Homework 6

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[✔] ADSI Problem 4.7: Coherence function

The magnitude square coherence function of two signals $x[n]$ and $y[n]$ is given by

$$\gamma_{xy}^2(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)S_{yy}(\omega)}.$$

Assume that $x[n]$ is the input to a given system, $y[n]$ is the output from the system, and the coherence function has been measured. What happens to the coherence function if

1) What happens to the coherence function if gain is increased from 1 to 2:

The gain setting of the amplifier in front of the ADC measuring $y[n]$ is increased from 1 to 2, i.e. $y[n] \rightarrow 2y[n]$?

To answer this question, we need to compute the new coherence function. Power Spectral Density $S_x(\omega)$ and Cross-Power Spectral Density $S_{\text{yx}}(\omega)$ are essentially the discrete-time Fourier Transform of the corresponding auto-correlation and cross-correlation.

So we need to compute those first. The autocorrelation for the input signal is the same because the input signal has not changed. We need to compute the autocorrelation for the new output signal.

Let $z(n) = 2y(n)$ denote the new output signal.

The autocorrelation for the new output signal $r_z(\ell)$ can be computed as follows:

$$r_z(\ell) = E[z(n)z(n-\ell)]$$

$$r_z(\ell) = E[2y(n)2y(n-\ell)]$$

$$r_z(\ell) = 4E[y(n)y(n-\ell)]$$

$$r_z(\ell) = 4r_y(\ell)$$

To find the PSD, we take the Fourier Transform:

$$S_z(\omega) = R_z(e^{j\omega}) = 4R_y(e^{j\omega}) = 4S_y(\omega)$$

This means that if the output gain is increased by 2, the PSD increases by a factor of 4.

Next, we need compute the cross-correlation $r_{\text{zx}}(\ell)$:

$$r_{\text{zx}}(\ell) = E[z(n)x(n-\ell)]$$

$$r_{\text{zx}}(\ell) = E[2y(n)x(n-\ell)]$$

$$r_{\text{zx}}(\ell) = 2E[y(n)x(n-\ell)]$$

$$r_{\text{zx}}(\ell) = 2r_{\text{yx}}(\ell)$$

To find the PSD, we take the Fourier Transform:

$$S_{\text{zx}}(\omega) = R_{\text{zx}}(e^{j\omega}) = 2R_{\text{yx}}(e^{j\omega}) = 2S_{\text{yx}}(\omega)$$

Finally, we can compute the coherence function:

$$|\gamma_{\text{zx}}(\omega)|^2 = \frac{|S_{\text{zx}}(\omega)|^2}{S_z(\omega)S_{\text{x}}(\omega)} = \frac{|2S_{\text{yx}}(\omega)|^2}{4S_y(\omega)S_{\text{x}}(\omega)} = \frac{|2|^2|S_{\text{yx}}(\omega)|^2}{4S_y(\omega)S_{\text{x}}(\omega)} = \frac{|S_{\text{yx}}(\omega)|^2}{S_y(\omega)S_{\text{x}}(\omega)}$$

The coherence function with the new output signal is the same the original coherence function.

This means that increasing the output gain by 2, nothing happens to the coherence function.

2) What happens to the coherence function if a delay happens in the measurement?

A delay happens in the measurement of $y[n]$ i.e. $y[n] \to y[n-D]$?

Let $z(n) = y(n-D)$

First, compute the autocorrelation for the new output signal $r_z(\ell)$:

$$r_z(\ell) = E[y(n-D)y(n-D-\ell)]$$

Let $m = n - D$ then we can write the above equation as:

$$r_z(\ell) = E[y(m)y(m-\ell)]$$

This expression is the same as $r_y(\ell)$ so the delayed output signal does not change the autocorrelation:

$$r_z(\ell) = r_y(\ell)$$

Next, we compute the cross-correlation:

$$r_{zx}(\ell) = E[z(n)x(n-\ell)]$$

$$r_{zx}(\ell) = E[y(n-D)x(n-\ell)]$$

$$r_{zx}(\ell) = E[y(n)x(n-\ell+D)]$$

$$r_{zx}(\ell) = r_{yx}(\ell+D)$$

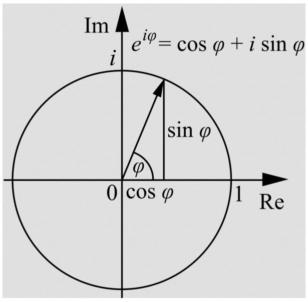
To find the Fourier Transform, we can use the Fourier pair: $f(t-t_0) \leftrightarrow F(e^{j\omega})e^{-j\omega t_0}$

$$R_{zx}(e^{j\omega}) = R_{yx}(e^{j\omega})e^{j\omega D} = S_{yx}(\omega)e^{j\omega D}$$

