

How to represent polynomials in Matlab?

Representation of polynomials in MATLAB Since most practical z -transforms are a ratio of polynomials, we start by explaining how MATLAB handles polynomials. In MATLAB polynomials are represented by *row* vectors containing the coefficients of the polynomial in decreasing order. For example, the polynomial

$$B(z) = 1 + 2z^{-1} + 3z^{-3}$$

is entered as `b=[1,2,0,3]`. We stress that even though the coefficient of the z^{-2} term

```
b = [1 2 0 3]
```

```
b = 1x4
     1     2     0     3
```

How to compute the roots of a polynomial?

```
b = [1 1.5 2];
z = roots(b)
```

```
z = 2x1 complex
    -0.750000000000000 + 1.198957880828180i
    -0.750000000000000 - 1.198957880828180i
```

How to compute partial fraction expansion?

Example 3.10 Partial fraction expansion using `residuez`

The following expansion:

$$X(z) = \frac{6 - 10z^{-1} + 2z^{-2}}{1 - 3z^{-1} + 2z^{-2}} = 1 + \frac{2}{1 - z^{-1}} + \frac{3}{1 - 2z^{-1}}, \quad (3.45)$$

is obtained by calling `residuez` with `b=[6,-10,2]` and `a=[1,-3,2]`. The reverse operation can be done using the same function as: `[b,a]=residuez(A,p,C)`.

```
b = [ 6 -10  2];
a = [ 1  -3  2];
```

```
% Partial fraction expansion using residuez
[A,p,C] = residuez(b, a)
```

```
A = 2x1
    3
    2
p = 2x1
    2
    1
C =
    1
```

```
% Reverse operation
[b, a] = residuez(A, p, C)
```

```
b = 1x3
    6   -10    2
a = 1x3
    1    -3    2
```

How to perform polynomial multiplication?

Polynomial multiplication in MATLAB The convolution theorem (3.52) shows that polynomial multiplication is equivalent to convolution. Therefore, to compute the product

$$\begin{aligned} B(z) &= (1 + 2z^{-2})(1 + 4z^{-1} + 2z^{-2} + 3z^{-3}) \\ &= 1 + 4z^{-1} + 4z^{-2} + 11z^{-3} + 4z^{-4} + 6z^{-5}, \end{aligned}$$

we use the function

```
>> b=conv([1 0 2],[1 4 2 3])
b =
    1     4     4    11     4     6
```

to find the coefficients of $B(z)$.

How to convert a transfer function $H(z)$ to its frequency response $H(e^{j\omega})$?

Suppose we have the following transfer function:

$$H(z) = \frac{0.05634(1 + z^{-1})(1 - 1.0166z^{-1} + z^{-2})}{(1 - 0.683z^{-1})(1 - 1.4461z^{-1} + 0.7957z^{-2})}$$

We can express the numerator and denominator as polynomial convolutions:

```
b0 = 0.05634;
b1 = [1 1];
b2 = [1 -1.0166 1];
```

```

a1 = [1 -0.683];
a2 = [1 -1.4461 0.7957];

b = b0*conv(b1,b2);
a = conv(a1,a2);

```

We can use the freqz function to get the frequency response as a vector. The second output variable w is the angular frequencies.

```

[H,w] = freqz(b,a);

```

How to plot the impulse response from a transfer function?

Suppose we have the following transfer function:

