College of Charleston
Computer Science Department

CSCI 350 Digital Logic and Computer Organization
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Spring 2020
Assignment 2- Due February 4, 2020

Course Objectives being addressed:
2. Use Boolean algebra relations to simplify Boolean expressions.

Assignment Problems

1. Verify the absorption 2 relations.

2. Show that $\bar{\bar{x}}$= x for any element x in an ***arbitrary*** Boolean algebra B (not necessarily the logic algebra, so you can't use truth tables).

3. Given elements A and B in an ***arbitrary*** Boolean algebra B (not necessarily the logic algebra so again, you can't use truth tables), show that if there exists an element C in this Boolean algebra such that

   ```
   A + C = 1
   A • C = 0
   B + C = 1
   B • C = 0
   ```

   then we must have A = B.

   Note:
   - You can't use cancellation as you can in algebra. We know, for example because by idempotency and and property 4 of the definition of a Boolean algebra we have 1+ 0 = 1 = 1 + 1, but 1 $\neq$ 0, so cancelling the 1's is not valid here.

   - This result can be a useful in practice to show that two logical expressions are equivalent. If instead of A and B we have two logical expressions that we want to show are equivalent, then suppose we can find a third expressions C that makes the four expressions above true, then we can conclude that the expressions represented by A and B are logically equivalent. This is the approach I recommend that you take for problem 4. below.

4. Verify DeMorgan's laws for ***any*** Boolean algebra (not necessarily the logic algebra). You may use the result of exercise 3 even if you don't know how to prove exercise 3.

5. Implement the following expressions with logic circuits in such a way that they use the fewest number of AND and OR gates with the fewest number of inputs. You may assume any of the results above results to simplify the given Boolean expressions first, even if you were not able to prove these results.

   a. A + $\bar{A}$ B
   b. AB + A$\bar{B}$
   c. $\bar{A}$BC + AC
   d. $\bar{X}$N + XN$\bar{C}$ + XNC
   e. $\overline{\overline{X + Y} + \overline{X + Y}}$
   f. $\overline{\overline{AA} + \overline{BB}}$

#1) Verify the absorption 2 relations.

$$x + \bar{x}y = x + y \qquad x(\bar{x} + y) = xy$$

rule 3

$$x\bar{x} + xy$$

Rule 3

$$\underbrace{(x+\bar{x})}_{1}(x+y) \quad \therefore x + y$$

rule 5

$$x \cdot \bar{x} = 0 \qquad \therefore xy$$

#2)

$$\begin{aligned} x &= \bar{\bar{x}} \\ &= \bar{\bar{x}} \cdot 1 \\ &= \bar{\bar{x}}(x + \bar{x}) \\ &= \bar{\bar{x}} \cdot x + \underbrace{(\bar{\bar{x}} \cdot \bar{x})}_{\varnothing} \\ &= \bar{\bar{x}} \cdot x \end{aligned}$$

$$\begin{aligned} x &= x \cdot 1 \\ &= x(\bar{x} + \bar{\bar{x}}) \\ &= \underbrace{x \cdot \bar{x}}_{\varnothing} + (x \cdot \bar{\bar{x}}) \\ &= \bar{\bar{x}} \cdot x \end{aligned}$$

Let's say that x is in Set N. By the definition of $\bar{x}$, $\bar{x}$ will be comprised of all the elements that are not in subset x. When you negate $\bar{x}$ to $\bar{\bar{x}}$ you end up with all the elements that are in subset X. This proves that x and $\bar{\bar{x}}$ are equivelent.

#3

$\overset{T}{A} + \overset{F}{C} = 1$

$\overset{T}{A} \cdot \overset{F}{C} = 0$

$\overset{T}{B} + \overset{F}{C} = 1$

$\overset{T}{B} \cdot \overset{F}{C} = 0$

$\therefore A = B$

3 Variable Input

$\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

$\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

2

* No cancelation

BCD
Clock

$A + C = B + C$

$T + F = T + F$

$T = T$

$A \cdot C = B \cdot C$

$T \cdot F = T \cdot F$

$F = F$

Rule 4

$A + 0 = B + 0$

$A = B$

$A \cdot 1 = B \cdot 1$

$A = B$

$$A = A \cdot 1 = A \cdot (B + C)$$
$$= AB + AC$$
$$= AB + 0$$
$$= AB$$

$$B = B \cdot 1 = B(A + C)$$
$$= BA + BC$$
$$= AB + 0$$
$$= AB$$

#4 DeMorgan's Law

$\overline{X+y} = \overline{X}\cdot\overline{y}$; $\overline{X\cdot y} = \overline{X}+\overline{y}$

then do this 2ndly

$A+C=1$
$A\cdot C=0$
$B+C=1$
$B\cdot C=0$

✓ $\overline{X+Y} + C = 1$
✓ $\overline{X+y} \cdot C = 0$
✓ $\overline{X}\cdot\overline{Y} + C = 1$
✓ $\overline{x}\cdot\overline{y}\cdot C = 0$

