Multirate Signal Processing

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Multirate Signal Processing

Discrete-time systems with different sampling rates at various parts of the system are called *multirate systems*.

The fundamental operations for changing the sampling rate are:

- decimation
- interpolation

Sampling rate conversion

Conceptually, the sampling rate conversion process can be regarded as a two-step operation:

1. the discrete-time signal is reconstructed into a continuous-time signal
2. the signal is resampled at a different sampling rate

In practice, the conversion is implemented using discrete-time signal processing without actual reconstruction of any continuous-time signal.

The term *resampling* is used to refer to the process of changing the *sampling rate* of a discrete-time signal without reconstructing the equavilent continous-time signal.

There are three resampling operations:

- $T_0 = \mathrm{DT}$, downsampling i.e., decreasing the sampling rate by an integer factor D
- $T_0 = \frac{T}{I}$, upsampling i.e., increasing the sampling rate by an integer factor I
- $T_0 = T\left(\frac{D}{I}\right)$, changing the sampling rate by non-integer factor

where T is the sampling rate of the original signal, T_0 is the new sampling rate, and D and I are integers.

Downsampling

[»] Problem 12.21: Resampling downsampled sequences

Using the `downsample` function, resample the following sequences using the given parameters D and the offset k. Using the `stem` function, plot the original and downsampled signals.

(a) $x[n] = \sin(0.2\pi n)$, $0 \leq n \leq 50$, $D = 4$, $k = 0$, and $k = 2$.

(b) $x[n] = \cos(0.3\pi n)$, $0 \leq n \leq 60$, $D = 3$, $k = 0$, and $k = 1$.

[»] Problem 12.22: Resample decimated sequences

Using the `decimate` function, resample the following sequences using the given parameters D. Using the `stem` function, plot the original and decimated signals. Obtain results using both the default IIR and FIR decimation filters and comment on the results.

(a) $x[n] = \cos(0.4\pi n)$, $0 \leq n \leq 100$, $D = 2$.

(b) $x[n] = \sin(0.15\pi n)$, $0 \leq n \leq 100$, $D = 3$.

[✔] Problem 12.26: Interpolation using MATLAB's interp function

```
clear variables;
```

Let $x[n] = 2\cos(0.1\pi n) + \sin(0.05\pi n)$, $0 \leq n \leq 60$.

(a) Using the `interp` function, interpolate using $I = 3$, $I = 6$, and $I = 9$. Stem plot the original and interpolated signals.

(b) Using the second output argument of the `interp` function, plot the frequency response of the lowpass filter used in each of the above interpolations.

a) Interpolate signal using MATLAB's interp function

(a) Using the `interp` function, interpolate using $I = 3$, $I = 6$, and $I = 9$. Stem plot the original and interpolated signals.

```
n = 0:60;
x = 2*cos(0.1*pi*n) + sin(0.05*pi*n);

stem(n, x)
title('Original signal')
stem(interp(x, 3))
title('Interpolated signal, I=3')
stem(interp(x, 6))
title('Interpolated signal, I=6')
stem(interp(x, 9))
title('Interpolated signal, I=9')
```

b) Plot the frequency response of the lowpass filter

(b) Using the second output argument of the `interp` function, plot the frequency response of the lowpass filter used in each of the above interpolations.

`[y,b] = interp(x,r,n,cutoff)` returns a vector, b, with the filter coefficients used for the interpolation.

```
[y, b] = interp(x, 3);
freqz(b, 1)
```

[»] Problem 12.19: Compare spectrum of a downsampled signal with the original

```
clear variables;
```

Consider the signal $x[n] = 0.9^n u[n]$. It is to be downsampled by a factor of $D = 3$ to obtain $x_D[n]$.

