The Foreign Exchange Carry Trade

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Abstract

I examine the foreign exchange (FX) carry trade, using both single currency and portfolio construction techniques. Chapter I looks at the background, the mechanics of constructing the FX carry trade, and the returns that it generates. In addition the behaviour of these returns is examined in the context of uncovered interest rate parity and several time series analysis aspects. Chapter II introduces both a theoretical stop-loss framework and a sample of hedge fund risk management policies obtained from industry sources. These stop-loss policies are then superimposed onto the FX carry trade. Although 'naive' returns to the FX carry trade, as documented elsewhere in the literature are strongly positive, allowing for stop-loss rules results in returns that are insignificantly different from zero. The ability to cash in on the much vaunted forward premium puzzle relies on being able to stay in the trade, which seems strongly at odds with industry risk management policies. The stop loss signals generated are modelled using available currency futures data and client segmentation categories, but fails to establish a meaningful relationship. Chapter III extends the already established link between the returns to the FX carry trade and historical volatility measures to that of the entire FX option implied volatility surface. The quotation conventions and mechanics of trading FX options are explained and despite being able to establish a strong contemporaneous relationship between FX carry trade returns and FX option implied volatility surfaces, this is not able to be extended to predicting future FX carry trade returns.

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Chapter I

Foreign Exchange Carry Explained

1 Introduction

The Foreign Exchange (FX) carry trade strategy involves borrowing in low interest rate currencies and investing in high interest rate currencies. Theoretically, according to uncovered interest rate parity (UIP), this profit from the interest differential between two countries will be eliminated by exchange rate movements. However, empirically the theoretical relationship fails to hold and the FX carry trade is a profitable strategy. This phenomenon is referred to as the forward premium puzzle and has attracted much attention from finance researchers (Bilson 1981, Fama 1984, Hansen and Hodrick 1980, Hsieh 1984) and given rise to much interest in the FX carry trade by the finance industry .

Several explanations have been proposed to explain the existence of the returns to the FX carry trade. Early work looked at the existence of time-varying risk premia (Engel 1984, Fama 1984). Empirically FX carry trades are susceptible to sudden 'currency crashes' (Brunnermeier et al. 2008). If investments in high interest rate currencies perform poorly during such crashes then FX carry trade excess returns can be interpreted as compensation for this risk.

Initial attempts at modelling this 'risk based' explanation were largely adopted from research on stock markets and used traditional factor models. Examples include CAPM, Fama-French three factor model, and the consumption CAPM. However, irrespective of how the FX carry trade is specified, these traditional models have failed to adequately explain the returns to the FX carry trade (Burnside et al. 2006, Lustig et al. 2011).

Increasingly research has focused on ad hoc factor models where the risk factors are derived from the currencies. Examples include sorting factors based on the size of the forward discount (Lustig et al. 2011), historical and implied currency volatility (Menkhoff et al. 2012(b)), historical currency skewness and implied currency skewness from option risk reversals (Brunnermeier et al. 2008), and the existence of a Peso problem (Burnside et al. 2011(a)). These models have been shown to have some success in explaining FX carry trade returns.

The theory of UIP relies firstly on the notion of covered interest rate parity (CIP). CIP is a no arbitrage condition that relates the FX forward price of a particular currency pair to the FX spot price and the interest rates of the two countries in question. The existence of forward exchange markets can be traced back to latter half of the nineteenth century. Following an increase in forward exchange activity post World War 1, it was Keynes (1922) who was the first professional economist to publish the CIP theory in an article in The Manchester Gaurdian. This was later revised and published in the Tract of Monetary Reform, Keynes (1923), in which he said " ..forward quotations for the purchase of the currency of the dearer money market tend to be cheaper than the spot quotations by the percentage per month equal to the excess on the interest rate which can be earned in a month in the dearer market over what can be earned in the cheaper..". Following this early work by Keynes the CIP condition has been extensively tested over the years and generally been confirmed to hold (Taylor 1989).

However history shows that the UIP condition was first formulated by Irving Fisher in 1896 who viewed it as the dual of the interest rate versus inflation relation (Dimand 1999). He saw both as examples of a general relationship between interest rates in different standards. The UIP condition, that the difference between interest rates expressed in two currencies is the expected rate of change of the exchange rate, was presented and tested in Appreciation and Interest, Fisher (1896), an American Economic Association monograph, and extended in The Rate of Interest, Fisher (1907), and The Theory of Interest, Fisher (1930). Despite history presenting UIP first, in essence the assumption of UIP adds a dynamic layer to the CIP condition, and if true for all horizons it means that the spot FX rate and the term structure of domestic and foreign interest rates can be used to infer the expected future path of the spot FX rate.

Following the collapse of the Bretton Woods system of fixed exchange rates among major currencies in March 1973, the move to floating rate currencies focused researchers attention on the UIP assumption. Shafer et al. (1983) note that "evidence began to accumulate as early as late 1976 that the uncovered interest parity condition might not hold or that expectations for which it held were not rational". By the early 1980s the failure of UIP was well documented and efforts by researchers to explain what was by then known as the forward premium puzzle began in earnest.

Not surprisingly, following this academic evidence of the existence of FX carry trade profits and the failure of UIP, the FX carry trade began to gain popularity in the investment community. From the early 1990s turnover in the global FX market began to grow markedly. Increased globalization, narrowing bid-offer spreads, increasing liquidity in emerging market currencies, and the growth of the hedge fund trading community all contributed to this growth. Charting the activity in the FX carry trade is inherently difficult due to the largely over-the-counter nature of the FX market. The BIS Triennial Central Bank Survey highlights the growth in turnover volume of low interest currencies and high interest rate currencies which can be inferred to be related to growth in the FX carry trade, a conclusion supported by the FX futures data available on the Chicago Mercantile Exchange (Galati et al. 2007). Hedge fund return data has also been used to highlight the growth in FX carry trade activity. By applying style regressions to hedge fund return data it has also been possible to show that they have significant exposure to the FX carry trade (McGuire and Upper 2007, Pojarliev and Levich 2010).

Having witnessed the growth in FX turnover by their hedge fund clients executing for the purposes of the FX carry trade, investment banks began to create their own FX carry indices. By 2008 all of the major global investment banks offered there own versions of FX carry indices for clients to invest in via various platforms.[1] This growth of FX indices able to be invested in by clients was part of a larger move from the investment banks to promote FX as

[1]FX Week 2008

an asset class in its own right (Levich and Pojarliev 2012, Secman 2012). As a proxy measure, the number of managers in the Barclay Currency Tracker Index has grown from 44 in 1993 to 145 in 2008.[2]

The availability of online FX execution platforms and access to investible FX carry indices meant retail clients could also now participate in the FX carry trade. In Japan where low domestic interest rates provided the catalyst for investors to look at returns available offshore the term "Mrs Watanabe" became a somewhat mythical reference to the army of Japanese housewives and their savings participating in the FX market.[3] It is now commonplace to see the term FX carry used in everyday news outlets.

In order to undertake any analysis of the FX carry trade the first requirement is to define how to construct the FX carry trade and then how to measure the returns. A description of the data used to construct the FX carry trade is presented in Section 2 followed by an explanation of the methodology behind an FX carry trade in Section 3. There are several approaches that can be taken to measure FX carry trade returns, the three most common being single currency FX Carry trades, naive equally weighted FX carry trades, and forward discount sorted portfolio based FX carry trades. To ensure that the results of any analysis are not specific to the choice of construction method, all three are presented. In addition, to provide the granularity of how the FX carry trade return is being generated a decom-

[2] https://www.barclayhedge.com/research/indices/cta/sub/curr.html
[3] https://www.ft.com/content/6c1a6eb2-fc8b-11dd-aed8-000077b07658

position into an interest rate component and an FX component is required. How to construct and measure returns of an FX carry trade, both in a single currency setting and in a portfolio setting, the interest rate return and FX return decomposition, and the mechanics of trade execution along with a worked example, are presented in Section 4. The returns of these various FX carry trades are presented in Section 5 and, consistent with the literature, confirm that over the available data period the FX carry trades considered generate on average excess returns.

These return results for the various FX carry trade construction methods considered form the basis upon which all future analysis is conducted. The remainder of Chapter 1 is devoted to gaining a better understanding of what is driving the FX carry trade returns, how they have behaved through time, how their inherent volatility states might be captured, and finally how timing of trade execution may effect returns. To shed light on what is driving the FX carry trade returns, the decomposition results from earlier are used to perform traditional UIP analysis using the Fama regression and making the distinction between low yielding and high yielding currencies. In the case of the low yielding currencies a look at the impact of the move to very low interest rates in some cases is also examined. This closer look at UIP in Section 6 shows that on average UIP is rejected and the low interest rate environment seen in some of the low yielding currencies has strengthened the evidence against UIP. To help understand how the FX carry trade has behaved through time a time series analysis of how the FX carry trade has behaved during three major historical economic events is undertaken in Section 7. The events considered are the equity market crash of 1987, the Russian financial crisis and LTCM episode of 1998, and the recent global financial crisis of 2007/2008. These historical events highlight the susceptibility of the FX carry trade to sudden drawdowns when periods of economic turmoil are encountered. What is also borne out is the differing recovery rates depending on how the FX carry trade is constructed, with the more diversified FX carry trades exhibiting a quicker recovery rate on average. The notion of the FX carry trade being in either a crisis state or normal state is formalised in Section 7.2 where a Markov-switching model is used to try and capture these two states. This simple approach shows that indeed the FX carry trade can be shown to experience long periods of high returns and low volatility which is interspersed by shorter periods of lower returns and higher volatility. The final aspect of FX carry trade returns examined is to test the sensitivity of returns to the time of execution. Out of convenience the literature tends to base its FX carry trade return calculations on either the first or last day of the month. However, when to execute FX carry trades has been an issue highlighted by banks offering investible FX carry indices. The results in Section 7.3 show that FX carry trade returns are not sensitive to the choice of execution date.

2 Data

Daily spot exchange rates and 1 month forward exchange rates versus the United States dollar (USD) for the following developed countries have been obtained from Datastream - Australia (AUD), Canada (CAD), Switzerland (CHF), Europe (EUR), United Kingdom (GBP), Japan (JPY), Norway (NOK), New Zealand (NZD), Sweden (SEK). According to the BIS Triennial Central Bank Survey this set of G10 currencies accounts for approximately 180% of total global turnover (200%). This data covers the period January 1976 until December 2012, with the sample varying slightly by currency. Prior to the introduction of the EUR in January 1999 the countries of Germany (DEM), France (FRF), and Italy (ITL) have been included.

The above data set is obtained by piecing together raw data in Datastream from several different sources. Spot and forward FX quotes for currencies versus the United States Dollar (USD) are available from WM/Reuters from December 1996 onwards, with the obvious exception of the Euro (EUR) which starts in December 1998 and correspondingly the German Mark (DEM), French Franc (FRF) and Italian Lira (ITL) end in December 1998. Prior to December 1996, with the exception of the Australian Dollar (AUD) and New Zealand Dollar (NZD), spot and forward FX quotes for currencies versus the British Pound (GBP) are available from WM/Reuters from Jan 1976, although the Japanese Yen (JPY) starts slightly later in June 1978. These quotes have been converted to quotes versus the USD by dividing the

USD/GBP quotes by the foreign currency units (FCU)/GBP quotes. In the case of AUD and NZD, prior to December 1996 I have augmented the data with quotes versus the USD from Barclays Bank, commencing in January 1985. The details of the Datastream mnemonics for all of the above data are in Tables 1 to 3.

Table 1: Datastream Mnemonics for WM/Reuters Spot FX and Forward FX Quotes Versus the British Pound

For each currency (versus the British Pound) the Datastream mnemonics for the WM/Reuters sourced FX Spot and 1 month FX Forward are shown, and the period for which the data is available. Quote refers to the convention by which the data is stored. For example, FCU/GBP means the quote is the number of foreign currency units per 1 unit of the British Pound (GBP). Note that from February 2007 onwards FX Forward quotes for CAD, CHF, JPY, NOK, SEK, and USD changed to UKCAD1F, UKCHF1F, UKJPY1F, UKNOK1F, UKSEK1F, and USGBP1F respectively. Note that the EUR FX Forward is quoted as GBP/FCU.

Currency	Reference	FX Spot	FX Forward	Period Available	Quote
Australian Dollar	AUD	AUSTDOL	UKAUD1F	Dec 1996 : Dec 2012	FCU/GBP
Canadian Dollar	CAD	CNDOLLR	CNDOL1F	Jan 1976 : Dec 2012	FCU/GBP
Swiss Franc	CHF	SWISSFR	SWISF1F	Jan 1976 : Dec 2012	FCU/GBP
German Mark	DEM	DMARKER	DMARK1F	Jan 1976 : Dec 1998	FCU/GBP
Euro	EUR	ECURRSP	UKEUR1F	Dec 1998 : Dec 2012	FCU/GBP
French Franc	FRF	FRENFRA	FRENF1F	Jan 1976 : Dec 1998	FCU/GBP
Italian Lira	ITL	ITALIRE	ITALY1F	Jan 1976 : Dec 1998	FCU/GBP
Japanese Yen	JPY	JAPAYEN	JAPYN1F	Jun 1978 : Dec 2012	FCU/GBP
Norwegian Krone	NOK	NORKRON	NORKN1F	Jan 1976 : Dec 2012	FCU/GBP
New Zealand Dollar	NZD	NZDOLLR	UKNZD1F	Dec 1996 : Dec 2012	FCU/GBP
Swedish Krone	SEK	SWEKRON	SWEDK1F	Jan 1976 : Dec 2012	FCU/GBP
United States Dollar	USD	USDOLLR	USDOL1F	Jan 1976 : Dec 2012	FCU/GBP

Daily interest rate data is also taken from Datastream, using 1 month interest rates from the London interbank market for the same set of currencies and time periods as the spot and forward exchange data above. This interest

Table 2: Datastream Mnemonics for WM/Reuters Spot FX and Forward FX Quotes Versus the United States Dollar

For each currency (versus the United States Dollar) the Datastream mnemonics for the WM/Reuters sourced FX Spot and 1 month FX Forward are shown, and the period for which the data is available. Quote refers to the convention by which the data is stored. For example, FCU/USD means the quote is the number of foreign currency units per 1 unit of the United States Dollar (USD). Note that the EUR FX Forward is quoted as USD/FCU.

Currency	Reference	FX Spot	FX Forward	Period Available	Quote
Australian Dollar	AUD	AUSTDO$	USAUD1F	Dec 1996 : Dec 2012	USD/FCU
Canadian Dollar	CAD	CNDOLL$	USCAD1F	Dec 1996 : Dec 2012	FCU/USD
Swiss Franc	CHF	SWISSF$	USCHF1F	Dec 1996 : Dec 2012	FCU/USD
German Mark	DEM	DMARKE$	USDEM1F	Dec 1996 : Dec 1998	FCU/USD
Euro	EUR	EUDOLLR	USEUR1F	Dec 1998 : Dec 2012	FCU/USD
French Franc	FRF	FRENFR$	USFRF1F	Dec 1996 : Dec 1998	FCU/USD
British Pound	GBP	USDOLLR	USGBP1F	Dec 1996 : Dec 2012	USD/FCU
Italian Lira	ITL	ITALIR$	USITL1F	Dec 1996 : Dec 1998	FCU/USD
Japanese Yen	JPY	JAPAYE$	USJPY1F	Dec 1996 : Dec 2012	FCU/USD
Norwegian Krone	NOK	NORKRO$	USNOK1F	Dec 1996 : Dec 2012	FCU/USD
New Zealand Dollar	NZD	NZDOLL$	USNZD1F	Dec 1996 : Dec 2012	USD/FCU
Swedish Krone	SEK	SWEKRO$	USSEK1F	Dec 1996 : Dec 2012	FCU/USD

Table 3: Datastream Mnemonics for BBI Australian & New Zealand Dollar Spot FX and Forward FX Quotes Versus the United States Dollar

For each currency (versus the United States Dollar) the Datastream mnemonics for the Barclays Bank sourced FX Spot and 1 month FX Forward are shown, and the period for which the data has been used. Quote refers to the convention by which the data is stored. For example, FCU/USD means the quote is the number of foreign currency units per 1 unit of the United States Dollar (USD).

Currency	Reference	FX Spot	FX Forward	Period Used	Quote
Australian Dollar	AUD	BBAUDSP	BBAUD1F	Jan 1985 : Dec 1996	FCU/USD
New Zealand Dollar	NZD	BBNZDSP	BBNZD1F	Jan 1985 : Dec 1996	FCU/USD

rate benchmark is commonly referred to as Libor - London Interbank Offered Rate. It represents the interest rates that contributor banks are prepared to lend money to one another and as such is the leading benchmark for short term interest rates. The details of the Datastream mnemonics for this interest rate data are shown in Table 4.

Table 4: Datastream Mnemonics for 1 Month Eurodollar Interest Rates

Country	Interest Rate 1mth	Period Available
Australia	ECAUD1M	Apr 1997 : Dec 2012
Canada	ECCAD1M	Jan 1976 : Dec 2012
Switzerland	ECSWF1M	Jan 1976 : Dec 2012
Germany	ECWGM1M	Jan 1976 : Dec 1998
Euro	ECEUR1M	Jan 1999 : Dec 2012
France	ECFFR1M	Jan 1976 : Dec 1998
Italy	ECITL1M	Jan 1976 : Dec 1998
Japan	ECJAP1M	Aug 1978 : Dec 2012
Norway	ECNOR1M	Apr 1997 : Dec 2012
New Zealand	ECNZD1M	Apr 1997 : Dec 2012
Sweden	ECSWE1M	Apr 1997 : Dec 2012
United Kingdom	ECUKP1M	Jan 1976 : Dec 2012
United States	ECUSD1M	Jan 1976 : Dec 2012

3 Methodology

In its simplest form an FX carry trade is constructed by borrowing funds in a low interest rate currency and investing those funds in a high interest rate currency with the objective of 'earning' the interest rate differential. Define :

S_t = spot exchange rate at time t, quoted as the local currency price per unit of foreign currency

i_t = local riskless interest rate at time t

i_t^* = foreign riskless interest rate at time t

Ignoring transactions costs, the local currency payoff from borrowing 1 unit of local currency and lending it in foreign currency is :

$$(1 + i_t^*)\frac{S_{t+1}}{S_t} - (1 + i_t) \tag{1}$$

Following from this, the payoff to the FX carry trade, x_{t+1} is then :

$$x_{t+1} = sign(i_t^* - i_t)\left[(1 + i_t^*)\frac{S_{t+1}}{S_t} - (1 + i_t)\right] \tag{2}$$

where $sign(i_t^* - i_t)$ captures whether the foreign riskless interest rate at time t (i_t^*) is greater(less) than the local riskless interest rate at time t (i_t) and as a result to construct the FX carry trade whether you lend(borrow) foreign currency versus borrow(lend) domestic currency.

Similarly the carry trade can be executed using forward exchange rates rather than local and foreign interest rates. When using forward exchange rates (F_t) to execute the carry trade you buy the foreign currency forward when it is at a forward discount ($F_t < S_t$) and sell the foreign currency forward when it is at a forward premium ($F_t > S_t$). In either case :

$$x_{t+1} = sign(F_t - S_t)Q_t(F_t - S_{t+1}) \tag{3}$$

where Q_t is the amount of foreign currency transacted.

Covered interest rate parity (CIP) states that :

$$(1 + i_t) = \frac{F_t}{S_t}(1 + i_t^*) \tag{4}$$

If the amount of foreign currency executed is set to $(1 + i_t)/F_t$ then :

$$x_{t+1} = sign(F_t - S_t)\frac{(1 + i_t)}{F_t}(F_t - S_{t+1}) \tag{5}$$

When CIP holds, Equations (2) and (5) can be shown to be equal. In practice, FX carry trades are executed in the FX forward markets (Lustig et al. 2011).

4 Measuring Carry Trade Returns

There are several approaches that can be taken to measure carry trade returns. The vast majority of the literature looks at single currency FX carry trades, naive equally weighted FX carry trades, and forward discount sorted portfolio based FX carry trades (Burnside et al. 2006, Brunnermeier et al. 2008, Burnside et al. 2008, Clarida et al. 2009, Burnside 2011, Burnside et al. 2011(a), Burnside et al. 2011(b), Lustig et al. 2011, Menkhoff et al. 2012(a), Menkhoff et al. 2012(b)). All three of these, the single currency method and the two portfolio methods, are considered in this paper. Another, less common, approach is to condition the inclusion of currencies within an FX carry portfolio on some form of volatility measure (Ackermann et al. 2012, Cenedese et al. 2014).

All calculations are from the perspective of a U.S. investor and all carry trade profits are expressed in USD's. All calculations are done on the first business day of each month and the forward period used is 1 month forward. This is by far the most commonly used period in the literature, with only one exception in the above references being Brunnermeier et al. (2008) which considers 1 week and 3 monthly terms.

In order to implement the FX carry trade, institutional investors typically use FX forwards and FX swaps due to their high volume and deep liquidity (BIS 2010 Triennial Central Bank Survey). An FX forward is a contract to buy or sell a specific amount of one currency versus another at a date in the future at a price fixed today. Note that no cash actually changes hands today. An FX swap is a contract to buy (sell) a specific amount of one currency versus another today at the spot price and a reverse transaction to sell (buy) that amount of currency versus another at a date in the future at the forward price. According to the BIS 2010 Triennial Central Bank Survey the daily turnover in FX swaps was $1765 billion. FX swaps are quoted in forward points which are just the difference between the FX spot price (S_t) and the FX forward price (F_t). Typically an FX forward is constructed by an FX spot transaction followed by an FX swap which reverses the FX spot transaction and creates the forward cash flows.

There are several order types that an investor can use to execute an FX spot transaction (including the FX spot leg of an FX forward). The 2 most common methods used by an investor to execute the FX spot leg of a FX carry trade are to simply ask a liquidity provider, typically a bank, for a price in a specific amount of the required currency pair, and execute on that price, or to leave an order for the specific amount of the required currency pair to be filled based on a fixing rate. The most common fixing rate used in the FX market is the WM/Reuters 4:00pm London fix (Chaboud et al. 2004).[4] This fixing is used by many client types, from corporates through to hedge funds, for portfolio valuations and performance measurements, and is also used by the majority of the main equity and bond index compilers. In addition many FX trading banks now provide a service to their clients whereby they guarantee to trade at the WM/Reuters rates. This execution method is commonly used by investors executing FX spot trades in order to implement the FX carry trade.

4.1 Single Currency Construction

Firstly the profitability of carry trades between each currency in the data set versus the USD are examined. In order to check whether a currency, i, is at a forward discount or a forward premium the following Discount Measure, DM_t^i, is calculated :

$$DM_t^i = \frac{(F_t^i - S_t^i)}{S_t^i} \tag{6}$$

[4] `http://www.wmcompany.com/wmr/index.htm`

where $i = 1, 2, .., n_t$ and n_t is the number of available foreign currencies versus the USD at time t.

If $DM_t^i < 0$ then the foreign currency i is at a forward discount and if $DM_t^i > 0$ then the foreign currency i is at a forward premium. In order to execute the single currency FX carry trade versus the USD check whether the respective foreign currency is at a forward discount(premium) using Equation (6) and then buy(sell) the foreign currency versus the USD 1 month forward. The bet size for each currency pair is scaled to 1 USD. The monthly profit for foreign currency i, in USD, for each carry trade is given by :

$$x_t^i = a^i * \frac{(F_{t-1}^i - S_t^i)}{F_{t-1}^i} \tag{7}$$

where $a^i = -1$ when $DM_{t-1}^i < 0$ and $a^i = 1$ when $DM_{t-1}^i > 0$.

At the end of each month whether currency i is still at a forward discount or a forward premium must be re-checked. If this has not changed then the position is 'rolled' forward, using an FX swap. If it has changed, the position is closed out in the foreign exchange spot market at the time and a new position, in the opposite direction, is executed at the prevailing forward exchange rate. An example of this can be found in Brunnermeier et al. (2008). Interest rate differentials tend to be 'persistent' and as a result there tends to be very infrequent switching of FX carry currencies. Due to this, trade positions are most commonly rolled over using FX swaps (Darvas 2009).

Putting aside any margin requirements and associated costs incurred in implementing the FX carry trade, these monthly FX carry trade returns, x_t^i, can be viewed as excess returns. Ignoring margin, the trade is self funding in that you exchange 1 USD for an equivalent amount of foreign currency. Generally the excess return of a particular investment strategy refers to a a rate of return over and above some benchmark interest rate. This benchmark interest rate represents what the capital in question could earn if it were deposited in a low risk interest bearing account, generally of bank or government grade credit. However, in the case of the FX carry trade this notion is redundant since the trade is self financing, margin considerations aside. Hence the returns are considered to be excess returns (Jurek 2014).

The monthly carry trade profit, x_t^i, can be decomposed into an interest rate component and a currency component. The interest rate component, $x_t^{i(Rate)}$, is given by :

$$x_t^{i(Rate)} = a^i * \frac{(F_{t-1}^i - S_{t-1}^i)}{F_{t-1}^i} \tag{8}$$

and the currency component, $x_t^{i(FX)}$, is given by :

$$x_t^{i(FX)} = a^i * \frac{(S_{t-1}^i - S_t^i)}{F_{t-1}^i} \tag{9}$$

and now :

$$x_t^i = x_t^{i(Rate)} + x_t^{i(FX)} \tag{10}$$

Note that the interest rate component is observable ex ante (Jurek 2014). Viewed in this context, when the FX carry trade is constructed at time $t-1$ the size of the forward discount or premium ($F_{t-1} - S_{t-1}$) is known which when normalized by the size of the trade (F_{t-1}) and scaled by the appropriate value of a yields Equation (8). The remaining variance in the profitability of the FX carry trade is the profit or loss generated by the movement in the spot FX rate between the time the FX carry trade was implemented (S_{t-1}), and when the return is realised (S_t). Again, when normalized by the size of the trade (F_{t-1}) and scaled by the appropriate value of a, this yields Equation (9). Koijen et al. (2013) use a similar approach in decomposing cross market carry trade returns.

4.1.1 Worked Example

Consider the following hypothetical example, ignoring bid/ask spreads, for NZDUSD. Suppose on the first day of the month the current NZDUSD spot rate is .8500, the 1 month forward rate is .8450, and by definition the 1 month FX swap points are -.0050. In order to implement the FX carry trade firstly check the discount measure :

$$DM_t^i = \frac{(F_t^i - S_t^i)}{S_t^i} = \frac{(.8450 - .8500)}{.8500} = -.0059$$

Since $DM_t^i < 0$, this implies that the foreign currency, NZD, is at a forward discount versus the home currency, USD. In interest rate terms this implies that the 1 month NZD interest rate is greater than the 1 month USD interest rate. Thus in order to implement the FX carry trade one needs to sell, say, 1,000,000 USD and buy 1,183,431.95 NZD (1,000,000/.8450) 1 month forward.

Assuming this 1 month forward FX trade was transacted with a bank, typically there would be two separate trades done within the bank on different interbank desks in order to facilitate this transaction. Firstly the FX spot trading desk would buy 1,183,431.95 NZD and sell 1,005,917.16 USD, at the current spot rate .8500, for value spot. The FX forward desk would then 'swap' this transaction out to the 1 month forward date via an FX swap. To do this they do two transactions. Firstly they would sell 1,183,431.95 NZD and buy 1,005,917.16 USD at .8500 for value spot, and secondly they would buy 1,183,431.95 NZD and sell 1,000,000 USD at .8450 (.8500-0.0050) for value in 1 month. The net result is that the investor implementing the FX carry trade receives one transaction facing the bank which is the 1 month FX forward transaction.

Suppose in 1 months time, on the first business day of the new month, the NZDUSD spot rate is unchanged at .8500 and the 1 month forward rate and FX swap points are also unchanged at .8450 and -.0050 respectively. The forward NZDUSD contract delivers 1,183,431.95 NZD and requires payment of 1,000,000 USD on this date. To close this position out the investor can sell the 1,183,431.95 NZD at the current spot rate of .8500, and receive 1,005,917.16 USD. This leaves the investor with no NZD position, and the resulting profit of the FX carry trade is 5,917.16 USD, or 0.5917%. Alternatively, using Equation (7) :

$$x_t^i = a^i * \frac{(F_{t-1}^i - S_t^i)}{F_{t-1}^i} = -1 * \frac{(.8450 - .8500)}{.8450} = 0.5917\%$$

For a single period FX carry trade this is in fact what an investor would do. However, for this analysis the FX carry trade is continued. In order to do so, on the first business day of the new month the return on the FX carry trade is measured as above. To implement the new trade, firstly check if $DM_t^i < 0$ or > 0. If in our example $DM_t^i < 0$ for the new month then rather than close out the the 'old' 1 month forward FX trade, that is about to settle, in the spot market, it can be 'rolled' using an FX swap for another 1 month period. However, if in our example DM_t^i had switched from being < 0 in the previous month to > 0 in the new month, then the 'old' 1 month forward FX trade that is about to settle would need to be closed out in the FX spot market, and a new 1 month FX forward transaction entered into whereby NZD is sold 1 month forward and USD is bought 1 month forward.

4.2 Portfolio Construction

There are two methods employed to construct FX carry trade portfolios.

Firstly, a portfolio including all currencies, weighted by equal USD amounts, using the same methodology as the single currency construction above is constructed. The USD amount bet on each individual currency is set to $(1/n_t)$

where n_t is the number of available currencies versus the USD at time t. Examples of this can be found in Darvas (2009) and Burnside et al. (2011(a)). The monthly profit for this equally weighted portfolio, x_t^{EW} is given by :

$$x_t^{EW} = \frac{1}{n_t} \sum_{i=1}^{n_t} x_t^i \tag{11}$$

Secondly, a series of portfolios based on the following ranking methodology are constructed. At the start of each month foreign currencies versus the USD are ranked based on their Discount Measure, DM_t^i, as defined in Equation (6). Portfolios are constructed by buying equal USD amounts of the k highest forward discount currencies and selling equal USD amounts of the k lowest forward discount currencies where $k = 1, 2, 3, 4$. So for each month 4 kxk portfolios, $1x1$, $2x2$, $3x3$, and $4x4$ are constructed. At the end of each month the currencies are again ranked based on their Discount Measure DM_t^i. For currencies staying 'in' the portfolio they are rolled for 1 month using an FX swap. For currencies coming out of the portfolio they are closed out in the foreign exchange spot market and the new currencies are executed accordingly using an FX forward. Examples of this can be found in Brunnermeier et al. (2008) and Clarida et al. (2009). By setting the total USD amount of the k foreign currencies bought (and sold) equal to 1 USD, the monthly profit for these kxk portfolios is given by :

$$x_t^{kxk} = \frac{1}{k} \sum_{i=1}^{k^{high}} x_t^i + \frac{1}{k} \sum_{i=1}^{k^{low}} x_t^i \tag{12}$$

where $k = 1, 2, 3, 4$, and k^{high} represent the k highest forward discount currencies bought versus the USD and k^{low} represent the k lowest forward discount currencies sold versus the USD. So for example when $k = 2$, the $2x2$ portfolio consists of buying equal USD amounts of the 2 highest forward discount foreign currencies versus the USD and selling equal USD amounts of the 2 lowest forward discount foreign currencies versus the USD. By definition these kxk portfolios do not have any USD position. Conversely, the equally weighted portfolio may have a USD position.

4.3 Summary Statistics

As seen, ignoring transactions costs, the payoff from selling(buying) 1 unit of local currency and buying(selling) it in foreign currency forward is x_t, calculated monthly. Because x_t is calculated based on 1 unit of local currency each month, it represents the monthly % return.

The mean monthly return over T months is :

$$\overline{x} = \frac{x_1 + x_2 + ... + x_T}{T} \tag{13}$$

The standard deviation estimate is :

$$s = \left[\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + ... + (x_T - \overline{x})^2}{(T-1)}\right]^{(1/2)} \tag{14}$$

The annualised mean return is :

$$\overline{x}^A = 12 * \overline{x} \tag{15}$$

The annualised standard deviation is :

$$s^A = s * \sqrt{12} \tag{16}$$

The annualised skewness estimate is :

$$Skewness = \frac{T}{(T-1)(T-2)} \sum_{t=1}^{T} \left(\frac{x_t - \bar{x}}{s} \right)^3 \left(\frac{1}{\sqrt{12}} \right) \tag{17}$$

The annualised sample excess kurtosis is :

$$Kurtosis = \left\{ \left(\frac{T(T+1)}{(T-1)(T-2)(T-3)} \sum_{t=1}^{T} \left(\frac{x_t - \bar{x}}{s} \right)^4 \right) - \frac{3(T-1)^2}{(T-2)(T-3)} \right\} \left(\frac{1}{12} \right) \tag{18}$$

To construct a carry trade strategy where the monthly amount of local currency starts at 1 unit and is then adjusted by monthly profits and losses, the cumulative payoff over T monthly periods can then be defined as :

$$X_T = (1 + x_1)(1 + x_2)......(1 + x_T) \tag{19}$$

The compounded annualised return is :

$$X^A = X_T^{(12/T)} - 1 \tag{20}$$

Alternatively, the compounded monthly return can be defined as :

$$X^m = X_T^{(1/T)} - 1 \tag{21}$$

And so :

$$X^A = (1 + X^m)^{12} - 1 \tag{22}$$

The annualised Sharpe ratio is :

$$SharpeRatio = \sqrt{12} * \frac{\overline{x}}{s} \tag{23}$$

5 Carry Trade Returns

Table 5 shows the results for the single currency FX carry trades. The average annual excess return across the single currency FX carry trades is 4.16%, with average annualised standard deviation of 11.1% and an annualised Sharpe ratio of 0.37. Consistent with the literature all currencies (with the exception of EURUSD) have negative skewness and positive excess kurtosis. However, despite all of the individual currency FX carry trades having positive excess returns, their performance varies greatly. The NZDUSD FX carry trade earns a high annualised excess return of 7.61%, whilst the CHFUSD FX carry trade earns a low annualised excess return of just 0.52%.

By way of comparison the U.S. stock market over the same period which

Table 5: Single Currency Annualised FX Carry Trade Returns

Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. Monthly carry trade returns for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. Monthly results are then annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. Sharpe Ratio = annualised Return / annualised Standard Deviation. annualised Skewness = $(1/\sqrt{12})$ * Monthly Skewness, annualised Excess Kurtosis = (1/12) * Monthly Excess Kurtosis. Cumulative Value of $1 is the end of sample period value of $1 invested at the start of the sample period for each currency pair, and the compounded return is the annualised return that equates to this value. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Months Data	Annualised Return	Standard Deviation	Sharpe Ratio	Skewness	Kurtosis	Cumulative Value $1	Compounded Return
AUDUSD	335	0.0738	0.1207	0.6112	-0.0773	0.1221	6.37	0.0685
CADUSD	443	0.0164	0.0679	0.2412	-0.1371	0.3606	1.68	0.0142
CHFUSD	443	0.0052	0.1264	0.0414	-0.1063	0.0638	0.90	-0.0028
DEMUSD	276	0.0065	0.1159	0.0565	-0.0429	0.0156	1.00	-0.0002
EURUSD	167	0.0617	0.1068	0.5773	0.0421	0.0466	2.18	0.0575
FRFUSD	276	0.0570	0.1110	0.5140	-0.0200	0.0235	3.22	0.0521
GBPUSD	443	0.0429	0.1068	0.4016	-0.0136	0.1602	3.94	0.0378
ITLUSD	276	0.0258	0.1116	0.2307	-0.1449	0.1121	1.56	0.0197
JPYUSD	413	0.0201	0.1202	0.1670	-0.1648	0.1424	1.55	0.0128
NOKUSD	443	0.0502	0.1041	0.4820	-0.0915	0.0626	5.20	0.0457
NZDUSD	335	0.0761	0.1298	0.5864	-0.0316	0.1707	6.59	0.0699
SEKUSD	443	0.0633	0.1107	0.5719	-0.1431	0.1694	8.21	0.0587

had an average excess return of 6.6%, a standard deviation of 15.6%, and a Sharpe ratio of 0.43.[5]

Table 6 shows the results for the portfolio FX carry trades. Constructing FX carry trades using portfolios of currencies clearly leads to improved performance. The simple equally weighted (EW) portfolio has an annualised excess return of 3.94%, standard deviation of 5.13%, and Sharpe ratio of .77. Note that the average of the annualised excess returns of the individual currencies does not equal the annualised excess return of the EW portfolio due to the data sample period varying by currency pair.

Likewise the kxk portfolios produce compelling numbers, with a corresponding reduction in the annualised standard deviation as k grows. Take for example the $2x2$ portfolio which produces an annualised excess return of 6.93%, standard deviation of 10.52%, and Sharpe ratio of 0.66. Consistent with findings from other authors, these portfolios exhibit negative skewness and positive excess kurtosis, both of which diminish as more currencies are added to the portfolios.

Table 7 provides some insight into what currencies make up the kxk portfolios over the sample period. Over the sample period the AUDUSD, ITLUSD, and NZDUSD are almost always in the top 4 high yielding currencies. Conversely the CHFUSD, DEMUSD, EURUSD, and JPYUSD are almost always in the bottom 4 low yielding currencies. There are several

[5]US stock market excess return data was sourced from Kenneth French's database. `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

Table 6: Portfolio Annualised FX Carry Trade Returns

Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. EW portfolio is an equal USD weighted portfolio of all available currencies where each month you are long(short) the foreign currency versus the USD if it is at a forward discount(premium). Monthly carry trade return for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. The kxk portfolios where $k = 1, .., 4$ are constructed by each month going long the k highest forward discount currencies and short the k lowest forward discount currencies, equal USD amounts. Monthly results are then annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. Sharpe Ratio = annualised Return / annualised Standard Deviation. annualised Skewness = $(1/\sqrt{12})$ * Monthly Skewness, annualised Excess Kurtosis = (1/12) * Monthly Excess Kurtosis. Cumulative Value of $1 is the end of sample period value of $1 invested at the start of the sample period for each currency pair, and the compounded return is the annualised return that equates to this value. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Months Data	Annualised Return	Standard Deviation	Sharpe Ratio	Skewness	Kurtosis	Cumulative Value $1	Compounded Return
EW	443	0.0394	0.0513	0.7677	-0.1743	0.2710	4.07	0.0388
$1x1$	443	0.0699	0.1419	0.4926	-0.3383	0.2822	8.96	0.0612
$2x2$	443	0.0693	0.1052	0.6586	-0.2002	0.1670	10.45	0.0656
$3x3$	443	0.0530	0.0865	0.6132	-0.1753	0.1627	6.15	0.0504
$4x4$	443	0.0446	0.0735	0.6075	-0.1563	0.1293	4.69	0.0427

currencies which are genuine switchers between the high yielding and low yielding groups over the sample period, namely CADUSD, FRFUSD and SEKUSD.

Table 7: kxk Portfolio Discount Factor % Ranking Analysis

This table shows the % of sample each currency versus the USD is in the k highest and k lowest, where $k = 1, .., 4$, rank of discount measure. Based on monthly, non overlapping data, the kxk portfolios are constructed by each month going long the k highest forward discount currencies and short the k lowest forward discount currencies. The forward discount measure for foreign currency i versus the USD at time t, DM_t^i, is equal to $(F_t^i - S_t^i)/S_t^i$, where S_t^i and F_t^i are the corresponding FX spot and 1 month FX forward rates at time t, quoted as the USD price per unit of foreign currency. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	1 Highest	2 Highest	3 Highest	4 Highest	4 Lowest	3 Lowest	2 Lowest	1 Lowest
AUDUSD	21.2%	46.3%	69.3%	84.8%	8.1%	5.4%	2.7%	0.3%
CADUSD	0.7%	2.7%	9.9%	27.8%	37.5%	17.6%	3.4%	0.0%
CHFUSD	0.0%	0.0%	0.0%	0.0%	97.7%	95.3%	90.1%	38.8%
DEMUSD	0.0%	0.0%	0.4%	4.0%	85.9%	80.1%	40.9%	2.5%
EURUSD	0.0%	0.0%	0.0%	3.6%	71.9%	31.7%	2.4%	0.0%
FRFUSD	1.8%	8.3%	21.7%	37.3%	36.6%	3.3%	0.0%	0.0%
GBPUSD	4.5%	21.2%	41.8%	57.1%	16.7%	4.5%	0.0%	0.0%
ITLUSD	50.4%	65.9%	76.8%	87.0%	0.7%	0.0%	0.0%	0.0%
JPYUSD	0.0%	0.0%	0.2%	0.7%	99.0%	98.3%	82.1%	63.2%
NOKUSD	12.4%	24.4%	47.0%	56.9%	17.4%	6.5%	0.2%	0.0%
NZDUSD	41.2%	74.6%	81.5%	84.2%	6.6%	4.2%	1.2%	0.3%
SEKUSD	2.7%	14.0%	25.5%	48.5%	23.5%	13.1%	0.5%	0.2%

Table 8 breaks down the monthly returns for the single currency and EW portfolio FX carry trades into their interest rate and FX components, as defined by Equation (8) and Equation (9). By definition the interest rate components of FX carry trades are positive. The interest rate component exhibits strong first-order autocorrelation of returns, in contrast to the FX component. As a result the variability in total FX carry trade profits is driven predominantly by exchange rate movements. None of the AR(1) coefficients for the total payoff of each currency are statistically significant at the 5% level.

Table 9 provides the same breakdown of monthly carry trade returns for the kxk FX carry trade portfolios. In addition the return of each portfolio, and its interest rate and FX components, is broken down by the high yielding currencies (k^{high}, long foreign currencies vs the USD) and low yielding currencies (k^{low}, short foreign currencies vs the USD), as defined by Equation (12). Note that when the total payoffs of each portfolio are split between the high yielding side and the low yielding side, both sides contribute positively in all portfolios. However, the vast majority of the total payoff is coming from the high yielding side of the portfolio in each case. For example in the $3x3$ portfolio the total monthly payoff is .0044, with the high yielding side of the portfolio contributing .0041 (93%) and the low yielding side the remainder of .0003.

The interest rate components are all positive, by construction, and as expected the contribution of the high side of each portfolio is larger than that of the low side of each portfolio. This reflects the greater interest rate differential, on average, of the high yielding side of the portfolio versus the USD than the low yielding side.

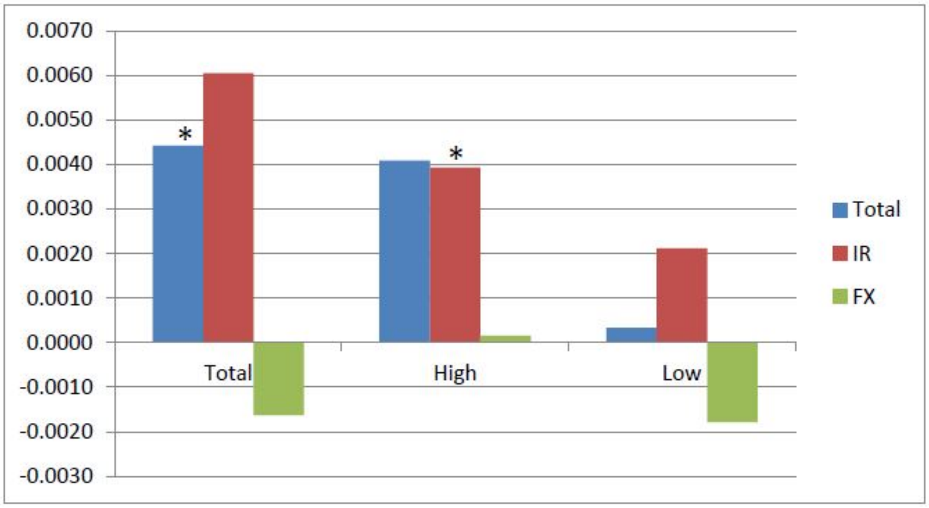
Other than the $1x1$ portfolio the FX components for each of the high

Table 8: Single Currency and EW Portfolio Monthly FX Carry Trade Return Decomposition

Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. Monthly carry trade returns for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. The EW portfolio is an equal USD weighted portfolio of all available currencies. The total carry trade return, x_t^i, is decomposed into an interest rate component, $x_t^{i(Rate)} = a * (F_{t-1}^i - S_{t-1}^i)/F_{t-1}^i$ and a foreign exchange component, $x_t^{i(FX)} = a * (S_{t-1}^i - S_t^i)/F_{t-1}^i$. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Total Payoff					Interest Rate Component					FX Component				
	Mean	Std Dev	Skewness	Kurtosis	AR(1)	Mean	Std Dev	Skewness	Kurtosis	AR(1)	Mean	Std Dev	Skewness	Kurtosis	AR(1)
AUDUSD	0.0061	0.0348	-0.2676	1.4655	0.0221	0.0028	0.0023	1.1341	0.9356	0.9135	0.0033	0.0347	-0.2592	1.5074	0.0200
CADUSD	0.0014	0.0196	-0.4749	4.3267	-0.0904	0.0012	0.0011	1.2490	0.9616	0.8276	0.0001	0.0195	-0.4333	4.3548	-0.0978
CHFUSD	0.0004	0.0365	-0.3682	0.7653	0.0127	0.0029	0.0023	1.2694	1.8335	0.9321	-0.0024	0.0362	-0.4231	0.8588	-0.0038
DEMUSD	0.0005	0.0334	-0.1485	0.1870	0.0152	0.0028	0.0017	0.8535	1.4010	0.8432	-0.0022	0.0332	-0.2080	0.1870	0.0016
EURUSD	0.0051	0.0308	0.1460	0.5589	-0.0365	0.0010	0.0007	0.2360	-0.9964	0.9449	0.0042	0.0309	0.1568	0.5808	-0.0375
FRFUSD	0.0048	0.0320	-0.0692	0.2824	-0.0088	0.0027	0.0026	2.4661	9.1540	0.5442	0.0021	0.0322	-0.1445	0.4757	-0.0107
GBPUSD	0.0036	0.0308	-0.0470	1.9219	0.0197	0.0020	0.0020	1.6231	3.9536	0.9171	0.0016	0.0306	-0.1129	2.0638	0.0102
ITLUSD	0.0021	0.0322	-0.5019	1.3453	0.1010	0.0051	0.0043	2.2740	6.8927	0.7907	-0.0030	0.0322	-0.5528	1.4527	0.0938
JPYUSD	0.0017	0.0347	-0.5710	1.7085	-0.0040	0.0028	0.0021	0.8847	1.0332	0.9386	-0.0012	0.0344	-0.6230	1.8604	-0.0210
NOKUSD	0.0042	0.0301	-0.3169	0.7517	0.0379	0.0027	0.0025	3.4476	23.3138	0.6875	0.0015	0.0301	-0.3696	0.8550	0.0333
NZDUSD	0.0063	0.0375	-0.1095	2.0490	-0.0017	0.0038	0.0042	4.7729	37.2780	0.7342	0.0025	0.0367	-0.2173	2.3087	-0.0206
SEKUSD	0.0053	0.0320	-0.4956	2.0327	0.0218	0.0025	0.0026	4.2685	29.5920	0.7024	0.0028	0.0324	-0.5582	2.2865	0.0327
EW	0.0033	0.0148	-0.6039	3.2517	0.0800	0.0027	0.0014	1.3110	2.2179	0.8694	0.0006	0.0149	-0.6462	3.5217	0.0862

Figure 1: $3x3$ FX Carry Portfolio Monthly Return Decomposition



This graph shows the decomposition of the total monthly FX carry return of the $3x3$ portfolio into its interest rate and FX components. Additionally, each of these return components are split between the 'high' side of the portfolio (long foreign currencies vs the USD) and the 'low' side of the portfolio (short foreign currencies vs the USD).

yielding sides of the portfolio are positive which is in direct contrast to UIP. That is, when high yielding currencies are bought forward, they do not tend to depreciate and offset the interest rate differentials, but actually tend to appreciate slightly, contributing positively to the carry trade profits. In contrast to this the low yielding sides of these portfolios all have negative average monthly FX components. This is in agreement with UIP. That is, for the low yielding sides of these portfolios which are short the foreign currencies against the USD, on average the foreign currencies are appreciating, as predicted by UIP. So the low yielding sides of these portfolios appear to exhibit a weak form of UIP. Weak in the sense that the contribution of the FX component is at least in the right direction, but in all 4 portfolios it is not quite enough to offset the interest rate component of the low yielding sides. These short currencies are effectively acting as the funding leg for the FX carry trade and this would seem to be the cost for doing so. For example in the $3x3$ portfolio the FX component of the low yielding side of the portfolio is -.0018, the interest rate component is .0021, with a net total payoff of .0003. See Figure 1.

