CNHR–MNHR Model
Formal Specification

Cost-Normalised Hourly Rate &
Marginal Net Hourly Rate

Integrated Mathematical, Statistical & Operational Framework
for Rideshare Driver Performance Analytics

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Specification & Analytical Reference

Contents

Abstract

This specification defines a comprehensive analytical framework for evaluating rideshare driver performance on a first-principles basis. The framework centres on two complementary metrics: the **Cumulative Net Hourly Rate** (CNHR, r_n), which measures aggregate profitability after fixed costs, and the **Marginal Net Hourly Rate** (MNHR, m_i), which measures the net economic contribution of individual trips. Both metrics are defined in dual-track variants—*paid* (using platform-reported engaged time only) and *true* (incorporating unpaid enroute time)—to capture the full spectrum from platform-comparable reporting to operational reality.

The specification introduces arithmetic and geometric mean aggregation of both CNHR and MNHR, establishing that the AM–GM gap serves as a model-free measure of rate volatility with direct economic interpretation. All parameters are empirically calibrated from a dataset of 635+ trips across 12 Uber weeks in London, with variance decomposition demonstrating that hours worked (not trip quality) constitute the binding constraint on weekly profitability.

The operational stopping framework is governed by a strict hierarchy: weekly hours targets take absolute priority over session-level stopping decisions. The four-state diagnostic matrix (SUSTAINED, ACCEL RECOVERY, DECELERATING, STALLED) provides session-level guidance that is explicitly subordinated to the weekly hours imperative established by the variance decomposition.

The framework is situated within the broader landscape of transport profitability analytics, demonstrating structural isomorphism with the aviation industry's CASM/RASM methodology, trucking's Cost Per Mile framework, and fleet management KPI systems. The MNHR directional signal, dual-track time accounting, and four-state diagnostic matrix are identified as contributions without parallel in existing transport economics.

The document is self-contained: it derives all formulae from first principles, provides complete algebraic proofs of key identities, specifies the statistical framework including regression and variance decomposition, defines the smoothing and diagnostic systems, and maps every computation to a concrete implementation.

Keywords: rideshare analytics, cost-normalised hourly rate, marginal analysis, enroute time, utilisation, CASM/RASM parallel, aviation economics, geometric mean, EMA smoothing, trip acceptance optimisation

Part I: Foundations

Introduction

Motivation

Rideshare platforms report driver earnings exclusively in terms of *engaged time*—the interval from passenger pickup to dropoff. This framing systematically overstates effective hourly earnings by excluding two categories of unpaid labour: *enroute time* (driving from the driver's current location to the passenger pickup point) and *deadhead time* (repositioning between trips without a fare). A driver observing a platform-reported rate of £32/hr may, in operational reality, be earning £19/hr on all time committed to trip fulfilment.

Furthermore, platform-reported earnings are gross figures. The driver's actual net position depends critically on fixed costs—vehicle rental, insurance, charging, telecommunications—that are incurred weekly irrespective of trip volume. A gross rate of £32/hr against weekly costs of £430 yields profitability only if sufficient hours are worked to amortise those costs.

This specification addresses both deficiencies by defining metrics that:

(i) Deduct fixed costs from earnings to yield net rates (CNHR, MNHR).

(ii) Incorporate unpaid working time to yield true rates alongside paid variants.

(iii) Provide both cumulative (CNHR) and marginal (MNHR) perspectives.

(iv) Aggregate using both arithmetic and geometric means to capture central tendency and volatility.

(v) Are grounded entirely in empirical data rather than theoretical assumptions.

Scope

This document specifies: the complete mathematical framework for CNHR and MNHR in paid and true variants; arithmetic and geometric mean aggregation with AM–GM gap interpretation; the enroute integration model and its extension to full deadhead accounting; empirical calibration from the operational dataset; regression analysis identifying the binding constraint on profitability; variance decomposition of weekly CNHR; smoothing framework with empirically determined parameters; the CNHR–MNHR directional relationship and four-state diagnostic matrix; the operational stopping hierarchy (week > session > trip); cross-industry comparison with aviation, trucking, and fleet management frameworks; time horizon definitions (Uber week, session, multi-week); and complete data schema and implementation specification.

Conventions

Throughout this document: all monetary values are in GBP (£); time is measured in decimal hours unless otherwise stated; the "Uber week" runs from Monday 04:00 UTC to Sunday 03:59 UTC; subscript n denotes the cumulative state after trip n within a week; subscript i denotes a per-trip quantity; superscript † denotes the "true" variant (including enroute time); the absence of † denotes the "paid" variant (engaged time only).

Axiomatic Foundations

Axiom 1 (Fixed Cost Periodicity)**.** The driver incurs a fixed cost C_w per Uber week, independent of trip volume, earnings, or hours worked. This cost is non-negotiable and non-deferrable.

Axiom 2 (Time as the Scarce Resource)**.** The driver's primary scarce resource is time. All costs are therefore allocated on a time-proportional basis. No alternative allocation basis (per-trip, per-mile) preserves the aggregation identities required for internal consistency.

Axiom 3 (Dual Time Accounting)**.** For each trip i, two time quantities exist: t_i (paid duration): platform-reported engaged time from pickup to dropoff; t_i^{en} (enroute time): unpaid time from trip acceptance/dispatch to pickup arrival. The total committed time for trip i is $t_i^\dagger = t_i + t_i^{\text{en}}$.

Axiom 4 (Earnings Invariance)**.** Trip earnings e_i are determined by the platform and are invariant to the driver's enroute time. The driver receives e_i regardless of how long the drive to pickup takes.

Axiom 5 (Target Threshold)**.** There exists a target net hourly rate r^* below which continued operation is economically unsustainable in the long run. This is an exogenous parameter set by the driver.

Primary State Variables

Definition 3.1 (Trip Earnings)**.** For trip i within an Uber week, $e_i \in \mathbb{R}_{>0}$ denotes the platform-reported portal earnings in GBP.

Definition 3.2 (Trip Duration — Paid)**.** $t_i \in \mathbb{R}_{>0}$ is the engaged (paid) duration of trip i in decimal hours.

Definition 3.3 (Enroute Time)**.** $t_i^{\text{en}} \in \mathbb{R}_{\geq 0}$ is the unpaid time from dispatch to pickup for trip i, in decimal hours. If enroute data is unavailable, $t_i^{\text{en}} = 0$ (conservative default).

Definition 3.4 (Trip Duration — True)**.**

$$t_i^\dagger = t_i + t_i^{\text{en}} \tag{1}$$

Definition 3.5 (Cumulative Earnings)**.**

$$E_n = \sum_{i=1}^{n} e_i \tag{2}$$

Definition 3.6 (Cumulative Hours — Paid)**.**

$$T_n = \sum_{i=1}^{n} t_i \tag{3}$$

Definition 3.7 (Cumulative Hours — True)**.**

$$T_n^\dagger = \sum_{i=1}^{n} t_i^\dagger = T_n + \sum_{i=1}^{n} t_i^{\text{en}} \tag{4}$$

Definition 3.8 (Trip Distance)**.** $d_i \in \mathbb{R}_{>0}$ is the trip distance in miles.

Definition 3.9 (Weekly Fixed Cost)**.** $C_w \in \mathbb{R}_{>0}$ is the total weekly fixed cost. Currently $C_w =$ £430/week, comprising vehicle rental (£220), insurance, EV charging, and telecommunications.

Definition 3.10 (Utilisation).
$$u_i = \frac{t_i}{t_i^\dagger} = \frac{t_i}{t_i + t_i^{\mathrm{en}}} \in (0, 1] \tag{5}$$

is the fraction of committed time that is paid for trip i. The weekly mean utilisation is:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i \tag{6}$$

and the time-weighted utilisation:
$$\bar{u}^{(T)} = \frac{T_n}{T_n^\dagger} \tag{7}$$

Part II: Core Metrics

Gross Hourly Rate (ϱ)

Per-Trip Gross Rate

Definition 4.1 (Trip Gross Rate — Paid).

$$\varrho_i = \frac{e_i}{t_i} \tag{8}$$

Definition 4.2 (Trip Gross Rate — True).

$$\varrho_i^\dagger = \frac{e_i}{t_i^\dagger} = \frac{e_i}{t_i + t_i^{\text{en}}} \tag{9}$$

Proposition 4.3 (Utilisation–Rate Relationship). $\varrho_i^\dagger = \varrho_i \cdot u_i$

Proof. $\varrho_i^\dagger = e_i/t_i^\dagger = (e_i/t_i) \cdot (t_i/t_i^\dagger) = \varrho_i \cdot u_i$. □

This identity is fundamental: the true rate is the paid rate discounted by utilisation. At 60% utilisation, the true rate is 60% of the paid rate—not a small correction, but a structural transformation.

Cumulative Gross Rate

Definition 4.4 (Cumulative Gross Rate — Paid). $\bar{\varrho}_n = E_n/T_n$

Definition 4.5 (Cumulative Gross Rate — True). $\bar{\varrho}_n^\dagger = E_n/T_n^\dagger$

Proposition 4.6. $\bar{\varrho}_n^\dagger = \bar{\varrho}_n \cdot \bar{u}^{(T)}$*, where* $\bar{u}^{(T)} = T_n/T_n^\dagger$.

Cost-Normalised Hourly Rate (CNHR)

Core Formula — Paid

Definition 5.1 (CNHR — Paid).

$$r_n = \frac{E_n - C_w}{T_n} \tag{10}$$

Core Formula — True

Definition 5.2 (CNHR — True).

$$r_n^\dagger = \frac{E_n - C_w}{T_n^\dagger} \tag{11}$$

Decomposition

Theorem 5.3 (CNHR Decomposition)**.**

$$r_n = \bar{\varrho}_n - \lambda_w \tag{12}$$
$$r_n^\dagger = \bar{\varrho}_n^\dagger - \lambda_w^\dagger \tag{13}$$

where $\lambda_w = C_w/T_n$ is the paid cost rate and $\lambda_w^\dagger = C_w/T_n^\dagger$ is the true cost rate.

Proof. $r_n = (E_n - C_w)/T_n = E_n/T_n - C_w/T_n = \bar{\varrho}_n - \lambda_w$. True variant identical. □

Remark 5.4. This decomposition is the central structural result. CNHR is the gap between gross rate and cost rate. Improving CNHR requires either increasing $\bar{\varrho}_n$ (earn more per hour) or decreasing λ_w (work more hours to dilute fixed costs). The variance decomposition (§13) determines which lever dominates empirically.

Relationship Between Paid and True CNHR

Proposition 5.5 (Paid–True CNHR Relationship)**.**

$$r_n^\dagger = r_n \cdot \bar{u}^{(T)} \tag{14}$$

Proof. $r_n^\dagger = (E_n - C_w)/T_n^\dagger = [(E_n - C_w)/T_n] \cdot [T_n/T_n^\dagger] = r_n \cdot \bar{u}^{(T)}$. □

Remark 5.6. At the empirical mean utilisation of $\bar{u}^{(T)} \approx 0.66$, the true CNHR is approximately two-thirds of the paid CNHR. The paid CNHR overstates net productivity by a factor of $1/\bar{u}^{(T)} \approx 1.52$.