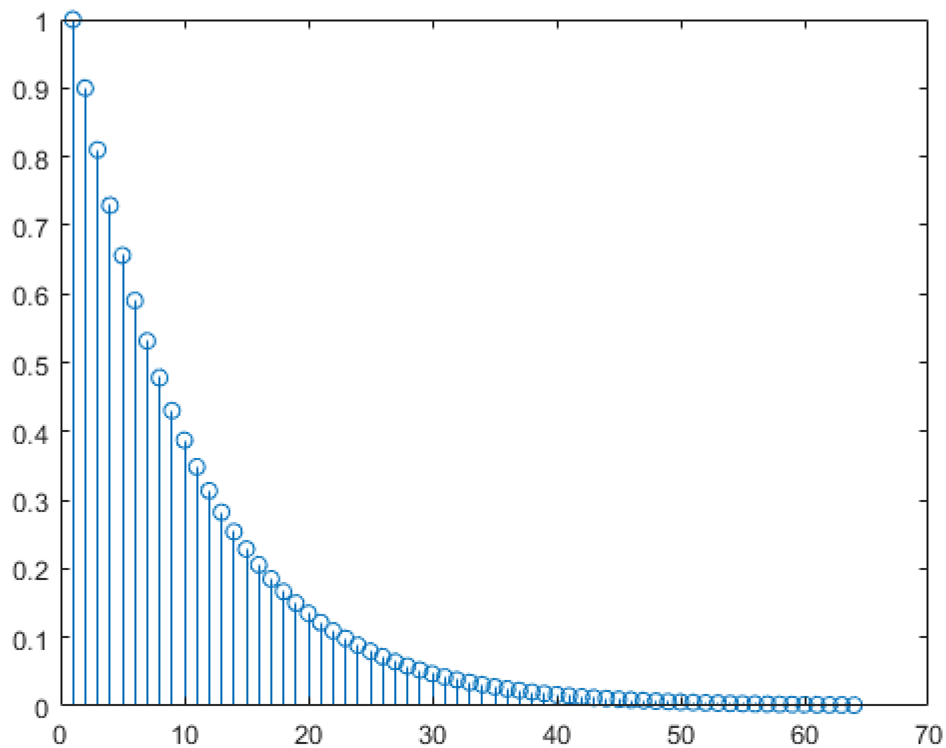
(a) Compute the spectrum of $x[n]$ and plot its magnitude.
(b) Compute the spectrum of $x_D[n]$ and plot its magnitude.
(c) Compare the two spectra.

The *unit step* sequence is given by

$$u[n] \triangleq \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

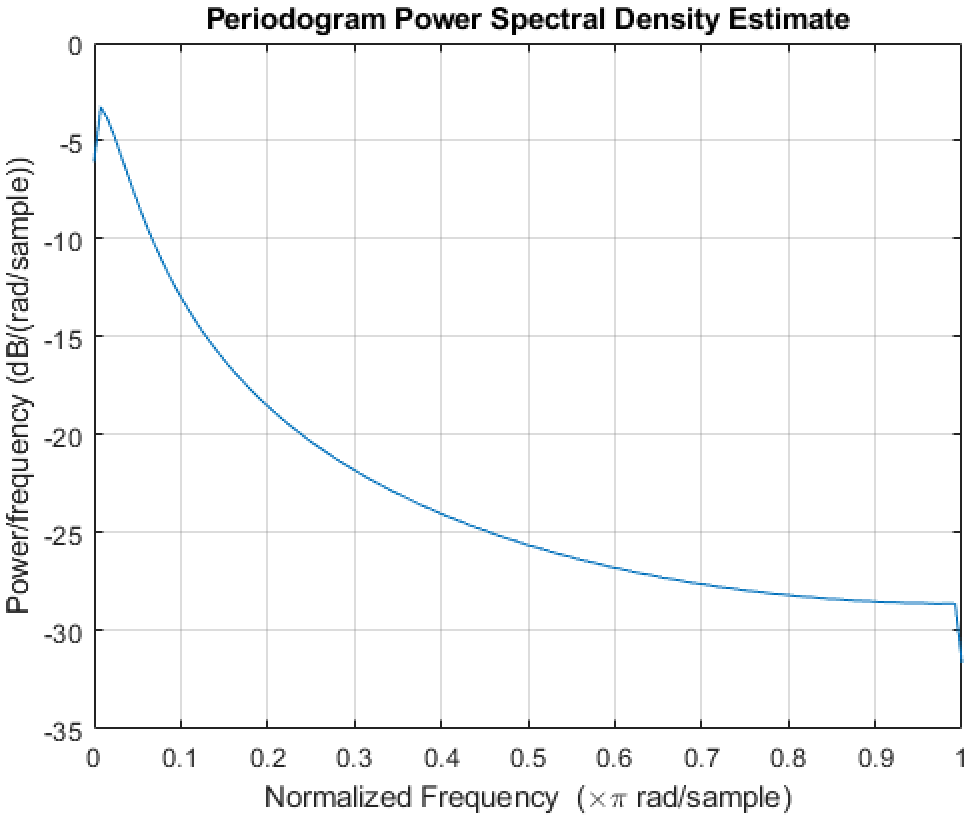
a) Compute the spectrum of the original signal and plot its magnitude

```
N = 64;
n = 0:N-1;
x = 0.9.^n;
% x = exp(-n);
stem(x)
```