Finally, we can compute the coherence function:

$$|\gamma_{zx}(\omega)|^2 = \frac{|S_{zx}(\omega)|^2}{S_z(\omega)S_x(\omega)} = \frac{\left|e^{j\omega D}S_{yx}(\omega)\right|^2}{S_y(\omega)S_x(\omega)} = \frac{\left|e^{j\omega D}\right|^2|S_{yx}(\omega)|^2}{S_y(\omega)S_x(\omega)}$$

The norm of the $e^{j\omega D}$ is 1 because this quantity describes a point on the unit circle:



Since the square of 1 is 1, the coherence function is the same:

$$|\gamma_{zx}(\omega)|^2 = \frac{|S_{yx}(\omega)|^2}{S_y(\omega)S_x(\omega)}$$

This means that a delay in the measurement does not effect the coherence function!

[✔] ADSI Problem 4.9: MA(q) processes

In this problem we will investigate the $\mathrm{MA}(q)$ process as defined by Eq. (13.132) by excluding the feedback part

$$y[n] = \sum_{k=0}^{q} b_k x[n-k]$$

where the input is a zero-mean Gaussian white noise process with unit variance.

1) Write out the full expressions

Write out the full expressions for $\mathrm{MA}(0)$, $\mathrm{MA}(1)$, $\mathrm{MA}(2)$ and $\mathrm{MA}(3)$ processes.

The general $\mathrm{ARMA}(p, q)$ is given by the difference equation:

$$y[n] = -\sum_{k=1}^{p} a_k y[n-k] + \sum_{k=0}^{q} b_k x[n-k], \qquad (13.132)$$

When the feedback part is excluded, all values of a_k is set to zero, we are left with:

$$y[n] = \sum_{k=0}^{q} b_k\, x[n-k]$$

We can write out the full expressions for the difference processes as follows:

$$\mathrm{MA}(0) \;\rightarrow\; y[n] = b_0\, x[n]$$

$$\mathrm{MA}(1) \;\rightarrow\; y[n] = b_0\, x[n] + b_1\, x[n-1]$$

$$\mathrm{MA}(2) \;\rightarrow\; y[n] = b_0\, x[n] + b_1\, x[n-1] + b_2\, x[n-2]$$

$$\mathrm{MA}(3) \;\rightarrow\; y[n] = b_0\, x[n] + b_1\, x[n-1] + b_2\, x[n-2] + b_3\, x[n-3]$$

2) Calcute the autocorrelation for the MA processes

Calculate the autocorrelation $r_{yy}[l]$ for the $\mathrm{MA}(0)$ to $\mathrm{MA}(3)$ processes.

The autocorrelation function of a random process is defined as:

$$r_{\mathrm{yy}}(\ell) = E\left[y(n)\, y(n-\ell)\right]$$

To compute the autocorrelation for the $\mathrm{MA}(0)$, just plug its difference equation into this equation:

$$r_{\mathrm{yy}}(\ell) = E\left[b_0\, x(n) \cdot b_0\, x(n-\ell)\right]$$

$$r_{yy}(\ell) = b_0^2 E[x(n) \cdot x(n-\ell)]$$

$$r_{yy}(\ell) = b_0^2\, r_{xx}(\ell)$$

The autocorrelation of a white noise signal $w(n)$ is: $r_{ww}(\ell) = \sigma_w^2 \delta(\ell)$ where σ_w^2 is the signal's variance (see week 6 notes)

We are given that the input signal $x(n)$ is Gaussian white noise with unit variance. This means $r_{xx}(\ell) = \delta(\ell)$

Therefore, the autocorrelation for MA(0) process is:

$$r_{yy}(\ell) = b_0^2\, \delta(\ell)$$

The autocorrelation for the $\mathrm{MA}(1)$ process is:

$$r_{yy}(\ell) = E[(b_0\, x[n] + b_1\, x[n-1]) \cdot (b_0\, x[n-\ell] + b_1\, x[n-\ell-1])]$$

Multiply the two factors:

$$r_{yy}(\ell) = E\left[b_0^2\, x[n]x[n-\ell] + b_0 b_1 x[n]x[n-\ell-1] + b_0 b_1\, x[n-1]\, x[n-\ell] + b_1^2 x[n-1]x[n-\ell-1]\right]$$

Rearrange the coefficients:

$$r_{yy}(\ell) = E\left[b_0^2\, x[n]x[n-\ell] + b_0 b_1 (x[n]x[n-\ell-1] + x[n-1]\, x[n-\ell]) + b_1^2 x[n-1]x[n-\ell-1]\right]$$

Split the expectation values and move constants outside $E[\cdot]$

$$r_{yy}(\ell) = b_0^2\, E[x[n]x[n-\ell]] + b_0 b_1\, E[(x[n]x[n-\ell-1] + x[n-1]\, x[n-\ell])] + b_1^2\, E[x[n-1]x[n-\ell-1]]$$

