Week 9

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Matched Filters

Matched filters can be useful to determine whether a recieved signal is either the reflected signal with additive noise or just noise. This is useful in a radar system:

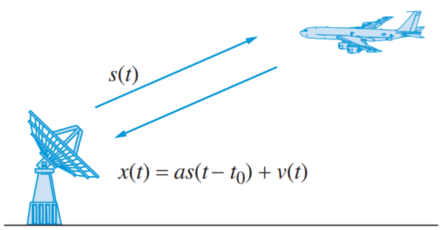


Figure 14.16 Principle of operation of a radar system.

where

- $s(t)$ is a deterministic signal of known form
- $x(t)$ is the signal measured by the radar if an object is present
- a is an attenuation factor
- t_0 is the round-trip delay
- $v(t)$ is random noise

QUIZ: What is the assumption of the received signal?

The above model for $x(t)$ is sometimes a bit too simple. What assumptions have been made in the model?

1. Does not take the Doppler effect into account.
2. The signal could be reflected from different surfaces and not only by the plane
3. Water particles in the air would scatter the signal (radar clutter)

The measured signal by the radar can be two things:

- If an object happens to be in the way then part of the signal is reflected plus noise
- If there is no object in the way of the transmitted pulse, the received signal is just noise

To help us determine whether an object is present, we pass the received signal into a $p-1$-tap FIR filter:

$$y[n] = \sum_{k=0}^{p-1} h[k]x[n-k] \qquad (14.89)$$

This figure shows what we are doing:

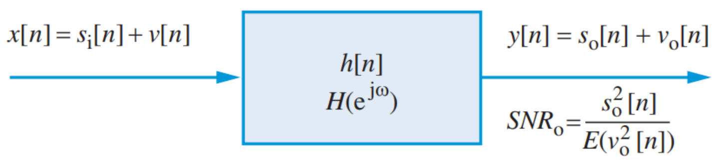


Figure 14.17 Input and output signals in a matched filter.

$a < 1$

QUIZ:

In our quest for a optimum filter that will help us decide on whether the signal $s[n]$ is present or not the book decides on the filter

$$y[n] = \sum_{k=0}^{p-1} h[k]x[n-k] \qquad (14.89)$$

Does the upper limit make sense?

or

What would change if we decrease or increase the number of taps in the filter?

We want to use a filter that is as long as the signal itself.

Our objective is to find the impulse $h[n]$ so that the output signal-to-noise ratio SNR_o is maximised:

$$\mathrm{SNR_o} = \frac{(\text{Value of filtered signal at } n = n_0)^2}{\text{Power of filtered noise}} = \frac{s_o^2[n_0]}{\mathrm{E}(v_o^2[n_0])}, \qquad (14.90)$$

n_0 is the

QUIZ: Why choose $n_0 = p + D - 1$? Can you think of a good reason for this particular choice?

at the decision time $n = n_0$. If we substitute the "signal present" case of (14.88) into (14.89) and set $n_0 = p + D - 1$, the output signal can be written as

$$y[n_0] = a\boldsymbol{h}^{\mathrm{T}}\boldsymbol{s} + \boldsymbol{h}^{\mathrm{T}}\boldsymbol{v}[n_0], \qquad (14.91)$$

The solution is known as a *matched filter.* The impulse response of the optimum filter is

$$\boldsymbol{h} = \kappa \boldsymbol{R}_v^{-1} \boldsymbol{s}. \qquad (14.97)$$

where

$\boldsymbol{R}_v$ is the autocorrelation matrix of the zero-mean wide-sense stationary noise $v[n]$

$$\boldsymbol{h} \triangleq \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[p-1] \end{bmatrix}, \ \boldsymbol{s} \triangleq \begin{bmatrix} s[p-1] \\ s[p-2] \\ \vdots \\ s[0] \end{bmatrix}$$

κ is a normalisation factor

Although the maximum SNR can be obtained by any choise of constant κ, we choose the constant by requiring that:

(a) $\boldsymbol{h}^{\mathrm{T}}\boldsymbol{s} = 1$, which yields $\kappa = 1/\boldsymbol{s}^{\mathrm{T}}\boldsymbol{R}_v^{-1}\boldsymbol{s}$

(b) $\mathrm{E}(v_o^2[n]) = \boldsymbol{h}^{\mathrm{T}}\boldsymbol{R}_v\boldsymbol{h} = 1$, which yields $\kappa = 1/\sqrt{\boldsymbol{s}^{\mathrm{T}}\boldsymbol{R}_v^{-1}\boldsymbol{s}}$.

QUIZ: Is it okay to choose $\kappa = 1$?

Yes, because κ is just a normalisation factor. We don't really care whether it is 1 or some other factor.

