A Risk-Oriented Model for Factor Timing Decisions

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The market conditions of recent years have challenged a lot of quantitative investment strategies. Traditionally, these strategies rely on (static) style/factor weightings, as if stock-selection factors were factual and permanent. Stock-selection factors, however, contain timing risks. Depending on the investment horizon and market conditions, risk may dominate return. This has been evident in many instances following the 2007 to 2008 episodes. Cross-asset de-levering/sell-off, overcrowded investment strategies, and challenging macroeconomic conditions have resulted in significant style/factor volatility and in sudden and severe shifts of factor predictability (see Khandani and Lo [2011], Li and Sullivan [2011]).[1] The new environment has thus made it crucial that quantitative investors consider more dynamic approaches to their style/factor selection and/or weighting.

In this article, we develop a framework for dynamic factor weighting that is designed to accommodate sudden changes in factor predictability. Factor efficacy is related periodically through time to factor portfolio risk. We facilitate this premise through an appropriate econometric methodology and a broad set of sensible risk indicators/independent variables. Risk in our framework contains the systematic and macroeconomic risk of the factor portfolio, as well as factor portfolio risk concentration. Recent works emphasise the importance of these facets of risk for monitoring equity market vulnerability (Berger and Pukthuanthong [2012], Kritzman et al. [2011], Sullivan and Xiong [2012]).

To quantify the effect of risk and other factor portfolio characteristics on factor predictability, we apply classification-tree analysis. This statistical approach determines the proper hierarchy and interaction of all independent variables, which is crucial for complex phenomena such as factor predictability. This complexity is evident in Daniel and Moskowitz [2013], who conclude that momentum crashes occur following market declines, when market volatility is high, and are contemporaneous with market rebounds.

Our study provides new and useful practical insights for active equity portfolio managers who use multi-factor equity selection models. We find that significant economic benefits accrue to dynamic factor weighting. A simple dynamic factor weighting approach results in an increase of the reward-to-risk ratio relative to a passive multi-factor portfolio from 0.12 to 0.33 in our sample, after transaction costs. Moreover, we find that the benefits can be magnified when dynamic factor weighting is pursued with a sophisticated model. The reward-to-risk ratio of a multi-factor portfolio that is constructed with the sophisticated model we advocate generates a reward-to-risk ratio of 0.52 after transaction costs, i.e., approximately four times that of a static approach and about one and a half times that of an alternative dynamic approach based on momentum.

Furthermore, our study extends the academic literature on factor timing. The majority of studies to date are focused on factor or style rotation. The perspective in these studies, as opposed to ours, is to reconcile the time-series behavior of segments of the stock markets. (Segments comprise stocks with common characteristics, such as book-to-price ratio.) These studies mainly have practical implications for top-down decisions; that is, to manage the exposure of a portfolio to different investment styles, or to generally pursue investments in particular segments of the market through style index funds, ETFs, or other investment vehicles. Indicative studies in this strand of literature include Kester [1990], Case and Cusimano [1995], Sorensen and Lazzara [1995], Kao and Shumaker [1999], Levis and Liodakis [1999], Ahmed et al. [2002], Bauer et al. [2004], Desrosiers et al. [2006], Arshanapalli et al. [2007], L'Her et al. [2007], and Miller et al. [2013]. Our article is related to these studies, but is distinct in that we focus on the time-varying, cross-sectional predictability of alpha factors and the practical implications for stock-selection decisions.

Finally, our article also contributes to an emerging strand of literature that emphasizes the importance of ex ante measures of risks for managing factor portfolio crashes. In this spirit are works by Barosso and Santa-Clara [2014] and Daniel and Moskowitz [2013]. Both investigate the price momentum factor. We complement these studies by providing further empirical evidence and insights on the type of risks and manner in which these risks are related to momentum predictability. Moreover, we extend this literature by studying the effect of risk on the predictability of other common alpha factors, and by suggesting a comprehensive modeling framework that can capture several stylized facts related to factor failure.

BACKGROUND ON MULTIPLE ALPHA MODELING AND FACTOR WEIGHTING

Our analysis considers a portfolio manager that uses multiple alpha models to forecast equity returns. To illustrate the basic ideas in multiple alpha modeling and factor weighting, let's denote with $\alpha_i^{F_k}$ the expected return of the stock of firm i at time t for time $t + 1$ that is attributable to the alpha factor F_k. Multi-factor alpha models aggregate multiple factor information into a single expected return. So if we use K alpha factors, the composite expected return for each stock, $\alpha_{i,t}$, is computed as:

$$\alpha_{i,t} = \sum_{k=1}^{K} w_k \alpha_{i,t}^{F_k} \tag{1}$$

In Equation (1) factor weights, w_k, are constant over time. This is consistent with the notion that the cross-sectional predictability of alpha factors is permanent. Factor timing builds on the idea that alpha factor predictability changes over time, and hence factor weights should also vary. We can thus re-write Equation (1) as:

$$\alpha_{i,t} = \sum_{k=1}^{K} w_{k,t} \alpha_{i,t}^{F_k} \tag{2}$$

where $w_{k,t}$ denotes the variable factor weight. Our objective is to measure and examine the benefits of varying factor weights. Factor weights in this context vary according to some model of factor predictability. For example, $w_{k,t}$ takes a positive value if the factor is expected to predict returns positively. We use the information coefficient (IC hereafter) to measure a factor's forecasting power. IC is defined as the cross-sectional correlation between the security return forecasts coming from a factor and the subsequent actual returns for securities (Grinhold [1989]).

DYNAMIC FACTOR-WEIGHTING MODELS

We determine factor predictability with two approaches: a simple approach that builds on factor predictability continuation, i.e., momentum, and a sophisticated statistical model that encompasses a broad array of novel, relevant variables.