The net result of this is that the total payoff in each portfolio closely resembles the interest rate component of the high side of the portfolios (bars labelled * in Figure 1), since the interest rate component of the low sides is mostly negated by the FX component of the low sides (weak form of UIP), and the contribution of the FX component of the high sides is negligible, albeit positive in most instances.

Similar to the single currency decomposition results, the kxk portfolio FX carry trade returns exhibit strong first-order autocorrelation of returns for the interest rate component, but none of the AR(1) coefficients for the total payoff of each portfolio are statistically significant at the 5% level. These somewhat benign correlation results for the total payoffs of both the single currency and kxk portfolio FX carry trade returns should offer some confidence in scaling the results to longer time periods (Diebold et al. 1997). This result is consistent with Lustig et al. (2011) who derive a 'slope factor' based on the difference in returns of a portfolio of high interest rate currencies and a portfolio of low interest rate currencies. They find that the high interest rate currencies load more on this slope factor than the low interest rate currencies.

6 UIP - A Closer Look

Given the results in Section 5, in particular for some of the low yielding currencies and the low yielding sides of the kxk portfolios where the FX component of the FX carry trade returns offset a large part of the interest component of the FX carry trade returns, it is worth examining UIP a little closer. The most common way to evaluate whether UIP holds has been to estimate the following Fama style regression :

$$\frac{(S_{t+1} - S_t)}{S_t} = \alpha + \beta \frac{(F_t - S_t)}{S_t} + \epsilon_t \tag{24}$$

Note that $(F_t - S_t)/S_t = DM_t$ as per Equation (6).

Under the null hypothesis that UIP holds, $\alpha = 0$ and $\beta = 1$. As noted earlier, the rejection of this hypothesis has been well documented.

6.1 Single Currency UIP Analysis

Table 10 shows the Fama regression results for each currency vs the USD in the data set. The average β estimate is -0.74, consistent with the literature, and in violation of UIP. A negative β estimate indicates that when a foreign currency is at a forward discount(premium), empirically it will tend to ap-

Table 9: kxk Portfolio Monthly FX Carry Trade Return Decomposition

Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. The kxk portfolios where $k = 1, .., 4$ are constructed by each month going long the k highest forward discount currencies (High) and short the k lowest forward discount currencies (Low), equal USD amounts. Monthly carry trade returns for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when you are long(short) the foreign currency versus the USD respectively, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. The total carry trade return, x_t^i, is decomposed into an interest rate component, $x_t^{i(Rate)} = a * (F_{t-1}^i - S_{t-1}^i)/F_{t-1}^i$ and a foreign exchange component, $x_t^{i(FX)} = a * (S_{t-1}^i - S_t^i)/F_{t-1}^i$. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

		Total Payoff					Interest Rate Component					FX Component				
		Mean	Std Dev	Skewness	Kurtosis	AR(1)	Mean	Std Dev	Skewness	Kurtosis	AR(1)	Mean	Std Dev	Skewness	Kurtosis	AR(1)
$1x1$	Total	0.0058	0.0410	-1.1719	3.3869	0.0087	0.0089	0.0055	2.3866	9.7868	0.7249	-0.0031	0.0410	-1.1949	3.3266	0.0119
	High	0.0051	0.0366	-0.3297	1.5366	0.0577	0.0057	0.0050	2.8590	13.5271	0.6936	-0.0005	0.0363	-0.4687	1.7565	0.0486
	Low	0.0007	0.0366	-0.4444	1.7846	-0.0048	0.0032	0.0026	0.6620	0.6220	0.9386	-0.0026	0.0362	-0.5165	1.9887	-0.0290
$2x2$	Total	0.0058	0.0304	-0.6936	2.0038	0.0582	0.0073	0.0041	1.6812	3.8541	0.8025	-0.0015	0.0304	-0.6916	2.0751	0.0698
	High	0.0049	0.0328	-0.1437	1.6765	0.0908	0.0046	0.0038	1.8939	5.2338	0.7708	0.0003	0.0326	-0.2280	1.7523	0.0836
	Low	0.0009	0.0309	-0.2579	0.8548	0.0135	0.0027	0.0023	0.6438	0.7963	0.9429	-0.0018	0.0306	-0.3480	0.9017	-0.0114
$3x3$	Total	0.0044	0.0250	-0.6071	1.9522	0.0860	0.0061	0.0033	1.3702	2.1305	0.8391	-0.0016	0.0252	-0.6216	1.9601	0.1085
	High	0.0041	0.0296	-0.2095	1.5078	0.0875	0.0039	0.0032	1.4909	3.2168	0.8194	0.0002	0.0295	-0.2580	1.5575	0.0808
	Low	0.0003	0.0285	-0.2027	0.4090	0.0272	0.0021	0.0022	0.6396	0.7815	0.9442	-0.0018	0.0282	-0.2911	0.4799	0.0071
$4x4$	Total	0.0037	0.0212	-0.5414	1.5512	0.0460	0.0049	0.0027	1.2783	1.6785	0.8441	-0.0012	0.0213	-0.5394	1.5512	0.0616
	High	0.0036	0.0278	-0.2193	1.4459	0.0681	0.0034	0.0028	1.2578	2.3276	0.8454	0.0002	0.0276	-0.2606	1.5257	0.0569
	Low	0.0002	0.0266	-0.0542	0.3481	0.0250	0.0016	0.0020	0.6259	0.9469	0.9404	-0.0014	0.0263	-0.1309	0.4293	0.0036

preciate(depreciate) which is in direct contrast to UIP which implies a move in the opposite direction. Consequently it is possible to reject $H_0 : \beta = 1$, at varying levels of significance, for all currency pairs versus the USD with the exception of FRFUSD, ITLUSD, and SEKUSD. For these 3 remaining currency pairs $H_0 : \beta = 1$ cannot be rejected, but the standard errors are sufficiently large and R^2 sufficiently small that it would be difficult to hypothesize an explanation in support of UIP. Chinn and Meredith (2004) document similar results for the FRFUSD and ITLUSD and, despite examining the implications of the 1992 ERM crisis, postulate that the stochastic process driving short term movements in these currencies has systematically differed from others.

The strong rejection of UIP for CHFUSD and JPYUSD, the 2 most dominant low yielding currency pairs in the sample, is interesting given the results of the FX carry trade return decomposition in Section 5. In Section 5 the FX carry trade returns, x_t^i, were decomposed into an interest rate component, $x_t^{i(Rate)}$ and an FX component, $x_t^{i(FX)}$, as defined by Equations (8) and (9). Note the similarity between these 2 components of the FX carry trade return and Equation (24). The explanatory variable in Equation (24) is $x_t^{i(Rate)}$ with denominator S_t instead of F_t, and the dependent variable in Equation (24) is $x_t^{i(FX)}$, scaled by -1 and again with denominator S_t instead of F_t. If UIP does hold the FX component of the FX carry trade return is expected to be negative, which as stated has been observed on average across the sample for some of the low yielding currencies and the low yielding sides of the kxk

Table 10: Single Currency Fama Regression Results

For each currency versus the USD OLS estimates of α and β (standard errors below) and the corresponding adjusted R^2 are shown for the regression:

$$\frac{(S_{t+1} - S_t)}{S_t} = \alpha + \beta \frac{(F_t - S_t)}{S_t} + \epsilon_t$$

where S_t and F_t are the FX spot and 1 month FX forward rates at time t quoted as the USD price per unit of foreign currency, based on monthly non overlapping data. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012. Asterisks ***, **, and * indicate rejection of $H_0 : \beta = 1$ at the significance levels of 1%, 5%, and 10% respectively.

	α	β	Adj R^2	Reject $H_0 : \beta = 1$
AUDUSD	-0.0018	-1.1790	0.0067	***
	0.0028	0.7889		
CADUSD	-0.0002	-0.5702	0.0018	**
	0.0010	0.6401		
CHFUSD	0.0057	-1.0837	0.0069	***
	0.0023	0.6203		
EURUSD	0.0017	-2.4549	0.0086	*
	0.0025	2.0533		
GBPUSD	-0.0023	-1.3490	0.0098	***
	0.0018	0.6470		
FRFUSD	0.0003	0.3188	0.0011	
	0.0022	0.5899		
DEMUSD	0.0033	-0.6697	0.0032	**
	0.0023	0.7176		
ITLUSD	-0.0005	0.4425	0.0035	
	0.0030	0.4509		
JPYUSD	0.0085	-2.2041	0.0225	***
	0.0025	0.7170		
NOKUSD	0.0006	0.1104	0.0001	*
	0.0017	0.4643		
NZDUSD	0.4739	-1.0755	0.0152	***
	0.0026	-0.0017		
SEKUSD	0.0009	0.8388	0.0068	
	0.0017	0.4819		

portfolios.

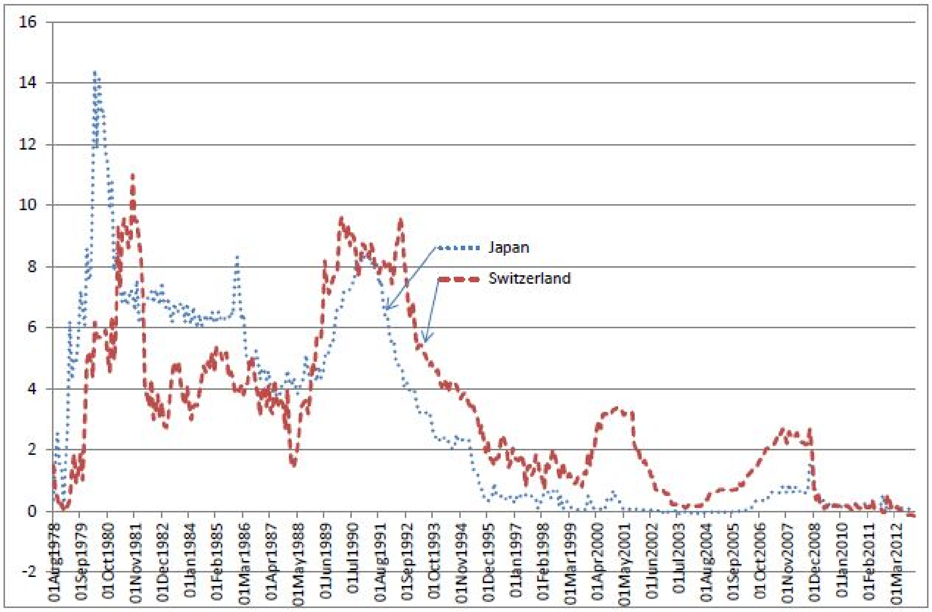
So despite the decomposition of the FX carry trade returns into an interest rate component and an FX component and these averages possibly pointing to some evidence in support of UIP for low yielding currencies such as the CHF and JPY, these results are not supported here. This result is consistent with Ichiue and Koyama (2011) who also observed that low interest rate currencies tend to depreciate which is on opposition to what is predicted by UIP.

6.1.1 Low Yielders : A Special Case ?

In order to look more closely at these low yielding currencies it is worth noting that since the mid 1990s there has been a move towards very low interest rates in some countries, with some effectively hitting the zero interest rate bound. How has this structural change in short term interest rates in some countries affected the level of evidence against UIP ? The recent global financial crisis has re-enforced this trend, with negative interest rates being observed. Figure 2 shows a time series plot of 1 month Libor rates for Switzerland and Japan.

Based on Figure 2, define post January 1996 as the 'low interest rate' environment and re-run the Fama regressions for three sample periods - the sample prior to January 1996, the sample post January 1996, and the sample post January 1996 but ending 30 June 2007 to eliminate any effect the 07/08 global financial crisis may be having on the results. The results are presented in Tables 11, 12, and 13. The results in Table 11 for the sample period prior

Figure 2: 1 Month Interest Rates : Japan and Switzerland



This graph plots 1 month Libor interest rates for Japan and Switzerland. Data is monthly, from Aug1978 to Dec2012, sourced from Datastream.

to January 1996 are broadly in line with the full sample results. The average β is -0.74 which is identical to the full sample results presented above. The results in Table 12 for the sample period post January 1996 offer a somewhat different picture than the full sample. The average β for all currency pairs is -1.19. However, removing DEMUSD, FRFUSD, and ITLUSD which have small samples given that January 1996 start date yields an average β of -1.40. However the ability to reject $H_0 : \beta = 1$ is less, with higher standard errors across all currencies in this sub sample. In the case of CHFUSD and JPYUSD the evidence is mixed. For JPYUSD, $H_0 : \beta = 1$ is not rejected in this sub sample, with the estimate of β now -1.0651 compared to -2.2041 in the full sample. However for CHFUSD, $H_0 : \beta = 1$ is still rejected, albeit at the 5% significance level, but the estimate of β is now -2.9875 compared to -1.0837 in the full sample. Looking at the results in Table 13 for the sample period January 1996 until June 2007 the evidence against UIP is considerably more damning. Eliminating the effect of the global financial crisis results in an average β (excluding DEMUSD, FRFUSD, and ITLUSD) for this sub sample of -3.05. This is consistent with market commentary that there was high levels of risk capital allocated to the FX carry trade leading into the global financial crisis and that this was unwound once the crisis was evident.

On balance it would seem that the low interest rate environment evident since the mid 1990s has resulted in stronger evidence against UIP, based on the average Fama regression β estimates.

6.2 Portfolio UIP Analysis

In order to extend the Fama regression analysis to the portfolios of FX carry trades a composite % change in spot prices and a composite discount measure based on what currencies make up the kxk portfolios at any given point in time are required. Define $\Delta S_t^{k,high}$ and $\Delta S_t^{k,low}$ as:

$$\Delta S_t^{k,high} = \frac{1}{k} \sum_{i=1}^{k} \left(\frac{S_{t+1}^i - S_t^i}{S_t^i} \right) \tag{25}$$

and :

Table 11: Single Currency Fama Regression Results - Pre January 1996

For each currency versus the USD, for data up to and including December 1995, OLS estimates of α and β (standard errors below) and the corresponding adjusted R^2 are shown for the regression:

$$\frac{(S_{t+1} - S_t)}{S_t} = \alpha + \beta \frac{(F_t - S_t)}{S_t} + \epsilon_t$$

where S_t and F_t are the FX spot and 1 month FX forward rates at time t quoted as the USD price per unit of foreign currency, based on monthly non overlapping data. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012. Asterisks ***, **, and * indicate rejection of $H_0 : \beta = 1$ at the significance levels of 1%, 5%, and 10% respectively.

	α	β	Adj R^2	Reject $H_0 : \beta = 1$
AUDUSD	-0.0075	-1.7565	0.0247	***
	0.0048	0.9683		
CADUSD	-0.0033	-1.4637	0.0243	***
	0.0013	0.6018		
CHFUSD	0.0073	-1.0352	0.0085	***
	0.0033	0.7228		
EURUSD				
GBPUSD	-0.0046	-1.7535	0.0197	***
	0.0029	0.8020		
FRFUSD	0.0014	0.5325	0.0027	
	0.0026	0.6606		
DEMUSD	0.0041	-0.6366	0.0031	**
	0.0025	0.7377		
ITLUSD	-0.0006	0.4290	0.0031	
	0.0035	0.4976		
JPYUSD	0.0108	-2.6869	0.0418	***
	0.0035	0.8939		
NOKUSD	0.0006	0.2599	0.0011	
	0.0024	0.5146		
NZDUSD	-0.0051	-1.3698	0.0645	***
	0.0038	0.4576		
SEKUSD	0.0027	1.2931	0.0216	
	0.0026	0.5636		

Table 12: Single Currency Fama Regression Results - Post January 1996

For each currency versus the USD, for data starting January 1996, OLS estimates of α and β (standard errors below) and the corresponding adjusted R^2 are shown for the regression:

$$\frac{(S_{t+1} - S_t)}{S_t} = \alpha + \beta \frac{(F_t - S_t)}{S_t} + \epsilon_t$$

where S_t and F_t are the FX spot and 1 month FX forward rates at time t quoted as the USD price per unit of foreign currency, based on monthly non overlapping data. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012. Asterisks ***, **, and * indicate rejection of $H_0 : \beta = 1$ at the significance levels of 1%, 5%, and 10% respectively.

	α	β	Adj R^2	Reject $H_0 : \beta = 1$
AUDUSD	-0.0013	-2.0979	0.0079	*
	0.0039	1.6596		
CADUSD	0.0020	-1.7317	0.0037	
	0.0017	2.0135		
CHFUSD	0.0069	-2.9875	0.0144	**
	0.0039	1.7408		
EURUSD	0.0017	-2.4549	0.0086	*
	0.0025	2.0533		
GBPUSD	0.0004	-0.1944	0.0001	
	0.0024	1.9283		
FRFUSD	-0.0163	7.8484	0.0172	
	0.0174	10.1595		
DEMUSD	0.0146	-9.9328	0.0081	
	0.0352	18.8145		
ITLUSD	0.0001	0.6434	0.0017	
	0.0055	2.6834		
JPYUSD	0.0044	-1.0651	0.0036	
	0.0040	1.2576		
NOKUSD	0.0003	-0.9096	0.0027	
	0.0025	1.2424		
NZDUSD	0.0022	0.1115	0.0000	
	0.0052	1.9528		
SEKUSD	0.0008	-1.5506	0.0045	
	0.0024	1.6266		

Table 13: Single Currency Fama Regression Results - January 1996 to July 2007

For each currency versus the USD, for data starting January 1996 and ending June 2007, OLS estimates of α and β (standard errors below) and the corresponding adjusted R^2 are shown for the regression:

$$\frac{(S_{t+1} - S_t)}{S_t} = \alpha + \beta \frac{(F_t - S_t)}{S_t} + \epsilon_t$$

where S_t and F_t are the FX spot and 1 month FX forward rates at time t quoted as the USD price per unit of foreign currency, based on monthly non overlapping data. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012. Asterisks ***, **, and * indicate rejection of $H_0 : \beta = 1$ at the significance levels of 1%, 5%, and 10% respectively.

	α	β	Adj R^2	Reject $H_0 : \beta = 1$
AUDUSD	-0.0034	-4.3633	0.0433	***
	0.0032	1.7594		
CADUSD	0.0026	-2.4153	0.0150	**
	0.0016	1.6762		
CHFUSD	0.0104	-4.5334	0.0389	***
	0.0050	1.9317		
EURUSD	0.0040	-4.3971	0.0483	***
	0.0028	1.9515		
GBPUSD	0.0009	-1.2186	0.0031	
	0.0026	1.8723		
FRFUSD	-0.0163	7.8484	0.0172	
	0.0174	10.1595		
DEMUSD	0.0146	-9.9328	0.0081	
	0.0352	18.8145		
ITLUSD	0.0001	0.6434	0.0017	
	0.0055	2.6834		
JPYUSD	0.0040	-1.3771	0.0037	
	0.0072	1.9371		
NOKUSD	0.0000	-2.0546	0.0232	***
	0.0024	1.1422		
NZDUSD	-0.0060	-3.7880	0.0346	***
	0.0044	1.7163		
SEKUSD	0.0019	-3.3426	0.0323	***
	0.0025	1.5687		

$$\Delta S_t^{k,low} = \frac{1}{k}\sum_{i=1}^{k}\left(\frac{S_{t+1}^i - S_t^i}{S_t^i}\right) \tag{26}$$

where $k = 1, 2, 3, 4$ representing the number of currency pairs versus the USD in each side of the FX carry portfolio as defined in Section 4. ΔS_t^{high} and ΔS_t^{low} are the equal weighted average of the % change in the spot prices of the currencies making up the high yielding side (long foreign currency versus the USD) and the low yielding side (short foreign currency versus the USD) of the FX carry trade portfolio respectively.

Similarly, define the portfolio discount measures, $DM_t^{k,high}$ and $DM_t^{k,low}$ as :

$$DM_t^{k,high} = \frac{1}{k}\sum_{i=1}^{k} DM_t^i \tag{27}$$

and :

$$DM_t^{k,low} = \frac{1}{k}\sum_{i=1}^{k} DM_t^i \tag{28}$$

where $k = 1, 2, 3, 4$ representing the number of currency pairs versus the USD in each side of the FX carry portfolio as defined in Section 4. $DM_t^{k,high}$ and $DM_t^{k,low}$ are the equal weighted average of the discount measures of the currencies making up the high yielding side (long foreign currency versus the USD) and the low yielding side (short foreign currency versus the USD) of the FX carry trade portfolio respectively.

Recall that in Section 5 the low yielding sides of the kxk portfolios all have negative average monthly FX components. That is, for the low yielding sides of these portfolios which are short the foreign currencies against the USD, on average, the foreign currencies are appreciating, as predicted by UIP. As with the single currencies, for the kxk FX carry trade portfolios the following Fama style regressions for the high and low yielding sides of the portfolio respectively are performed and the estimates of the slope coefficient β to used to evaluate the degree to which UIP holds, or not as the case may be. So for the high yielding side :

$$\Delta S_t^{k,high} = \alpha + \beta DM_t^{k,high} + \epsilon_t \tag{29}$$

and correspondingly for the low yielding side :

$$\Delta S_t^{k,low} = \alpha + \beta DM_t^{k,low} + \epsilon_t \tag{30}$$

The results for all 4 portfolios are in Table 14. Once again the evidence against UIP is strong and $H_0 : \beta = 1$ is rejected for all of the high and low yielding sides of the kxk FX carry trade portfolios. As with the single currency analysis, for the low yielding sides of the kxk portfolios despite the decomposition of FX carry returns having a negative average FX component across the sample in all 4 kxk portfolios, there is no evidence of UIP holding.

Table 14: kxk Portfolio Fama Regression Results

For each kxk portfolio OLS estimates of α and β (standard errors below) and the corresponding adjusted R^2 are shown for the regressions:

$$\Delta S_t^{k,high} = \alpha + \beta DM_t^{k,high} + \epsilon_t \quad \text{and} \quad \Delta S_t^{k,low} = \alpha + \beta DM_t^{k,low} + \epsilon_t$$

for the high yielding and low yielding sides respectively, where $\Delta S_t^{k,high} = 1/k \sum_{i=1}^{k} \left((S_{t+1}^i - S_t^i)/S_t^i\right)$, $\Delta S_t^{k,low} = 1/k \sum_{i=1}^{k} \left((S_{t+1}^i - S_t^i)/S_t^i\right)$, $DM_t^{k,high} = 1/k \sum_{i=1}^{k} DM_t^i$, $DM_t^{k,low} = 1/k \sum_{i=1}^{k} DM_t^i$, and $k = 1, 2, 3, 4$ and $DM_t^i = (F_t^i - S_t^i)/S_{ti}$. The kxk portfolios where $k = 1, .., 4$ are constructed by each month going long the k highest forward discount currencies and short the k lowest forward discount currencies, equal USD amounts, where S_t^i and F_t^i are the FX spot and 1 month FX forward rates at time t quoted as the USD price per unit of foreign currency i, based on monthly non overlapping data. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012. Asterisks ***, **, and * indicate rejection of $H_0 : \beta = 1$ at the significance levels of 1%, 5%, and 10% respectively.

		α	β	Adj R^2	Reject $H_0 : \beta = 1$
$1x1$	High	0.0000	0.0974	0.0002	***
		0.0026	0.3486		
	Low	0.0084	-1.7893	0.0161	***
		0.0028	0.6651		
$2x2$	High	0.0004	0.0223	0.0000	**
		0.0025	0.4190		
	Low	0.0059	-1.5322	0.0138	***
		0.0022	0.6167		
$3x3$	High	0.0006	0.1073	0.0001	**
		0.0022	0.4487		
	Low	0.0042	-1.0870	0.0071	***
		0.0019	0.6101		
$4x4$	High	0.0000	-0.0591	0.0000	**
		0.0020	0.4724		
	Low	0.0034	-1.2801	0.0096	***
		0.0016	0.6183		

7 FX Carry Time Series Analysis

In order to gain a better understanding of how the returns of the FX carry trade have behaved through time this section looks at the impact of several major historical economic events on the FX carry trade returns. These events highlight the susceptibility of the FX carry trade to sudden drawdowns when periods of economic turmoil are encountered. If the excess returns of the FX carry trade do indeed represent a time varying risk premia then it is these crisis periods that they are compensating. Next a two state Markov-switching model is used to model these two distinct states, a crisis state and a normal state. Finally the sensitivity of the returns of the FX carry trade to the choice of execution date is examined.

7.1 FX Carry and Major Economic Events

The recent global financial crisis highlighted many risk management short comings within the global financial system. Within a bank context, the use of Value at Risk (VaR) as a risk measurement tool was found to have failed dismally. In response to this the Basel capital adequacy rules were rehashed for a third time in the form of Basel III. The core of Basel III is the requirement for banks to hold more capital against the risk that they run. With respect to VaR, Basel III introduced the concept of Stress VaR, which is a new mandatory reporting requirement. The idea, since VaR failed so badly, is to take the current risk positions on a trip back through time to see how they would have coped in prior 'financial market meltdowns'. History has shown us that the impact of these prior 'financial market meltdowns' was simply not captured by VaR reporting. History has also shown us that these events keep occurring at a frequency much higher than VaR modelling would have us believe. There is one critical assumption with Stress VaR - it still relies on the future resembling the past (BIS 2011).

With no particular inclusion criteria, a list of financial market 'meltdowns' includes the likes of :

- 1987 equity market crash (Black Monday)
- 1992 ERM crisis
- 1994 bond market crash
- 1998 Russian financial crisis and failure of LTCM
- 2000 dot-com bubble bursts
- 2007 sub-prime mortgage turbulence leads to global financial crisis

How has the FX carry trade performed during these periods of financial market turmoil ?

The measure used to analyse the downside performance of the FX carry trade is drawdown (DD_t), which is defined as :

$$DD_t = \begin{cases} 0 & if \quad X_t = X_t^{max} \\ X_t - X_t^{max} & if \quad X_t < X_t^{max} \end{cases} \tag{31}$$

where X_t is the cumulative value of 1 unit invested from the start of the sample period until time t, as defined by Equation (19), and X_t^{max} is the maximum cumulative value from the start of the sample period until time t. So if the current cumulative value, X_t, sets a new high water mark then DD_t will be 0, otherwise DD_t will be a negative number being the difference between the current cumulative value and the current high water mark (Chekhlov et al. 2000, Goldberg and Mahmoud 2016). The drawdown percentage loss from the previous high watermark is defined as :

$$DD_t^{\%} = \frac{DD_t}{X_t^{max}} \tag{32}$$

7.1.1 Equity Market Crash - 1987

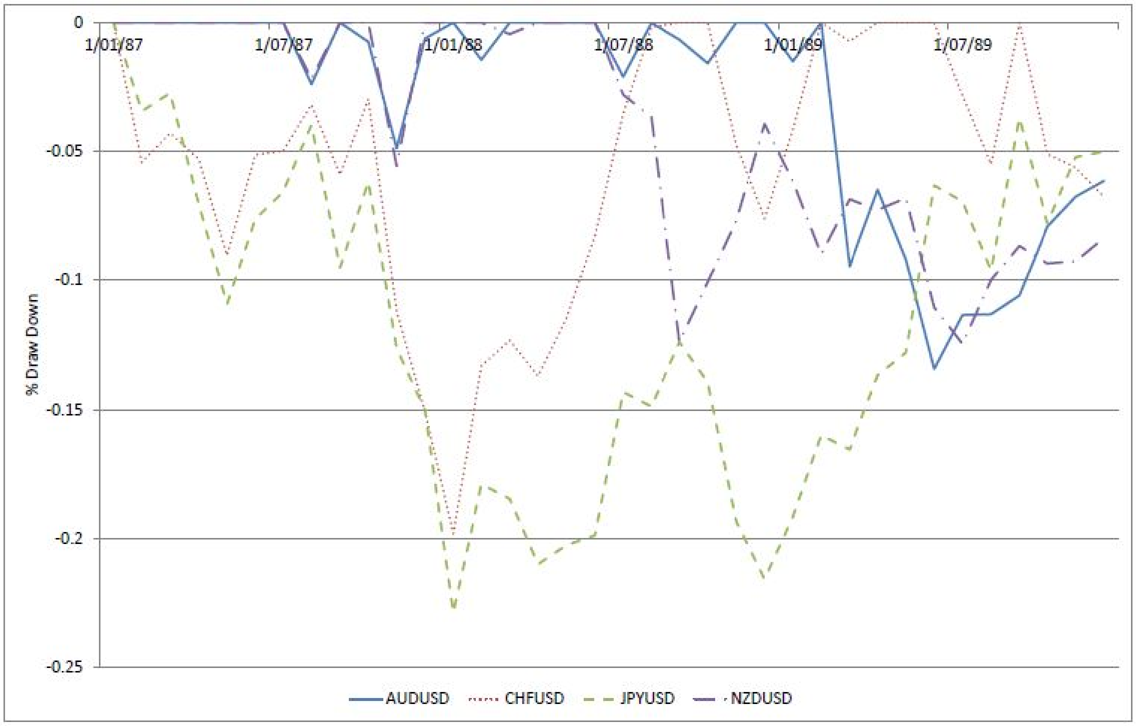
For several years prior to October 1987 global equity markets had performed well. However, by mid 1987 the global macroeconomic picture was becoming a little clouded. Interest rates began rising globally and in the case of the United States a growing trade deficit and a declining US dollar was leading to concerns about inflation and the requirement for higher interest rates going forward (Metz 1988).

From Wednesday October 14 1987 to Friday October 15 1987, the S&P 500 lost over 10%, the largest 3 day decline since World War II (McKeon and Netter 2009). Reports that the US House of Representatives had filed legislation to mitigate tax benefits from merger financing and the announcement of a worse than expected trade deficit are two events commonly pointed at as precipitating the decline on Wednesday 14th. Once in decline of this magnitude, other risk management factors came into play, particularly the selling required from program selling strategies and margin call requirements. To compound global issues, markets in London were effectively shut due to a storm that was wrecking havoc and made travel to work in the City of London impossible for most.

On Monday 19 October 1987 the S&P 500 declined over 20%. Throughout the day many trading systems struggled to cope with the volatility and volume of trades. This remains the largest single day percentage loss in history and is often referred to as Black Monday (although in New Zealand due to the time zone difference it is Black Tuesday). Prior to the US markets opening on Tuesday 20 October 1987 the Federal Reserve released a statement reaffirming its willingness to provide liquidity if required to support the stability of the financial system, a statement followed up by the subsequent open market operations they carried out. By the end of October 1987 global stock markets had declined markedly from their peak values earlier in 1987, with one of the worst affected being New Zealand which had fallen approximately 60% from its 1987 peak. Carlson (2007) provides a detailed summary of these events.

Figure 3 shows the drawdown percentages of the single currency FX carry trades from January 1987 through until December 1989 for NZDUSD, AUDUSD, CHFUSD, and JPYUSD which were the two highest and two lowest yielding currencies respectively at the start of the period in question.

Figure 3: Single Currency FX Carry Trade Drawdowns January 1987 - December 1989



This graph shows the drawdown percentages of the single currency FX carry trades from January 1987 through until December 1989 for NZDUSD, AUDUSD, CHFUSD, and JPYUSD

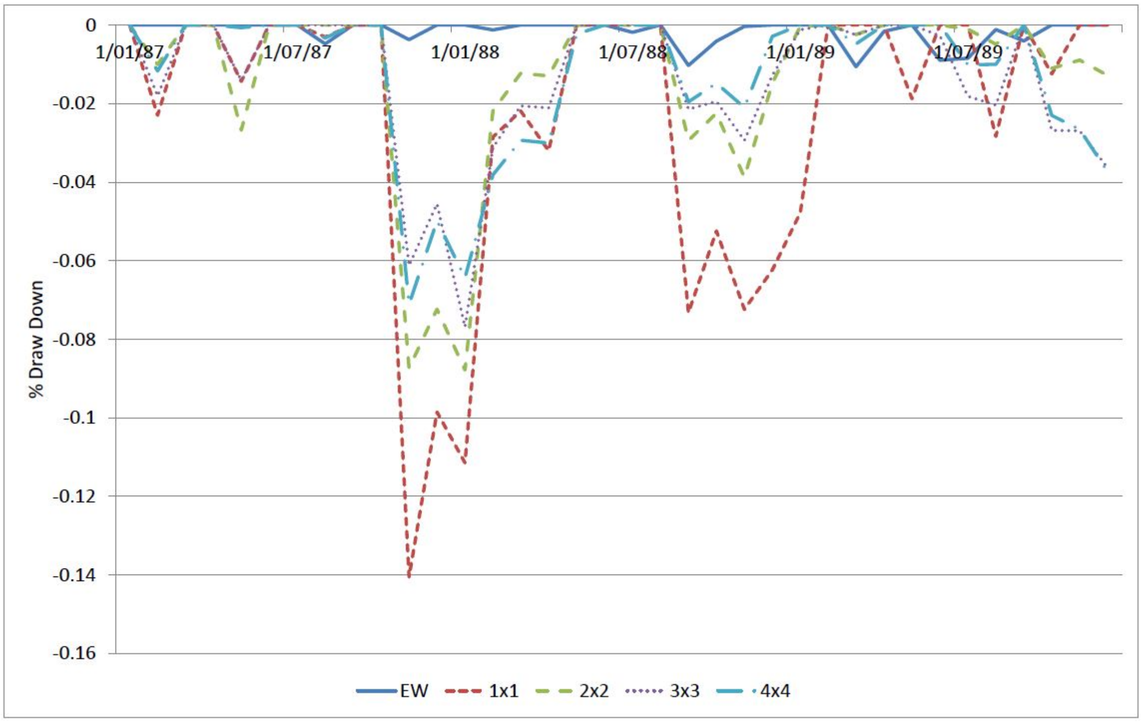
After the US led global stock market crash in October 1987 the FX carry trades of two high yielding currencies in Figure 3, AUDUSD and NZDUSD, experienced a very modest drawdown of approximately 5% and in each case they regained their high water marks by the start of 1988. On the other hand the FX carry trades of the two lowest yielding currencies at the time didn't fare so well. Both the CHFUSD and JPYUSD FX carry trades, from the start of 1987, were in a drawdown of 5%-10% already, when the October 1987 stock market crash occurred. Post the crash the drawdowns for both of these FX carry trades increased to be down approximately 20% by the end of 1987. The CHFUSD FX carry trade recovered its high watermark by late 1998, but in the case if the JPYUSD FX carry trade, it was yet recover by the end of 1989.

Figure 4 shows the drawdown percentages of the EW and kxk portfolio FX carry trades from January 1987 through until December 1989. All of the four kxk FX carry portfolios incurred drawdowns immediately following the stock market turmoil in October 1987. Not surprisingly the $1x1$ portfolio suffered the largest drawdown, of 14%, with the larger number of currencies in the remaining kxk portfolios resulting in smaller drawdowns. By mid 1998 all of the kxk portfolios had recovered their high water mark. Impressively the EW portfolio FX carry trade was largely insulated from the stock market events, due to its short USD exposure at the time.

7.1.2 Russian Financial Crisis and LTCM - 1998

Leading up to early 1997, the US dollar had appreciated due mainly to rising interest rates. Asia's 'Tiger' economies which had been touted as success stories were starting to show signs of overheating. Across the region currency pegs to the US dollar had been introduced to reduce risk for both foreign lenders and local borrowers. However, with their currencies pegged to the US dollar these emerging Asian export led economies became increasingly less

Figure 4: Portfolio FX Carry Trade Drawdowns January 1987 - December 1989



This graph shows the drawdown percentages of the EW and kxk portfolio FX carry trades from January 1987 through until December 1989.

competitive. As their currency reserves diminished, nervous investors began to withdraw their funds, and speculators added to this with short selling, the pressure on the dollar pegs became too much. Across the region in mid 1997 local currencies crashed and volatility rose sharply.

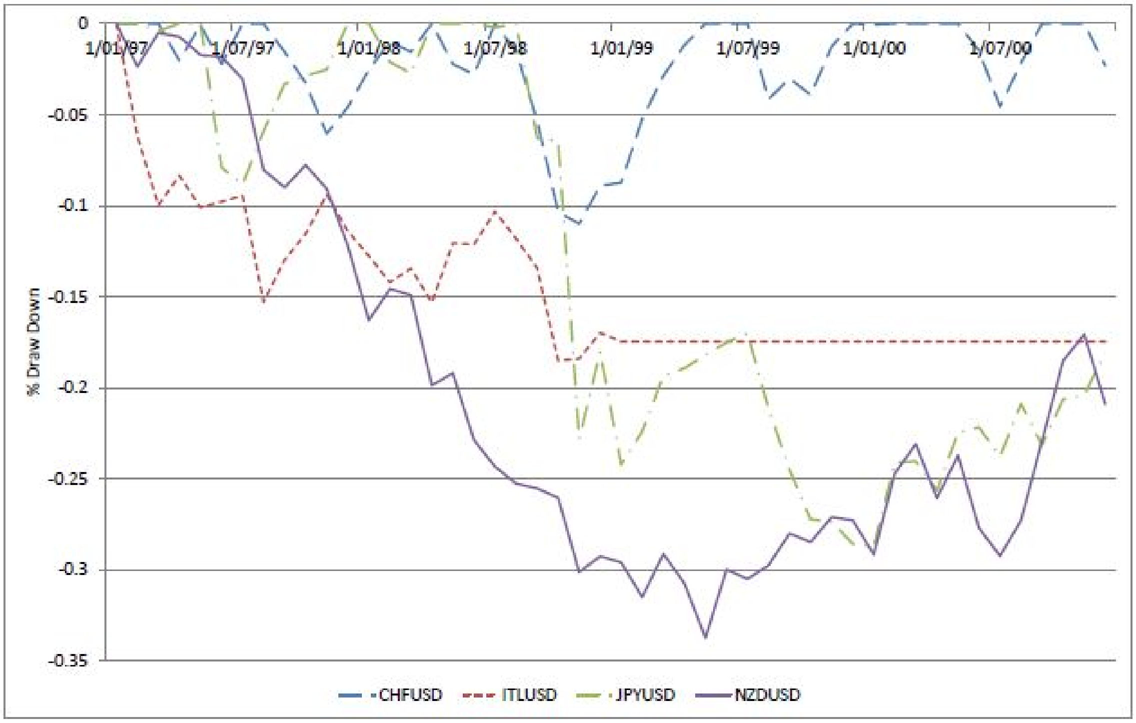
At the time the other 'big' trade in Emerging markets was in Russia. Like Asia, its currency the Ruble was pegged to the US dollar. This gave investors the confidence, and mitigated their foreign exchange risk provided the peg held, to buy short dated high yielding Russian bonds, called GKO's. Expectations were that these high yields would be a short term phenomenon and so foreign investors were eager to be involved in this trade. However, the majority of foreign investors did not physically buy the Russian bond. Rather, they bought a simple derivative from a bank called a total return swap, or TRS. This derivative had the same return had they physically bought the Russian bonds, less fees charged by the arranging bank, but crucially it did not have the same cash flows. In a TRS, the client does not pay the bank the full notional amount of the bond upfront, but instead pays a small margin. The bank uses its own capital to purchase the Russian bonds. This is risky business for the bank because if Russia ever defaulted, the Russian bond that they own would be worthless but the client would still owe the bank the principal (and interest) of the implicit loan. If the clients did not have this money then the bank is left out of pocket. In addition, by enabling clients to trade this product on a margin basis they were able to leverage their available capital.

One of the most prolific investors in the Russian trade was a hedge fund called Long Term Capital Management, LTCM. LTCM was setup in 1993 by John Meriwether who had been a star trader at Salomon Brothers. LTCM assembled some of the smartest minds in finance to join them, most notably Fischer Black and Robert Merton, both famous for their contributions to modern day finance and in particular option pricing. Wealthy investors provided \$1.25 billion in start up capital. By mid 1997 this initial capital had grown to approximately \$4.7 billion, representing an average annual return of around 40%. LTCM traded in enormous size and with enormous leverage. Lowenstein (2002) provides a colourful expose of the LTCM story and global environment at the time.

By June of 1998 the Asian economies had gone into deep recession. As a result of this, the price of oil was declining, a huge concern for Russia given its primary export status. GKO yields started 1997 at 30% and quickly fell to 17% by mid that year. They were now back at 50%. On 17 August 1998 the Russian government devalued the ruble, defaulted on domestic debt, and declared a moratorium on repayment of foreign debt. The ruble lost 80% of its value in days and Russian bonds were worthless. On 2 September, LTCM announced that the fund had lost 44% in August alone and that they were unwinding all of their positions. The problem was the banks had the same positions and LTCM were simply too big to get out. Concerns over what their possible failure might mean for the stability of the financial system and for their counterparty banks were very real. In late September 1998, the US Federal Reserve managed to convince a group of banks to inject \$4 billion of capital into LTCM, which gradually saw the crisis abate.

Figure 5 shows the drawdown percentages of the single currency FX carry trades from January 1997 through until December 2000 for NZDUSD, ITLUSD, CHFUSD, and JPYUSD which were the two highest and two lowest yielding currencies respectively at the time. Note that in the case of ITLUSD the series ends in January 1999 when the Euro was introduced.

Figure 5: Single Currency FX Carry Trade Drawdowns January 1997 - December 2000



This graph shows the drawdown percentages of the single currency FX carry trades from January 1997 through until December 2000 for NZDUSD, ITLUSD, CHFUSD, and JPYUSD

The two high yielding currencies in Figure 5, NZDUSD and ITLDUSD, suffer initial carry trade drawdowns of 10% - 15% in 1997 as the Asian economic situation was playing out. However in 1998 the NZDUSD FX carry trade entered a sustained drawdown period which by early 1999 saw it down almost 35%, from the start of 1997. Of the low yielding FX carry trades depicted, CHFUSD remained relatively resilient through out the period with a maximum drawdown of approximately 10% in late 1998. The same can be said for JPYUSD until September 1998 at which point it entered into a sustained drawdown of close to 30% by early 2000. However, this may in part have been attributable to the domestic issues Japan was experiencing at the time, particularly in the banking sector.

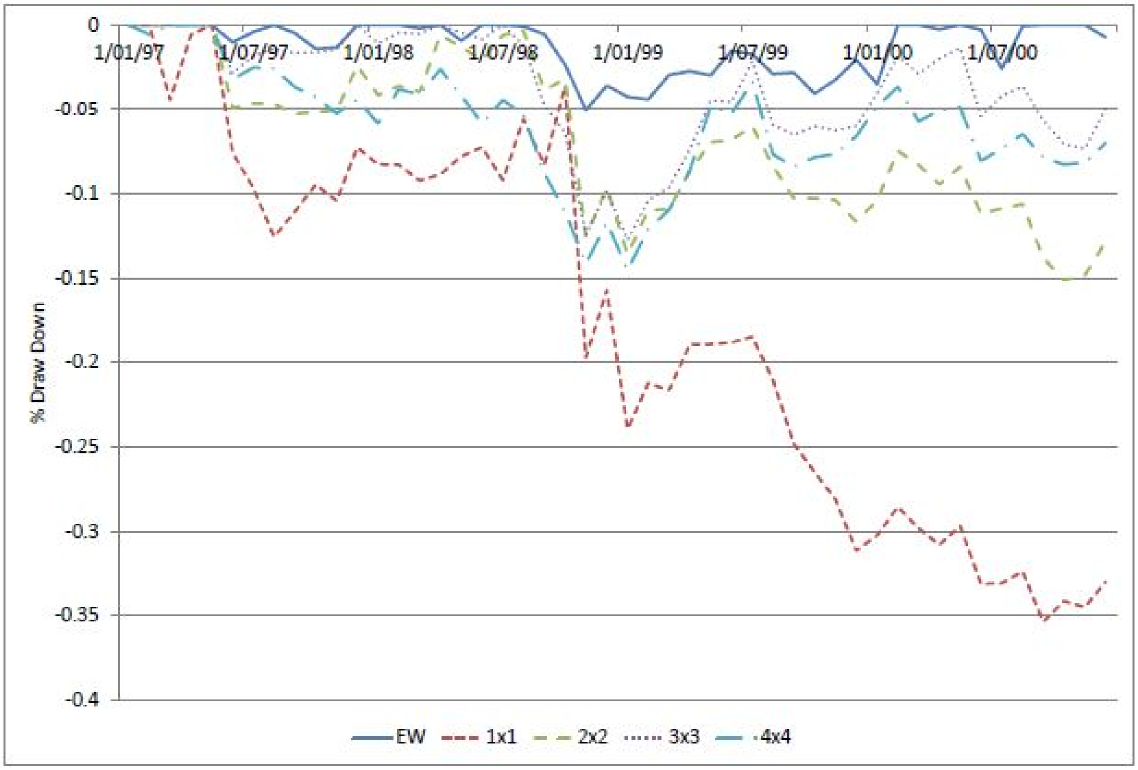
Figure 6 shows the drawdown percentages of the EW and kxk portfolio FX carry trades from January 1997 through until December 2000. With the exception of the $1x1$ FX carry trade, the remaining portfolio carry trades in Figure 6 are relatively resilient through out the period of the Russian financial crisis and LTCM, experiencing a maximum drawdown of almost 15% by late 1998, early 1999. The $1x1$ FX carry trade experiences a much larger and sustained drawdown from late 1998 due mainly to the influence of JPYUSD in the portfolio.