$C = x+y$

$C = x\cdot y$

✓ $\overline{X\cdot y} + C = 1$
✓ $\overline{X\cdot y}\cdot C = 0$
$\overline{X}+\overline{y}+C = 1$
$\overline{X}+\overline{y}\cdot C = 0$

$1 = \overline{X}+\overline{y}+(X\cdot y)$
$= \overline{X}+(\overline{y}+X)\underbrace{(\overline{y}+y)}_{1}$
$= \overline{y}+\underbrace{(\overline{x}+x)}_{1}$
$= 1$

$1 = \overline{X}\cdot\overline{y}+(X+y)$
$= (\overline{X}\,\overline{y}+x)+y$
$= \underbrace{(X+\overline{X})}_{1}\cdot(X+\overline{y})+y$
$= (x+\overline{y})+y$
$= x+\underbrace{(\overline{y}+y)}_{1}$
$= X+1$
$= 1$

$\emptyset = \overline{X}\cdot\overline{y}\cdot(X+y)$
$= \underbrace{\overline{X}\,\overline{y}X}_{\emptyset} + \overline{X}\underbrace{\overline{y}y}_{\emptyset}$
$= (\emptyset)\overline{y}+\overline{x}(\emptyset)$
$= \emptyset+\emptyset$
$= \emptyset$

$\emptyset = \overline{X}+\overline{y}\cdot(x\cdot y)$
$= \underbrace{\overline{X}x}_{\emptyset}y + \overline{y}\underbrace{xy}_{\emptyset}$
$= y(\emptyset)+x(\emptyset)$
$= \emptyset+\emptyset$
$= \emptyset$

$$\overline{X+y} = \overline{X}\cdot\overline{y}$$
$$= \overline{X+y}\cdot 1$$
$$= \overline{X+y}\cdot(\overline{X}\cdot\overline{y}+C)$$
$$= \overline{X+y}\cdot(\overline{X}\cdot\overline{y}) + \underbrace{\overline{X+y}\cdot C}_{\emptyset}$$
$$= (\overline{X+y})\,\overline{Xy}$$

$$\overline{X}\cdot\overline{y} = \overline{Xy}\cdot 1$$
$$= \overline{Xy}\cdot(\overline{X+y}+C)$$
$$= \overline{Xy}(\overline{X+y}) + \underbrace{\overline{Xy}\cdot C}_{\emptyset}$$
$$= \overline{xy}(\overline{X+y})$$

$$\overline{X\cdot y} = \overline{X}+\overline{y}$$
$$= \overline{X\cdot y}\cdot 1$$
$$= \overline{X\cdot y}(\overline{x}+\overline{y}+C)$$
$$= \overline{xy}(\overline{x}+\overline{y}) + \underbrace{\overline{xy}\cdot C}_{\emptyset}$$
$$= \overline{xy}(\overline{x}+\overline{y})$$

$$\overline{X}+\overline{y} = \overline{X}+\overline{y}\cdot 1$$
$$= \overline{X}+\overline{y}(\overline{xy}+C)$$
$$= \overline{X}+\overline{y}(\overline{xy}) + \underbrace{\overline{X}+\overline{y}(C)}_{\emptyset}$$
$$= (\overline{X}+\overline{y})\,\overline{xy}$$

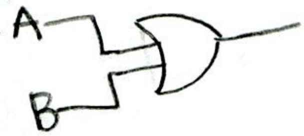
#4 for all cases

#5

(a) $a+\bar{a}b$

*absorption 2

$a+b$



(b) $ab+a\bar{b}$

$a\underbrace{(b+\bar{b})}$

*rule 5

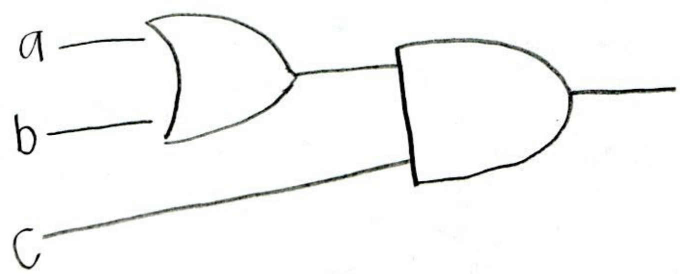
$a(1)$

a

(c) $\bar{a}bc+ac$

$c(\bar{a}b+a)$

$c(a+b)$



(d) $\bar{X}N+(XN\bar{C}+XNC)$

$\bar{X}N+XN\underbrace{(\bar{C}+C)}_{1}$

$N\underbrace{(\bar{X}+X)}_{1}$

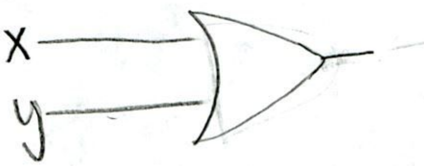
N

(e) $\overline{\overline{X+Y} + \overline{X+Y}}$

$\overline{\overline{x+y}} \cdot \overline{\overline{x+y}}$

$(x+y) \cdot (x+y)$

$x+y$



(f) $\overline{\overline{AA} + \overline{BB}}$

$\overline{\overline{AA}} \cdot \overline{\overline{BB}}$

$AA \cdot BB$

$A \cdot B$