Phase Classification

Definition 5.7 (Three-Phase Classification)**.**

$$\text{Phase}(r_n) = \begin{cases} \textsc{deficit} & \text{if } r_n < 0 \\ \textsc{recovery} & \text{if } 0 \le r_n < r^* \\ \textsc{target} & \text{if } r_n \ge r^* \end{cases} \tag{15}$$

Inflection Points

Definition 5.8 (Break-Even Trip)**.** $n_{\text{BE}} = \min\{n : E_n \ge C_w\}$. At n_{BE}, the phase transitions from DEFICIT to RECOVERY.

Definition 5.9 (Target Trip)**.** $n^* = \min\{n : r_n \ge r^*\}$.

The Cost Rate (λ)

Retrospective Cost Rate

Definition 6.1 (Retrospective Cost Rate — Paid)**.** $\lambda_w^{(\text{retro})} = C_w/T_w$

Definition 6.2 (Retrospective Cost Rate — True)**.** $\lambda_w^{(\text{retro})\dagger} = C_w/T_w^\dagger$

The retrospective rate is exact but available only after the week concludes.

Prospective Cost Rate

Definition 6.3 (Prospective Cost Rate)**.**

$$\hat{\lambda} = \frac{C_w}{\bar{H}}, \qquad \bar{H} = \frac{1}{W}\sum_{w=1}^{W} T_w \tag{16}$$

Remark 6.4. This is the operational cost rate used for real-time MNHR computation. As more weeks accumulate, $\bar{H}$ converges to the driver's typical weekly hours, and $\hat{\lambda}$ converges to the typical cost rate.

Empirical Calibration

Parameter	Paid	True
$\bar{H}$ (mean weekly hours)	20.05 h	≈30.38 h*
$\hat{\lambda}$ (cost rate)	£21.45/hr	£14.15/hr
Min weekly λ	£13.93/hr	—
Max weekly λ	£62.32/hr	—
CV(λ)	57.6%	—

*Estimated at mean utilisation 66%.

Marginal Net Hourly Rate (MNHR)

The Cost Allocation Problem

To assign a "net" value to each trip, the weekly fixed cost C_w must be distributed across trips. The choice of allocation basis has algebraic consequences for whether the resulting per-trip metric aggregates consistently to the weekly CNHR.

Time-Proportional Allocation (Adopted)

Definition 7.1 (MNHR — Paid)**.**

$$m_i = \varrho_i - \hat{\lambda} \tag{17}$$

Definition 7.2 (MNHR — True)**.**

$$m_i^\dagger = \varrho_i^\dagger - \hat{\lambda} \tag{18}$$

Algebraic Consistency

Theorem 7.3 (CNHR–MNHR Aggregation Identity)**.** *Under time-proportional cost allocation:*

$$r_n = \frac{\sum_{i=1}^{n} m_i \cdot t_i}{T_n} \tag{19}$$

where $m_i = \varrho_i - C_w/T_n$ uses the retrospective cost rate. The identity holds exactly.

Proof. $\sum m_i t_i/T_n = \sum(\varrho_i - C_w/T_n)t_i/T_n = (\sum e_i - C_w)/T_n = r_n$. □

Alternative Allocation Bases (Rejected)

Under per-trip allocation ($c_i = C_w/n$), c_i/t_i varies inversely with trip duration, creating perverse incentives. Per-mile allocation introduces distance dependence that decouples the metric from time-based profitability. Time-proportional allocation is the unique basis preserving the aggregation identity (see Appendix C).

Arithmetic and Geometric Means

Definitions

Definition 8.1 (Arithmetic Mean of Gross Rate). $\mathrm{AM}(\varrho) = \frac{1}{n}\sum_{i=1}^{n} \varrho_i$

Definition 8.2 (Geometric Mean of Gross Rate). $\mathrm{GM}(\varrho) = \exp\bigl(\frac{1}{n}\sum_{i=1}^{n} \ln \varrho_i\bigr)$

True-variant means are computed analogously over trips with enroute data.

Derived MNHR Means

$\mathrm{AM}(m) = \mathrm{AM}(\varrho) - \hat{\lambda}$ and $\mathrm{GM}(m) := \mathrm{GM}(\varrho) - \hat{\lambda}$ (notational convention).

The AM–GM Inequality and Its Interpretation

Definition 8.3 (AM–GM Gap). $\Delta_{\mathrm{AG}} = \mathrm{AM}(\varrho) - \mathrm{GM}(\varrho) \geq 0$

Proposition 8.4 (Volatility Interpretation). *For log-normal rates with log-variance σ^2:*

$$\frac{\mathrm{GM}}{\mathrm{AM}} = \exp\bigl(-\sigma^2/2\bigr) \approx 1 - \sigma^2/2 \tag{20}$$

Hence $\Delta_{\mathrm{AG}} \approx \mathrm{AM} \cdot \sigma^2/2$.

Operational thresholds: $\Delta_{\mathrm{AG}} > £4$/hr (high variance); $[£2, £4]$/hr (moderate); $< £2$/hr (disciplined).

Remark 8.5 (Distributional Caveat). Proposition 8.1 assumes log-normal rates for the $\sigma^2/2$ approximation. Rideshare rates are not strictly log-normal: they are bounded below by platform base fares and bounded above by algorithmic surge caps, producing a distribution with compressed tails relative to log-normal. The approximation therefore slightly overestimates the GM/AM ratio for extreme-variance weeks and underestimates it for low-variance weeks. However, Δ_{AG} itself—the raw gap—is *distribution-free* and exact. Only the *interpretation* via $\sigma^2/2$ is approximate. The operational thresholds above are calibrated from empirical data and do not depend on the log-normal assumption.

Multi-Week Aggregation

$$\mathrm{AM}_{\mathrm{agg}}(\varrho) = \frac{\sum_{w=1}^{W} n_w \cdot \mathrm{AM}_w(\varrho)}{\sum_{w=1}^{W} n_w} \tag{21}$$

$$\mathrm{GM}_{\mathrm{agg}}(\varrho) = \exp\left(\frac{\sum_{w=1}^{W} n_w \cdot \ln \mathrm{GM}_w(\varrho)}{\sum_{w=1}^{W} n_w}\right) \tag{22}$$

Part III: Enroute and Deadhead Integration

The Unpaid Time Problem

Taxonomy of Driver Time

Category	Symbol	Paid?	Observable?
Engaged (pickup→dropoff)	t_i	Yes	Yes (platform data)
Enroute (dispatch→pickup)	t_i^{en}	No	Yes (paired trip data)
Deadhead (inter-trip repositioning)	t_i^{dh}	No	Requires GPS logging
Idle (waiting for dispatch)	t_i^{idle}	No	Requires session logging

Enroute Time: Data Source and Coverage

Enroute time is obtained by pairing consecutive trips: if trip i ends at time τ_i^{end} and trip $i+1$ begins at time $\tau_{i+1}^{\text{start}}$, then $t_{i+1}^{\text{en}} = \tau_{i+1}^{\text{start}} - \tau_i^{\text{end}}$.

Coverage Metric	Value
Total trips (12 weeks)	635+
Trips with enroute data	77–91% per week
Mean enroute time	15.8 min
Mean utilisation ($\bar{u}$)	66%
Utilisation range (per trip)	3.3% – 100%

Impact Quantification

Theorem 9.1 (Enroute Penalty on CNHR). *At utilisation $\bar{u}^{(T)}$: $r_n^\dagger = r_n \cdot \bar{u}^{(T)}$. At $\bar{u}^{(T)} = 0.66$, the paid CNHR overstates the true CNHR by a factor of $1/0.66 = 1.52$, approximately 52%.*

Remark 9.2 (Empirical Validation). From the Week 09–16 Feb dataset (90 trips): $E_n = £1{,}261.40$, $T_n = 39.58\,\text{h}$, $\bar{u}^{(T)} = 60.5\%$, $r_n = £31.87/\text{hr}$, $r_n^\dagger = £19.26/\text{hr}$ (39.5% reduction). Matches theoretical prediction: $31.87 \times 0.605 = 19.28$.

CNHR with Enroute: Implementation

Trips without enroute data default to $t_i^{\text{en}} = 0$ (utilisation $= 1.0$), which is conservative: it understates the enroute penalty, biasing those trips favourably rather than penalising for missing data.

Recalibrated Constants for True Variant

Constant	Paid	True
$\bar{H}$	20.05 h	≈30.38 h
$\hat{\lambda}$	£21.45/hr	£14.15/hr
r^*	£15.00/hr	£9.90/hr*

*Mechanically equivalent to paid r^* at 66% utilisation: $15.00 \times 0.66 = 9.90$.

Deadhead Extension Framework

Definition 10.1 (Deadhead Time). t_i^{dh} for trip i is the time spent repositioning without a fare between the end of the previous trip's dropoff and the start of the next trip's enroute drive.

Full Committed Time

With deadhead data: $t_i^{\ddagger} = t_i + t_i^{\text{en}} + t_i^{\text{dh}}$ and $r_n^{\ddagger} = (E_n - C_w)/T_n^{\ddagger}$, where $T_n^{\ddagger} = \sum_{i=1}^{n} t_i^{\ddagger}$.

Data Requirements

Deadhead time requires continuous GPS or session logging—it cannot be inferred from paired trip timestamps alone because the gap between trips may include idle time, personal breaks, or charging stops.

Three-Tier Metric Hierarchy

Tier	Denominator	Data Required	Status	Bias Direction
Paid (r_n)	$\sum t_i$	Platform data	Implemented	Overstates (ignores all unpaid)
True ($r_n^{\dagger}$)	$\sum(t_i + t_i^{\text{en}})$	Paired trip times	Implemented	Overstates (ignores DH + idle)
Full ($r_n^{\ddagger}$)	$\sum(t_i + t_i^{\text{en}} + t_i^{\text{dh}})$	GPS/session logging	Pending Sentinel	Unbiased

Each tier shares the same algebraic structure—differing only in the denominator—ensuring that all identities, decompositions, and diagnostic relationships hold uniformly.

Remark 10.2 (Residual Overstatement in the True Variant). The true variant ($r_n^{\dagger}$) is the current operational metric. It corrects for enroute time (the largest unpaid component, mean 15.8 min/trip) but excludes deadhead repositioning and idle waiting. The resulting overstatement is bounded: if f_{DH} denotes the fraction of total working time spent in deadhead and idle, then:

$$r_n^{\ddagger} = r_n^{\dagger} \cdot \frac{T_n^{\dagger}}{T_n^{\ddagger}} = r_n^{\dagger} \cdot (1 - f_{\text{DH}}) \tag{23}$$

If deadhead and idle constitute 15–20% of committed time beyond enroute (a plausible range for urban night-shift PHV), the true variant overstates the full rate by 15–20%. This is a known, directional, and bounded bias—preferable to the 52% overstatement of the paid variant, but not negligible.