Split the expectation value in the second term:

$$r_{yy}(\ell) = b_0^2\, E[x[n]x[n-\ell]] + b_0 b_1\, (E[x[n]x[n-\ell-1]] + E[x[n-1]\, x[n-\ell]]) + b_1^2\, E[x[n-1]x[n-\ell-1]]$$

Replace expectation values with autocorrelation functions:

$$r_{yy}(\ell) = b_0^2\, r_{xx}(\ell) + b_0 b_1\, (r_{xx}(\ell-1) + r_{xx}(\ell+1)) + b_1^2\, r_{xx}(\ell)$$

Rearrange the terms

$$r_{yy}(\ell) = \left(b_0^2 + b_1^2\right) r_{xx}(\ell) + b_0 b_1\, (r_{xx}(\ell-1) + r_{xx}(\ell+1))$$

As before $r_{xx}(\ell) = \delta(\ell)$ because the input signal is a white noise with unit variance. Substitute:

$$r_{yy}(\ell) = \left(b_0^2 + b_1^2\right) \delta(\ell) + b_0 b_1\, (\delta(\ell-1) + \delta(\ell+1))$$

The autocorrelation for an MA(2) process is:

$$r_{yy}(\ell) = \left(b_0^2 + b_1^2 + b_2^2\right)\delta(\ell) + (b_0 b_1 + b_1 b_2)\delta(\ell+1) + (b_0 b_1 + b_1 b_2)\delta(\ell-1) + b_0 b_2 \delta(\ell+2) + b_0 b_2 \delta(\ell-2)$$

The autocorrelation for an MA(3) process is:

$$r_{yy}(l) = (b_0^2 + b_1^2 + b_2^3 + b_3^2)\delta(l) + (b_0 b_1 + b_1 b_2 + b_2 b_3)\delta(l+1) + (b_0 b_1 + b_1 b_2 + b_2 b_3)\delta(l-1)$$
$$+ (b_0 b_2 + b_1 b_3)\delta(l+2) + (b_0 b_2 + b_1 b_3)\delta(l-2) + b_0 b_3 \delta(l+3) + b_0 b_3 \delta(l-3)$$

3) Calculate power density spectra

Calculate the general expressions for power density spectra of MA(0), MA(1) and MA(2) processes using Eq. (13.112).

The power spectral density (PSD) is defined as:

$$S_{xx}(\omega) \triangleq \sum_{\ell=-\infty}^{\infty} r_{xx}[\ell] e^{-j\ell\omega}. \qquad (13.112)$$

In 2) we found that the autocorrelation for $\mathrm{MA}(0)$ is $r_{yy}(\ell) = b_0^2 \delta(\ell)$, so plug it in this PSD formula:

$$S_{yy}^{\mathrm{MA}(0)}(\omega) = \sum_{\ell=-\infty}^{\infty} r_{yy}(\ell) e^{-j\ell\omega} = \sum_{\ell=-\infty}^{\infty} b_0^2 \delta(\ell) e^{-j\ell\omega}$$

We notice that $r_{yy}(\ell)$ has non-zero value only when $\ell = 0$. Therefore, the terms for when $\ell \neq 0$ are zero. We can, therefore, remove the infinite sum. Since $\delta(0) = 1$ and $e^{-j0\omega} = 1$, we have:

$$S_{yy}^{\mathrm{MA}(0)}(\omega) = b_0^2 \delta(0) e^{-j0\omega} = b_0^2 \cdot 1 \cdot 1 = b_0^2$$

We do similar computation to find the power density spectrum of MA(1) process.

In 2) we found that the autocorrelation for MA(1) is:

$$r_{yy}(\ell) = \left(b_0^2 + b_1^2\right)\delta(\ell) + b_0 b_1 \left(\delta(\ell-1) + \delta(\ell+1)\right)$$

There are three non-zero values of the autocorrelation function so the infinite sum has only three terms

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \sum_{\ell=-\infty}^{\infty} \left(\left(b_0^2 + b_1^2\right)\delta(\ell) + b_0 b_1 \left(\delta(\ell-1) + \delta(\ell+1)\right)\right) e^{-j\ell\omega}$$

There are three non-zero values of the autocorrelation function so the infinite sum has only three terms:

- When $\ell = 0$: $\left(b_0^2 + b_1^2\right)\delta(0) e^{-j(0)\omega} = \left(b_0^2 + b_1^2\right) \cdot 1 \cdot 1 = \left(b_0^2 + b_1^2\right)$
- When $\ell = -1$: $b_0 b_1 \,\delta(0) e^{-j(-1)\omega} = b_0 b_1 \,(1) e^{j\omega} = b_0 b_1 \, e^{j\omega}$
- When $\ell = 1$: $b_0 b_1 \,\delta(0) e^{-j(1)\omega} = b_0 b_1 \,(1) e^{-j\omega} = b_0 b_1 \, e^{-j\omega}$

Therefore, we have:

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \left(b_0^2 + b_1^2\right) + b_0 b_1 e^{j\omega} + b_0 b_1 e^{-j\omega}$$

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \left(b_0^2 + b_1^2\right) + b_0 b_1 (e^{j\omega} + e^{-j\omega})$$

Since $\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$ and $2 \cdot \cos(\theta) = e^{j\theta} + e^{-j\theta}$, we write:

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \left(b_0^2 + b_1^2\right) + 2b_0 b_1 \cos(\omega)$$

In similar fashion, the PSD of $\mathrm{MA}(2)$ is:

$$S_{yy}^{\mathrm{MA}(2)}(\omega) = \left(b_0^2 + b_1^2 + b_2^2\right) + 2(b_0 b_1 + b_1 b_2)\cos(\omega) + 2b_0 b_1 \cos(2\omega)$$

4) Plot the power density spectra given coefficients

Plot the power density spectra for the two processes defined by

q	b_0	b_1	b_2
1	3	2	
2	3	2	1

We are given the coefficients for $S_{yy}^{\mathrm{MA}(1)}(\omega)$ and $S_{yy}^{\mathrm{MA}(2)}(\omega)$. We use results from 3) to plot the power density spectra for the two processes.

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \left(b_0^2 + b_1^2\right) + 2b_0 b_1 \cos(\omega)$$

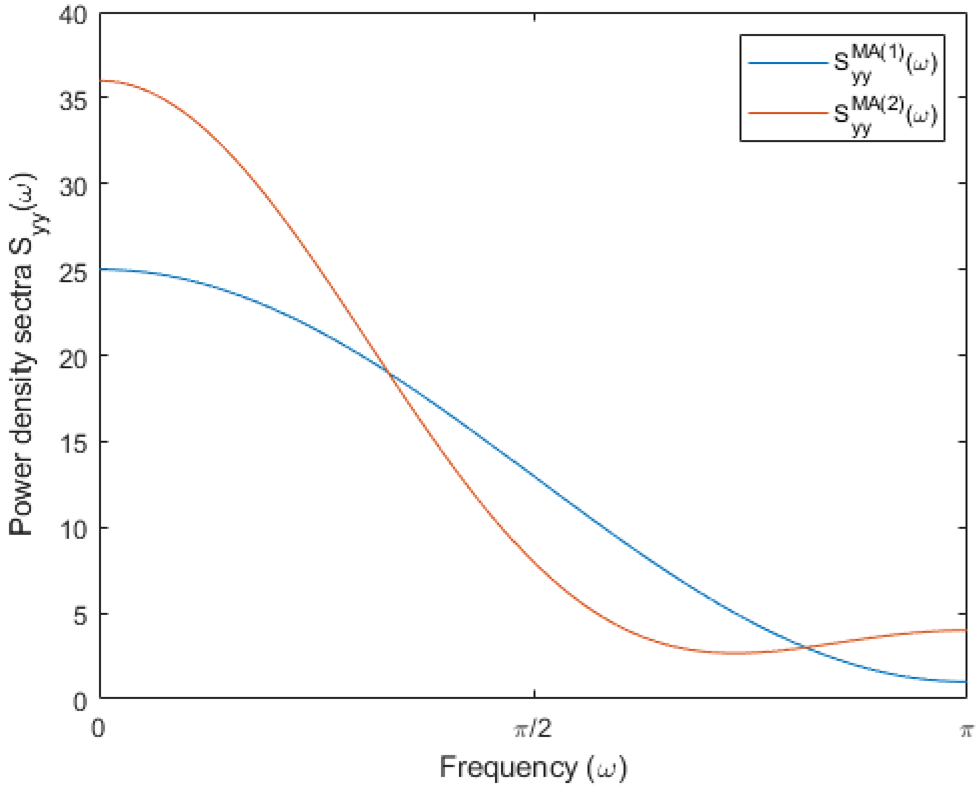
$$S_{yy}^{\mathrm{MA}(2)}(\omega) = \left(b_0^2 + b_1^2 + b_2^2\right) + 2(b_0 b_1 + b_1 b_2)\cos(\omega) + 2b_0 b_1 \cos(2\omega)$$

```
w=0:0.001:2*pi;

% Define the coefficients
b0 = 3; b1 = 2; b2 = 1;

% The power desnity spectrum for the MA(1) process
S1 = (b0^2 + b1^2) + 2*b0*b1*cos(w);
% The power desnity spectrum for the MA(2) process
S2 = (b0^2 + b1^2 + b2^2) + 2*(b0*b1 + b1*b2)*cos(w) + 2*b0*b2*cos(2*w);

plot(w, S1, w, S2);
legend('S_{yy}^{MA(1)}(\omega)','S_{yy}^{MA(2)}(\omega)');
xlabel('Frequency (\omega)');
ylabel('Power density sectra S_{yy}(\omega)');
set(gca,'XTick',0:pi/2:2*pi)
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'})
xlim([0,pi])
```



