ for $n > 0$; $x(0) = 0$

$= x(n-2) + (n-1) + n$

$= x(n-3) + (n-2) + (n-1) + n$

$= x(n-i) + (n-i+1) + (n-i+2) + n$

$= x(0) + 1 + 2 + \dots + n$

$= \frac{n(n+1)}{2}$

d) $x(n) = x(\frac{n}{2}) + n$ for $n > 1$; $x(1) = 1$ solve for $n = 2^k$

exponent rule

$x(2^k) = x(\frac{2^k}{2}) + 2^k$

$= x(2^{k-1}) + 2^k$

K	2^k	$x(2^k)$
2	4	5
3	8	13
4	16	24
5	32	56

$x(2^2) = x(2^{2-1}) + 2^2$

$= x(2^1) + 4$

$= \underbrace{x(1)}_{1} + 4$

$x(2^3) = x(2^{3-1}) + 2^3$

$= \underbrace{x(2^2)}_{5} + 8$

$x(2^4) = x(2^{4-1}) + 2^4$

$= \underbrace{x(2^3)}_{8} + 16$

$x(2^5) = x(2^4) + 2^5$

$24 + 32$

2.3

$\log_2 999 = 500$

1. (a) $1+3+5+7+\ldots+999$

$$\sum_{i=1}^{500} 2i-1 = 2\sum_{i=1}^{500} i - \sum_{i=1}^{500} 1$$

$$\frac{500(500+1)}{2} - (500-1+1)$$

$$2(125250) - 500 =$$

$$250500 - 500 = 250{,}000$$

(b) $2+4+8+16+\ldots 1024$

$\log_2 1024 = 10$

$$\sum_{i=0}^{10} 2^i = \frac{2^{10+1}-1}{2-1} = 2046$$

(c)
$$\sum_{i=3}^{n+1} 1 = (n+1)-3+1 = (n+1)-2 = n-1$$

(d)
$$-3+\sum_{i=3}^{n+1} i = \frac{(n+1)(n+1\ \ +1)}{2} = \frac{(n+1)(n+2)}{2} = \frac{n^2+2n+n+2}{2} - \underline{3}$$

$$= \frac{n^2+3n+2}{2} + \frac{6}{2} = \frac{n^2+3n+4}{2}$$

(e)

$$\sum_{i=0}^{n-1} i(i+1) = \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i$$

$$\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$\frac{(n-1)((n-1)+1)(2(n-1)+1)}{6} + \frac{n^2+n}{2}$$

$$(n-1)(n)(2n-2+1)$$

$$\frac{(n-1)(n)(2n-1)}{6} + \frac{n^2+n}{2}$$

$$(n-1)(2n^2-2n+n)$$

$$\frac{2n^3 - 2n^2 + n^2 - 2n^2 + \overbrace{2n - n}^{n}}{6} + \frac{n^2+n}{2}$$

$$\frac{2n^3 - 3n^2 + n}{6} + \frac{3n^2+3n}{6} \approx \frac{(n^2-1)n}{3}$$

(f)

$$\sum_{j=1}^{n} 3^{j+1} = 3^1 \sum_{j=1}^{n} 3^j = 3\left(\frac{3^{n+1}-1}{3-1}\right) = \frac{3^{n+2}-9}{2}$$

(g) $$\sum_{i=1}^{n} \sum_{j=1}^{n} ij = \sum_{i=1}^{n} i \sum_{j=1}^{n} j = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} i \to \frac{n(n+1)}{2} \qquad \sum_{j=1}^{n} j \to \frac{n(n+1)}{2}$$

(h) $$\sum_{i=0}^{n-1} \frac{1}{i^2+i} = \sum_{i=0}^{n-1} \frac{1}{i^2} + \sum_{i=0}^{n-1} \frac{1}{i}$$

$$\frac{1}{(n-1)^2} + \frac{1}{(n-1)}$$

$$\frac{1}{i} - \frac{1}{i+1} = \frac{1}{i(i+1)}$$

$$\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \frac{1}{3}\right) + \dots$$