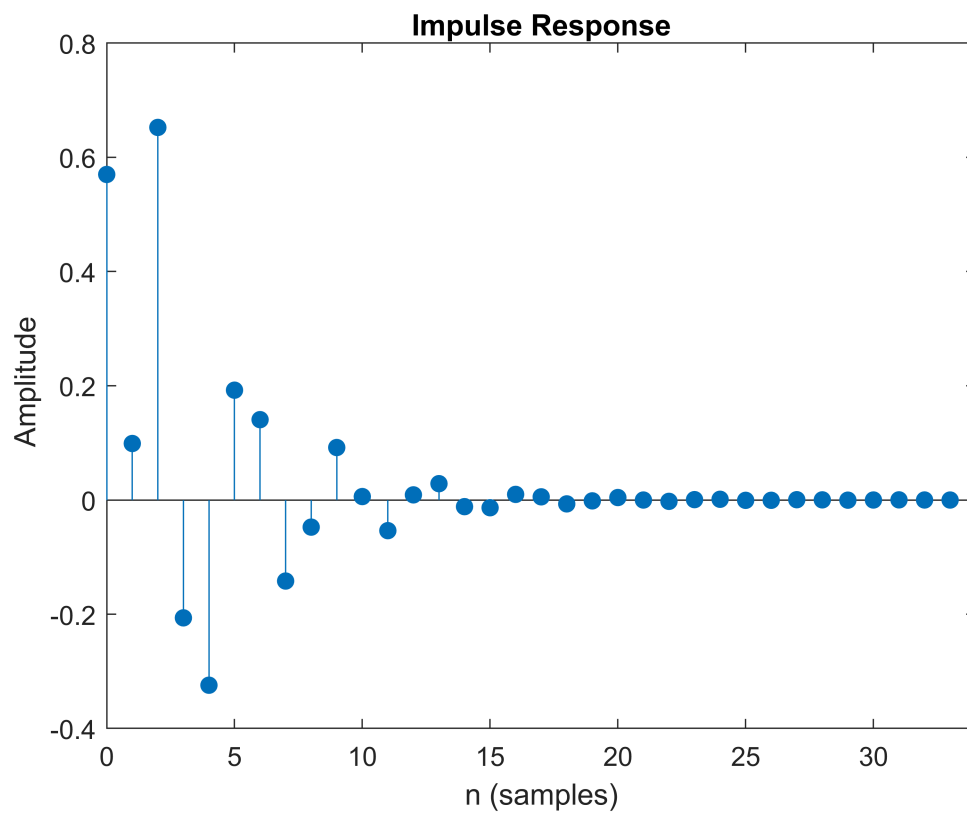
$$H(z) = \frac{0.57 + 0.23z^{-1} + z^{-2}}{1 + 0.23z^{-1} + 0.57z^{-2}}$$

We can use the impz function:

```

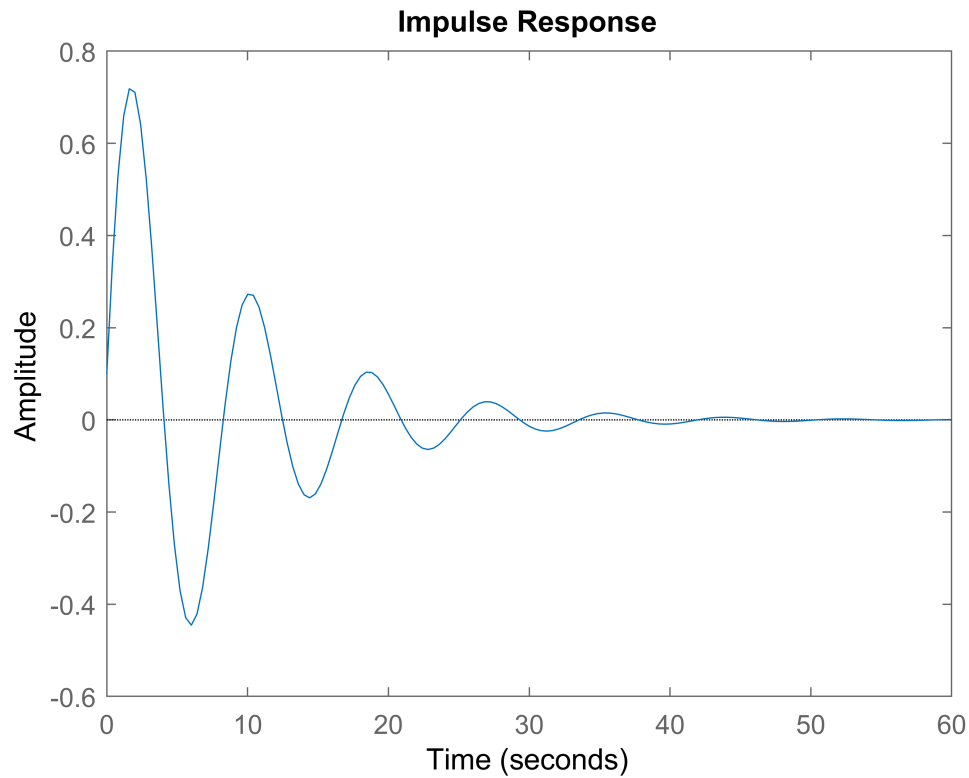
b = [0.57, 0.23, 1];
a = [1, 0.23, 0.57];
impz(b, a);