7.1.3 Global Financial Crisis - 2007/2008

Much has been written about the global financial crisis of 2007/08. In 2006 there were concerns being voiced about the state of the US mortgage market. A Business Week [6] cover story, 'Nightmare Mortgages', in September 2006 quoted a housing economist describing the option adjustable rate mortgages as being "like a neutron bomb, its going to kill all the people but leave the houses standing". The initial active phase of the crisis, which largely manifested itself as a credit crisis, can be dated back to 9 August 2007 when BNP

[6] `http://www.bloomberg.com/news/articles/2006-09-10/nightmare-mortgages`

Figure 6: Portfolio FX Carry Trade Drawdowns January 1997 - December 2000



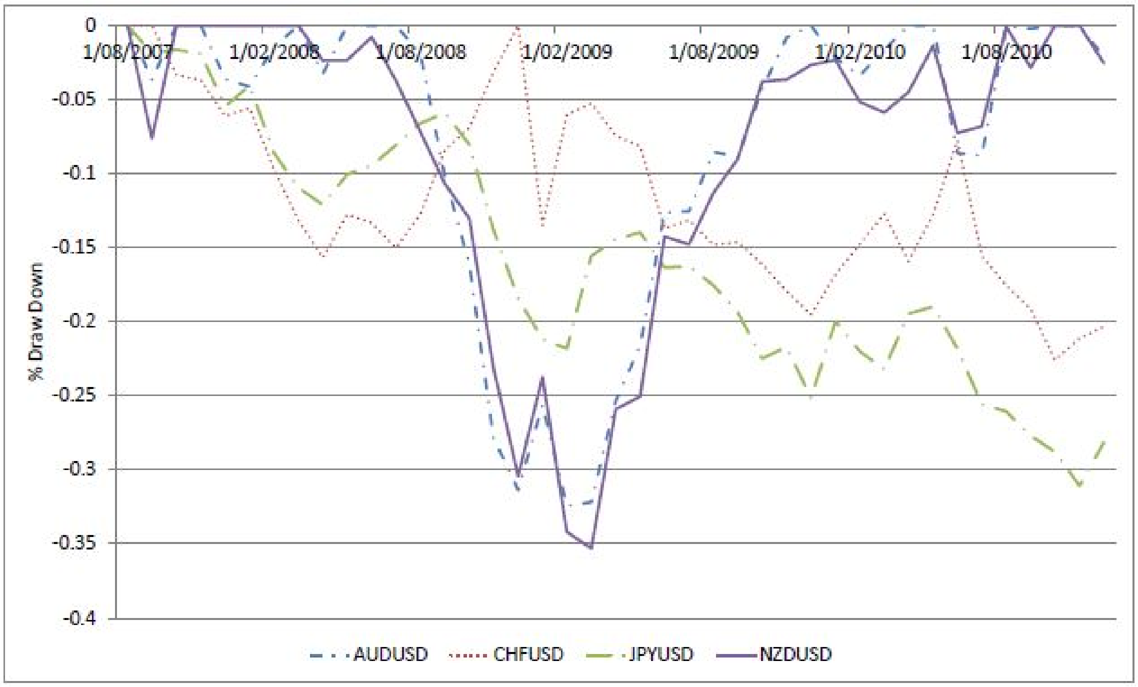
This graph shows the drawdown percentages of the EW and kxk portfolio FX carry trades from January 1997 through until December 2000.

Paribas terminated withdrawals from three of its hedge funds citing a "complete evaporation of liquidity". As the sub-prime mortgage market began to unravel and banks struggled to fund themselves, the fragility of these organizations, due to their excessive leverage, became apparent. In September 2008 Lehman Brothers was allowed to fail, the largest bankruptcy in US history. Among other failures, AIG needed bailing out by the US government, Bear Sterns was sold under duress to JP Morgan Chase, and Merrill Lynch merged with Bank of America to prevent its demise. In October 2008 the US government introduced the Troubled Asset Relief Program (TARP) which effectively guaranteed US banks with a $700 billion rescue facility. Taylor (2009) discusses the GFC and policy response.

Figure 7 shows the drawdown percentages of the single currency FX carry trades from August 2007 through until December 2010 for AUDUSD, NZDUSD, CHFUSD, and JPYUSD which were the two highest and two lowest yielding currencies respectively at the time.

Initially the four single currency FX carry trades in Figure 7 are relatively resilient to the onset of the GFC. In early 2008 the two low yielding carry trades begin to suffer with a drawdown of 10% - 15% but then in August 2008 when the GFC was in full swing the two high yielding carry trades go into free fall eventually incurring a drawdown of approximately 35%. By early 2010 the two high yielding carry trades have recovered from this drawdown. However, in stark contrast to this the two low yielding carry trades, despite a brief recovery in late 2008, settle into a protracted drawdown of 20% - 30%. Figure 8 shows the drawdown percentages of the portfolio FX carry trades from August 2007 through until December 2010. The portfolio FX carry trades experience moderate drawdowns prior to mid 2008 when the GFC was in full swing. At that point they all experience drawdowns for the remainder of 2008 ranging from 10% for the EW portfolio through to a whopping 50% for the $1x1$ portfolio. By the end of the period analysed, although some are

Figure 7: Single Currency FX Carry Trade Drawdowns August 2007 - December 2010



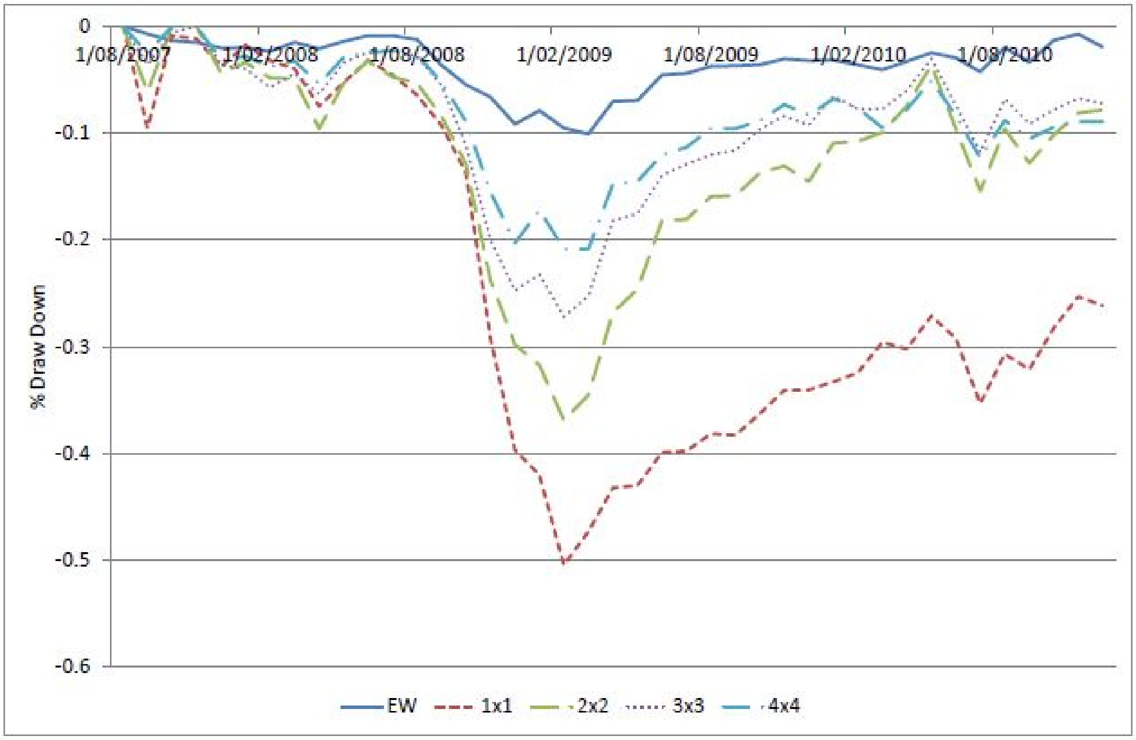
This graph shows the drawdown percentages of the single currency FX carry trades from August 2007 through until December 2010 for AUDUSD, NZDUSD, CHFUSD, and JPYUSD

close none of the portfolio FX carry trades have recovered their high water mark.

7.1.4 Conclusions

Simple visual inspection of the drawdown graphs for the 3 major economic events discussed in Section 7.1 show that the FX carry trade is indeed susceptible to sudden drawdowns when periods of economic turmoil are encoun-

Figure 8: Portfolio FX Carry Trade Drawdowns August 2007 - December 2010



This graph shows the drawdown percentages of the portfolio FX carry trades from August 2007 through until December 2010.

tered. The Economist [7] (2007) likened the carry trade to "picking up nickels in front of steamrollers: you have a long run of small gains but eventually get squashed". This is certainly borne out in the examination of these periods of economic crisis. However, what is striking is that across the 3 events discussed the $2x2$, $3x3$, and $4x4$ portfolios recover from such drawdowns relatively well when you consider the time frame to retrace to previous high

[7] `http://www.economist.com/node/8742054`

water marks. The single currency and $1x1$ portfolio are prone to currency specific effects which seem to result in more varied recovery periods. So it would seem that on average diversification matters both in terms of minimising the drawdown during the period of turmoil and also in the time required to retrace the drawdown.

7.2 FX Carry - Markov switching regime analysis

In Section 7.1 several major economic events from the past were analysed and it was shown that the FX carry trade is susceptible to significant drawdowns and increased volatility during such events. This notion of there being 'crisis' states and 'non-crisis' states raises the question of whether a regime switching model can statistically capture their existence. Engel and Hamilton (1990) were the first to use a Markov-switching model in the context of the FX markets where they found evidence of long swings in exchange rate dynamics. This precipitated others to follow suit and apply various versions of Markov-switching regimes to the FX markets. Colavecchio (2008) provides a summary of work in this field. Likewise there has been significant work applying the Markov switching framework to equity markets (Cecchetti et al. 1990, Hamilton and Lin 1996, Ryden et al. 1998, Schaller and Van Norden 1997).

In the case of FX carry, can a Markov-switching model capture different 'states' or 'regimes', denoted by R_t, within the series of FX carry trade returns, and if so whether these 'states' encapsulate the types of economic events outlined in Section 7.1. Consider the following 2 state Markov-switching model :

$$x_t = I_{(R_t=1)}\mu_1 + I_{(R_t=2)}\mu_2 + \epsilon_t \tag{33}$$

where $\epsilon_t \sim N(0, \sigma_t^2)$, and :

$$\sigma_t^2 = I_{(R_t=1)}\sigma_1^2 + I_{(R_t=2)}\sigma_2^2 \tag{34}$$

and :

$$I_{(R_t=i)} = \begin{cases} 1 & R_t = i \\ 0 & R_t \neq i \end{cases} \tag{35}$$

where $i \in (1, 2)$. This is the Markov-switching mixture of normal distributions described by Hamilton (1990).

So FX carry trade returns are modelled as having 2 states, each with their own intercept term and variance. In a Markov-switching process it is not known in which state the process is but the probabilities of being in either state can be estimated. The one-step ahead transition probabilities are denoted $p_{R_t,R_{t+1}}$, so for example p_{11} is the probability of staying in state 1 in the next period given that the process is in state 1 in the current period. The parameters of the above model are estimated by an iterative expectation maximization algorithm (Hamilton 1989, 1990) which is an iterative method originally developed by Dempster et al. (1977) and adjusted for Markov switching models. The results for the portfolio FX carry trade returns are shown in Table 15.

Table 15: Portfolio FX Carry Trades Markov Switching Results

Using a 2 state Markov switching model :

$$x_t = I_{(R_t=1)}\mu_1 + I_{(R_t=2)}\mu_2 + \epsilon_t$$

where ϵ_t $N(0, \sigma_t^2)$, and :

$$\sigma_t^2 = I_{(R_t=1)}\sigma_1^2 + I_{(R_t=2)}\sigma_2^2$$

and :

$$I_{(R_t=i)} = \begin{cases} 1 & R_t = i \\ 0 & R_t \neq i \end{cases}$$

where $i \in (1, 2)$. Constant 1 and 2 are the coefficient estimates in states 1 and 2 respectively, $p_{R_t,R_{t+1}}$ are the one step ahead transition probabilities, and Duration 1 and 2 are the expected duration (in months) in states 1 and 2 respectively. The kxk and EW portfolio FX carry trade returns are defied in Section 4. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Constant 1	Constant 2	p11	p12	p21	p22	Duration 1	Duration 2
$1x1$	-0.0312	0.0113	0.6326	0.3674	0.0539	0.9461	2.72	18.56
$2x2$	-0.0086	0.0101	0.7724	0.2276	0.0691	0.9309	4.39	14.47
$3x3$	-0.0023	0.0066	0.8742	0.1258	0.0428	0.9572	7.95	23.35
$4x4$	-0.0046	0.0058	0.8215	0.1785	0.0463	0.9537	5.60	21.60
EW	0.0022	0.0037	0.8976	0.1024	0.0413	0.9587	9.76	24.24

Constant 1 and 2 are the coefficient estimates in states 1 and 2 respectively, $p_{R_t,R_{t+1}}$ are the one step ahead transition probabilities, and Duration 1 and 2 are the expected duration (in months) in states 1 and 2 respectively. In each case state 1 is the 'crisis' state. Across the board state 2 exhibits a high level of persistence with p_{22} values all above 0.9 and consequently the expected duration of months spent in state 2 is larger than those of state 1.

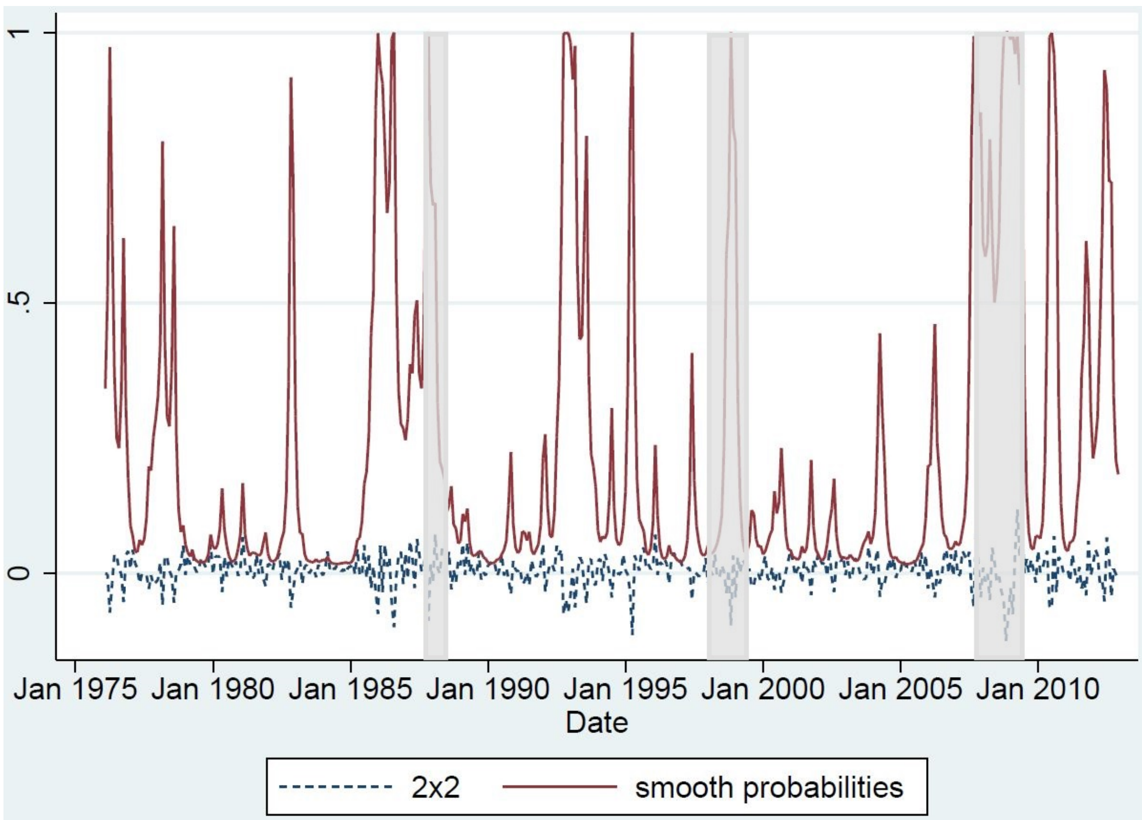
Figure 9 shows the smoothed one-step ahead probabilities for the $2x2$ portfolio FX carry trade. The smoothing process uses the entire sample to calculate the one-step ahead probability of being in state 1 rather than just the data up until the date at which the one-step ahead forecast is being made. Considering our economic events discussed in Section 7.1 it is clear that the approximate dates of 1987, 1998, and 2007/08 are captured by state 1, or 'crisis' state, by the Markov switching model employed.

This quick look at the the FX carry trade using a Markov switching model certainly supports the notion that FX carry trade goes through long periods of high returns and low volatility (state 2 in the above example) which is interspersed by shorter periods of lower returns and higher volatility (state 1 in the above example).

7.3 FX Carry - When to Execute ?

Out of convenience the literature tends to base its FX carry trade return calculations on the first day of a month or the last day of a month. The issue remains though, are the returns to the FX carry trade dependent on when in a month the trade is executed ? For publicly investible FX carry indices this is a very real issue for two reasons. Firstly, the question above, does the time of month the FX carry trade is executed influence their returns, and secondly, is there execution risk by making the mechanics of the trade

Figure 9: $2x2$ FX Carry Trade Markov Switching Results



This graph plots the monthly returns to the $2x2$ FX carry trade and the smoothed one-step ahead probabilities from the 2 state Markov switching model defined in Equations (33) to (35). The shaded areas represent the three periods discussed in Section 7.1.

execution publicly available and opening yourself up to market 'front running' ?

Providers of investible FX carry indices have constructed trade execution rules to try and negate the second issue of execution risk. For instance, Deutsche Bank [8] has a 'roll window' of five business days and executes the required FX transactions on two of those five days. These two execution days are not made public until after the 'roll window' expires in an effort to mitigate their execution risk around known market orders.

For investible FX carry indices the trade execution dates tend to be month start or month end dates, or for those requiring quarterly execution dates, IMM dates are often used in an effort to align their products with exchange traded products. IMM (International Money Market) dates are the third Wednesday of March, June, September, and December.

The FX carry trade returns presented in Section 5 were based on trade execution occurring on the first business day of the month. To answer the question of whether the returns to the FX carry trade are dependent on the time of month the trade is executed these returns are recalculated using execution dates of the sixth, eleventh, and sixteenth business days of each month as proxy's for executing at the start of the second, third, and fourth weeks of each month respectively. The results are presented in Table 16
Table 16 shows the differences between the annualised returns and standard deviations of the respective FX carry trades executed in the second, third,

[8] `http://globalmarkets.db.com/new/docs/dbCurrencyReturns_March2009.pdf`

Table 16: Weekly Execution Results for FX Carry Trades

The differences between the annualised returns and standard deviations of the respective FX carry trades executed in the second, third, and fourth weeks of each month and those of the FX carry trades executed in the first week of each month are shown. The single currency, kxk, and EW portfolio FX carry trade returns are defied in Section 4. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Week 2 Differences		Week 3 Differences		Week 4 Differences	
	Return	Std Dev	Return	Std Dev	Return	Std Dev
AUDUSD	-0.0072	-0.0019	0.0009	-0.0128	-0.0002	-0.0013
CADUSD	-0.0057	-0.0034	-0.0026	-0.0100	0.0066	-0.0028
CHFUSD	-0.0030	0.0009	-0.0015	-0.0017	-0.0034	-0.0029
DEMUSD	0.0017	0.0037	0.0072	0.0000	0.0015	-0.0067
EURUSD	-0.0100	0.0016	-0.0064	-0.0028	-0.0211	0.0038
FRFUSD	0.0064	0.0024	0.0081	0.0018	0.0142	-0.0058
GBPUSD	-0.0037	0.0000	-0.0039	-0.0040	0.0003	-0.0048
ITLUSD	-0.0013	0.0044	0.0012	-0.0013	0.0049	-0.0056
JPYUSD	-0.0007	0.0015	-0.0026	0.0029	0.0017	-0.0072
NOKUSD	-0.0030	0.0027	-0.0021	-0.0030	0.0009	0.0012
NZDUSD	0.0064	-0.0005	0.0056	-0.0088	0.0089	-0.0075
SEKUSD	-0.0093	0.0003	-0.0158	-0.0071	-0.0116	-0.0002
EW	-0.0024	0.0013	-0.0014	-0.0014	0.0006	-0.0005
$1x1$	-0.0041	-0.0016	0.0052	-0.0058	0.0120	-0.0044
$2x2$	-0.0080	0.0056	-0.0187	-0.0010	-0.0001	0.0023
$3x3$	-0.0076	0.0051	-0.0113	-0.0029	-0.0062	0.0023
$4x4$	-0.0019	0.0014	-0.0059	-0.0029	-0.0039	0.0007

and fourth weeks of each month and those of the FX carry trades executed in the first week of each month, presented earlier. So for example for the AUDUSD FX carry trade the week 2 difference for return of -.0072 means the annualised return to the AUDUSD FX carry trade executed in the second week of each month is -.0072 less than when it is executed in the first week of each month. Using a dependent group t-test, testing the differences between the monthly return series for each FX carry trades for week 2, week 3, and week 4 all versus week 1 showed that none of the average monthly returns were significantly different from the week 1 numbers. This suggests that statistically the returns to the FX carry trades are not a function of when you execute the trades with in the month.

Chapter II

FX Carry and General Superposition Strategies

8 Introduction

When characterizing the FX carry trade reference is often made of its inherent volatility, and in particular to its susceptibility to have large drawdowns (Beranger et al. 1999, Cairns et al. 2007, Galati et al. 2007, Gagnon and Chaboud 2007, Brunnermeier et al. 2008), as shown in the selected examples in Section 7.1. Additionally, Darvas (2009) acknowledges the difficulty investors might face in staying in highly volatile carry trades in a behavioral context, "Due to psychological factors it is hard to assume that any investor is capable of sticking consistently to a single strategy if he witnesses the loss of say, 75% of its wealth within a short period of time".

In the research on FX carry there have been several papers that have adequately addressed the practical issues of allowing for bid offer spreads (Burnside et al. 2006, Lustig et al. 2011, Menkhoff et al. 2012(b)) and leverage (Darvas 2009) in constructing FX carry trade portfolios.

Non dealer financial trading institutions, particularly hedge funds, are heavily involved in the FX carry trade (Rime and Schrimpf 2013, King and Rime 2010, Galati and Heath 2007, Becker and Clifton 2007, Galati and Melvin 2004). At a hedge fund (and indeed other financial trading institutions), irrespective of the overall firm level risk management policies, at an individual trader level traders are governed by individual risk management frameworks which typically take the form of a series of stop-loss rules (Drobny 2006, Belmont 2011).

The aim of this chapter is to examine, within an institutional trading setting, what impact applying money management rules, in particular stop-losses, to the FX carry trade has on the available returns and on survivorship. The superposition strategy is this application of the stop-loss rules to the FX carry trade. This will enable the following question to be addressed: is the forward premium puzzle in fact smaller in an institutional setting than what is traditionally presented in the literature which is implicitly an unconstrained risk management environment? For clarity I am not saying anything about how institutional traders might be trading FX carry. I am simply taking the FX carry trade, as it is most commonly specified in the literature, and imposing stop-loss risk management policies on it to see what impact they have on the return properties.

Previous work on stop-loss rules has largely focused on stock markets and the 'alternative' investment in interest rate markets (Kaminski and Lo 2008). It is plausible that by overlaying these stop-loss rules on the FX carry trade, the available returns are materially different and in fact the trade may not survive. This will provide insight into how the FX carry trade is implemented within institutional trading establishments and the degree to which the forward premium puzzle exists from their perspective.

The main result of this chapter is to show that when a stop-loss risk management policy is imposed on the FX carry trade, the available returns are reduced and in some cases the trade does not survive the sample period. That is to say, the forward premium puzzle is smaller under these institutional risk management policies compared to an unconstrained risk management environment.

9 Stop-Loss Rules

Stop-loss rules are standard practice in the professional trading community. Individual traders at hedge funds and bank proprietary trading desks are governed by some sort of stop-loss policy, and generally some sort of maximum position size policy (Drobny 2006, Kaminski and Lo 2008, Belmont 2011). The details of these rules are often central to contract negotiations with traders.

Despite their widespread use in the industry, there has been surprising little research done on stop-loss rules. Several reasons have been put forward to explain the scarcity of academic work in this area. Scherer (2008) contends that the reason for this has been the popularity of the random walk hypothesis in academic finance and explains that a stop-loss policy is just a random market timing strategy which Samuelson (1994) proves to underperform a buy and hold strategy. Hachemian et al. (2013) agrees that the popularity of the random walk hypothesis has contributed to the "paucity" of academic studies, but also notes that it is primarily an empirical matter.

In its simplest form a stop-loss rule serves to reduce a portfolio's exposure after its cumulative losses reach some pre-defined level. In practice the period over which cumulative losses are measured can vary from monthly, to the financial year, and to lifetime profit and loss at the firm. Likewise the measure of losses can be either losses below zero profit or losses from the point of maximum profit above zero, if indeed there has been one. Additionally, there may also be the issue of what the re-entry trigger is and at what point do you start increasing a portfolio's exposure after it has reached one of these pre-defined stop-loss levels and reduced risk accordingly.

From a theoretical perspective, adopting a stop-loss policy and expecting it to have a positive benefit on expected returns hinges on the assumption that returns display some form of positive autocorrelation. That is, large losses are expected to be followed by further losses, so the imposition of a stop-loss policy will have a benefit to expected returns. This autocorrelation may be across the entire sample or conditional in the sense that after a certain threshold level of losses autocorrelation is accentuated, and hence the benefit of imposing a stop-loss policy.

The literature has reached various conclusions about the effectiveness of stop-loss rules. Dybvig (1988), Gollier (1997), Vanstone (2008), and Erdestam and Stangenberg (2008) present cases for the inefficiency of stop-loss rules in the sense that any reduction in return volatility, as a result of using a stop-loss policy, is more than offset by a corresponding negative impact on expected returns. Ma et al. (2008) and Macrae (2005) find that the use of stop-loss rules alters the return distribution in an unexpected manner by creating a 'kink' on the negative side of the distribution. Additionally Ma et al. (2008) also find instances where stop-losses can significantly reduce volatility. Lei and Li (2009) examine the impact of stop-loss strategies on individual stocks and find that they neither reduce nor increase investors losses relative to a buy and hold strategy when security returns are extended to all future possible paths. They do find a unique stop-loss mechanism that reduces risk suggesting that the benefits of stop-loss strategies may come from risk reduction rather than return improvement. James and Yang (2010) take a different approach and instead of determining whether stop-loss rules add value to a trading strategy they determine a procedure for actually setting stop-loss levels. Scherer (2008) applies stop-loss rules to some popular FX trading strategies but finds negative conditional autocorrelation, which of course do not support stop-loss policies. Hachemian et al. (2013) find evidence that stop-loss policies can prevent severe losses but the results are more ambiguous when the losses are within expected limits. Kaminski and Lo (2008) develop a simple stop-loss policy and re-entry rule. They conclude that the benefit of a stop-loss policy, in terms of its ability to improve risk adjusted returns, is dependent on the type of process that is driving the underlying return series. They show that under the random walk hypothesis or a mean reversion process, stop-loss policies always reduce a trading strategy's expected return, but in the presence of momentum, stop-loss rules can add value. They go on to show empirically that their stop-loss and re-entry policy added value to a buy and hold strategy involving US equities and long term government bonds.

For clarity, there are several other branches of study where the notion of stop-loss is encountered, that are not addressed in this paper. There is a growing literature on market microstructure and in particular limit order and optimal order selection which often discuss the notion of stop-loss order types (Biais et al. 1995, Handa and Schwartz 1996, Harris and Hasbrouck 1996, Seppi 1997, Lo et al. 2002) . Likewise, the study of technical analysis trading rules often refers to the use of stop-loss rules. In behavioral finance there is work that looks at the relationship between human decision making and stop-loss decisions (Odean 1998).

10 Theoretical Superposition Framework

In this section a theoretical superposition framework is applied to the FX carry trades defined in Section 4 and the impact this framework has on the returns of the FX carry trade are evaluated.

10.1 Stop-Loss Policy Definition

The following is an adaptation of the theoretical stop-loss framework presented in Kaminski and Lo (2008).

In order to implement a stop-loss policy on the FX carry trade investment strategy cumulative returns must be tracked. Define the realised cumulative returns of the FX carry trade strategy over a window of J months [9] :

[9] As in Kaminski and Lo (2008) I have ignored compounding effects and defined cumulative returns to be the sum of simple returns for the purposes of determining the stop-loss policy trigger.

$$X_t(J) = \sum_{j=1}^{J}(x_{t-j+1}) \tag{36}$$

A simple stop-loss framework, l_t, is then applied to the cumulative returns. Returns are now defined as :

$$x_t^l = l_t x_t \tag{37}$$

And so cumulative returns of the stop-loss imposed monthly FX carry trade returns is now defined as :

$$X_t^l(J) = \sum_{j=1}^{J}(x_{t-j+1}^l) \tag{38}$$

The stop-loss is triggered when the cumulative returns over J months fall below some specified level γ, and the re-entry is triggered when the last period return is above some specified value δ :

$$l_t = \begin{cases} 0 \quad if \quad X_{t-1}^l(J) & < \quad -\gamma \quad and \quad l_{t-1} = 1 \quad (exit) \\ 1 \quad if \quad x_{t-1} & \geq \quad \delta \quad and \quad l_{t-1} = 0 \quad (re-enter) \\ 1 \quad if \quad X_{t-1}^l(J) & \geq \quad -\gamma \quad and \quad l_{t-1} = 1 \quad (stay\ in) \\ 0 \quad if \quad x_{t-1} & < \quad \delta \quad and \quad l_{t-1} = 0 \quad (stay\ out) \end{cases} \tag{39}$$

So the stop-loss policy is a dynamic binary asset-allocation rule, l_t, between the FX carry trade and no investment. The cumulative returns, $X_t^l(J)$, under the stop-loss framework l_t is a rolling sum of J periods, whether or not you are in the trade. If you exit the strategy ($X^l_{t-1}(J) < -\gamma$ and $l_{t-1} = 1$) then for the next periods cumulative sum the previous months return will be 0 since you are not in the trade ($x^l_t = l_t x_t$ and $l_t = 0$). This will remain the case until you re-enter the trade ($x_{t-1} \geq \delta$ and $l_{t-1} = 0$).

To measure the impact of applying this stop-loss framework to the FX carry trades the difference between the annualised return of the FX carry trade under the stop-loss framework and the original FX carry trades annualised return are calculated :

$$\triangle\ \overline{x}^A = \overline{x}^{A,l} - \overline{x}^A \tag{40}$$

where $\overline{x}^{A,l}$ is the annualised return of the FX carry trade after the stop-loss policy has been imposed :

$$\overline{x}^{A,l} = 12 * \overline{x}^l \tag{41}$$

and $\overline{x}^l$ is the average monthly FX carry trade return under the stop-loss policy :

$$\overline{x}^l = \frac{x^l_1 + x^l_2 + ... + x^s_T}{T} \tag{42}$$

The difference between the annualised standard deviation of the FX carry trade under the stop-loss framework and the annualised standard deviation of the original FX carry trade is :

$$\triangle\ s^A = s^{A,l} - s^A \tag{43}$$

where $s^{A,l}$ is the annualised standard deviation of the FX carry trade after the stop-loss policy has been imposed :

$$s^{A,l} = s^l * \sqrt{12} \tag{44}$$

where the monthly standard deviation estimate under the stop-loss policy is :

$$s^l = \left[\frac{(x_1^l - \overline{x}^l)^2 + (x_2^l - \overline{x}^l)^2 + ... + (x_T^l - \overline{x}^l)^2}{(T-1)}\right]^{(1/2)} \tag{45}$$

10.2 Results

10.2.1 Single Currency FX Carry Trades

Tables 17 to 20 present the annualised returns ($\overline{x}^{A,l}$) and standard deviations ($s^{A,l}$) for the single currency FX carry trades with the theoretical stop-loss policy imposed on them for $J = 12$ months, $\gamma = 0, 0.05, 0.1, 0.15, 0.2$, and $\delta = 0, 0.01, 0.02, 0.03$. For each currency versus the USD the lower panel also shows the difference between the annualised return ($\triangle\ \overline{x}^A$) and annualised standard deviation ($\triangle\ s^A$) of the FX carry trade under the stop-loss framework and the annualised return and annualised standard deviation of the original FX carry trade.

Table 17: Single Currency Stop-Loss Policy Results

For each currency versus the USD annualised returns and standard deviations achieved by imposing the stop-loss policy are shown in the upper panels, and the difference between these stop-loss impacted results and the original FX carry trade returns and standard deviations are presented in the lower panels. The stop-loss policy l_t is imposed onto the FX carry trade returns x_t so the new stop-loss impacted returns are defined as $x_t^l = l_t x_t$ where l_t is defined as :

$$l_t = \begin{cases} 0 & if \quad X_{t-1}^l(J) < -\gamma \quad and \quad l_{t-1} = 1 \quad (exit) \\ 1 & if \quad x_{t-1} \geq \delta \quad and \quad l_{t-1} = 0 \quad (re-enter) \\ 1 & if \quad X_{t-1}^l(J) \geq -\gamma \quad and \quad l_{t-1} = 1 \quad (stay\ in) \\ 0 & if \quad x_{t-1} < \delta \quad and \quad l_{t-1} = 0 \quad (stay\ out) \end{cases}$$

where $X_t^l(J) = \sum_{j=1}^{J}(x_{t-j+1}^l)$ is cumulative stop-loss impacted returns over J months. So, the stop-loss is triggered when the cumulative returns over J months fall below some specified level γ, and the re-entry is triggered when the previous months unconstrained return is above some specified value δ. Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data and annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. Monthly carry trade returns for currency i at time t, $x_t^i = a*(F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively. The stop-loss policy is imposed for J (cumulative return window) = 12,6,3 months, γ (cumulative stop-loss trigger) = 0,0.05,0.1,0.15,0.2 and δ (re-entry trigger) = 0,0.01,0.02,0.03. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

AUDUSD Stop-Loss Policy Results : J=12

Gamma(γ)	Delta(δ) 0	0.01	0.02	0.03	Delta(δ) 0	0.01	0.02	0.03
	Return				Standard Deviation			
0	0.0581	0.0278	0.0201	0.0385	0.1022	0.0906	0.0857	0.0879
0.05	0.0626	0.0555	0.0491	0.0518	0.1133	0.1083	0.1083	0.1072
0.1	0.0668	0.0589	0.0549	0.0572	0.1185	0.1136	0.1141	0.1134
0.15	0.0695	0.0597	0.0577	0.0600	0.1188	0.1153	0.1152	0.1144
0.2	0.0691	0.0597	0.0585	0.0585	0.1189	0.1154	0.1153	0.1153
	Return Difference				Standard Deviation Difference			
0	-0.0156	-0.0460	-0.0537	-0.0352	-0.0185	-0.0301	-0.0350	-0.0327
0.05	-0.0112	-0.0182	-0.0247	-0.0219	-0.0074	-0.0124	-0.0124	-0.0134
0.1	-0.0069	-0.0148	-0.0189	-0.0166	-0.0022	-0.0070	-0.0066	-0.0073
0.15	-0.0042	-0.0140	-0.0161	-0.0138	-0.0019	-0.0053	-0.0055	-0.0063
0.2	-0.0046	-0.0140	-0.0153	-0.0153	-0.0018	-0.0053	-0.0054	-0.0054

Table 18: Single Currency Stop-Loss Policy Results cont'd

	Delta(δ)				Delta(δ)			
Gamma(γ)	0	0.01	0.02	0.03	0	0.01	0.02	0.03
CADUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.01256	0.01281	0.00754	-0.00145	0.05459	0.04811	0.04286	0.03415
0.05	0.01391	0.01246	0.0118	0.01004	0.06017	0.05617	0.052	0.04926
0.1	0.01088	0.01227	0.01373	0.01535	0.06467	0.06017	0.05864	0.06302
0.15	0.01089	0.01319	0.01219	0.00968	0.06579	0.06146	0.05997	0.06236
0.2	0.01688	0.01688	0.01536	0.01536	0.06765	0.06765	0.06733	0.06733
	Return Difference				Standard Deviation Difference			
0	-0.00383	-0.00357	-0.00884	-0.01783	-0.01333	-0.0198	-0.02505	-0.03376
0.05	-0.00247	-0.00392	-0.00459	-0.00634	-0.00774	-0.01174	-0.01591	-0.01865
0.1	-0.0055	-0.00412	-0.00266	-0.00103	-0.00324	-0.00774	-0.00927	-0.00489
0.15	-0.00549	-0.00319	-0.00419	-0.0067	-0.00212	-0.00645	-0.00794	-0.00555
0.2	0.0005	0.0005	-0.00102	-0.00102	-0.00026	-0.00026	-0.00058	-0.00058
CHFUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0163	0.0085	0.0071	0.0060	0.1057	0.1005	0.0938	0.0845
0.05	0.0253	0.0182	0.0128	0.0103	0.1132	0.1083	0.1027	0.1001
0.1	0.0179	0.0159	0.0149	0.0087	0.1171	0.1131	0.1088	0.1064
0.15	0.0108	0.0085	0.0052	-0.0005	0.1207	0.1170	0.1142	0.1116
0.2	0.0071	0.0099	0.0075	0.0057	0.1221	0.1205	0.1175	0.1164
	Return Difference				Standard Deviation Difference			
0	0.0111	0.0033	0.0019	0.0007	-0.0206	-0.0259	-0.0326	-0.0419
0.05	0.0201	0.0130	0.0075	0.0051	-0.0131	-0.0181	-0.0237	-0.0263
0.1	0.0127	0.0107	0.0097	0.0035	-0.0093	-0.0133	-0.0176	-0.0199
0.15	0.0056	0.0032	0.0000	-0.0058	-0.0057	-0.0093	-0.0122	-0.0148
0.2	0.0018	0.0047	0.0023	0.0005	-0.0043	-0.0058	-0.0089	-0.0100
DEMUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0209	0.0182	0.0178	0.0142	0.0952	0.0955	0.0905	0.0836
0.05	0.0077	0.0033	0.0071	0.0029	0.1018	0.1005	0.0965	0.0911
0.1	0.0145	0.0058	0.0009	0.0128	0.1067	0.1052	0.1047	0.1007
0.15	0.0128	0.0094	0.0096	0.0145	0.1100	0.1093	0.1093	0.1050
0.2	0.0031	0.0031	0.0064	0.0163	0.1115	0.1115	0.1125	0.1080
	Return Difference				Standard Deviation Difference			
0	0.0144	0.0117	0.0112	0.0077	-0.0207	-0.0204	-0.0253	-0.0323
0.05	0.0012	-0.0032	0.0005	-0.0036	-0.0141	-0.0154	-0.0193	-0.0247
0.1	0.0080	-0.0007	-0.0056	0.0063	-0.0091	-0.0106	-0.0112	-0.0151
0.15	0.0063	0.0028	0.0030	0.0079	-0.0059	-0.0066	-0.0066	-0.0108
0.2	-0.0035	-0.0035	-0.0002	0.0098	-0.0043	-0.0043	-0.0033	-0.0079
EURUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0582	0.0553	0.0476	0.0465	0.0948	0.0833	0.0839	0.0778
0.05	0.0489	0.0570	0.0490	0.0623	0.0920	0.0967	0.0909	0.0930
0.1	0.0625	0.0616	0.0664	0.0649	0.1066	0.1063	0.1050	0.0999
0.15	0.0617	0.0617	0.0617	0.0617	0.1068	0.1068	0.1068	0.1068
0.2	0.0617	0.0617	0.0617	0.0617	0.1068	0.1068	0.1068	0.1068
	Return Difference				Standard Deviation Difference			
0	-0.0034	-0.0064	-0.0141	-0.0152	-0.0121	-0.0235	-0.0229	-0.0290
0.05	-0.0128	-0.0046	-0.0127	0.0006	-0.0148	-0.0101	-0.0159	-0.0139
0.1	0.0008	-0.0001	0.0047	0.0032	-0.0002	-0.0005	-0.0018	-0.0069
0.15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 19: Single Currency Stop-Loss Policy Results cont'd

Gamma(γ)	Delta(δ) 0	0.01	0.02	0.03	Delta(δ) 0	0.01	0.02	0.03
	FRFUSD Stop-Loss Policy Results : J=12							
	Return				Standard Deviation			
0	0.0522	0.0471	0.0359	0.0078	0.0941	0.0924	0.0904	0.0722
0.05	0.0436	0.0487	0.0259	0.0173	0.0987	0.1006	0.0962	0.0900
0.1	0.0545	0.0549	0.0509	0.0529	0.1061	0.1056	0.1064	0.1051
0.15	0.0496	0.0529	0.0529	0.0527	0.1085	0.1088	0.1088	0.1077
0.2	0.0570	0.0570	0.0570	0.0570	0.1110	0.1110	0.1110	0.1110
	Return Difference				Standard Deviation Difference			
0	-0.0048	-0.0100	-0.0211	-0.0493	-0.0169	-0.0186	-0.0206	-0.0388
0.05	-0.0135	-0.0084	-0.0311	-0.0397	-0.0123	-0.0104	-0.0148	-0.0209
0.1	-0.0026	-0.0021	-0.0061	-0.0042	-0.0049	-0.0054	-0.0045	-0.0059
0.15	-0.0074	-0.0041	-0.0041	-0.0044	-0.0025	-0.0022	-0.0022	-0.0033
0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	GBPUSD Stop-Loss Policy Results : J=12							
	Return				Standard Deviation			
0	0.0240	0.0217	0.0221	0.0212	0.0921	0.0890	0.0801	0.0758
0.05	0.0197	0.0170	0.0277	0.0170	0.0969	0.0955	0.0922	0.0853
0.1	0.0285	0.0269	0.0337	0.0317	0.1011	0.1006	0.1011	0.0998
0.15	0.0418	0.0418	0.0415	0.0411	0.1066	0.1066	0.1065	0.1064
0.2	0.0429	0.0426	0.0426	0.0423	0.1068	0.1067	0.1067	0.1066
	Return Difference				Standard Deviation Difference			
0	-0.0189	-0.0212	-0.0208	-0.0217	-0.0147	-0.0177	-0.0266	-0.0310
0.05	-0.0232	-0.0259	-0.0152	-0.0259	-0.0098	-0.0113	-0.0145	-0.0215
0.1	-0.0144	-0.0159	-0.0091	-0.0112	-0.0057	-0.0062	-0.0057	-0.0070
0.15	-0.0011	-0.0011	-0.0014	-0.0018	-0.0002	-0.0002	-0.0003	-0.0004
0.2	0.0000	-0.0003	-0.0003	-0.0006	0.0000	-0.0001	-0.0001	-0.0002
	ITLUSD Stop-Loss Policy Results : J=12							
	Return				Standard Deviation			
0	0.0183	0.0297	0.0216	0.0187	0.0928	0.0910	0.0845	0.0793
0.05	0.0202	0.0228	0.0141	0.0158	0.0987	0.0972	0.0967	0.0917
0.1	0.0215	0.0240	0.0230	0.0247	0.1034	0.1028	0.1028	0.1008
0.15	0.0199	0.0192	0.0232	0.0184	0.1101	0.1097	0.1065	0.1053
0.2	0.0252	0.0252	0.0264	0.0246	0.1116	0.1116	0.1111	0.1107
	Return Difference				Standard Deviation Difference			
0	-0.0074	0.0040	-0.0042	-0.0070	-0.0188	-0.0206	-0.0271	-0.0323
0.05	-0.0055	-0.0030	-0.0116	-0.0099	-0.0129	-0.0144	-0.0149	-0.0199
0.1	-0.0043	-0.0018	-0.0028	-0.0010	-0.0082	-0.0088	-0.0088	-0.0108
0.15	-0.0058	-0.0066	-0.0025	-0.0073	-0.0015	-0.0019	-0.0051	-0.0063
0.2	-0.0006	-0.0006	0.0007	-0.0012	0.0000	0.0000	-0.0006	-0.0009
	JPYUSD Stop-Loss Policy Results : J=12							
	Return				Standard Deviation			
0	0.0140	0.0229	0.0222	0.0094	0.0977	0.0932	0.0891	0.0841
0.05	0.0180	0.0284	0.0265	0.0190	0.1019	0.1012	0.0999	0.0976
0.1	0.0245	0.0250	0.0267	0.0242	0.1094	0.1085	0.1069	0.1064
0.15	0.0239	0.0220	0.0254	0.0199	0.1140	0.1119	0.1124	0.1113
0.2	0.0240	0.0231	0.0231	0.0231	0.1173	0.1157	0.1157	0.1157
	Return Difference				Standard Deviation Difference			
0	-0.0060	0.0028	0.0022	-0.0107	-0.0226	-0.0270	-0.0311	-0.0361
0.05	-0.0021	0.0083	0.0064	-0.0011	-0.0184	-0.0190	-0.0203	-0.0226
0.1	0.0044	0.0049	0.0066	0.0041	-0.0108	-0.0117	-0.0133	-0.0138
0.15	0.0038	0.0019	0.0053	-0.0001	-0.0062	-0.0083	-0.0078	-0.0089
0.2	0.0039	0.0030	0.0030	0.0030	-0.0029	-0.0045	-0.0045	-0.0045

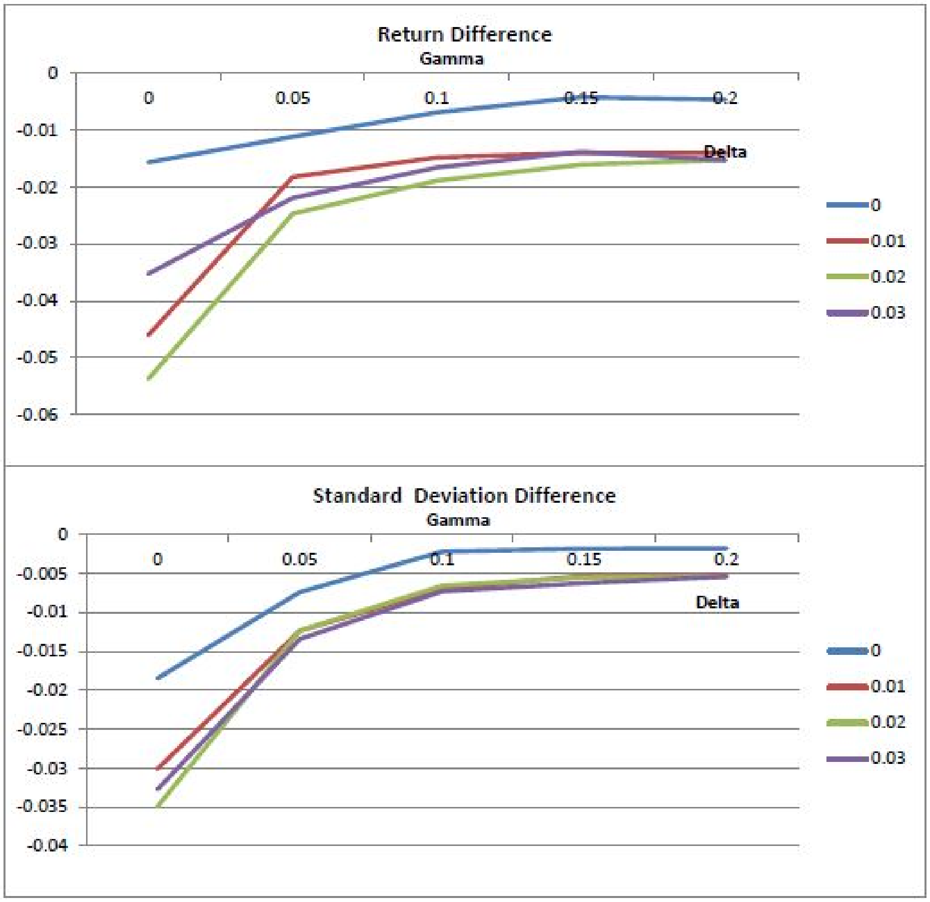
Table 20: Single Currency Stop-Loss Policy Results cont'd

Gamma(γ)	Delta(δ) 0	0.01	0.02	0.03	Delta(δ) 0	0.01	0.02	0.03
NOKUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0394	0.0460	0.0455	0.0390	0.0843	0.0820	0.0786	0.0773
0.05	0.0409	0.0463	0.0443	0.0377	0.0891	0.0888	0.0877	0.0864
0.1	0.0483	0.0504	0.0521	0.0486	0.0978	0.0978	0.0967	0.0963
0.15	0.0439	0.0439	0.0443	0.0429	0.1008	0.1008	0.1002	0.0998
0.2	0.0421	0.0421	0.0422	0.0422	0.1019	0.1019	0.1019	0.1019
	Return Difference				Standard Deviation Difference			
0	-0.0108	-0.0042	-0.0047	-0.0112	-0.0198	-0.0221	-0.0255	-0.0268
0.05	-0.0093	-0.0039	-0.0059	-0.0124	-0.0150	-0.0153	-0.0164	-0.0177
0.1	-0.0019	0.0003	0.0019	-0.0016	-0.0063	-0.0063	-0.0074	-0.0078
0.15	-0.0063	-0.0063	-0.0058	-0.0073	-0.0033	-0.0033	-0.0039	-0.0043
0.2	-0.0081	-0.0081	-0.0080	-0.0080	-0.0022	-0.0022	-0.0022	-0.0022
NZDUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0607	0.0575	0.0520	0.0443	0.1051	0.1006	0.0962	0.0917
0.05	0.0684	0.0609	0.0583	0.0469	0.1112	0.1089	0.1075	0.1045
0.1	0.0649	0.0678	0.0583	0.0512	0.1170	0.1170	0.1154	0.1154
0.15	0.0658	0.0667	0.0649	0.0649	0.1197	0.1193	0.1188	0.1188
0.2	0.0621	0.0638	0.0637	0.0637	0.1219	0.1215	0.1212	0.1212
	Return Difference				Standard Deviation Difference			
0	-0.0154	-0.0186	-0.0241	-0.0319	-0.0247	-0.0292	-0.0336	-0.0381
0.05	-0.0078	-0.0152	-0.0178	-0.0292	-0.0186	-0.0209	-0.0223	-0.0253
0.1	-0.0112	-0.0083	-0.0179	-0.0249	-0.0128	-0.0128	-0.0144	-0.0144
0.15	-0.0103	-0.0094	-0.0113	-0.0112	-0.0101	-0.0105	-0.0111	-0.0111
0.2	-0.0140	-0.0124	-0.0125	-0.0125	-0.0079	-0.0083	-0.0086	-0.0086
SEKUSD Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0451	0.0496	0.0452	0.0414	0.0916	0.0912	0.0900	0.0878
0.05	0.0595	0.0618	0.0598	0.0572	0.0983	0.0984	0.0970	0.0955
0.1	0.0611	0.0624	0.0614	0.0584	0.1037	0.1025	0.1021	0.1015
0.15	0.0604	0.0604	0.0598	0.0580	0.1049	0.1049	0.1041	0.1037
0.2	0.0582	0.0582	0.0581	0.0564	0.1058	0.1058	0.1054	0.1050
	Return Difference				Standard Deviation Difference			
0	-0.0182	-0.0138	-0.0182	-0.0220	-0.0191	-0.0195	-0.0208	-0.0230
0.05	-0.0038	-0.0016	-0.0035	-0.0061	-0.0125	-0.0123	-0.0137	-0.0152
0.1	-0.0023	-0.0009	-0.0020	-0.0049	-0.0071	-0.0082	-0.0086	-0.0093
0.15	-0.0029	-0.0029	-0.0036	-0.0053	-0.0058	-0.0058	-0.0066	-0.0071
0.2	-0.0051	-0.0051	-0.0052	-0.0069	-0.0050	-0.0050	-0.0053	-0.0058

So for example for the AUDUSD FX carry trade with the stop-loss policy defined above imposed on it with $J = 12$ months, a cumulative stop-loss trigger (γ) of 0.10 (so the trigger is -10% as per Equation (39)), and a re-entry trigger (δ) of 0.02, the annualised return is now 5.49% with standard deviation 11.41%. Looking at the lower panel the difference in annualised return for the AUDUSD FX carry trade with the stop-loss policy imposed and the original trade is -1.89%. The difference in standard deviation is -0.66%. This means that by imposing the stop-loss policy on the AUDUSD FX carry trade, the annualised return has decreased by almost 2% which has been accompanied by a small reduction in its standard deviation. Examining the lower panel for AUDUSD it is clear that for $J = 12$ months and for all combinations of γ and δ, imposing the theoretical stop-loss policy on the AUDUSD FX carry trade results in a reduction in annualised return and a reduction in annualised standard deviation. Figure 10 plots the annualised return and standard deviation differences for $J = 12$ months, $\gamma = 0, 0.05, 0.1, 0.15, 0.2$, and $\delta = 0, 0.01, 0.02, 0.03$ for AUDUSD.