$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

2.3

(2) Find the order of growth

(a) $$\sum_{i=0}^{n-1} (i^2+1)^2 = (i^2+1)(i^2+1) = i^4 + i^2 + i^2 + 1 = i^4 + 2i^2 + 1$$

largest power

$\Theta(n^5)$

$$\sum_{i=0}^{n-1} i^4 + 2\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 1$$

(b) $$\sum_{i=2}^{n-1} \log_2 i^2 = \sum_{i=2}^{n-1} 2\log_2 i = 2\sum_{i=2}^{n-1} \log_2 i = 2\sum_{i=1}^{n} \log_2 i - 2\log_2 n$$

$\Theta(n \log n)$ appendix p.476

$x(0) = 0$

$x(1) = x(1-1) + 1 = x(0) + 1 = 1$

$x(2) = x(2-1) + 2 = x(1) + 2 = 3$

$x(3) = x(3-1) + 3 = x(2) + 3 = 6$

$x(4) = x(4-1) + 4 = x(3) + 4 = 10$

(c) $x(n) = x(n-1) + n$ for $n > 0$; $x(0) = 0$

$$= x(n-2) + (n-1) + n$$

$$= x(n-3) + (n-2) + (n-1) + n$$

$$= \underbrace{x(n-i)} + \underbrace{(n-i+1)} + \underbrace{(n-i+2)} + n$$

$$= x(0) + 1 + 2 + \ldots + n$$

$$= \frac{n(n+1)}{2}$$

(d) $x(n) = x\left(\frac{n}{2}\right) + n$ for $n > 1$; $x(1) = 1$ solve for $n = 2^k$

exponent rule

$$x(2^k) = x\left(\frac{2^k}{2}\right) + 2^k$$

$$= x(2^{k-1}) + 2^k$$

$$x(2^2) = x(2^{2-1}) + 2^2$$

$$= x(2^1) + 4$$

$$= \underbrace{x(1)}_{1} + 4$$

K	2^K	$x(2^K)$
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$$x(2^3) = x(2^{3-1}) + 2^3$$

$$= \underbrace{x(2^2)}_{5} + 8$$

$$x(2^4) = x(2^{4-1}) + 2^4$$

$$= \underbrace{x(2^3)}_{8} + 16$$

$$x(2^5) = x(2^4) + 2^5$$

$$24 + 32$$

2.3

$\log_2 999 = 500$

1. (a) $1+3+5+7+\ldots+999$

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(c)

$$\sum_{i=3}^{n+1} 1 = (n+1)-3+1 = (n+1)-2 = n-1$$

(d)

$$-3+\sum_{i=3}^{n+1} i = \frac{(n+1)(n+1\ \ +1)}{2} = \frac{(n+1)(n+2)}{2} = \frac{n^2+2n+n+2}{2} - \underline{3}$$

$$= \frac{n^2+3n+2}{2} + \frac{6}{2} = \frac{n^2+3n+4}{2}$$

2.4

2. Create a recurrance relation for the # of calls made by F(n), computing n!

// base case

$C(0) = 1$ // 1 call

$C(n) = \underbrace{C(n-1)}_{\text{recursive call back}} + \underbrace{1}_{\#\text{ of operations} = *}$ // pos int