The Simple Model

In the simple model, our benchmark for dynamic factor weighting, the IC forecast is obtained as the time-series average of realized ICs over a predetermined period of time. This approach is ad-hoc in nature, but anecdotal evidence suggests that it is popular among practitioners. It is also consistent with the style return momentum idea that has been discussed in prior academic works (e.g., Clare et al. [2010], Chen and De Bondt [2004]). We can thus write the time t expected IC as:

$$E_t^{IC}(IC_{t+1}) = \frac{1}{N}\sum_{t-N+1}^{t} IC_t \tag{3}$$

where N is the number of months over which ICs are averaged, and IC is computed with monthly data. Therefore we conjecture that an alpha factor positively predicts future returns if it has on average positively predicted returns in a selected period of time in the past. This approach is myopic with respect to the economic environment that prevails and the variables that may cause the cyclicality of factor predictability or factor failures.

The Sophisticated Model

The sophisticated model we advocate uses classification-tree analysis. Classification decision-tree analysis (DT hereafter) is a multivariate statistical technique that explores conditional relationships between a dependent variable and a set of independent variables.[2] Relative to standard linear regression models, the DT methodology is not subject to strict assumptions of linearity and normality, it is robust in handling outliers, and it is capable of uncovering multiple structures in the data set. These features are particularly relevant for the problem at hand. In our setting, we do not know a priori which variables are important; furthermore, we do not know whether the structure of the relationship between the relevant variables and factor predictability is single or conditionally multiple; and last, we know that factors often experience extreme events (fat tails). Sophisticated models, with the capacity to accommodate a wider array of possible outcomes and non-normal distributions, are also advocated in recent studies as essential tools in the new market environment (e.g., Li and Sullivan [2011]).

Although the statistical technique that we adopt is important, the model's predictive variables are equally important. One of the contributions of our article is in fact the set of the variables we use. The full set of independent variables we consider comprises a group of micro- and a group of macro/market-oriented variables. The micro-oriented variables measure the fundamental characteristics of the factor portfolio. The macro/market-oriented variables measure the systematic risk, as well as the risk concentration of the factor portfolio. In unreported analysis we document that macroeconomic risk has recently become extremely important and in particular has accounted for more than 50% of the total explained risk in 2010 to 2012. All independent variables are measured at the factor portfolio level. The following example discusses the nature and intuition of the variables of our model.

Assume we want to determine if the book-to-price ratio at time t will positively or negatively predict equity returns at time $t + 1$ for S&P 500 stocks. We apply the DT model with a set of independent variables, all measured at time t for the book-to-price ratio factor portfolio, i.e., a portfolio of stocks that is long the S&P 500 stocks whose book-to-price ratio is in the top 20% and short the S&P 500 stocks whose book-to-price ratio is in the bottom 20%. These variables contain bottom-up relative characteristics of the factor portfolio (fundamental characteristics) and a number of risk measures (macroeconomic and market exposures of factors).

One of the characteristics is momentum (a full list follows). In our example, this is calculated as the equally weighted 12 months (excluding the last month) performance of every stock in the top portfolio of the book-to-price ratio factor, divided by the same measure for the bottom portfolio of the book-to-price ratio factor. Our prior is that, if stocks in the top portfolio of the book-to-price factor exhibit stronger momentum, the book-to-price ratio will predict returns positively over the next period. With respect to the risk measures, we calculate (among others) the risk concentration of the factor portfolio. We hypothesize that if the risk of the factor portfolio is concentrated, i.e., a large proportion of the total variability can be explained by a set of systematic factors, the factor portfolio becomes vulnerable[3] and hence book-to-price ratio may fail to predict returns over the next period. The DT model will determine whether these measures take priority over others and/or how and whether they matter at all for determining the predictability of book-to-price ratio ratio.

Exhibit 1 tabulates all independent variables. The first column of Exhibit 1 lists the bottom-up measures, which can generally be classified in four categories, valuation (e.g., earnings yield), growth (e.g., earnings

EXHIBIT 1
Predictive Variables

Fundamental Characteristics (Bottom-Up Measures)	Factor Macroeconomic and Market Exposures (Betas of Factor Portfolio Returns) and Risk Concentration	
Earnings Yield	Inflation Shock	Betas
Earnings Growth Rate	Long-Term Interest Rates	Betas
Return on Equity	Short-Term Interest Rates	Betas
Book Yield	Credit Spread	Betas
Dividend Yield	Oil Price	Betas
Historical Volatility	Dollar Exchange Rate	Betas
Momentum	Market	Betas
Earnings Revisions	Small Cap Premium	Betas
Forward Earnings Yield	Growth/Value Premium	Betas
Market Beta	Variance Macro/Total Variance	Risk Concentration
	Variance of Size/Total Variance	Risk Concentration
	Variance of Style/Total Variance	Risk Concentration
	Variance of Market and Sectors/Total Variance	Risk Concentration

growth rate), momentum (e.g., earnings revisions), and risk (e.g., market beta). They are all calculated as the ratio of the respective equally weighted characteristic of every stock in the top portfolio of the factor, divided by the equally weighted characteristic of every stock in the bottom portfolio of the factor. We compute the fundamental characteristics of the factor portfolio monthly.

The second column of Exhibit 1 presents the macroeconomic and market exposure measures that can potentially correlate with factor predictability. We obtain these measures from Citi's U.S. Risk Attribute Model (USRAM). The USRAM is a highly regarded risk analysis model that was first introduced in 1989 and has since been widely used by equity portfolio professionals. It is a macroeconomic time-series risk model. Intuitively, the USRAM model can be viewed as a Fama and French [1993] three-factor model augmented with macroeconomics risk factors. We use USRAM to obtain risk attributes of the factor portfolio. These attributes include risk exposures, (i.e., betas) of the factor portfolio to the variables, and measures of factor risk concentration, i.e., the percentage of the total factor return variance that can be explained by groups of factors (macro, style, size, market/sectors). We thus measure both directional factor portfolio systematic risk exposures, as well as factor portfolio risk concentration, which some have argued is a measure of systemic risk (Kritzman et al. [2011]). Both sets of risk attributes are calculated monthly through risk analysis of the factor portfolio.