Remark 10.3 (Sentinel as the Tier-3 Enabler). Progression from Tier 2 (True) to Tier 3 (Full) is gated entirely on the Sentinel observation pipeline. Distinguishing deadhead from idle from personal breaks requires continuous state classification:

Inter-Trip State	Detection Method	Sentinel Capability
Deadhead (repositioning)	GPS: vehicle moving, no active trip	GPS + screen state
Idle (waiting for dispatch)	GPS: stationary, app showing "waiting"	GPS + OCR
Personal break	GPS: stationary, app offline or paused	GPS + screen state
Charging	GPS: at known charger location	GPS + geofence

Until this pipeline is operational, $r_n^{\dagger}$ remains the best available metric with the bias characterised above.

Part IV: Empirical Analysis

Dataset

Scope

Metric	Value
Total trips	635+
Uber weeks	12
Date range	7 Dec 2025 – 16 Feb 2026
Total portal earnings	£8,500+
Total engaged hours	260+ h
Total miles	4,900+ mi
Platform	Uber (London, UK)
Vehicle	Kia e-Niro EV (PHV)
Typical shift	19:00–07:00 (night)

Data Fields per Trip

Each trip record contains: sequential number, datetime, day of week, service type, earnings (£), duration (hours), distance (miles), pickup location, dropoff location, and (where available) enroute time derived from paired trip timestamps.

Constants

Symbol	Value	Description
C_w	£430/wk	Fixed weekly costs
r^*	£15/hr	Target CNHR
α	0.15	EMA smoothing factor
$\bar{H}$	20.05 h	Baseline weekly hours
$\hat{\lambda}$	£21.45/hr	Prospective cost rate

Regression Analysis

Weekly-Level: What Drives Earnings?

Three OLS models were fitted to the 11-week dataset.

Model 1: Hours Only

$E_w = \beta_1 H_w + \beta_0$: $\beta_1 = £33.15/\text{hr}$, $\beta_0 = -£4.97$, $R^2 = 0.8875$. Hours alone explain 89% of variance.

Model 2: Hours + Trip Count

$E_w = \beta_1 H_w + \beta_2 N_w + \beta_0$: $\beta_1 = £24.55/\text{hr}$, $\beta_2 = £4.51/\text{trip}$, $\beta_0 = -£55.79$, $R^2 = 0.9128$.

Model 3: Hours + Trips + Miles

$E_w = \beta_1 H_w + \beta_2 N_w + \beta_3 D_w + \beta_0$: $\beta_1 = £8.42$/hr, $\beta_2 = £4.81$/trip, $\beta_3 = £0.69$/mi, $\beta_0 = -£10.51$, $R^2 = 0.9794$.

Trip-Level: Earnings Structure

$e_i = \beta_1 t_i + \beta_2 d_i + \beta_0$: $\beta_1 = £13.91$/hr, $\beta_2 = £0.65$/mi, $\beta_0 = £2.64$ (base fare), $R^2 = 0.8555$.

This confirms Uber's pricing is a hybrid of time, distance, and base fare. For cost allocation, however, the critical question is which basis preserves the CNHR–MNHR aggregation identity. The answer (time-proportional) was established in Theorem 7.3.

Variance Decomposition

Since $r_n = \bar{\varrho}_n - \lambda_w$ (Theorem 5.3):

$$\mathrm{Var}(r_n) = \mathrm{Var}(\bar{\varrho}) + \mathrm{Var}(\lambda_w) - 2\,\mathrm{Cov}(\bar{\varrho}, \lambda_w) \tag{24}$$

Results

Component	**Value**	**Share of** $\mathrm{Var}(r_n)$
$\mathrm{Var}(r_n)$	651.63	100%
$\mathrm{Var}(\bar{\varrho})$	18.98	2.9%
$\mathrm{Var}(\lambda_w)$	590.99	90.7%
$-2\,\mathrm{Cov}(\bar{\varrho}, \lambda_w)$	45.82	7.0%

Correlation Structure

Pair	**Correlation**
$\mathrm{Corr}(r_n, H)$	+0.8239
$\mathrm{Corr}(r_n, \bar{\varrho})$	+0.3579
$\mathrm{Corr}(r_n, N)$	+0.7341

The Binding Constraint

Theorem 13.1 (Hours as the Binding Constraint)**.** *The cost rate $\lambda_w = C_w/T_w$ accounts for 90.7% of week-to-week CNHR variance. Gross hourly rate $\bar{\varrho}$ contributes only 2.9%. Therefore:*

(i) The primary lever for improving CNHR is increasing hours worked, not improving trip quality.

(ii) Gross rate $\bar{\varrho}$ is relatively stable across weeks ($\mathrm{CV} \approx 13\%$), while cost rate λ_w is highly variable ($\mathrm{CV} \approx 57.6\%$).

(iii) $\mathrm{Corr}(r_n, H) = +0.82$ confirms that longer weeks mechanically produce better CNHR through cost dilution.

Remark 13.2. This result has profound operational implications. A driver obsessing over individual trip quality addresses only 2.9% of CNHR variance. Working an additional 5 hours per week addresses 90.7%. The correct strategic priority: **maximise hours, then optimise trip quality at the margin.**

Hours Required at Current Gross Rate

Given $\bar{\varrho} \approx £32.91/\mathrm{hr}$:

Weekly Hours	λ_w (£/hr)	**CNHR** (£/hr)	**Phase**
15 h	28.67	4.24	Recovery
20 h	21.50	11.41	Recovery
24 h	17.92	14.99	Recovery (borderline)
25 h	17.20	15.71	Target
30 h	14.33	18.58	Target

Break-even for target: $C_w/(\bar{\varrho} - r^*) = 430/(32.91 - 15) = 24.0\,\mathrm{h/wk}$.

Cost Rate Analysis: Retrospective vs Prospective

	Retrospective (C_w/T_w)	**Prospective** ($C_w/\bar{H}$)
Mean	£24.30/hr	£21.45/hr (constant)
Min	£13.93/hr	—
Max	£62.32/hr	—
CV	57.6%	0%
Usage	Post-hoc analysis	Real-time MNHR

Part V: Smoothing and Diagnostics

The Smoothing Problem

Raw per-trip MNHR values exhibit high variance (individual trips range from $-£15$/hr to $+£60$/hr). For the MNHR to serve as a directional signal, it must be smoothed to suppress idiosyncratic trip noise while preserving genuine trend information. The quality criterion is *directional accuracy*: the fraction of trips where the smoothed MNHR correctly predicts whether CNHR will rise or fall at the next trip.

Smoothing Methods Evaluated

Exponential Moving Average (EMA)

Definition 15.1 (EMA of MNHR)**.**

$$\tilde{m}_0 = m_0 \tag{25}$$
$$\tilde{m}_n = \alpha \cdot m_n + (1-\alpha) \cdot \tilde{m}_{n-1} \tag{26}$$

Dual computation for paid and true variants:

$$\tilde{m}_n = \alpha \cdot m_n + (1-\alpha) \cdot \tilde{m}_{n-1} \tag{27}$$
$$\tilde{m}_n^\dagger = \alpha \cdot m_n^\dagger + (1-\alpha) \cdot \tilde{m}_{n-1}^\dagger \tag{28}$$

Rolling Mean (Fixed Window)

$$\tilde{m}_n^{(\mathrm{roll},k)} = \frac{1}{\min(n,k)} \sum_{j=\max(1,n-k+1)}^{n} m_j \tag{29}$$

Rolling Median

$\tilde{m}_n^{(\mathrm{med},k)} = \mathrm{median}\{m_j : j \in [\max(1, n-k+1), n]\}$

Hybrid: Median Pre-filter + EMA

Apply a rolling median of window k to reject outliers, then smooth the filtered series with EMA.

Time-Decay EMA

Modify the EMA factor by the inter-trip time gap to give less weight to trips separated by long intervals.

Empirical Calibration

Evaluation Metric

Definition 16.1 (Directional Accuracy)**.**

$$\mathrm{DA} = \frac{1}{n-1}\sum_{i=1}^{n-1} \mathbf{1}[\mathrm{sign}(\tilde{m}_i - r_i) = \mathrm{sign}(r_{i+1} - r_i)] \tag{30}$$

Grid Search Results

Method	Optimal Parameters	DA
EMA	$\alpha = 0.15$	92.5%
Rolling Mean	$k = 7$	89.3%
Rolling Median	$k = 5$	87.1%
Hybrid (Median + EMA)	$k = 3$, $\alpha = 0.15$	91.8%
Time-Decay EMA	$\alpha_0 = 0.15$, $\tau = 2\,\mathrm{h}$	91.2%

Key Findings

Simple EMA with $\alpha = 0.15$ achieves the highest directional accuracy at 92.5%. The median pre-filter adds complexity without accuracy gain. The time-decay variant is marginally worse and harder to implement. The optimal $\alpha = 0.15$ implies an effective window of $2/\alpha - 1 \approx 12$ trips ($\approx$1.5 sessions).

Recommended Configuration

Adopted Smoothing: EMA with $\alpha = 0.15$**.** Compute raw $m_i = \varrho_i - \hat{\lambda}$ and $m_i^\dagger = \varrho_i^\dagger - \hat{\lambda}$. Apply EMA: $\tilde{m}_n = 0.15 \cdot m_n + 0.85 \cdot \tilde{m}_{n-1}$. No median filter, no hybrid, no CNHR smoothing. CNHR is displayed unsmoothed (its cumulative nature provides inherent smoothing).

The CNHR–MNHR Directional Relationship

Theorem 17.1 (Directional Theorem)**.** *Let* $r_n = (E_n - C_w)/T_n$. *Then:*

$$m_n > r_n \implies r_{n+1} > r_n \quad \textit{(CNHR rises)} \tag{31}$$

$$m_n < r_n \implies r_{n+1} < r_n \quad \textit{(CNHR falls)} \tag{32}$$

$$m_n = r_n \implies r_{n+1} = r_n \quad \textit{(CNHR unchanged)} \tag{33}$$

where $m_n = \varrho_n - C_w/T_n$ *uses the retrospective cost rate.*

Proof.

$$\begin{aligned} r_{n+1} - r_n &= \frac{E_{n+1} - C_w}{T_{n+1}} - \frac{E_n - C_w}{T_n} \\ &= \frac{e_{n+1} \cdot T_n - (E_n - C_w) \cdot t_{n+1}}{T_{n+1} \cdot T_n} \\ &= \frac{t_{n+1}}{T_{n+1}}\left(\frac{e_{n+1}}{t_{n+1}} - \frac{E_n - C_w}{T_n}\right) = \frac{t_{n+1}}{T_{n+1}}(\varrho_{n+1} - r_n) \end{aligned}$$

Since $t_{n+1}/T_{n+1} > 0$, $\mathrm{sign}(r_{n+1} - r_n) = \mathrm{sign}(\varrho_{n+1} - r_n)$. □

Remark 17.2. The smoothed variant $\tilde{m}_n > r_n$ predicts CNHR direction with 92.5% accuracy. The 7.5% failure rate has two distinct sources: EMA lag at trend inflection points, and the prospective–retrospective cost rate gap analysed in §17.1.