[✔] ADSI Problem 4.10: MA processes and phase properties

Consider the following two MA(q) systems and assume that they are excited by zero-mean white Gaussian noise with unit variance.

$$y_1[n] = 2x[n] - x[n-2] \quad \text{and} \quad y_2[n] = x[n] - 2x[n-2]$$

1) What is the order q of the processes

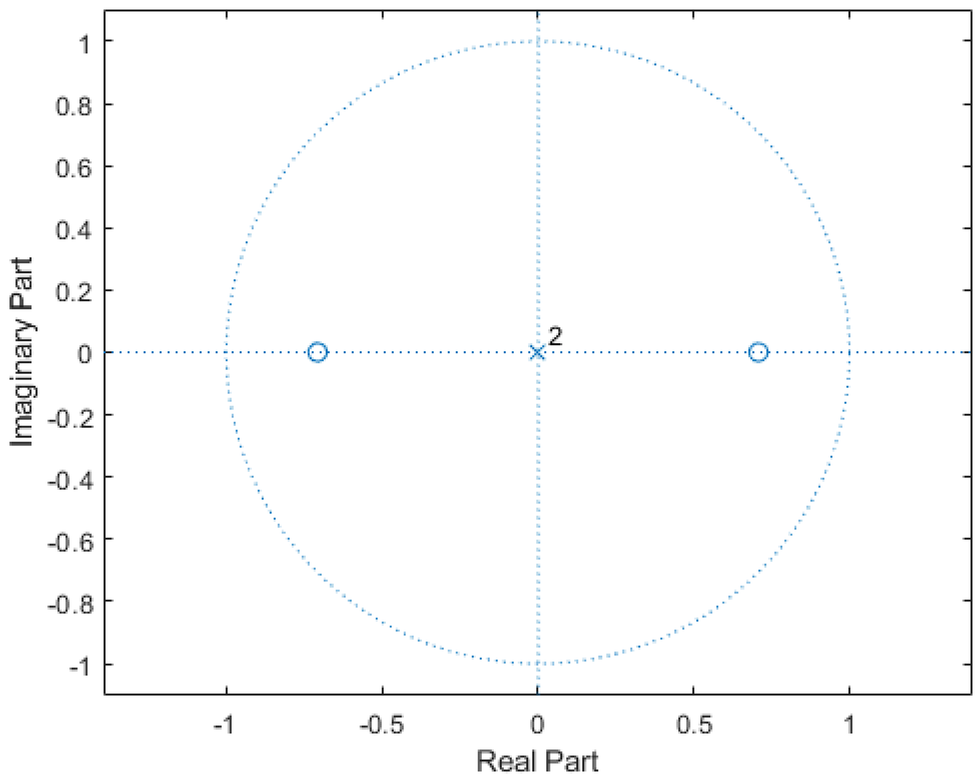
The order q of an $\mathrm{MA}(q)$ process is given by the longest delay, i.e., $x[n-q]$. Therefore, both systems are of order 2.

2) Compare the phase properties of the two systems

One important phase property of a system is whether it is invertible. For a MA(q) system, this implies that the all zeros should be inside the unit circle. If all zeros are located within the unit circle, then we have a minimum-phase system.

We can use zero-pole plot in MATLAB:

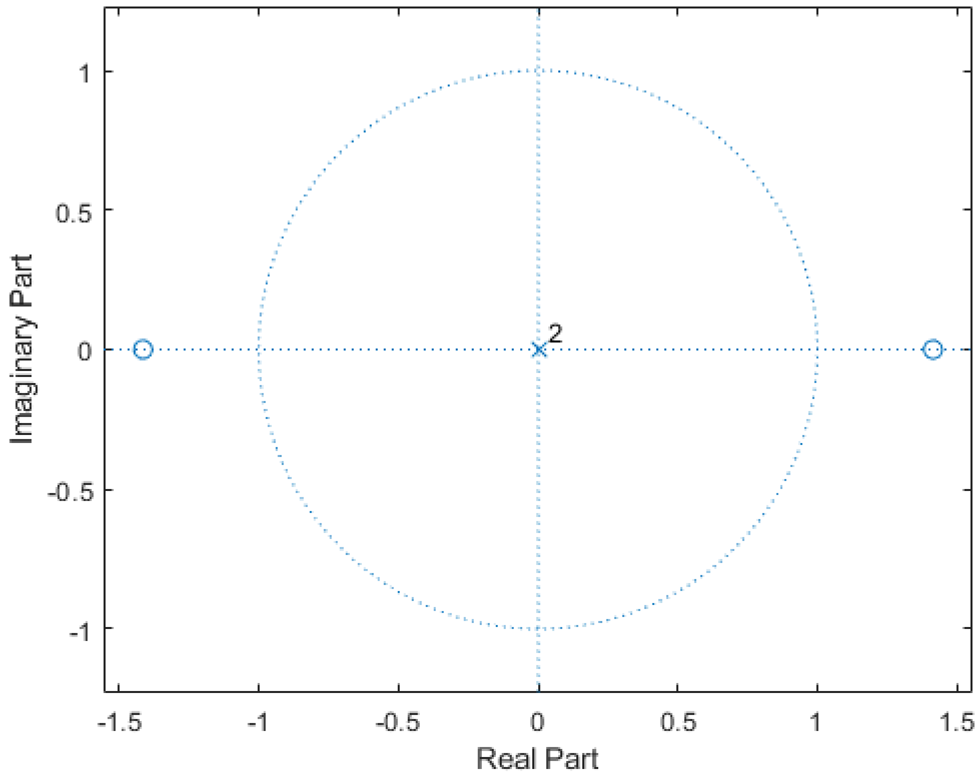
```
h1 = [2, 0, -1]; % Impulse response for y1[n]
zplane(h1);
```