In our implementation of the DT model, the dependent variable is (as in Equation (3)) the expected IC, $E_t^{DT}(IC_{t+1})$ and the independent variables are a set of variables that we consider potentially important for predicting factor failure. We can thus write the time t expected IC, as

$$\mathrm{E}_t^{DT}(\mathrm{IC}_{t+1}) = f(\mathrm{Fundamentals}_t, \mathrm{Risk}_t) \tag{4}$$

where $f(\cdot)$ denotes the DT model. As highlighted in Exhibit 2, the DT analysis determines the optimal sequence for screening with these variables, as well as the optimal screening criteria.

Data and Portfolio Construction

Our empirical analysis is conducted with stocks from the S&P 500 for the period December 1978 to August 2012. Every month we rank stocks independently on the basis of their past 12 months (excluding the last month) of performance, four-week change in FY1 estimates, book-to-price ratios, and earnings-to-price ratios. The former two alpha factors are typical in the implementation of momentum strategies, while the latter are commonly used in value strategies. Based on these rankings, we formulate four common quantitative equity alpha factors: momentum, earnings revision, book yield, and earnings yield, respectively.[4]

Our baseline analysis involves monthly rebalancing. To determine which stocks enter the long and short legs of the investment portfolio, we need two inputs. One is the cross-sectional z-score of each stock with respect to a factor (we assume that z-scores translate to expected return). The other input is the IC forecast of each factor. In our baseline analysis we assign positive weights to a factor when it is predicted to have a positive IC, and zero otherwise. For example, suppose that the cross-sectional z-scores for a stock are 0.76, 0.73, −0.20, and −0.56, with respect to its past 12 months (excluding last month) of performance, four-week change in FY1 estimates, book-to-price ratio ratios, and earnings-to-price ratios. Suppose also that the predicted ICs for momentum, earnings revisions, book-to-price, and earnings-to-price are +ve, −ve, −ve, and +ve, respectively. The predictions for momentum and earnings-to-price ratio suggest that stocks with high (low) past performance and high (low) earnings-to-price ratios are expected to post high (low) future returns. The predictions for earnings revisions and book-to-price ratio ratios suggest that—contrary to the stylized facts—stocks with high (low) earnings momentum and high (low) book-to-price ratios are expected to post low (high) future returns. To compute the composite z-score, we dismiss the counter-intuitive information and make the following calculation: $[0.76 + (-0.56)]/2 = 0.10$, i.e., the composite z-score is an equally weighted z-score of the ICs with positive prediction. This example illustrates how views on factor predictability are translated (together with views on the stock's expected return) into an expected return for each stock. We pursue this calculation for all stocks in the S&P 500 every month. The investment strategy we test buys stocks in the top 20% of the composite z-score and shorts stocks in the bottom 20%.

We formally express the weighting schemes as follows:

$$w_{k,t} = \begin{cases} 1/K, & \text{EQ-Weighted} \\ \mathrm{E}_t^{IC}\left(\mathrm{IC}_{t+1}^k\right)^+ / \sum^{K^+} \mathrm{E}_t^{IC}\left(\mathrm{IC}_{t+1}^k\right)^+, \quad \text{if } \mathrm{E}_t^{IC}\left(\mathrm{IC}_{t+1}^k\right)^+, & \text{IC-Weighted} \\ 1/K^+, \quad \text{if } \mathrm{E}_t^{DT}\left(\mathrm{IC}_{t+1}^k\right)^+, & \text{DT} \end{cases} \tag{5}$$

where K equals four and K^+ denotes the number of factors with positive predicted IC. That is, we consider a strategy that assumes factors are permanent and invests passively on all alpha factors (EQ-W). This is intuitively a strategy with equal weights on value and momentum, a simple dynamic strategy that weights factors with positive predicted IC and gives a value of zero to factors with negative ICs, with predictions obtained through Equation (3) using a window of 12 months (IC-W). (The weights are the scaled contribution of the positive factor IC to the sum of all positive ICs). It is a sophisticated dynamic strategy that also uses equal weighting of factors with positive predicted IC and a value of zero for factors with negative ICs, with predictions obtained through Equation (4) (DT). We use monthly data from December 1978 to December 1998 to estimate Equation (4) for our first out-of-sample prediction. The model is recalibrated monthly using an expanding window. Our out-of-sample analysis covers the period January 1999 to August 2012.

In additional analysis, we investigate the impact of transaction costs. We consider a constant two-way transaction cost of 20 basis points, which we believe is in the normal range for a diversified equity basket with S&P 500 stocks. We also consider a more conservative cost of 40 basis points. Furthermore, we examine an alternate investment horizon by performing a similar backtest with quarterly rebalance. Finally, we test the benefits from shorting a factor when it is predicted to be out of favor.

EMPIRICAL ANALYSIS

A Static Tree

Before proceeding to the discussion of the portfolio construction results, in Exhibit 2 we present decision trees that analyse the relationship between the one-month-ahead IC of the four common alpha factors and the full set of independent variables (tabulated in Exhibit 1).[5] We focus on the tree estimated for book-to-price ratio (Exhibit 2, panel C) to elaborate the causalities and the intuition of the estimated relationships.

