ICPC Team of 3

Midterm
HW 60 - 70% = C

Snefdli Σ
due ~~Monday~~ Tues

[2.5 (2, 3, 4)]

HW3: [3.1 (✓2, ✓6, ✓8, ✓9)] [3.2 (✓1, ✓4)] [3.4 (✓1, ✓2, ✓4)] [3.5 (✓6, 8)] - definition

3.1

2 ⓐ what is the time efficiency of the brute force alg for computing $a^n$ as a function of n?

$a^n = a * \underbrace{a * \ldots a,}_{n \text{ times}}$ $\Theta(n)$

as a function of the number of $\underbrace{\text{bits,}}_{8}$ in the binary representation of n?

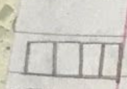
$\Theta(n)$ $2^{\text{binary}}$

ⓑ If you are to compute $a^n$ mod m where $a > 1$ and n is a large positive integer, how would you circumvent the problem of a very large magnitude of $a^n$?

$a^n$ mod m ← principal component of a leading encryption alg

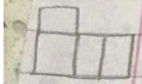
6. Tetromino tilings

w/out overlapping 8×8 chessboard

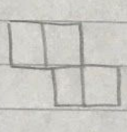


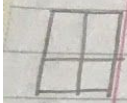
a. Straight? 16 yes

e. Z? no

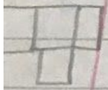


c. L tetromino? 16 yes

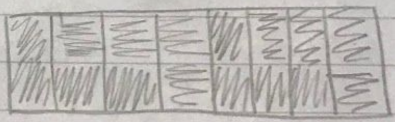




b. square? 16 yes



d. T? 16 yes



▌ means sorted

8. Sort the list $[\overset{2}{E}, \overset{6}{X}, \overset{1}{A}, \overset{4}{M}, \overset{5}{P}, \overset{3}{L}, \overset{2}{E}]$ by selection sort

* find the smallest & add to the end

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | X | $\overset{\downarrow}{A}$ | M | P | L | E | |
| A ▌ | X | E | M | P | L | E | |
| A | E ▌ | X | M | P | L | E | |
| A | E | E ▌ | M | P | L | X | |
| A | E | E | L ▌ | P | M | X | |
| A | E | E | L | M ▌ | P | X | |
| A | E | E | L | M | P ▌ | X | |
| A | E | E | L | M | P | X ▌ | ] dont need to check |

9. is selection sort stable?

n-1 comparisons

yes

preserves the relative order of any 2 equal elements in its input

ex if an input contains 2 = elements in pos i & j where i < j then they have to be in pos i' j'

3.2

1 # of comparisons in sentinel version of sequencial

ⓐ worst case = $\Theta(n)$ check the entire list = n+1

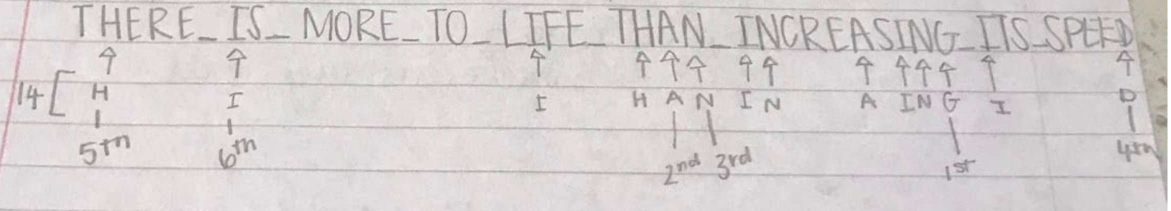
★ ⓑ avg case = $\Theta(n)$ = total number of permutations n/2 between n≥1

best case = $\Theta(1)$ it is the 1st element

```
A[n] = key
 i = 0
while (A[i] ≠ key){
        i = i+1;              // keep incrementing till the element in A=k
}
if ( i < n ){                 // n = total
      return i;        // position of key in array
}else{
      return -1;       // not in list
}
```

4 # of char comparisons brute force alg → GANDHI

len = 47 char



47 - 6 + 1 = 42

↑ self companison

43

↑ # of comp

3.4

1 ⓐ Assuming that each tour can be generated in constant time, what will be the efficency class of the exhaustive - search alg outlined for the traveling salesman, problem?

find the shortest tour through a given set of n cities that visit each city exactly once before returning to where it started

$\frac{1}{2} n! \, 10^{-9} \leq t$

n+1 cities bc you return to where you started

n additions for total len

n! because of the definition

ⓑ 1 hr = 15 24hr = 16 1yr = 18 1 century = 20

2 Outline an exhaustive search alg for Hamiltonian curcit

Sequence of n+1 adjacent verticies

\# of permutations $\frac{1}{2}(n-1)!$

n vertecies = 1st v } check every pair of v current permutation if connected by an edge ✓

else: recursively call

4 Complete the app of exhaustive search to the instance of the assignmen problem

$$\begin{array}{c|cccc|l} \text{Row} & 1 & 2 & 3 & 4 & \text{Place} \\ \hline 1 & 9 & 2 & 7 & 8 & \\ 2 & 6 & 4 & 3 & 7 & \\ 3 & 5 & 8 & 1 & 8 & \\ 4 & 7 & 6 & 9 & 4 & \end{array}$$

