Homework 2

ADSI Problem 1.4: Filter decomposition

Let the system function for a FIR filter be given by

$$H(z) = 1 - 3z^{-1} + \frac{5}{2}z^{-2} - z^{-3}$$

1. Decompose the system function into a product of a minimum-phase filter and an all-pass filter, $H(z) = H_{min}(z)H_{ap}(z)$.

Any system funcrtion can be decomposed into a product of a minimum-phase filter and an all-pass filter using the following formula:

$$H(z) = H_{\min}(z)H_{\text{ap}}(z)$$

The minimum phase filter can be computed as follows:

$$H_{\min}(z) = -\frac{1}{a^*}H_1(z)(1 - az^{-1})$$

where $H_1(z)$ corresponds to the part of the transfer function where zeros are inside the unit circle and $a = \frac{1}{{z_0}^*}$ and z_0 is the zero outside the unit circle.

The allpass filter can be calculated:

$$H_{\text{ap}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$

The filter decomposition algorithm has following steps:

1. convert transfer function into pole-zero representation in order to find the zeros that are outside the unit circle
2. compute a and its conjugate a^*
3. find $H_1(z)$ which corresponds to the part of the transfer function where zeros are inside the unit circle
4. plugin the numbers for the formula for the minimum-phase filter
5. plugin the numbers for the formula for the allpass filter
6. put everything together:

Step 1: convert transfer function into pole-zero representation in order to find the zeros that are outside the unit circle

```
zeros = roots([1, -3, 5/2, -1])
```

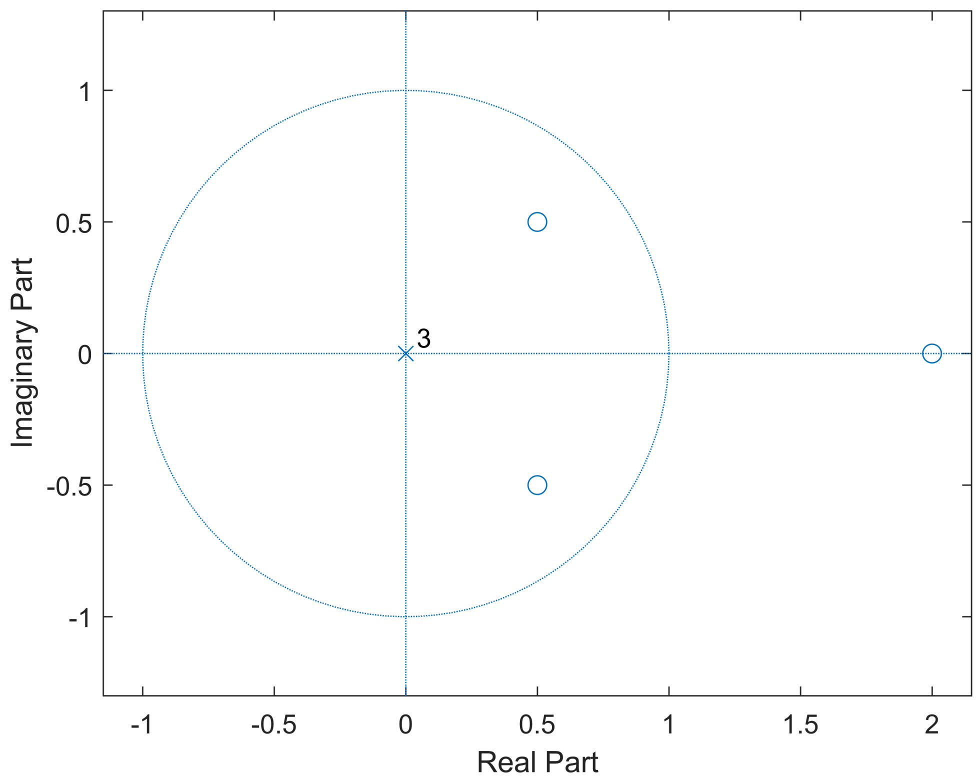
```
zeros = 3×1 complex
   2.0000 + 0.0000i
   0.5000 + 0.5000i
   0.5000 - 0.5000i
```

The zero-pole representation of the transfer function is:

$$H(z) = (1)(1 - 2z^{-1})(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})$$
$$H(z) = 1(1 - 2z^{-1})(0.5 - 0.5\jmath\, z^{-1})(0.5 + 0.5\jmath z^{-1})$$

To find the zero z_0 that is outside the unit circle, we can plot zplane:

```
zplane([1, -3, 5/2, -1]);
```



In this exercise, the zero outside the unit circle is $z_0 = 2$.

