Homework 10

Table of Contents

ADSI Problem 6.2: The autocorrelation of an AR(1) process (proof)

Consider an AR(1) process given by $\quad y(n) = -ay(n-1) + x(n)$

with $-1 < a < 1$ and $x(n)\ WN(0, \sigma_x^2)$.

1. Show that the autocorrelation of the AR(1) process is given by

$$r_{yy}(l) = \frac{\sigma_x^2}{1-a^2}(-a)^{|l|}$$

Hint: Use equation (13.138) and (13.140).

ADSI Problem 6.3: Wiener FIR Filtering, minimum square error

Consider a signal $x(n) = s(n) + w(n)$ where $s(n)$ is an AR(1) process that satisfies the difference equation

$$s(n) = 0.8s(n-1) + v(n)$$

where $\{v(n)\}$ is a white noise sequence with variance $\sigma_v^2 = 0.49$ and $\{w(n)\}$ is a white noise sequence with variance $\sigma_w^2 = 1$. The processes $\{v(n)\}$ and $\{w(n)\}$ are uncorrelated.

1) Determine the autocorrelation sequences for x(n) and s(n)

Determine the autocorrelation sequences $\{r_s(l)\}$ and $\{r_x(l)\}$.

2) Design a Wiener filter of length M=2 to estimate s(n)

Design a Wiener filter of length $M = 2$ to estimate $\{s(n)\}$.

3) Determine the minimum mean square error for M=2

ADSI Problem 6.4: Linear interpolation, estimate missing samples

Sometimes it happens that a datapoint is missing from some signal acquistion due to sensor failure, transmission errors etc. Assume that we have a long stationary sequence $\{x[n]\}_{n=0}^{N-1}$ where the j'th sample is missing i.e.

$$\{x[n]\} = \{x[0],\ x[1],\ \ldots\ x[j-1]\ x[j+1],\ x[j+2],\ \ldots x[N-2],\ x[N-1]\}$$

We want estimate the missing datapoint as a linear combination of the two neighbouring samples

$$\hat{x}[j] = c_1 x[j-1] + c_2 x[j+1]$$

1. Use our standard mean square error approach to derive equations for c_1 and c_2 based on the autocorrelation $r_{xx}(l)$.

1) Use mean square error to derive coefficients based on the ACRS

ADSI Problem 6.5: Levinson-Durbin by Hand

The aim of this problem is to get a finger-tip feeling of the flow of the Levinson-Durbin recursion. Assume that the following autocorrelation function values have been determined from an unknown random process $\{x(n)\}$

l	$r_{xx}(l)$
0	5
1	4
2	3
3	2
4	1

Work through the Levinson-Durbin recursion by hand and find the optimum linear predictors for m=1, 2 and 3 as well as the corresponding minimum mean square errors J_m's and reflection coefficients k_m's.

1) Work though Levinson-Durbin by hand to find the optimum linear predictors for m=1,2,3

2) Find the corresponding minimum mean square errors

3) Find the corresponding reflection coefficients

ADSI Problem 6.6: Levinson-Durbin and linear prediction

The autocorrelation function of an AR(2) process with two complex conjugated poles at $p = r_p e^{\pm j\omega_p}$ can be calculated analytically and is given by

$$r_{xx}(l) = \frac{r_p^l\big(\sin((l+1)\omega_p) - r_p^2 \sin((l-1)\omega_p)\big)}{(1-r_p^2)\sin(\omega_p)(1-2r_p^2\cos(2\omega_p)+r_p^4)} \quad \text{for } l \geq 0$$

Assume that $r_p = 0.9$ and $\omega_p = \pi/16$.

1) Plot the autocorrelation function.

2) Compute reflection coefficients for m'th order optimum linear predictors

Use the above autocorrelation function and the Levinson-Durbin recursion to calculate reflection coefficients and minimum mean square errors for m'th order optimum linear predictors for $m = 1$ to $m = 6$. Are the results in agreement with your anticipations and Eq. (14.149)?

$$J_{m+1} = J_m + \beta_{m+1}k_{m+1} = (1 - k_{m+1}^2)J_m. \qquad (14.149)$$

ADSI Problem 6.7: Linear prediction

This problem addresses linear prediction on a simple harmonic signal where the results can be compared with our intuitive understanding.

Let a discrete time signal be given by

$$x(n) = \sqrt{2}\sin(\omega_0 n + \phi)$$

Where the phase ϕ is uniformly distributed between 0 and 2π.

1) Determine the autocorrelation function for *x*(*n*)

2) Determine the 2nd order forward linear prediction filter

Write down the normal equation for the forward linear prediction filter and determine the filter coefficients for a 2nd order filter. For mathematical convinience we set $\omega_0 = \frac{\pi}{3}$.

3) Find the system function for the filter and locate the zeros

Find the system function $H(z)$ for the filter and locate the zeros.

4) Determine the frequency response, plot it and comment on the result

Determine the frequency response $H(\omega)$. Plot it and comment on the result.

5) Calculate the prediction error

ADSI Problem 6.8: Autocorrelation function and linear prediction

Assume that for a given sequence of data $\{x(n)\}$ the autocorrelation function has been calculated and used to solve the normal equations so that the optimum p'th order linear predictor was found. Now, an amplifier is placed in the signal chain so that the signal is $\{c \cdot x(n)\}$. How does the autocorrelation function and the linear predictor change?

Functions