From the zero-pole plot, we observe that all the zeros are within the unit circle. Therefore, we can conclude that the system $y_1[n]$ is a minimum-phase.

Let us make the zero-plot for the system $y_2[n] = x[n] - 2x[n-2]$

```
h2 = [1, 0, -2]; % Impulse response for y2[n]
zplane(h2);
```



Since both zeros are outside the unit circle, we can conclude the system $y_2[n]$ is a maximum-phase system.

3) Compute and plot the power density spectra of the two processes

We are given two MA(q) systems:

$$y_1(n) = 2x(n) - x(n-2)$$

$$y_2(n) = x(n) - 2x(n-2)$$

where the input is $x(n) \sim \mathrm{WGN}(0, 1)$ i.e., white Gaussian noise with unit variance.

We are asked to compute the power density spectra of both systems.

Before we can compute the power density spectrum, we need to find the autocorrelation.

Since we know impulse response, we can use Eq. (13.104) to compute the autocorrelation:

$$r_{yy}[\ell] = r_{hh}[\ell] * r_{xx}[\ell], \qquad (13.104)$$

where

$$r_{hh}[m] = h[-m] * h[m], \qquad (13.103)$$

The autocorrelation of white noise is $\sigma_x^2\delta(\ell)$ so the ACRS of the input signal:

$r_{xx}(\ell) = \sigma_x^2\delta(\ell) = 1 \cdot \delta(\ell)$

Since $r_{xx}(\ell) = \delta(\ell)$ then Eq. (13.104) becomes:

$r_{yy}(\ell) = r_{hh}(\ell)$

This means that if we compute $r_{hh}(\ell)$ then we have the autocorrelation.

We can use MATLAB to compute $r_{h1}(\ell)$ and $r_{h2}(\ell)$:

```
r_h1 = conv(fliplr(h1), h1)
```

```
r_h1 = 1×5
    -2     0     5     0    -2
```

```
r_h2 = conv(fliplr(h2), h2)
```

```
r_h2 = 1×5
    -2     0     5     0    -2
```

Now, we have the autocorrelation of both systems.

$\lvert\ell\rvert$	r_{y1}	r_{y2}
0	5	5
1	0	0
2	−2	−2
≥ 3	0	0

We observe that they are the same. This means that the power spectrum density will be the same.

Whenever we need to compute the PSD based on an autocorrelation sequence, we use Eq. (13.119):

$$S_{xx}(\omega) = r_{xx}[0] + 2\sum_{\ell=1}^{\infty} r_{xx}[\ell]\cos\omega\ell, \qquad (13.119)$$