These results seem intuitively plausible. From Table 5, over the sample the AUDUSD FX carry trade has an annualised return of 7.38% with an annualised standard deviation of 12.07%. Figure 10 shows that by imposing the stop-loss policy, as γ increases the difference between the annualised return and annualised standard deviation decreases. Recall that γ is the cumulative stop-loss trigger which needs to be breached to exit the carry trade. So the larger this threshold, the less frequently the stop-loss policy

Figure 10: AUDUSD Stop-Loss Annualised Return & Standard Deviation Differences : J=12 months



For AUDUSD, J=12 months, the left panel plots $\triangle\, \overline{x}^A = \overline{x}^{A,l} - \overline{x}^A$ where $\overline{x}^{A,l}$ is the annualised return of the FX carry trade after the stop-loss policy has been imposed, for different combinations of γ and δ, and $\overline{x}^A$ is the annualised return of the original FX carry trade. The right panel plots $\triangle\, s^A = s^{A,l} - s^A$ where $s^{A,l}$ is the annualised standard deviation of the FX carry trade after the stop-loss policy has been imposed, for different combinations of γ and δ, and s^A is the annualised standard deviation of the original FX carry trade.

takes you out of the FX carry trade, and hence the more the stop-loss imposed FX carry trade resembles the original FX carry trade.

The results are broadly what you would expect for δ as well. Recall that δ is the required last month return to re-enter the FX carry trade if you have been stopped out previously. So for smaller values of δ you are more likely to re-enter the trade and hence the differences between the stop-loss impacted FX carry trade returns and standard deviation, and the original FX carry trade returns and standard deviation should be smaller. This is what Figure 10 shows.

Similar to the AUDUSD results, it is clear that imposing the stop-loss policy leads to reduced annualised returns for almost all combinations of γ and δ for CADUSD, FRFUSD, GBPUSD, ITLUSD, NOKUSD, NZDUSD, and SEKUSD. This result is broadly in line with Kaminski and Lo (2008) who show that if the underlying return series follows a random walk, then imposing a stop-loss policy will reduce the expected returns. A quick look at the properties of these single currency FX carry trade return series reveals the inability to reject the null hypothesis of white noise using autocorrelations up to specified lag periods and like wise using augmented Dickey-Fuller tests the null hypothesis of a unit root is rejected.

In contrast to these results, imposing the stop-loss policy on the CHFUSD FX carry trade has very different consequences. For almost all combinations of γ and δ the stop-loss policy results in higher annualised returns, the largest being 2.5% when $J = 12, \gamma = 0.05, \delta = 0$. When one considers that the original CHFUSD FX carry trade had a return of just 0.5%, this is a significant increase.

In addition to the CHFUSD results, it is clear that imposing the stop-loss policy leads to increased annualised returns for almost all combinations of γ and δ for DEMUSD and JPYUSD.

The effect of the stop-loss policy on the EURUSD FX carry trade is less clear. On balance it results in a reduction in annualised returns, but the results are not strong enough to fit the 'across the board' generalization above, perhaps not helped by the smaller sample size.

The results of increased annualised returns under the stop-loss framework for CHFUSD, DEMUSD, and JPYUSD and reduced annualised returns for all other currencies is interesting when you consider Table 7. CHFUSD, DEMUSD, and JPYUSD are the 3 dominant 'low' yielding currencies. As seen in Section 5, when the FX carry trade returns were decomposed into an interest rate component and an FX component (Table 8) the CHFUSD, DEMUSD, and JPYUSD FX carry trades all had negative FX components. The only other currency to do so was the ITLUSD. The decomposition results also highlighted that the return volatility of the FX carry trade is driven predominantly from the FX component, as opposed to the interest rate component. So contributing to the annualised return of the stop-loss impacted FX carry trade being lower for these 3 currencies is that the stop-loss policy takes you out of the market on occasions and hence you avoid the, on average, negative FX component returns. Of course you also are not earning the on average positive interest rate component returns as well, but that is no different to all the currencies.

Overall it would seem that imposing this theoretical stop-loss framework on consistently 'low' yielding currencies results in an increase in annualised returns whilst imposing it on generally 'high' yielding currencies results in a reduction in annualised returns.

The effect of the stop-loss policy on annualised standard deviations is much clearer. For all currencies the stop-loss policy results in lower annualised standard deviations for all combinations of γ and δ. It's worth noting that the 3 low yielders that had increased annualised returns, CHFUSD, DEMUSD, and JPYUSD, had 3 of the 4 biggest reductions in average annualised standard deviation across all combinations of γ and δ. The other biggest reduction in average annualised standard deviation was NZDUSD.

10.2.2 Portfolio FX Carry Trades

The impact of the stop-loss policy on the series of portfolio FX carry trades is mixed. The results are presented in Tables 21 to 22.

In the case of the EW portfolio there is a clear reduction in annualised return for all combinations of γ and δ. This makes sense when you consider the results for the single currency FX carry trades above for which the imposition of the stop-loss policy resulted in a reduction in annualised returns for the majority of currencies. As with all the single currency FX carry trades the annualised standard deviations are lower under the stop-loss framework.

Table 21: EW and kxk Portfolio Stop-Loss Policy Results

For each FX carry portfolio, annualised returns and standard deviations achieved by imposing the stop-loss policy are shown in the upper panels, and the difference between these stop-loss impacted results and the original FX carry trade returns and standard deviations are presented in the lower panels. The stop-loss policy l_t is imposed onto the FX carry trade returns x_t so the new stop-loss impacted returns are defined as $x_t^l = l_t x_t$ where s_t is defined as :

$$l_t = \begin{cases} 0 & if \quad X_{t-1}^l(J) < -\gamma \quad and \quad l_{t-1} = 1 \quad (exit) \\ 1 & if \quad x_{t-1} \geq \delta \quad and \quad l_{t-1} = 0 \quad (re-enter) \\ 1 & if \quad X_{t-1}^l(J) \geq -\gamma \quad and \quad l_{t-1} = 1 \quad (stay\ in) \\ 0 & if \quad x_{t-1} < \delta \quad and \quad l_{t-1} = 0 \quad (stay\ out) \end{cases}$$

where $X_t^l(J) = \sum_{j=1}^{J}(x_{t-j+1}^l)$ is cumulative returns over J months. So, the stop-loss is triggered when the cumulative returns over J months fall below some specified level γ, and the re-entry is triggered when the last period return is above some specified value δ. Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data and annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. EW portfolio is an equal USD weighted portfolio of all available currencies where each month you are long(short) the foreign currency versus the USD if it is at a forward discount(premium). Monthly carry trade return for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. The kxk portfolios where $k = 1, .., 4$ are constructed by each month going long the k highest forward discount currencies and short the k lowest forward discount currencies, equal USD amounts. The stop-loss policy is imposed for J (cumulative return window) = 12,6,3 months, γ (cumulative stop-loss trigger) = 0,0.05,0.1,0.15,0.2 and δ (re-entry trigger) = 0,0.01,0.02,0.03. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

EW Stop-Loss Policy Results : J=12

	Delta()				Delta()			
Gamma()	0	0.01	0.02	0.03	0	0.01	0.02	0.03
	Return				Standard Deviation			
0	0.0323	0.0245	0.0045	0.0063	0.0434	0.0412	0.0337	0.0332
0.05	0.0382	0.0374	0.0356	0.0306	0.0490	0.0488	0.0480	0.0450
0.1	0.0358	0.0360	0.0342	0.0291	0.0500	0.0498	0.0490	0.0466
0.15	0.0364	0.0364	0.0356	0.0356	0.0501	0.0501	0.0494	0.0494
0.2	0.0394	0.0394	0.0394	0.0394	0.0513	0.0513	0.0513	0.0513
	Return Difference				Standard Deviation Difference			
0	-0.0071	-0.0149	-0.0349	-0.0331	-0.0079	-0.0101	-0.0176	-0.0182
0.05	-0.0012	-0.0020	-0.0038	-0.0088	-0.0024	-0.0026	-0.0033	-0.0064
0.1	-0.0036	-0.0035	-0.0052	-0.0103	-0.0014	-0.0016	-0.0023	-0.0048
0.15	-0.0030	-0.0030	-0.0038	-0.0038	-0.0012	-0.0012	-0.0019	-0.0019
0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 22: EW and kxk Portfolio Stop-Loss Policy Results cont'd

	Delta(δ)				Delta(δ)			
Gamma(γ)	0	0.01	0.02	0.03	0	0.01	0.02	0.03
$1x1$ Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0899	0.0857	0.0895	0.0751	0.1211	0.1202	0.1199	0.1154
0.05	0.0800	0.0797	0.0797	0.0828	0.1265	0.1265	0.1252	0.1201
0.1	0.0845	0.0844	0.0806	0.0763	0.1290	0.1290	0.1284	0.1259
0.15	0.0771	0.0739	0.0729	0.0700	0.1332	0.1328	0.1327	0.1304
0.2	0.0779	0.0779	0.0773	0.0765	0.1355	0.1355	0.1355	0.1340
	Return Difference				Standard Deviation Difference			
0	0.0200	0.0158	0.0196	0.0052	-0.0208	-0.0217	-0.0220	-0.0266
0.05	0.0101	0.0098	0.0098	0.0128	-0.0155	-0.0155	-0.0167	-0.0219
0.1	0.0146	0.0145	0.0107	0.0064	-0.0129	-0.0129	-0.0135	-0.0160
0.15	0.0071	0.0040	0.0029	0.0001	-0.0088	-0.0091	-0.0092	-0.0115
0.2	0.0080	0.0080	0.0074	0.0066	-0.0064	-0.0064	-0.0065	-0.0080
$2x2$ Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0653	0.0703	0.0719	0.0715	0.0912	0.0892	0.0844	0.0826
0.05	0.0734	0.0725	0.0677	0.0653	0.0967	0.0958	0.0942	0.0930
0.1	0.0738	0.0736	0.0722	0.0701	0.0995	0.0988	0.0985	0.0981
0.15	0.0737	0.0737	0.0737	0.0716	0.1023	0.1023	0.1023	0.1021
0.2	0.0730	0.0730	0.0730	0.0710	0.1029	0.1029	0.1029	0.1027
	Return Difference				Standard Deviation Difference			
0	-0.0040	0.0010	0.0026	0.0022	-0.0141	-0.0160	-0.0208	-0.0227
0.05	0.0041	0.0032	-0.0016	-0.0040	-0.0085	-0.0094	-0.0110	-0.0122
0.1	0.0045	0.0043	0.0029	0.0008	-0.0058	-0.0065	-0.0068	-0.0072
0.15	0.0043	0.0043	0.0043	0.0023	-0.0029	-0.0029	-0.0029	-0.0032
0.2	0.0037	0.0037	0.0037	0.0017	-0.0023	-0.0023	-0.0023	-0.0026
$3x3$ Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0426	0.0454	0.0466	0.0203	0.0739	0.0726	0.0665	0.0581
0.05	0.0517	0.0499	0.0439	0.0319	0.0810	0.0807	0.0781	0.0651
0.1	0.0564	0.0561	0.0551	0.0447	0.0817	0.0817	0.0816	0.0736
0.15	0.0493	0.0489	0.0485	0.0446	0.0847	0.0847	0.0846	0.0789
0.2	0.0524	0.0524	0.0524	0.0481	0.0854	0.0854	0.0854	0.0827
	Return Difference				Standard Deviation Difference			
0	-0.0104	-0.0076	-0.0064	-0.0327	-0.0126	-0.0139	-0.0200	-0.0284
0.05	-0.0014	-0.0031	-0.0092	-0.0211	-0.0055	-0.0058	-0.0084	-0.0214
0.1	0.0034	0.0030	0.0021	-0.0084	-0.0048	-0.0048	-0.0049	-0.0129
0.15	-0.0037	-0.0041	-0.0045	-0.0085	-0.0018	-0.0018	-0.0019	-0.0076
0.2	-0.0007	-0.0007	-0.0007	-0.0049	-0.0011	-0.0011	-0.0011	-0.0038
$4x4$ Stop-Loss Policy Results : J=12								
	Return				Standard Deviation			
0	0.0391	0.0386	0.0232	0.0119	0.0623	0.0597	0.0536	0.0410
0.05	0.0440	0.0375	0.0284	0.0296	0.0687	0.0663	0.0644	0.0628
0.1	0.0446	0.0401	0.0331	0.0338	0.0722	0.0710	0.0690	0.0680
0.15	0.0450	0.0412	0.0391	0.0399	0.0723	0.0710	0.0703	0.0694
0.2	0.0446	0.0446	0.0446	0.0446	0.0735	0.0735	0.0735	0.0735
	Return Difference				Standard Deviation Difference			
0	-0.0055	-0.0061	-0.0214	-0.0327	-0.0111	-0.0137	-0.0199	-0.0324
0.05	-0.0007	-0.0071	-0.0162	-0.0150	-0.0048	-0.0071	-0.0090	-0.0106
0.1	0.0000	-0.0045	-0.0116	-0.0108	-0.0012	-0.0025	-0.0044	-0.0055
0.15	0.0004	-0.0034	-0.0055	-0.0047	-0.0012	-0.0024	-0.0031	-0.0041
0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

However for the $1x1$ portfolio the impact of the stop-loss policy is quite the opposite, with an across the board increase in annualised returns. Considering the currencies that generally make up this portfolio this result makes sense. As seen earlier, on the 'low' yielding side the currencies likely to be in this portfolio have an increase in annualised return when the stop-loss policy is imposed. On the 'high' yielding side, according to Table 7, ITLUSD, NZDUSD, and AUDUSD are the 3 most frequent currencies in this portfolio. Recall from Table 9 that both the 'high' and 'low' yielding side of this $1x1$ portfolio has a negative FX component return to the FX carry trade. It would seem that the benefit of this 'saving' when the stop-loss policy takes you out of this trade is more than offsetting the loss in not earning the interest rate component, and hence this increase in annualised return across the stop-loss policy parameters. It is worth noting that of all the original single and portfolio FX carry trades constructed this is the carry trade with the highest annualised standard deviation, 14.19%. Again, the impact of the stop-loss policy on the annualised standard deviations is to reduce them across the board. But in this case the average reduction for all combinations of γ and δ is a significant 1.4%.

Likewise with the $2x2$ portfolio, the stop-loss policy results in an increase in annualised returns and a reduction in annualised standard deviations, but both are of smaller magnitudes than that of the $1x1$ portfolio. So the 'saving' of not incurring the on average losing FX component when the stop-loss policy takes you out of the trade is being diluted, but still enough to result in an increase in return under the stop-loss policy.

For the $3x3$ and the $4x4$ portfolios the impact of the stop-loss policy is similar to that of the EW portfolio, with clear reductions in the annualised returns and annualised standard deviations for all combinations of γ and δ. The small number of 'low' yielding currencies with negative FX component returns of the FX carry trade are now dominated by the larger number of currencies with positive FX component returns that are being included in these larger portfolios and the net result is a decrease in annualised returns when the stop-loss policy is imposed. Note that there are some scenarios where with $\gamma = 0.20$ there is no difference between the numbers of the original FX carry trades and those with the stop-loss policy imposed because the cumulative loss threshold is never breached. In such a case the return series under the stop-loss policy is identical to that of the original trade.

In addition results for $J = 6$ and 3 months are available upon request. They are broadly consistent with the results presented above for $J = 12$ months.

11 Hedge Fund Superposition Strategies

In this section a collection of 'industry specific' superposition frameworks are applied to the FX carry trades defined in Section 4 and the impact these frameworks have on the returns of the FX carry trades are evaluated. These are proprietary rules obtained from industry participants.

11.1 Hedge Fund Risk Management Policy Examples

Whilst the theoretical stop-loss framework outlined in Section 10 is an adequate starting point, the real acid test is what do real world stop-loss policies look like and what are their impact on the returns to the FX carry trade ? The difficulty in answering this question historically has been obtaining the details of the real world stop-loss policies. Fortunately, through industry contacts, I have been able to interview traders and partners, past and present, who have worked at hedge funds and bank proprietary trading desks (prior to the introduction of the Volcker Rule [10]). They have provided a clear insight into what risk management frameworks the traders at these institutions are contractually obligated to operate under. This is a very unique proprietary data set which is not generally available to people outside of the respective firms. In terms of mandate the traders are all Global Macro (Drobny 2006) style traders. The total assets under management from the firms which the traders have responded on is approximately 80 billion USD. Details of the policies that have no impact on their implementation to the FX carry trade have been omitted. Often the policies have position size limits but unless these change at various profit and loss (P&L) levels they do not impact the analysis. Where required the stop-loss policies have been amended slightly so that they fit with the monthly manner in which the FX carry trade re-

[10] The Volcker Rule is part of the Dodd-Frank Wall Street Reform and Consumer Protection Act implemented after the global financial crisis of 2007-2010 and serves to largely ban proprietary risk taking at commercial banks. This has resulted in an exodus of proprietary traders from banks to hedge funds.

turns are constructed. The results in Section 7.3 which showed that the FX carry trade returns were not sensitive to the time of month the trades were executed support this.

Whilst there are some similarities between the following 8 real world stop-loss policies and that of Kaminski and Lo's (2008), there are also some very important differences :

- Triggering the maximum drawdown from zero limit in the hedge fund examples results in the traders employment being terminated where as in Kaminski and Lo (2008) the drawdown from zero trigger means trading ceases until the re entry trigger is satisfied

- The Kaminski and Lo (2008) re entry trigger is for the last period return to be greater than some threshold value. Instead, the hedge fund examples generally have a finite stand down period of no trading. This is generally 1 month in our framework.

- The theoretical stop-loss framework does not have drawdown from maximum profit limits

11.1.1 Example A

Each trader is allocated a notional capital amount.

The drawdown parameters below are based on lifetime (P&L) at the firm .

Drawdown from zero limits :

- 5% of allocated capital, stopped out, no trading the following month
- then 1% per month, so if down 6% stopped out again and no trading the following month
- get your 5% limit back when your P&L is back above zero
- when down 10%, all positions are closed out and your employment ceases

Drawdown from peak capital limits :

- same rules apply
- 5% drawdown from peak capital (maximum life to date P&L), stopped out, no trading the following month
- then 1% per month, so if down 6% stopped out again and no trading the following month
- get your 5% limit back when your P&L is back above your old peak capital, thus making a new peak
- when down 10%, all positions closed out and your employment ceases

11.1.2 Example B

The drawdown parameters are based on your P&L in the current financial year.

Annual stop-loss of 10% of notional capital from zero, at which point all positions are closed out and employment ceases.

This is allocated on a monthly basis as 30% of 'live' equity where :

$$\text{'live' equity} = (\text{Annual drawdown limit } (10\%) + \text{Year to date P\&L } (\%)).$$

If you hit the monthly drawdown limit all positions are closed out and no trading the following month.

11.1.3 Example C

The drawdown parameters are based on your P&L in the current financial year.

There is a Maximum Position Size Limit (MPSL) that changes as described below.

Drawdown from zero limits :

- down 5%, close positions, no trading the following month, no change in MPSL
- down 10%, close positions, no trading the following month, MPSL reduced by 1/3rd
- down 15%, close positions, no trading the following month, MPSL reduced by another 1/3rd (of the starting MPSL)
- down 20%, close all positions and employment ceases

Drawdown from peak capital limits :

- 5% drawdown from peak capital , close positions, no trading the following month
- 5% from new ‘start’ level, whilst above zero, close positions, no trading the following month, and so on
- When P&L is back below zero the drawdown from zero limits apply

11.1.4 Example D

The drawdown parameters are based on your P&L in the current financial year.

Annual stop-loss of 12% from zero, at which point all positions are closed out and employment ceases.

Monthly drawdown limit of 4%, close positions, no trading the following month.

11.1.5 Example E

The drawdown parameters are based on your P&L in the current financial year.

Annual stop-loss of 10% from zero, at which point all positions are closed out and employment ceases.

Drawdown from peak capital is :

Max Drawdown = (Annual drawdown limit (10%) + 50% Max Year to date P&L (%)).

If the drawdown from peak capital limit is reached, then all positions are closed out and employment ceases.

11.1.6 Example F

The drawdown parameters below are based on lifetime P&L at the firm.

Anytime stop-loss of 7% from zero, at which point all positions are closed out and employment ceases.

Monthly drawdown limit of 3%, close positions, no trading the following month.

11.1.7 Example G

The drawdown parameters below are based on lifetime P&L at the firm.

Anytime stop-loss of 10% from zero, at which point all positions are closed out and employment ceases.

Drawdown from peak capital is :

$$\text{Max Drawdown} = (\text{Drawdown limit } (10\%) + 30\% \text{ Max life to date P\&L } (\%)).$$

If the max drawdown limit is reached then all positions are closed out and employment ceases.

In addition there is :

- a rolling 90 day stop-loss of -5%, close positions, no trading the following month
- at -7% from zero, close positions, no trading the following month, risk limits halved (i.e. notional capital halved) when trading resumes. Original risk limits are re-instated if P&L goes back above zero.

11.1.8 Example H

The drawdown parameters are based on your P&L in the current financial year.

Drawdown from zero limits :

- Trigger 1 - down 2.5%. Noted, but risk remains unchanged.
- Trigger 2 - down 5%. Close positions, no trading the following month. Risk limit reduced to 50% of original limit when resume trading.
- Trigger 3 - down 7.5%. Close positions, no trading the following month. Risk limit reduced to 25% of original limit when resume trading.
- Trigger 4 - down 10%. Close positions, employment terminated.

Drawdown from peak limits :

From peak year to date profit the same triggers apply but with a buffer of 5%. So for example if you were up 7.5%, the 2.5% drawdown trigger would occur if you go back to 0%.

In both the drawdown from zero and the drawdown from peak limits, once 5% or 7.5% triggers are activated you get the original capital back once you stay above Trigger 1 for 3 months.

11.2 Results

This section presents the results from applying the 8 'hedge fund' stop-loss policies in Section 11.1 to the FX carry trade. In doing so there is an implied assumption that the 'trader' at each of the 8 institutions is fully invested in the FX carry trade and has no other risk. This may or may not be the case in reality but recall the objective is to determine what effect the institutions stop-loss policy has on the FX carry trade returns.

It is also worth noting that the FX spot and forward rates used to calculate the FX carry trade returns implicitly have hedge fund FX activity, including their FX carry trade activity, embedded in them. According to the BIS 2013 Triennial Central Bank Survey, global FX turnover climbed to $5.3 trillion per day in 2013 from $4.0 trillion in 2010. Non-dealer financial institutions (such as lower tier banks, institutional investors and hedge funds) now account for $2.8 trillion per day, so the FX flow of this client sector is an important determinant of FX rates.

The results are presented in Tables 23 to 25. The first column shows the respective results of the simple unconstrained FX carry trades. Each of the 8 stop-loss policy examples are applied to the 12 single currency FX carry trades, the EW portfolio FX carry trade, and to the 4 kxk FX carry trade portfolios, as defined in Section 4. For each of the stop-loss policies and FX carry trades the % of the sample months that the strategy remained live is reported as % Sample Live. If % Sample Live is less than 100% then this indicates that the stop-loss policy resulted in the FX carry trade being stopped out permanently. Note that if the stop-loss policy has a stand down period but allows the trader to re-enter the FX carry trade at a later date then these stand down months are counted as live months. Obviously if % Sample Live equals 100% then this implies that the FX carry trade in question was not stopped out permanently and the trader was able to stay in the trade for the entire sample period. The summary statistics have been calculated as per Section 4.3. However, in examples where the FX carry trade has been stopped out permanently, making comparisons between annualised statistics may not be particularly insightful. As such the terminal value of $1 over the period for which the FX carry trade is live is reported as Terminal Value $1. If the stop-loss policy results in the trader being stopped out permanently before the end of the sample then the Terminal Value $1 will be the cumulative value of $1 invested up to the point at which the trader was stopped out.

11.2.1 Example A

Applying this stop-loss policy to the FX carry trade results in the strategy being stopped out permanently in every case. Of the 17 FX carry trade strategies analysed, all but 4 were stopped out due to the drawdown from peak capital limit being breached. Quite simply the FX carry trade is prone to drawdowns that are too big to survive this stop-loss policy. Encouragingly the EW and $4x4$ portfolios survived close to half the sample period corresponding to their lower standard deviation. In addition EURUSD was

Table 23: Single Currency and EW Portfolio Annualised Hedge Fund Stop-Loss Policy Results

For each currency versus the USD the results of the simple unconstrained FX carry trade are shown (Carry Trade). Stop-loss rules A-H are the results obtained after superimposing the hedge fund stop-loss policies onto the FX carry trade. % Sample Live measures the percentage of the total sample period that the hedge fund policy enabled the trader to remain in the FX carry trade. A number less than 100% indicates that the stop-loss policy resulted in the trader being stopped out permanently prior to the end of the sample period. Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. Monthly carry trade returns for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency. Monthly results are then annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. EW portfolio is an equal USD weighted portfolio of all available currencies where each month you are long(short) the foreign currency versus the USD if it is at a forward discount(premium). Terminal Value of $1 is the end of sample period value of $1 invested at the start of the sample period for each currency pair. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Carry Trade	Stop-Loss Rule A	B	C	D	E	F	G	H
					AUDUSD				
Return	0.0738	0.0198	0.0310	0.0706	0.0192	0.0027	0.0652	0.0804	0.0027
Standard Deviation	0.1207	0.0538	0.0787	0.1123	0.0538	0.0402	0.1127	0.1136	0.0402
% Sample Live		16%	59%	100%	16%	6%	100%	100%	6%
Terminal Value $1	6.37	1.67	2.18	6.00	1.64	1.06	5.15	7.83	1.06
					CADUSD				
Return	0.0164	-0.0006	0.0105	0.0174	0.0144	0.0214	-0.0021	-0.0013	0.0152
Standard Deviation	0.0679	0.0119	0.0334	0.0640	0.0602	0.0483	0.0128	0.0136	0.0537
% Sample Live		5%	46%	100%	91%	81%	6%	6%	91%
Terminal Value $1	1.68	0.98	1.44	1.76	1.59	2.11	0.92	0.95	1.66
					CHFUSD				
Return	0.0052	-0.0029	-0.0077	0.0050	-0.0072	-0.0030	-0.0030	-0.0029	-0.0069
Standard Deviation	0.1264	0.0114	0.0223	0.0666	0.0226	0.0137	0.0137	0.0113	0.0192
% Sample Live		5%	6%	28%	6%	5%	5%	5%	6%
Terminal Value $1	0.90	0.89	0.74	1.11	0.76	0.89	0.89	0.90	0.77
					DEMUSD				
Return	0.0065	-0.0057	-0.0104	0.0128	-0.0119	-0.0097	-0.0034	-0.0051	0.0172
Standard Deviation	0.1159	0.0174	0.0245	0.1071	0.0250	0.0237	0.0140	0.0153	0.0683
% Sample Live		9%	12%	100%	12%	12%	7%	9%	42%
Terminal Value $1	1.00	0.87	0.78	1.18	0.75	0.80	0.92	0.89	1.41
					EURUSD				
Return	0.0617	0.0479	0.0688	0.0560	0.0660	0.0572	0.0639	0.0495	0.0548
Standard Deviation	0.1068	0.0683	0.1048	0.1015	0.1048	0.0713	0.1046	0.1012	0.0945
% Sample Live		60%	100%	100%	100%	57%	100%	100%	100%
Terminal Value $1	2.18	1.88	2.41	2.03	2.32	2.14	2.25	1.85	2.01

Table 24: Single Currency and EW Portfolio Annualised Hedge Fund Stop-Loss Policy Results cont'd

	Carry Trade	Stop-Loss Rule A	B	C	D	E	F	G	H
					FRFUSD				
Return	0.0570	0.0090	0.0108	0.0487	0.0098	0.0053	-0.0035	0.0057	0.0330
Standard Deviation	0.1110	0.0530	0.0556	0.1054	0.0550	0.0275	0.0105	0.0553	0.0838
% Sample Live		28%	28%	100%	28%	14%	3%	30%	66%
Terminal Value $1	3.22	1.19	1.24	2.70	1.21	1.12	0.92	1.10	1.97
					GBPUSD				
Return	0.0429	-0.0025	-0.0038	0.0359	0.0262	-0.0032	-0.0025	-0.0032	-0.0027
Standard Deviation	0.1068	0.0124	0.0152	0.0991	0.0662	0.0131	0.0124	0.0132	0.0121
% Sample Live		1%	2%	100%	36%	1%	1%	2%	2%
Terminal Value $1	3.94	0.91	0.86	3.13	2.42	0.89	0.91	0.89	0.90
					ITLUSD				
Return	0.0258	-0.0044	-0.0044	0.0186	-0.0074	-0.0044	-0.0044	-0.0044	-0.0044
Standard Deviation	0.1116	0.0213	0.0213	0.1014	0.0255	0.0213	0.0213	0.0213	0.0213
% Sample Live		0%	0%	100%	1%	0%	0%	0%	0%
Terminal Value $1	1.56	0.90	0.90	1.36	0.84	0.90	0.90	0.90	0.90
					JPYUSD				
Return	0.0201	0.0114	0.0136	0.0164	0.0168	0.0060	-0.0023	0.0074	0.0061
Standard Deviation	0.1202	0.0428	0.0583	0.1114	0.0580	0.0356	0.0137	0.0574	0.0543
% Sample Live		10%	21%	100%	21%	6%	0%	22%	21%
Terminal Value $1	1.55	1.43	1.51	1.42	1.68	1.20	0.92	1.22	1.17
					NOKUSD				
Return	0.0502	0.0104	0.0073	0.0453	0.0057	0.0098	0.0484	0.0069	0.0060
Standard Deviation	0.1041	0.0343	0.0399	0.0982	0.0401	0.0354	0.0963	0.0403	0.0381
% Sample Live		19%	24%	100%	24%	18%	100%	25%	24%
Terminal Value $1	5.20	1.43	1.27	4.45	1.20	1.40	5.01	1.25	1.22
					NZDUSD				
Return	0.0761	0.0060	0.0060	0.0525	0.0060	0.0098	-0.0025	0.0617	0.0017
Standard Deviation	0.1298	0.0407	0.0407	0.1157	0.0407	0.0453	0.0119	0.1166	0.0334
% Sample Live		4%	4%	100%	4%	4%	1%	100%	4%
Terminal Value $1	6.59	1.16	1.16	3.59	1.16	1.28	0.93	4.62	1.03
					SEKUSD				
Return	0.0633	0.0015	0.0140	0.0560	0.0193	0.0059	0.0598	0.0055	0.0039
Standard Deviation	0.1107	0.0419	0.0581	0.1024	0.0614	0.0449	0.1036	0.0450	0.0430
% Sample Live		19%	37%	100%	42%	19%	100%	22%	19%
Terminal Value $1	8.21	1.02	1.57	6.48	1.90	1.20	7.42	1.18	1.12
					EW				
Return	0.0394	0.0174	0.0407	0.0376	0.0392	0.0171	0.0400	0.0396	0.0387
Standard Deviation	0.0513	0.0282	0.0504	0.0505	0.0505	0.0286	0.0493	0.0503	0.0499
% Sample Live		42%	100%	100%	100%	42%	100%	100%	100%
Terminal Value $1	4.07	1.87	4.28	3.81	4.05	1.85	4.18	4.11	3.98

Table 25: *kxk* Portfolio Annualised Hedge Fund Stop-Loss Policy Results

For each *kxk* portfolio the results of the simple unconstrained FX carry trade are shown (Carry Trade). Stop-loss rules A-H are the results obtained after superimposing the hedge fund stop-loss policies onto these *kxk* FX carry trades. % Sample Live measures the percentage of the total sample period that the hedge fund policy enabled the trader to remain in the FX carry trade. A number less than 100% indicates that the stop-loss policy resulted in the trader being stopped out permanently prior to the end of the sample period. Returns are measured as United States Dollar (USD) per 1 USD bet, calculated using monthly, non overlapping data. Monthly carry trade return for currency i at time t, $x_t^i = a * (F_{t-1}^i - S_t^i)/F_{t-1}^i$ where $a = -1(1)$ when the foreign currency versus the USD is at a forward discount(premium) at time $t-1$, S_t^i and F_{t-1}^i are the corresponding FX spot and 1 month FX forward rates at time t and $t-1$ respectively, quoted as the USD price per unit of foreign currency.The *kxk* portfolios where $k = 1, .., 4$ are constructed by each month going long the k highest forward discount currencies and short the k lowest forward discount currencies, equal USD amounts. Monthly results are then annualised accordingly - annualised Return = 12 * Average Monthly Return, annualised Standard Deviation = $\sqrt{12}$ * Monthly Standard Deviation. Terminal Value of $1 is the end of sample period value of $1 invested at the start of the sample period for each portfolio. Currency data versus the USD is sourced from Datastream. DEM, FRF, and ITL is available Jan1976:Dec1998 and EUR is available Dec1998:Dec2012. AUD and NZD are available Jan1985:Dec2012, JPY is available Jan1978:Dec2012, all other currencies are available Jan1976:Dec2012.

	Carry Trade	Stop-Loss Rule							
		A	B	C	D	E	F	G	H
					$1x1$				
Return	0.0699	-0.0029	-0.0029	0.0208	0.0223	-0.0029	-0.0029	-0.0029	-0.0029
Standard Deviation	0.1419	0.0150	0.0150	0.0595	0.0588	0.0150	0.0150	0.0150	0.0150
% Sample Live		1%	1%	28%	27%	1%	1%	1%	1%
Terminal Value $1	8.96	0.90	0.90	2.01	2.14	0.90	0.90	0.90	0.90
					$2x2$				
Return	0.0693	-0.0020	0.0205	0.0648	0.0381	0.0216	-0.0020	0.0551	0.0187
Standard Deviation	0.1052	0.0231	0.0461	0.0952	0.0719	0.0521	0.0119	0.0953	0.0431
% Sample Live		6%	28%	100%	52%	29%	1%	100%	29%
Terminal Value $1	10.45	0.92	2.05	9.20	3.70	2.11	0.93	6.44	1.92
					$3x3$				
Return	0.0530	-0.0016	0.0102	0.0494	0.0112	-0.0028	-0.0023	0.0466	0.0114
Standard Deviation	0.0865	0.0179	0.0376	0.0802	0.0383	0.0106	0.0099	0.0783	0.0351
% Sample Live		6%	29%	100%	28%	1%	1%	100%	28%
Terminal Value $1	6.15	0.94	1.42	5.48	1.47	0.90	0.92	4.97	1.49
					$4x4$				
Return	0.0446	0.0216	0.0442	0.0448	0.0241	0.0118	-0.0020	0.0418	0.0115
Standard Deviation	0.0735	0.0458	0.0638	0.0702	0.0492	0.0333	0.0098	0.0688	0.0326
% Sample Live		46%	89%	100%	48%	29%	1%	100%	29%
Terminal Value $1	4.69	2.13	4.72	4.76	2.33	1.51	0.93	4.28	1.50

relatively successful, surviving for 60% of the sample period.

11.2.2 Example B

The EURUSD and EW FX carry trades survive the entire sample period when this stop-loss policy is applied, and in fact the Terminal Value \$1 is higher under the stop-loss policy than the original FX carry trades. The $4x4$ FX carry trade survives for 89% of the sample period and also has a Terminal Value \$1 higher than the original FX carry trade. The remaining 14 FX carry trades are stopped out permanently and have lower corresponding returns than the original trades.

11.2.3 Example C

Under stop-loss policy C all but the CHFUSD and $1x1$ FX carry trades survived the entire sample period. Of the 15 survivors, the CADUSD, DEMUSD, and $4x4$ FX carry trades had higher returns under the stop-loss policy than the original trade. The high rate of survival under this policy is due to the combination of the relatively large drawdown from zero limits and there being no terminal peak to trough drawdown limit. The drawdown from peak capital limit is aimed at slowing the drawdown process but if it continues then ultimately the drawdown from zero limits come into play.

11.2.4 Example D

Applying stop-loss policy D to the FX carry trades results in lower returns for all FX carry trades other than EURUSD. Additionally the EW FX carry trade survives the entire sample period but has a slightly lower return under the stop-loss policy. Under this stop-loss policy the annual stop-loss limit of 12% is, in most cases, too tight for the FX carry trade to survive.

11.2.5 Example E

Stop-loss policy E results in all FX carry trades being stopped out permanently, with the drawdown from zero limit and the drawdown from peak capital limit contributing roughly equally to the demise of the FX carry trades. CADUSD was the only FX carry trade which resulted in a higher return, having survived 81% of the sample period. Otherwise all returns were lower under the stop-loss policy. The annual drawdown limit of 10% and the drawdown from peak capital limit (10% + 50% Max ytd P&L) is simply too tight for the FX carry trades to survive.

11.2.6 Example F

Stop-loss policy F resulted in some extreme results. The AUDUSD, EURUSD, NOKUSD, SEKUSD, and EW FX carry trades survived the entire sample period under this stop-loss policy. All other FX carry trades were stopped out permanently, and in most of these cases survived for very few months with correspondingly much lower returns than the original trades. In the case of the EURUSD and EW FX carry trades the returns were slightly higher under the stop-loss policy than the original trade. This stop-loss policy is characterized by having a very tight drawdown from zero limit. So although it is based on life to date P&L at the institution the FX carry trades survival is clearly dependent on accumulating positive P&L in the 'early' months.

11.2.7 Example G

This stop-loss policy was relatively successful in that 7 of the 17 FX carry trades survived the entire sample period without getting stopped out, and in the case of the AUDUSD FX carry trade had higher returns under the stop-loss policy than the original trade. However, there were still a number of FX carry trades that were stopped out relatively early. Of the 10 FX carry trades that were stopped out permanently, the drawdown from zero limit and drawdown from peak capital limit contributed equally to their demise.

11.2.8 Example H

Applying stop-loss policy H to the FX carry trades resulted in all of them being stopped out permanently other than the EURUSD and EW trades. The majority of FX carry trades were stopped out due to triggering the drawdown from zero limit of 10%, which has been highlighted in other stop-loss examples as being too tight for most of the FX carry trades to survive.

11.3 Summary

In the single currency FX carry trades 8 hedge fund stop-loss frameworks were applied to 12 currency pairs. Of the resulting 96 return series generated, there were only 21 FX carry trades that survived their respective sample period under the respective stop-loss framework. However of those 21, only 6 FX carry trades resulted in a higher terminal value of $1 than the original FX carry trades without any stop-loss policy imposed.

The EW FX carry trade fared slightly better. Of the 8 stop-loss frameworks imposed, it survived 6 of them for the entire sample period and 3 of those resulted in a higher terminal value of $1 invested than the original EW FX carry trade. The relatively lower standard deviation of the EW FX carry trade is clearly beneficial in helping it survive the stop-loss frameworks.

The 4 kxk FX carry trades had a low survival rate across the 8 stop-loss frameworks. Of the 32 return series generated, there were only 6 FX carry trades that survived their respective sample period under the respective stop-loss framework. However, of those 6 only 1 FX carry trade resulted in a higher terminal value of $1 than the original FX carry trades without any stop-loss policy imposed.

Additionally, across all the FX carry trades, it is worth noting that there were 5 cases where the respective stop-loss framework resulted in the FX carry trade being stopped out permanently, but at this point the terminal value of $1 was higher under the respective stop-loss framework than the original FX carry trade. Perhaps little consolation given that your employment has been terminated.

Overall it is clear that in the majority of cases the FX carry trades are simply too volatile to survive the hedge fund stop-loss frameworks surveyed.

