22/06/2025

Chapter 02

Number System and Codes

Binary number system : (Base 2)

3	2	1	0	$\rightarrow$ Position number
2^3	2^2	2^1	2^0	$\rightarrow$ Position weight
8	4	2	1	

Binary to Decimal:

$(\overset{8}{1}\overset{4}{1}\overset{2}{0}\overset{1}{1})_2 = (13)_{10}$

$(101)_2 = (5)_{10}$

Decimal to Binary:

$(9)_{10} = (\overset{8}{1}\overset{4}{1}\overset{2}{0}\overset{1}{1})_2$

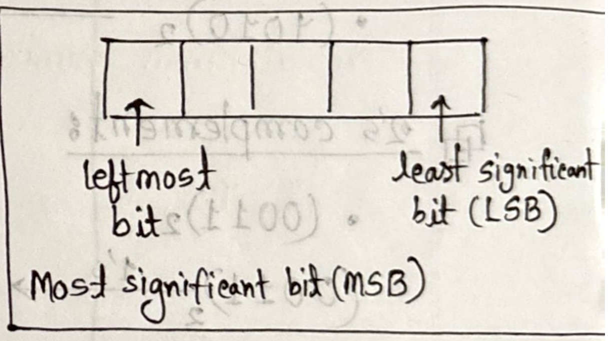
$(7)_{10} = (0111)_2$

Truth table combination:

$n = 4$

$2^n = 2^4 = 16$

	Binary 8	4	2	1	Decimal
1	0	0	0	0	0
2	0	0	0	1	1
3	0	0	1	0	2
4	0	0	1	1	3
5	0	1	0	0	4
6	0	1	0	1	5
7	0	1	1	0	6
8	0	1	1	1	7
9	1	0	0	0	8
10	1	0	0	1	9
11	1	0	1	0	10
12	1	0	1	1	11
13	1	1	0	0	12
14	1	1	0	1	13
15	1	1	1	0	14
16	1	1	1	1	15



*** 4 bit যোগ/বিয়োগ করে যোগফল / বিয়োগফল 4 bit হবে।

Binary addition :

- 4+8

$$\begin{array}{r} 4 \rightarrow 0100 \\ 8 \rightarrow 1000 \\ \hline 12 \rightarrow 1100 \end{array}$$

- 5+12

$$\begin{array}{r} 5 \rightarrow 0101 \\ 12 \rightarrow 1100 \\ \hline 17 \rightarrow 10001 \end{array}$$

1's complement : (Number exchange)

- $(10110)_2 \xrightarrow{\text{1's Complement}} (01001)_2$
- $(1010)_2 \xrightarrow{\text{1's complement}} (0101)_2$

2's complement :

- $(0011)_2$

$(0011)_2 \xrightarrow{\text{1's}} (1100)_2$

$$\begin{array}{r} 1100 \\ +1 \\ \hline (1101)_2 \end{array}$$

$(0011)_2 \xrightarrow{\text{2's}} (1101)_2$

- Method 1

Steps

① 1's complement করতে হবে

② তার সাথে 1 যোগ করতে হবে

Method 2 • Steps: ① ডান থেকে 1 search করব, 1 আসার আগে যা থাকবে তাই বসবে।

② 1st time 1 পেলে same থাকবে, 2nd time 1 থাকলে exchange হতে থাকবে (Complement)

Scanned with CamScanner

2's complement Signed number System: (+, -)

Range: -2^{n-1} to $+(2^{n-1}-1)$

$(n=4)$, $\Rightarrow -2^{4-1}$ to $+(2^{4-1}-1)$

$\Rightarrow -8$ to $+7$

Sign Bit
0 = Positive
1 = negative

Binary:	-8	4	2	1	Decimal
	0	0	0	0	+0
	0	0	0	1	+1
	0	0	1	0	+2
	0	0	1	1	+3
	0	1	0	0	+4
	0	1	0	1	+5
	0	1	1	0	+6
	0	1	1	1	+7
	1	0	0	0	-8
	1	0	0	1	-7
	1	0	1	0	-6
	1	0	1	1	-5
	1	1	0	0	-4
	1	1	0	1	-3
	1	1	1	0	-2
	1	1	1	1	-1

* neg value কে 2's complement করলে +ve হবে, (Vice Versa)

$(+5)_{10} = (0101)_2 \xrightarrow{2\text{'s complement}} (1011)_2 = (-5)_{10}$

$(-3)_{10} = (1101)_2 \xrightarrow{2\text{'s complement}} (0011)_2 = (+3)_{10}$

Algebric Addition :

$x+y$	
x	y
+	+
+	−
−	+
−	−

(+,+) ① $x = 3,\ y = 2\ (3+2)$

$3 \rightarrow 0011$
$2 \rightarrow 0010$
$+5 \rightarrow 0101$

(+,+) ② $x = 4,\ y = 5$

$4 \rightarrow 0100$
$5 \rightarrow 0101$
$+9 \rightarrow 1001$ (overflow)

The decimal number of $(1001)_2 = (-7)_{10}$ [Range এর positive value থেকে বেশি হলে overflow.

(+,−) ③ $x = +3,\ y = -2$

$+3 \rightarrow 0011$
$-2 \rightarrow 1110$
$+1 \rightarrow 10001$

Discard

④ $x = +3,\ y = -4$

$+3 \rightarrow 0011$
$-4 \rightarrow 1110$
$-1 \rightarrow 1111$

*** 4 bit এর সংখ্যা যোগ/বিয়োগ করলে যোগফল / বিয়োগফল 4 bit এ হবে।

(−,−)

(−,+) ⑤ $x = -3,\ y = +4$

$-3 = 1101$

$+4 = 0100$

$+1 = 10001$

↓ Discard

⑥ $x = -3,\ y = -4$

$-3 \rightarrow 1101$

$-4 \rightarrow 1100$

$-7 \rightarrow 11001$

↓ Discard

(−,−) ⑦ $x = -4,\ y = -5$

$-4 \longrightarrow 1100$

$-5 \longrightarrow 1011$

$-9 \longrightarrow 10111$ (underflow)

↓ Discard

$\therefore (0111)_2$ decimal number is +7.

Subtraction:

$x - y = x + (-y)$, where,

$-y$ = 2's complement of y

x	y
+	+
+	−
−	+
−	−

(+,+) ① $x = 4,\ y = 2$

$x - y = 4 - 2 = 4 + (-2)$

$4 \longrightarrow 0100$

$-2 \longrightarrow 1110$

$+2 \longrightarrow 10010$

↓ Discard

② $x = 2,\ y = 4$

$x - y = 2 - 4 = 2 + (-4)$

$2 \rightarrow 0010$

$-4 \rightarrow 1100$

$-2 \rightarrow 1110$

$(-,+)$

(+,-) ③ $x = 3, y = -2$

$x - y = 3 - (-2) = 3 + 2$

$3 \longrightarrow 0011$

$2 \longrightarrow 0010$

$+5 \longrightarrow 0101$

(+,-) ④ $x = 4, y = -5$

$x - y = 4 - (-5) = 4 + 5$

$4 \rightarrow 0100$

$5 \rightarrow 0101$

$9 \longrightarrow 1001$ (overflow)

Decimal value is (-7)

(-,+) ⑤ $x = -3, y = +2$

$x - y = -3 - (+2) = (-3) + (-2)$

$-3 \longrightarrow 1100$

$-2 \longrightarrow 1110$

$+5 \longrightarrow 11010$

Discard

⑥ $x = -5, y = 4$

$x - y = -5 - (4)$

$= -5 + (-4)$

$-5 \longrightarrow 1011$

$-4 \longrightarrow 1100$

$-9 \rightarrow 10111$ (underflow)

discard

Decimal $(+7)$

$(-,-)$

⑦ $x = -3, y = -2$

$x - y = -3 - (-2) = -3 + 2$

$-3 \longrightarrow 1101$

$2 \longrightarrow 0010$

$-1 \longrightarrow 1111$

⑧ $x = -3, y = -4$

$x - y = (-3) - (-4)$

$= -3 + 4$

$-3 \rightarrow 1101$

$4 \rightarrow 0100$

$1 \rightarrow 10001$

Discard.