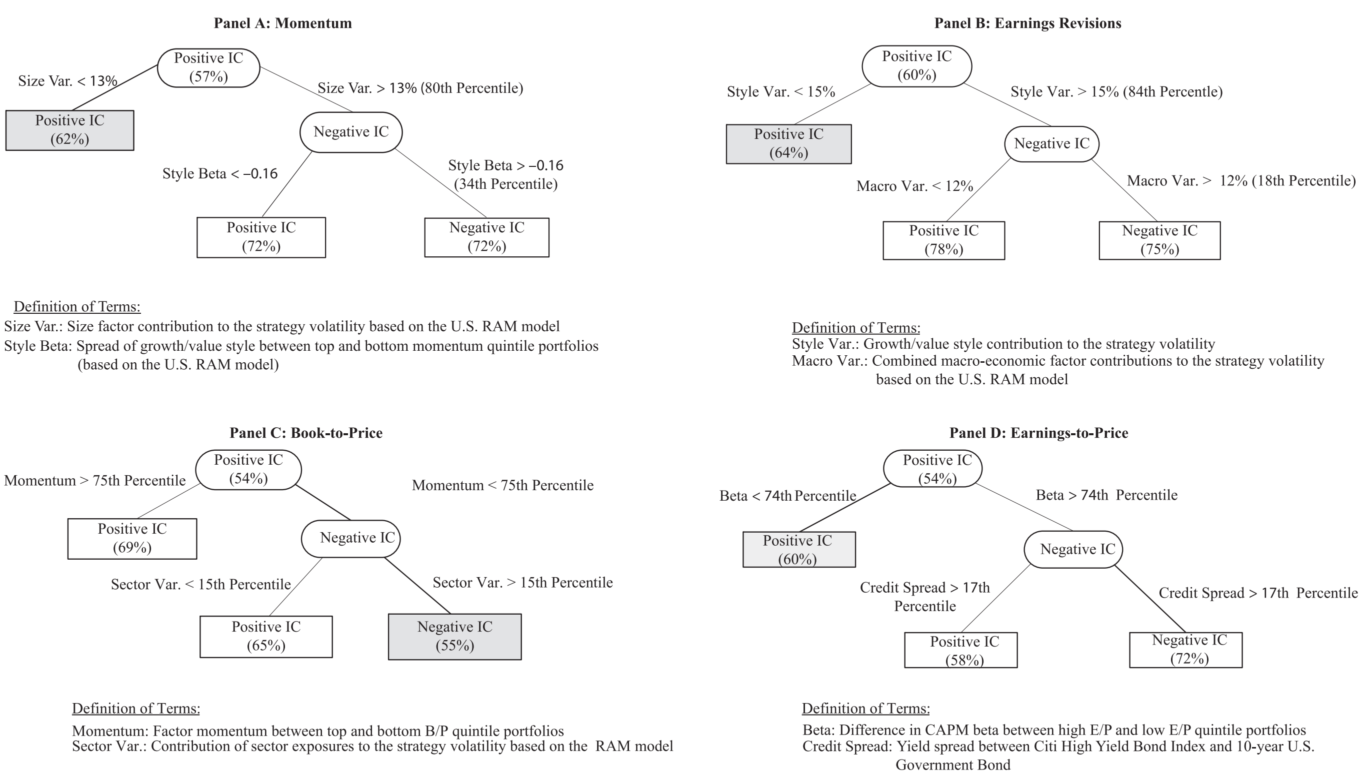
At the top of the tree, the primary variable is the expected IC of book-to-price ratio. It is indicated that 54% of the time, book-to-price ratio exhibited positive IC. The partitioning algorithm determined that, of all possible variables in Exhibit (1), momentum (i.e., the relative momentum of stocks in the top and bottom baskets of the factor portfolio, as discussed earlier) is the single most powerful variable in determining the book-to-price ratio's IC on a historical basis. According to this outcome, when the relative momentum of the top basket is high (i.e., in the top 25% of its historical values), there is an even higher likelihood (69% versus 54%) that the book-to-price ratio IC is positive.

There is a further improvement of historical predictability with secondary splits on the right hand side of the tree. Sector var (i.e., sector risk contribution to the total factor portfolio risk) and a splitting threshold at the 15th percentile of its historical values are determined to form the variable threshold that mostly differentiates positive and negative IC for the subgroup of samples that end on the right. Having conditioned on momentum, sector var has incremental predictive power. In particular, low momentum firstly and high-risk concentration will lead to even greater likelihood of negative IC (55% versus 46%). The DT analysis has additionally confirmed our intuition regarding the possible effect of the two variables on IC. Most importantly, it has identified which one takes over the other in the determination of IC, i.e., momentum first, followed by risk concentration.

Two general observations from panels A to C in Exhibit 2 deserve further attention. First, occurrences of the predicted scenario increase from the top of the

EXHIBIT 2

Decision Tree for Determining the Direction of the IC



tree to the bottom, and this increase is particularly high for negative IC. This observation suggests that the conditioning variables can increase our ability to predict future IC—and negative IC in particular. Second, the model identifies as relevant conditioning variables those that measure the factor's risk and risk concentration. This observation suggests that, regardless of the fact that factors' fundamental characteristics are considered as potentially relevant, in the majority of the cases the model identifies the risk attributes of the factor portfolio as more important.

Baseline Results

In Exhibit 3, we show the historical monthly performance of the models from January 1999 through August 2012. We plot the cumulative total-return index level of the portfolios obtained through the EQ-W, the IC-W, and the DT approaches. We assigned a value of 100 to each index at the end of December 1998. The results suggest that both approaches that engage dynamic factor weighting outperform the passive strategy. The IC-W portfolio presents with a terminal index value of 208, which constitutes an improvement of the terminal wealth over the EQ-W portfolio of about 70%. Moreover, the terminal index value of the DT approach reaches 344, which is about 65% higher than that of the IC-W, and about 180% higher than the EQ-W portfolio.