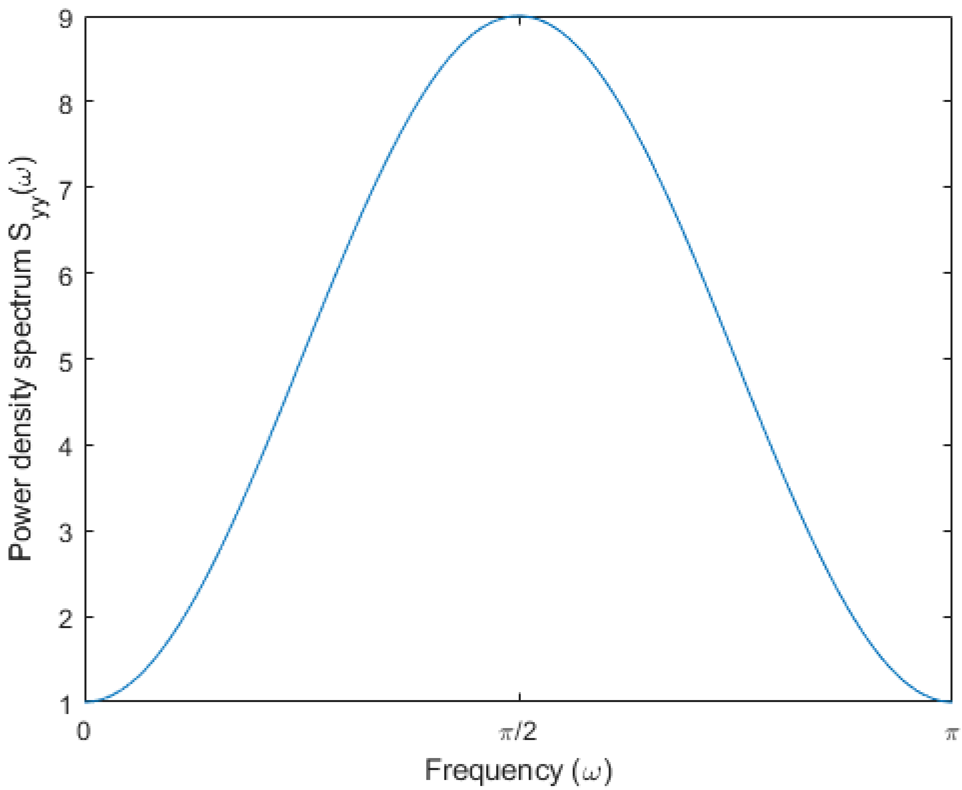
Plugging in our values, we get:

$S_{yy}(\omega) = 5 + 2(0 + (-2)\cos(2\omega))$

$S_{yy}(\omega) = 5 - 4\cos(2\omega)$

We can plot it in MATLAB:

```
w=0:0.001:2*pi;
S_yy = 5 - 4*cos(2*w);
plot(w, S_yy);
xlabel('Frequency (\omega)');
ylabel('Power density spectrum S_{yy}(\omega)');
set(gca,'XTick',0:pi/2:2*pi)
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'})
xlim([0, pi])
```



[✔] ADSI Problem 4.12: MA(q) spectral estimation from autocorrelation

The autocorrelation function for a MA(1) process has been found to be

$\lvert l \rvert$	$r_{yy}(l)$
0	2
1	1
$\lvert l \rvert \geq 2$	0

1) Compute and plot the power density spectrum

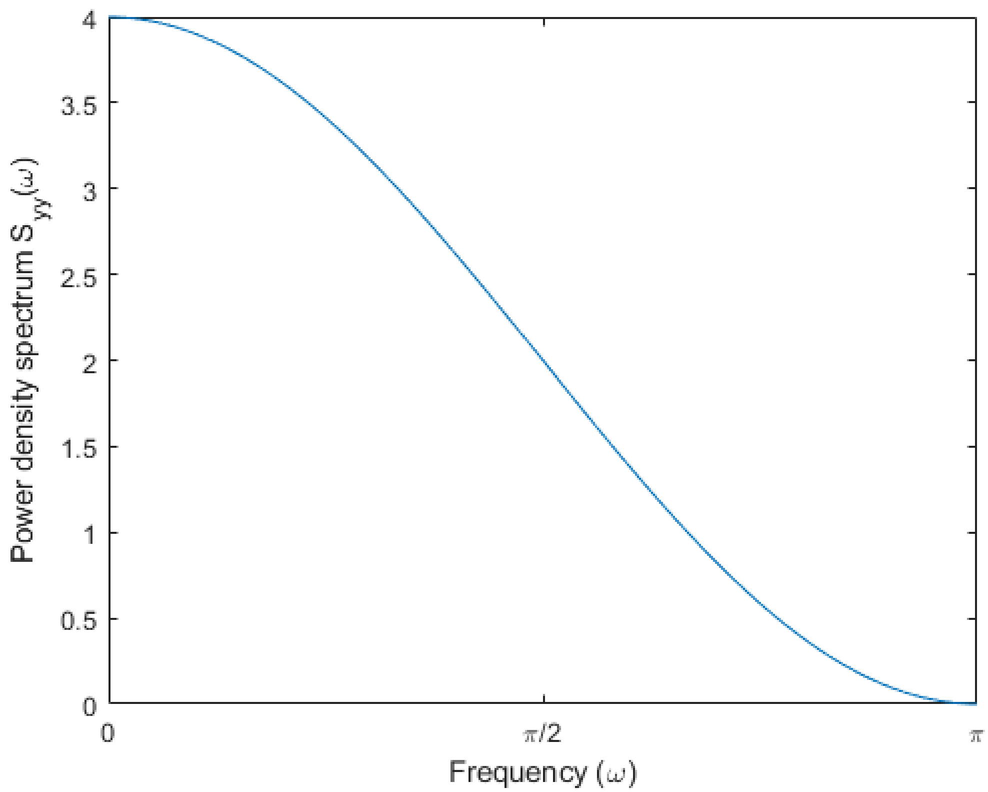
Whenever we need to compute the PSD based on an autocorrelation sequence, we use Eq. (13.119):

$$S_{xx}(\omega) = r_{xx}[0] + 2\sum_{\ell=1}^{\infty} r_{xx}[\ell]\cos\omega\ell, \qquad (13.119)$$

