**Introduction:**

In 1860, James Clerk Maxwell introduced a distribution known as Maxwell distribution which was further modified by Ludwig Boltzmann (P.M. Morse, 1969).According to the name of these two pioneers the distribution is named as Maxwell- Boltzmann distribution or simply Maxwell distribution. Since the invention of this distribution it is used in lifetime modelling, chemistry as well as statistical mechanics (Malik, 2012) (Panwar, 2015) (S. M. A. Kazmi, 2011).In statistics we have different types of distributions to model the life time data such as weibull distribution, gamma distribution, exponential distribution extreme value distribution etc. But in real life we got different kinds of situations where these type of popular distributions are not suitable. For this we have to use other statistical distributions to fit the life time data .Tyagi and Bhattacharya (Bhattacharya, 1989) used Maxwell distribution as a possible lifetime distribution. From the literature we also know that airbags that is used to help the passenger reduce their speed in crash without getting injured which contains a mixture of different gases where the gas speed follows Maxwell distribution (Islam, 2015). Indeed Maxwell distributions plays a vital role in different fields of statistics. So we need a more efficient Maxwell distribution for this purpose we introduce the Inverse Maxwell distribution. Inverse distributions arise in particular in the [Bayesian](https://en.wikipedia.org/wiki/Bayesian_inference) context of [prior distributions](https://en.wikipedia.org/wiki/Prior_distribution) and [posterior distributions](https://en.wikipedia.org/wiki/Posterior_distribution) for [scale parameters](https://en.wikipedia.org/wiki/Scale_parameter) If a random variable follows Maxwell distribution then the distribution of the inverse of that random variable is called Inverse Maxwell distribution.

**Literature review:**

In 1915, Schrodinger proposed inverse Gaussian distribution which is also known as Wald distribution that is used for Brownian motion (wikipedia.org).From then the era of the inverse distributions was started. In statistics and probability theory, there are different kindly of inverse distributions are available such as inverse uniform distribution, inverse t-distribution, inverse Cauchy distribution, inverse gamma distributions etc (wikipedia.org). In 2014 Kusum Lata Singh and R.S. Srivastava provided the pdf, cdf, survival and hazard function and the MLE estimation of the scale parameter of the inverse Maxwell distribution. They also show the graph of the pdf, cdf, survival and hazard function but there are some problems in the graphs. Also they didn’t use the standard form of Maxwell distribution to drive the inverse Maxwell distribution (Kusum Lata Singh, Inverse Maxwell Distribution as a Survival Model, Genesis and Parameter, 2014).In another paper both of them are discussed about the size-biased inverse Maxwell distribution where they show the Bayesian estimation of an inverse Maxwell distribution via size-biased sampling, bayes estimator of the scale parameter of the inverse Maxwell distribution under squared error, entropy and another two loss functions for using quasi-prior (Kusum Lata Singh, Estimation of the Parameter in the Size-Biased, 2014).There is no further research on inverse Maxwell distribution are available.

**Problem statement:**

Problem 1: From the literature we know that only the few properties of the inverse Maxwell distribution is known so we need to find out the other properties of the inverse Maxwell distribution by using the standard form of the Maxwell distribution.

Problem 2: In the literature we know that the actual graph of the pdf, cdf, survival and hazard functions are not correctly drawn. So we need to find the actual figure of the graphs.

Problem 3: There is no real life applications of the inverse Maxwell distribution is available in the literature. So we need to find out the applications and compare this with the Maxwell distributions by doing this we can find out that which distribution gives better result on life time data.

Problem 4: we known that for small sample size the ranked set sampling is gives better result than the simple random sampling. So here we will find out the MLE of the scale parameter under the ranked set sampling and compare it with simple random sampling. Then we will find that which sampling technique works better under the inverse Maxwell distribution.

**Research methodology:**

(i) Jacobian transformation: In order to the change the variable in an integral we usually jacobian transformation. It is denoted by J. Let, . Then the jacobian will be |J|=||=|=.

(ii) Probability density function: If R is a continuous random variable, then the function f(r), defined on the continuous sample space Ω with domain the real line and counter domain the interval [0,1], is called probability density function or simply density function.

(iii) Cumulative distribution function: If R is a continuous random variable with density function f(r), the cumulative distribution function of R, denoted by F(r), is defined by

(iv)Raw Moment: For a sample of size n, consider a set of observations to be r1,r2,….rn..The sth raw moment is given by,

(v) Central Moment: Central moments are denoted by It can be found from the raw moments by the following relationships,

=-3+2

= – 3

(vi)Skewness: The asymmetric property of the curve is known as skewness. It is denoted by and defined by,

(vii)Kurtosis: The height characteristics of frequency curve is known as kurtosis. It is denoted by and defined by

(viii) survival function: Let, R be a non-negative random variable representing the life time of individuals in some population. Let, f(r) be the pdf of R and let the cdf be,

The probability of an individual surviving to time r is given by the survival function,

S(r) is a monotonic decreasing function with S(0)=1 and S()=0.

(ix)Hazard function: A very important concept with life time distribution is the hazard function h(t), defined as

(x)Fisher Information: Fisher information is denoted by and defined by

(xi)Cramer-Rao Lower bound: From the Cramer- Rao lower bound theorem we know that it is

defined by,

(xii)Entropy: Entropy is a measure of lack of order or predictability or uncertainty. It is usually denoted by H(R) and defined by,

(xiii)Moment generating function: If R is a continuous random variable then the moment generating function about origin denoted by and defined by

(ixv)Characteristics function: If R is a continuous random variable then the Characteristics function about origin denoted by and defined by

We also use statistical software R to plot the graph of the distribution and MS excel to estimate the scale parameter of the distribution.

**Preliminary results:**

(i)we know that, The Maxwell distribution with scale parameter σ has the following probability density function (pdf)

; r>0

So, the probability density function (pdf) of the inverse Maxwell distribution is given by

(ii) we know that, The Maxwell distribution with scale parameter σ has the following cumulative distribution function(cdf)

So, the cumulative distribution function(cdf) of the inverse Maxwell distribution is given by

(iii)The Survival function of the Inverse Maxwell distribution is

(iv)The Hazard function of the Inverse Maxwell distribution is

**Possible applications:**

In a vertical boring machine Manufacturing Company, the lifetimes of all the  
vertical boring machines produced by manufacturer may be considered as a population. We know that this data follows Maxwell distribution which is a positively skewed distribution and so the inverse Maxwell distribution is also positively skewed. For this we hope that this will also follow inverse Maxwell distribution.

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