In Exhibit 4 we present several performance metrics that overall provide strong support for the sophisticated factor model we adopt. The IC-W model outperforms the EQ-W in terms of average arithmetic-annualized return by almost 4% per year. The outperformance of the DT over the EQ-W is about 8% per year. Comparing the DT and the IC-W portfolios with the EQ-W portfolios provides favorable assessments for the former for almost all metrics we report. The magnitude of the difference is even higher for the DT portfolios. In particular, the reward-to-risk ratio of the DT model more than triples relative to that of the EQ-W, from 0.18 to 0.58. The hit ratio improves by about an absolute 10%, from 55.49% to 64.02%, which is more than 15% in relative terms.

To rule out the possibility that the results of our analysis are concentrated in certain periods that make up for poor performance in other periods, we carry out a sub-period analysis. We split the sample in two almost equal sub-samples, i.e., from January 1999 to December 2005 and from January 2006 to August 2012, and repeat the analysis. These results, also reported in Exhibit 4, should be interpreted cautiously, given the relatively short time series they represent. The results indicate that the conclusions reached in the previous section for the

Exhibit 3

Historical Performance of Dynamic Factor Rotation Strategies

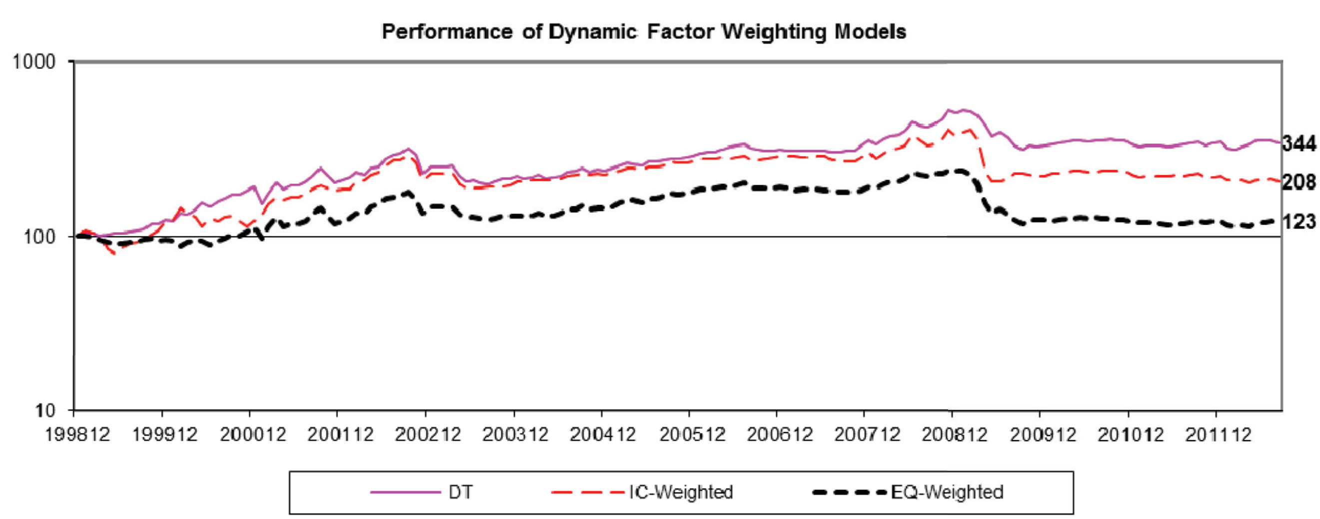


Exhibit 4

Descriptive Statistics of Multi-Factor Timing Strategies

Moderate Transaction Costs (20 basis points)

	EQ-W	IC-W	DT	EQ-W	IC-W	DT	EQ-W	IC-W	DT
	JAN 1999–AUG 2012			JAN 1999–DEC 2005			JAN 2006–AUG 2012		
Geometric Mean Return (Annualized)	1.49	5.37	9.06	8.41	14.13	15.19	–5.72	–3.77	2.66
Arithmetic Mean Return (Annualized)	3.03	7.80	10.89	10.29	16.52	17.66	–4.60	–1.35	3.79
P-value (H_0: Arithmetic Mean ≤ 0)	0.26	0.09	0.02	0.08	0.02	0.02	0.79	0.57	0.26
Standard Deviation (Annualized)	17.26	21.36	18.89	19.39	21.75	21.87	14.50	20.75	15.03
Return/Risk (Annualized)	0.18	0.37	0.58	0.53	0.76	0.81	–0.32	–0.07	0.25
P-value (H_0: Return/Risk ≤ 0)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
Hit Ratio	55.49	60.37	64.02	60.71	64.29	70.24	50.00	56.25	57.50
Average Return When Correct	3.23	3.88	3.79	4.07	4.70	4.42	2.17	2.90	2.97
Average Return When Wrong	–3.46	–4.41	–4.21	–4.11	–4.61	–5.49	–2.93	–4.22	–3.27
Turnover	448.18	374.28	524.74	440.08	303.03	508.97	456.58	448.20	541.09

whole period hold true in the sub-periods. We observe that all models perform better in the first sub-sample; however, the relative ranking of the models remains intact in the second sub-period. In fact, in the second sub-period we observe that the economic benefits of the DT model are more pronounced; it is the only model that produces positive average returns.

In unreported analysis we use a binomial distribution (and assumptions about its normality) to assess the statistical significance of the incremental value of the DT model over the IC-W model, as well as over the EQ-W model, with respect to their hit rates. In the full sample, we concluded that the DT model's incremental value over the IC-W was marginally statistically significant at the 5% significance level (*t*-statistic = 1.96). We also found that the DT model's incremental value over the EQ-W was highly statistically significant (*t*-statistic = 4.43). In the sub-samples, the respective statistics are 2.38 and 3.66 for the first sub-period and 1.79 and 0.45 for the second sub-period. We also tested similar hypotheses with respect to the mean returns of the three strategies and obtained qualitatively similar results. Given the relatively small number of observations in the sample and the effect of this on the calculated statistics, we suggest interpreting the statistics with caution.

