Simplex Solver

September 5, 2018

Problem

Given the following linear system and objective function, find the optimal solution.

$$\max 2x_1 + 3x_2$$
$$\begin{cases} x_1 \leq 30 \\ x_2 \leq 20 \\ x_1 + 2x_2 \leq 54 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} x_1 + s_1 = 30 \\ x_2 + s_2 = 20 \\ x_1 + 2x_2 + s_3 = 54 \end{cases}$$

Create the initial tableau of the new linear system.

$$\begin{array}{cc} \left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 1 & 0 & 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 1 & 0 & 20 \\ 1 & 2 & 0 & 0 & 1 & 54 \\ \hline -2 & -3 & 0 & 0 & 0 & 0 \end{array}\right] & \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{cc} \left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 1 & 0 & 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 1 & 0 & 20 \\ 1 & 0 & 0 & -2 & 1 & 14 \\ \hline -2 & 0 & 0 & 3 & 0 & 60 \end{array}\right] & \begin{array}{c} \\ s_1 \\ x_2 \\ s_3 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_1 and the departing variable is s_3.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c} \left[\begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 0 & 0 & 1 & 2 & -1 & 16 \\ 0 & 1 & 0 & 1 & 0 & 20 \\ 1 & 0 & 0 & -2 & 1 & 14 \\ \hline 0 & 0 & 0 & -1 & 2 & 88 \end{array}\right] \begin{array}{c} \\ s_1 \\ x_2 \\ x_1 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is s_2 and the departing variable is s_1.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c} \left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 0 & 0 & 1/2 & 1 & -1/2 & 8 \\ 0 & 1 & -1/2 & 0 & 1/2 & 12 \\ 1 & 0 & 1 & 0 & 0 & 30 \\ \hline 0 & 0 & 1/2 & 0 & 3/2 & 96 \end{array}\right] \begin{array}{c} \\ s_2 \\ x_2 \\ x_1 \\ \\ \end{array} \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 0, s_2 = 8, s_3 = 0, x_1 = 30, x_2 = 12, z = 96$$