Retrospective Identity vs Prospective Approximation

The Directional Theorem (Theorem 17.1) is an unconditional identity: $\operatorname{sign}(r_{n+1} - r_n) = \operatorname{sign}(\varrho_{n+1} - r_n)$. Note that the sign condition involves only the *gross* trip rate ϱ_{n+1} and the *net cumulative* rate r_n. No cost rate appears in the comparison.

However, the operational MNHR signal tests $\tilde{m}_n > r_n$, where $\tilde{m}_n$ is the EMA of $m_i = \varrho_i - \hat{\lambda}$ and $\hat{\lambda} = C_w/\bar{H}$ is the prospective cost rate. Rearranging:

$$\tilde{m}_n > r_n \iff \mathrm{EMA}(\varrho_i) > r_n + \hat{\lambda} \tag{34}$$

The exact identity requires comparing ϱ_{n+1} against r_n directly. The operational test instead compares an EMA-smoothed gross rate against $r_n + \hat{\lambda}$. This introduces two sources of discrepancy:

Source 1: Cost rate mismatch. The retrospective cost rate within the current week is $\lambda_w = C_w/T_n$, which varies continuously as hours accumulate. The prospective rate $\hat{\lambda} = C_w/\bar{H}$ is a fixed historical estimate. The gap between them is:

$$\hat{\lambda} - \frac{C_w}{T_n} = C_w\left(\frac{1}{\bar{H}} - \frac{1}{T_n}\right) \tag{35}$$

This gap is zero only when $T_n = \bar{H}$, i.e., when the current week's cumulative hours exactly equal the historical mean. The bias is *systematic and predictable*:

- **Early in week** ($T_n \ll \bar{H}$): $C_w/T_n \gg \hat{\lambda}$, so r_n is deeply negative while $\hat{\lambda}$ is moderate. The operational threshold $r_n + \hat{\lambda}$ is *lower* than the exact threshold r_n, making the signal *too optimistic*—it predicts CNHR improvement more readily than warranted.
- **Late in a long week** ($T_n > \bar{H}$): $C_w/T_n < \hat{\lambda}$, so the threshold is *higher* than exact, making the signal too conservative.
- **At** $T_n = \bar{H}$: The gap vanishes. Prospective and retrospective coincide.

Source 2: EMA smoothing lag. The identity requires comparing the *current* trip's gross rate ϱ_{n+1}. The EMA $\tilde{m}_n$ is a weighted average of the past ≈12 trips' MNHRs, creating inherent delay at inflection points.

Quantifying the combined error. The 92.5% directional accuracy reflects both sources jointly. The EMA lag dominates at trend reversals (sudden demand shifts); the cost rate gap dominates in the early-week DEFICIT phase where C_w/T_n diverges sharply from $\hat{\lambda}$.

Remark 17.3 (Design Rationale). The prospective rate $\hat{\lambda}$ is used despite this gap because the retrospective rate C_w/T_n is *undefined* until the first trip (division by zero) and pathologically large for the first few trips, rendering it unusable for real-time signalling. The prospective rate provides a stable baseline. The 7.5% accuracy cost is accepted as the price of a computable real-time signal. This is an engineering trade-off, not a mathematical deficiency: the identity is exact; the implementation is an approximation.

Convergence Behaviour

As $n \to \infty$, r_n converges to $\bar{\varrho} - C_w/T_\infty$, approaching $\bar{\varrho}$ from below. If $\tilde{m}_\infty > 0$, the marginal trip is net-positive and CNHR continues to improve.

Four-State Diagnostic Matrix

Threshold Definitions

Level threshold: $r_n \gtrless r^*$. Momentum threshold: $\tilde{m}_n^\dagger \gtrless r_n$. The true-variant EMA ($\tilde{m}_n^\dagger$) is used for state classification because it captures the full cost of trip commitment including enroute time.

State Table

	$\tilde{m}_n^\dagger > r_n$ (Improving)	$\tilde{m}_n^\dagger \le r_n$ (Deteriorating)
$r_n \ge r^*$	SUSTAINED	DECELERATING
$r_n < r^*$	ACCEL RECOVERY	STALLED

State Descriptions

SUSTAINED: At or above target and still improving. Optimal state. ACCEL RECOVERY: Below target but improving—momentum favourable. DECELERATING: At target but declining—recent marginal trips below CNHR. STALLED: Below target and worsening—recent trips net-negative at the margin.

Phase Transition Events

A transition event occurs when the state changes between consecutive trips. These are logged and displayed in per-week commentary.

Operational Stopping Framework

The variance decomposition (Theorem 13.1) establishes that weekly hours account for 90.7% of CNHR variance. This result imposes an absolute constraint on all stopping decisions: **no session-level stopping decision may reduce projected weekly hours below H_{target} without the driver's conscious, informed override.**

The STALLED state describes the *current session's demand environment*, not the week's remaining potential. Rideshare demand is stochastic and non-stationary. The framework has no basis to assert that future sessions will replicate a current session's poor conditions.

The Operational Hierarchy

All operational decisions are governed by strict priority:

Priority	Level	Decision	Constraint
1 (Highest)	Week	Are cumulative hours tracking toward H_{target} (24 h)?	If not, schedule additional shifts. **Non-negotiable.**
2	Session	Has this session entered STALLED with demand collapse?	End this session. Redeploy hours to a better demand window.
3 (Lowest)	Trip	Is this offer MNHR-positive?	Decline if demand supports a replacement offer.

Revised State Actions

State	Action	Scope	Constraint
SUSTAINED	Continue	Session	—
ACCEL RECOVERY	Continue	Session	—
DECELERATING	Monitor; end session if persistent	Session only	Plan replacement session if hours deficit exists
STALLED	End session; redeploy hours to better window	Session only	Weekly hours constraint remains binding

Contextual Examples

Scenario	State	Action	Rationale
STALLED at 04:30 Tue, 15 hrs this week	STALLED	End session. Drive Wed + Thu nights.	Demand collapsed. 9 hrs still needed.
STALLED at 22:30 Fri, 10 hrs this week	STALLED	Reposition to high-demand zone.	Peak hours—STALLED likely locational. Do not stop.
STALLED at 03:00 Sun, 26 hrs this week	STALLED	End session. Week target met.	H_{target} exceeded. Safe to stop.
DECEL at 01:00 Sat, 18 hrs this week	DECEL	Monitor. Continue.	Above target but eroding. 6 hrs still needed.

Remark 18.1 (Weekly Hours Invariant). The weekly hours imperative is an invariant of the framework. It cannot be overridden by any session-level signal. STALLED means "this session's demand environment is producing net-negative marginal trips." It does not mean "stop working for the week."

Scope Boundary: Endogenous Offer Dynamics

The CNHR–MNHR framework evaluates realised performance conditional on the observed trip sequence. It does not model how driver actions—acceptance decisions, geographic repositioning, cancellation behaviour—alter the conditional distribution of future trip offers under platform dispatch algorithms.

Three causal feedback channels exist outside the framework's scope:

(i) **Acceptance filtering.** Declining low-MNHR offers (which the framework encourages) may alter the platform's dispatch priority, offer frequency, or offer quality for the driver. The acceptance rate becomes a state variable affecting future ϱ_i, but the framework treats it as exogenous.

(ii) **Geographic repositioning.** A driver who repositions after a STALLED signal changes the generating process of future offers. The framework captures the *outcome* (improved ϱ_i in subsequent trips) but not the *mechanism* by which repositioning alters the conditional offer distribution.

(iii) **Algorithmic path dependence.** The platform's matching algorithm is not stateless. A driver's acceptance history, completion rate, and geographic patterns may feed into offer ranking. The sequence $\varrho_1, \varrho_2, \ldots, \varrho_n$ is not i.i.d.—it is a stochastic process whose transition kernel depends on the driver's prior actions.

Distinction: instantaneous vs. policy optimality. The Directional Theorem (Theorem 17.1) guarantees that if $\varrho_{n+1} > r_n$, then $r_{n+1} > r_n$. This is conditionally exact for any realised trip. But it does not guarantee that an acceptance policy based on MNHR positivity maximises the expected future stream. The framework solves *instantaneous* optimality (is this trip helping?). It does not solve *policy* optimality (what acceptance strategy maximises long-run earnings given platform feedback dynamics?). The latter constitutes a partially observable Markov decision process (POMDP) and lies outside the present specification.

Design rationale. The framework deliberately excludes platform-response modelling because: (a) the dispatch algorithm is unobservable, non-stationary, and possibly personalised, making any structural model speculative; (b) the causal effects are empirically testable using CNHR–MNHR metrics retrospectively without requiring a forward model; (c) cost accounting and reinforcement learning are different analytical categories, and conflating them would compromise the algebraic purity that is this framework's principal strength.

Empirical testing. The following hypotheses are testable within the existing data structure without extending the framework:

- Does acceptance rate in session k correlate with mean ϱ_i in session $k+1$?
- Does repositioning after STALLED improve subsequent MNHR relative to continuing in place?
- Is there serial dependence in offer quality conditional on recent acceptance history?

If policy optimisation is subsequently required, the CNHR–MNHR metrics provide the natural reward signal ($R_n = r_{n+1} - r_n$) for a reinforcement learning layer. This would constitute a separate specification (see Appendix, Extension E9).

Part VI: Time Horizons

Uber Week (Primary Horizon)

The Uber week (Monday 04:00 – Sunday 03:59 UTC) is the natural primary horizon for CNHR because: C_w is a weekly cost aligning with the cost accounting period; Uber reports and settles earnings on a weekly basis; the cumulative structure of r_n naturally resets at week boundaries.

Within each week, r_n traces a trajectory from $-\infty$ (first trip: $r_n \to -C_w/t_1$) through break-even ($r_n = 0$) and potentially to target ($r_n = r^*$).

Daily (Session) Horizon

Daily CNHR is not recommended as a primary metric because: C_w is a weekly cost and distributing it across days requires arbitrary apportionment; $C_w/7$ per day penalises the first day; proportional apportionment creates circular dependency.

Instead, daily analysis uses: daily gross rate $\bar{\varrho}_{\text{day}} = E_{\text{day}}/T_{\text{day}}$, daily MNHR trajectory (the EMA series within the day), and daily contribution to weekly CNHR tracked via cumulative r_n at day boundaries.