ADDITIONAL RESULTS

Transaction-Costs Analysis

We carry out analysis in the full sample, but also in sub-samples, to examine the impact of transaction costs. We conduct two analyses: one that assumes a moderate transaction cost of 20 basis points each way and a more conservative one that assumes 40 basis points per round trip. Detailed transaction costs would have provided a more accurate representation of real-life returns. However, consultation with traders suggested that in the universe of stocks we analyse, these figures will provide a meaningful estimate of the first-order transaction-cost effect for our backtest.

Exhibit 5 tabulates the results of this analysis. The results are qualitatively similar to those presented in Exhibit 4. The relative ranking of models remains unchanged. The reward-to-risk ratio of the DT model more than quadruples relative to the EQ-W, from 0.12 to 0.52 with transaction costs at 20 basis points, and becomes an even higher multiple when the assumed transaction cost is 40 basis points, i.e., going from 0.07 to 0.47. Similarly, there is an improvement relative to the IC-W model by a factor of more than 1.5—from 0.33 to 0.52 and from 0.30 to 0.47 for transaction costs of 20 basis points and 40 basis points, respectively. This analysis suggests that the benefits of dynamic factor weighting with the sophisticated model documented in Exhibit 4 do not come at a significant implementation cost and remain economically significant.

Quarterly Rebalance

Factor risk is typically relatively stable, and hence monthly rebalance based on factor risk may seem unrealistic. The premise of the sophisticated model, however, is that it can accommodate structural changes in the relationship between IC and the independent variables. Frequent rebalancing will be meaningful only if such

EXHIBIT 5

Multi-Factor Timing Strategies After Transaction Costs – Monthly Rebalance

	EQ-W	IC-W	DT	EQ-W	IC-W	DT	EQ-W	IC-W	DT
	JAN 1999–AUG 2012			JAN 1999–DEC 2005			JAN 2006–AUG 2012		
Panel A: Moderate Transaction Costs (20 Basis Points)									
Geometric Mean Return (Annualized)	0.60	4.62	8.01	7.53	13.53	14.18	–6.63	–4.66	1.58
Arithmetic Mean Return (Annualized)	2.13	7.06	9.84	9.41	15.92	16.64	–5.51	–2.25	2.70
P-value (H_0: Arithmetic Mean ≤ 0)	0.32	0.11	0.03	0.10	0.03	0.02	0.84	0.61	0.32
Standard Deviation (Annualized)	17.24	21.35	18.88	19.37	21.74	21.85	14.47	20.73	15.01
Return/Risk (Annualized)	0.12	0.33	0.52	0.49	0.74	0.77	–0.38	–0.11	0.18
P-value (H_0: Return/Risk ≤ 0)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
Hit Ratio	53.05	58.54	63.41	58.33	63.10	69.05	47.50	53.75	57.50
Average Return When Correct	3.30	3.94	3.73	4.16	4.74	4.41	2.20	2.95	2.87
Average Return When Wrong	–3.35	–4.14	–4.23	–3.94	–4.51	–5.36	–2.86	–3.84	–3.36
Turnover	448.18	374.28	524.74	440.08	303.03	508.97	456.58	448.20	541.09
Panel B: Conservative Transaction Costs (40 Basis Points)									
Geometric Mean Return (Annualized)	–0.29	3.87	6.96	6.65	12.92	13.16	–7.54	–5.56	0.49
Arithmetic Mean Return (Annualized)	1.24	6.31	8.79	8.54	15.31	15.62	–6.42	–3.14	1.62
P-value (H_0: Arithmetic Mean ≤ 0)	0.40	0.14	0.04	0.12	0.03	0.03	0.87	0.65	0.39
Standard Deviation (Annualized)	17.22	21.34	18.86	19.35	21.73	21.84	14.45	20.70	14.99
Return/Risk (Annualized)	0.07	0.30	0.47	0.44	0.71	0.72	–0.45	–0.15	0.11
P-value (H_0: Return/Risk ≤ 0)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
Hit Ratio	52.44	57.93	62.20	57.14	63.10	66.67	47.50	52.50	57.50
Average Return When Correct	3.26	3.92	3.71	4.17	4.69	4.48	2.12	2.95	2.78
Average Return When Wrong	–3.38	–4.14	–4.17	–3.89	–4.56	–5.06	–2.94	–3.81	–3.44
Turnover	448.18	374.28	524.74	440.08	303.03	508.97	456.58	448.20	541.09

changes have a significant effect on portfolio performance. To test this conjecture, we consider an investor with quarterly rebalancing and pursue the analysis with the transaction-cost assumptions of the earlier section. Exhibit 6 presents the results of this analysis.

We observe that, even for quarterly rebalancing, the IC-W model outperforms the EQ-W in terms of annualized return. The improvement is economically sizable, but not of the magnitude we documented earlier. This suggests that there are benefits to monitoring factor risk in shorter investment horizons, regardless of the cost of the frequent rebalancing. Furthermore, we observe that the difference between the DT model and the IC-W model, by means of the return and risk of the respective portfolios, is only modest. However, even in this instance, we document a significant difference in the volatility of the two portfolios from 2006 to 2012: 13.79% for the DT versus 21.65% for the IC-W portfolio. This supports the argument that the DT model is effective in predicting factor failures that cause significant volatility and drawdowns in the constructed portfolio.