[?] How do you compute the spectrum of the signal?

```
periodogram(x)
```



a) Compute the spectrum of the signal and plot its magnitude

b) Compute the spectrum of the downsampled signal and plot its magnitude

c) Compare the two spectra

ADSI Problem 8.1: Upsampling with linear interpolation

```
clear variables;
```

Consider a signal given by:

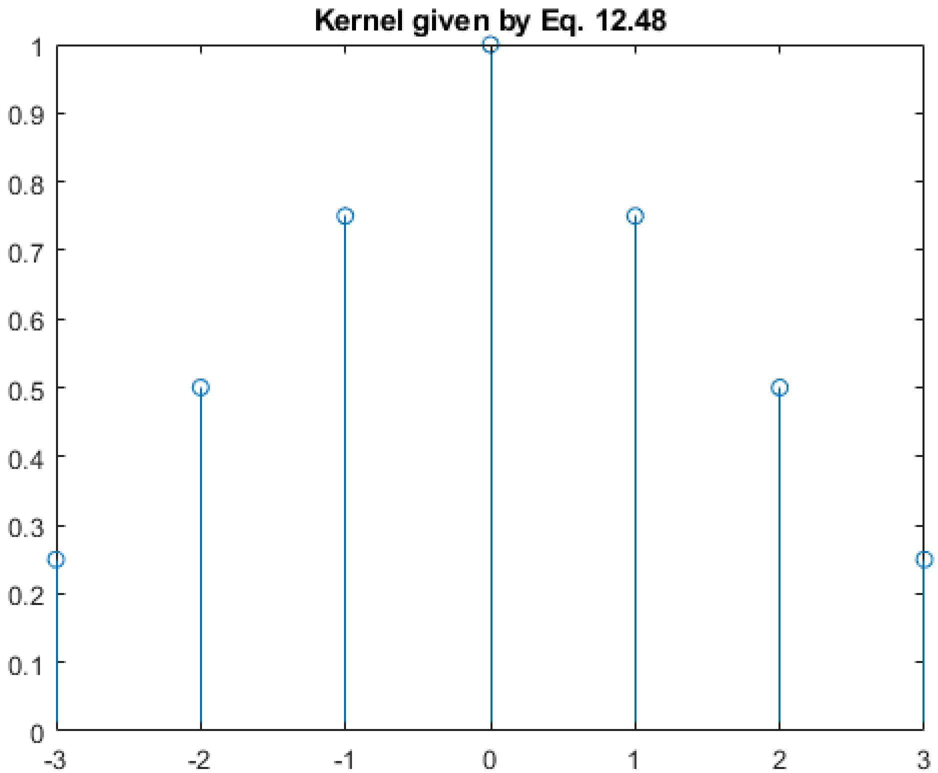
$$x[n] = \begin{cases} -1 & n=1 \\ 3 & n=2 \\ 0 & \text{elsewhere} \end{cases}$$

```
N = 10;
n = 1:N;
x = zeros(N,1);
x(1) = -1;
x(2) = 3;
```

1. Upsample the signal using *I* = 4 and the linear interpolation kernel given by Eq. 12.48.

$$g_{\text{lin}}[n] \triangleq \begin{cases} 1 - \dfrac{|n|}{I}, & -I < n < I \\ 0, & \text{otherwise} \end{cases} \qquad (12.48)$$