```



Alternatively,

```
b = [0.57, 0.23, 1];  
a = [1, 0.23, 0.57];  
h = tf(b, a);  
impzplot(h);
```



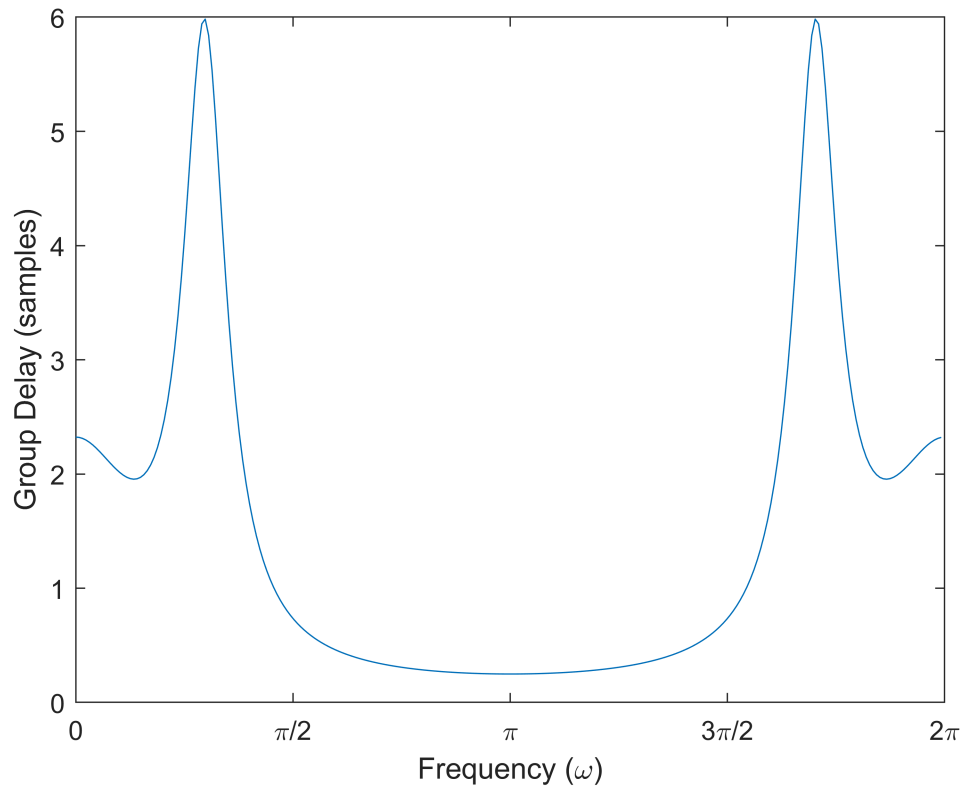
How to compute the group delay?

Suppose we have the following transfer function:

$$H(z) = \frac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}},$$

```
b = [1, 1.655, 1.655, 1];
a = [1, -1.57, 1.264, -0.4];
[gd, w] = grpdelay(b, a, 255, 'whole');

plot(w, gd);
set(gca, 'XTick', 0:pi/2:2*pi)
set(gca, 'XTickLabel', {'0', '\pi/2', '\pi', '3\pi/2', '2\pi'})
xlabel('Frequency (\omega)')
ylabel('Group Delay (samples)')
xlim([0, 2*pi]);
```