11.4 Monthly Hedge Fund Analysis

In Section 11.2 the impact of imposing the 8 hedge fund stop-loss frameworks defined in Section 11.1 on the FX carry trades starting in January 1976 were evaluated. However, given the nature of the stop-loss policies their impact on the FX carry trade returns is essentially path dependent on the unconstrained FX carry trade returns themselves. So our results in Section 11.2 are essentially based on one observation, an FX carry trade starting in January 1976. As it turns out April 1976 was a particularly poor month for a number of the FX carry trades which resulted in some of them being stopped out permanently under the hedge fund stop-loss frameworks. Given the path dependent nature of the stop-loss frameworks surveyed, could it be the case that for example an FX carry trade starting in May 1976 would have had sufficient time to build up a positive return buffer and survive a stop-loss framework much longer resulting in a constrained return much closer to the unconstrained FX carry trade return ?

To address this issue and check the robustness of our conclusions in Section 11.3, FX carry trades starting on a monthly basis are used. Firstly the unconstrained return series is created for each of the FX carry trades using monthly start dates. Recall that the data series goes from January 1976 through until December 2012. So, the first unconstrained return series runs from February 1976 (recall that the FX carry trade constructed is for 1 month - the trade is put on in January 1976 and then the first return is realised in February 1976) through until December 2012. This series is the 'one data sample' used in Section 11.2. The next unconstrained return series will start 1 month later, from March 1976 through until December 2012, and so on. Continuing with a 1 month shorter start date until the final series which is from January 2008 through until December 2012 results in 384 return series, each with start dates 1 month later than the previous series.

Next the 8 hedge fund stop-loss policies are imposed on the same 384 return series for each of the FX carry trades. This enables the difference in performance summary statistics between the unconstrained FX carry trades and those with the hedge fund stop-loss policies imposed on them to be calculated. In addition to the performance summary statistics already utilized in earlier sections annualised skewness and annualised kurtosis (sample excess kurtosis) are presented as are the minimum, median, and maximum of the 384 series annualised returns. Survival success is the percentage of the 384 series that are not permanently stopped out when the respective hedge fund stop-loss policy is imposed. Trade is "live" is the average, across the 384 series, percentage of months where the trade is not temporarily or permanently stopped out. Under-perform is the percentage of series where the stop-loss trade has a strictly lower terminal value than the simple unconstrained FX carry trade. The results of this monthly analysis for the 4 kxk FX carry trade portfolios are shown in Tables 26 and 27. The results for the single currency FX carry trades and the EW portfolio FX carry trade are available upon request.

11.4.1 Results

Tables 26 and 27 show that the average annual return, across the 384 series, for the simple unconstrained $1x1$, $2x2$, $3x3$, and $4x4$ FX carry trades are 5.31%, 5.73%, 5.01%, and 3.90% respectively. Of the 8 stop-loss policies applied to these 4 FX carry portfolios there is only 1 instance where the average annual constrained return is greater than the unconstrained return, stop-loss rule C applied to the $4x4$ portfolio (4.15% versus 3.90%). All other 31 stop-loss constrained series have lower average annual returns than their respective unconstrained FX carry portfolios. In addition this reduction in average annual return across the stop-loss constrained series is not compensated for by a corresponding reduction in volatility. In the case of the $1x1$ FX carry portfolio, all 8 stop-loss constrained series have a lower Sharpe ratio than the unconstrained series, and in the case of the $2x2$, $3x3$, and $4x4$ FX carry portfolios, 7 of the 8 stop-loss constrained series have a lower Sharpe ratio than the unconstrained series with stop-loss rule C being the exception in each case. The stop-loss rules also result in greater negative skewness and excess kurtosis in all cases except once again for stop-loss rule C applied to the $2x2$, $3x3$, and $4x4$ FX carry portfolios.

The main source of these differences in performance statistics between

Table 26: kxk FX Carry Trade Returns and Stop-Loss Rules: Multiple Starting Dates

This table compares the annualised returns from 384 simple unconstrained kxk portfolio FX carry trades with the returns from the corresponding trades subject to the eight stop-loss rules described in Section 11.1. The first trade begins in January 1976 (realised in February 1976), the second in February 1976, and the last in December 2008 (realised in January 2009), all terminate (if not previously stopped-out), in December 2012, with returns calculated monthly on a USD basis. The sample moments (mean, standard deviation, skewness, excess kurtosis) are all averages across the 384 annualised series. The additional parameters (maximum, minimum, median) are based on the 384 series. Survival Success is the percentage of series that are not permanently stopped-out. Trade is "Live" is the average percentage of months where the trade is not temporarily or permanently stopped out. Under-Perform is the percentage of series where the stop-loss trade has a strictly lower terminal value than the simple unconstrained trade. Other details appear in Table 25.

	Carry Trade	Stop-Loss Rule							
		A	B	C	D	E	F	G	H
					$1x1$				
Mean Return	0.0531	0.0164	0.0088	0.0346	0.0156	0.0120	0.0450	0.0129	0.0186
Standard Deviation	0.1471	0.0464	0.0597	0.0904	0.0629	0.0468	0.0907	0.0616	0.0550
Sharpe Ratio	0.3665	0.2067	0.0946	0.3544	0.1803	0.1749	0.2942	0.0847	0.2572
Skewness	-0.3767	-1.1413	-1.2424	-0.5744	-0.9408	-1.1619	-1.2533	-1.4221	-1.1439
Kurtosis	0.3159	6.4186	5.2698	2.1931	4.1919	5.6964	4.5827	5.7562	5.3764
Minimum	-0.0411	-0.0247	-0.0670	-0.0675	-0.0551	-0.0269	-0.0450	-0.0450	-0.0340
Median	0.0583	0.0070	0.0069	0.0399	0.0135	0.0084	0.0723	0.0043	0.0143
Maximum	0.0816	0.0892	0.0690	0.0788	0.0802	0.0693	0.1090	0.0931	0.0878
Survival Success		0%	0%	0%	0%	0%	60%	9%	0%
Trade is "Live"		16%	20%	48%	23%	15%	56%	24%	20%
Under-Perform		76%	99%	45%	74%	94%	37%	91%	72%
					$2x2$				
Mean Return	0.0573	0.0131	0.0164	0.0501	0.0186	0.0099	0.0444	0.0129	0.0119
Standard Deviation	0.1134	0.0382	0.0539	0.0969	0.0620	0.0515	0.0833	0.0521	0.0438
Sharpe Ratio	0.5151	0.2360	0.2762	0.5222	0.2512	0.1538	0.4017	0.1250	0.2023
Skewness	-0.1530	-0.7724	-0.4935	-0.1332	-0.4659	-0.7887	-0.6218	-0.9117	-0.5828
Kurtosis	0.1610	4.0219	2.0432	0.1198	1.3409	2.9353	2.2562	3.3436	2.8487
Minimum	0.0036	-0.0423	-0.0423	-0.0104	-0.0447	-0.0506	-0.0340	-0.0298	-0.0358
Median	0.0603	0.0068	0.0186	0.0517	0.0235	0.0074	0.0570	0.0041	0.0089
Maximum	0.0761	0.0608	0.0479	0.0726	0.0481	0.0478	0.0805	0.0638	0.0514
Survival Success		0%	0%	100%	0%	0%	75%	14%	0%
Trade is "Live"		18%	31%	93%	40%	25%	70%	33%	25%
Under-Perform		100%	100%	98%	100%	100%	40%	100%	100%

Table 27: kxk FX Carry Trade Returns and Stop-Loss Rules: Multiple Starting Dates cont'd

	Carry Trade	Stop-Loss Rule A	B	C	D	E	F	G	H
					$3x3$				
Mean Return	0.0501	0.0142	0.0212	0.0481	0.0094	0.0053	0.0357	0.0155	0.0202
Standard Deviation	0.0901	0.0359	0.0513	0.0833	0.0449	0.0419	0.0630	0.0499	0.0491
Sharpe Ratio	0.5627	0.2888	0.3224	0.5828	0.1476	0.0879	0.4070	0.1899	0.2675
Skewness	-0.1647	-0.4023	-0.5527	-0.0957	-0.7439	-0.7750	-0.6283	-0.5802	-0.5050
Kurtosis	0.1700	2.6000	1.6332	0.1406	2.3961	2.8267	2.1404	2.2927	2.0185
Minimum	0.0122	-0.0392	-0.0358	0.0121	-0.0342	-0.0421	-0.0237	-0.0228	-0.0125
Median	0.0516	0.0094	0.0209	0.0486	0.0057	0.0022	0.0537	0.0060	0.0105
Maximum	0.0655	0.0598	0.0533	0.0641	0.0490	0.0377	0.0640	0.0612	0.0713
Survival Success		0%	0%	100%	0%	0%	70%	30%	0%
Trade is "Live"		26%	48%	96%	33%	26%	67%	44%	42%
Under-Perform		100%	100%	71%	100%	100%	45%	100%	69%
					$4x4$				
Mean Return	0.0390	0.0123	0.0307	0.0415	0.0129	0.0057	0.0308	0.0218	0.0197
Standard Deviation	0.0768	0.0402	0.0589	0.0726	0.0464	0.0359	0.0610	0.0557	0.0461
Sharpe Ratio	0.5131	0.2555	0.4919	0.5746	0.2423	0.1194	0.3978	0.3008	0.3229
Skewness	-0.1260	-0.1939	-0.1383	-0.0837	-0.2710	-0.5571	-0.4341	-0.2670	-0.3288
Kurtosis	0.1080	1.7201	0.2283	0.1064	1.2761	2.2662	1.4068	1.1244	1.8663
Minimum	0.0032	-0.0297	-0.0305	0.0086	-0.0321	-0.0321	-0.0231	-0.0235	-0.0088
Median	0.0412	0.0148	0.0359	0.0430	0.0176	0.0049	0.0399	0.0262	0.0122
Maximum	0.0498	0.0306	0.0485	0.0522	0.0392	0.0301	0.0500	0.0493	0.0552
Survival Success		0%	0%	100%	0%	0%	79%	52%	0%
Trade is "Live"		31%	72%	97%	39%	26%	76%	63%	49%
Under-Perform		100%	68%	2%	100%	100%	52%	99%	60%

the unconstrained and stop-loss constrained FX carry portfolios is much the same as discussed in Section 11.3, survival rates. Firstly, looking at the survivor success numbers in the case of the $1x1$ FX carry portfolio 6 of the 8 stop-loss rules never have one instance across the 384 samples of the stop-loss constrained portfolio surviving until December 2012, and in the case of the $2x2$, $3x3$, and $4x4$ FX carry portfolios 5 of the 8 stop-loss constrained series never survive until December 2012. Temporary suspensions also play a significant role with only stop-loss constrained portfolios C and F being "live" for more than half the time available, across the 4 kxk portfolios. In the case of the $4x4$ FX carry portfolio, stop-loss constrained portfolios B and G also achieve "live" status greater than half the available time, due largely to the lower average volatility of the $4x4$ FX carry portfolio. Finally, in the majority of cases the terminal values of the stop-loss constrained series are lower than those of the unconstrained FX carry portfolios. Of the 32 stop-loss constrained portfolios 27 of them under perform their respective unconstrained FX carry portfolio greater than 50% of the time.

Once again, as discussed in Section 11.2, the effect of stop-loss rules C and F are somewhat muted in comparison to the others. In the case of stop-loss policy C the terminal drawdown from zero limit of 20% based on financial year P&L combined with there being no terminal peak to trough drawdown limit ensures that it has sufficient survivorship and consequently its results are closer to the unconstrained FX carry portfolios, particularly in the case of the $2x2$, $3x3$, and $4x4$ FX carry portfolios. In the case of F the crucial distinction is that drawdown thresholds are based on lifetime P&L, meaning that trades that get off to a good star are able to ride out bad months in subsequent years, and in addition there is no terminal drawdown from peak capital threshold, just a temporary stand down one.

It is clear that the strong negative impact of industry stop-loss rules on FX carry trade returns does not appear to be a timing quirk. Regardless of when such carry trades are implemented between 1976 and 2008, the stop-loss rules result in lower returns and weaker performance in almost all cases examined.

11.5 Formal Testing Issue

In Section 11.4.1, using a variety of summary statistics it was shown that imposing the industry standard stop-loss rules on the 4 kxk portfolios had a strong negative impact on the available returns, irrespective of when the FX carry trades were implemented. Ideally there would be a formal test that can be used to test the differences between the unconstrained FX carry trade returns and the hedge fund stop-loss constrained FX carry trade returns. There are several issues that need to be overcome.

Firstly there is the issue of overlapping data. The 384 unconstrained FX carry trade return series are constructed by using a start date 1 month later in each case which results in series that have observations in common. The overlapping of observations creates a moving average error term and thus ordinary least squares parameter estimates would be inefficient and hypothesis tests biased (Hansen and Hodrick 1980). For a given hedge fund stop-loss rule this is then applied to each of these unconstrained series to produce the corresponding constrained series, which again face the same overlapping data issue. Unless disaggregated data is used, as was the case in Section 11.2, in order to perform inferences between the unconstrained and constrained series of FX carry trade returns, this overlapping data issue must be allowed for.

The next consideration is how to deal with a constrained series that is stopped out permanently. Recall that the imposition of a given hedge fund stop-loss policy can result in the FX carry trade being stopped out temporarily or permanently. In the case that it is stopped out temporarily this will mean that for the following month the monthly return in the constrained series will be 0, and then the constrained series will resume trading again the next month. However when imposing the hedge fund-stop loss policy results in the constrained FX carry trade being stopped out permanently this means the return for the next month and all remaining months of the time series will be set to 0 representing the fact that the trader has effectively lost their job. Of course the unconstrained series continues generating returns for the remainder of the time series. In this scenario where a constrained series has been stopped out permanently if inferences are drawn using an average return for the entire sample period, the average for the constrained series will include a sequence of 0 monthly returns from the month the strategy is stopped out until the end of the sample period. Alternatively, testing a measure like terminal value, over the entire sample period, may be more insightful.

Whilst not being the focus of this thesis, but nevertheless important, a formal testing procedure that accommodates these issues provides an opportunity for future work.

12 Hedge Fund Stop-Loss Policies and CFTC Flow Analysis

In Section 11 the difficulty hedge fund traders would have surviving the FX carry trade due to the stop-loss policies they are governed by was illustrated. If this is the case, what types of traders are involved in the FX carry trade on an ongoing basis ? It would seem that in order to survive the inherent volatility of the FX carry trade you need to have the flexibility to withstand periods of significant drawdown. Does this mean that small retail investors, who do not have a risk management policy imposed upon them, are the client type involved in the FX carry trade on an ongoing basis ? In order to look at this question CFTC data on the relevant currency futures contracts can be used. For each of the single currency FX carry trades and the hedge fund stop-loss policies, described in Section 11 a time series of signals s_t is generated. $s = 1$ indicates being in the trade, $s = 0$ indicates being stopped out of the trade, either temporarily or permanently if the trader has reached the point of having his/her employment terminated. For each of these dates the CFTC data can provide a relative measure of the level of open interest of small investors to large investors. This will enable a test on the relative futures position of small investors to large investors under the 2 signal states, in the trade or out of the trade.

12.1 CFTC Data

The Commitment of Traders (CoT) report produced weekly by the CFTC [11] classifies clients into the following categories :

- Commercial Traders
- Non-commercial Traders
- Non-reportable Positions

The CFTC classify Non-commercial traders as those that are not using futures for hedging purposes, rather they are used for speculative purposes. For this exercise Non-commercial Traders are our proxy for Large Investors and Non-reportable Positions are our proxy for Small Investors. A positive futures position is equivalent to a currency trade in which the foreign currency has been purchased versus selling the USD.
For Large Investors the following futures data is available :

- NonComm_Positions_Long_All

[11] http://www.cftc.gov/MarketReports/CommitmentsofTraders/index.htm

- NonComm_Positions_Short_All
- NonComm_Positions_Spread_All

These provide the inputs to calculate an open interest measure for Large Investors :

$$\begin{aligned} &Large\_Investors\_Open\_Interest_t = \\ &\qquad NonComm\_Positions\_Long\_All_t + \\ &\qquad NonComm\_Positions\_Short\_All_t + \\ &\qquad (NonComm\_Positions\_Spread\_All_t * 2) \end{aligned} \quad (46)$$

For small investors the following futures data is available :

- NonRept_Positions_Long_All
- NonRept_Positions_Short_All

These provide the inputs to calculate an open interest measure for Small Investors :

$$\begin{aligned} &Small\_Investors\_Open\_Interest_t = \\ &\qquad NonRept\_Positions\_Long\_All_t + \\ &\qquad NonRept\_Positions\_Short\_All_t \end{aligned} \quad (47)$$

The ratio of Small Investors Open Interest to Large Investors Open Interest is then calculated as :

$$Ratio\_Small\_Large_t = \frac{Small\_Investors\_Open\_Interest_t}{Large\_Investors\_Open\_Interest_t} \qquad (48)$$

The CFTC data is available from different dates for respective currency pairs. There is no futures data for NOKUSD and SEKUSD and the legacy European currencies, DEMUSD, FRFUSD, and ITLUSD have been removed as the start date of the futures data is too close to the introduction of EURUSD. CFTC data start dates vary by currency, from February 1986 for CADUSD, CHFUSD, GBPUSD, and JPYUSD, from January 1993 for AUDUSD, from February 1999 for EURUSD, and from January 2006 for NZDUSD. From these respective dates the single currency FX carry trades under each of the 8 benchmark stop-loss frameworks in Section 11 have been calculated to obtain the corresponding time series of trade signals, s_t. $s = 1$ when institutional investors are invested in the FX carry trade and $s = 0$ when they are stopped out of the FX carry trade.

12.2 Analysis

Firstly, a test of whether the average of *Ratio_Small_Large* is equal when institutional traders are in or out of the trade respectively is performed, for each currency versus the USD, and for each of the 8 hedge fund stop-loss policies.

Define $\bar{R}_0$ = the average of $Ratio\_Small\_Large_t$ when $s_t = 0$ and $\bar{R}_1$ = the average of $Ratio\_Small\_Large_t$ when $s_t = 1$. As such the hypothesis that to be tested are :

$$\begin{aligned} H_0 &: \bar{R}_0 = \bar{R}_1 \\ H_1 &: \bar{R}_0 \neq \bar{R}_1 \end{aligned} \tag{49}$$

In order to do this firstly check whether the variances are equal. If so, then the Pooled t test is used, otherwise the Satterthwaite t test is used.

The results in Table 28 show that it was only possible to reject H_0 on 14 of the 56 occasions tested. On 12 of those 14 occasions $\bar{R}_1 > \bar{R}_0$. This is difficult to interpret. $\bar{R}_1 > \bar{R}_0$ suggests that when institutional investors are involved in the FX carry trade, the level of retail or small investor involvement is proportionally higher than when the institutions are not involved in the FX carry trade based on the series of stop signals generated.

There are clearly limitations to this analysis as the choice of start date, determined by the availability of the CFTC data for each currency, clearly effects the number of signals generated under each of the hedge fund stop-loss policies. In addition the categories within the CFTC client classifications are very broad, and in the global currency markets futures trading represents a very small part of it, with the vast majority of trading being conducted on an over the counter basis. It is also unknown what constraints retail investors involved in the FX carry trade might also be facing when the institutional stop-loss signals are being generated. In some instances they may also be facing margin or leverage constraints, but without knowing the details of how individuals are trading and what their self imposed loss parameters are this is impossible to predict.

Secondly, a probit regression analysis is used to model each trade signal series and test the resulting parameter estimates. The equation estimated for each currency, i, versus the USD, and for each of the 8 hedge fund stop-loss policies, ϕ is :

$$s_t^{i,\phi} = \alpha + \beta Ratio\_Small\_Large_t^i + \epsilon_t \tag{50}$$

Let $\widehat{\beta}^{i,\phi}$ be the probit estimate of β. The hypothesis that to be tested are :

$$\begin{aligned} H_0 :& \widehat{\beta}^{i,\phi} = 0 \\ H_1 :& \widehat{\beta}^{i,\phi} \neq 0 \end{aligned} \tag{51}$$

The results in Table 29 show that it was only possible to reject H_0 on 5 of the 56 occasions tested. On each of those 5 occasions the probit regression coefficients are all positive. In each of the probit procedures the probability modelled is state 1. So a positive regression coefficient can be interpreted as saying an increase in the $Ratio\_Small\_Large_t^i$ leads to an increase the predicted probability of state 1, that is institutional investors being invested in the FX carry trade. Again, this result is difficult to make sense of, in the same way as the results of testing the average of $Ratio\_Small\_Large$ is equal

Table 28: CFTC t Test Results $\bar{R}_0 = \bar{R}_1$

From the CFTC futures data for each of the currencies below versus the USD I calculate :

$$\begin{aligned} Large\_Investors\_Open\_Interest_t &= NonComm\_Positions\_Long\_All_t \\ &+ NonComm\_Positions\_Short\_All_t \\ &+ (NonComm\_Positions\_Spread\_All_t * 2) \\ Small\_Investors\_Open\_Interest_t &= NonRept\_Positions\_Long\_All_t \\ &+ NonRept\_Positions\_Short\_All_t \\ Ratio\_Small\_Large_t &= \frac{Small\_Investors\_Open\_Interest_t}{Large\_Investors\_Open\_Interest_t} \end{aligned}$$

Define $\bar{R}_0$ = the average of $Ratio\_Small\_Large_t$ when $s_t = 0$ (institutional investors stopped out of the FX carry trade) and $\bar{R}_1$ = the average of $Ratio\_Small\_Large_t$ when $s_t = 1$ (institutional investors invested in the FX carry trade). Results for t Test of $H_0 : \bar{R}_0 = \bar{R}_1$ vs $H_1 : \bar{R}_0 \neq \bar{R}_1$ are shown below. Asterisks ***, **, and * indicate rejection of H_0 at the significance levels of 1%, 5%, and 10% respectively. o indicates there are insufficient $s_t = 0$ observations to perform the test, x indicates there are insufficient signal observations in total from the date which the CFTC data is available to perform the test. CFTC data used from Feb1986 for CADUSD, CHFUSD, GBPUSD, JPYUSD, from Jan1993 for AUDUSD, from Feb1999 for EURUSD, and from Jan2006 for NZDUSD. Trade signal series, s_t, generated from these same dates for the respective currencies under each of the 8 hedge fund stop-loss policies, A-H.

	A	B	C	D	E	F	G	H
AUDUSD		**		*	o			**
CADUSD			***	*	o		***	***
CHFUSD	*	*		*	o	x	*	o
EURUSD				**	o			
GBPUSD					o			
JPYUSD	x				x		x	x
NZDUSD	o		*	o	o	*	o	o

when traders are in or out of the trade was.

12.3 CFTC Results

In summary it is difficult to draw any meaningful conclusions from the CFTC data as to when institutional and retail investors are involved in the FX carry trade. The significant data limitations, both in terms of the CFTC data's lack of client type granularity and the limited number of trade signals generated under many of the hedge fund stop-loss policies, are likely to be impeding the results. So the question remains, based on the hedge fund stop-loss policies surveyed, when and how are these traders invested in the FX carry trade ?

13 Conclusions

The impact of imposing both a theoretical stop-loss framework and a sample of hedge fund stop-loss frameworks on the available returns to the FX carry trade has been examined.

In the case of the theoretical framework, in the majority of cases it resulted in lower annualised returns and in all cases resulted in lower annualised standard deviations of returns. The exception to this was the low yielding currencies which had higher annualised returns to the FX carry trade when the stop-loss policy was imposed. In contrast to the hedge fund policies I sampled, the theoretical framework of Kaminski and Lo (2008) had some key differences :

Table 29: CFTC Probit Regression Results

From the CFTC futures data for each of the currencies below versus the USD I calculate :

$$\begin{aligned}
Large_Investors_Open_Interest_t &= NonComm_Positions_Long_All_t \\
&+ NonComm_Positions_Short_All_t \\
&+ (NonComm_Positions_Spread_All_t * 2) \\
Small_Investors_Open_Interest_t &= NonRept_Positions_Long_All_t \\
&+ NonRept_Positions_Short_All_t \\
Ratio_Small_Large_t &= \frac{Small_Investors_Open_Interest_t}{Large_Investors_Open_Interest_t}
\end{aligned}$$

CFTC data used from Feb1986 for CADUSD, CHFUSD, GBPUSD, JPYUSD, from Jan1993 for AUDUSD, from Feb1999 for EURUSD, and from Jan2006 for NZDUSD. Trade signal time series, s_t, generated from these same dates for the respective currencies under each of the 8 hedge fund stop-loss policies, A-H. $s = 1$ when institutional investors are invested in the FX carry trade and $s = 0$ when they are are stopped out of the FX carry trade. Using probit regression the equation I estimate for each currency, i, versus the USD, and for each of the 8 hedge fund stop-loss policies, ϕ is :

$$s_t^{i,\phi} = \alpha + \beta Ratio\_Small\_Large_t^i + \epsilon_t$$

Let $\widehat{\beta}^{i,\phi}$ be the probit estimate of β. The hypothesis that I wish to test are $H_0 : \widehat{\beta}^{i,\phi} = 0 vs \mathrm{H}_1 : \widehat{\beta}^{i,\phi} \neq 0$. Asterisks ***, **, and * indicate rejection of H_0 at the significance levels of 1%, 5%, and 10% respectively. x indicates the MLE does not exist due to there being too few trade signal observations.

	A	B	C	D	E	F	G	H
AUDUSD								
CADUSD			***	*			***	***
CHFUSD	x	x		x		x	x	x
EURUSD				*				
GBPUSD								
JPYUSD	x				x		x	x
NZDUSD	x			x	x		x	x

- Triggering the maximum drawdown from zero limit in the hedge fund examples results in the traders employment being terminated where as in Kaminski and Lo (2008) the drawdown from zero trigger means trading ceases until the re entry trigger is satisfied

- The re entry trigger is for the last period return to be greater than some threshold value. Instead, the hedge fund examples generally have a finite stand down period of no trading. This is generally 1 month in our framework.

- The theoretical stop-loss framework does not have drawdown from maximum profit limits

The sample of hedge fund stop-loss policies resulted in a low survival rate for the FX carry trade. In most cases permanent stop-loss events were triggered and in those that did survive the full sample period most resulted in a lower return than the original FX carry trade. Overall it is clear that in the majority of cases the FX carry trades that I have specified are simply too volatile to survive the hedge fund stop-loss frameworks surveyed. These results highlight that when a stop-loss risk management policy is overlayed onto the FX carry trade, the degree to which the forward premium puzzle can be exploited is significantly less than in an unconstrained environment with no risk management policy.

Chapter III

FX Option Implied Volatility and the FX Carry Trade

14 Introduction

The Foreign Exchange (FX) options market is one of the largest and most liquid over-the-counter (OTC) derivatives markets in the world. Table 30 shows the Bank of International Settlements (BIS) triennial FX survey market turnover results split by instrument type. In the 2013 survey, FX options had a daily average turnover of 337 billion US dollars.

Table 30: Global Foreign Exchange Market Turnover

Daily averages in April, in billions of US dollars.

Instrument	1998	2001	2004	2007	2010	2013
Foreign Exchange Instruments	1,527	1,239	1,934	3,324	3,971	5,345
Spot transactions	568	386	631	1,005	1,488	2,046
Outright forwards	128	130	209	362	475	680
Foreign exchange swaps	734	656	954	1,714	1,759	2,228
Currency swaps	10	7	21	31	43	54
Options and other products	87	60	119	212	207	337
Exchange traded derivatives	11	12	26	80	155	160

Source - BIS Triennial Central Bank Survey.

It is also worth noting that almost all of the turnover in the FX market is done on an OTC basis. The volumes traded on organised exchanges, as shown in Table 30, are negligible. This fact along with the decentralized structure of the FX market where liquidity is fragmented across different geographical regions and trading platforms, makes quantitative insights into market activity difficult. The FX market is unique in other ways. Despite the recent attention from regulators post the global financial crisis, the FX market has essentially evolved by being self regulated, developing its own conventions and protocols.

The OTC conventions by which FX options are quoted and traded between interbank counterparties is unique to the FX market. Unlike equity markets which generally quote option strike-price or option strike-implied volatility pairs, the FX OTC options market quotes implied volatility and delta (as opposed to strike) pairs. In addition it is convention to quote implied volatilities for several combinations of options.

The aim of this chapter is to provide readers with a thorough understanding of the quoting conventions used for FX options and how they are traded. The implied FX volatility surfaces that underlie the FX options market are then used to predict future FX carry trade returns.

15 Option Basics

Option contracts give the holder the right but not the obligation to buy or sell some underlying asset at a specified price at, or up to and including, a specified time on the date of expiry of the option. There are two types of option expiries. A European option can only be exercised at a certain time on the day of expiry. An American option can be exercised any time up to and including a specified time on the day of expiry. It is convention in the interbank FX options market to quote and trade European options. American options give rise to some interesting early exercise conditions in the case of FX options (Garman and Kohlhagen 1983).[12] This chapter looks solely at European options.

There are 2 basic types of option contracts, calls and puts. The holder of a call option has the right but not the obligation to buy an underlying asset at a specified price (the strike price) at a specified time in the future (the expiry date). Conversely, the holder of a put option has the right but not the obligation to sell an underlying asset at a specified price at a specified time in the future. The difference in time between the date an option contract is entered into and the expiry date is referred to as the time to expiry or maturity.

In the case of FX options the underlying asset of an option contract is a currency pair. So for example the holder of a NZDUSD call option has the right but not the obligation to buy NZD and sell USD at a certain strike price and expiry date. So in this case a NZD call is by definition a USD put. Conversely the holder of a NZDUSD put option has the right but not the obligation to sell NZD and buy USD at a certain strike price and expiry date.

To better understand options it is worth considering their payoff profiles at maturity. Figure 11 shows the payoff profile at maturity for the holder of a European call option. Recall that the holder of a call option has the right but not the obligation to buy an underlying asset at a specified strike

[12] In certain circumstances it will be optimal to exercise an American option before expiry. This is generally a consideration for deep in the money options when the call currency has high interest rates relative to the put currency. See Fabozzi et al. (1990).

price at the maturity of the option. If the price of the underlying asset at the time of the option maturity is greater than the strike price then the option holder would exercise the option and gain the difference between the current market price of the asset and the strike price of the option. If the price of the underlying asset at the time of the option maturity is sufficiently higher than the strike price of the option such that premium paid for the option is offset, then the option becomes profitable. As shown in Figure 11 the underlying asset price at which this occurs is referred to as the breakeven price. On the other hand if the price of the underlying asset at the time of the option maturity is less than the strike price of the option then it is not profitable to exercise the option and it would expire worthless. In this case the holder of the call option would lose the premium that was paid for the option, as shown in Figure 11. Note that the call option holders loss is limited to the amount of the premium that was paid for the option whilst their potential upside is unlimited.

Define :

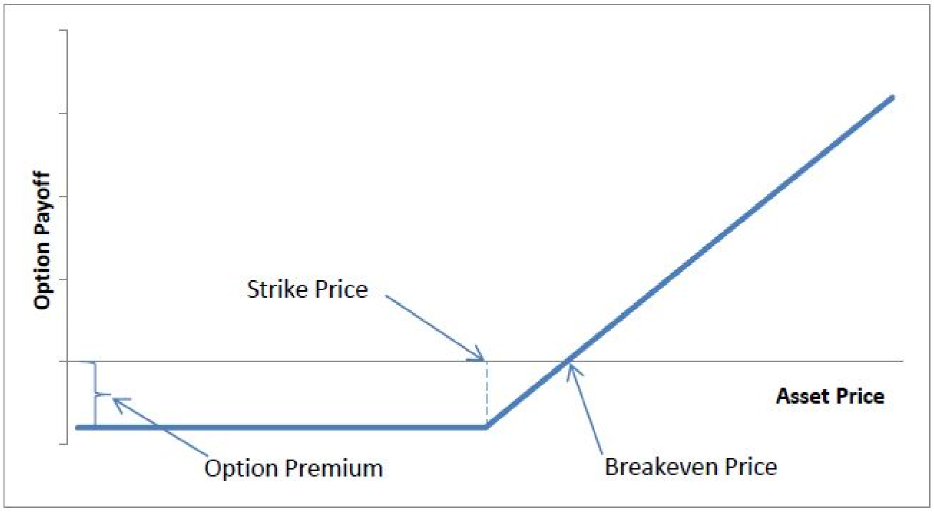
c = FX European vanilla call option premium

S_T = spot exchange rate at option maturity time T, quoted as the local currency price per unit of foreign currency

K = strike price of the option.

Figure 11, the call option payoff at maturity, can be defined as :

Figure 11: Call Option Payoff Profile

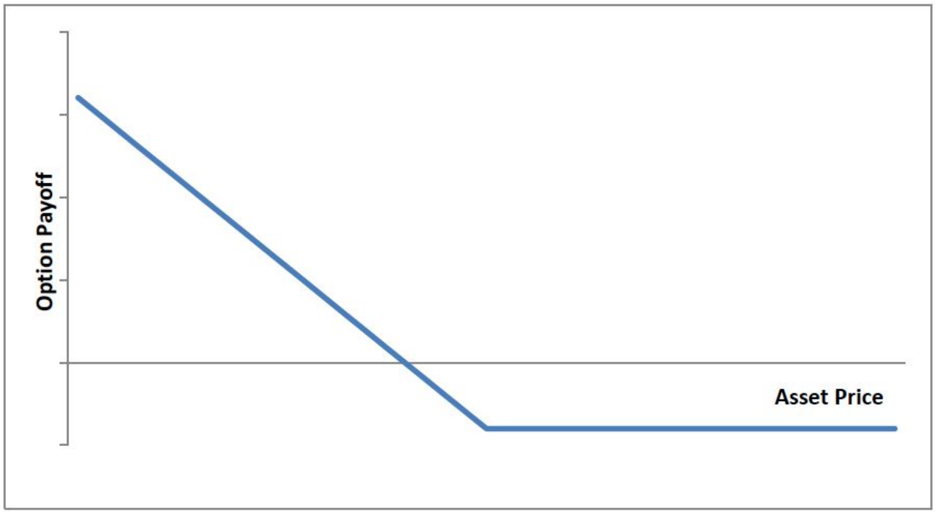


This graph shows the payoff profile at maturity for the owner of a call option. Option payoff is plotted on the y axis and the asset price of the underlying asset of the call option is plotted on the x axis.

$$Call\ Payoff\ = max\ (0,\ S_T - K) - c \tag{52}$$

Conversely, for the holder of a put option it would be profitable to exercise the option at maturity if the market price of the underlying asset is less than the strike price of the option. Figure 12 shows the payoff profile at maturity for the holder of a European put option. If instead the market price of the underlying asset is greater than the strike price of the option then it is not profitable to exercise the option and it would expire worthless, meaning that the holder loses the option premium that they had paid. Again, the option holders loss is limited to the amount of the premium that was paid for the

Figure 12: Put Option Payoff Profile



This graph shows the payoff profile at maturity for the owner of a put option. Option payoff is plotted on the y axis and the asset price of the underlying asset of the put option is plotted on the x axis.

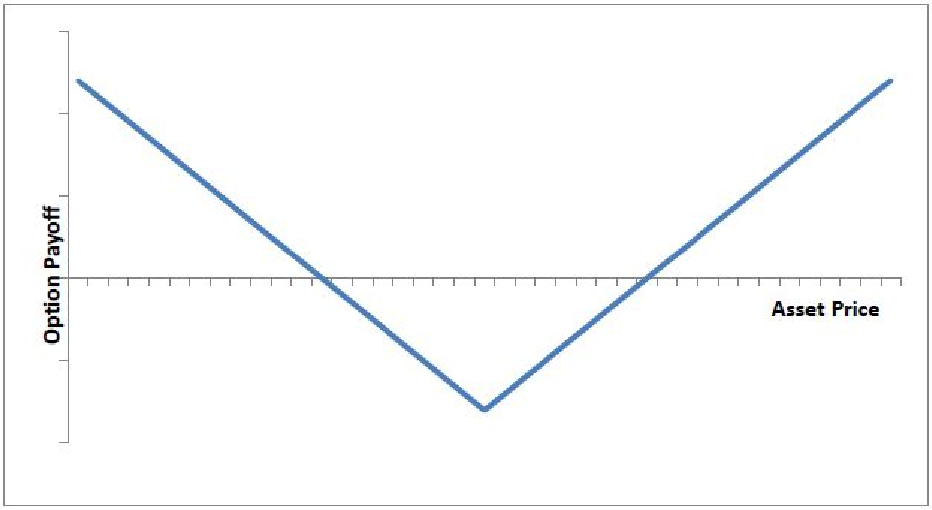
option whilst their potential upside is restricted only by any limits as to how far the price of the underlying asset is able to fall.

Similarly, let p = FX European vanilla put option premium, Figure 12, the put option payoff at maturity, can be defined as :

$$Put\ Payoff\ = max\ (0,\ K - S_T) - p \tag{53}$$

So far four different option positions have been discussed, long a call, short a call, long a put, and short a put. For a given option maturity by combining different combinations of call and put options it is possible to create various payoff profiles at maturity. Firstly, the simultaneous purchase

Figure 13: Straddle Payoff Profile



This graph shows the payoff profile at maturity for the owner of a straddle. Option payoff is plotted on the y axis and the asset price of the underlying asset of the straddle is plotted on the x axis.

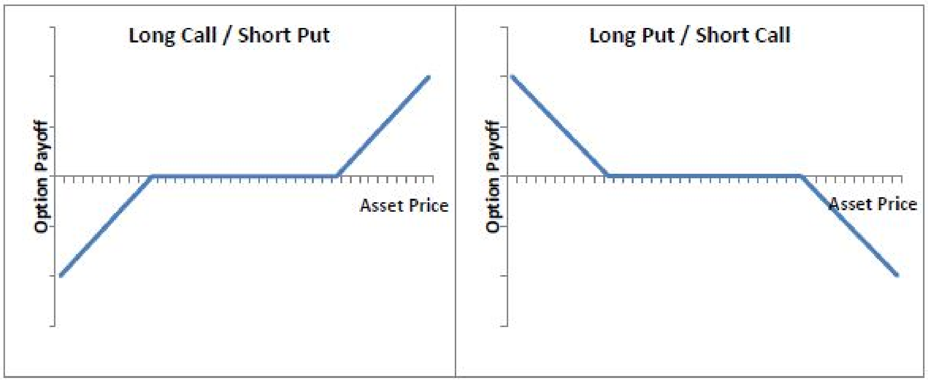
of a call and a put option with the same strike price, K, is referred to as a straddle. Figure 13 shows the payoff at maturity profile for the holder of a straddle. The more the price of the underlying asset deviates from the strike price of the options the more profitable the straddle potentially becomes. From a directional perspective this type of option strategy is appropriate when a trader expects there to be a large move in the price of the underlying asset, but does not know in what direction the move is likely to be.

Figure 13, the purchased straddle option payoff at maturity, can be defined as :

$$
\begin{aligned}
Straddle\ Payoff &= (max\ (0,\ S_T - K) + max\ (0,\ K - S_T)) - (c+p) \\
&= max(S_T - K, K - S_T) - (c+p) \\
&= |S_T - K| - (c+p)
\end{aligned}
\tag{54}
$$

A risk reversal can be constructed by buying (selling) a call option with a high strike price and selling (buying) a put option with a low strike price. Assuming that the option premiums for each option were equal, meaning the total premium for the risk reversal is zero, Figure 14 shows the payoff profile at maturity for buying a call and selling a put, and secondly for buying a put and selling a call. From a directional perspective this combination of options is appropriate when a trader has a strong directional view about the future price of the underlying asset. For instance in the case of the left panel in Figure 14 where the trader is long the call option and short the put option this would be consistent with a directional view that the price of the underlying asset is likely to appreciate in the future. Alternatively, in the case of a hedger who has an underlying exposure to the price of the asset in question, this type of option strategy may be appropriate to hedge their risk. For instance if a hedger owned the underlying asset in question then they would naturally have a positive exposure to increases in the price of the underlying asset and a negative exposure to decreases in the price of the underlying asset. If they were concerned about a short term decrease in the

Figure 14: Risk Reversal Payoff Profile



These graphs shows the payoff profile at maturity for a risk reversal. The left panel is the payoff for a long call / short put risk reversal, and the right panel is the payoff for a long put / short call risk reversal. Option payoff is plotted on the y axis and the asset price of the underlying asset of the risk reversals is plotted on the x axis.

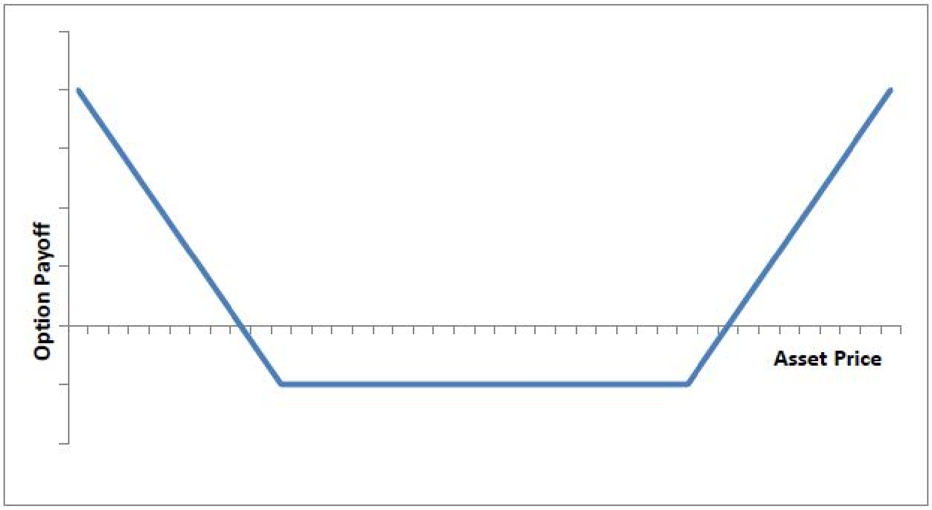
price of the underlying asset and wanted to hedge some of this risk then they could do so by trading the risk reversal in the right panel of Figure 14 by buying a put option and selling a call option.

Let K_h be the high strike call option strike and K_l be the low strike put option strike and assume $c = p$, then the left hand panel of Figure 14 can be represented as :

$$\begin{aligned} RR\ Payoff &= (max\ (0,\ S_T - K_h) - c) - (max\ (0,\ K_l - S_T) - p) \\ &= max\ (0,\ S_T - K_h) - max\ (0,\ K_l - S_T) \end{aligned} \tag{55}$$

A strangle can be constructed by buying a call option with a high strike

Figure 15: Strangle Payoff Profile



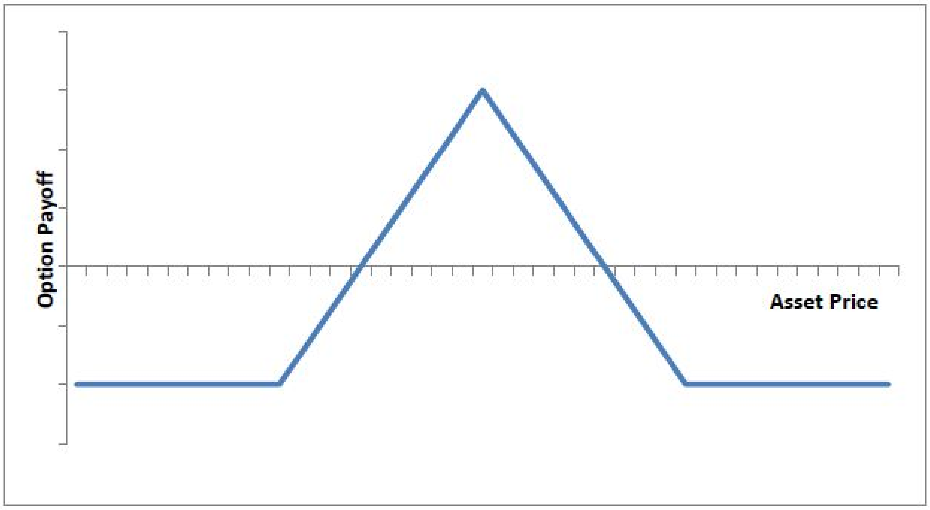
This graph shows the payoff profile at maturity for the owner of a strangle. Option payoff is plotted on the y axis and the asset price of the underlying asset of the strangle is plotted on the x axis.

and a put option with a low strike. Figure 15 shows the payoff at maturity for the holder of a strangle. Much like the straddle in Figure 13, from a directional perspective a strangle is also an appropriate option strategy when a trader expects there to be a large move in the price of the underlying asset but does not have a view on what direction the move might be.

Likewise, Figure 15 can be represented as :

$$\begin{aligned} Strangle\ Payoff &= (max\ (0,\ S_T - K_h) + max\ (0,\ K_l - S_T)) - (c + p) \\ &= max(S_T - K_h, K_l - S_T, 0) - (c + p) \end{aligned} \tag{56}$$

Figure 16: Butterfly Payoff Profile



This graph shows the payoff profile at maturity for the owner of a butterfly. Option payoff is plotted on the y axis and the asset price of the underlying asset of the butterfly is plotted on the x axis.

It is possible to combine a straddle and a strangle to create a butterfly. Figure 16 shows the payoff profile of being long a butterfly which is constructed by buying a strangle and selling a straddle. From a directional perspective this type of option strategy is appropriate when a trader has a view that the price of the underlying asset is going to remain central, or at least in very tight range, around the strike price of the straddle which occurs at the peak of the payoff profile in Figure 16.

16 Option Pricing in the FX Market

16.1 Black-Scholes Option Pricing

In 1973 the now famous Black-Scholes option pricing formula was developed for pricing non-dividend paying stocks (Black and Scholes (1973)). Garman and Kohlhagen (1983) extended this work to price currency options by incorporating the foreign interest rate. However, it remains common practice to refer to their formula as the Black-Scholes formula. For those with an equity back ground you will notice the similarity to the Black-Scholes model for stocks paying continuous dividends, sometimes referred to as the Black-Scholes-Merton model.

The Black-Scholes formula for pricing an FX European vanilla call option (c) is :

$$c = Se^{-r_f T} N(d_1) - Ke^{-r_d T} N(d_2) \tag{57}$$

and an FX European vanilla put option (p) is :

$$p = Ke^{-r_d T} N(-d_2) - Se^{-r_f T} N(-d_1) \tag{58}$$

where,

$$d_1 = \frac{\ln(S/K) + (r_d - r_f + v^2/2)T}{v\sqrt{T}}, \quad d_2 = d_1 - v\sqrt{T} \tag{59}$$

where,

S = spot exchange rate, quoted as the local currency price per unit of foreign currency

K = strike price of the option

r_d = domestic interest rate, per annum, for period T

r_f = foreign interest rate, per annum, for period T

T = time in years until the option expires

v = implied volatility for strike K and period T

$N(.)$ = standard normal cumulative distribution

Note that c and p are expressed in domestic currency on an option notional of one unit of foreign currency. Also for simplicity the difference between the actual settlement date of the initial premium and delta hedge (details of which are explained in Section 16.4) and the date when an option is transacted have been ignored. Generally the option premium and delta hedge are both due to be settled on what is referred to as 'spot date' which for most currencies is 2 business days after the transaction date of the option. Likewise at the expiry if the option, if it is exercised , then the resulting spot transaction is settled on the spot date, that is, 2 business days after the expiry date of the option.

