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Optimization
- One-dimensional Search Methods

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Optimization of 1D Unimodal function

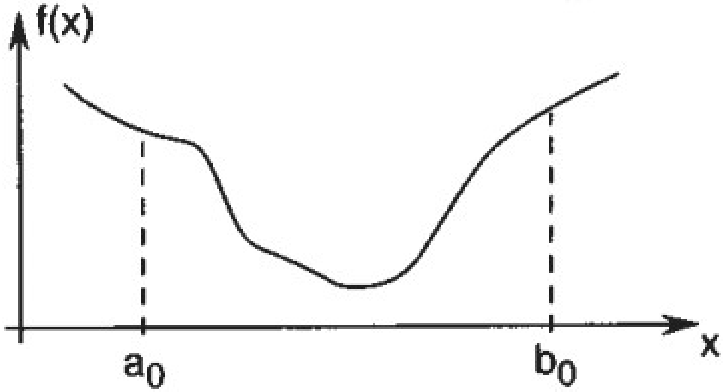

Figure 7.1 Unimodal function.

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Optimization, Golden Section Search in 1D

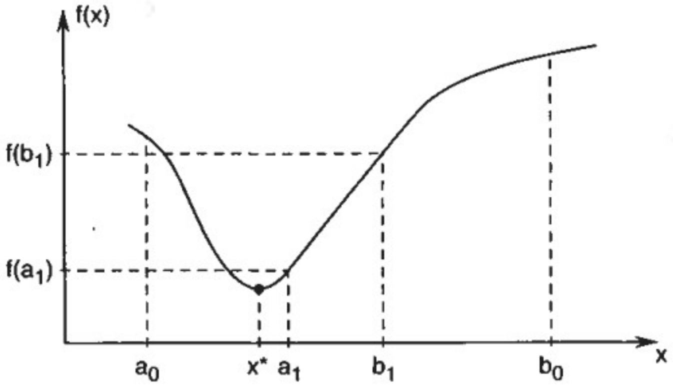

Figure 7.3 The case where $f(a_1) < f(b_1)$; the minimizer $x^* \in [a_0, b_1]$.

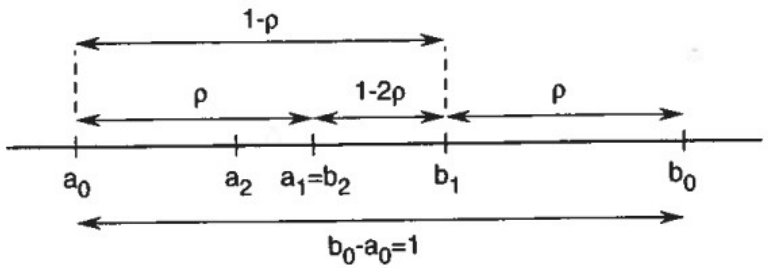

Figure 7.4 Finding value of ρ resulting in only one new evaluation of f.

Optimization, Golden Section Search in 1D

Example 7.1 Use the golden section search to find the value of x that minimizes

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x$$

in the range $[0, 2]$ (this function comes from an example in [19]). Locate this value of x to within a range of 0.3.

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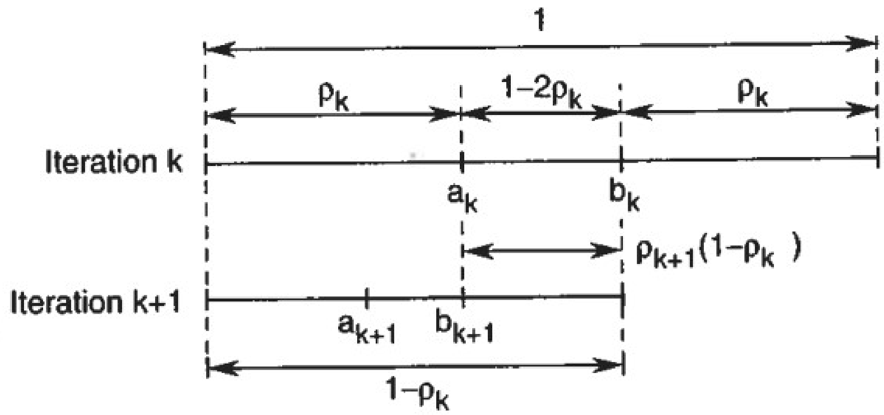
Optimization, Fibonacci Search 1D



Minimization of uncertainty range: $(1-\rho_1)(1-\rho_2)\cdots(1-\rho_N).$

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Optimization, Fibonacci Serach 1D

Minimization of uncertainty range: $(1-\rho_1)(1-\rho_2)\cdots(1-\rho_N).$

Solution: Rations from Fibonnaci sequnce:

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
1	2	3	5	8	13	21	34

$$\rho_1 = 1 - \frac{F_N}{F_{N+1}},$$
$$\rho_2 = 1 - \frac{F_{N-1}}{F_N},$$
$$\vdots$$
$$\rho_k = 1 - \frac{F_{N-k+1}}{F_{N-k+2}},$$
$$\vdots$$
$$\rho_N = 1 - \frac{F_1}{F_2},$$

$$(1-\rho_1)(1-\rho_2)\cdots(1-\rho_N) = \frac{F_N}{F_{N+1}}\frac{F_{N-1}}{F_N}\cdots\frac{F_1}{F_2} = \frac{F_1}{F_{N+1}} = \frac{1}{F_{N+1}}.$$

Optimization, Fibonacci Search 1D

Example 7.2 Consider the function

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x.$$

Use the Fibonacci search method to find the value of x that minimizes f over the range $[0, 2]$. Locate this value of x to within the range 0.3.

