Week 9

Table of Contents

Matched Filters

Matched filters can be useful to determine whether a recieved signal is either the reflected signal with additive noise or just noise. This is useful in a radar system:

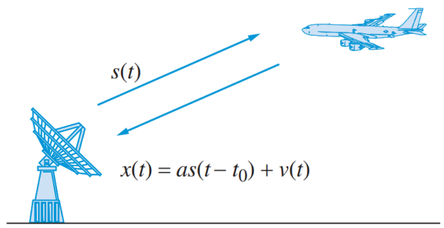

Figure 14.16 Principle of operation of a radar system.

where

- $s(t)$ is a deterministic signal of known form
- $x(t)$ is the signal measured by the radar if an object is present
- a is an attenuation factor
- t_0 is the round-trip delay
- $v(t)$ is random noise

The measured signal by the radar can be two things:

- If an object happens to be in the way then part of the signal is reflected plus noise
- If there is no object in the way of the transmitted pulse, the received signal is just noise

To help us determine whether an object is present, we pass the received signal into a $p-1$-tap FIR filter:

$$y[n] = \sum_{k=0}^{p-1} h[k]x[n-k] \qquad (14.89)$$

This

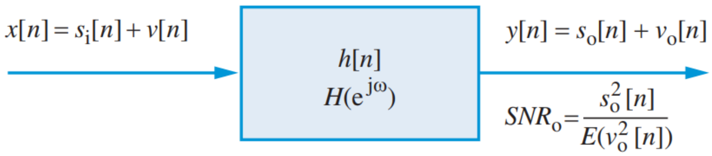

Figure 14.17 Input and output signals in a matched filter.

Our objective is to find the impulse $h[n]$ so that the output signal-to-noise ratio SNR_o is maximised:

$$\mathrm{SNR}_\mathrm{o} = \frac{(\text{Value of filtered signal at } n = n_0)^2}{\text{Power of filtered noise}} = \frac{s_\mathrm{o}^2[n_0]}{\mathrm{E}(v_\mathrm{o}^2[n_0])}, \qquad (14.90)$$

The solution is known as a *matched filter.* The impulse response of the optimum filter is

$$\boldsymbol{h} = \kappa \boldsymbol{R}_v^{-1} \boldsymbol{s}. \qquad (14.97)$$

where

$\boldsymbol{R}_v$ is the autocorrelation matrix of the zero-mean wide-sense stationary noise $v[n]$

$$\boldsymbol{h} \triangleq \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[p-1] \end{bmatrix}, \ \boldsymbol{s} \triangleq \begin{bmatrix} s[p-1] \\ s[p-2] \\ \vdots \\ s[0] \end{bmatrix}$$

κ is a normalisation factor

Although the maximum SNR can be obtained by any choise of constant κ, we choose the constant by requiring that:

(a) $\boldsymbol{h}^\mathrm{T}\boldsymbol{s} = 1$, which yields $\kappa = 1/\boldsymbol{s}^\mathrm{T}\boldsymbol{R}_v^{-1}\boldsymbol{s}$

(b) $\mathrm{E}(v_\mathrm{o}^2[n]) = \boldsymbol{h}^\mathrm{T}\boldsymbol{R}_v\boldsymbol{h} = 1$, which yields $\kappa = 1/\sqrt{\boldsymbol{s}^\mathrm{T}\boldsymbol{R}_v^{-1}\boldsymbol{s}}$.

The maximum possible value of the output SNR is given by:

$$\mathrm{SNR}_\mathrm{o} = a^2\tilde{\boldsymbol{s}}^\mathrm{T}\tilde{\boldsymbol{s}} = a^2\boldsymbol{s}^\mathrm{T}\boldsymbol{R}_v^{-1}\boldsymbol{s}. \qquad (14.98)$$

In summary, to design af matched filter we need two information:

- the autocorrelation sequence of the noise $r_{vv}[\ell]$
- the transmitted signal $s[n]$

White noise process

If the noise if white

The autocorrelatiom matrix of white noise is $\boldsymbol{R}_v = \sigma_v^2 \boldsymbol{I}$. Then equations (14.97) and (14.98) are simplified to:

$$\boldsymbol{h}_w = \frac{\kappa}{\sigma_v^2}\boldsymbol{s}, \quad \text{SNR}_w = \frac{a^2}{\sigma_v^2}\sum_{k=0}^{p-1} s^2[k] \triangleq a^2 \frac{E_s}{\sigma_v^2}. \tag{14.99}$$