The maximum possible value of the output SNR is given by:

$$\text{SNR}_o = a^2\tilde{\boldsymbol{s}}^T\tilde{\boldsymbol{s}} = a^2\boldsymbol{s}^T\boldsymbol{R}_v^{-1}\boldsymbol{s}. \tag{14.98}$$

In summary, to design af matched filter we need two information:

- the autocorrelation sequence of the noise $r_{vv}[\ell]$
- the transmitted signal $s[n]$

QUIZ: How can we interpret in the frequency domain?

We found the impulse response of the optimum filter

$$\mathbf{h}_{opt} = \kappa R^{-1}\mathbf{s}$$

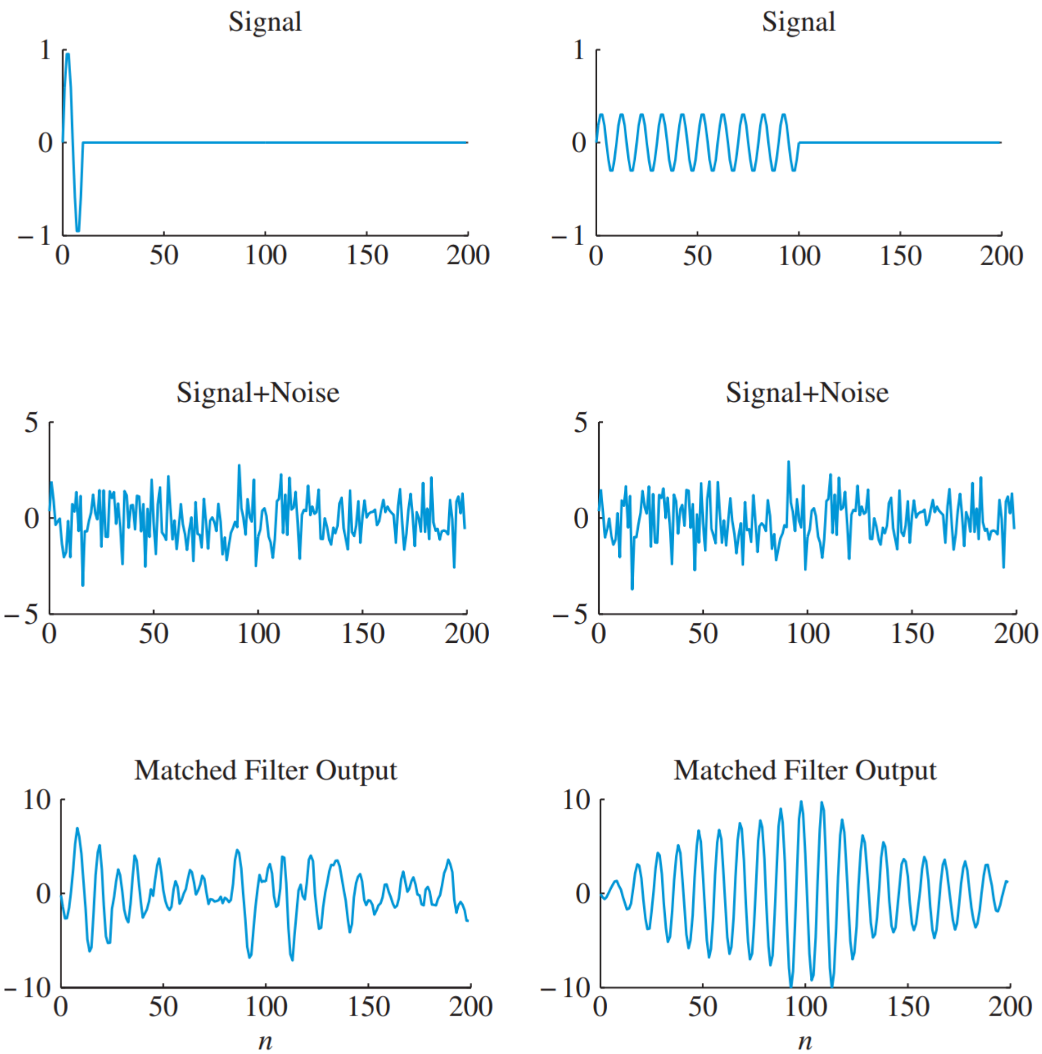
How do we interpret this in the frequency domain, i.e. what is $H_{opt}(e^{j\omega})$?

White noise process

If the noise if white

The autocorrelatiom matrix of white noise is $\boldsymbol{R}_v = \sigma_v^2\boldsymbol{I}$. Then equations (14.97) and (14.98) are simplified to:

$$\boldsymbol{h}_w = \frac{\kappa}{\sigma_v^2}\boldsymbol{s}, \quad \text{SNR}_w = \frac{a^2}{\sigma_v^2}\sum_{k=0}^{p-1} s^2[k] \triangleq a^2\frac{E_s}{\sigma_v^2}. \tag{14.99}$$


Signal
Signal
Signal+Noise
Signal+Noise
Matched Filter Output
Matched Filter Output
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