H.W: addition (8 bit)

① 7   5

② -5   -7

substraction

① 48, -23   ③ 48, -(-23)

② 23, -48   ④ -48, -23

24/06/2025

Chapter 03

Boolean Algebra

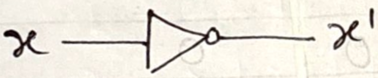
Binary number system এ

set of values, $B = \{0, 1\}$

set of operation $= \{NOT, AND, OR\}$

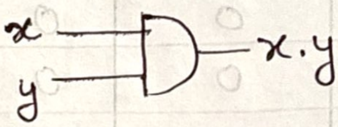
$\{NOT \rightarrow x',\ AND \rightarrow x \cdot y,\ OR \rightarrow x+y\}$

NOT



x	x'
0	1
1	0

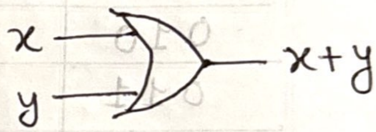
AND



x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

$x \cdot y$ কে xy লিখা যাবে (without dot)

OR



x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

Postulate 2: (OR, AND)

(a) $x + 0 = x$

(b) $x \cdot 1 = x$

Postulate 3: (Commutative law)

(a) $x + y = y + x$

(b) $xy = yx$

□ Postulate 4: (Distributive Law)

(a) $x(y+z) = xy+xz$

(b) $x+yz = (x+y).(x+z)$

□ Prove (a): (not important for exam) # exam এ prove আসবে না

xyz	$(y+z)$	$x(y+z)$	xy	xz	$xy+xz$
000	0	0	0	0	0
001	1	0	0	0	0
010	1	0	0	0	0
011	1	0	0	0	0
100	0	0	0	0	0
101	1	1	0	1	1
110	1	1	1	0	1
111	1	1	1	1	1

□ Postulate 5: (a) $x+x' = 1$

(b) $x.x' = 0$

• Theorem 1: (a) $x+x = x$

(b) $x.x = x$

• Theorem 2: (a) $x+1 = 1$

(b) $x.0 = 0$

• Theorem 3: Involution law $(x')' = x$

• Theorem 4: (Associative law) (a) $x+(y+z) = (x+y)+z$

(b) $x(yz) = (xy)z$

• Theorem 5: (De Morgan's Law)

(a) $(x+y)' = x' \cdot y'$

(b) $(x+y+z)' = x' \cdot y' \cdot z'$

(c) $(xy)' = x' + y'$

(d) $(ABC)' = A' + B' + C'$

Operator Precedence:

1. Parentheses
2. NOT
3. AND
4. OR

□ $xy' + z(x+y)$ for $x = 0$, $y = 1$ and $z = 0$

$xy' + z(x+y)$

$= 0.1' + 0(0+1)$

$= 0.0 + 0.1$

$= 0 + 0$

$= 0$

□ $F_1 = xyz'$, $F_2 = x + y'z$, $F_3 = x'y'z + x'yz + xy'$,

$F_4 = xy' + x'z$

Truth table for F_1,

x y z	z'	xy	xyz'
0 0 0	1	0	0
0 0 1	0	0	0
0 1 0	1	0	0
0 1 1	0	0	0
1 0 0	1	0	0
1 0 1	0	0	0
1 1 0	1	1	1
1 1 1	0	1	0

Another easy method to solve truth table of F1 is :-

$\therefore F_1 = xyz'$

$= 110$ [prime থাকলে 0 ধরবো]

xyz	F_1	F_2	F_3	F_4
000	0	0	0	0
001	0	1	1	1
010	0	0	0	0
011	0	0	1	1
100	0	1	1	1
101	0	1	1	1
110	1	1	0	0
111	0	1	0	0

Other functions,

$\therefore F_2 = x + y'z$

1-- + -01

100 001

101 101

110

111

$\therefore F_3 = x'y'z + x'yz + xy'$

001 011 10-

100

101

$\therefore F_4 = xy' + x'z$

10- 0-1

100 001

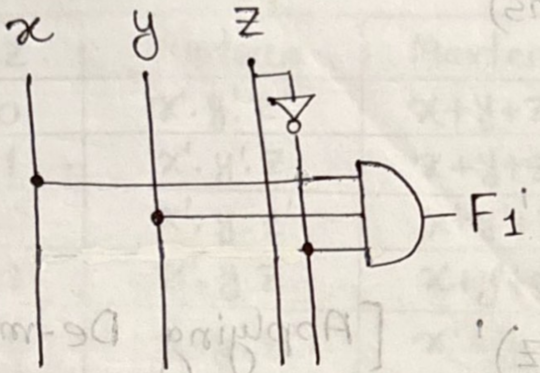
101 011

Here, F_3 and F_4 functions are same but their expressions are different

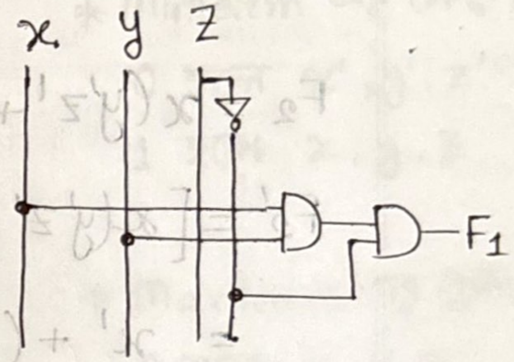
$$\begin{aligned} F_3 &= x'y'z + x'yz + xy' \\ &= x'z(y'+y) + xy' \\ &= x'z.1 + xy' \\ &= x'z + xy' \\ &= xy' + x'z \\ &= F_4 \end{aligned}$$

□ Logic design diagram:-

• $F_1 = xyz'$

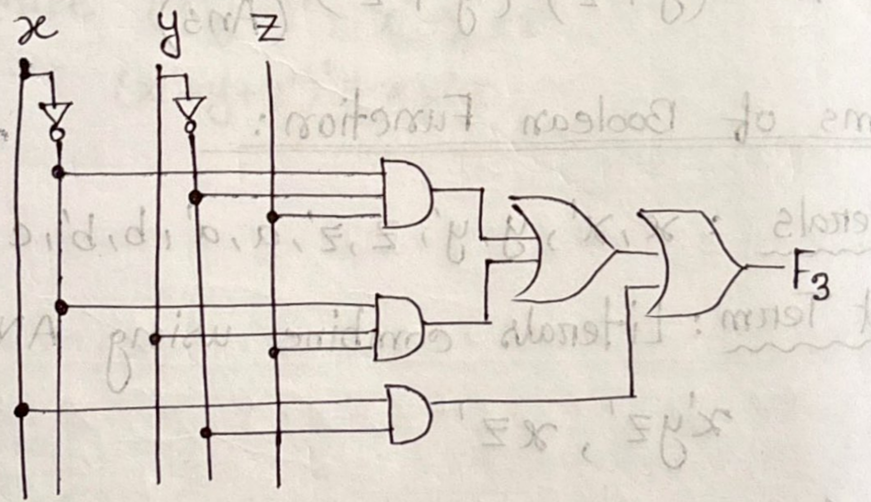


৩ ইনপুটের জন্য diagram



২ ইনপুটের জন্য diagram

• $F_3 = x'y'z + x'yz + xy'$



□ Complement of Boolean Function:

$F_1 = x'yz' + x'y'z$

$F_1' = (x'yz' + x'y'z)'$

Applying De-morgan's law, $F_1' = (x'yz' + x'y'z)'$

$= (x'yz')' . (x'y'z)'$

[P.T.O]

$= [(x')' + y' + (z')'] . [(x')' + (y')' + z]$

$= (x + y' + z) . (x + y + z')$

(Ans)

$F_2 = x(y'z' + yz)$

$F_2' = [x(y'z' + yz)]'$

$= x' + (y'z' + yz)'$ [Applying De-morgan's Law]

$= x' + [(y'z')' . (yz)']$

$= x' + \{(y')' + (z')'\} . (y' + z')$

$F_2' = x' + (y + z) . (y' + z')$ (Ans)

Forms of Boolean Function:

- Literals : $x, x', y, y', z, z', a, a', b, b', c, c'$

- Product Term: Literals combine using AND operation.