Plugging in our values, we get:

$$S_{yy}(\omega) = 2 + 2(1\cos(1\omega)) = 2 + 2\cos(\omega)$$

```
w=0:0.001:2*pi;
S_yy = 2 + 2*cos(w);
plot(w, S_yy);
xlabel('Frequency (\omega)');
ylabel('Power density spectrum S_{yy}(\omega)');
set(gca,'XTick',0:pi/2:2*pi)
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'})
xlim([0, pi])
```



2) Calculate the model parameters

We are asked to find b_0 and b_1 for an MA(1) process given its autocorrelation sequence.

In 1) we found that:

$$S_{yy}(\omega) = 2 + 2\cos(\omega)$$

In ADSI Problem 4.9, we found that:

$$S_{yy}^{\mathrm{MA}(1)}(\omega) = \left(b_0^2 + b_1^2\right) + 2b_0 b_1 \cos(\omega)$$

This implies that:

$$b_0^2 + b_1^2 = 2 \quad \text{and} \quad b_0 b_1 = 1$$

Rewriting the second equation:

$$b_0 = \frac{1}{b_1}$$

We can substitute in the first equation:

$$\frac{1}{b_1^2} + b_1^2 = 2$$

From the equation above, we see that $b_1 = \pm 1$. We turn our attention the second equation:

$b_0 = 1$ if $b_1 = 1$

$b_0 = -1$ if $b_1 = -1$

So the model parameters are $b_0 = 1, b_1 = 1$ or $b_0 = -1, b_1 = -1$.

```
% Verify in MATLAB
syms b0 b1
solution = solve([b0^2 + b1^2 == 2, b0*b1 == 1], [b0, b1]);
solution.b0
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
solution.b1
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

ADSI Problem 4.15: AR(p) signal modelling

Consider the following $\mathrm{AR}(2)$ process

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n]$$

where $x[n]$ is a zero-mean WGN process with variance equal to 2.

1) Calculate and plot the theoretical power density spectrum.

2) Create a 1024 samples long realization of the process and calculate the autocorrelation

Create a 1024 samples long realization of the process and calculate the autocorrelation with `xcorr`.

3) Fit AR(4) and AR(6) models

Fit AR(4) and AR(6) models to the experimentally derived autocorrelation function values. Are the retrieved model parameters in concordance with your expectations.

4) Compare the power density spectra of the AR(4) and AR(6) model with the result from question 1.

Compare the power density spectra of the AR(4) and AR(6) model with the result from question 1.

ADSI Problem 4.17: Signal modelling of speech

One of the reasons for modelling signals is that the model gives us the ability to describe a given signal with a few parameters rather than being forced to deal with the entire signal. This is used in e.g. compression of speech signals and recognition of speech. In the problem we will create all-pole models of speech signals. The hypothesis is that a signal $x(n)$ can be modelled as Gaussian white noise $w(n)$ sent through a Pth order all-pole filter i.e.

$$x(n) = -\sum_{k=1}^{P} a_k x(n-k) + w(n).$$

We can not expect to have an exact match between the signal and the model. Instead, the model and the signal should have the same statistical properties.

1) Record your voice

Use the microphone in your pc to record a few sequences where you use both voiced (i.e. 'a', 'b' or 'd') and unvoiced (i.e. 'f', 's' or 't') sounds. Try also to create (white) noise with your voice.

2) Can a short sequence of your signal be described with an AR(1) model?

The model is built on the assumption that the signals are stationary. Obviously, a speech signal is non-stationary, but if we only consider short sequences of 20-25 ms duration these sequences are quite close to being stationary.

An AR(1) model driven by zero mean white noise with variance σ_w^2 is described by

$$x(n) = -ax(n-1) + w(n)$$

The autocorrelation function of the AR(1) process is given by

$$r_{xx}(l) = \frac{\sigma_w^2}{1-a^2}(-a)^{|l|}$$

Use Matlabs `xcorr` command and the above equation to decide whether a short sequence of your signal can be described with an AR(1) model.

3) Create an AR(2) model using Yule-Walter equations (13.149)

4) Whiten your speech signal

One way of testing whether the assumed model is good is to use the model coefficients to whiten the signal. If the result of the whitening is white noise, the model is good.

$$w(n) = \sum_{k=0}^{P} a_k x(n-k) \quad \text{with } a_0 \equiv 1$$

Whiten your speech signal with the AR(2) coefficients and test whether the output is white.

5) Increase the order of the all-pole filter

Increase the order of the all-pole filter by using (13.142) and see if you can get a good model of the signal.

6) Repeat the above questions for voiced, unvoiced and noisy speech signals.

Repeat the above questions for voiced, unvoiced and noisy speech signals.