4 1 2 3 = 8 + 6 + 8 + 9 = 31
4 1 3 2 = 8 + 6 + 1 + 6 = 21
4 2 1 3 = 8 + 4 + 5 + 9 = 26
4 2 3 1 = 8 + 4 + 1 + 7 = 20
4 3 1 2 = 8 + 3 + 5 + 6 = 22
4 3 2 1 = 8 + 3 + 8 + 7 = 26

1, 2, 3, 4 = 9 + 4 + 1 + 4 = 18
1, 2, 4, 3 = 9 + 4 + 8 + 9 = 30
1, 3, 2, 4 = 9 + 3 + 8 + 4 = 24
1, 3, 4, 2 = 9 + 3 + 8 + 6 = 26
1, 4, 2, 3 = 9 + 7 + 8 + 9 = 33
1, 4, 3, 2 = 9 + 7 + 1 + 6 = 23

2, 1 3 4 = 2, + 6 + 1 + 4 = 13
2, 1 4 3 = 2 + 6 + 8 + 9 = 25
2, 3 1 4 = 2 + 3 + 5 + 4 = 14
2, 3 4 1 = 2 + 3 + 8 + 7 = 20
2, 4 1 3 = 2 + 7 + 5 + 9 = 23
2, 4 3 1 = 2 + 7 + 1 + 7 = 17

3, 1, 2, 4 = 7 + 6 + 8 + 4 = 25
3, 1, 4, 2 = 7 + 6 + 8 + 6 = 27
3, 2, 1, 4 = 7 + 4 + 5 + 4 = 20
3, 2, 4, 1 = 7 + 4 + 8 + 7 = 26
3, 4, 1, 2 = 7 + 7 + 5 + 6 = 25
3, 4, 2, 1 = 7 + 7 + 8 + 7 = 29

2.5

2 rabits how many pairs will be there in a year
if initaly 1 M & 1 F new born
give birth @ end of every month

$R(0) = 1$
$R(1) = 1$
$R(n) = R(n-1) + \overset{\text{born @ end}}{n}$
$R(n) = R(n-1) + R(n-2)$ fon $n > 1$ $= F(n+1)$
*from in class

3 Find the number of diff ways to climb a n-staircase if each step is either/or 2 stairs

3 stair 1-1-1
2-1
1-2

$$\left.\begin{matrix} S(n-1) + S(n-2) \\ S(n-2) + S(n-1) \end{matrix}\right) n = 3 \qquad \begin{matrix} S(1) = 1 \\ S(2) = 2 \end{matrix}$$

$$\downarrow$$

$S(n) = \text{Fibonacci}(n+1)$ n is positive

4 how many even # are in the first n Fib #'s?

$F(0) = 0$
$F(1) = 1$
$F(n) = F(n-1) + F(n-2)$

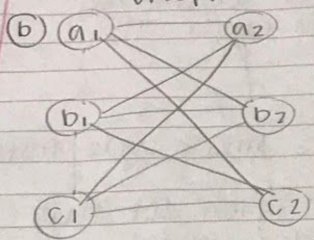
sequence e, o, o, e, o, o ∴ $n/3$, every 3rd

3.5

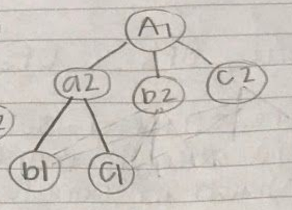
#8 bipartite - all its vertecies can be partitioned onto 2 disjoint subsets X & Y such that every edge connects a vertex in X w/ vertex in Y

Graph

(b)



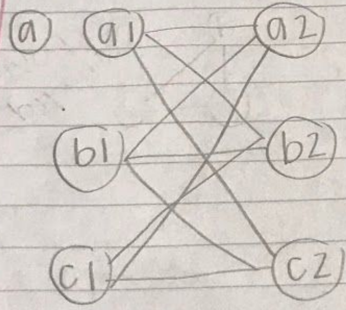
BFS



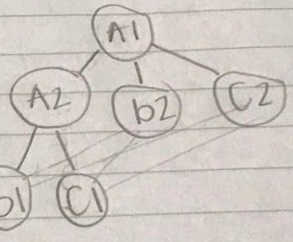
goal of the corse

If there is not an edge between a node at level x where x is any level in the BFS tree and a node in the same level x ± 2m

(a)



DFS



If there is not an edge between a node @ level x where x is any level in the DFS tree & a node in the same level x ± 2m

3.5

#6 ⓐ explain how one can check a graphs acyclity by using breadth first search

through the use of a topological sorting alg if the directed graph has a cycle the alg will not be sucessful

ⓑ do either traversals BFS vs DFS find a cycle faster

DFS is the more efficent solution for this answer - you are able to mark/unmark nodes faster which in conclusion would show if there is a cycle or not

STACK