$x'yz', xz'$

- Minterm : Product terms where all the variables are present. (no missing variable)

$x'yz', xyz'$

- SumTerm: Literals combined using OR operation

$(x+y), (x+y+z)$

- **Max Term** : Sum Term where all the variables are present

$(x'+y'+z)$, $(x'+y+z)$

Variables

xyz	Minterm	Maxterm
000	$x'.y'.z'$	$x+y+z$
001	$x'.y'.z$	$x+y+z'$
010	$x'.y.z'$	$x+y'+z$
011	$x'.y.z$	$x+y'+z'$
100	$x.y'.z'$	$x'+y+z$
101	$x.y'.z$	$x'+y+z'$
110	$x.y.z'$	$x'+y'+z$
111	$x.y.z$	$x'+y'+z'$

* Minterm এর ক্ষেত্রে,

0 হলে x', y', z'

1 হলে x, y, z

* Maxterm এর ক্ষেত্রে,

0 হলে x, y, z

1 হলে x', y', z'

Minterm এর complement Maxterm. (viceversa)

H.W Practice $(x'yz)' = x+y'+z'$

$(x'+y+z')' = xy'z$

26/06/25

Canonical Forms

Sum of Product (SOP)

$$f = xy' + x'yz + xyz$$

no missing variable → **Canonical sum of product (CSOP) / Sum of minterm**

$$f_1 = x'y'z + xy'z' + xyz$$

$$\quad 001 \quad 100 \quad 111$$

$$f_2 = x'yz + xy'z + xyz' + xyz$$

$$\quad 011 \quad 101 \quad 110 \quad 111$$

xyz	f_1	f_2
000	0	0
001	1	0
010	0	0
011	0	1
100	1	0
101	0	1
110	0	1
111	1	1

Short notation of Sum of Minterm

$$f_1(x,y,z) = \Sigma(1,4,7)$$

$$f_2(x,y,z) = \Sigma(3,5,6,7)$$

$$\quad 011 \quad 101 \quad 110 \quad 111$$

Product of Sum (POS)

$$f = (x+y'z)(x+z)(y'+z)$$

Canonical Product of Sum (CPOS) / Product of Maxterm

$$f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$

$$\quad 000 \quad 010 \quad 011 \quad 101 \quad 110$$

$$f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$$

$$\quad 000 \quad 001 \quad 010 \quad 100$$

Short notation of Product of Maxterm

xyz	f_1	f_2
000	0	0
001	1	0
010	0	0
011	0	1
100	1	0
101	0	1
110	0	1
111	1	1

$$f_1(x,y,z) = \pi(0,2,3,5,6)$$

$$\quad 000 \quad 010 \quad 011 \quad 101 \quad 110$$

$$f_2(x,y,z) = \pi(0,1,2,4)$$

pie sign

Non-canonical Forms

Sum of product (SOP)

Non-canonical form কে canonical form এ নেওয়া যায়।

Algebric way

$$f = y' + xy + x'yz'$$

$$= 1.y'.1 + x.y.1 + x'yz'$$

$$= (x+x').y'.(z+z') + x.y(z+z') + x'yz'$$ [We know $x + x' = 1$]

$$= (xy' + x'y')(z+z') + xyz + xyz' + x'yz'$$

$$f = xy'z + xy'z' + x'y'z + x'y'z' + xyz + xyz' + x'yz'$$

Canonical form

Another method: truth table

$$f = y' + xy + x'yz'$$

$$= \_0\_ + 11\_ + 010$$

000	110
001	111
100	
101	

x y z	f
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Same answer

H.W Practice: $F(A, B, C) = A + B'C$ (Sum of product)

Non-canonical form

Product of sum (POS)

Algebric way

$$F = (x'+y)(x+z)(y+z)$$

$$= (x'+y+0)(x+0+z)(0+y+z)$$

$$= (x'+y+z.z')(x+y.y'+z)(x.x'+y+z)$$ [We know, $x.x' = 0$]

$$= (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)(x+y+z)(x'+y+z$$

$$F = (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)$$

(* যদি কোন term repeat হয় তাহলে একবার লিখবো।)

Another method: From Truth table

$F = (x'+y)(x+z)(y+z)$

$(x'+y)$	$(x+z)$	$(y+z)$
10–	0–0	–00
100	000	000
101	010	100

xyz	F
000	0
001	1
010	0
011	1
100	0
101	0
110	1
111	1

Same as previous. (Ans).

H.W Practice: $F(x,y,z) = x(y'+z)(x'y+z')$

Non-standard Forms :-

$$F(A,B,C,D) = \underbrace{(AB+CD)}_{\text{Sum of product}} \cdot \underbrace{(A'B'+C'D')}_{\text{Sum of product}}$$

SOP, CSOP, POS, CPOS } Standard Form

Non-standard form কে standard form এ নেওয়া যায়,

(a) $x(y+z) = \underbrace{xy+xz}_{SOP}$ → SOP বের করার জন্য এই formula

(b) $x+yz = \underbrace{(x+y)\cdot(x+z)}_{POS}$ → POS " " " " "

Conversion of Non-standard forms into standard form.

for SOP: $F(A,B,C,D) = \underbrace{(AB+CD)}_{x}(\underbrace{A'B'}_{y} + \underbrace{C'D'}_{z})$

$= (AB+CD)\cdot(A'B') + (AB+CD)\cdot(C'D')$

$= \underbrace{ABA'B'}_{0} + CDA'B' + ABC'D' + \underbrace{CDC'D'}_{0}$ [We know, $x \cdot x' = 0$]

$= CDA'B' + ABC'D'$

$\therefore F = CDA'B' + ABC'D'$ [CSOP]

*** for POS: $F(A,B,C,D) = (\underbrace{AB}_{x}+\underbrace{C}_{y}\underbrace{D}_{z})(\underbrace{A'B'}_{x}+\underbrace{C'}_{y}\underbrace{D'}_{z})$

$= (\underbrace{A}_{y}\underbrace{B}_{z}+\underbrace{C}_{x})(AB+D)\cdot(A'B'+C')(A'B'+D')$

$F = (A+C)(B+C)(A+D)(B+D)(A'+C')(B'+C')(A'+D')(B'+D')$ [POS]

(* Ques এ SOP/POS থেকে canonical form এ বের করতে বলতে পারে)

29/06/25

Conversion between canonical Forms

① $F(A,B,C) = \Sigma(1,4,5,6,7)$

② $F(x,y,z) = \pi(0,2,4,5)$

① $F(A,B,C) = \Sigma(1,4,5,6,7)$

$= \pi(0,2,3)$

কারণ minterm ও maxterm একে অপরের complement

② $F(x,y,z) = \pi(0,2,4,5)$

$= \Sigma(1,3,6,7)$

$F(A,B,C,D) = (AB+CD)(A'B'+C'D')$

11-- --11 00-- --00

ABCD	(AB+CD)	(A'B'+C'D')	F
0000	0	1	0
0001	0	1	0
0010	0	1	0
0011	1	1	1
0100	0	1	0
0101	0	0	0
0110	0	0	0
0111	1	0	0
1000	0	1	0
1001	0	0	0
1010	0	0	0
1011	1	0	0
1100	1	1	1
1101	1	0	0
1110	1	0	0
1111	1	0	0

$AB+CD$

11-- --11

1100 0011

1101 0111

1110 1011

1111 1111

$A'B'+C'D'$

00-- --00

0000

0001

0010

0011

Incompletely Specified Boolean Functions :

x y z	F
0 0 0	1
0 0 1	0
0 1 0	X
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	X
1 1 1	1

- Don't care (x) output

∴ sum of mean term :

$F(x,y,z) = \Sigma(0,4,7) + \Sigma_{d.c}(2,6)$

∴ Product of maxterm :

$F(x,y,z) = \pi(1,3,5) . \pi_{d.c}(2,6)$

Truth table থেকে short notation লিখা যায়। (Vice versa)

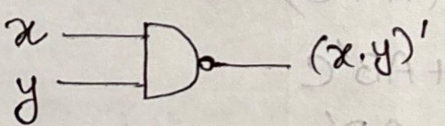
Exercise no :

3.2, 3.3, 3.4, 3.5, 3.6, 3.7(a,d,f), 3.8, 3.12

Chapter-4

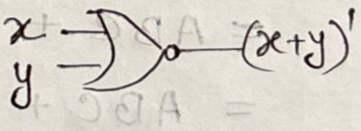
Logic Gates

- **NAND**



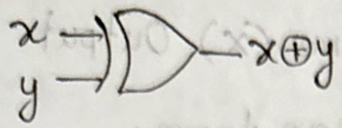
x y	(x.y)'
0 0	1
0 1	1
1 0	1
1 1	0

- **NOR**



x y	(x+y)'
0 0	1
0 1	0
1 0	0
1 1	0

• EXOR ⊕



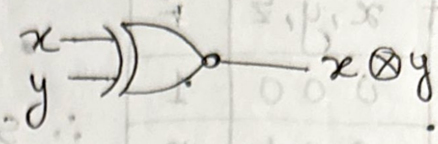
x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

odd no. 1 থাকলে 1 হবে

SOP :

$$F = x'y + xy'$$
$$= x \oplus y$$

• EXNOR



x	y	$x \otimes y$
0	0	1
0	1	0
1	0	0
1	1	1

SOP :

$$F = x'y' + xy$$
$$= x \otimes y$$

Chapter 5

Simplification of Boolean function :-

Ex 1 :

$$F = ABC + AB''(A'C')'$$
$$= ABC + AB'(A+C)$$
$$= ABC + AB' + AB'C$$
$$= AC(B+B') + AB'$$
$$= AC \cdot 1 + AB'$$
$$= AC + AB'$$

Algebric way of simplification.