16.2 Option Greeks

The sensitivity of the value of an option, as defined by the Black-Scholes formula, to changes in the respective inputs into the pricing formula are generally referred to as the option greeks. Knowing and understanding these relationships for individual options and portfolios of options is is critical for FX option traders, both in terms of pricing options and risk management of option portfolios.

Delta measures the sensitivity of the option price to changes in the price of the underlying asset. In the case of FX options, the delta of a call option (Δ_c) is defined as :

$$\Delta_c = \frac{dc}{dS} = e^{-r_f T} N(d_1) \tag{60}$$

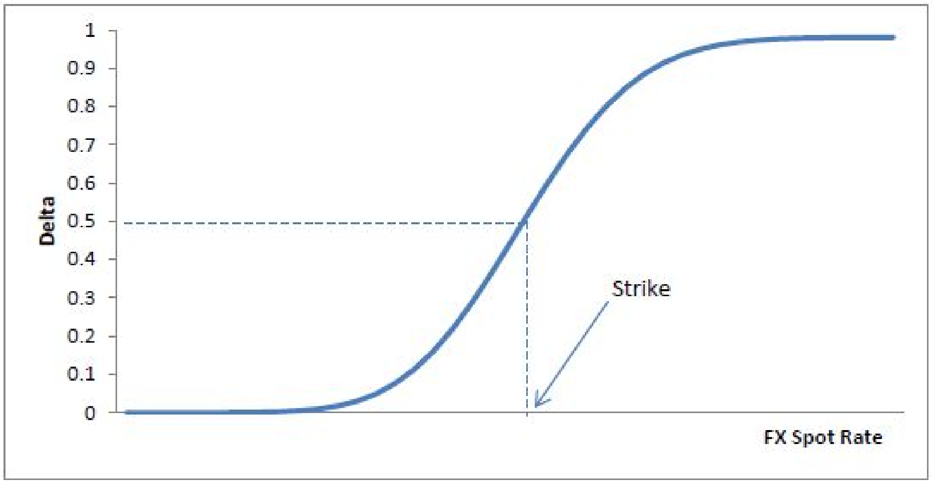
and the delta of a put option (Δ_p) is defined as :

$$\Delta_p = \frac{dp}{dS} = -e^{-r_f T} N(-d_1) \tag{61}$$

where d_1 is defined by Equation (59).

In the FX options market an at-the-money option strike refers to the strike of a zero delta straddle. This is the strike for which the delta of the call option (Δ_c) is equal to the delta of the put option (Δ_p) and will be approximately equal to the forward price for the time to maturity of the option. An at-the-money option has a delta of around 50%. Options that have a delta of less than 50% are said to be out-of-the-money and those with

Figure 17: European FX Option Call Delta vs FX Spot



This graph plots the delta of a long European FX option versus the underlying FX spot rate.

deltas greater than 50% are said to be in-the-money. Figure 17 shows this relationship between the delta of a European FX call option and the FX spot rate.

The relationship between delta and the time to maturity of an option is dependent on whether the option is in-the-money or out-of-the-money. For an in-the-money option, all else being equal, as the time to maturity decreases (eventually to 0 as maturity is reached) the delta of the option increases to finally be 100%. Conversely for an out-of-the-money option, all else being equal, as the time to maturity decreases the delta of the option decreases to finally be 0%.

Vega measures the sensitivity of the option price to changes in the implied volatility of the option. In the case of FX options, the vega of a call option

(V_c) is defined as :

$$V_c = \frac{dc}{dv} = S\sqrt{T}e^{-r_f T}n(d_1) \tag{62}$$

and the vega of a put option (V_p) is also defined as :

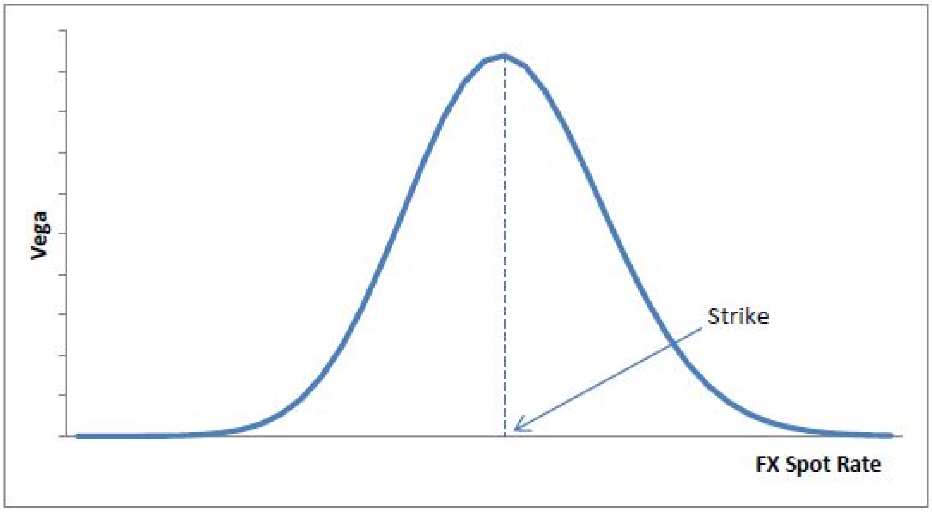
$$V_p = \frac{dp}{dv} = S\sqrt{T}e^{-r_f T}n(d_1) \tag{63}$$

where $n(.)$ is the standard normal probability density function.

As can be seen, all else being equal an increase in implied volatility will increase the value of an option. There are several second order relationships worth mentioning. The longer the time to maturity of an option, all else being equal, the higher its vega will be. The vega of an option is greatest when the price of the underlying asset is equal to the strike price of the option (at-the-money), all else being equal. As the price of the underlying asset moves away from the strike price, in either direction, the vega of the option decreases. Figure 18 shows the relationship between the vega of a European FX call option and the FX spot rate. For an option with a given delta, all else being equal, the longer the time until maturity the higher the vega of the option will be.

Gamma, although strictly a second order greek, measures the sensitivity of delta to changes in the price of the underlying asset. In the case of FX options, the gamma of a call option Γ_c is defined as :

Figure 18: European FX Option Call Vega vs FX Spot



This graph plots the vega of a long European FX option versus the underlying FX spot rate.

$$\Gamma_c = \frac{d^2c}{dS^2} = \frac{e^{-r_f T} n(d_1)}{Sv\sqrt{T}} \tag{64}$$

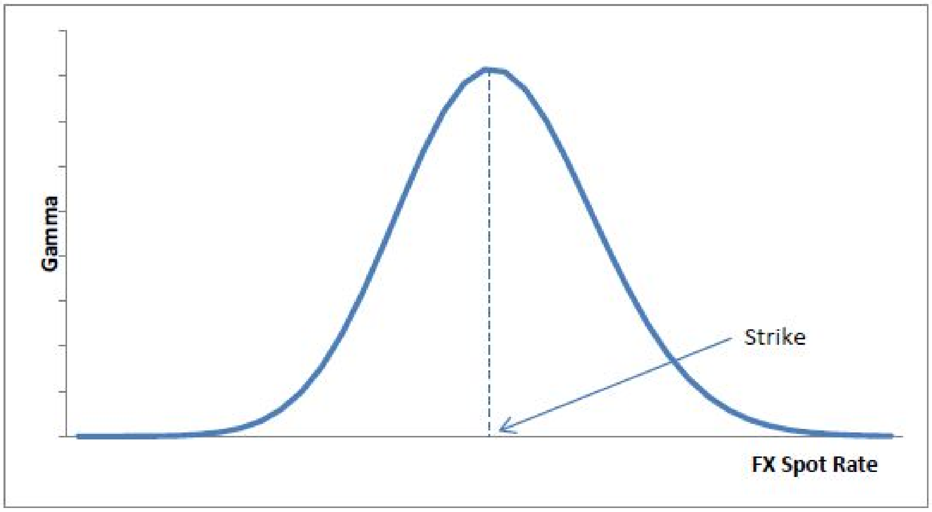
and the gamma of a put option Γ_p is also defined as :

$$\Gamma_p = \frac{d^2p}{dS^2} = \frac{e^{-r_f T} n(d_1)}{Sv\sqrt{T}} \tag{65}$$

The gamma of an option, all else being equal is highest when the option is at-the-money. Gamma decreases the further in-the-money or out-of-the-money an option becomes. Figure 19 shows the relationship between the gamma of a European FX call option and the FX spot rate.

The relationship between gamma and time to maturity of an option is

Figure 19: European FX Option Call Gamma vs FX Spot



This graph plots the gamma of a long European FX option versus the underlying FX spot rate.

once again dependent on whether the option in question is in-the-money, out-of-the-money, or at-the-money. For in-the-money and out-of-the-money options as time to maturity decreases, gamma tends to increase gradually, before falling away sharply to 0 at expiry. For an at-the-money option, all else being equal, gamma increase sharply as time to maturity decreases until just prior to expiry.

Theta measures the sensitivity of the option price to changes in the time to maturity of the option. In the case of FX options, the theta of a call option (θ_c) is defined as :

$$\theta_c = \frac{dc}{dT} = \frac{-e^{-r_f T} S n(d_1) v}{2\sqrt{T}} + r_f S e^{-r_f T} N(d_1) - r_d K e^{-r_d T} N(d_2) \qquad (66)$$

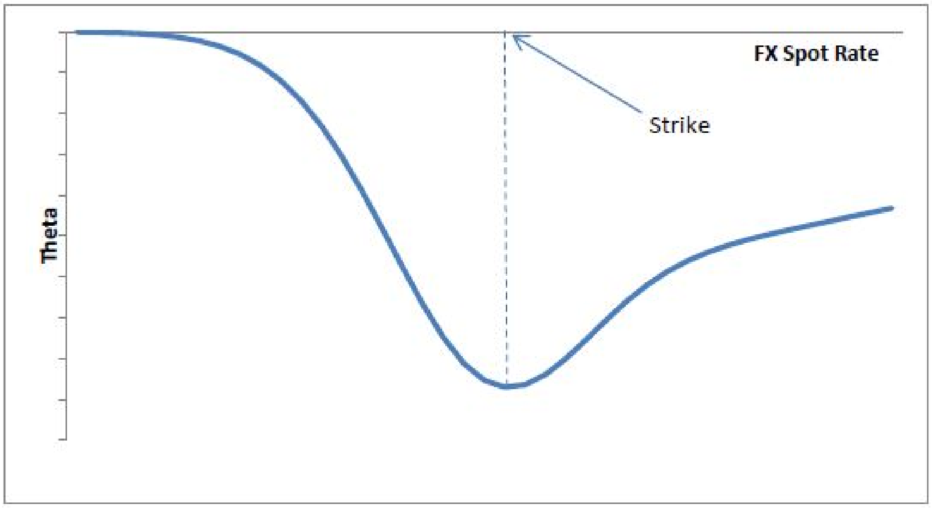
and the theta of a put option (θ_p) is defined as :

$$\theta_p = \frac{dp}{dT} = \frac{-e^{-r_f T} S n(d_1) v}{2\sqrt{T}} - r_f S e^{-r_f T} N(-d_1) + r_d K e^{-r_d T} N(-d_2) \qquad (67)$$

Generally FX option traders refer to theta as the time decay of an option, or portfolio of options, per calender day of time. All else being equal if the time to maturity of an option is decreased by one day then its value will decrease. If you are the owner of this option then you effectively pay this time decay and you are said to have negative theta. Generally speaking shorter dated options have higher time decay than longer dated options. The relationship between theta and the underlying FX spot rate is similar to that of vega. All else being equal, the theta of an option is highest when it is at-the-money and it decreases the further FX spot moves away from the strike of the option in either direction. Figure 20 shows the daily theta of a European FX call option versus the FX spot rate.

The relationship between theta and time to maturity is very similar to that of gamma, albeit with the signs reversed. So, for an owner of an option if it is at-the-money, all else being equal, as time to maturity decreases the negative theta becomes increasingly negative until just prior to expiry.

Figure 20: European FX Option Call Theta vs FX Spot



This graph plots the theta of a long European FX option versus the underlying FX spot rate.

It is possible to decompose the value of an option into the sum of its intrinsic value and time value. Intrinsic value is the degree to which the option is in-the-money which can be viewed as the financial gain if it was able to be exercised now. For out-of-the-money or at-the-money options this is zero and for in-the-money options this is positive. The time value of an option exists because there is the probability that the underlying FX spot rate can move to make the option more in-the-money prior to it's expiration. For an out-of-the money option its value consists solely of this time value component. On the date of maturity an option will have a value equal to its intrinsic value since there is zero time left meaning time value is zero. Theta effectively captures the decay of this time value over the life of the option.

Rho measures the sensitivity of the option price to changes in either the domestic or foreign interest rate and is generally expressed per basis point. So in the case of FX options there are two measures of rho since the underlying asset is a currency pair. The domestic rho of a call option (ρ_c^d) is defined as :

$$\rho_c^d = \frac{dc}{dr_d} = Texp^{-r_d T} KN(d_2) \tag{68}$$

and the domestic rho of a put option (ρ_p^d) is defined as :

$$\rho_p^d = \frac{dp}{dr_d} = -Texp^{-r_d T} KN(-d_2) \tag{69}$$

Conversely, the foreign rho of a call option (ρ_c^f) is defined as :

$$\rho_c^f = \frac{dc}{dr_f} = -Texp^{-r_f T} SN(d_1) \tag{70}$$

and the foreign rho of a put option (ρ_p^f) is defined as :

$$\rho_p^f = \frac{dp}{dr_f} = Texp^{-r_f T} SN(-d_1) \tag{71}$$

16.3 Implied Volatility

In FX options markets implied volatilities are quoted for specific deltas and option maturities. These implied volatility values are what interbank FX option traders are responsible for making markets in and trading and essentially represents the markets best guess about future underlying volatility.

These are the implied volatility values that the Black-Scholes option pricing formula requires, for an option with strike K and for maturity T. This is markedly different to equity markets where implied volatility is backed out to match observed option prices for specific strikes and option maturities.

The market implied volatility for a given option contract is determined, like any freely traded market, by the simultaneous buying and selling between market makers. But how do individual FX option traders form their own expectations about what the level of implied volatility should be ?

In forming their view on implied volatility, individual FX option traders incorporate the following (James et al. 2012):

- expectations of future volatility, time averaged over the term of the respective option
- expectations about the underlying asset price versus implied volatility dynamic
- expected term structure of volatility
- the pricing of higher order option 'greeks' and implied forward volatilities
- historical volatility
- FX option supply and demand considerations
- past experience and gut feel.

One of the inputs that FX option traders consider when forming their expectations of future volatility is the time series of realised historical volatility. In this sense historical volatility is referred to as backward looking where as implied volatility is referred to as forward looking. How these two time series compare has been the subject of much work over the years. This is discussed further in Section 18.

The notion of volatility being an asset class has grown in popularity over recent years, although not without some debate (Svirschi 2012). As a result there are now a range of volatility products that can be traded in the FX market. These include volatility swaps, variance swaps, and forward volatility agreements.

One of the advantages of quoting OTC FX options in terms of deltas and implied volatilities is that traders do not have to change their quote every time the spot exchange rate moves, unless of course their view on the spot exchange versus implied volatility dynamic and/or expected future volatility requires them to. At the point at which an option transaction is conducted the Black-Scholes formula provides a one-to-one non linear mapping between the volatility-delta space in which the quotes are made, and the strike price-option price space in which the OTC contract is actually confirmed.

16.4 FX Option Quoting Conventions

In the interbank FX option market, the standard maturities for which implied volatilities are quoted are overnight (1 business day options), 1 week, 1 month, 2 month, 3 month, 6 month, 1 year, and 2 year. Quotes for longer maturities are available on a request basis and vary slightly by currency. Note that on a Friday an overnight option expires on Monday, so it is 3 calender days long. Typically the implied volatility for these options is adjusted down to reflect this, and ensure that the option price is consistent with 1 business day of optionality (Clark 2011). Implied volatilities for specific option contracts that fall between these standard dates are available on request, and of course clients of FX option market makers are generally able to obtain implied volatilties for option expiry dates of their choosing.

As discussed, a European FX option gives the holder the right to buy or sell a particular currency against another at a specified strike price on a certain day. The FX market is a global market and has official opening hours from 5:00 am Sydney, Australia time on a Monday morning and closes at 5:00 pm New York, USA time on a Friday evening. As a result there are several times throughout the day which FX option contracts actually expire and can potentially be exercised. The 2 most common expiry times for options are referred to as Tokyo cut and New York cut. Tokyo cut options expire at 3:00 pm Tokyo time, and New York cut options expire at 10:00 am New York time. The New York cut expiry time is increasingly becoming the most common option expiry time.

When FX options, or combinations of FX options, are traded in the interbank FX options market they are done so on what is referred to as a delta neutral basis. This means that a delta hedge is exchanged between the two option trading counterparties to offset the delta of the option(s) being traded at the time the trade is executed. So for example if Counterparty A was buying a 50 delta NZD call / USD put from Counterparty B, then Counterparty A would simultaneously sell NZDUSD spot to Counterparty B for an amount approximately equal to 50% of the notional amount of the option trade. When added together the delta of the FX option and the FX spot transaction equal zero for both counterparties, at the time of the trade. This further supports the notion of trading volatility as an asset class as one of the key 'greeks', delta, is hedged at the point of trade, and has been proposed as one of the reasons that FX options OTC market has such deep liquidity (Svirschi 2012).

In Section 16.1 the standard Black-Scholes option pricing formula for European vanilla FX options was presented. In order to proceed the Black-Scholes formulas for European call and put options, Equations (57) and (58), can be rewritten as :

$$O = \phi e^{-r_d T}(FN(\phi d_1) - KN(\phi d_2)) \tag{72}$$

where,

O = the black-scholes option premium

ϕ = +1 for calls and -1 for puts

F = the forward FX rate where :

$$F = Se^{(r_d - f_f)T} \tag{73}$$

and d_1 can now be rewritten as :

$$d_1 = \frac{\ln(F/K) + (v^2/2)T}{v\sqrt{T}} \tag{74}$$

The calculated option premiums are in domestic currency per unit of foreign currency (df) which I denote as O^{df}. It is also possible to quote the prices of FX options in a number of other formats. They can be quoted in foreign currency per unit of domestic currency (fd), or they can be quoted in percentage of notional in either foreign ($\%f$) or domestic ($\%d$) terms. Once a notional amount of the FX option is specified, the option premiums can also be expressed in absolute terms, in either foreign (f) or domestic (d) currency.

So in the case of a European FX option the possible FX option price quote methods are :

$$\begin{aligned}
O^{df} &= \phi e^{-r_d T}(FN(\phi d_1) - KN(\phi d_2)) \\
O^{\%f} &= \frac{O^{df}}{S_0} \\
O^{\%d} &= \frac{O^{df}}{K} \\
O^{fd} &= \frac{O^{df}}{S_0 K} \\
O^{d} &= N_F O^{df} \\
O^{f} &= \frac{N_F O^{df}}{S_0}
\end{aligned} \tag{75}$$

where N_f is the notional amount of the option in foreign currency.

In addition, FX option premiums can be quoted in FX pips. An FX pip, or 'price interest point', is a measure of the smallest amount of possible change in the exchange rate for a specific currency pair. The monetary value of an FX pip depends on the currency pair being traded, the exchange rate, and the size of the trade. For example in AUDUSD a move from .8500 to .8501 is an increase of 1 pip, in EURUSD a move from 1.1155 to 1.1154 is a decrease of 1 pip, and in USDJPY a move from 110.20 to 110.21 is an increase of 1 pip. So what is a pip worth ? In our examples for 1 million AUD the move in the AUDUSD FX rate from .8500 to .8501 is worth 100 USD. Likewise, for 1 million EUR the move from 1.1155 to 1.1154 is worth 100 USD. However, in the case of USDJPY for 1 million USD the move from 110.20 to 110.21 is worth 10,000 JPY. These pip values can be converted to the alternate currency by using the new exchange rate. In the case of the FX option price quote methods above O^{df} and O^{fd} are equivalent to quoting in domestic pips (dp) and foreign pips (fp). So :

$$\begin{aligned} O^{dp} &\equiv O^{df} \\ O^{fp} &\equiv O^{fd} \end{aligned} \tag{76}$$

Reiswich and Wystup (2010) provide a good summary of the different premium quotations while Clark (2011) provides a good technical discussion on the calculation of the different possible option premiums. Section 17.3 has a worked example showing the different quote methods.

In Section 16.2 it was shown that delta measures the sensitivity of the option price to changes in the price of the underlying asset. Given that there are several ways in which option premiums can be calculated there are also several ways in which option deltas can be calculated. In addition, when dealing with FX options it is possible to make the distinction between a spot delta and a forward delta. A spot delta involves doing a delta hedge in the FX spot market, whereas a forward delta involves doing a delta hedge in the FX forward market to match the maturity of the FX option in question. The standard delta is a quantity in percent of the foreign currency. However, if the option premium is denominated in the foreign currency then the standard delta hedge would need to be adjusted by this amount to remain delta neutral at the point of trade of the option.

Define the spot pips delta (Δ_S^p) as the ratio of the change in the value of the option to the change in spot :

$$\begin{aligned} \Delta_S^p &= \frac{dO^{dp}}{dS_0} \\ &= \phi e^{-r_f T} N(\phi d_1) \end{aligned} \tag{77}$$

So this is a % of foreign currency and is the amount of foreign currency I need to buy or sell to hedge an option per unit of foreign notional and per K units of domestic notional.

Define the premium adjusted spot pips delta (Δ_S^{pa}) as the ratio of the change in the value of the option, in % foreign terms, to the change in spot, in % foreign terms :

$$
\begin{aligned}
\Delta_S^{pa} &= \frac{d(O^{dp}/S_0)}{d(lnS_0)} \\
&= \Delta_S^p - O^{\%f} \\
&= \phi e^{-r_d T}\frac{K}{S_o}N(\phi d_2)
\end{aligned} \tag{78}
$$

Section 17.3 has a worked example showing these different delta calculations. See Clark (2011) for the derivation of these and for details of possible forward deltas.

In the interbank FX options market there are a set of delta conventions for G10 currencies. These are summarized in Table 31.

Table 31: G10 FX Option delta Conventions

For G10 currency pairs the standard interbank premium currency and delta convention for European vanilla FX options are shown.

Currency Pair	Premium Currency	Delta Convention
AUDUSD	USD	Pips
EURUSD	USD	Pips
GBPUSD	USD	Pips
NZDUSD	USD	Pips
USDCAD	USD	Pips adjusted
USDCHF	USD	Pips adjusted
USDJPY	USD	Pips adjusted
USDNOK	USD	Pips adjusted
USDSEK	USD	Pips adjusted

Looking at non USD currency pairs, as a rule of thumb there is a hierarchy of which currencies dominate being the premium currency :

$$USD > EUR > GBP > AUD > NZD > CAD > CHF > \{NOK, SEK\} >$$

JPY

Clients of FX option liquidity providers are generally able to request option quotes in the currency of their choice with the corresponding deltas.

For each of the standard option maturities, within the interbank FX option market it is convention to quote implied volatilities for the following option contracts :

- at-the-money straddle $(v_{ATM,T})$
- 25 delta risk reversal $(v_{25RR,T})$
- 25 delta butterfly $(v_{25BF,T})$
- 10 delta risk reversal $(v_{10RR,T})$
- 10 delta butterfly $(v_{10BF,T})$

This results in a 5 point smile that can then be interpolated using one of a number of available methods to construct a smooth and continuous implied volatility smile for each option maturity.

Firstly, lets consider the at-the-money straddle volatility $(v_{ATM,T})$. Not surprisingly, defining an at-the-money option is not straight forward. At-the-money could refer to at-the-money spot, at-the-money forward, or at-the-money delta neutral straddle such that for a given strike the call option delta equals the negative put option delta. Historically for G10 FX options with maturities of 1 year and under the default at-the-money convention has been that of an at-the-money straddle, the strike of which ensures that there is no spot FX exposure. So for maturities 1 year and under, in the FX market if you are quoted an at-the-money volatility you are in fact quoted an at-the-money delta neutral straddle. If you chose to buy this at-the-money delta neutral straddle you would be buying equal notional amounts of both a call option and a put option, with the same strike, and the combined spot delta exposure of these two options would be zero. The at-the-money strike for the delta neutral straddle for pips delta (K^{p}_{DNS}) is :

$$K^{p}_{DNS} = F^{T}_{0} exp\left(\frac{1}{2}vATM, T^{2}T\right) \tag{79}$$

and the at-the-money strike for the delta neutral straddle for pips adjusted delta (K^{pa}_{DNS}) is :

$$K^{pa}_{DNS} = F^{T}_{0} exp\left(-\frac{1}{2}v^{2}_{ATM,T}T\right) \tag{80}$$

The at-the-money straddle volatility, ($v_{ATM,T}$), for a given option maturity, T, can be defined as :

$$v_{ATM,T} = \frac{v_{50c,T} + v_{50p,T}}{2} \tag{81}$$

where $v_{50c,T}$ and $v_{50p,T}$ are the implied volatilities for 50 delta calls and puts respectively, for maturity T. Put-Call parity ensures that $v_{50c,T} = v_{50p,T}$, that is the implied volatility used to price a call option and a put option with the same strike, and all other details being the same, must be equal.

Put-call parity is a simple distribution free arbitrage relationship that exists between the price of a European call option and the price of a European put option on the same underlying asset, with the same time to maturity, and the same strike price. Put simply a portfolio of a long call option and a short put option is equivalent to (has the same value) as a forward contract at the same strike price and maturity. That is :

$$c - p = (F - K)e^{r_d T} \tag{82}$$

For longer dated option maturities and for most emerging market currencies, it is customary to use forward deltas as opposed to spot deltas when an option is transacted. By exchanging a forward delta hedge at the point of transacting an FX option this means that delta hedge is valued at the maturity date of the option and settles the same day as the spot transaction that results if the option is exercised, generally two business days after the maturity date. From a risk management perspective this has the additional benefit of ensuring that your rho risk is also hedged at the transaction date of the option. Conversely when you execute a spot delta hedge this means that your rho risk with the associated forward delta is not hedged at the transaction date of the FX option. The reason forward deltas tend to be used for longer dated options and emerging market currencies is due primarily to the increased interest rate risk and generally larger differences in interest rates. It is also worth noting that since the global financial crisis in 2008 that the use of forward deltas has at times become more prevalent even for short dated G10 options. This is due to interbank counterparties having difficulty in the aftermath of the financial crisis agreeing what the respective interest rates are, and the resulting discount factor, in order to discount the forward delta back to a spot delta.

The next type of option for which implied volatilities are quoted for in the interbank FX option market is risk reversals. In the FX option market it is convention to quote 25 delta and 10 delta risk reversals for each maturity. In the case of the 25 delta risk reversal, this would mean buying (selling) a 25 delta call option and selling (buying) a 25 delta put option for a given option maturity. Assuming the options in question were written on a G10 currency pair and with option maturity 1 year or less then the strikes corresponding to the 25 delta call and put options would be based on spot deltas. Market convention is to quote the risk reversal as the difference between the implied volatility of the call option and the put option.

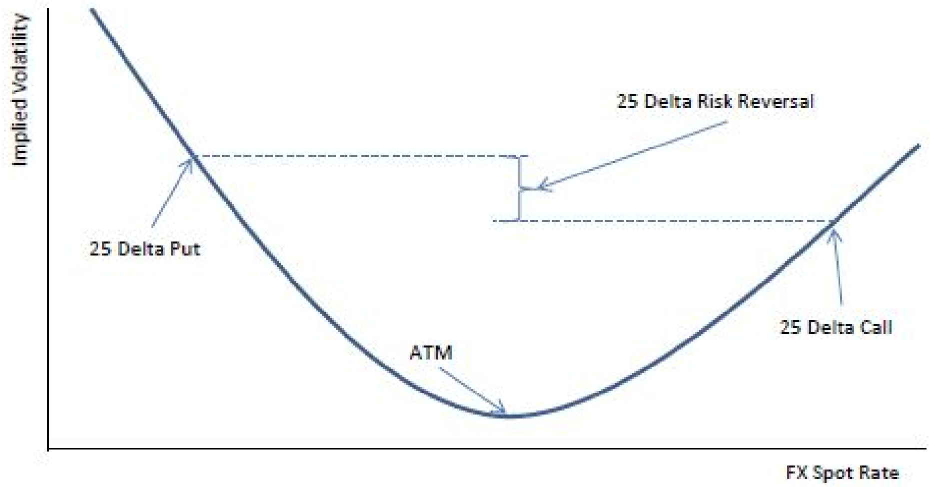
Using the same notation conventions, the 25 delta risk reversal ($v_{25RR,T}$) and 10 delta risk reversal ($v_{10RR,T}$) can be defined as :

$$v_{25RR,T} = v_{25c,T} - v_{25p,T} \tag{83}$$

and :

$$v_{10RR,T} = v_{10c,T} - v_{10p,T} \tag{84}$$

Figure 21: 25 Delta Risk Reversal



This graph plots FX option implied volatility versus the underlying FX spot rate for a given option maturity, commonly referred to as a volatility smile. The implied volatilities for the 25 delta call and 25 delta put that combine to form the 25 delta risk reversal are highlighted.

Risk reversals can be interpreted as measuring the skewness of the implied volatility smile. Figure 21 illustrates a 25 delta risk reversal on a typical volatility smile.

The final type of option for which implied volatilities are quoted for in the interbank FX option market is a butterfly. Once again it is standard practice to quote implied volatilities for 25 delta and 10 delta butterfly's. In the case of the 25 delta butterfly this would involve buying (selling) a 25 delta call option and a 25 delta put option (otherwise known as a 25 delta strangle) and selling (buying) an at-the-money straddle. Again, assuming the options in question were written on a G10 currency pair and with option maturity 1 year or less then the strikes corresponding to the 25 delta call and put options would be based on spot deltas and would offset each other, and as discussed the at-the-money straddle, by construction, has no spot delta. Once again, market convention is to quote the butterfly as the difference between the implied volatility of the strangle and the at-the-money straddle. So a butterfly quote represents the spread above the at-the-money volatility that a strangle would trade. This implicitly means that the call and put that makes up the strangle each have the same implied volatility.

The 25 delta butterfly ($v_{25BF,T}$) and the 10 delta butterfly ($v_{10BF,T}$) can be defined as:

$$v_{25BF,T} = v_{25S,T} - v_{ATM,T} \tag{85}$$

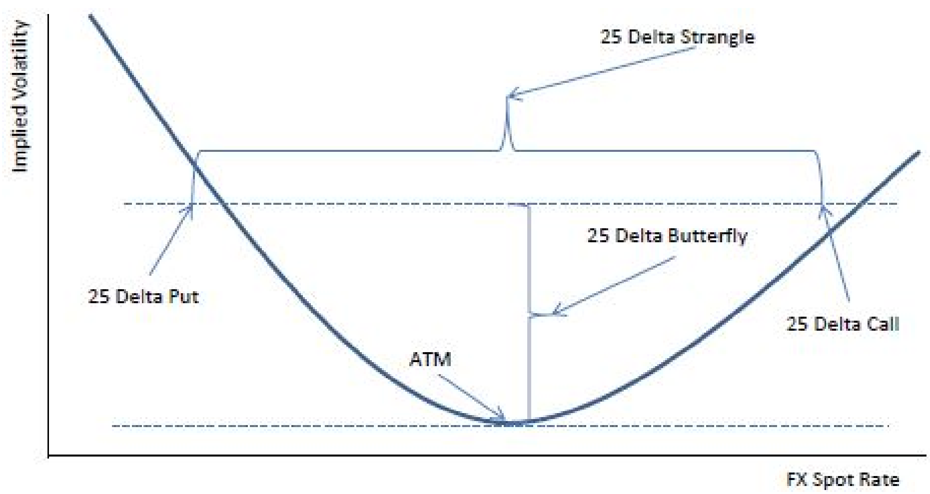
and :

$$v_{10BF,T} = v_{10S,T} - v_{ATM,T} \tag{86}$$

where $v_{25S,T}$ and $v_{10S,T}$ represent the 25 and 10 delta strangle volatilities. Figure 22 shows a market quoted 25 delta butterfly and how it relates to the volatility smile.

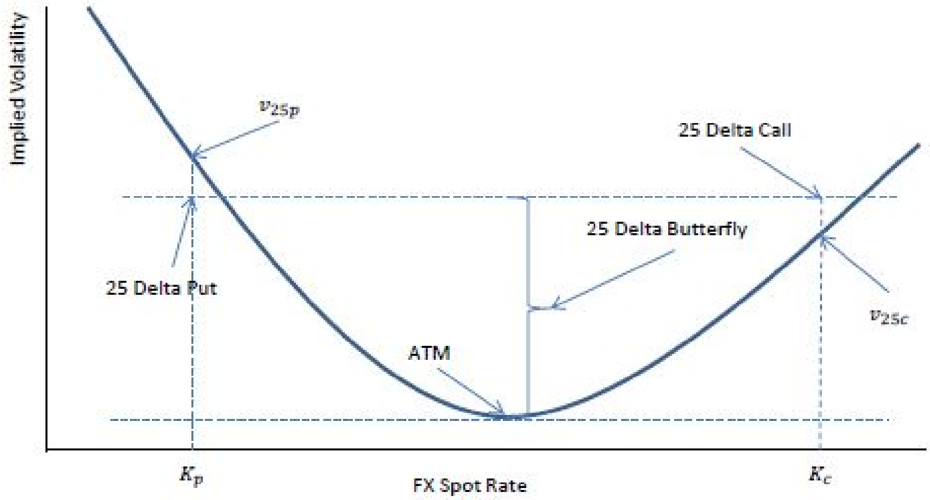
Taking the simplified approach presented in Malz (1997), if $(v_{25c,T} + v_{25p,T})/2$ and $(v_{10c,T} + v_{10p,T})/2$ represent the 25 delta strangle ($v_{25S,T}$) and 10 delta strangle ($v_{25S,T}$) implied volatilties respectively then :

Figure 22: 25 Delta Market Quoted Butterfly



This graph plots FX option implied volatility versus the underlying FX spot rate for a given option maturity, commonly referred to as a volatility smile. The implied volatilities for the 25 delta call and 25 delta put that combine to form the 25 delta strangle are highlighted.

Figure 23: 25 Delta Butterfly - Simple Approach



This graph plots FX option implied volatility versus the underlying FX spot rate for a given option maturity, commonly referred to as a volatility smile. The implied volatilities for the 25 delta call and 25 delta put derived using the simple approach from the 25 delta risk reversal and 25 delta butterfly are highlighted.

$$v_{25BF,T} = \frac{(v_{25c,T} + v_{25p,T})}{2} - v_{ATM,T} \tag{87}$$

and :

$$v_{10BF,T} = \frac{(v_{10c,T} + v_{10p,T})}{2} - v_{ATM,T} \tag{88}$$

Figure 23 illustrates this.

With this simple approach it is now possible to solve for the 25 delta call ($v_{25c,T}$) and 25 delta put ($v_{25p,T}$) implied volatilites using Equations (83) and (87), and likewise for the 10 delta call ($v_{10c,T}$) and 10 delta put ($v_{10p,T}$)

implied volatilites using Equations (84) and (88) :

$$v_{25c,T} = v_{ATM,T} + v_{25BF,T} + \frac{v_{25RR,T}}{2} \quad (89)$$

$$v_{25p,T} = v_{ATM,T} + v_{25BF,T} - \frac{v_{25RR,T}}{2} \quad (90)$$

$$v_{10c,T} = v_{ATM,T} + v_{10BF,T} + \frac{v_{10RR,T}}{2} \quad (91)$$

$$v_{10p,T} = v_{ATM,T} + v_{10BF,T} - \frac{v_{10RR,T}}{2} \quad (92)$$

Combined with the at-the-money straddle volatility ($v_{ATM,T}$) this yields the 5 points which define the implied volatility smile for a given option maturity.

However, these resulting implied volatilities are approximate due to the difference between the 'market' butterfly that is described above and the 'smile' butterfly that results once the inputs to the 5 point smile are fitted. As explained the market butterfly quote represents a spread to the at-the-money volatility for both the call and put implied volatility for a given delta. So in the case of a 25 delta butterfly in order to calculate the 25 delta strangle strikes you would add the 25 delta butterfly spread to the at-the-money volatility and using this same resulting volatility then solve for the corresponding call and put strikes for the strangle. In Figure 23 these are shown as K_c and K_p respectively. These strikes will each be 25 delta based on this single volatility. However, this single volatility ignores the risk reversal, or skew, of the implied volatility smile. In order to calculate a market consistent implied volatility surface the market butterfly quotes require that the sum of the option valuations for the call and put based on this single volatility (defined as a spread to the at-the-money) must equal the sum of the option values for the same call and put options but priced using implied volatilities from the implied volatility surface, i.e. incorporating the risk reversal. These implied volatilities are shown as v_{25c} and v_{25p} in Figure 23. However once these 'correct' volatilities from the volatility smile are used to value this market strangle then this will change the deltas and provided the risk reversal is not 0 the call and put will no longer each be 25 delta. This is why this simplified approach produces approximate volatilities.

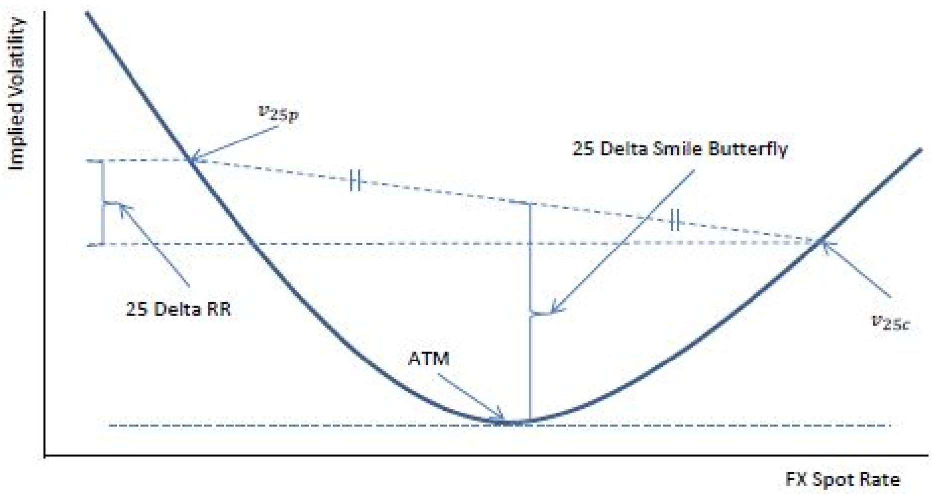
The smile butterfly on the other hand is the difference between the average of the actual 25 delta call and 25 delta put implied volatilities from the implied volatility smile and the at-the-money volatility as shown in Figure 24. If this smile butterfly is used in Equations (89) to (92) instead of the market butterfly then a market consistent implied volatility smile is ensured. If the risk reversal in question is 0 then this is not an issue. The difference between the two butterfly measures is exacerbated the larger the risk reversal is.

This is a some what technical issue that is often misunderstood. Malz (1997) simply refers to the strangle. What he is actually referring to is the smile strangle but he goes onto describe what option traders quote, that is the market strangle. Clark (2011) discusses this confusion in more detail. Where this issue originates from seems largely to simply be an historical convention. When the FX options market was in its infancy many years ago the smile tended to be more symmetric and as such it was convenient to quote a strangle as a volatility spread to at-the-money options. This is the market strangle quote style that is seen today. However over time the risk reversals for some currencies became significant and the resulting implied FX option volatility smile became more asymmetric which highlighted the issue discussed above. Quoting the market strangle certainly has advantages from an ease of trade and price comparison perspective but from an option traders perspective it is imperative that their respective pricing and risk management platform adequately distinguishes between the two versions of strangle quotes to ensure they have consistency in their pricing. For the purposes of this paper the simple approach above using the market quoted butterfly will suffice. For a detailed description of this issue see Reiswich and Wystup (2009), Reiswich and Wystup (2010), and Clark (2011).

Convention in the FX option market is to trade a butterfly on a vega neutral basis. This means that the total vega of buying (selling) the strangle (low delta put and low delta call) is equal to the total vega of selling (buying) the at-the-money straddle. Because the vega of a low delta option is less than the vega of an at-the-money option, in order for the butterfly to be traded on a vega neutral basis the notional amount of the low delta call and low delta put (strangle) must be greater than the notional amount of the at-the-money call and put (Castagna and Mercurio 2006).

To allow the reader to gain a better understanding of these quoting con-

Figure 24: 25 Delta Smile Butterfly



This graph plots FX option implied volatility versus the underlying FX spot rate for a given option maturity, commonly referred to as a volatility smile. The implied volatilities for the 25 delta call and 25 delta put derived using the smile approach from the 25 delta risk reversal and 25 delta butterfly are highlighted.

ventions and option combinations, a worked example of the common combinations of options that are traded in the interbank FX options market is presented in Section 17.3.

16.5 Implied Volatility Surface Calibration Techniques

Once the five standard FX option interbank quoted implied volatility inputs are known for each standard maturity it is then possible to construct a complete implied volatility surface in delta / time to maturity space. The first challenge to do this is for a given maturity a market consistent smile function must be fitted. There is no unique solution to this problem and it turns out that FX market participants use a variety of methods.

It is possible to use a simplified parabolic interpolation where the implied volatility smile for a given maturity is constructed as a second order parabola in delta space that matches the at-the-money and risk reversal implied volatility inputs. In addition one can specify a convexity control parameter that allows the volatility smile to match the market butterfly quote (Reiswich and Wystup 2012). Another approach is the SABR model introduced in Hagan et al. (2002) in which closed form approximations for implied volatilities are derived under a particular type of stochastic volatility model. Another possible approach is a vanna-volga interpolation. In this context vanna is the derivative of the vega with respect to spot and volga is the derivative of the vega with respect to volatility. Castagna (2010) and Castagna and Mercurio (2006) show that owing to the ability of being able to hedge these second order effects by three options it is possible to specify the full smile function from three implied volatility inputs. Reiswich (2011) provides a summary of these approaches and examines their empirical pros and cons.

The next challenge in constructing a complete implied volatility surface is, for a given delta, to interpolate through time to maturity and fill in the gaps between the standard maturity nodes that are quoted. Although it is convention in the interbank FX options market to quote and trade the standard option maturities, in practice FX option liquidity providers are also required to provide implied volatility quotes for specific option maturities that fall between these standard tenors.

In addition, once an FX option counterparty has traded FX options they are generally required to value them periodically, in some cases daily, on a marked to market basis for accounting purposes. In order to do this they will require implied volatility points from the implied volatility surface that fall between standard quoted maturities as time passes and the time to maturity of their options changes accordingly. Once again this turns out to be non trivial. Liner interpolation of implied volatility is never used due to producing non sensible forward implied volatilities. Likewise linear interpolation of variance is also prone to producing non sensible forward volatilities. What tends to be used in practice is a variation of linear interpolation in integrated variance. This has the effect of interpolating on flat forward volatility. Another issue to consider, particularly for short dated options, is the effect of holidays and weekends. For example if you trade an overnight option on a

Friday, which expires on Monday, it has three calender days until maturity. However it really only has one business day of volatility and so the implied volatility quote will be adjusted down to reflect this by essentially zeroing out the forward volatilities for the non business days, when compared to an overnight option that has one calender day until maturity (Clark 2011). Likewise this concept can also be extended to intra day effects where certain times of the 24 hour day are deemed to be worth more in terms of volatility than others.

The above summary of methods that can be used to construct an implied volatility surface has been housed in the context of European FX options. Interbank FX option trading establishments of course have not only European FX options on their books but also a large range of exotic options. As such they tend to have complex risk management systems which have many additional layers of complexity to constructing implied volatility surfaces than presented above. For instance it is common practice to use a blended stochastic / local volatility model whereby the calibration process is performed on the European vanilla implied volatility inputs discussed and a range of observed exotic option market prices. Understandably these calibration processes are complex and computationally burdensome (Clark 2011). The remainder of our work focuses on the inputs to the implied volatility surface of European vanilla FX options so these more complex models are not relevant to us.

17 Data

The option data set was kindly provided by Deutsche Bank. It consists of daily data for the five standard FX option interbank quoted inputs :

- at-the-money straddle ($v_{ATM,T}$)
- 25 delta risk reversal ($v_{25RR,T}$)
- 25 delta butterfly ($v_{25BF,T}$)
- 10 delta risk reversal ($v_{10RR,T}$)
- 10 delta butterfly ($v_{10BF,T}$)

for 1 week, 1 month, 3 month, 6 month, and 1 year maturities. It covers the G10 currency set, quoted against the United States Dollar. The series within the data set have slightly varying start dates ranging from October 2003 to January 2006 which is the common start date across all currencies and implied volatility inputs. The data goes through to December 2012.

17.1 Implied Volatility Surface

For a given currency pair, using the five standard FX option interbank quoted volatility inputs for each option maturity it is now possible to construct an implied volatility surface.

Table 32 shows the volatility surface inputs for AUDUSD on 2 June 2008.

Table 32: AUDUSD Implied Volatility Inputs 2 June 2008

For AUDUSD as at 2 June 2008 for option maturities of 1 week, 1 month, 3 month, 6 month, and 12 month, the five standard interbank quoted implied volatility nodes of at-the-money, 25 delta risk reversal, 25 delta butterfly, 10 delta risk reversal, and 10 delta butterfly are shown. Risk reversal numbers are quoted as AUD call / USD put minus AUD put / USD call. Butterfly numbers are quoted as the spread which the strangle options trade above the at-the-money option.

	ATM	25RR	25BF	10RR	10BF
1 week	10.00	-0.70	0.10	-0.60	0.50
1 month	10.47	-0.73	0.25	-1.00	0.85
3 month	10.89	-1.05	0.35	-1.45	1.23
6 month	11.45	-1.10	0.40	-1.85	1.48
12 month	11.63	-1.38	0.45	-2.45	1.73

Using Equations (89) to (92) it is possible to calculate the implied volatilities for 25 and 10 delta calls and puts for each maturity. The implied volatilities are shown in Table 33.