Factor Shorting

In this section we investigate the benefits that can accrue to investors when they use the proposed model to short alpha factors. To implement this idea, we simply set $w_{k,t} = -1$ when the factor is predicted to have a negative IC, and $w_{k,t} = 1$ when the factor is predicted to have a positive IC. For an example, refer to the cross-sectional z-scores for a stock we cited earlier: 0.76, 0.73, −0.20, and -0.56, with respect to its past 12 months (excluding last month) of performance, four-week change in FY1 estimates, book-to-price ratios, and earnings-to-price ratios. With predicted ICs for momentum, earnings revisions, book-to-price ratio, and earnings-to-price ratio, +ve, −ve, −ve, and +ve, respectively, the composite z-score for this stock becomes $[0.76 + (-0.73) + 0.20 + (-0.56)]/4 = -0.08$ (a value that was 0.10 in our baseline analysis). We find that, with transaction costs of 20 basis points and monthly rebalancing, when factors are allowed to be shorted in the form we describe earlier, the average return of the DT portfolio rises from 9.84% to 13.95% and the return to risk ratio from 0.52 to 0.74. The results are qualitatively similar with the alternate

EXHIBIT 6
Multi-Factor Timing Strategies After Transaction Costs – Quarterly Rebalance

	EQ-W	IC-W	DT	EQ-W	IC-W	DT	EQ-W	IC-W	DT
	JAN 1999–AUG 2012			JAN 1999–DEC 2005			JAN 2006–AUG 2012		
Panel A: Moderate Transaction Costs (20 Basis Points)									
Geometric Mean Return (Annualized)	–0.96	1.35	1.41	6.07	6.97	6.70	–8.30	–4.51	–4.11
Arithmetic Mean Return (Annualized)	0.36	3.86	2.98	7.70	9.35	8.81	–7.35	–1.91	–3.16
P-value (H_0: Arithmetic Mean ≤ 0)	0.47	0.26	0.26	0.13	0.13	0.13	0.92	0.59	0.72
Standard Deviation (Annualized)	16.00	21.74	17.42	18.04	21.84	20.22	13.29	21.65	13.79
Return/Risk (Annualized)	0.02	0.18	0.17	0.43	0.43	0.44	–0.56	–0.09	–0.23
P-value (H_0: Return/Risk ≤ 0)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
Hit Ratio	50.00	56.10	54.27	55.95	61.90	59.52	43.75	50.00	48.75
Average Return When Correct	3.13	4.03	3.52	4.03	4.31	4.19	1.93	3.66	2.66
Average Return When Wrong	–3.07	–4.41	–3.64	–3.66	–4.96	–4.35	–2.59	–3.98	–3.04
Turnover	174.65	186.58	206.50	166.64	156.86	198.20	182.97	217.39	215.11
Panel B: Conservative Transaction Costs (40 Basis Points)									
Geometric Mean Return (Annualized)	–1.30	0.99	1.00	5.74	6.65	6.30	–8.66	–4.93	–4.53
Arithmetic Mean Return (Annualized)	0.01	3.49	2.57	7.36	9.03	8.41	–7.71	–2.33	–3.57
P-value (H_0: Arithmetic Mean ≤ 0)	0.50	0.28	0.29	0.14	0.14	0.14	0.93	0.61	0.75
Standard Deviation (Annualized)	15.97	21.71	17.40	18.00	21.81	20.19	13.27	21.62	13.79
Return/Risk (Annualized)	0.00	0.16	0.15	0.41	0.42	0.42	–0.58	–0.11	–0.26
P-value (H_0: Return/Risk ≤ 0)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
Hit Ratio	50.00	56.10	54.27	55.95	61.90	59.52	43.75	50.00	48.75
Average Return When Correct	3.10	3.99	3.48	3.99	4.27	4.16	1.89	3.62	2.62
Average Return When Wrong	–3.10	–4.44	–3.67	–3.68	–4.97	–4.38	–2.61	–4.01	–3.08
Turnover	174.65	186.58	206.50	166.64	156.86	198.20	182.97	217.39	215.11

transaction-cost fees; the average return of the DT portfolio rises from 8.79% to 12.76% and the return to risk ratio from 0.47 to 0.65.

Collectively, we find that the economic benefits from dynamic factor weighting are significant, especially when the timing of factors is pursued conditional on factor risks. These benefits are pervasive and robust to transaction costs. Furthermore, dynamic factor weighting appears also attractive for investors with quarterly portfolio rebalancing; however, it is more meaningful for investors with shorter investment horizons, i.e., monthly rebalancing.

Why Dynamic Factor Weighting with DT Outperforms?

In this section we explain why we believe the DT model we adopt is a powerful framework for managing the exposures to stock-selection factors. We argue that both the statistical technique that underlies the model and the set of predictive variables we use jointly drive the outperformance we document in the empirical analysis. We start our discussion with some observations with respect to the predictive variables and the statistical framework, and then highlight how our empirical evidence ties with these attributes of the model.

The model's most important novel element is the integration of a set of broad risk variables in the set of independent variables. Recent evidence suggests that systemic risk measures are important for market-timing purposes (Kritzman et al. [2011]), as well as for identifying instances of increasing likelihood of market crashes (Berger and Pukthuanthong [2012]). The potential effect of systemic risk on factor predictability is recognised in Khandani and Lo [2011], who argue that the quant meltdown of August 2007 was largely a consequence of systemic risks posed by the hedge-fund industry. Somewhat in this spirit, Barosso and Santa-Clara [2014] and Daniel and Moskowitz [2013] also document that a broad measure of risk, i.e., momentum volatility, is related to momentum failures.