Ex 2: $F = ABC + ABC' + AB'C$

$= ABC + ABC' + ABC + AB'C$

$= AB(C+C') + AC(B+B')$

$= AB \cdot 1 + AC \cdot 1$

$= AB + AC$

[We know, $x + x = x$
$ABC + ABC = ABC$]

যেই variable এর value vary করে, সেই variable simplification এর পরে থাকে না।

$ABC + ABC' = AB$
111 110 11_

$ABC + AB'C = AC$
111 101 1_1

1-bit Variation

To reduce circuit cost :-

- Reduce number of Product terms (AND gates)
- Reduce inputs of AND gates

Karnaugh Map Method (K-MAP)

- Structure of K-map

A \ B	0	1
0	00	01
1	10	11

← cell

2-variable K-map

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

— Decimal Value

3-variable K-map

Use Gray code sequence for BC

00
01
11
10

Adjacent
0, 4 | 1, 5 | 3, 7 | 2, 6
0, 2 | 4, 6

Adjacent
0,1 | 1,3 | 3,2
4,5 | 5,7 | 7,6

ABC	ABC
011	001
010	101

Adjacent :-

- row-wise : 0,1 | 2,3 | 6,7 | 4,5
- column-wise : 0,2 | 2,6 | 6,4
  1,3 | 3,7 | 7,5
- Corner-wise : 0,4 | 1,5

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

01/07/25

□ 4-variable K-map :

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Adjacent :-

- row-wise 0,1 | 1,3 | 3,2
  4,5 | 5,7 | 7,6
  12,13 | 13,15 | 15,14
  8,9 | 9,11 | 11,10
- column-wise : 0,4 | 4,12 | 12,8
  1,5 | 5,13 | 13,9
  3,7 | 7,15 | 15,11
  2,6 | 6,14 | 14,10
- Corner-wise : 0,2 | 4,6 | 12,14 | 8,10
  0,8 | 1,9 | 3,11 | 2,10

Truth table to K-MAP :-

ABC	F
000	0
001	1
010	1
011	0
100	X
101	0
110	X
111	1

A \ BC	00	01	11	10
0	0	1	0	1
1	X	0	1	X

Expression to K-MAP :-

- **CSOP :** $F = A'B'C + A'BC' + AB'C' + ABC$

  001   010   100   111

A \ BC	00	01	11	10
0		1		1
1	1		1	

- **SOP :** $F = A'C + A'B + AB'C + BC$

$A'C$	$A'B$	$AB'C$	BC
0_1	01_	101	_11
001	010		011
010	011		111

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

- **CPOS :** $F = (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C)$

  000   011   101   110

A \ BC	00	01	11	10
0	0		0	
1		0		0

- **POS :** $F = (B+C)(A'+B)(A+B'+C')$

$(B+C)$	$(A'+B)$	$(A+B'+C')$
_00	10_	011
000	100	
100	101	

A \ BC	00	01	11	10
0	0		0	
1	0	0		

Short Notation to K-MAP :-

Ex-1: $F(A,B,C,D) = \Sigma(0,2,5,7,8,10,15)$

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11			1	
10	1			1

→ $A'BD$

→ BCD

→ $B'D'$

Procedure:

1. Find largest group.
2. Find next largest group. Include atleast a new 1. Overlap can be done.

P.T.O. →

Theory

Principle:

1. Largest possible group.
   - Group size $= 2^m$
   $$= 2^0, 2^1, 2^2, 2^3, 2^4 \quad [m = 0, 1, 2, 3, 4]$$
   $$= 1, 2, 4, 8, 16$$

2. Smallest number of groups.

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

1 group of 8 cells

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

2 groups of 4 cells

Procedure:

1. Find largest group.
2. Find next group. Include at least a new 1. Overlap can be done.

Ex1 — Continue . . . . (Previous table)

AB	CD
01	01
	11

$A'BD$

AB	CD
01	11
11	

BCD

AB	CD
00	00
10	10

$B'D'$

$$\therefore F = A'BD + BCD + B'D'$$

SOP Simplification:

Ex-2: $F(A,B,C,D) = \Sigma(1,4,5,6,7,9,10,11,13,14)$

AB \ CD	00	01	11	10
00		1		
01	1	1	1	1
11		1		1
10		1	1	1

$\rightarrow A'B$, $\rightarrow ACD'$, $\rightarrow AB'C$, $C'D$

$F = A'B + C'D + AB'C + ACD'$

Literal সংখ্যা ১০

Another solution:

AB \ CD	00	01	11	10
00		1		
01	1	1	1	1
11		1		1
10		1	1	1

$\rightarrow A'B$, $\rightarrow BCD'$, $\rightarrow AB'C$, $C'D$

$F = A'B + C'D + AB'C + BCD'$

Literal সংখ্যা ১০

Another solution:

AB \ CD	00	01	11	10
00		1		
01	1	1	1	1
11		1		1
10		1	1	1

$\rightarrow A'B$, $\rightarrow ACD'$, $C'D$, $AB'D$

$F = A'B + C'D + ACD' + AB'D$

Literal সংখ্যা ১০

∴ যেহেতু group সংখ্যা ও Literal সংখ্যা ৩টারই same তাই ৩টাই Best Solution. So, কিছু লিখার দরকার নাই।

Note: যেটাতে কম সংখ্যক group বা কম সংখ্যক literal থাকবে সেটাই Best solution হবে। যদি best solution পাওয়া যায়, তাহলে লিখতে হবে যে, কোনটা best.

C.W
08/07/2023

Example-3: $F(A,B,C,D) = \Sigma(0,1,3,7,8,12) + \Sigma_{d.c}(5,10,13,14)$

AB \ CD	00	01	11	10
00	1	1	1	
01		X	1	
11	1	X		X
10	1			X

$A'B'C'$ $A'D$ AD'

* শুধু don't care কোন group হবে না।
* don't care কে 1 এর সাথে group করলে group size যদি বড় হয়, তখন don't care কে গ্রুপে নিব।

$$F = A'D + AD' + A'B'C'$$

Another solution:

AB \ CD	00	01	11	10
00	1	1	1	
01		X	1	
11	1	X		X
10	1			X

$A'D$ AD' $B'C'D'$

$$F = A'D + AD' + B'C'D'$$

Ex-4: $F(A,B,C,D) = \Sigma(1,5,6,7,11,12,13,15)$

AB \ CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

$A'C'D$ $A'BC$ BD ~~X~~ invalid হয়ে যাবে ACD ABC'

AB \ CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

$A'C'D$ $A'BC$ ABC' ACD

$$F = A'BC + ACD + ABC' + A'C'D$$

H.W Exercise – 5.3

POS Simplification :

$F(A,B,C,D) = \pi(2,4,6,9,11,15) \cdot \pi_{dc}(5,10,13,14)$

AB \ CD	00	01	11	10
00				0
01	0	X		0
11		X	0	X
10		0	0	X

$A+B'+C$

$(C'+D)$

$(A'+D')$

$F = (C'+D)(A'+D')(A+B'+C)$

Another Soln:

AB \ CD	00	01	11	10
00				0
01	0	X		0
11		X	0	X
10		0	0	X

$(C'+D)$

$(A+B'+D)$

$(A'+D')$

$F = (C'+D)(A'+D')(A+B'+D)$

$F(A,B,C,D) = \Sigma(0,1,2,3,5,7,8,9,11,14)$
Find the simplified POS expression using K-map method.