Multi-Week (Aggregate) Horizon

$$r_{\text{agg}} = \frac{\sum_{w=1}^{W} E_w - C_w \cdot W}{\sum_{w=1}^{W} T_w} \tag{36}$$

Segmented CNHR (Display-Layer Extension)

The base CNHR treats all time as fungible: a minute at 08:30 contributes identically to a minute at 02:30. This is algebraically necessary for the aggregation identity but economically lossy, because demand intensity, charging costs, and congestion exposure vary systematically across time-of-day, geographic zone, and shift type.

Definition 22.1 (Segmented CNHR)**.** For a segment filter $\mathcal{F}$ that selects a subset of trips $\{i : \mathcal{F}(i) = \text{true}\}$:

$$r_n^{(\mathcal{F})} = \frac{\sum_{\mathcal{F}(i)} e_i^*}{\sum_{\mathcal{F}(i)} t_i} \tag{37}$$

where $e_i^* = e_i - c_i$ uses net trip earnings. No fixed cost is deducted because fixed costs cannot be attributed to individual segments without arbitrary allocation.

Remark 22.2. $r_n^{(\mathcal{F})}$ is a *gross segment rate*, not a net rate. It answers: "what is the average net-of-variable-cost earnings rate for trips matching this filter?" Cross-segment comparison ($r^{(\text{night})}$ vs. $r^{(\text{day})}$) reveals where the driver's time is most productive, enabling informed shift allocation.

Useful segment filters include:

Segment	**Filter**	**Operational Question**
Night shift	19:00–07:00	How productive are nights vs. days?
Day shift	07:00–19:00	Is the congestion penalty worth the demand?
Zone (e.g., SW1)	Pickup in zone	Which zones yield highest $\bar{\varrho}^*$?
Peak hours	22:00–02:00	Is the late-night premium real after costs?
Surge trips	$\sigma_i > 1.0$	Does surge genuinely improve net rates?

All segment-level metrics (AM, GM, Δ_{AG}, MNHR distributions) are computed identically to their whole-week counterparts, applied to the filtered subset. This requires no modification to the core specification—the algebra applies within any subset of trips.

Horizon Summary

Horizon	**Metric**	**Cost Treatment**	**Purpose**
Uber Week	r_n, $\tilde{m}_n$	C_f (single week)	Primary analysis
Session (Day)	$\bar{\varrho}_{\mathrm{day}}$, MNHR trajectory	None (gross only)	Shift-level diagnostics
Segment	$r_n^{(\mathcal{F})}$	Variable only	Cross-segment comparison
Multi-Week	r_{agg}	$C_f \times W$	Long-run viability

Part VII: Operational Applications

Shift Planning

The variance decomposition (Theorem 13.1) establishes hours as the primary lever:

$$H_{\text{target}} = \frac{C_w}{\bar{\varrho} - r^*} = \frac{430}{32.91 - 15} = 24.0 \text{ hours/week} \tag{38}$$

At 8-hour shifts, this requires 3 shifts per week.

Trip Acceptance Decision Support

For a proposed trip with estimated earnings $\hat{e}$, duration $\hat{t}$, and enroute time $\hat{t}^{\text{en}}$:

$$\hat{m}^\dagger = \frac{\hat{e}}{\hat{t} + \hat{t}^{\text{en}}} - \hat{\lambda} \tag{39}$$

Condition	Recommendation
$\hat{m}^\dagger > r_n$	Accept: trip pulls CNHR upward
$0 < \hat{m}^\dagger \leq r_n$	Accept if in RECOVERY: still net-positive
$\hat{m}^\dagger \leq 0$	Decline if alternatives exist: net cost-negative

NLW Statement Reconciliation

Uber issues National Living Wage statements that may include top-up payments, tips, and holiday pay not captured in portal earnings: Gap = Statement Total $- E_n$.

Part VIII: Generalised Cost Structure

This section replaces the single-parameter cost model ($C_w = £430$) with a decomposed framework that separates fixed, variable, and regime-dependent costs while preserving all existing algebraic identities.

The Limitation of the Single-Parameter Model

The specification to this point treats C_w as an atomic constant. In reality, the £430 conflates components with fundamentally different economic structures:

Component	Current Value	Fixed?	Scales With
Vehicle hire	£220/wk	Yes (contract)	—
Insurance	£50/wk	Yes (annual)	—
Telecommunications	£10/wk	Yes (contract)	—
EV charging	£30/wk (est.)	**No**	Miles driven, price tier
Tyre wear	Absorbed in hire	**No**	Miles driven

This conflation is harmless *as long as the driver's operational pattern is stable.* When any of the following changes occur, the single-parameter model breaks:

(i) Vehicle regime change (hire → ownership, or vehicle swap).

(ii) Charging strategy change (off-peak vs. emergency, different networks).

(iii) Shift pattern change (night-only → mixed, affecting per-trip variable costs).

(iv) Maintenance events (owned vehicle repairs, tyre replacement).

Decomposed Cost Model

The Two-Component Structure

Definition 28.1 (Generalised Weekly Cost). Total cost incurred across n trips within an Uber week:

$$\mathcal{C}(n) = C_f + \sum_{i=1}^{n} c_i \tag{40}$$

where:

- $C_f \in \mathbb{R}_{>0}$ is the **fixed weekly cost**: incurred in full regardless of whether any trips are completed. This includes insurance, telecommunications, vehicle finance or depreciation (amortised weekly), and licensing costs.
- $c_i \in \mathbb{R}_{\geq 0}$ is the **variable cost of trip** i: incurred only because trip i was undertaken. This includes energy (charging), distance-proportional wear, and any per-session costs attributable to that trip.

Remark 28.2 (Backward Compatibility). The original model is the special case where $c_i = 0$ for all i and $C_f = C_w = £430$. No existing result is invalidated; every formula in Parts I–X remains valid under this substitution.

Net Trip Earnings

Definition 28.3 (Net Trip Earnings).

$$e_i^* = e_i - c_i \tag{41}$$

is the trip earnings after deducting the variable cost directly attributable to trip i.

Definition 28.4 (Cumulative Net Earnings).

$$E_n^* = \sum_{i=1}^{n} e_i^* = \sum_{i=1}^{n} (e_i - c_i) = E_n - \sum_{i=1}^{n} c_i \tag{42}$$

Generalised CNHR

Theorem 28.5 (Generalised CNHR). *Under the decomposed cost model:*

$$r_n = \frac{E_n^* - C_f}{T_n} = \frac{E_n - C_f - \sum_{i=1}^{n} c_i}{T_n} \tag{43}$$

This is algebraically identical to the original CNHR formula (10) with:

- e_i *replaced by* $e_i^* = e_i - c_i$
- C_w *replaced by* C_f

All existing identities, decompositions, and theorems carry through under this substitution.

Proof. Direct expansion:

$$r_n = \frac{\sum_{i=1}^{n}(e_i - c_i) - C_f}{T_n} = \frac{\sum_{i=1}^{n} e_i^* - C_f}{T_n}$$

This has the same algebraic form as $(E_n - C_w)/T_n$ with $E_n \to E_n^*$ and $C_w \to C_f$. □

Generalised CNHR Decomposition

Theorem 28.6 (Generalised Decomposition).

$$r_n = \bar{\varrho}_n^* - \lambda_f \tag{44}$$

where $\bar{\varrho}_n^* = E_n^*/T_n$ *is the* ***net cumulative gross rate*** *and* $\lambda_f = C_f/T_n$ *is the* ***fixed cost rate****.*

Remark 28.7. The decomposition now cleanly separates two levers:

(i) $\bar{\varrho}_n^*$ (net gross rate): affected by both platform earnings *and* variable cost management (charging strategy, route efficiency).

(ii) λ_f (fixed cost rate): affected only by hours worked and the fixed cost base.

Under the old model, charging strategy was invisible. Under the generalised model, a driver who charges at £0.23/kWh instead of £0.40/kWh sees a direct improvement in $\bar{\varrho}_n^*$, making charging optimisation a visible lever on CNHR.

Generalised MNHR

Definition 28.8 (Generalised MNHR — Paid).

$$m_i = \varrho_i^* - \hat{\lambda}_f \tag{45}$$

where $\varrho_i^* = e_i^*/t_i = (e_i - c_i)/t_i$ is the **net trip gross rate** and $\hat{\lambda}_f = C_f/\bar{H}$ is the prospective fixed cost rate.

Definition 28.9 (Generalised MNHR — True).

$$m_i^\dagger = \varrho_i^{*\dagger} - \hat{\lambda}_f, \qquad \varrho_i^{*\dagger} = \frac{e_i - c_i}{t_i^\dagger} \tag{46}$$

Preservation of the Aggregation Identity

Theorem 28.10 (Generalised Aggregation Identity)**.** *Under time-proportional allocation of fixed costs:*

$$r_n = \frac{\sum_{i=1}^{n} m_i \cdot t_i}{T_n} \tag{47}$$

where $m_i = \varrho_i^* - C_f/T_n$ uses the retrospective fixed cost rate. The identity is algebraically identical to Theorem 7.3 and holds exactly.

Proof.

$$\frac{\sum m_i t_i}{T_n} = \frac{\sum(\varrho_i^* - C_f/T_n)t_i}{T_n} = \frac{\sum e_i^* - C_f}{T_n} = r_n$$

□

Remark 28.11 (Why This Works). Variable costs are absorbed into per-trip net earnings *before* the CNHR framework is applied. The framework then operates on e_i^* exactly as it previously operated on e_i. This is not an approximation—it is an exact algebraic substitution. The Directional Theorem, four-state matrix, EMA smoothing, variance decomposition, and all other results carry through without modification.

Variable Cost Components

Charging Cost Model

Definition 29.1 (Per-Trip Charging Cost)**.**

$$c_i^{(\text{charge})} = d_i^\dagger \cdot \eta \cdot p_i \tag{48}$$

where:

- $d_i^\dagger$ is the **total committed distance** for trip i (including enroute driving, not just the paid trip distance). If enroute distance is unavailable, d_i (paid distance) is used as a lower bound.
- η is the **energy consumption rate** (kWh/mile). For the Kia e-Niro, empirically $\eta \approx 0.28$ kWh/mile (3.6 mi/kWh).
- p_i is the **effective electricity price** for trip i (£/kWh). This is the blended price of the charging session that fuelled this trip.

Remark 29.2 (Charging Price Assignment). The price p_i is not determined at trip time—it is determined at *charging* time. A single charging session may fuel multiple trips. The practical implementation assigns a uniform $p_i = \bar{p}$ (session average price) to all trips fuelled by that session. Where session-level data is unavailable, the weekly average price $\bar{p}_w$ is used.