If these values are plotted in implied volatility, option maturity, and option delta space this shows the implied option volatility surface. Figure 25 shows a 3 dimensional image of the implied volatility surface for AUDUSD on 2 June 2008, with linear joins between the data input nodes. Despite the convention in the interbank FX option market of quoting implied volatilities for the five options outlined in Section 16.4, it is possible, and indeed clients demand it, to trade FX options with maturities anywhere on the implied volatility surface and with deltas in the range of approximately 0 to 100. In order to facilitate this, interbank FX option traders use varying interpolation and extrapolation methods to construct a 'smooth' implied volatility surface

Table 33: AUDUSD Implied Volatility Surface 2 June 2008

For AUDUSD as at 2 June 2008 using the standard FX option interbank quoted nodes from Table 32 the implied volatilities for 25 delta and 10 delta AUD calls and AUD puts are shown. These are calculated using the simple approach of :

$$\begin{aligned}
v_{25c,T} &= v_{ATM,T} + v_{25BF,T} + \frac{v_{25RR,T}}{2} \\
v_{25p,T} &= v_{ATM,T} + v_{25BF,T} - \frac{v_{25RR,T}}{2} \\
v_{10c,T} &= v_{ATM,T} + v_{10BF,T} + \frac{v_{10RR,T}}{2} \\
v_{10p,T} &= v_{ATM,T} + v_{10BF,T} - \frac{v_{10RR,T}}{2}
\end{aligned}$$

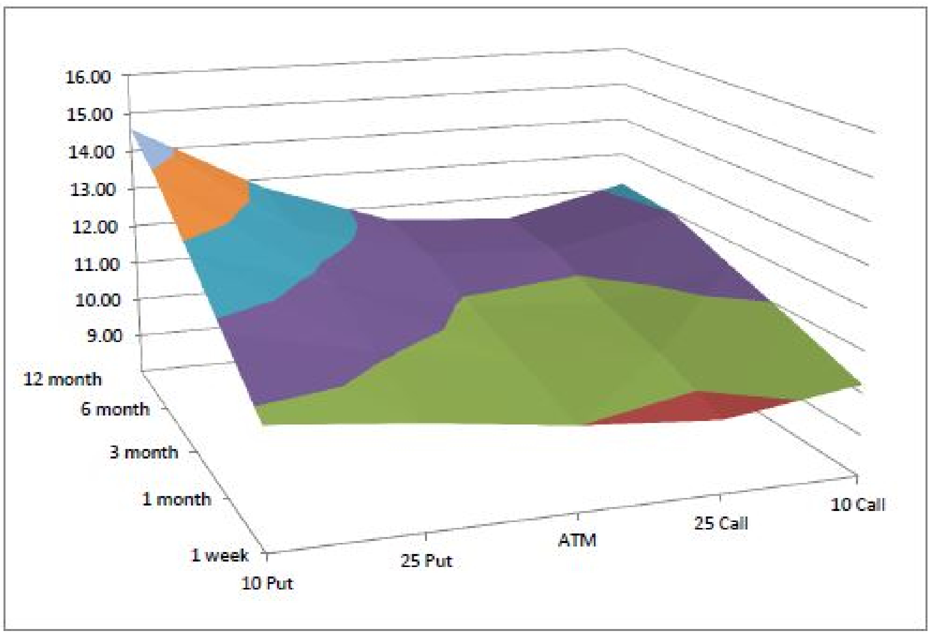
	10 Put	25 Put	ATM	25 Call	10 Call
1 week	10.80	10.45	10.00	9.75	10.20
1 month	11.82	11.08	10.47	10.35	10.82
3 month	12.84	11.77	10.89	10.72	11.39
6 month	13.85	12.40	11.45	11.30	12.00
12 month	14.58	12.78	11.63	11.39	12.13

off which they are happy to trade at any point on, as discussed in Section 16.5.

In Figure 25 there are two notable features of this volatility surface that are common throughout the G10 currency set. Firstly, for a given maturity the volatility surface exhibits a smile across different deltas, and secondly for a given delta the volatility surface has term structure, or slope, across different maturities.

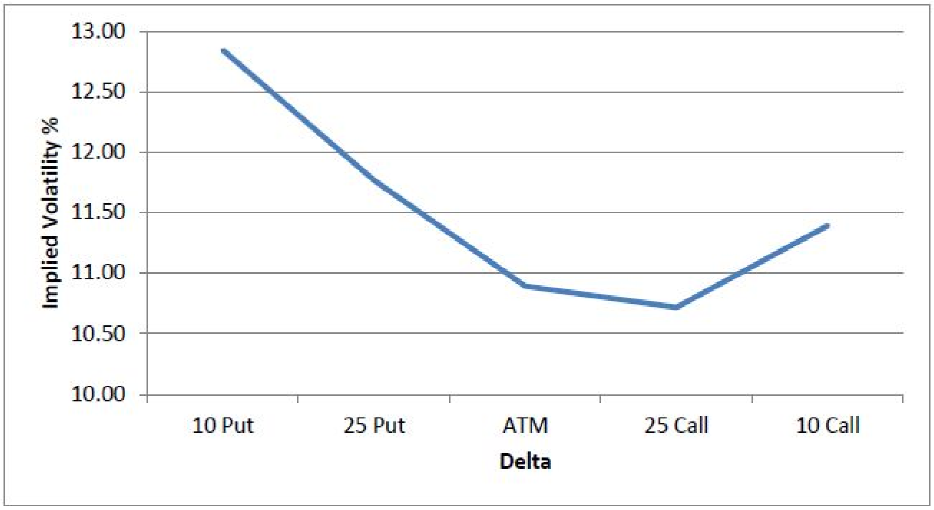
Figure 26 shows the implied volatility smile for the 3 month tenor for AUDUSD as at 2 June 2008. It is in effect the view obtained by 'slicing' the 3-dimensional implied volatility surface in Figure 25 at the 3 month maturity node. In this example the 10 delta AUD puts (USD calls) trade

Figure 25: AUDUSD Implied Volatility Surface 2 June 2008



This graph plots FX option implied volatility surface for AUDUSD as at 2 June 2008. The horizontal axis plot standard option maturities versus option delta and the vertical axis plots implied volatility .

Figure 26: AUDUSD 3 Month Implied Volatility Smile 2 June 2008



This graph plots the FX option implied volatilities for 3 month AUDUSD FX options as at 2 June 2008. The x axis plots option deltas and the y axis plots FX option implied volatilities.

at a higher volatility than the 10 delta AUD calls (USD puts). The lowest implied volatility input in this example is the 25 delta AUD calls (USD puts), around which the surface 'smiles'. Equations (83) and (84) define the 25 delta and 10 delta risk reversals respectively. As inputs into the implied volatility surface, these risk reversals effectively define the 'slope' of the volatility smile. Equations (87) and (88) define the 25 delta and 10 delta butterflies. As inputs into the implied volatility surface, these butterflies effectively define the amount of curvature in the volatility smile.

Figure 27 shows the implied volatility term structure for AUDUSD at-the-money options as at 2 June 2008. In this example the implied volatility term structure for at-the-money options is clearly upward sloping.

Figure 27: AUDUSD at-the-money Implied Volatility Term Structure 2 June 2008

This graph plots the FX option implied volatility term structure for at-the-money AUDUSD FX options as at 2 June 2008. The x axis plots standard option maturities and the y axis plots implied volatility .

One interpretation of the implied volatility surface is that it represents the forward looking range of uncertainty of market expectations for a given currency pair. Based on an implied volatility surface it is possible to extract, using one of several available methods, a probability density function that can be interpreted as measuring the entire expected distribution of future exchange rates. Once such a probability density function is derived it is possible to calculate the standard measures that describe its shape - standard deviation, skewness, and kurtosis. In Section 16.4 it was explained that it is convention in the interbank FX option market to quote 3 types of options - at-the-money options, risk reversals, and butterflies. Each of these option types contain information about the shape of a probability density function derived from an implied volatility surface. At-the-money implied volatility contains information about the width of a probability density function. For example the higher the level of implied option volatility the wider the implied probability density function will be. Risk reversals contain information about the skewness of such a probability density function. In the AUDUSD example above, Figure 26 shows that for a specific delta AUD puts / USD calls trade at a higher implied volatility than AUD calls / USD puts. This suggests that the implied probability density function will be negatively skewed. Butterflies contain information about the kurtosis of such a density function. The higher the value of the butterfly, the more kurtotic the implied probability density function will be (Malz 1997).

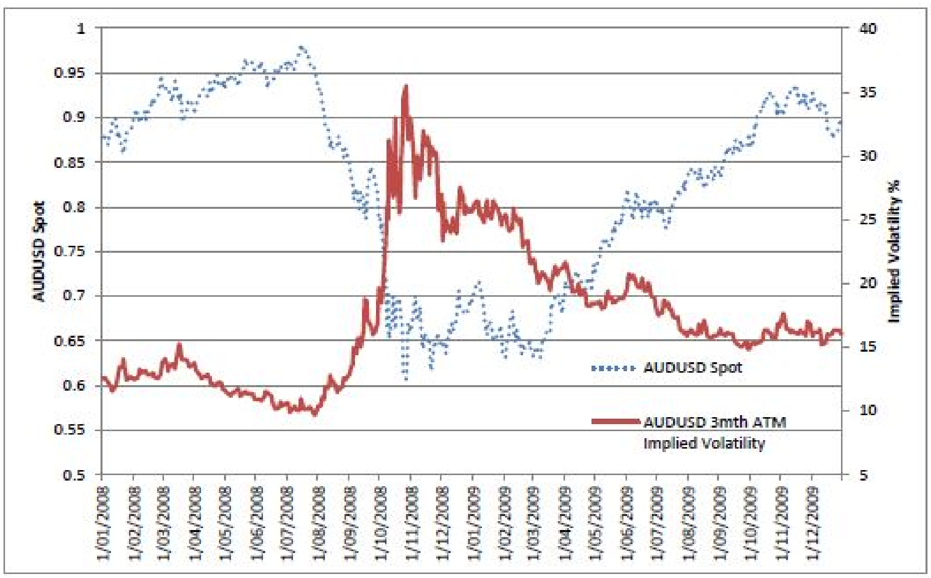
17.2 Implied Volatility Surface Dynamics - Example

To help gain a better understanding of how the implied volatility surface can move when there are large moves in the underlying FX spot rate firstly lets look at the AUDUSD implied volatility surface for the period that encapsulates the global financial crisis.

Figure 26 implies if the AUDUSD spot rate depreciates then the market expects that implied volatility will increase. Figure 28 plots the AUDUSD FX spot rate versus the 3 month AUDUSD at-the-money implied volatility for 2008 and 2009 which incorporates the global financial crisis. Commencing in July 2008 there is a sharp depreciation in the AUDUSD FX spot rate from a high of .9825 on 15 July 2008 down to a low of .6072 on 27 October 2008, a fall of 38%. Correspondingly 3 month at-the-money implied volatility increased from a low of 9.7% on 29 July 2008 to 35.5% on 27 October 2008. In terms of sign this is the FX spot / implied volatility dynamic that is implied by the implied volatility surface.

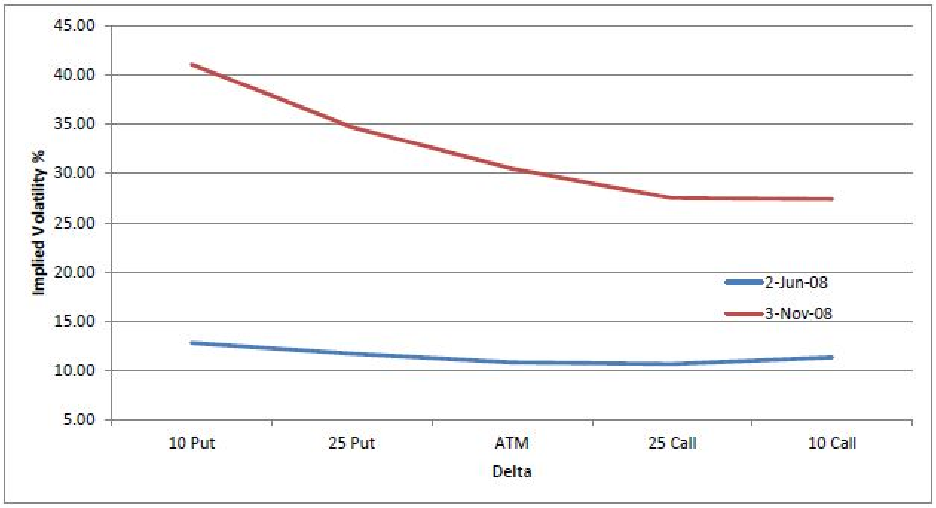
Turning now to how the implied volatility smile behaved during the global financial crisis, Figure 29 plots the AUDUSD 3 month implied volatility smile for 2 different dates, 2 June 2008 and 3 November 2008. For reference the AUDUSD FX spot rates for the two dates were .9549 and .6790 respectively. As discussed above, 3 month at-the-money implied volatility increased markedly throughout this period. This can be seen in Figure 29 with the 3 November 2008 implied volatility smile sitting higher than the 2 June 2008 volatility smile. The second observation is that the implied volatility smile

Figure 28: AUDUSD Spot & 3 Month at-the-money Implied Volatility - GFC



This graph plots the time series of AUDUSD FX spot rate and AUDUSD 3 month at-the-money implied FX option volatility for the period January 2008 to December 2009.

Figure 29: AUDUSD 3 Month Implied Volatility Smile - GFC

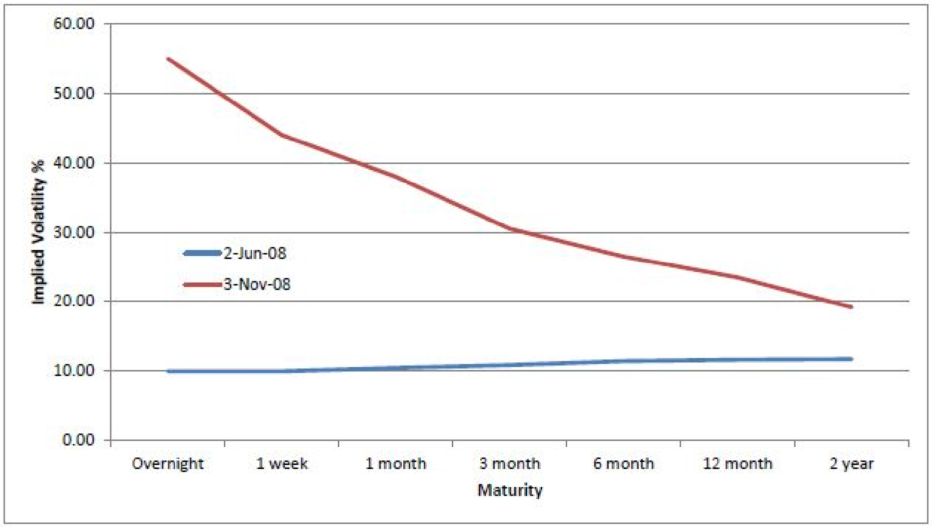


This graph plots the FX option implied volatilities for 3 month AUDUSD FX options as at 2 June 2008 and as at 3 November 2008. The x axis plots option deltas and the y axis plots FX option implied volatilities.

has become noticeably steeper in favour of AUD puts / USD calls. Using the terminology introduced by Equations (83) and (84) the risk reversal has become more negative between these two dates.

Finally, looking at the third dimension of the FX option implied volatility surface the term structure, Figure 30 plots the AUDUSD at-the-money implied volatility term structure for the same two dates, 2 June 2008 and 3 November 2008. On 2 June 2008 the volatility term structure is mildly upward sloping from shorter maturities to longer maturities. However, after the sharp depreciation of the AUDUSD spot rate the term structure on 3 November is now sharply negative sloping from shorter to longer option maturities, and as already discussed the over-all level of the implied volatility surface

Figure 30: AUDUSD at-the-money Implied Volatility Term Structure - GFC



This graph plots the FX option implied volatility term structure for at-the-money AUDUSD FX options as at 2 June 2008 and as at 3 November 2008. The x axis plots standard option maturities and the y axis plots implied volatility .

is higher so it sits above the earlier date. So the increased uncertainty as a result of the sharp depreciation in the AUDUSD FX spot rate has resulted in a sharp increase in shorter dated option implied volatilities relative to longer dated option implied volatilities. This characteristic of shorter dated option maturities having higher 'volatility of volatility' is evident in the data, where the standard deviation of the at-the-money implied volatility series gradually decreases the longer the option maturity. For instance in the case of AUDUSD in the data set 1 week at-the-money implied volatility has a standard deviation of 5.45, whereas 1 year at-the-money implied volatility has a standard deviation of 3.24.

In summary, as a result of the sharp depreciation in the AUDUSD FX spot rate during the global financial crisis the implied volatility surface :

- moved up to have an overall higher level of implied volatility
- for a given option maturity, had increased slope in favour of AUD puts / USD calls, that is the risk reversal became more negative
- the term structure became sharply negative from shorter dated option maturities to longer dated option maturities

17.3 FX Option Pricing - Example

To further explain the common combinations of options that are traded in the interbank FX options market consider the following worked example for 3 month AUDUSD using market data from 2 June 2008 :

- $S_t = 0.95485$
- $r_d = 2.725\%$
- $r_f = 7.73\%$
- $T = 91/365$ years (91 days)

where S_t, r_d, r_f, and T are defined in Equations (57) and (58). Once the strike price of the option (K) and the implied volatility (v) are specified and assuming that r_d and r_f are continuously compounded interest rates, it is then possible to use Equations (57) and (58) to price European call and put options respectively. Any impact settlement date effects may have on the option pricing are ignored.

Table 32 shows that as at 2 June 2008 the 3 month AUDUSD at-the-money straddle implied volatility is 10.89%. The market convention for AUDUSD FX options is to calculate the option premiums in FX pips (Table 31) and for implied volatility quotes to reference spot deltas out to and including 1 year maturities. Consider and example to buy 1 million AUD per leg of the 3 month at-the-money straddle in the interbank FX options market. Using the implied volatility of 10.89% the strike K, for both the AUD call / USD put and the AUD put / USD call, such that the straddle is delta neutral on a FX spot basis, must be determined. Using Equation (79) this strike turns out to be $K = .9444$. So to buy 1 million AUD per leg of the 3 month at-the-money straddle requires transacting the following two options :

- Buy 1 million AUD call / USD put $K = 0.9444$

  Spot Delta $(\Delta_c) = 0.49$

  Premium (c) = 196.5 pips = \$ 19,650 USD

- Buy 1 million AUD put / USD call $K = 0.9444$

  Spot Delta $(\Delta_c) = 0.49$

  Premium (c) = 210.3 pips = \$ 21,030 USD

In order to buy 1 million AUD per leg of the 3 month at-the-money straddle a total premium of $ 40,680 USD is payable and by definition there would be no FX spot delta hedge transacted given that the strikes of the the two options were chosen such that this was 0.

Earlier in Section 16.4 the 6 possible ways in which an FX option premium could be quoted were presented. Using the AUD call / USD put above with strike of 0.9444 as an example the different quotation methods in Equation (75) yield the following results :

- $O^{df} = 0.01965$
- $O^{fd} = 0.02179$
- $O^{\%d} = 2.0804\%$
- $O^{\%f} = 2.0576\%$
- $O^{d} = 19{,}650$ USD
- $O^{f} = 20{,}576$ AUD

Earlier in Section 16.4 the different deltas for an FX option depending on the quotation method used for the premium of the FX option were presented. Once again, using the above example of the AUD call / USD put with strike of 0.9444 it has a spot pips delta of 0.490489 (49.0489%). This means that for face value of 1 million AUD if an interbank option counterparty was to buy this option they would simultaneously sell 490,489 AUD versus 468,343

USD for spot settlement with the seller of the option. However, if the option premium was instead to be paid in the foreign currency, in this case 20,576 AUD, then the spot delta hedge transacted between the two counterparties would be different to reflect this. Using Equation 78 the spot delta hedge would now be 469,913 AUD versus 448,696 USD. Note that in this case the sum of the AUD premium adjusted delta hedge and the AUD premium equals the original spot pips delta hedge.

Table 32 shows that as at 2 June 2008 the 3 month AUDUSD 25 delta butterfly is 0.35%. This means that the 25 delta AUD call / USD put and 25 delta AUD put / USD call, when traded as part of the 25 delta butterfly, will both trade at an implied volatility 0.35% above that of the implied volatility of the at-the-money straddle. So in this case these two 25 delta options will trade at an implied volatility of 11.24%. Using implied volatility of 11.24% the next step is to work out what strikes correspond to a 25 delta AUD call / USD put and 25 delta AUD put / USD call respectively. Given that the market convention for AUDUSD FX options is to calculate the option premiums in FX pips, this requires solving Equation (77) for K (Reiswich and Wystup 2010):

$$K = Fexp(-\phi N^{-1}(\phi exp(r_f T)\Delta_S^p)v\sqrt{T} + 0.5v^2T) \qquad (93)$$

These strikes turn out to be 0.9801 and 0.9102 respectively. When trading butterflies in the interbank FX option market it is standard practice to trade them on a vega neutral basis. In order for this to be the case, a larger amount of the out-of-the-money AUD call / USD put and out-of-the-money AUD put / USD call needs to be traded than the at-the-money straddle because the vega of an out-of-the-money option is less than the vega of an at-the-money option. In this example this ratio of vegas turns out to be 1.24273. So to buy 1 million AUD per leg of the 3 month butterfly requires transacting the following four options :

- Sell 1 million AUD call / USD put $K = 0.9444$

  Spot Delta (Δ_c) $= 0.49$

  Premium (c) $=$ 196.5 pips $=$ \$ 19,650 USD

- Sell 1 million AUD put / USD call $K = 0.9444$

  Spot Delta (Δ_p) $= -0.49$

  Premium (c) $=$ 210.3 pips $=$ \$ 21,030 USD

- Buy 1.24273 million AUD call / USD put $K = 0.9801$

  Spot Delta (Δ_c) $= 0.25$

  Premium (c) $=$ 78.2 pips $=$ \$ 9,718 USD

- Buy 1.24273 million AUD put / USD call $K = 0.9102$

  Spot Delta (Δ_p) $= -0.25$

  Premium (c) $=$ 82.8 pips $=$ \$ 10,289 USD

In this case the net premium of \$ 20,673 USD is received and once again there is no delta hedge exchanged as the sum of the delta's of the four options is by construction zero.

Table 32 shows that as at 2 June 2008 the 3 month AUDUSD 25 delta risk reversal was -1.05%. This means that 25 delta AUD calls / USD puts trade at an implied volatility 1.05 below 25 delta AUD puts / USD calls. Combined with 3 month AUDUSD at-the-money implied volatility of 10.89% and the 3 month AUDUSD 25 delta butterfly of 0.35% it is possible to use Equations (89) and (90) to calculate the implied volatility of the 25 delta AUD call / USD put and 25 delta AUD put / USD call. This yields implied volatilities of 10.72% and 11.77% respectively. Using these implied volatilites the next step is to work out what strikes correspond to a 25 delta AUD call / USD put and 25 delta AUD put / USD call. Using Equation (93), these strikes turn out to be 0.9783 and 0.9087 respectively. In the interbank FX option market it is standard practice to trade options, or combinations of options, on a delta neutral basis. For example to buy the AUD call / USD put and sell the AUD put / USD call then this combination of options has a spot delta of 0.5 by construction. So in order for the overall trade to be delta neutral a spot delta FX hedge of -0.5 of the face value of the risk reversal would be exchanged. So, to buy 1 million AUD per leg of the 3 month 25 delta risk reversal requires transacting the following two options and delta hedge :

- Buy 1 million AUD call / USD put $K = 0.9783$

Spot Delta (Δ_c) = 0.25

Premium (c) = 74.6 pips = \$ 7,460 USD

- Sell 1 million AUD put / USD call $K = 0.9087$

  Spot Delta (Δ_p) = -0.25

  Premium (c) = 86.7 pips = \$ 8,670 USD

- Sell 0.5 million AUDUSD FX spot at .95485

In this case the net premium receivable is \$ 1,210 USD for buying the AUD call / USD put and selling the AUD put / USD call.

18 Volatility and the FX Carry Trade

With a focus on equity markets there is a large amount of work looking at what information may be contained within the implied volatility surfaces for the purposes of predicting future equity returns. One of the earlier papers, Bates (1991), looks at the stock market crash of 1987 and asks the question did option prices contain information to indicate whether a crash was expected ? Looking at the S&P 500, he finds that out-of-the-money puts were unusually expensive relative to out-of-the-money calls in the year preceding the crash but concedes that there was no strong fears of a crash immediately preceding October 1987. Doran et al. (2007) look at options on the S&P 100 and conclude that the shape of the skew (for which they use several measures) can reveal with significant probability when the market will crash or spike. However they find that the magnitude of the spike prediction is not significant. Van Buskirk (2011) looks at individual firm data and looks at whether implied volatility skew predicts extreme negative returns. He concludes that volatility skew predicts crashes, but only those crashes occurring in earnings announcement periods. Xing et al. (2010) use a risk reversal type measure (in their case defined as the difference in implied volatility between an out-of-the-money put and an at-the-money call) at the firm level and find that the shape of this volatility 'smirk' has significant cross-sectional predictive power for future equity returns. Jin et al. (2012) obtain similar results and also look at whether this predictive ability derives from option traders information advantage, and if so when and how they gain such an advantage.

Clarida et al. (2009) examine the link between changes in 1 month at-the-money implied FX volatility and FX carry trade returns and find that increases in implied volatility lead, on average, to lower returns on the carry trade. Likewise they find that declines in implied volatility are associated with an appreciation of the high carry currency against the low carry currency and they conclude that " it highlights the way changes in forward looking volatilities from the option market are contemporaneously associated with returns from the carry trade".

Some recent literature has used FX options to examine the importance of disasters in FX markets. Bhansali (2007) tests a trading strategy of implementing the FX carry trade by buying at-the-money forward call options on high yielding currencies and rolling them (equivalent to buying the underlying spot FX and buying an at-the-money put option as protection). This simple strategy delivers small profits over time. Brunnermeier et al. (2008) use 1 month FX option implied volatility 25 delta risk reversals as a measure of the price of crash risk. They find that after controlling for interest rate differentials the relationship between risk reversals and future skewness is negative, i.e. after a crash speculators are willing to pay more for insurance even though the future crash risk is lower. Jurek (2014) and Farhi et al. (2009) look at hedging the FX carry trade by purchasing out-of-the-money put options on the high yielding currencies as protection and conclude that the crash risk premia accounts for 30%-40% of the excess returns.

There is a small branch of work that looks at the ability of FX option implied volatility risk reversals to predict future FX spot movements. Given that the FX carry trade can be decomposed into an interest rate and an FX component this has relevance. Dunis and Lequeux (2003) look at 1 month FX option implied volatility risk reversals and conclude that although there is a strong contemporaneous relationship between risk reversals and FX spot, risk reversals do not help in assessing the future evolution of exchange rates. Similar results are obtained by Gudhus (2003) and Eitrheim et al. (1999).

A much larger body of work exists trying to use the moments of the implied probability density functions from FX option implied volatility surfaces to explain FX spot returns, or in some cases FX carry trade returns. Chen and Gwati (2013) establish that for a given maturity the FX option implied moments can explain a significant proportion of the FX carry trade returns.

They extend their analysis to include the term structure of the FX option implied volatility surface and using 3 month FX carry returns they show that the term structure of the implied option moments has additional explanatory power.

The link between historical FX volatility and the returns to the FX carry trade have been explored in the literature. Historically the FX carry trade is susceptible to having large drawdowns coinciding with periods of increased volatility (Beranger et al. 1999, Cairns et al. 2007, Galati et al. 2007, Gagnon and Chaboud 2007, Brunnermeier et al. 2008). Menkhoff et al. (2012(b)) look directly at the relationship between historical FX volatility and FX carry trade returns and find that carry trades perform poorly during periods of market turmoil.

Lustig et al. (2014) provides a literature review of recent work that has looked at the predictability of FX carry returns. Of these papers, Bakshi and Panayotov (2013) are the only authors to use a volatility measure , in this case a measure of historical FX volatility, among others, to predict FX carry returns.

The question remains then if there is an established contemporaneous link between historical volatility and the returns to the FX carry trade then to what extent can the inputs to the implied FX volatility surface explain and predict the returns to the FX carry trade ? After all implied volatility is a better predictor of future realised volatility than historical volatility so it is reasonable to expect that the inputs to the implied volatility surface will offer more explanatory power towards explaining FX carry returns than historical volatility.

The issue of whether implied volatility is a better predictor of future realised volatility than historical volatility has been examined in the literature for some time. The majority view is that implied volatility is a superior predictor of future realised volatility but this topic has certainly been contentious (Christensen and Prabhala (1998), Christensen and Hansen (2002), Shu and Zhang (2003), Li and Yang (2009), Han and Park (2013), Shaikh and Padhi (2013), Mishra and Panda (2016)). However there are some authors that question the superiority of implied volatility over historical volatility for the purposes of forecasting future realised volatility (Canina and Figlewski (1993), Filis (2009)), whilst others show that by using high frequency data, historical volatility outperforms implied volatility in explaining future realised volatility for short forecast periods. However, for longer forecast periods implied volatility remains superior to historical volatility (Anderson and Bollerslev (1998), Anderson et al. (2003), Pong et al. (2004)). And finally there are some authors who show that historical volatility outperforms implied volatility in predicting future realised volatility (Jackwerth and Rubinstein (1996), Koopman et al. (2005)).

So motivated by firstly the established contemporaneous link between historical FX volatility and the returns to the FX carry trade, and secondly the generally accepted superiority of implied volatility over historical volatility for forecasting future realised volatility the research question proposed is to what extent can the ex-ante inputs into the FX option implied volatility surface explain the ex-post returns to the FX carry trade ?

19 Single Currency FX Carry Trade Returns and FX Option Implied Volatility

Recall in Section 4.1 a method for constructing a single currency FX carry trade and measuring its returns was presented. Using a time frame of 1 month, for each currency versus the USD, a discount measure was calculated to determine if that currency was at a forward premium or forward discount. This determines whether the currency is sold or bought versus the USD for 1 month forward. At the end of the 1 month period the return on the FX carry trade can be calculated and the process is repeated. Recall that x_{t+T} is the return to the FX carry trade implemented at time t, for the period T, realised at time $t+T$. Once again, the following analysis is performed for 1 month FX carry trades, $T = 1$ month. Using Equation (7) 1 month FX carry trade returns are calculated on a daily basis. This is in contrast to the earlier work in Section 4 which used non overlapping FX carry trade returns. The move to daily calculations of FX carry trade returns for this chapter is in order to use all the available implied volatility data.

In line with the literature (Clarida et al. 2009) to understand the contemporaneous relationship between the maturity matched inputs of the implied volatility surface and corresponding FX carry trade consider the following specification :

$$x^i_{t+T} = \gamma^i_0 + \gamma^i_1 v^i_{t+T,1mATM} + \gamma^i_2 v^i_{t+T,1m25RR} + \gamma^i_3 v^i_{t+T,1m25BF} + \epsilon^i_{t+T} \qquad (94)$$

where $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD). So for currency pair i, regress 1 month at-the-money implied volatility at time $t+T$, 1 month 25 delta risk reversal at time $t+T$, and 1 month 25 delta butterfly at time $t+T$ on the 1 month FX carry trade returns realised at time $t+T$. Table 34 shows the regression coefficients for each of the 9 single currency FX carry trades.

The coefficient estimates for 1 month at-the-money implied volatility for each currency pair are negative in all cases with the exception of JPYUSD and EURUSD. This is in line with the literature which shows that increases (decreases) in volatility, largely irrespective of what type of volatility measure you use, results in periods of poor (good) performance of the FX carry trade. The two exceptions, JPYUSD and EURUSD, are interesting in that they tend to be low yielding, or funding, currencies (see Table 7). It is reasonable to postulate that in periods of general financial distress and increases in implied volatility that there is a 'flight to quality' effect which sees the USD benefit from inward flows, and assuming these FX carry trades are short the JPY or EUR leg then the FX carry trade benefits. You could also make the same argument for CHFUSD which in Table 34 has a negative coefficient

Table 34: Single Currency FX Carry Trade Returns and Maturity Matched Implied Volatility Inputs

The coefficient estimates and adjusted R^2 of the following equation are shown

$$x^i_{t+T} = \gamma^i_0 + \gamma^i_1 v^i_{t+T,1mATM} + \gamma^i_2 v^i_{t+T,1m25RR} + \gamma^i_3 v^i_{t+T,1m25BF} + \epsilon^i_{t+T}$$

where $i \in$ (AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), x^i_{t+T} is the FX carry trade return for currency pair i executed at time t and realised at time $t+T$, $v^i_{t+T,1mATM}$ is the 1 month at-the-money implied volatility at time $t+T$ for currency pair i, $v^i_{t+T,1m25RR}$ is the 1 month 25 delta risk reversal at time $t+T$ for currency pair i, and $v^i_{t+T,1m25BF}$ is the 25 delta butterfly at time $t+T$ for currency pair i and $T = 1$ month. Asterisks ***, **, and * indicate the coefficient estimates are significant at 1%, 5%, and 10% respectively using White standard errors. Data start dates vary between October 2003 to January 2006 by currency pair and all end at December 2012.

Currency	Intercept	atm1m	25rr1m	25bf1m	Adj R^2
AUDUSD	0.0053 **	-0.0010 ***	0.0193 ***	0.1421 ***	0.23
CADUSD	0.0014	-0.0023 ***	0.0007	0.0944 ***	0.03
CHFUSD	0.0032	-0.0015 ***	0.0178 ***	0.0498 ***	0.17
EURUSD	-0.0208 ***	0.0033 ***	0.0047 ***	-0.0422 ***	0.05
GBPUSD	0.0057 ***	-0.0016 ***	0.0207 ***	0.0933 ***	0.19
JPYUSD	0.0142 ***	0.0011 ***	0.0092 ***	-0.0534 ***	0.22
NOKUSD	0.0099 ***	-0.0025 ***	-0.0051 ***	0.1080 ***	0.05
NZDUSD	0.0157 ***	-0.0002	0.0191 ***	0.0694 ***	0.21
SEKUSD	0.0131 ***	-0.0017 ***	0.0022 **	0.0273 ***	0.02

estimate. However the flip side to this argument is that during periods of financial distress, the CHF and JPY, and to a lesser extent the EUR, are generally viewed as 'safe haven' currencies (Ranaldo and Soderlind 2010) which makes generalizations difficult to draw.

Interpreting the coefficient estimates of the 1 month 25 delta risk reversals is slightly less straight forward. Recall that the risk reversal data is quoted as currency 1 call minus currency 1 put. So in the case of AUDUSD, EURUSD, GBPUSD, and NZDUSD the risk reversal is quoted as currency 1 call / USD put minus currency 1 put / USD call. For example for AUDUSD it is quoted AUD call / USD put minus AUD put / USD call. These 4 currency pairs traditionally have a negative risk reversal meaning for example an AUD put / USD call trades at a higher implied volatility than a AUD call / USD put. The coefficient estimates for the 1 month 25 delta risk reversals for these four currency pairs are positive which seems intuitively plausible. If the risk reversal for these pairs increases, or becomes less negative, then based on the expected implied volatility surface dynamic this is due to a likely reduction in implied volatility and / or an appreciation of these currencies against the USD. Such periods of stability tend to be positive for FX carry trade returns. Conversely, if the risk reversal for these currency pairs decreases, or becomes more negative, then the implied volatility surface dynamic suggests this is due to an increase in volatility and / or a depreciation of these currencies against the USD. As as been documented, such periods of increased volatility tend to have a negative impact on FX carry trade returns. In the interbank FX option market CAD, NOK, and SEK are quoted as USDCAD, USDSEK, and USDNOK. So in these cases given that our FX option data set quotes the risk reversals as currency 1 call minus currency 1 put this means these pairs are quoted as USD call / currency 2 put minus USD put / currency 2 call. These 3 currency pairs tend to have a positive risk reversal so for example in the case of USDSEK this means that a USD call / SEK put tends to trade at a higher implied volatility than a USD put / SEK call. The coefficient estimates for the 1 month 25 delta risk reversals for CAD and SEK are both positive which lends itself to the same interpretation as above for AUD, GBP, and NZD. The coefficient estimate for NOK is however negative which is somewhat difficult to understand. For the 2 most consistently low yielding currencies, CHF and JPY, they are also quoted in the interbank FX option market as USDCHF and USDJPY. In our data set these 2 currency pairs tend to have negative risk reversals, so for instance in the case of USDJPY, USD puts / JPY calls tend trade at a higher implied volatility than USD calls / JPY puts. However, it is worth noting that in the case of USDCHF there has been increased volatility in the risk reversals since 2009 when the Swiss National Bank embarked on a series of FX intervention policies to stem the appreciation of the CHF. The coefficient estimates for the 1 month 25 delta risk reversals for both USDJPY and USDCHF are positive, which again seems intuitively plausible. If the risk reversal for these pairs increases, or becomes less negative, then based on the expected implied volatility surface dynamic this is due to a likely reduction in implied volatility and / or a depreciation of these currencies against the

USD. Such periods of stability tend to be positive for FX carry trade returns. Conversely, if the risk reversal for these currency pairs decreases, or becomes more negative, then the implied volatility surface dynamic suggests this is due to an increase in volatility and / or an appreciation of these currencies against the USD which historically as a negative impact on FX carry trade returns. It is worth noting the symmetry between the CHF and JPY, as traditional funding currencies, and the remaining currencies as implied by the dynamics of the implied volatility surface (Farhi et al. 2009). For the CHF and JPY, USD calls are the relatively 'cheap' side of the risk reversal, whereas for the other currencies USD calls are the relatively 'expensive' side of the risk reversal.

The coefficient estimates for the 1 month 25 delta butterfly's are all positive with the exception of EURUSD and CHFUSD. This is difficult to reconcile as one would expect that as butterflies increase this is due to increased volatility and/or expected future volatility which would be expected to have a negative impact on FX carry trade returns. The volatility of butterflies is relatively low in some cases so perhaps their re-pricing is slow and in effect 'after the event' ?

So far the maturity matched inputs of the implied volatility surface and the FX carry trade have been considered. However, the implied volatility data set has nodes from 1 week to 1 year from which the entire surface can be constructed. Can this entire implied volatility surface explain the FX carry trade returns ?

In order to evaluate whether the FX carry trade returns, x_{t+T}, can be explained by the 5 inputs per maturity to the FX implied volatility surface consider the following equation :

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} v^{i,j}_{t+T,ATM} + \sum_j \gamma^i_{2,j} v^{i,j}_{t+T,25RR} + \sum_j \gamma^i_{3,j} v^{i,j}_{t+T,10RR} + \sum_j \gamma^i_{4,j} v^{i,j}_{t+T,25BF} + \sum_j \gamma^i_{5,j} v^{i,j}_{t+T,10BF} + \epsilon^i_{t+T} \quad (95)$$

where $T = 1$ month and where $j \in (1wk, 1mth, 3mth, 6mth, 12mth)$ and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD). So the FX carry trade returns for currency pair i implemented at time t and realised at time $t+T$ are regressed on the inputs to the FX option implied volatility surface available at time $t+T$

One obvious issue with this regression is the large number of explanatory variables. For each of the 5 maturity nodes on the implied volatility surface there are 5 inputs to the surface. Lets consider how to achieve a more parsimonious specification.

Equation (95) uses the 25 inputs to the implied volatility surface as explanatory variables. It is reasonable to assume that there is some degree of correlation between these 25 inputs to the implied volatility surface.

Firstly, lets consider the results for AUDUSD. Table 35 shows the correlations between the the 5 at-the-money implied volatility inputs for AUDUSD.

It can be seen that the closer the two nodes are together, in terms of maturity, the more correlated they are, and the further apart they are the less correlated they tend to be.

Table 35: AUDUSD At-The-Money Correlations

For AUDUSD the correlations between between the at-the-money implied volatility inputs for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	atm1m	atm3m	atm6m	atm1y
atm1w	1.00				
atm1m	0.97	1.00			
atm3m	0.91	0.97	1.00		
atm6m	0.84	0.92	0.98	1.00	
atm1y	0.76	0.86	0.95	0.99	1.00

Table 36 shows the correlations between the 25 and 10 delta risk reversals for each maturity node for AUDUSD. For a given maturity node the 25 delta and 10 delta risk reversals are highly correlated with each other. In Section 16.5 three common implied volatility smile construction approaches for a given maturity were presented. In the FX option market the vanna-volga method is commonly used by practitioners, although it is possible to show that for deltas other than extremely low or high deltas the results are very similar to the SABR specification (Castagna and Mercurio 2007). Under the vanna-volga approach it is possible to derive an implied volatility for any delta based on the at-the-money straddle and the risk reversal and butterfly quotes for a given delta. So for instance, based on the at-the-money straddle, 25 delta risk reversal and 25 delta butterfly it is possible to derive the implied volatilities for 10 a delta call and put. This is achieved by calculating a three option hedge which ensures that the derived volatilities are consistent in terms of the cost of dVega/dSpot (Vanna) and dVega/dVol (Volga). Based on the derived implied volatilities for the 10 delta call and put it is then possible to back out the derived 10 delta risk reversal and 10 delta butterfly that are consistent with it. Conversely, this derivation could be done the other way, starting with the at-the-money straddle, 10 delta risk reversal and 10 delta butterfly and then calculating the 25 delta risk reversal and 25 delta butterfly (Castagna and Mercurio 2007). This internal consistency in constructing the implied volatility smile gives rise to the strong correlation between the 25 and 10 delta risk reversals. The 1 week node is the lowest at .93, while the other 4 nodes are all above .98. Similar to Table 1, the further apart nodes are in terms of maturity the less correlated they are.

Table 36: AUDUSD Risk Reversal Correlations

For AUDUSD the correlations between between the 25 delta and 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	25rr1w	10rr1w	25rr1m	10rr1m	25rr3m	10rr3m	25rr6m	10rr6m	25rr1y	10rr1y
25rr1w	1.00									
10rr1w	0.93	1.00								
25rr1m	0.88	0.92	1.00							
10rr1m	0.86	0.92	0.99	1.00						
25rr3m	0.75	0.77	0.93	0.93	1.00					
10rr3m	0.73	0.79	0.94	0.95	0.98	1.00				
25rr6m	0.67	0.70	0.88	0.89	0.99	0.98	1.00			
10rr6m	0.65	0.72	0.89	0.90	0.98	0.99	0.99	1.00		
25rr1y	0.60	0.63	0.83	0.84	0.97	0.96	0.99	0.98	1.00	
10rr1y	0.60	0.66	0.85	0.86	0.96	0.97	0.98	0.99	0.99	1.00

Table 37 shows the correlations between the 25 and 10 delta butterflies for each maturity node for AUDUSD. Overall the correlations are a touch lower than the risk reversals but still remain high. This is to be expected given how the implied volatility smile is constructed, as discussed above. Once again other than the 1 week node the correlations between the 25 delta and 10 delta butterflies for a given maturity are very high, +.90.

Table 37: AUDUSD Butterfly Correlations

For AUDUSD the correlations between between the 25 delta and 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	25bf1w	10bf1w	25bf1m	10bf1m	25bf3m	10bf3m	25bf6m	10bf6m	25bf1y	10bf1y
25bf1w	1.00									
10bf1w	0.63	1.00								
25bf1m	0.61	0.74	1.00							
10bf1m	0.62	0.78	0.90	1.00						
25bf3m	0.43	0.71	0.91	0.80	1.00					
10bf3m	0.42	0.71	0.86	0.90	0.91	1.00				
25bf6m	0.35	0.66	0.86	0.76	0.98	0.89	1.00			
10bf6m	0.35	0.69	0.83	0.86	0.92	0.98	0.92	1.00		
25bf1y	0.28	0.60	0.79	0.68	0.95	0.86	0.98	0.90	1.00	
10bf1y	0.30	0.65	0.80	0.81	0.92	0.97	0.93	0.99	0.92	1.00

Table 38 shows the correlations between the at-the-money volatilities and the 10 delta risk reversals for the 5 maturity nodes for AUDUSD. The signs of the correlations between the risk reversals and the at-the-money nodes are negative which is expected. As implied volatility goes up the risk reversals will become steeper and since they are measured here as call delta minus put delta this means they become more negative.

Table 38: AUDUSD At-The-Money and Risk Reversal Correlations

For AUDUSD the correlations between between the at-the-money implied volatility inputs and the 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	10rr1w	atm1m	10rr1m	atm3m	10rr3m	atm6m	10rr6m	atm1y	10rr1y
atm1w	1.00									
10rr1w	-0.69	1.00								
atm1m	0.97	-0.67	1.00							
10rr1m	-0.67	0.92	-0.68	1.00						
atm3m	0.91	-0.62	0.97	-0.69	1.00					
10rr3m	-0.60	0.79	-0.65	0.95	-0.70	1.00				
atm6m	0.84	-0.57	0.92	-0.68	0.98	-0.73	1.00			
10rr6m	-0.56	0.72	-0.61	0.90	-0.68	0.99	-0.73	1.00		
atm1y	0.76	-0.51	0.86	-0.67	0.95	-0.75	0.99	-0.76	1.00	
10rr1y	-0.53	0.66	-0.59	0.86	-0.67	0.97	-0.72	0.99	-0.77	1.00

Table 39 shows the correlations between the at-the-money volatilities and the 10 delta butterflies for the 5 maturity nodes for AUDUSD. The signs of the correlations between the butterflies and the at-the-money nodes are positive which is to be expected.

Table 40 shows the correlations between the 10 delta risk reversals and the 10 delta butterflies for each maturity node for AUDUSD. The signs of the correlations between the risk reversals and butterflies for a given maturity node are negative which is expected. The absolute level of the correlation between the risk reversals and butterflies appears higher for the longer dated maturities.