Ans: $F(A,B,C,D) = \Sigma(0,1,2,3,5,7,8,9,11,14)$
$= \pi(4,6,10,12,13,15)$

AB \ CD	00	01	11	10
00				
01	0			0
11	0	0	0	
10				0

$(A+B'+D)$

$B'+C+D$

$(A'+B+C'+D)$

$(A'+B'+D')$

$F = (A'+B'+D')(B'+C+D)(A+B'+D)(A'+B+C'+D)$

Here, we will find out SOP expression (not part of the question)

AB \ CD	00	01	11	10
00	1	1	1	1
01		1	1	
11				1
10	1	1	1	

$A'B'$

$A'D$

$ABCD'$

$B'C'$

$B'D$

$\therefore F = A'B' + A'D + B'C' + B'D + ABCD'$

HW
Exercise – 5.4

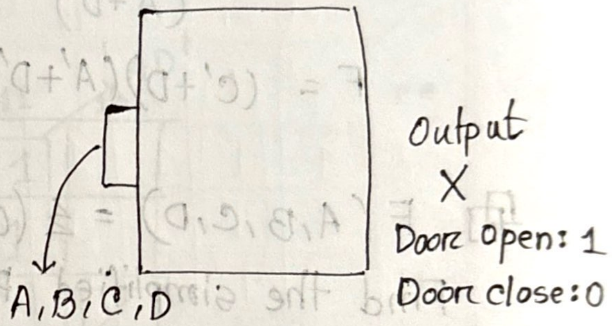
Chapter-6

Combinational logic Design :—

- Analysis
- Design

Steps of Design :—

1. Problem statement
2. Truth Table
3. K-map
4. Logic Diagram



- Absent: 0
  Present: 1

☐ **Conditions: (to open door)**

(1) If A is absent,
B, C, D should be present

(2) If A is present,
B, C, D এর atleast একজনকে present থাকতে হবে

☐ **Step-1:**

A B C D	X
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	1
1 0 1 1	1
1 1 0 0	1
1 1 0 1	1
1 1 1 0	1
1 1 1 1	1

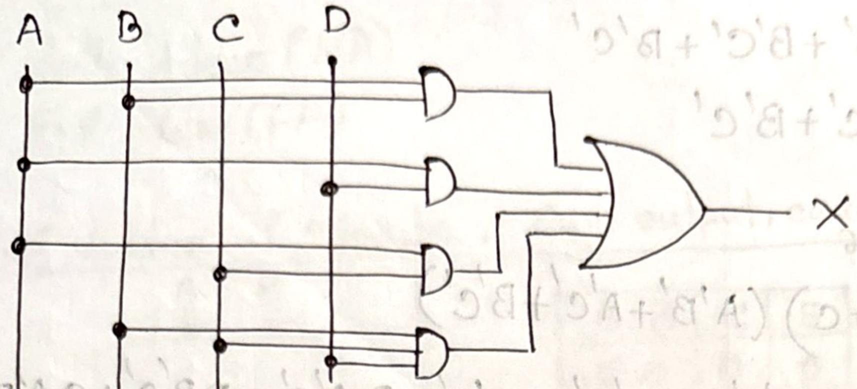
Step-2:

AB \ CD	00	01	11	10
00				
01			1	
11	1	1	1	1
10		1	1	1

BCD
AB
AC
AD

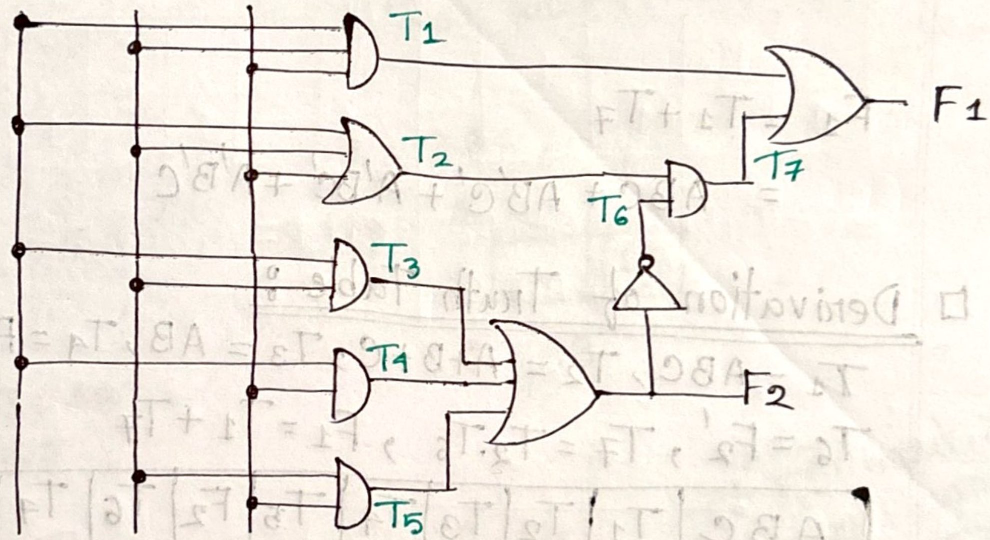
$X = AB + AD + AC + BCD$

Step-3: Logic Diagram:-



Analysis of combinational Circuit:- (Design এর উল্টা)

Analysis:



Method-1: Derivation of Boolean Expressions:-

$T_1 = ABC$

$T_2 = A+B+C$

$T_3 = AB$

$T_4 = AC$

$T_5 = BC$

$F_2 = T_3 + T_4 + T_5$

$F_2 = AB + AC + BC$

$T_6 = F_2'$

$= (AB+AC+BC)'$

$= (AB)'.(AC)'.(BC)'$

$= (A'+B')(A'+C')(B'+C')$

$= (A' + A'C' + A'B' + B'C')(B'+C')$

$= (A'(1+C'+B') + B'C')(B'+C')$

* কোনো gate এর output থেকে অন্য gate এর input এ গেলে তাকে wire করবো

$= (A'+B'C')(B'+C')$

$= A'B' + A'C' + B'C' + B'C'$

$T_6 = A'B' + A'C' + B'C'$

$T_7 = T_2 . T_6$

$= (A+B+C)(A'B' + A'C' + B'C')$

$= \underbrace{AA'B'}_{0} + \underbrace{AA'C'}_{0} + AB'C' + \underbrace{BA'B'}_{0} + BA'C' + \underbrace{BB'C'}_{0} + CA'B' + \underbrace{CA'C'}_{0} + \underbrace{CB'C'}_{0}$

$= AB'C' + A'BC' + A'B'C$

$T_7 = AB'C' + A'BC' + A'B'C$

$F_1 = T_1 + T_7$

$= ABC + AB'C' + A'BC' + A'B'C$

□ Derivation of Truth Table :

$T_1 = ABC$, $T_2 = A+B+C$, $T_3 = AB$, $T_4 = AC$, $T_5 = BC$

$T_6 = F_2'$, $T_7 = T_2 . T_6$, $F_1 = T_1 + T_7$

A B C	T_1	T_2	T_3	T_4	T_5	F_2	T_6	T_7	F_1
0 0 0	0	0	0	0	0	0	1	0	0
0 0 1	0	1	0	0	0	0	1	1	1
0 1 0	0	1	0	0	0	0	1	1	1
0 1 1	0	1	0	0	1	1	0	0	0
1 0 0	0	1	0	0	0	0	1	1	1
1 0 1	0	1	0	1	0	1	0	0	0
1 1 0	0	1	1	0	0	1	0	0	0
1 1 1	1	1	1	1	1	1	0	0	1

Quiz - Chapter 3, Analysis (Logic Diagram দেওয়া থাকবে AND-OR, NOT, NAND সব Gate থাকবে)

C.W 13/07/2025

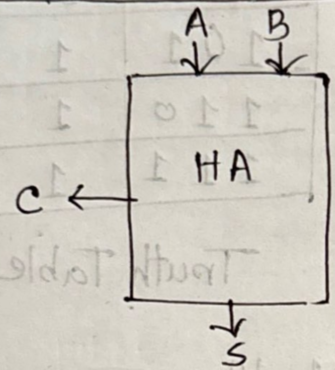
Chapter-7

Arithmetic and Comparator circuits

Adders:-

1. Half-Adder (HA)
2. Full-Adder (FA)

(1) Half Adder (2 inputs, sum output, carry output):-



Block Diagram

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Truth Table

S: K-map:

A \ B	0	1
0		1
1	1	

$A'B$

AB'

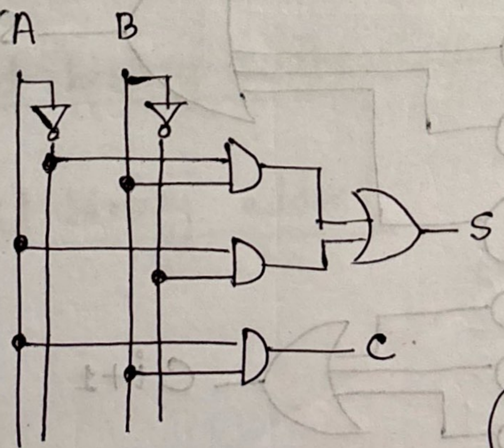
$S = A'B + AB'$
$= A \oplus B$

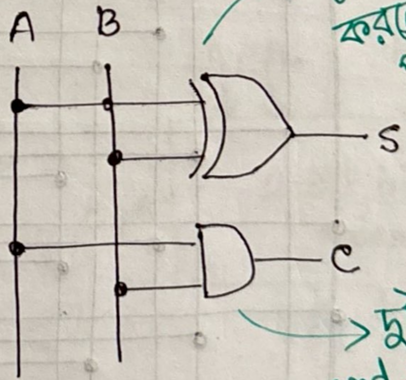
C: K-map:

A \ B	0	1
0		
1		1

$C = AB$

Logic Diagram:



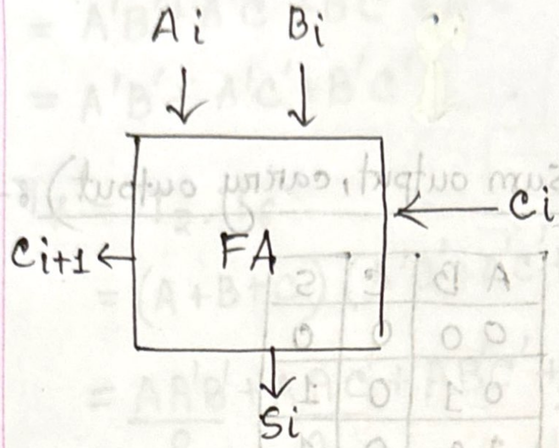


দুটো input XOR করলে sum output পাওয়া যায়

দুটো input and করলে carry output পাওয়া যায়

(দুইটা Diagram ই আঁকা যাবে)

2) Full-Adder (FA) (3 inputs):



Block Diagram

$A_iB_iC_i$	C_{i+1}	S_i
000	0	0
001	0	1
010	0	1
011	1	0
100	0	1
101	1	0
110	1	0
111	1	1

Truth Table

S_i:

A_i \ B_iC_i	00	01	11	10
0		1		1
1	1		1	

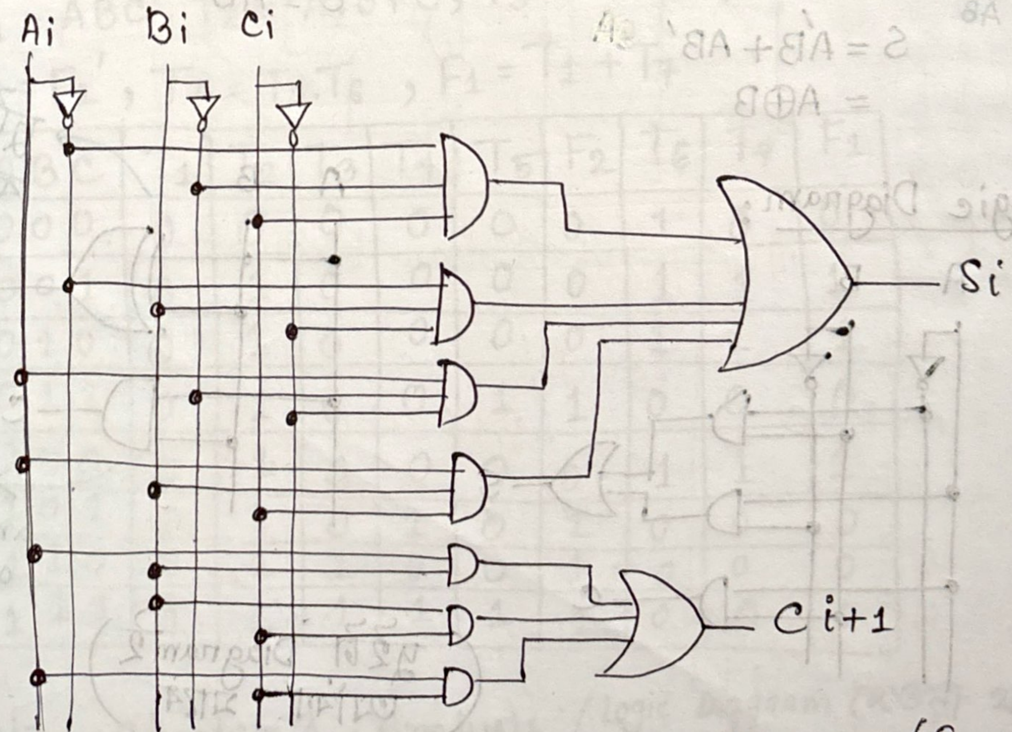
$$S_i = A_i'B_i'C_i + A_i'B_iC_i' + A_iB_i'C_i' + A_iB_iC_i$$

C_{i+1}:

A_i \ B_iC_i	00	01	11	10
0			1	
1		1	1	1

$$C_{i+1} = A_iB_i + B_iC_i + A_iC_i$$

Logic Diagram:



Full adder AND-OR circuit design (from Kmap)

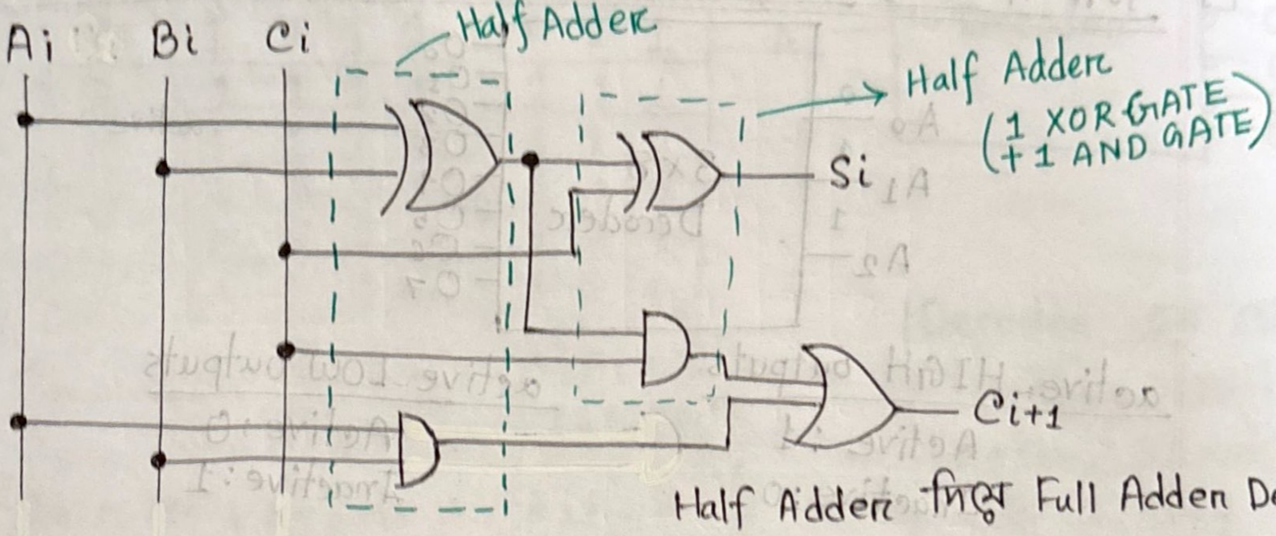
From truth table:

$S_i = A_i \oplus B_i \oplus C_i$

$C_{i+1} = A_i'B_iC_i + A_iB_i'C_i + A_iB_iC_i' + A_iB_iC_i$

$= C_i(A_i'B_i + A_iB_i') + A_iB_i(C_i' + C_i)$

$= (A_i \oplus B_i)C_i + A_iB_i$



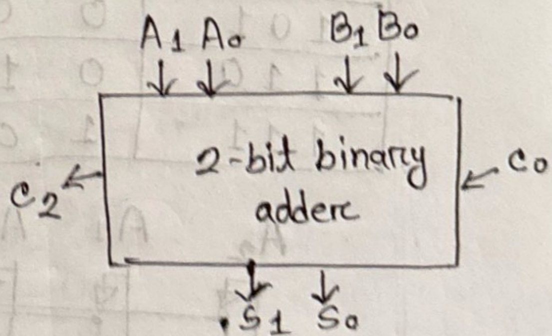
Half Adder দিয়ে Full Adder Design

	$A_3 A_2 A_1 A_0$	
$A_i \rightarrow$	1 0 1 1	
	$B_3 B_2 B_1 B_0$	
$B_i \rightarrow$	0 1 1 0	
	$C_3 C_2 C_1 C_0$	
$C_i \rightarrow$	1 1 0 0	(initial carry always 0)
$S_i \rightarrow$	1 0 0 0 1	
	$C_4 S_3 S_2 S_1 S_0$	

4 bit এর জন্য এখানে i এর value 3
$C_{i+1} = C_{3+1} = C_4$

8 bit এর জন্য i = 7 হবে
তখন $C_{i+1} = C_{7+1} = C_8$ হবে

□ **2-bit binary adder:** →



□ **5-bit binary adder:**

C.W
15/07/2025

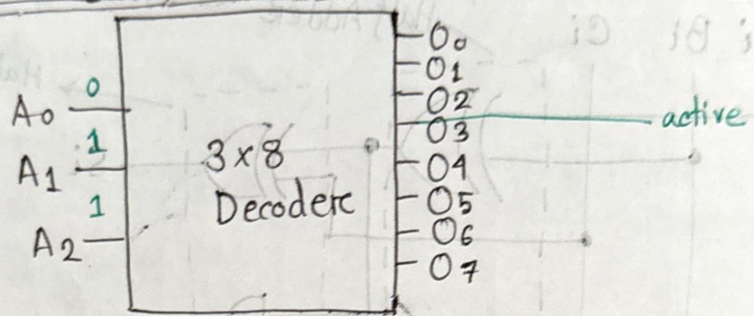
Chapter-8

Decoder

Input : n

Output : 2^n

□ for $n = 3$, 3×8 decoder (with active HIGH outputs) by :-



active-HIGH outputs

Active : 1

Inactive : 0

active-LOW outputs

Active : 0

Inactive : 1

Inputs A_2, A_1, A_0	Outputs O_7	O_6	O_5	O_4	O_3	O_2	O_1	O_0
0 0 0	0	0	0	0	0	0	0	1
0 0 1	0	0	0	0	0	0	1	0
0 1 0	0	0	0	0	0	1	0	0
0 1 1	0	0	0	0	1	0	0	0
1 0 0	0	0	0	1	0	0	0	0
1 0 1	0	0	1	0	0	0	0	0
1 1 0	0	1	0	0	0	0	0	0
1 1 1	1	0	0	0	0	0	0	0

$O_0 = A_2' A_1' A_0'$

$O_1 = A_2' A_1' A_0$

$O_2 = A_2' A_1 A_0'$

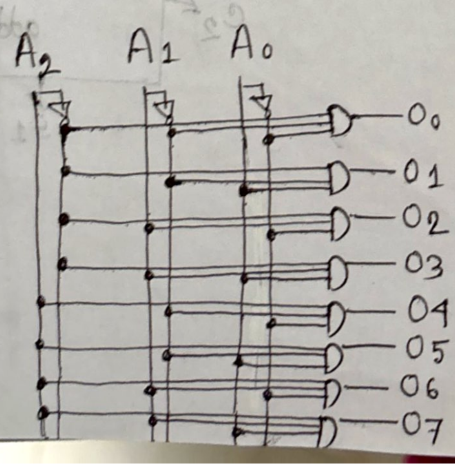
$O_3 = A_2' A_1 A_0$

$O_4 = A_2 A_1' A_0'$

$O_5 = A_2 A_1' A_0$

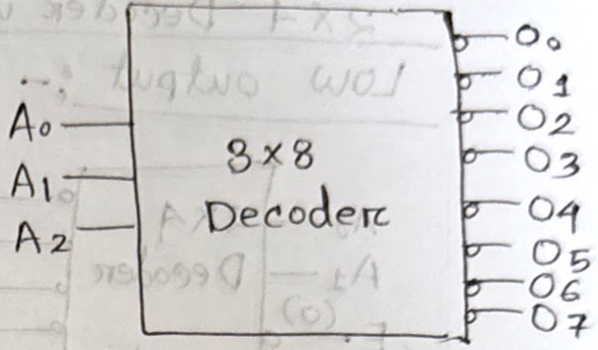
$O_6 = A_2 A_1 A_0'$

$O_7 = A_2 A_1 A_0$



3×8 Decoder with active-LOW outputs:-

Input $A_0\ A_1\ A_2$	Outputs O_7	O_6	O_5	O_4	O_3	O_2	O_1	O_0
0 0 0	1	1	1	1	1	1	1	0
0 0 1	1	1	1	1	1	1	0	1
0 1 0	1	1	1	1	1	0	1	1
0 1 1	1	1	1	1	0	1	1	1
1 0 0	1	1	1	0	1	1	1	1
1 0 1	1	1	0	1	1	1	1	1
1 1 0	1	0	1	1	1	1	1	1
1 1 1	0	1	1	1	1	1	1	1

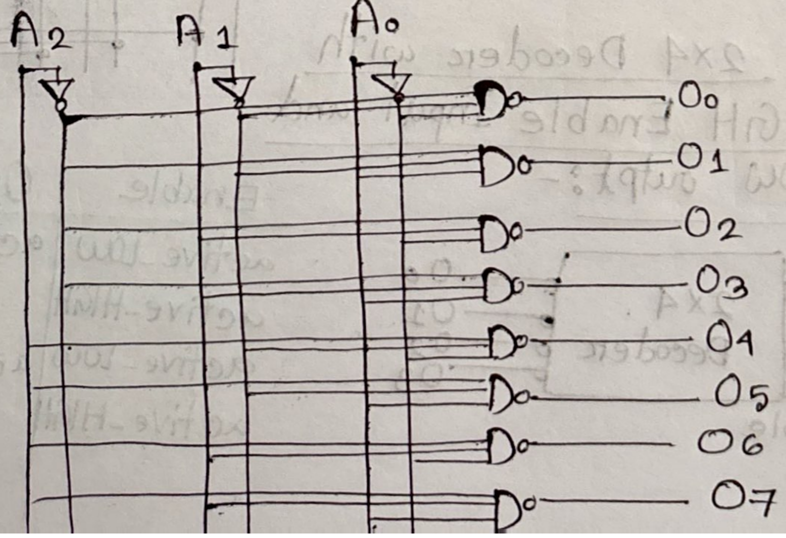


$$O_0 = A_2 + A_1' + A_0 = (A_2' A_1' A_0')'$$
$$O_1 = A_2 + A_1' + A_0' = (A_2' A_1' A_0)'$$
$$O_2 = A_2 + A_1 + A_0 = (A_2' A_1 A_0')'$$
$$O_3 = A_2 + A_1 + A_0' = (A_2' A_1 A_0)'$$
$$O_4 = A_2' + A_1' + A_0 = (A_2 A_1' A_0')'$$
$$O_5 = A_2' + A_1' + A_0' = (A_2 A_1' A_0)'$$
$$O_6 = A_2' + A_1 + A_0 = (A_2 A_1 A_0')'$$
$$O_7 = A_2' + A_1 + A_0' = (A_2 A_1 A_0)'$$

Decoder এর ক্ষেত্রে, active-HIGH output active-LOW output একে অপরের complement.

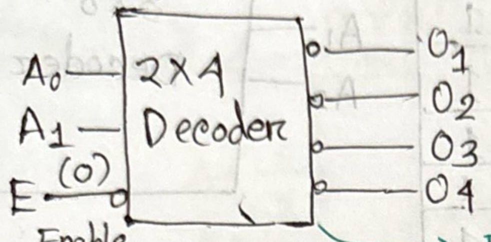
Practice
2×4
4×16

∴ 3×8 Decoder is also called 'Binary-to-Octal' Decoder.