Charging Price Tiers

From empirical data (January 2026), Tesla Supercharger off-peak pricing:

Tier	**Locations**	**Price (£/kWh)**	**Window**
Off-peak (best)	Bluewater, Sidcup, Dorking	0.23	23:00–09:00
Off-peak (good)	Stevenage, Waddon, Grays	0.24–0.25	22:00–09:00
Off-peak (moderate)	Cheshunt, Guildford, Gatwick	0.26–0.27	22:00–09:00
Peak / emergency	Various	0.40–0.79	Daytime

The difference is material. For 38.4 kWh (20–80% charge on the e-Niro): £8.83 at £0.23/kWh vs. £15.36 at £0.40/kWh—a 74% cost premium for emergency charging. Over a year, disciplined off-peak charging saves approximately £1,700 versus habitual peak charging.

Impact on Net Trip Rate

At $\eta = 0.28$ kWh/mi and typical trip distance of 7.5 miles:

Charging Tier	$c_i^{(\text{charge})}$	**Effect on** ϱ_i^*
£0.23/kWh (off-peak)	£0.48/trip	−£1.45/hr on ϱ^*
£0.40/kWh (peak)	£0.84/trip	−£2.52/hr on ϱ^*
£0.79/kWh (emergency)	£1.66/trip	−£4.97/hr on ϱ^*

The emergency charging penalty (£4.97/hr) is substantial—it erodes nearly one-third of the target CNHR (£15/hr). Under the old model, this was invisible. Under the generalised model, it appears directly in ϱ_i^* and therefore in every downstream metric.

Wear and Maintenance Cost Model

Definition 29.3 (Per-Trip Wear Cost)**.**

$$c_i^{(\text{wear})} = d_i^{\dagger} \cdot w \tag{49}$$

where w (£/mile) is the wear rate encompassing tyre degradation, brake wear, suspension, and consumables.

Regime-Dependent Wear Rate

Regime	**Wear Rate** w	**Rationale**
Vehicle hire	$w \approx 0$	Wear is hire company's liability
Owned (new/warranty)	$w \approx £0.03–£0.05$/mi	Tyres + consumables only
Owned (post-warranty)	$w \approx £0.06–£0.10$/mi	Add repair contingency

Remark 29.4 (Maintenance Events). Major maintenance (tyre replacement, brake pads, unexpected repairs) can be modelled two ways:

(a) **Amortised into** w: Estimate annual maintenance cost, divide by annual miles, add to per-mile rate. This smooths the cost but may underestimate.

(b) **Lump-sum event**: Treat major repairs as a one-off addition to C_f for the week in which they occur. This creates a spike in that week's CNHR but is honest about the cash flow impact.

For regular wear (tyres, brakes), amortisation into w is correct. For unpredictable repairs (collision damage, component failure), lump-sum treatment is more accurate.

Per-Session Costs

Definition 29.5 (Per-Session Fixed Costs)**.** Costs incurred per driving session, not per trip:

- Congestion charge (£15/day if entering the zone)
- Parking fees at charging locations (if applicable)
- ULEZ charge (if non-compliant—not applicable for EV)

These are allocated equally across all trips within the session:

$$c_i^{(\text{session})} = \frac{C_{\text{session}}}{n_{\text{session}}} \tag{50}$$

Total Variable Cost Per Trip

Definition 29.6 (Composite Variable Cost)**.**

$$c_i = c_i^{(\text{charge})} + c_i^{(\text{wear})} + c_i^{(\text{session})} \tag{51}$$

Vehicle Regime Model

The generalised cost structure enables seamless transition between operational regimes without modifying the CNHR/MNHR framework. Only the parameter vector changes.

Regime Definitions

Definition 30.1 (Cost Regime)**.** A cost regime $\mathcal{R}$ is a parameter tuple:

$$\mathcal{R} = (C_f,\ \eta,\ \bar{p},\ w) \tag{52}$$

specifying the fixed weekly cost, energy consumption rate, expected charging price, and per-mile wear rate.

Hire Regime (Current)

Parameter	Value	Basis
C_f	£280/wk	Hire (£220) + insurance (£50) + phone (£10)
η	0.28 kWh/mi	e-Niro empirical
$\bar{p}$	£0.23–0.25/kWh	Off-peak Tesla SC
w	£0/mi	Wear is hire company's liability

Implied weekly variable cost at 400 mi/wk: $400 \times 0.28 \times 0.24 = £26.88$/wk.
Total weekly cost: $£280 + £26.88 \approx £307$/wk.

Remark 30.2. This is lower than the current $C_w = £430$ because the old model included charging within C_w as a fixed estimate. The generalised model makes the charging component explicit and variable, reducing the apparent fixed base while making total cost more accurate.

Ownership Regime (Hypothetical)

Parameter	Value	Basis
C_f	£130/wk (est.)	Finance (£70) + insurance (£40) + phone (£10) + servicing/MOT amortised (£10)
η	0.28 kWh/mi	Same vehicle assumed
$\bar{p}$	£0.23–0.25/kWh	Same charging strategy
w	£0.06/mi	Tyres + brakes + contingency

Implied weekly variable cost at 400 mi/wk: $400 \times (0.28 \times 0.24 + 0.06) = £26.88 + £24.00 = £50.88$/wk.
Total weekly cost: $£130 + £50.88 \approx £181$/wk.

Regime Comparison

Regime	C_f (£/wk)	**Variable** (£/wk)	**Total** (£/wk)	H_{target}
Hire (current)	280	∼27	∼307	17.1 h*
Ownership (est.)	130	∼51	∼181	10.1 h*

*$H_{\text{target}} = C_f/(\bar{\varrho}^* - r^*)$, where $\bar{\varrho}^*$ adjusts for variable costs. These are illustrative estimates; actual values require calibration from operational data under each regime.

Remark 30.3 (Strategic Implication). The ownership regime approximately halves the total weekly cost and reduces H_{target} from ∼17 to ∼10 hours. This transforms the binding constraint: at 10 hours/week required, even a two-shift week comfortably reaches target. The hours-dominance finding from the variance decomposition (Theorem 13.1) remains structurally true but its *severity* is dramatically reduced—the driver gains operational flexibility.

Shift Variability and Mixed-Pattern Weeks

The Variable Shift Problem

PHV driving has no fixed schedule. A given week may include any combination of night shifts, day shifts, partial shifts, and rest days. The single-parameter model treats all hours as equivalent. The generalised model recognises that the *cost profile* of an hour worked varies with context:

Shift Type	Charging Price	Demand	Congestion?	Net Impact
Night (19:00–07:00)	Off-peak (£0.23)	Moderate–High	No	Best ϱ^*
Day (07:00–19:00)	Peak (£0.40+)	Moderate	Yes (£15)	Worst ϱ^*
Split/Mixed	Blended	Variable	Partial	Middle

Why the Framework Handles This Without Modification

Because variable costs are allocated per-trip via c_i, a night trip and a day trip naturally have different net earnings e_i^* even if their platform earnings e_i are identical. The CNHR accumulates e_i^* without requiring any knowledge of shift boundaries.

A week with two night shifts and one day shift simply accumulates:

- Night trips: $e_i^* = e_i - c_i^{(\text{night})}$ where $c_i^{(\text{night})}$ uses off-peak charging
- Day trips: $e_i^* = e_i - c_i^{(\text{day})}$ where $c_i^{(\text{day})}$ uses peak charging + congestion allocation

The CNHR at week-end reflects the blended reality. No special-casing or conditional logic is required.

Shift-Type MNHR Comparison

The generalised MNHR enables direct comparison of shift profitability:

$$\text{AM}(m^*_{\text{night}}) \gtrless \text{AM}(m^*_{\text{day}}) \tag{53}$$

If night shifts yield consistently higher $\text{AM}(m^*)$, the driver has a quantitative basis for shift allocation beyond intuition. This comparison was not possible under the old model because costs were invisible at the trip level.

Revised Axiom and Parameter Definitions

The generalised model requires a revision to Axiom 1:

Axiom 6 (Decomposed Cost Structure (Revised))**.** The driver incurs two classes of cost per Uber week:

(a) A **fixed cost** C_f incurred in full regardless of trip volume.

(b) A **variable cost** c_i per trip, proportional to the committed distance and dependent on the prevailing energy price and wear rate.

The total cost $\mathcal{C}(n) = C_f + \sum_{i=1}^{n} c_i$ replaces the single parameter C_w.

Definition 32.1 (Revised Prospective Fixed Cost Rate)**.**

$$\hat{\lambda}_f = \frac{C_f}{\bar{H}} \tag{54}$$

This replaces $\hat{\lambda} = C_w/\bar{H}$ in all MNHR computations.

Definition 32.2 (Variable Cost Rate (Informational))**.**

$$\bar{v} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} d_i^{\dagger}} \tag{55}$$

is the realised variable cost per committed mile. This is a diagnostic metric, not a parameter in the CNHR formula (which uses per-trip c_i directly).

Impact on Variance Decomposition

Under the generalised model, the CNHR decomposition becomes:

$$r_n = \bar{\varrho}_n^* - \lambda_f \tag{56}$$

The variance decomposition now measures:

$$\mathrm{Var}(r_n) = \mathrm{Var}(\bar{\varrho}^*) + \mathrm{Var}(\lambda_f) - 2\,\mathrm{Cov}(\bar{\varrho}^*, \lambda_f) \tag{57}$$

Two effects:

(i) $\mathrm{Var}(\lambda_f)$ is *reduced* relative to $\mathrm{Var}(\lambda_w)$ because $C_f < C_w$—the fixed base is smaller, so the cost rate varies less with hours.

(ii) $\mathrm{Var}(\bar{\varrho}^*)$ may *increase* slightly because variable costs introduce trip-level variation not present in gross $\bar{\varrho}$.

The net effect is that the hours-dominance finding (90.7%) likely reduces to perhaps 75–85%, giving more weight to the net-rate lever. This means variable cost management (charging strategy, route efficiency) becomes a *more significant lever* than it was under the old model—precisely the visibility gain this extension is designed to provide.

Remark 33.1 (Recalibration Required). The exact variance shares under the generalised model require recalibration from data that includes per-trip variable costs. Until that data is collected, the 90.7% finding remains the best available estimate, with the understanding that it likely overstates hours-dominance by absorbing variable cost variance into the residual.

Implementation Notes

Data Requirements

The generalised model adds one field per trip:

Field	**Type**	**Description**
`var_cost`	float	Total variable cost c_i (£) for this trip
`charge_rate`	float	Effective £/kWh for the charging session

All existing fields remain unchanged. The `t_earn` field continues to represent platform gross earnings. A new computed field `net_earn` = `t_earn` − `var_cost` replaces `t_earn` in all CNHR/MNHR computations.

Sentinel Integration

For the Sentinel app's trip acceptance logic, the projected MNHR becomes:

$$\hat{m}^\dagger = \frac{\hat{e} - \hat{c}}{\hat{t} + \hat{t}^{\text{en}}} - \hat{\lambda}_f \tag{58}$$

where $\hat{c} = \hat{d}^\dagger \cdot \eta \cdot \bar{p} + \hat{d}^\dagger \cdot w$ is the estimated variable cost. This requires Sentinel to estimate trip distance at offer time (available from the Uber offer screen).