Tables 41 to 46 show the same correlation results for JPYUSD as Tables 35 to 40 do for AUDUSD. In contrast to AUD, JPY is generally a 'low

Table 39: AUDUSD At-The-Money and Butterfly Correlations

For AUDUSD the correlations between between the at-the-money implied volatility inputs and the 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	10bf1w	atm1m	10bf1m	atm3m	10bf3m	atm6m	10bf6m	atm1y	10bf1y
atm1w	1.00									
10bf1w	0.42	1.00								
atm1m	0.97	0.51	1.00							
10bf1m	0.75	0.78	0.82	1.00						
atm3m	0.91	0.60	0.97	0.86	1.00					
10bf3m	0.71	0.71	0.79	0.90	0.86	1.00				
atm6m	0.84	0.64	0.92	0.85	0.98	0.89	1.00			
10bf6m	0.65	0.69	0.73	0.86	0.82	0.98	0.87	1.00		
atm1y	0.76	0.65	0.86	0.82	0.95	0.90	0.99	0.90	1.00	
10bf1y	0.61	0.65	0.69	0.81	0.78	0.97	0.84	0.99	0.88	1.00

Table 40: AUDUSD Risk Reversal Butterfly Correlations

For AUDUSD the correlations between between the 10 delta risk reversals and the 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	10rr1w	10bf1w	10rr1m	10bf1m	10rr3m	10bf3m	10rr6m	10bf6m	10rr1y	10bf1y
10rr1w	1.00									
10bf1w	-0.36	1.00								
10rr1m	0.92	-0.50	1.00							
10bf1m	-0.67	0.78	-0.75	1.00						
10rr3m	0.79	-0.57	0.95	-0.73	1.00					
10bf3m	-0.70	0.71	-0.84	0.90	-0.89	1.00				
10rr6m	0.72	-0.58	0.90	-0.70	0.99	-0.89	1.00			
10bf6m	-0.64	0.69	-0.81	0.86	-0.89	0.98	-0.91	1.00		
10rr1y	0.66	-0.57	0.86	-0.68	0.97	-0.88	0.99	-0.90	1.00	
10bf1y	-0.62	0.65	-0.80	0.81	-0.89	0.97	-0.91	0.99	-0.92	1.00

yielding' currency (Table 7). The correlation results for JPYUSD are similar to those for AUDUSD. Correlation results for the remaining currency pairs are available on request.

Table 41: JPYUSD At-The-Money Correlations

For JPYUSD the correlations between between the at-the-money implied volatility inputs for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	atm1m	atm3m	atm6m	atm1y
atm1w	1.00				
atm1m	0.97	1.00			
atm3m	0.89	0.96	1.00		
atm6m	0.78	0.88	0.97	1.00	
atm1y	0.61	0.72	0.86	0.96	1.00

In light of the above results consider a re-specification of Equation (95) which only uses as explanatory variables the the 6 input nodes that effectively define the 'corners' of the FX option implied volatility surface. Using Equations (91) and (92) for the 1week and 1 year maturities these nodes are 1 week at-the-money straddles, 1 week 10 delta risk reversals, 1 week 10 delta butterflies, 1 year at-the-money straddles, 1 year 10 delta risk reversals, and 1 year 10 delta butterflies. So Equation (95) now becomes :

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} v^{i,j}_{t+T,ATM} + \sum_j \gamma^i_{2,j} v^{i,j}_{t+T,10RR} + \sum_j \gamma^i_{3,j} v^{i,j}_{t+T,10BF} + \epsilon^i_{t+T} \tag{96}$$

Table 42: JPYUSD Risk Reversal Correlations

For JPYUSD the correlations between between the 25 delta and 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	25rr1w	10rr1w	25rr1m	10rr1m	25rr3m	10rr3m	25rr6m	10rr6m	25rr1y	10rr1y
25rr1w	1.00									
10rr1w	0.99	1.00								
25rr1m	0.95	0.95	1.00							
10rr1m	0.93	0.94	0.99	1.00						
25rr3m	0.89	0.90	0.98	0.98	1.00					
10rr3m	0.89	0.90	0.98	0.98	1.00	1.00				
25rr6m	0.85	0.87	0.96	0.96	0.99	0.99	1.00			
10rr6m	0.84	0.86	0.95	0.95	0.99	0.99	1.00	1.00		
25rr1y	0.81	0.82	0.92	0.92	0.98	0.97	0.99	0.99	1.00	
10rr1y	0.80	0.82	0.92	0.92	0.97	0.97	0.99	0.99	0.99	1.00

Table 43: JPYUSD Butterfly Correlations

For JPYUSD the correlations between between the 25 delta and 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	25bf1w	10bf1w	25bf1m	10bf1m	25bf3m	10bf3m	25bf6m	10bf6m	25bf1y	10bf1y
25bf1w	1.00									
10bf1w	0.68	1.00								
25bf1m	0.83	0.69	1.00							
10bf1m	0.48	0.88	0.62	1.00						
25bf3m	0.79	0.63	0.96	0.57	1.00					
10bf3m	0.40	0.83	0.57	0.97	0.53	1.00				
25bf6m	0.78	0.57	0.93	0.48	0.96	0.44	1.00			
10bf6m	0.38	0.79	0.54	0.94	0.51	0.98	0.44	1.00		
25bf1y	0.69	0.48	0.83	0.42	0.88	0.40	0.91	0.41	1.00	
10bf1y	0.35	0.75	0.51	0.91	0.49	0.96	0.41	0.99	0.39	1.00

Table 44: JPYUSD At-The-Money and Risk Reversal Correlations

For JPYUSD the correlations between between the at-the-money implied volatility inputs and the 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	10rr1w	atm1m	10rr1m	atm3m	10rr3m	atm6m	10rr6m	atm1y	10rr1y
atm1w	1.00									
10rr1w	-0.75	1.00								
atm1m	0.97	-0.73	1.00							
10rr1m	-0.82	0.94	-0.83	1.00						
atm3m	0.89	-0.60	0.96	-0.73	1.00					
10rr3m	-0.84	0.90	-0.87	0.98	-0.79	1.00				
atm6m	0.78	-0.42	0.88	-0.57	0.97	-0.65	1.00			
10rr6m	-0.83	0.86	-0.87	0.95	-0.82	0.99	-0.69	1.00		
atm1y	0.61	-0.18	0.72	-0.36	0.86	-0.44	0.96	-0.49	1.00	
10rr1y	-0.82	0.82	-0.87	0.92	-0.83	0.97	-0.71	0.99	-0.52	1.00

Table 45: JPYUSD At-The-Money and Butterfly Correlations

For JPYUSD the correlations between between the at-the-money implied volatility inputs and the 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	atm1w	10bf1w	atm1m	10bf1m	atm3m	10bf3m	atm6m	10bf6m	atm1y	10bf1y
atm1w	1.00									
10bf1w	0.37	1.00								
atm1m	0.97	0.44	1.00							
10bf1m	0.63	0.88	0.69	1.00						
atm3m	0.89	0.48	0.96	0.69	1.00					
10bf3m	0.69	0.83	0.75	0.97	0.75	1.00				
atm6m	0.78	0.47	0.88	0.65	0.97	0.70	1.00			
10bf6m	0.72	0.79	0.77	0.94	0.76	0.98	0.70	1.00		
atm1y	0.61	0.45	0.72	0.57	0.86	0.60	0.96	0.61	1.00	
10bf1y	0.73	0.75	0.78	0.91	0.77	0.96	0.70	0.99	0.60	1.00

Table 46: JPYUSD Risk Reversal and Butterfly Calculations

For JPYUSD the correlations between between the 10 delta risk reversals and the 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year are shown. Data for the period January 2006 until December 2012 is used.

	10rr1w	10bf1w	10rr1m	10bf1m	10rr3m	10bf3m	10rr6m	10bf6m	10rr1y	10bf1y
10rr1w	1.00									
10bf1w	-0.26	1.00								
10rr1m	0.94	-0.43	1.00							
10bf1m	-0.52	0.88	-0.69	1.00						
10rr3m	0.90	-0.49	0.98	-0.73	1.00					
10bf3m	-0.59	0.83	-0.75	0.97	-0.79	1.00				
10rr6m	0.86	-0.50	0.95	-0.73	0.99	-0.79	1.00			
10bf6m	-0.61	0.79	-0.75	0.94	-0.80	0.98	-0.80	1.00		
10rr1y	0.82	-0.50	0.92	-0.72	0.97	-0.78	0.99	-0.79	1.00	
10bf1y	-0.61	0.75	-0.75	0.91	-0.80	0.96	-0.81	0.99	-0.81	1.00

where $T = 1$ month and where $j \in (1wk, 12mth)$ and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD). So in this case there are 6 explanatory variables as opposed to the 25 in Equation (95) The regression results of Equation (96) are shown in Table 47. The adjusted R^2's are greater than those from the simple maturity matched regression (Equation (94)) for all currency pairs with AUDUSD having the highest adjusted R^2 of 0.31. Overall the results are encouraging in that the simplified implied volatility surface specification has explanatory power for the contemporaneous single currency FX carry trade returns in most cases.

In addition this specification is checked and supported on average for each currency pair using Stepwise Regression. Using the backward elimination technique whereby all explanatory variables are included in the model to

Table 47: Single Currency FX Carry Trade Returns and 1 Week and 1 Year 10 Delta Implied Volatility Input Nodes

The coefficient estimates and adjusted R^2 of the following equation are shown

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} v^{i,j}_{t+T,ATM} + \sum_j \gamma^i_{2,j} v^{i,j}_{t+T,10RR} + \sum_j \gamma^i_{3,j} v^{i,j}_{t+T,10BF} + \epsilon^i_{t+T}$$

where $T = 1$ month and where $j \in (1wk, 12mth)$ and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), x^i_{t+T} is the FX carry trade return for currency pair i executed at time t and realised at time $t+T$, $v^{i,j}_{t+T,ATM}$ is the at-the-money implied volatility at time $t+T$ for currency pair i with option maturity j, $v^{i,j}_{t+T,10RR}$ is the 10 delta risk reversal at time $t+T$ for currency pair i with option maturity j, and $v^{i,j}_{t+T,10BF}$ is the 10 delta butterfly at time $t+T$ for currency pair i with option maturity j. Asterisks ***, **, and * indicate the coefficient estimates are significant at 1%, 5%, and 10% respectively using White standard errors. Data start dates vary between October 2003 to January 2006 by currency pair and all end at December 2012.

Currency	Intercept	atm1w	atm1y	10rr1w	10rr1y	10bf1w	10bf1y	Adj R^2
AUDUSD	-0.0082 **	-0.0034 ***	0.0045 ***	0.0033 ***	0.0083 ***	0.0002	0.0226 ***	0.31
CADUSD	0.0168 ***	0.0005	-0.0059 ***	0.0108 ***	-0.0038 ***	0.0255 ***	0.0157 ***	0.12
CHFUSD	0.0286 ***	-0.0002	-0.0037 ***	0.0166 ***	-0.0054 ***	-0.0001	0.0096 ***	0.19
EURUSD	-0.0197 ***	0.0013 ***	0.0004	0.0026 ***	-0.0003	-0.0043 ***	0.0020	0.05
GBPUSD	0.0068 **	-0.0005	-0.0027 ***	0.0145 ***	0.0013 **	-0.0018 **	0.0275 ***	0.25
JPYUSD	0.0354 ***	-0.0004	-0.0027 ***	0.0057 ***	-0.0025 ***	-0.0003	-0.0024	0.22
NOKUSD	-0.0203 ***	-0.0060 ***	0.0057 ***	0.0012	-0.0041 ***	0.0093 ***	0.0162 ***	0.18
NZDUSD	0.0101 **	-0.0012 ***	0.0004	0.0085 ***	0.0083 ***	-0.0005	0.0295 ***	0.28
SEKUSD	0.0042	-0.0024 ***	0.0003	-0.0023 **	0.0023 ***	-0.0006	0.0126 ***	0.06

start with and then the variables are deleted from the model one by one until all the variables remaining in the model produce F statistics significant at a specified level. At each step the variable showing the smallest contribution to the model is deleted. Summing the occurrences of 'kept' variables across all currency pairs shows that the 10 delta risk reversals and butterflies are kept more times than the 25 delta risk reversals and butterflies.

19.1 Forecasting Single Currency FX Carry Trade Returns

Having established the contemporaneous relationship between the returns to the single currency FX carry trade and the inputs to the FX option implied volatility surface can this be extended to forecast single currency FX carry trade returns. Firstly, adding a lag structure to Equation 96, which now becomes :

$$x^i_{t+T} = \gamma^i_0 + \sum_{j,l} \gamma^i_{1,j,l} v^{i,j}_{t-l,ATM} + \sum_{j,l} \gamma^i_{2,j,l} v^{i,j}_{t-l,10RR} + \sum_{j,l} \gamma^i_{3,j,l} v^{i,j}_{t-l,10BF} + \epsilon^i_{t+T} \tag{97}$$

where $T = 1$ month and where $j \in (1wk, 12mth)$ and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD) and where $l = 0, 1, 2, 3$ represents the number of monthly lagged observations in the regression. So in the case where $l = 0$ the FX carry trade returns for currency pair i implemented at time t and realised at time $t + T$ are regressed on the 1 week and 1 year at-the-money, 10 delta risk reversals, and 10 delta butterfly inputs to the FX option implied volatility surface available at time t, that is when the carry trade was implemented. When $l = 1, 2, 3$ the monthly lagged inputs to the FX option implied volatility surface are introduced as explanatory variables.

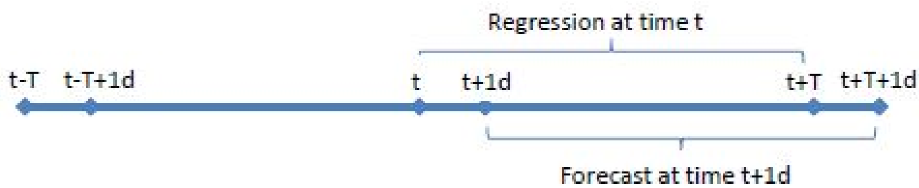
Equation (97) enables the one step ahead prediction errors to be calculated. By using all data up to and including period t, for currency pair i parameter estimates $\widehat{\gamma}_0$ and $\widehat{\gamma}_{e,j,l,t}$ where $e = 1, 2, 3$, $j \in (1wk, 12mth)$, $l = 0, 1, 2, 3$ as above, are obtained. These parameter estimates enable 1 observation ahead ($1d$) forecasts to be calculated, $\widehat{x}_{t+T+1d}$, and then the corresponding one step ahead forecast error, x^{error}_{t+T+1d} :

$$x^{error}_{t+T+1d} = \widehat{x}_{t+T+1d} - x_{t+T+1d} \tag{98}$$

By proceeding in this manner, adding one sample point each time until all time series observations are used, yields a series of one step ahead forecast errors. Figure 31 shows this in the case of $l = 0$.

Recall that the common start date across the currencies and the implied volatility input nodes is January 2006. Starting the 1 observation ahead forecasts in January 2007 ensures that the initial parameter estimates are based on sufficient data. This yields a series of 1543 1 step ahead forecasts and

Figure 31: Timeline Forecast Methodology



forecast errors. To evaluate the performance of these 1 step ahead forecasts the following summary statistics are calculated. The Root Mean Square Error (RMSE) defined as :

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\widehat{x}_{t+T+1d} - x_{t+T+1d})^2} \tag{99}$$

and the Mean Absolute Deviation (MAD) defined as :

$$MAD = \frac{\sum_{i=1}^{n} |\widehat{x}_{t+T+1d} - x_{t+T+1d}|}{n} \tag{100}$$

and the Mean Absolute Percentage Error (MAPE) defined as :

$$MAPE = \frac{\sum_{i=1}^{n} \left|\frac{(\widehat{x}_{t+T+1d} - x_{t+T+1d})}{x_{t+T+1d}}\right|}{n} \tag{101}$$

In addition to calculating the these summary statistics for the 1 step ahead forecast errors as defined in Equation (98) the exercise is repeated where the 1 step ahead forecast is simply equal to the previous observations value. That is, the forecast FX carry trade return for the next observation is set equal to the FX carry trade return of the current observation. In this case, Equation (98) becomes :

$$x^{error}_{t+T+1d} = x_{t+T} - x_{t+T+1d} \tag{102}$$

19.1.1 Results

Tables 48 and 49 show the one step ahead forecast results using Equation (97). The first column of Tables 48 and 49 show the average monthly FX carry trade returns and standard deviations for each currency, based on daily calculations. The second column, lagged forecast error, shows the forecast performance statistics when the previous observation is used as the forecast value. The final 4 columns, regression forecast error, show the forecast performance statistics for lagged explanatory variable structures $l = 0$ to $l = 3$ in Equation (97).

Overall the forecast results for all currencies are very poor. In all cases the addition of lagged explanatory variables adds virtually no predictive power at all. Disappointingly the forecasting ability of Equation (97) is worse than the naive 'no change' forecast for all currency pairs. This is somewhat surprising given the seemingly robust contemporaneous results. Perhaps the time lag between using observed implied volatility inputs at time t when the FX carry trade is implemented and the FX carry trade returns at time $t + T$, where T=1 month, is a step too far to enable any sort of forecasting performance.

Table 48: Single Currency FX Carry Trade Return One Step Ahead Forecast Results

One step ahead forecasts using the coefficient estimates from :

$$x^i_{t+T} = \gamma^i_0 + \sum_{j,l} \gamma^i_{1,j,l} v^{i,j}_{t-l,ATM} + \sum_{j,l} \gamma^i_{2,j,l} v^{i,j}_{t-l,10RR} + \sum_{j,l} \gamma^i_{3,j,l} v^{i,j}_{t-l,10BF} + \epsilon^i_{t+T}$$

where $T = 1$ month and where $j \in (1wk, 12mth)$ and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), $l = 0, 1, 2, 3$ represents the number of monthly lagged observations, x^i_{t+T} is the FX carry trade return for currency pair i executed at time t and realised at time $t+T$, $v^{i,j}_{t-l,ATM}$ is the at-the-money implied volatility at time $t-l$ for currency pair i with option maturity j, $v^{i,j}_{t-l,10RR}$ is the 10 delta risk reversal at time $t-l$ for currency pair i with option maturity j, and $v^{i,j}_{t-l,10BF}$ is the 10 delta butterfly at time $t-l$ for currency pair i with option maturity j. Regression forecast errors $x^{error}_{t+T+1d} = \widehat{x}_{t+T+1d} - x_{t+T+1d}$ are calculated for $l = 0, 1, 2, 3$ which enables the summary statistics RMSE, MAD, MAPE to be calculated. Lagged forecast errors where the forecast value is equal to the previous observed value, $x^{error}_{t+T+1d} = x_{t+T} - x_{t+T+1d}$, are also calculated for comparison. Average monthly cash flow returns and standard deviations are shown. Daily data for the period January 2006 until December 2012 is used.

		Carry trade return	Lagged forecast error	Regression forecast error			
				l =0	$l = 1$	$l = 2$	$l = 3$
AUDUSD	Average CF	0.0079					
	Std Dev CF	0.0451					
	RMSE		0.0149	0.0447	0.0451	0.0445	0.0432
	MAD		0.0104	0.0319	0.0326	0.0326	0.0318
	MAPE		155.91	295.46	318.66	330.13	338.64
CADUSD	Average CF	-0.0022					
	Std Dev CF	0.0308					
	RMSE		0.0143	0.0311	0.0305	0.0306	0.0303
	MAD		0.0085	0.0221	0.0220	0.0220	0.0222
	MAPE		290.71	417.74	453.45	469.16	629.30
CHFUSD	Average CF	-0.0041					
	Std Dev CF	0.0366					
	RMSE		0.0122	0.0339	0.0337	0.0337	0.0334
	MAD		0.0081	0.0254	0.0248	0.0250	0.0250
	MAPE		177.92	328.09	276.44	431.81	521.53
EURUSD	Average CF	-0.0008					
	Std Dev CF	0.0340					
	RMSE		0.0146	0.0364	0.0361	0.0353	0.0353
	MAD		0.0082	0.0258	0.0257	0.0249	0.0248
	MAPE		129.23	197.31	219.59	241.15	266.67
GBPUSD	Average CF	-0.0030					
	Std Dev CF	0.0299					
	RMSE		0.0159	0.0304	0.0317	0.0309	0.0309
	MAD		0.0080	0.0221	0.0225	0.0220	0.0224
	MAPE		234.51	353.45	369.61	366.65	397.54

Table 49: Single Currency FX Carry Trade Return One Step Ahead Forecast Results cont'd

		Carry trade return	Lagged forecast error	Regression forecast error $l=0$	$l=1$	$l=2$	$l=3$
JPYUSD	Average CF	-0.0042					
	Std Dev CF	0.0290					
	RMSE		0.0104	0.0299	0.0298	0.0295	0.0292
	MAD		0.0076	0.0231	0.0232	0.0229	0.0229
	MAPE		406.67	237.16	410.97	454.19	556.85
NOKUSD	Average CF	-0.0006					
	Std Dev CF	0.0371					
	RMSE		0.0130	0.0369	0.0373	0.0374	0.0393
	MAD		0.0096	0.0281	0.0287	0.0287	0.0283
	MAPE		191.34	288.06	297.92	353.27	305.08
NZDUSD	Average CF	0.0061					
	Std Dev CF	0.0458					
	RMSE		0.0146	0.0458	0.0468	0.0455	0.0454
	MAD		0.0106	0.0337	0.0340	0.0340	0.0333
	MAPE		125.65	215.68	207.23	229.84	263.57
SEKUSD	Average CF	-0.0008					
	Std Dev CF	0.0402					
	RMSE		0.0222	0.0412	0.0410	0.0413	0.0427
	MAD		0.0113	0.0297	0.0299	0.0299	0.0297
	MAPE		157.50	417.95	399.53	407.16	375.71

19.2 Principal Component Approach

In Section 19 a simplified parametrization, Equation (96), of the implied volatility surface was used to contemporaneously model the returns to the single currency FX carry trades. This specification was arrived at by analysing the correlations of the available inputs to the implied FX volatility surface. In Section 19.1 this parametrization was then used to forecast the one step ahead single currency FX carry trade returns.

Another approach available to us is to use principal component analysis. Principal component analysis is a non parametric technique which is able to reduce the number of original variables to a smaller set of derived variables which are liner combinations of the original variables. This statistical method of dimension reduction is able to reduce the complexity of a large data set while minimizing the information loss (Jolliffe 2002). Principal components are uncorrelated variables derived from interrelated variables in the original data set. The technique involves calculating eigenvectors from the covariance matrix and then ordering them by their eigenvalue, highest to lowest. This ranking reflects the significance of the components in being able to explain the original data. Given the complex nature of financial markets, applying principal component analysis has been a natural focus of research (Driesson et al. 2003, Perignon et al. 2007, Kritzman et al. 2011, Chen and Gwati 2013).

Recall that the full implied volatility data set consists of five maturity nodes (1 week, 1 month, 3month, 6 month, 1 year) and for each maturity node there is an at-the-money volatility and 25 delta and 10 delta risk reversals and butterflies. This full set of variables, as explanatory variables to explaining the single currency FX carry trade returns is shown by Equation (95). Using principal component analysis results in a more parsimonious specification that improves on the forecasting performance in Section 19.1 of predicting the one step ahead FX carry trade returns. Specifically is there a set of principal components that can adequately capture the information contained in each of the term structure of at-the-money volatilities, the term structure of risk reversals (both 25 and 10 delta's), and the term structure of butterflies (both 25 and 10 delta's). Equation (95) now becomes :

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} PC^{i,j}_{t+T,ATM} + \sum_j \gamma^i_{2,j} PC^{i,j}_{t+T,RR} + \sum_j \gamma^i_{3,j} PC^{i,j}_{t+T,BF} + \epsilon^i_{t+T} \tag{103}$$

where $T = 1$ month, $j =$ number of principal components, $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), $PC^{i,j}_{t+T,ATM}$ is the $j'th$ principal component for the term structure of at-the-money implied volatilities for currency i at time $t+T$, $PC^{i,j}_{t+T,RR}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta risk reversals for currency i at time $t+T$, and $PC^{i,j}_{t+T,BF}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta butterflies for currency i at time $t+T$.

The regression results of Equation (103) for each currency are shown in Table 50. For each currency, and for each of the at-the-money, risk reversal, and butterfly term structure of inputs the number of principal components was set at $j = 3$. On average for each of the term structure of at-the-money volatilities, risk reversals, and butterflies, 3 factors was able to explain mid to high ninety percent of the total variation. The adjusted R^2's for each currency pair are higher than those of the simplified implied volatility surface model results in Table 47 which suggests that any additional information content in the full term structure of inputs to the implied volatility surface, and the 25 and 10 delta inputs to the surface at each maturity node, is being captured by the principal component approach. 5 of the 9 currency pairs now have adjusted R^2's greater than 0.25.

Extending this principal component framework to forecast one step ahead FX carry trade returns and evaluate it's forecast performance requires Equation (103) to be rewritten as :

$$x^i_{t+T} \; = \; \gamma^i_0 + \sum_j \gamma^i_{1,j} PC^{i,j}_{t,ATM} + \sum_j \gamma^i_{2,j} PC^{i,j}_{t,RR} + \sum_j \gamma^i_{3,j} PC^{i,j}_{t,BF} + \epsilon^i_{t+T} \tag{104}$$

where $T = 1$ month, $j =$ (1,2,3) the number of principal components, $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), $PC^{i,j}_{t,ATM}$ is the $j'th$ principal component for the term

Table 50: Single Currency FX Carry Trade Return Principal Component Regression Results

The coefficient estimates and adjusted R^2 of the following equation are shown

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} PC^{i,j}_{t+T,ATM} + \sum_j \gamma^i_{2,j} PC^{i,j}_{t+T,RR} + \sum_j \gamma^i_{3,j} PC^{i,j}_{t+T,BF} + \epsilon^i_{t+T}$$

where $T = 1$ month, $j =$ number of principal components, $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), $PC^{i,j}_{t+T,ATM}$ is the $j'th$ principal component for the term structure of at-the-money implied volatilities for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time $t+T$, $PC^{i,j}_{t+T,RR}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time $t+T$, and $PC^{i,j}_{t+T,BF}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time $t+T$. Asterisks ***, **, and * indicate the coefficient estimates are significant at 1%, 5%, and 10% respectively using White standard errors. Data for the period January 2006 until December 2012 is used.

Currency	Intercept	PC^1_{ATM}	PC^2_{ATM}	PC^3_{ATM}	PC^1_{RR}	PC^2_{RR}	PC^3_{RR}	PC^1_{BF}	PC^2_{BF}	PC^3_{BF}	Adj R^2
AUDUSD	0.0142 ***	0.0000	-0.0093 ***	-0.0036 ***	0.0321 ***	-0.0012	-0.0023 ***	0.0203 ***	-0.0050 ***	0.0021 *	0.32
CADUSD	0.0059 ***	-0.0150 ***	0.0021 **	0.0008	0.0017	0.0069 ***	0.0044 ***	0.0156 ***	0.0037 ***	-0.0008	0.16
CHFUSD	-0.0035 ***	-0.0117 ***	0.0010	-0.0005	0.0116 ***	0.0136 ***	-0.0061 ***	0.0114 ***	-0.0015 **	-0.0008	0.25
EURUSD	-0.0029 ***	0.0085 ***	-0.0002	-0.0007	-0.0004	0.0030 ***	-0.0063 ***	-0.0034 *	-0.0038 ***	0.0063 ***	0.11
GBPUSD	0.0024 ***	-0.0074 ***	0.0008	0.0008	0.0189 ***	0.0065 ***	-0.0047 ***	0.0211 ***	-0.0026 ***	-0.0029 ***	0.26
JPYUSD	0.0010 **	0.0015	-0.0067 ***	-0.0064 ***	0.0301 ***	0.0073 ***	-0.0053 ***	0.0041 ***	-0.0138 ***	0.0002	0.38
NOKUSD	-0.0028 ***	-0.0022	-0.0101 ***	0.0038 ***	-0.0113 ***	-0.0005	0.0081 ***	0.0099 ***	-0.0015	0.0001	0.22
NZDUSD	0.0173 ***	-0.0051 ***	-0.0011	0.0024	0.0413 ***	0.0073 ***	-0.0020 **	0.0319 ***	-0.0056 ***	-0.0066 ***	0.32
SEKUSD	0.0001	-0.0086 ***	-0.0028 ***	0.0007	0.0014	-0.0027 ***	0.0015	0.0091 ***	-0.0028 **	0.0023 **	0.06

structure of at-the-money implied volatilities for currency i at time t, $PC_{t,RR}^{i,j}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta risk reversals for currency i at time t, and $PC_{t,BF}^{i,j}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta butterflies for currency i at time t. So in this case the explanatory variables are observed at time t when the FX carry trade is constructed, and not at time $t+T$ when the FX carry trade returns are realised, as in the contemporaneous example.

Using the same methodology as Section 19.1 a series of one observation ahead forecasts are generated, and the same performance statistics are calculated. Note that for each iteration the principal components must be regenerated as the data set expands by one observation each time. The results for each currency pair are shown in Table 51.

On average the results are largely similar to those presented earlier in Tables 48 and 49, based on the simplified specification of the implied volatility surface in Equation 97. So whilst it seems that the use of principal component methodology is able to adequately capture the variation in the term structure of the at-the-money implied volatilities, risk reversals, and butterflies, when used as explanatory variables for the purposes of one step ahead forecasting of single currency FX carry trade returns, they are not able to outperform the naive 'no change' forecast.

Table 51: Single Currency FX Carry Trade Return Principal Component One Step Ahead Forecast Results

One step ahead forecasts using the coefficient estimates from :

$$x^i_{t+T} = \gamma^i_0 + \sum_j \gamma^i_{1,j} PC^{i,j}_{t,ATM} + \sum_j \gamma^i_{2,j} PC^{i,j}_{t,RR} + \sum_j \gamma^i_{3,j} PC^{i,j}_{t,BF} + \epsilon^i_{t+T}$$

where T = 1 month, j = number of principal components, i ∈(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD), $PC^{i,j}_{t,ATM}$ is the $j'th$ principal component for the term structure of at-the-money implied volatilities for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time t, $PC^{i,j}_{t,RR}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta risk reversals for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time t, and $PC^{i,j}_{t,BF}$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta butterflies for option maturities of 1 week, 1 month, 3 month, 6 month, and 1 year for currency i at time t. Regression forecast errors $x^{error}_{t+T+1d} = \widehat{x}_{t+T+1d} - x_{t+T+1d}$ are calculated which enables the summary statistics RMSE, MAD, MAPE to be calculated. Lagged forecast errors where the forecast value is equal to the previous observed value, $x^{error}_{t+T+1d} = x_{t+T} - x_{t+T+1d}$, are also calculated for comparison. Data for the period January 2006 until December 2012 is used.

		Lagged forecast error	PC Regression forecast error
AUDUSD	RMSE	0.0149	0.0438
	MAD	0.0104	0.0314
	MAPE	155.91	312.54
CADUSD	RMSE	0.0143	0.0311
	MAD	0.0085	0.0221
	MAPE	290.71	498.93
CHFUSD	RMSE	0.0122	0.0337
	MAD	0.0081	0.0258
	MAPE	177.92	301.10
EURUSD	RMSE	0.0146	0.0341
	MAD	0.0082	0.0254
	MAPE	129.23	214.45
GBPUSD	RMSE	0.0159	0.0308
	MAD	0.0080	0.0220
	MAPE	234.51	247.94
JPYUSD	RMSE	0.0104	0.0291
	MAD	0.0076	0.0228
	MAPE	406.67	364.99
NOKUSD	RMSE	0.0130	0.0371
	MAD	0.0096	0.0283
	MAPE	191.34	309.20
NZDUSD	RMSE	0.0146	0.0452
	MAD	0.0106	0.0329
	MAPE	125.65	221.13
SEKUSD	RMSE	0.0222	0.0400
	MAD	0.0113	0.0296
	MAPE	157.50	331.64

20 Portfolio FX Carry Trade Returns and FX Option Implied Volatility

For the EW and kxk portfolio FX carry trades to what extent can the inputs to the implied volatility surfaces explain the kxk and EW portfolio FX carry trade returns ? Extending Equation 95 to the kxk and EW portfolios and including as explanatory variables the implied volatility input nodes for all candidate currencies yields :

$$x^k_{t+T} = \gamma^k_0 + \sum_{i,j} \gamma^k_{1,i,j} v^{i,j}_{t+T,ATM} + \sum_{i,j} \gamma^k_{2,i,j} v^{i,j}_{t+T,25RR} + \sum_{i,j} \gamma^k_{3,i,j} v^{i,j}_{t+T,10RR} + \sum_{i,j} \gamma^k_{4,i,j} v^{i,j}_{t+T,25BF} + \sum_{i,j} \gamma^k_{5,i,j} v^{i,j}_{t+T,10BF} + \epsilon^k_{t+T} \quad (105)$$

where x^k_{t+T} is the FX carry trade return for the kxk and EW portfolio implemented at time t and realised at time $t+T$, $T = 1$ month, and where $j \in$(1wk, 1mth, 3mth, 6mth, 12mth) and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD) and $k =$(1,2,3,4,EW) representing the kxk and EW portfolios.

Equation (95) has a large number of explanatory variables. In Section 19 a more parsimonious specification was achieved by firstly looking at the correlation between inputs to the implied volatility surface for a given currency pair. However in this portfolio context there is the additional issue of correlations of the inputs to the implied volatility surface across currency pairs. To proceed, the same principal component methodology as used in Section 19.2 is adopted, so Equation (105) now becomes :

$$x_{t+T}^{k} = \gamma_0^k + \sum_j \gamma_{1,j}^k PC_{t+T,ATM}^j + \sum_j \gamma_{2,j}^k PC_{t+T,RR}^j + \sum_j \gamma_{3,j}^k PC_{t+T,BF}^j + \epsilon_{t+T}^k \tag{106}$$

where $T = 1$ month, $j =$ number of principal components, $PC_{t+T,ATM}^j$ is the $j'th$ principal component for the term structure (1wk, 1mth, 3mth, 6mth, 12mth) of at-the-money implied volatilities derived from all currencies, i, at time $t+T$, $PC_{t+T,RR}^j$ is the $j'th$ principal component for the term structure (1wk, 1mth, 3mth, 6mth, 12mth) of 25 delta and 10 delta risk reversals derived from all currencies, i, at time $t+T$, and $PC_{t+T,BF}^j$ is the $j'th$ principal component for the term structure (1wk, 1mth, 3mth, 6mth, 12mth) of 25 delta and 10 delta butterflies derived from all currencies, i, at time $t+T$, and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD).

The regression results of Equation (106) are shown in Table 52. Once again the number of principal components was set at $j = 3$. The contemporaneous regression results for each of the 4 kxk portfolios have encouraging adjusted R^2's, ranging from .36 to .41, with the EW portfolio slightly lower at 0.34.

Table 52: Portfolio FX Carry Trade Principal Component Regression Results

The coefficient estimates and adjusted R^2 of the following equation are shown

$$x^k_{t+T} = \gamma^k_0 + \sum_j \gamma^k_{1,j} PC^j_{t+T,ATM} + \sum_j \gamma^k_{2,j} PC^j_{t+T,RR} + \sum_j \gamma^k_{3,j} PC^j_{t+T,BF} + \epsilon^k_{t+T}$$

where x^k_{t+T} is the FX carry trade return for the kxk portfolio implemented at time t and realised at time $t+T$, k =(1,2,3,4,EW) representing the kxk and EW portfolios, T = 1 month, j = number of principal components, $PC^j_{t+T,ATM}$ is the $j'th$ principal component for the term structure (1 week, 1, 3, 6, 12 month) of at-the-money implied volatilities derived from all currencies, i, at time $t+T$, $PC^j_{t+T,RR}$ is the $j'th$ principal component for the term structure(1 week, 1, 3, 6, 12 month) of 25 delta and 10 delta risk reversals derived from all currencies, i, at time $t+T$, and $PC^j_{t+T,BF}$ is the $j'th$ principal component for the term structure (1 week, 1, 3, 6, 12 month) of 25 delta and 10 delta butterflies derived from all currencies, i, at time $t+T$, and i ∈(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD). Asterisks ***, **, and * indicate the coefficient estimates are significant at 1%, 5%, and 10% respectively using White standard errors. Data for the period January 2006 until December 2012 is used.

kxk	Intercept	PC1 ATM	PC2 ATM	PC3 ATM	PC1 RR	PC2 RR	PC3 RR	PC1 BF	PC2 BF	PC3 BF	Adj R^2
$1x1$	-0.0007	-0.0054 **	0.0009	-0.0097 ***	-0.0219 ***	0.0300 ***	0.0020	0.0262 ***	-0.0055 ***	0.0015	0.36
$2x2$	0.0017	-0.0082 ***	0.0004	-0.0086 ***	-0.0252 ***	0.0261 ***	0.0040 ***	0.0324 ***	-0.0099 ***	-0.0002	0.38
$3x3$	0.0030 ***	-0.0079 ***	-0.0015	-0.0060 ***	-0.0180 ***	0.0186 ***	0.0024 ***	0.0256 ***	-0.0070 ***	0.0017 *	0.41
$4x4$	0.0033 ***	-0.0080 ***	-0.0020 **	-0.0029 ***	-0.0160 ***	0.0150 ***	0.0016 **	0.0238 ***	-0.0048 ***	0.0003	0.37
EW	0.0007	-0.0052 ***	0.0001	-0.0010 **	-0.0117 ***	0.0118 ***	0.0030 ***	0.0182 ***	-0.0040 ***	-0.0017 ***	0.34

Extending this principal component framework to forecast one step ahead portfolio FX carry trade returns and evaluate it's performance, as in Section 19.1, requires Equation (106) to be rewritten as :

$$x_{t+T}^{k} = \gamma_0^k + \sum_j \gamma_{1,j}^k PC_{t,ATM}^j + \sum_j \gamma_{2,j}^k PC_{t,RR}^j + \sum_j \gamma_{3,j}^k PC_{t,BF}^j + \epsilon_t^k \quad (107)$$

where $T = 1$ month, $j =$ number of principal components, $PC_{t,ATM}^j$ is the $j'th$ principal component for the term structure of at-the-money implied volatilities derived from all currencies, i, at time t, $PC_{t,RR}^j$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta risk reversals derived from all currencies, i, at time t, and $PC_{t,BF}^j$ is the $j'th$ principal component for the term structure of 25 delta and 10 delta butterflies derived from all currencies, i, at time t, and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD).

Using the same methodology as Section 19.1 a series of one observation ahead forecasts are generated by re-calculating a new set of principal components to use in Equation (107) for each iteration. The same forecast performance statistics are calculated, the results of which are shown in Table 53.

Unfortunately once again the one observation ahead forecast results for the five portfolios considered are very underwhelming. In each case the forecasted portfolio FX carry trade returns underperformed the naive 'no change'

Table 53: Portfolio FX Carry Trade Principal Component Forecast Results

One step ahead forecasts using the coefficient estimates from :

$$x_{t+T}^{k} = \gamma_{0}^{k} + \sum_{j} \gamma_{1,j}^{k} PC_{t,ATM}^{j} + \sum_{j} \gamma_{2,j}^{k} PC_{t,RR}^{j} + \sum_{j} \gamma_{3,j}^{k} PC_{t,BF}^{j} + \epsilon_{t}^{k}$$

where $T = 1$ month, $j =$ number of principal components, $PC_{t,ATM}^{j}$ is the $j'th$ principal component for the term structure (1 week, 1, 3, 6, 12 month) of at-the-money implied volatilities derived from all currencies, i, at time t, $PC_{t,RR}^{j}$ is the $j'th$ principal component for the term structure (1 week, 1, 3, 6, 12 month) of 25 delta and 10 delta risk reversals derived from all currencies, i, at time t, and $PC_{t,BF}^{j}$ is the $j'th$ principal component for the term structure (1 week, 1, 3, 6, 12 month) of 25 delta and 10 delta butterflies derived from all currencies, i, at time t, and $i \in$(AUDUSD, CADUSD, CHFUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, SEKUSD). Regression forecast errors $x_{t+T+1d}^{error} = \widehat{x}_{t+T+1d} - x_{t+T+1d}$ are calculated which enables the summary statistics RMSE, MAD, MAPE to be calculated. Lagged forecast errors where the forecast value is equal to the previous observed value, $x_{t+T+1d}^{error} = x_{t+T} - x_{t+T+1d}$, are also calculated for comparison. Average monthly cash flow returns and standard deviations are shown. Daily data for the period January 2006 until December 2012 is used.

		Carry trade return	Lagged forecast error	PC Regression foreast error
$1x1$	Average CF	0.0005		
	Std Dev CF	0.0511		
	RMSE		0.0213	0.0408
	MAD		0.0136	0.0311
	MAPE		214.38	977.60
$2x2$	Average CF	0.0023		
	Std Dev CF	0.0439		
	RMSE		0.0161	0.0348
	MAD		0.0110	0.0269
	MAPE		134.13	322.15
$3x3$	Average CF	0.0022		
	Std Dev CF	0.0312		
	RMSE		0.0115	0.0245
	MAD		0.0079	0.0189
	MAPE		301.46	667.55
$4x4$	Average CF	0.0013		
	Std Dev CF	0.0252		
	RMSE		0.0095	0.0205
	MAD		0.0065	0.0154
	MAPE		132.88	287.74
EW	Average CF	-0.0002		
	Std Dev CF	0.0175		
	RMSE		0.0070	0.0147
	MAD		0.0046	0.0112
	MAPE		241.56	733.74

forecast. This is a disappointing result in light of the strong contemporaneous results, albeit consistent with the single currency FX carry trade return results in Section 19.2.

21 Conclusions

The contemporaneous regression results for the single currency and portfolio FX carry trade returns in Sections 19.1 and 20 were encouraging and consistent with the literature. Earlier work has established the contemporaneous relationship between the returns to the FX carry trade and FX volatility albeit with the main body of previous work using historical based measures for FX volatility. In addition, given the consensus view that implied FX volatility is a superior indicator of future FX volatility than historical volatility, and the fact that the dynamics of the implied FX volatility surface were implicitly introduced into the explanatory variables, the contemporaneous results seemed to support this. In the case of the single currency FX carry trades two approaches to specifying the inputs to the FX option implied volatility surface were considered. The first, based on examining the correlations of the inputs in the FX option implied volatility surface resulted in an average contemporaneous adjusted R^2 of 0.18, whilst the second approach using principal component analysis resulted in an average contemporaneous adjusted R^2 of 0.25. In the case of the portfolio FX carry trade returns the principal component specification resulted in an average contemporaneous adjusted R^2

of 0.37.

The one step ahead forecasting of the FX carry trade returns yielded poor results for both single currencies and portfolios of FX carry trades. This was a disappointing result, especially in light of the above contemporaneous results. There are several points worth considering in reviewing these results. Firstly, the data sample for which implied FX volatility surface data was available had a common start date across all currencies considered of January 2006, running through until December 2012. This period borders the global financial crisis which, as seen in Section 7.1, was a period of significant volatility for the FX carry trade. This was also a period of significant volatility for FX implied volatility and the inputs to the implied volatility surface, as seen briefly in Section 17.2. This period was so difficult for the FX carry trade that in the case of the single currency FX carry trades the average monthly returns based on daily data for the sample period in question (Tables 48 and 49) were in fact negative for all currencies other than AUDUSD and NZDUSD. This is in contrast to the average returns for the longer term spot data sample that was used in Section 5, whose results are displayed in Tables 5 and 6, where all currencies had positive average returns.

The other point to keep in mind is that in order to accurately forecast FX carry trade returns this essentially requires the ability to forecast FX spot rates. As seen in Section 4 it is possible to decompose the returns to the FX carry trade into an interest rate component and an FX component. To accurately forecast FX carry trade returns requires the accurate forecasting of this FX component, albeit with a positive or negative scalar depending on the direction of the FX carry trade. However, traditional macroeconomic modelling of exchange rates has struggled to outperform a random walk model (Meese and Rogoff 1983, Meese 1990). Recently, a microstructure branch of literature has emerged that has provided some promising results. These papers have demonstrated a link between the daily exchange rate movements and order flow, where order flow is defined as the net of buyer and seller initiated currency transactions (Evans and Lyons 2002, 2005). Data access issues aside, looking at the interplay between high frequency FX order flow and implied volatility surfaces would be an interesting exercise to see if this can add some forecasting ability to the implied volatility approach considered here.

So regrettably despite establishing the contemporaneous link between implied FX option volatility and returns to the FX carry trade, there appears to be essentially no ability for the ex-ante inputs into the FX option implied volatility surface to explain the ex-post returns to the FX carry trade.

Chapter IV

Concluding Remarks

Chapter 1 discussed the background of the FX carry trade and looked in detail at the various methods by which it can be constructed. Based on both single currency and portfolio construction methods the returns to the FX carry trade were calculated and analysed. The FX carry trade returns were then decomposed into their respective interest rate components and FX components. In the context of UIP this decomposition yielded some interesting results, pointing to possible support of UIP for typically low yielding currencies. However, further investigation showed that UIP was formally rejected. A look back in history at some of the major economic events revealed that the FX carry trade is susceptible to significant drawdowns during periods of economic turmoil. This notion of there being a 'normal' state and a 'crisis' state was supported by applying a Markov switching regime. Finally, in light of concerns expressed by providers of investible FX carry indices about execution risk a comparison of FX carry trade returns for several execution dates within a month revealed that returns were not sensitive to the choice of execution date within a month.

Chapter 2 examined the impact of imposing both a theoretical stop-loss framework and a sample of hedge fund stop-loss frameworks on the available returns to the FX carry trade, as defined in Chapter 1. Imposing the theoretical stop-loss framework on the FX carry trade resulted, in general, in a reduction in annualised returns and lower annualised standard deviations of returns. The hedge fund stop-loss policies were obtained by interviewing industry participants and the application of these policies to the FX carry trade represents a new contribution to the literature. The general result was that the FX carry trade was found to be too volatile to to survive the hedge fund stop-loss policies surveyed. This result casts serious doubt on the ability of hedge fund traders to earn the returns reportedly available in the FX carry trade or alternatively hedge fund traders are implementing the FX carry trade in a manner different to how the FX carry trade is presented in the academic literature. Unfortunately using CFTC futures data did not provide any significant ability to model the series of stop loss signals generated from the imposition of the hedge fund stop-loss policies. If it was possible to obtain OTC FX flow data, split by client type, from a major market making bank the natural extension to this work would be to see if such a data set could explain the hedge fund stop-loss triggers within the FX carry trade.

Chapter 3 examined the ability of the of the FX option implied volatility surface to explain and predict the FX carry trade returns. After explaining the theory and market conventions of FX options, a strong contemporaneous relationship between the FX option implied volatility surface and FX carry trade returns was confirmed. This is consistent with the literature which has used various volatility measures to help explain the FX carry trade returns. Disappointingly the one step ahead forecasting performance of using the FX option implied volatility surface to predict FX carry trade returns was very poor, under performing the naive no change prediction. Whilst on the surface this result is disappointing it can be rationalised somewhat given that the period considered captures the global financial crisis and baring in mind that to accurately forecast the FX carry trade essentially requires the ability to forecast FX spot rates. Once again if it were possible to obtain FX flow data, as described above, this micro structure approach to modelling FX spot rates in conjunction with the the FX option implied volatility surfaces would would provide a potentially interesting approach to forecasting FX carry trade returns.

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