The DT model encompasses variables that are potentially able to capture two different facets of risk:[6] first, the systematic risk and second, the risk concentration of a factor. Increases in systematic risk are a possible indication of trading commonality and hence market vulnerability (Sullivan and Xiong [2012]). Pojarliev and Levich [2011] in fact measure factor crowdness based on this premise. Risk concentration is another facet of systemic risk for which we proxy through variables that measure the fraction of the total factor return variance that is explained by groups of factors, i.e., macro, style, size, and market/sectors. We therefore expect that such variables can improve our ability to predict factor failure.

Equally critical is the statistical technique that facilitates the relationship between the predictive variables and the IC of the alpha factor. We acknowledge that parametric approaches, e.g., predictive regressions, are more attractive from an implementation standpoint. However, parametric representations may not be a good approximation of the reality of markets, especially in their current, complex and adaptive form. The evidence in the literature suggests that the DT approach can offer significant benefits in stock-selection decisions, both in normal times (Sorensen et al. [2000]), but also in more turbulent times (Zhu et al. [2012]). Most authors attribute the outperformance of the DT approach primarily to its ability to deal with non-normality in equity returns (i.e., skewness and fat tails), which is common in shorter time horizons, and the non-linearity in the relationship between dependent and independent variables. Factor returns may exhibit nonlinear, asymmetric payoffs, as Daniel and Moskowitz [2013] find for momentum. We thus expect that the DT model will sufficiently accommodate complex relationships and extreme market conditions.

The empirical evidence we present is clearly in line with our conjectures and explains why the DT model is a powerful framework for factor timing. Exhibit 2 indicates that, for the sample period for which the trees were built, the important determinants of factor predictability, i.e., positive versus negative IC, were primarily risk concentration variables. The variables that were identified as important by the DT model do not change much over other periods of recalibrations of the model. Moreover, the model identifies multiple structures in the data. For example, in panel B we see the statement that that risk concentration on value/growth is only conditionally (on high macro-risk concentration) an indicator of negative IC. Intuitively, this suggests that only when risk is largely concentrated on value/growth and macro variables is the earnings-revision alpha factor likely to fail. It is not possible to represent this conditional relationship with a standard linear model.

The evidence in Exhibit 4 and Exhibit 5 shows how the DT can be of particular importance in more turbulent periods. The empirical analysis indicates that, over calmer periods, the benefits over a simple model may be marginal, although whether that is driven solely by shorter periods of trending markets must be further investigated. In the second half of our sample, however, when evidently the predictability of alpha factors exhibited severe and sudden reversals, the DT model is far better than the two alternative models. It presents with much higher returns and much lower volatility than the simple factor-weighting model, even though the models' hit ratios are similar. Strikingly, it is the only model that manages to generate positive average returns with relatively low volatility. Barosso and Santa-Clara [2013] and Daniel and Moskowitz [2013] also document significant outperformance over the same period.

Overall, we conclude that the DT approach works better than the other approaches we test because of both the relevance of the predictive variables it encompasses and the statistical technique that it uses to combine them to forecast possible factor failure. The benefits are particularly present in market conditions that we have observed recently—i.e., a macro-driven environment with vulnerable markets—which many argue are likely to prevail in the future (Li and Sullivan [2011]).

CONCLUSION

Alpha factors are built to perform well over time, on average. There are instances when they do not, and knowing these instances ex ante can be a significant source of added value for investors. We argue that factor failure is a function of its broad risk and propose appropriate variables to measure it. We are interested in both a factor's systematic risk and its risk concentration. We combine these variables with a nonparametric model that uses classification-tree analysis to determine the optimal screening criteria, as well as the optimal sequence for screening with the conditioning variables. This approach is well behaved in modeling highly dynamic systems.

Our empirical analysis reaches the conclusion that investors can benefit significantly by weighting factors according to this approach. In particular, the reward-to-risk ratio of a multi-factor portfolio that is constructed with the model we advocate generates a reward-to-risk ratio of 0.52, i.e., approximately four times that of a static approach and about one and a half times that of an alternative dynamic approach based on momentum. The results we produce are robust in the sub-periods we examined. In relative terms, our results are better in the second half of our sample, which includes the global financial crisis as well as other severe episodes in the financial markets.

ENDNOTES

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[1]In unreported analysis of the four common alpha factors we use in this study, for the period March 2009 to August 2012, we document between four and six changes in factor predictability during the 10 sub-periods we examine.

[2]DT analysis was first introduced in financial research by Frydman et al. [1985] and Marais et al. [1984]. DT has since been used by a number of other authors in various areas of financial research. An excellent treatment of the DT methodology in the context of investment management can be found in Sorensen et al. [2000]. Several other authors highlight the merits of the DT methodology for investment management applications (Kao and Shumaker [1999], L'Her et al. [2007], Zhu et al. [2011], Zhu et al. [2012], Miller et al. [2013]).

[3]Kritzman et al. [2011] analyse aggregate equity market vulnerability in a similar context. They argue that a high fraction of total market return variability explained by the variability of a set of latent factors is an indication of high systemic risk and hence an indication of market vulnerability.

[4]This set of factors represents commonly used alpha factors. Khandani and Lo [2011] use similar quantitative equity factors in their analysis (i.e., book-to-market ratio, earnings-to-price ratio, cash flow-to-market ratio, price momentum, and earnings momentum). The entire analysis has been carried out with a large cross-section of other alpha factors and the results we obtain are qualitatively similar. These results are available from the authors on request.

[5]These trees are built for the entire sample period. They are potentially somewhat different from the trees we build at each point of time in our out-of-sample analysis. Our out-of-sample analysis considers trees that are built based only on the data known at the time the model is calibrated. We note, however, that the structures evolve very slowly provided that they are also estimated with an expanding window.

[6]Bisias et al. [2012], in their comprehensive survey of systemic risk analytics, argue that systemic risk is complex and adaptive, and hence more than one measure is needed to capture it.

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