**Table of Contents**



# Homework 3

## ADSI Problem 2.1: Implementation of All-zero lattice filters

Work you way through the MATLAB code in Figure 9.26 and make sure you understand what is going on.

Code can be found in azlatfilt.m.

## ADSI Problem 2.2: Find impulse reponse of an all-zero lattice filter

Consider an all-zero (FIR) lattice filter with $k_1 = 0.65$, $k_2 = -0.34$ and $k_3$=0.8.

1. Find the impulse response of the filter by tracing an impulse $x(n) = \delta(n)$ through the filter.

## ADSI Problem 2.3: Find the reflection coefficients for an all-zero lattice filter

An all-zero lattice filter has the following system function

$$H(z) = 1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}$$

1. Find the reflection coefficients $k_1$, $k_2$ and $k_3$ for the corresponding lattice filter.

## ADSI Problem 2.7: Find system function an all-pole filter from reflection coefficients

Determine the system function for an all-pole filter with the reflection coefficients $k_1 = 0.6$, $k_2 = 0.3$, $k_3 = 0.5$ and $k_4 = 0.9$.

## ADSI Problem 2.10: Linear prediction and lattice filters

The idea behind linear prediction is than the signal $\hat{x}(n)$ can be estimated as a weighted linear combination of the $p$ previous samples

$$\hat{x}(n) = -\sum_{k=1}^{p} a_p(k)x(n-k)$$

The file `signal1.dat` contains 64 samples from an artificial signal.

1. Load `signal1.dat` into MATLAB. Create an autocorrelation of the signal using `xcorr` and plot it. What does the autocorrelation tell you about the signal?

Based on the autocorrelation values we can calculate the coefficients for the optimum $p$th order linear predictor with the normal equations (which we will derive later in the course)

$$r_x(l) = -\sum_{k=1}^{p} a_p(k)r_x(l-k), \qquad l = 1, \ldots, p$$

Let $p = 2$.

2. Compute $a_2(1)$ and $a_2(2)$.

Next, consider a two-stage all-zero lattice filter.

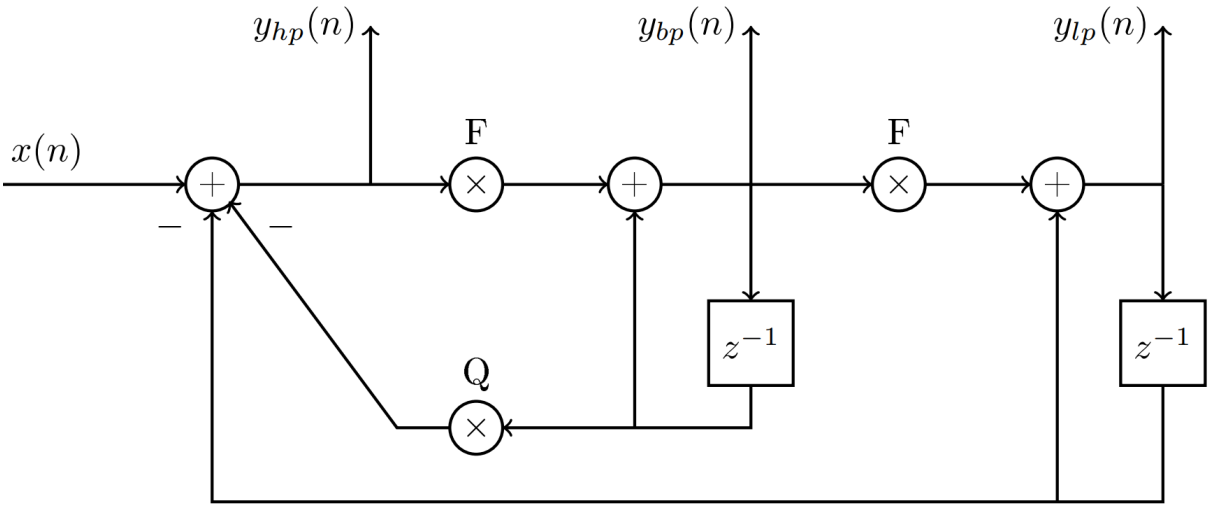
3. What is the relation between the output of the filter, $f_2(n)$, and the above linear prediction?

4. Compute the reflection coefficients $k_1$ and $k_2$ from the $a_2(1)$ and $a_2(2)$ values found above.

5. Send the signal through a 2nd order all-zero lattice filter and analyze the prediction error. Is the signal predictable? Repeat for a higher order predictor.

## ADSI Problem 2.11 (Optional): The digital state variable filter

One interesting filter structure is the digital state variable filter which has its roots in the analog state variable filter. It can be considered as a standard second order filter, but with the valuable feature that the center frequency and Q-factor are independently tuneable. The structure of the filter is shown here




Where $F = 2\sin(\pi F_c/F_s)$ with $F_c$ the filter corner frequency, $F_c$ is the sampling frequency and $Q$ is the $Q$-factor of the filter. From the structure, updating equations for the three state variables, $y_{hp}(n)$, $y_{bp}(n)$ and $y_{lp}(n)$ can easily be written down. Index hp, bp and lp denotes highpass, bandpass and lowpass respectively. For $y_{hp}(n)$ the equation becomes

$$y_{hp}(n) = x(n) - y_{lp}(n-1) - Qy_{bp}(n-1).$$

1. Write down the updating equations for the two other state variables.

2. Calculate the transfer function $H_{LP}(z)$ from the input to the lowpass output.

3. Implement the state variable filter as a function in Matlab. Make sure the three equations are updated in the correct order.

4. Test the implementation by sending white noise sampled at 48000 Hz through the filter. Use Fc=1000 Hz and Q=2. Plot the spectra of the three outputs on the same graph and compare with the expected spectra. Repeat for Q=10.

The filter is described in P. Dutilleux "Simple to operate digital time varying filters" AES 86th Convention, Hamburg (1989) In this paper Dutillex shows that the digital state variable filter fails if $F_c$ becomes to large. A rule of thumb is to keep $F_c < F_s/10$.

5. Demonstrate that this is true by using $Fc$=20000 Hz in the above implementation.

6. A band reject output is also easily obtained with the digital state variable filter. Extend the above drawing with a $y_{br}(n)$ output.