Step 2: compute a and its conjugate a^*

$$a = \frac{1}{z_0} = \frac{1}{2}$$

$$a^* = \left(\frac{1}{z_0}\right)^* = \left(\frac{1}{2}\right)^* = \frac{1}{2}$$

Step 3: find $H_1(z)$ which corresponds to the part of the transfer function where zeros are inside the unit circle

The zero-pole representation of the transfer function is:

$H(z) = (1)(1 - 2z^{-1})(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})$

therefore $H_1(z)$ is

$H_1(z) = (1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})$

Step 4: plugin the numbers for the formula for minimum-phase filter

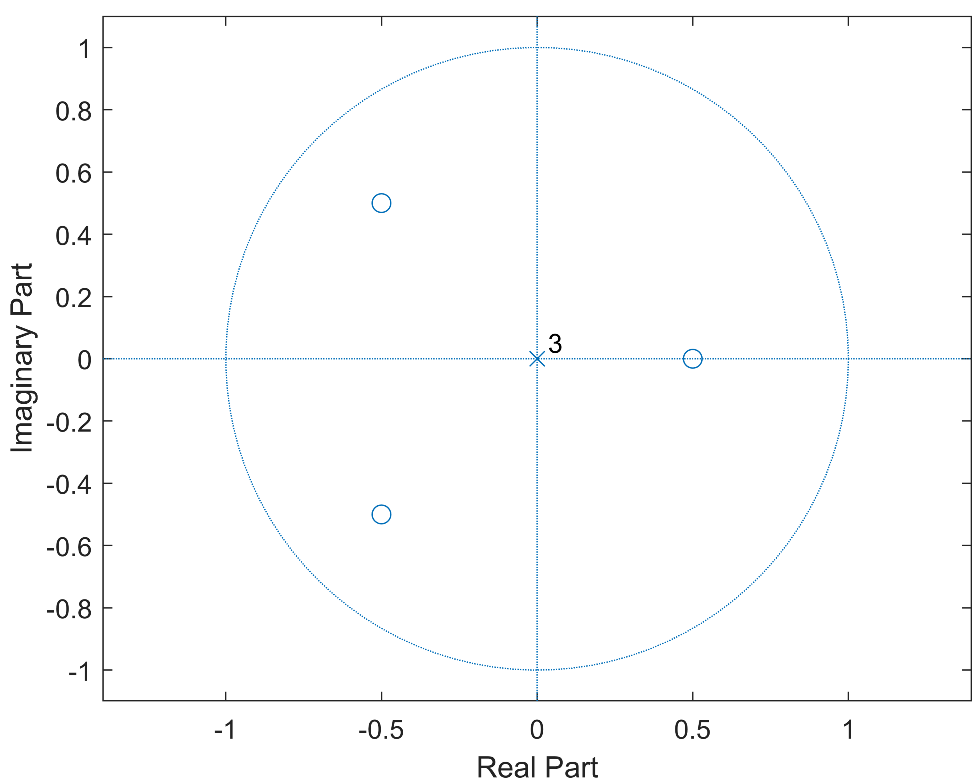
$H_{\min}(z) = -\frac{1}{a^*}H_1(z)(1 - az^{-1})$

$H_{\min}(z) = -\frac{1}{\left(\frac{1}{2}\right)}(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})(1 - 0.5z^{-1})$

$H_{\min}(z) = -2(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})(1 - 0.5z^{-1})$

The minimim-phase filter should have all its zeros inside the unit cirlce:

```
b0 = -2;
b1 = [1, 0.5+0.5i];
b2 = [1, 0.5-0.5i];
b3 = [1,-0.5];
b = b0 * conv(conv(b1, b2), b3);
zplane(b);
```



We can use Matlab function `isminphase(b,a)` to determine whether a filter is a minimum-phase filter.

```
isminphase(b)
```

```
ans = logical
   1
```

Step 5: compute the allpass filter :

$$H_{\mathrm{ap}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$

$$H_{\mathrm{ap}}(z) = \frac{z^{-1} - 0.5}{1 - 0.5z^{-1}}$$

Step 6: put everything together:

$$H_{\min}(z) = -2(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})(1 - 0.5z^{-1})$$

$$H_{\mathrm{ap}}(z) = \frac{z^{-1} - 0.5}{1 - 0.5z^{-1}}$$

$$H(z) = H_{\min}(z)H_{\mathrm{ap}}(z)$$

$$H(z) = -2(1 - (0.5 + 0.5\jmath)z^{-1})(1 - (0.5 - 0.5\jmath)z^{-1})(1 - 0.5z^{-1})\left(\frac{z^{-1} - 0.5}{1 - 0.5z^{-1}}\right)$$