Optimization, Golden Search

Try out the first 2 iteration of the golden search method for the function:

$$f(x) = x^2 - 2x + 2$$

on the interval [0;2]. How much is the uncertainty reduces in 2 iterations?

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Optimization, Newtons Method 1D

Minimization of $f(x)$:

$$x^{(k+1)} = x^{(k)} - \frac{f'(x^{(k)})}{f''(x^{(k)})}.$$

Example 7.3 Using Newton's method, find the minimizer of

$$f(x) = \frac{1}{2}x^2 - \sin x.$$

The initial value is $x^{(0)} = 0.5$. The required accuracy is $\epsilon = 10^{-5}$, in the sense that we stop when $|x^{(k+1)} - x^{(k)}| < \epsilon$.

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Optimization, Newtons Method 1D

Tangent method for root finding in g(x):

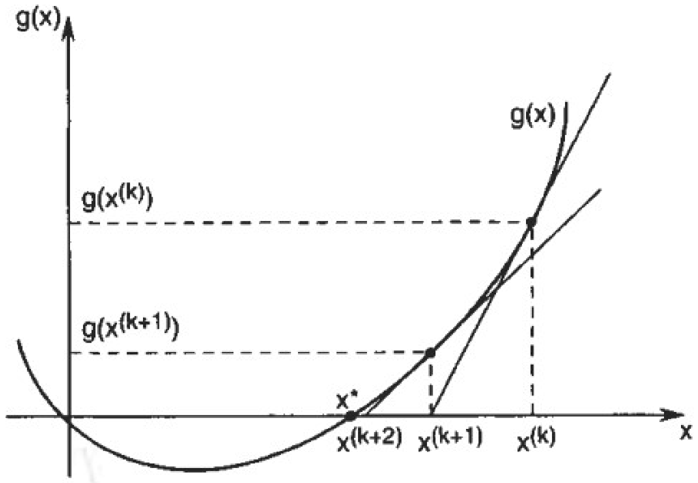

Figure 7.8 Newton's method of tangents.

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})}.$$

Optimization, Newtons Method 1D

Tangent method for root finding in g(x):

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})}.$$

Example 7.4 We apply Newton's method to improve a first approximation, $x^{(0)} = 12$, to the root of the equation

$$g(x) = x^3 - 12.2x^2 + 7.45x + 42 = 0.$$

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Optimization, Secant method 1D

Minimization of $f(x)$:

$$x^{(k+1)} = x^{(k)} - \frac{x^{(k)} - x^{(k-1)}}{f'(x^{(k)}) - f'(x^{(k-1)})} f'(x^{(k)}),$$

or:

$$x^{(k+1)} = \frac{f'(x^{(k)})x^{(k-1)} - f'(x^{(k-1)})x^{(k)}}{f'(x^{(k)}) - f'(x^{(k-1)})}.$$

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Optimization, Secant method 1D

Secant method for root finding:

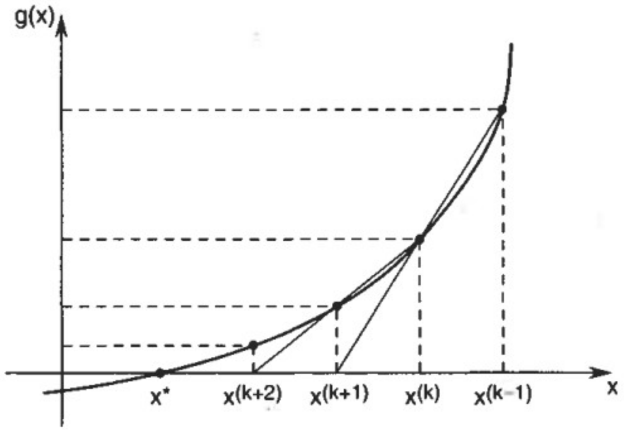

Figure 7.10 Secant method for root finding.

$$x^{(k+1)} = x^{(k)} - \frac{x^{(k)} - x^{(k-1)}}{g(x^{(k)}) - g(x^{(k-1)})} g(x^{(k)}),$$

$$x^{(k+1)} = \frac{g(x^{(k)})x^{(k-1)} - g(x^{(k-1)})x^{(k)}}{g(x^{(k)}) - g(x^{(k-1)})}.$$

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Optimization, Secant method 1D

Secant method for root finding:

Example 7.5 We apply the secant method to find the root of the equation

$$g(x) = x^3 - 12.2x^2 + 7.45x + 42 = 0.$$

We perform two iterations, with starting points $x^{(-1)} = 13$ and $x^{(0)} = 12$. We obtain

$$x^{(1)} = 11.40,$$
$$x^{(2)} = 11.25.$$

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Example 7.6 Suppose that the voltage across a resistor in a circuit decays according to the model $V(t) = e^{-Rt}$, where $V(t)$ is the voltage at time t and R is the resistance value.

Given measurements $V_1, \ldots, V_n$ of the voltage at times $t_1, \ldots, t_n$, respectively, we wish to find the best estimate of R. By the *best estimate* we mean the value of R that minimizes the total squared error between the measured voltages and the voltages predicted by the model.