Configuration

The regime is specified as a configuration block:

```
var REGIME = {
  Cf:    280,          // fixed weekly cost (GBP)
  eta:   0.28,         // kWh per mile
  p_bar: 0.24,         // default charge price (GBP/kWh)
  w:     0.00,         // wear rate (GBP/mile), 0 for hire
  label: 'HIRE_ENIRO' // regime identifier
};
```

Switching regimes (e.g., hire → ownership) requires only updating this block.

Part IX: Cross-Industry Comparison

The Universal Structure of Transport Profitability

Every transport sector uses a common economic skeleton:

$$\text{Net Rate} = \text{Gross Revenue Rate} - \text{Cost Rate} \tag{59}$$

The CNHR decomposition ($r_n = \bar{\varrho}_n - \lambda_w$) is algebraically identical to this universal identity.

Concept	Aviation	Trucking	Bus/Fleet	CNHR–MNHR
Unit of production	ASM	Mile	Vehicle mile	Engaged hour
Revenue metric	RASM	Rev/mile	Rev/bus mile	$\bar{\varrho}$
Cost metric	CASM	CPM	Cost/bus mile	λ
Profitability test	RASM−CASM>0	Rev/mi−CPM>0	Rev−Cost>0	$\bar{\varrho} - \lambda > 0$
Utilisation	Load factor	Loaded vs DH	% in rev. svc	$u = t/t^\dagger$
Marginal signal	None	Informal	None	MNHR + Thm

The Aviation Parallel: CASM/RASM

The airline industry's CASM/RASM framework is the most mature transport profitability system in existence.

Structural Isomorphism

Aviation	Formula	CNHR Equiv.	Formula
CASM	Op. Cost / ASM	λ	C_w/T_n
RASM	Revenue / ASM	$\bar{\varrho}$	E_n/T_n
Operating Margin/ASM	RASM − CASM	CNHR	$\bar{\varrho}_n - \lambda_w$
Load Factor	RPM/ASM	Utilisation	$T_n/T_n^\dagger$
Block Hours	Gate-to-gate	True Time	$t^\dagger$
Airborne Hours	In-flight only	Paid Time	t
BE Load Factor	CASM/Yield	BE Hours	$C_w/\bar{\varrho}$

Remark 36.1. The isomorphism is not metaphorical. The CNHR decomposition is algebraically identical to airline Operating Margin per ASM = RASM − CASM.

Block Hours vs. True Time

Aviation distinguishes *block hours* (gate-to-gate, including taxi and idle) from *airborne hours* (wheels-off to wheels-on). Operating costs are computed per block hour; revenue generated only during airborne operation.

Aviation Phase	Rideshare Phase	Paid?
Taxi out	Enroute drive to pickup	No
Airborne	Engaged (pickup to dropoff)	Yes
Taxi in	Deadhead after dropoff	No
Gate idle	Idle waiting for dispatch	No

At $\bar{u}^{(T)} = 0.66$, the paid CNHR overstates by factor 1.52—the precise analogue of an airline computing profitability on airborne hours only while ignoring taxi time.

The Volume Imperative

The variance decomposition finding (90.7% from hours) has a direct aviation parallel. Airlines with high fixed costs achieve profitability through *aircraft utilisation* (block hours per aircraft per day), not per-flight yield. A low-cost carrier flying 12 block hours/day at moderate yield outperforms a carrier flying 8 hours/day at high yield. Same finding: **in high-fixed-cost transport, volume dominates per-unit revenue quality.**

The Trucking Parallel: Cost Per Mile

Owner-operator trucking presents the closest structural analogue. Both are independent operators with high fixed costs, variable revenue per unit, and the fundamental accept/reject decision.

An owner-operator computes CPM by dividing total expenses by total miles. A proposed load is profitable if revenue-per-mile exceeds CPM—structurally identical to $m_i > 0$. Truckers also explicitly distinguish loaded miles (revenue) from deadhead miles (repositioning empty)—the precise parallel to paid/true.

The critical difference: truckers apply this test informally, without cumulative tracking, directional theorem, EMA smoothing, or diagnostic state matrix.

Contributions Beyond Existing Frameworks

The Marginal Directional Signal (MNHR)

No transport framework provides a per-unit marginal signal predicting cumulative direction. Theorem 17.1 ($m_n > r_n \Rightarrow r_{n+1} > r_n$) is a mathematical identity with no equivalent. Combined with EMA (92.5% accuracy), this provides a real-time decision signal no other framework offers.

Dual-Track Time Accounting with Quantified Gap

While aviation distinguishes block/airborne and trucking distinguishes loaded/deadhead, no framework simultaneously tracks both in real time with a quantified utilisation coefficient. $r_n^\dagger = r_n \cdot \bar{u}^{(T)}$ provides a precise, continuously-updated measure of platform overstatement.

The Four-State Diagnostic Matrix

Standard transport KPI frameworks present individual metrics without state classification. The SUSTAINED/ACCEL RECOVERY/DECELERATING/STALLED matrix, combining level and momentum, has no precedent in the surveyed literature.

Comparative Assessment Summary

Contribution	**Nearest Analogue**	**Advancement**
MNHR directional thm	Trucker's informal BE check	Mathematical identity; 92.5% DA
Dual-track time	Aviation block/airborne	Continuous real-time + coeff.
Four-state matrix	Fleet KPI dashboards	Level + momentum combined
Variance decomposition	Industry volume intuition	Formal: 90.7% cost rate
AM–GM volatility	No equivalent	Model-free consistency metric

Within single-agent operational optimisation under partial information, the CNHR–MNHR framework is more rigorous than anything documented in rideshare literature or comparable owner-operator sectors.

Part X: Data Schema and Implementation

Week Object Schema

Field	Type	Description
`label`	string	Week label (e.g., "9 Dec – 15 Dec")
`key`	string	ISO week key
`total_n`	int	Trip count
`total_e`	float	Total portal earnings (£)
`total_h`	float	Total engaged hours
`total_mi`	int	Total miles
`final_rho`	float	Final cumulative ϱ
`final_r_n`	float	Final CNHR (paid)
`final_r_n_true`	float	Final CNHR (true)
`final_mnhr_ema`	float	Final MNHR EMA (paid)
`final_mnhr_true_ema`	float	Final MNHR EMA (true)
`final_state`	string	Last trip's four-state classification
`be_trip`	int/null	Break-even trip number
`target_trip`	int/null	Target trip number
`am_rho_paid`	float	AM(ϱ)
`gm_rho_paid`	float	GM(ϱ)
`am_rho_true`	float	AM($\varrho^{\dagger}$)
`gm_rho_true`	float	GM($\varrho^{\dagger}$)
`mean_util`	float	Average utilisation (0–1)
`mean_enroute`	float	Average enroute time (minutes)
`n_true`	int	Trips with paired enroute data
`trips`	array	Trip objects

Trip Object Schema

Field	Type	Description
`n`	int	Sequential trip number within week
`day`	string	Day of week
`time`	string	Start time HH:MM
`dt`	string	ISO datetime
`service`	string	Uber service type
`t_earn`	float	Trip earnings (£)
`t_dur`	float	Trip duration (hours)
`t_rho`	float	Trip gross rate: $\varrho_i = e_i/t_i$
`t_dist`	float	Trip distance (miles)
`mnhr`	float	MNHR (paid): $m_i = \varrho_i - \hat{\lambda}$
`mnhr_ema`	float	EMA of MNHR (paid)
`mnhr_true`	float	MNHR (true): $m_i^\dagger$
`mnhr_true_ema`	float	EMA of MNHR (true)
`cum_e`	float	E_n
`cum_h`	float	T_n (paid)
`cum_h_true`	float	$T_n^\dagger$ (true)
`r_n`	float	CNHR (paid)
`r_n_true`	float	CNHR (true)
`rho_cum`	float	$\bar{\varrho}_n$
`four_state`	string	Diagnostic state
`util`	float	u_i (utilisation)
`enroute_min`	float	Enroute minutes (0 if unavailable)

MNHR Computation (ES5 JavaScript)

```
// Constants
var CW = 430;
var ALPHA = 0.15;
var LAMBDA_HAT = CW / MEAN_H;  // prospective cost rate

// Per-trip computation within weekly loop
for (var j = 0; j < w.trips.length; j++) {
  var t = w.trips[j];

  // Gross rate
  t.t_rho = t.t_earn / t.t_dur;

  // MNHR (paid)
  t.mnhr = t.t_rho - LAMBDA_HAT;

  // True variant (if enroute available)
  var t_true = t.t_dur + t.enroute_min / 60;
  t.rho_true = t.t_earn / t_true;
  t.mnhr_true = t.rho_true - LAMBDA_HAT;

  // Cumulative
  t.cum_e = (j === 0) ? t.t_earn : w.trips[j-1].cum_e + t.t_earn;
  t.cum_h = (j === 0) ? t.t_dur : w.trips[j-1].cum_h + t.t_dur;
  t.cum_h_true = (j === 0) ? t_true : w.trips[j-1].cum_h_true + t_true;

  // CNHR
  t.r_n = (t.cum_e - CW) / t.cum_h;
  t.r_n_true = (t.cum_e - CW) / t.cum_h_true;
```

```
  // EMA
  if (j === 0) {
    t.mnhr_ema = t.mnhr;
    t.mnhr_true_ema = t.mnhr_true;
  } else {
    t.mnhr_ema = ALPHA * t.mnhr + (1 - ALPHA) * w.trips[j-1].mnhr_ema;
    t.mnhr_true_ema = ALPHA * t.mnhr_true
                    + (1 - ALPHA) * w.trips[j-1].mnhr_true_ema;
  }

  // Four-state
  var atTarget = t.r_n >= R_STAR;
  var improving = t.mnhr_true_ema > t.r_n;
  if (atTarget && improving) t.four_state = 'SUSTAINED';
  else if (atTarget && !improving) t.four_state = 'DECELERATING';
  else if (!atTarget && improving) t.four_state = 'ACCEL_RECOVERY';
  else t.four_state = 'STALLED';
}
```

AM/GM Computation

```
// After all trips processed
var sum_rho = 0, sum_ln_rho = 0;
var sum_rho_true = 0, sum_ln_rho_true = 0;
var n_true = 0;

for (var j = 0; j < w.trips.length; j++) {
  var t = w.trips[j];
  sum_rho += t.t_rho;
  sum_ln_rho += Math.log(t.t_rho);
  if (t.enroute_min > 0) {
    sum_rho_true += t.rho_true;
    sum_ln_rho_true += Math.log(t.rho_true);
    n_true++;
  }
}

w.am_rho_paid = sum_rho / w.trips.length;
w.gm_rho_paid = Math.exp(sum_ln_rho / w.trips.length);
if (n_true > 0) {
  w.am_rho_true = sum_rho_true / n_true;
  w.gm_rho_true = Math.exp(sum_ln_rho_true / n_true);
}
```

Offer Object Schema (Sentinel Extension)

The CNHR–MNHR specification operates on *accepted* trips. For the empirical testing of endogenous offer dynamics (§18.6) and future RL extensions (E9), the Sentinel application must also record *all offers seen*, including those declined. This schema is not required for CNHR computation but provides the data foundation for extensions.