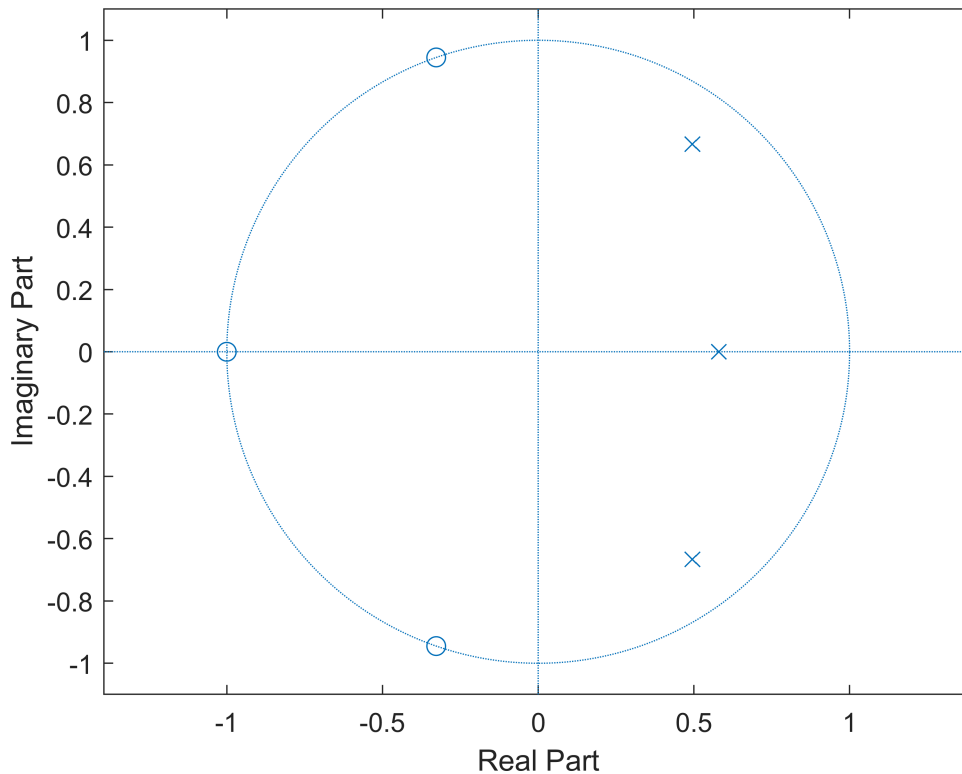


How to plot the zeros and poles?

Suppose we have the following transfer function:

$$H(z) = \frac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}},$$

```
b = [1, 1.655, 1.655, 1];
a = [1, -1.57, 1.264, -0.4];
zplane(b, a);
```



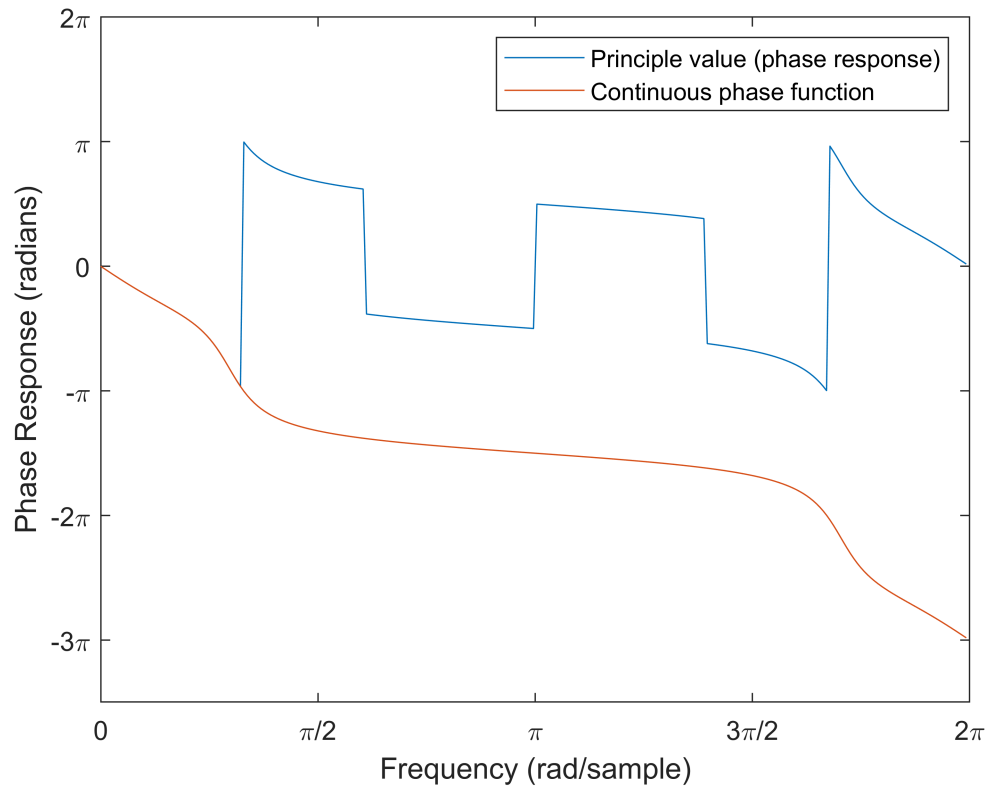
How to plot phase response (principle value) and continuous phase function?