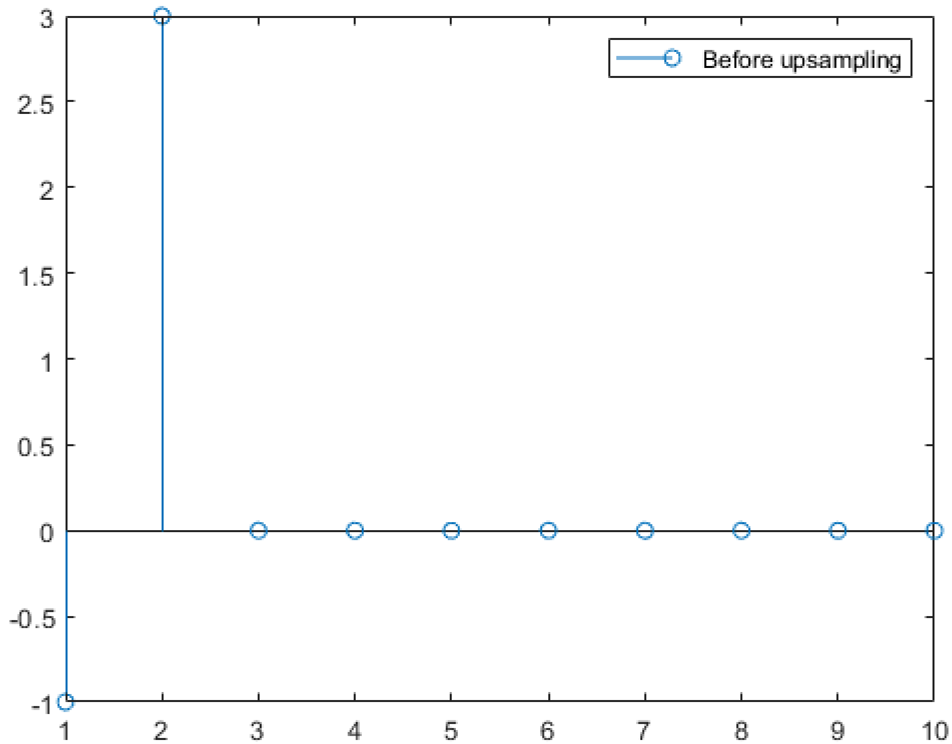
```
I = 4;
gn = -I+1:1:I-1;
g = 1 - abs(gn)/I;  % g = triang(I*2-1)

stem(gn, g);
title('Kernel given by Eq. 12.48')
```



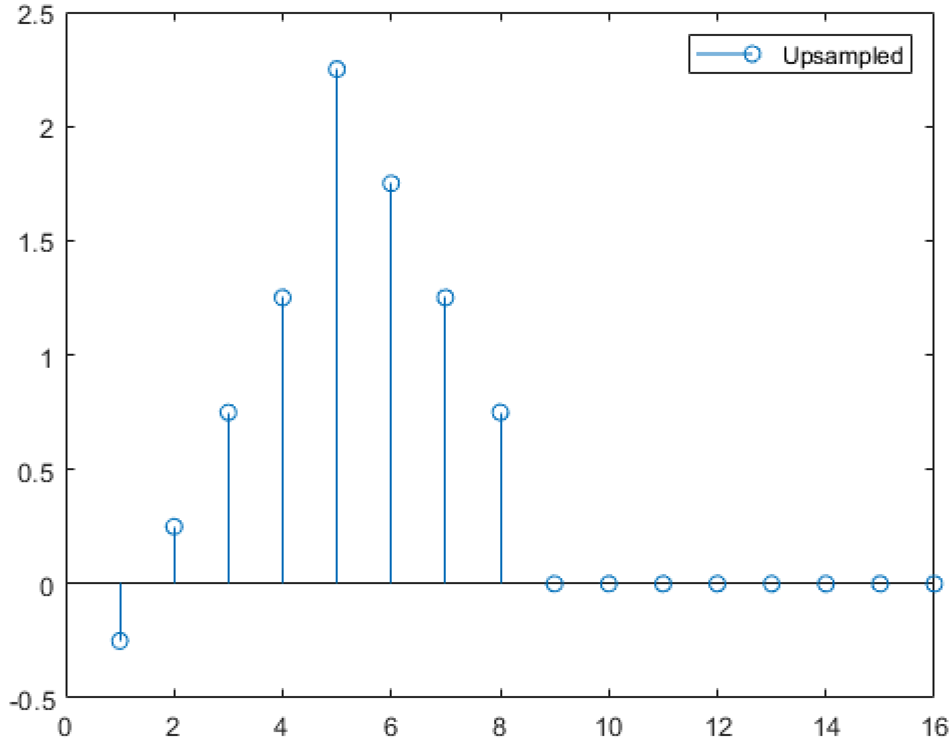

```
x_upsampled = conv(x, g);
```

2. Sketch the signal before and after the upsampling and interpolation.

```
stem(n, x)
legend('Before upsampling')
```




```
stem(x_upsampled)
legend('Upsampled')
```


Upsampled
2.5
2
1.5
1
0.5
0
-0.5
0
2
4
6
8
10
12
14
16