Field	Type	Description
`offer_id`	string	Unique offer identifier
`dt`	string	ISO datetime of offer presentation
`action`	enum	`accepted` | `declined` | `expired` | `cancelled`
`est_earn`	float	Estimated earnings shown on offer screen (£)
`est_dur`	float	Estimated trip duration (hours)
`est_dist`	float	Estimated trip distance (miles)
`est_enroute`	float	Estimated enroute time (minutes)
`est_rho`	float	Estimated ϱ_i = `est_earn / est_dur`
`est_mnhr`	float	Projected MNHR: $\hat{m}^{\dagger}$
`pickup_zone`	string	Pickup area/zone identifier
`dropoff_zone`	string	Dropoff area/zone identifier
`surge_mult`	float	Surge multiplier (1.0 = no surge)
`service_type`	string	UberX, Comfort, Green, etc.
`driver_lat`	float	Driver latitude at offer time
`driver_lon`	float	Driver longitude at offer time
`session_accept_rate`	float	Running acceptance rate this session
`bonus_context`	object	Quest/consecutive bonus state (if active)

Remark 44.1. The offer schema captures the *decision environment*, not just the outcome. This enables retrospective analysis of: offer quality conditional on acceptance history, zone-dependent offer distributions, surge regime effects on MNHR, and the empirical hypotheses specified in §18.6. Minimum recommended data collection period before analysis: 8–12 weeks of offer-level data.

Data Integrity Invariants

The following invariants must hold for any valid dataset:

(i) $E_n = \sum_{i=1}^{n} e_i$ (cumulative earnings consistency).

(ii) $T_n = \sum_{i=1}^{n} t_i$ (cumulative hours consistency).

(iii) $r_n = (E_n - C_w)/T_n$ (CNHR formula).

(iv) $|\tilde{m}_n - [\alpha \cdot m_n + (1-\alpha) \cdot \tilde{m}_{n-1}]| < \epsilon$ (EMA recurrence).

(v) $\mathrm{AM}(\varrho) \geq \mathrm{GM}(\varrho)$ (AM–GM inequality).

(vi) Phase transitions are monotonically ordered: DEFICIT $\to$ RECOVERY $\to$ TARGET.

Part XI: Visualisation and Display

Architecture

The dashboard renders as a single-file HTML document with embedded JavaScript (ES5) and CSS. No external dependencies, no build step, no server. All data is embedded as a JavaScript object literal.

Chart System

Chart Modes

Mode	X-Axis	Series
CNHR (Dual-Track)	Trip #	r_n (paid), $r_n^\dagger$ (true), r^*
MNHR (EMA)	Trip #	$\tilde{m}_n$ (paid), $\tilde{m}_n^\dagger$ (true), r_n
Utilisation	Trip #	u_i per trip, $\bar{u}$ running mean

CNHR Chart (Dual-Track Enhancement)

The CNHR chart displays both paid and true variants simultaneously, with the area between them shaded to visualise the enroute penalty. The target line r^* and break-even line ($r_n = 0$) are displayed as horizontal references.

AM/GM Metrics Panel

For each week, display: $\mathrm{AM}(\varrho)$, $\mathrm{GM}(\varrho)$, Δ_{AG}, $\mathrm{AM}(\varrho^\dagger)$, $\mathrm{GM}(\varrho^\dagger)$, $\Delta^\dagger_{\mathrm{AG}}$. Colour-coded: green if $\Delta_{\mathrm{AG}} < £2/\mathrm{hr}$, amber if £2–£4/hr, red if $> £4/\mathrm{hr}$.

Colour Palette

Element	Colour	Hex
CNHR Paid	Amber	`#f59e0b`
CNHR True	Cyan	`#06b6d4`
MNHR EMA (Paid)	Purple	`#a855f7`
MNHR EMA (True)	Teal	`#14b8a6`
Target line	Green	`#059669`
Break-even	Grey	`#64748b`
Background	Slate-900	`#0f172a`

Appendices

Notation Reference

Symbol	Description
e_i	Trip earnings (£)
t_i	Trip duration, paid (hours)
t_i^{en}	Enroute time (hours)
$t_i^{\dagger}$	True committed time: $t_i + t_i^{\text{en}}$
$t_i^{\ddagger}$	Full committed time: $t_i + t_i^{\text{en}} + t_i^{\text{dh}}$
d_i	Trip distance (miles)
ϱ_i	Per-trip gross rate (paid): e_i/t_i
$\varrho_i^{\dagger}$	Per-trip gross rate (true): $e_i/t_i^{\dagger}$
E_n	Cumulative earnings after n trips
T_n	Cumulative paid hours
$T_n^{\dagger}$	Cumulative true hours
C_w	Weekly fixed cost (£430)
$\bar{H}$	Mean weekly hours (expanding window)
λ_w	Retrospective cost rate: C_w/T_w
$\hat{\lambda}$	Prospective cost rate: $C_w/\bar{H}$
r_n	CNHR (paid): $(E_n - C_w)/T_n$
$r_n^{\dagger}$	CNHR (true): $(E_n - C_w)/T_n^{\dagger}$
m_i	MNHR (paid): $\varrho_i - \hat{\lambda}$
$m_i^{\dagger}$	MNHR (true): $\varrho_i^{\dagger} - \hat{\lambda}$
$\tilde{m}_n$	EMA-smoothed MNHR (paid)
$\tilde{m}_n^{\dagger}$	EMA-smoothed MNHR (true)
α	EMA smoothing factor (0.15)
r^*	Target CNHR (£15/hr paid)
$\bar{\varrho}_n$	Cumulative gross rate (paid): E_n/T_n
u_i	Trip utilisation: $t_i/t_i^{\dagger}$
$\bar{u}^{(T)}$	Time-weighted utilisation: $T_n/T_n^{\dagger}$
n_{BE}	Break-even trip number
n^*	Target achievement trip number
Δ_{AG}	AM–GM gap
H_{target}	Weekly hours target (24 h)

Glossary

CNHR

Cost-Normalised Hourly Rate. Net earnings per hour after deducting weekly fixed costs.

MNHR

Marginal Net Hourly Rate. Per-trip net value above the time-proportional cost rate.

EMA

Exponential Moving Average. A smoothing filter with parameter α.

Utilisation
Fraction of committed time that is paid/engaged.

Enroute
Unpaid driving time from dispatch to passenger pickup.

Deadhead
Unpaid repositioning time between trips.

Break-Even
The trip at which cumulative earnings first cover weekly costs.

Target
The trip at which CNHR first reaches r^*.

Phase
One of DEFICIT, RECOVERY, or TARGET based on CNHR value.

Four-State
Diagnostic matrix combining CNHR level and MNHR direction.

Uber Week
Monday 04:00 to Sunday 03:59 UTC.

PHV
Private Hire Vehicle (UK licensing category).

NLW
National Living Wage (UK statutory minimum).

AM Arithmetic Mean.

GM Geometric Mean.

AM–GM Gap
The non-negative difference AM $-$ GM, a volatility indicator.

CASM
Cost per Available Seat Mile (aviation).

RASM
Revenue per Available Seat Mile (aviation).

CPM
Cost Per Mile (trucking).

Block Hours
Gate-to-gate time in aviation, including taxi and idle.

Airborne Hours
Wheels-off to wheels-on time in aviation.

Proofs and Derivations

Proof: Uniqueness of Time-Proportional Allocation

Theorem C.1. *Time-proportional allocation is the unique allocation basis under which:*

(a) Each trip's cost burden is $c_i = \lambda \cdot t_i$.

(b) The per-trip MNHR $m_i = \varrho_i - \lambda$ is constant in the cost rate.

(c) The CNHR is the time-weighted average of MNHRs (Equation 19).

Proof. Suppose an allocation basis assigns cost $c_i = f(x_i) \cdot C_w / \sum_j f(x_j)$ for some function f of trip characteristics x_i. Define the MNHR as:

$$m_i^{(f)} = \frac{e_i - c_i}{t_i} = \frac{e_i}{t_i} - \frac{f(x_i) \cdot C_w}{t_i \cdot \sum_j f(x_j)}$$

For the aggregation identity $\sum m_i^{(f)} \cdot t_i / T_n = r_n$ to hold, we need $\sum f(x_i) / \sum f(x_j) = 1$, which holds trivially. However, for the MNHR to have a constant cost rate (i.e., c_i/t_i independent of i), we need $f(x_i)/t_i = k$ for all i, which forces $f(x_i) = k \cdot t_i$. This is precisely time-proportional allocation. □

Proof: AM–GM Gap Approximation for Log-Normal Rates

If $\ln \varrho_i \sim N(\mu, \sigma^2)$, then:

$$\mathrm{AM}(\varrho) = \exp(\mu + \sigma^2/2) \tag{60}$$

$$\mathrm{GM}(\varrho) = \exp(\mu) \tag{61}$$

Therefore:

$$\frac{\mathrm{GM}}{\mathrm{AM}} = \exp(-\sigma^2/2) \approx 1 - \sigma^2/2$$

and $\Delta_{\mathrm{AG}} = \mathrm{AM} - \mathrm{GM} = \mathrm{AM}(1 - \exp(-\sigma^2/2)) \approx \mathrm{AM} \cdot \sigma^2/2$.

Future Extensions

E1: Deadhead integration
Requires GPS/session logging. Algebraic framework ready (§10); only data source pending.

E2: Sentinel integration
Real-time MNHR computation within the Sentinel iOS app for trip acceptance recommendations.

E3: Time-of-day analysis
Decompose MNHR by hour-of-day to identify optimal shift timing.

E4: Geospatial analysis
Correlate enroute times with pickup locations to identify high-utilisation zones.

E5: Multi-platform
Extend to Bolt, FreeNow by generalising cost structure.

E6: Dynamic cost rate
Replace static $\hat{\lambda}$ with Bayesian estimate updating as current week's hours accumulate.

E7: Confidence intervals
Bootstrap-based uncertainty bounds on CNHR and MNHR trajectories.

E8: Surge/demand overlay
Correlate MNHR patterns with Uber surge pricing and demand data.

E9: Reinforcement learning layer

Formulate the acceptance/repositioning decision as a POMDP with state $S_n = (r_n, \tilde{m}_n, A_n, G_n)$, action $a_n \in \{\text{accept}, \text{decline}, \text{reposition}\}$, and reward $R_n = r_{n+1} - r_n$. The CNHR–MNHR framework provides the reward signal and state evaluation; the RL layer would optimise the policy. Requires empirical estimation of platform dispatch response to acceptance behaviour (see §18.6).

End of Specification