3. $S(n) = 1^3 + 2^3 + \dots + n^3$

input: pos int n
output: Σ first n cubes

```
S(n){
  if (n=1){
     return 1;
  }else{
     return S(n-1)+n*n*n;
  }
```

basic opp

a. Recurrance Relation

$C(1) = 0$ // no operations

$C(n) = C(n-1) + 2$ // basic op = **

$C(1) = 0$

$C(2) = C(2-1) + 2 = C(1)+2 = 2$

$C(3) = C(3-1)+2 = C(2)+2 = 4$

$C(4) = C(4-1)+2 = C(3)+2 = 6$

$C(5) = C(5-1)+2 = C(4)+2 = 8$

$2(n-1)+2$

b.
```
Snon(n){
  S=1
  for (int i=2; i<n; i++){
        S = S + i* i* i;
  }
  return S;
}
```

it preforms the same number of operations

$2^0 = 1$

2.4

8. design a recursive alg that computes 2^n based on $2^n = 2^{n-1} + 2^{n+1}$

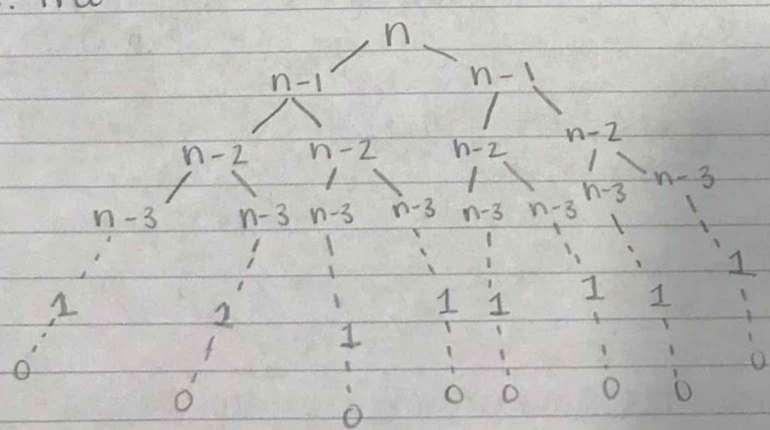
a.
```
twoToThe(n){
    if(n == 0){
        return 1
    }else{
        return twoToThe(n-1) + twoToThe(n-1)
```
not +1 bc will never reach base

b. $C(0) = 0$

$C(n) = 2C(n-1) + \underbrace{1}_{\text{addition}}$

n	C(n)	
1	1 $\times 2 + 1$	$= 2C(1-1) + 1 = 2\overbrace{C(0)}^{2(0)} + 1$
2	3 $\times 2 + 1$	$= 2C(2-1) + 1 = 2C(1) + 1$
3	7 $\times 2 + 1$	$= 2C(3-1) + 1 = 2C(2) + 1$
4	15	$= 2C(4-1) + 1 = 2C(3) + 1$
		$= 2^{n-1}$

c. tree



d. no because you are doing more work than the simple multiplication 2^n

$2^{n+1} + 2^{n-1}$

2.3

* 12 $1 > 4 > 8 > 12$

n	von(n)
0	1
1	$4 + 4(1) = 5$
2	$8 \quad = 13$
4· 3	12
4	16

$4n = \text{von}(n)$

$S(0) = 1$

$S(1) = S(0) + 4(1) = 5$

$S(2) = S(1) + 4(2) = 13$

$S(3) = S(2) + 4(3) = 25$

$S(4) = S(3) + 4(4) = 41$

$2n^2 + 2n + \underbrace{1}_{\text{odd}}$

2.4

12 Find the # of cells in the von Neumann neighborhood for range n → solve RR

$C(0) = 1$
$C(n) = C(n-1) + 4n$

n	C(n)
1	5
2	13
3	25
4	41

$C(1) = C(1-1) + 4(1) = \overbrace{C(0)}^{1} + 4$ +4
$C(2) = C(2-1) + 4(2) = C(1) + 8$ +4
$C(3-1) + 4(3) = C(2) + 12$ +4
$C(4-1) + 4(4) = \underbrace{C(3)}_{n-1} + \underbrace{16}_{4n}$

$C(0) = 1$
$C(1) = C(1-1) + 4(1) = C(0) + 4 = 4$
$C(2) = C(2-1) + 4(2) = C(1) + 8 = 12$
$C(3) = C(3-1) + 4(3) = C(2) + 12 = 24$
$C(4) = C(4-1) + 4(4) = C(3) + 16 = 40$