Suppose we have the following transfer function:

$$H(z) = \frac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}}$$

```
b = [1, 1.655, 1.655, 1];
a = [1, -1.57, 1.264, -0.4];

[gd, w] = grpdelay(b, a, 255, 'whole');
[H, w] = freqz(b, a, 255, 'whole');
plot(w, angle(H), w, contphase(gd, w));
legend('Principle value (phase response)', 'Continuous phase function')
set(gca, 'XTick', 0:pi/2:2*pi)
set(gca, 'XTickLabel', {'0', '\pi/2', '\pi', '3\pi/2', '2\pi'})
set(gca, 'YTick', -3*pi:pi:3*pi)
set(gca, 'YTickLabel', {'-3\pi', '-2\pi', '-\pi', '0', '\pi', '2\pi', '3\pi'})
xlabel('Frequency (rad/sample)')
ylabel('Phase Response (radians)')
xlim([0, 2*pi]);
ylim([-3.5*pi, 2*pi]);
```

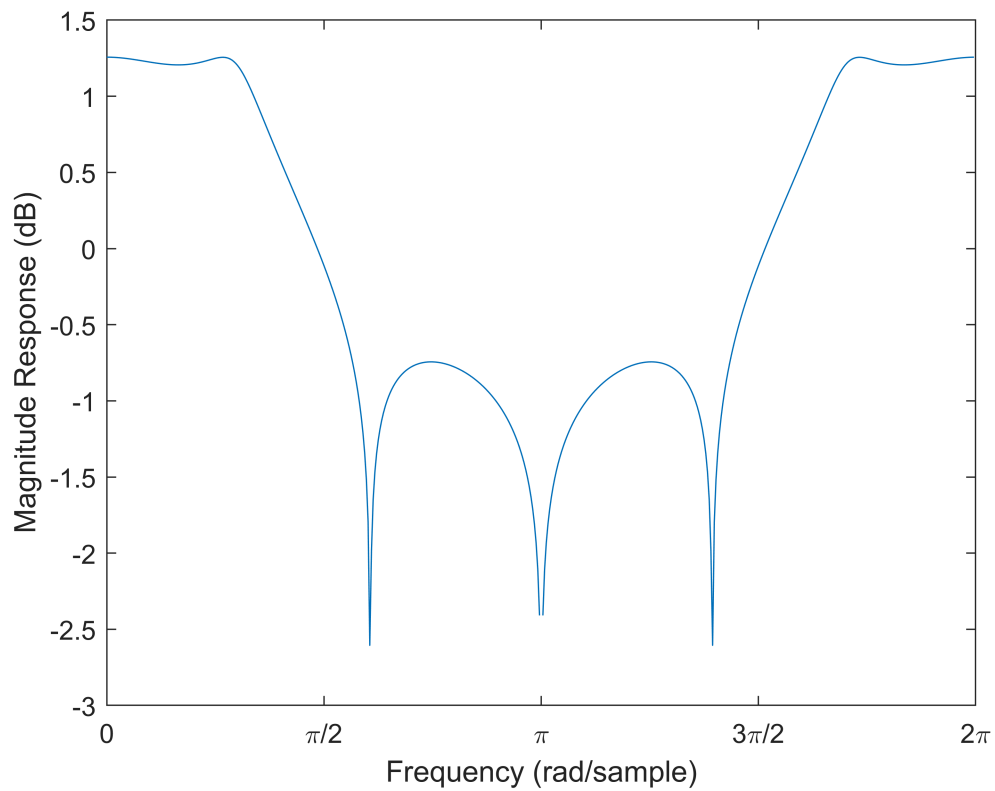


How to plot the magnitude response?

Suppose we have the following transfer function:

$$H(z) = \frac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}},$$

```
[H, w] = freqz(b, a, 'whole');
plot(w, log10(abs(H)));
set(gca, 'XTick', 0:pi/2:2*pi)
set(gca, 'XTickLabel', {'0', '\pi/2', '\pi', '3\pi/2', '2\pi'})
xlabel('Frequency (rad/sample)')
ylabel('Magnitude Response (dB)')
xlim([0, 2*pi]);
```

```
function cph=contphase(grd,om)
% Computation of continuous phase function
% from equidistant values of group delay
N=length(om);
dom=om(2)-om(1);
p(1)=0;
for k=2:N
    p(k)=p(k-1)+dom*(grd(k-1)+grd(k))/2;
end
cph=-p;
end
```