Decoder with Enable Input:

2X4 Decoder with Active-LOW Enable Input and active-LOW output :-



Decoder এর operation control করে

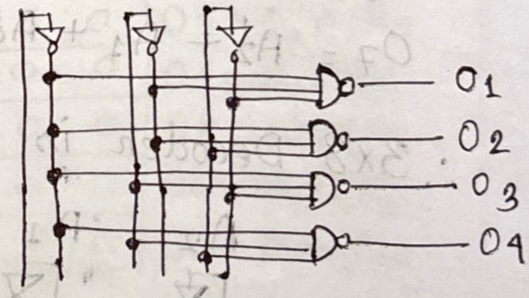
E
active_LOW ⇒ E = 0
active_HIGH ⇒ E = 1

E = 0 হলে কোনো active output পাবো।
E = 1 হলে কোনো active output পাবো না।

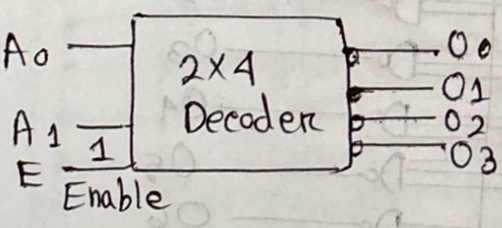
Inputs			Outputs			
E	A_1	A_0	O_3	O_2	O_1	O_0
0	0	0	1	1	1	0
0	0	1	1	1	0	1
0	1	0	1	0	1	1
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

active output পাবো না E = 1

$O_0 = E + A_1 + A_0 = (E' A_1' A_0')'$
$O_1 = E + A_1 + A_0' = (E' A_1' A_0)'$
$O_2 = E + A_1' + A_0 = (E' A_1 A_0')'$
$O_3 = E + A_1' + A_0' = (E' A_1 A_0)'$



~~Decoder~~ 2X4 Decoder with active-HIGH Enable Input and active-LOW output :-



Enable	Output
active_LOW	active_LOW
active_HIGH	"
active_LOW	active_HIGH
active_HIGH	"

Inputs			Outputs			
E	A_1	A_0	O_3	O_2	O_1	O_0
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1

C.W
20/07/2025
Sunday

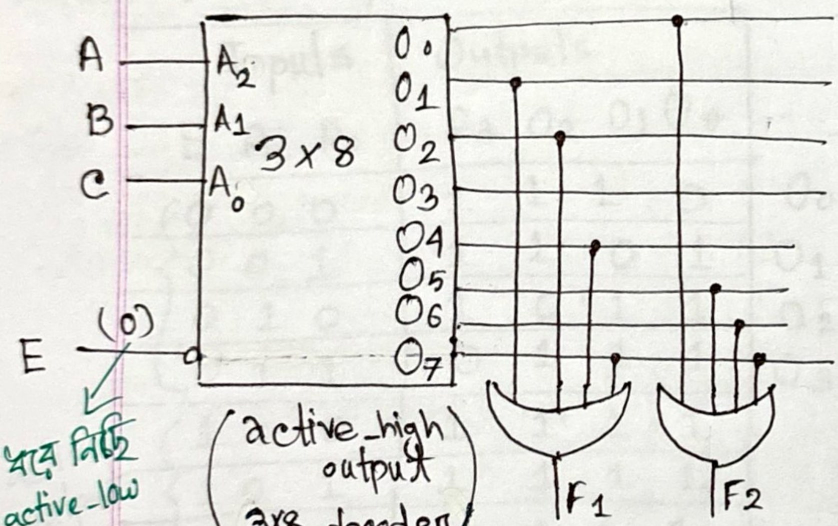
active-low = bubble আছে
active-high = bubble ছাড়া

Combinational Circuit Implementation Using Decoder:

CSOP: $F_1(\overset{A_2}{A},\overset{A_1}{B},\overset{A_0}{C}) = \Sigma(1,2,4,7)$

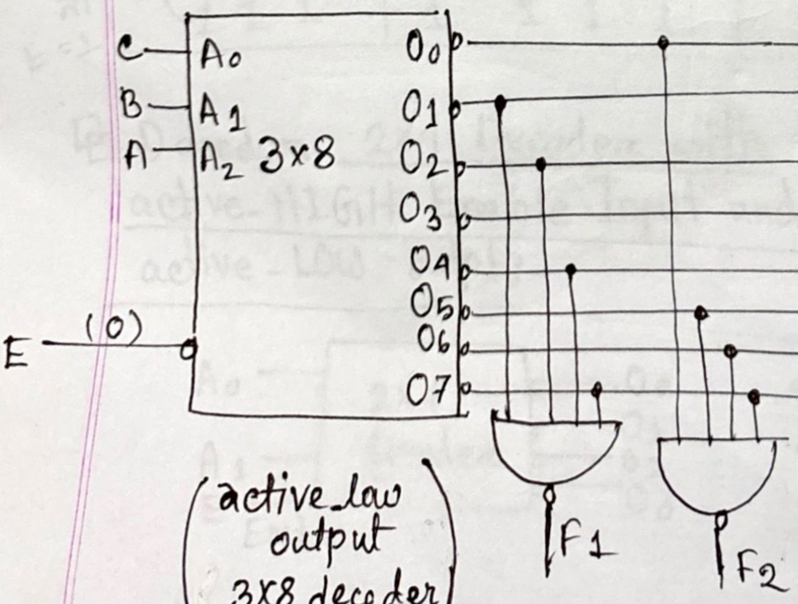
$F_2(A,B,C) = \Sigma(0,5,6,7)$

fixed for all type of functions (ex- CSOP, CPOS etc)
- * active-HIGH outputs generate Minterms (m). $\boxed{m' = M}$
- * active-LOW outputs generate Maxterms (M) $\boxed{M' = m}$



ধরে নিচ্ছি active-low

(active_high output 3x8 decoder)



(active_low output 3x8 decoder)

যদি logic gate মনে থাকে তাহলে conversion দরকার নেই

Conversion

$F_1(A,B,C)$

$= \Sigma(1,2,4,7)$

$= m_1 + m_2 + m_4 + m_7$

$= ((m_1+m_2+m_4+m_7)')'$

$= (m_1' \cdot m_2' \cdot m_4' \cdot m_7')'$

$= (M_1 \cdot M_2 \cdot M_4 \cdot M_7)'$

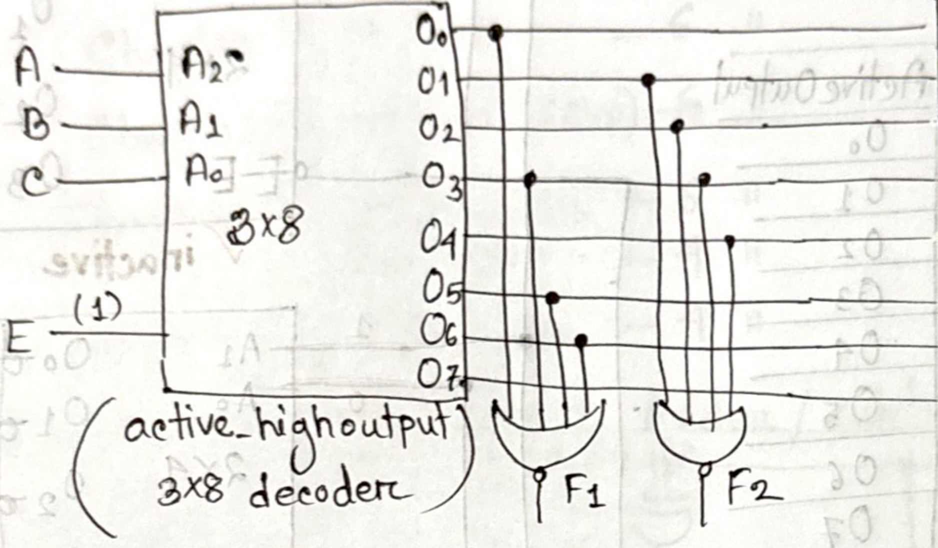
$F_2 = (M_0 \cdot M_5 \cdot M_6 \cdot M_7)'$

Not necessary if logic gate is remembered

$A_2 A_1 A_0$

CPOS: $F_1(A,B,C) = \pi(0,3,5,6)$

$F_2(A,B,C) = \pi(1,2,3,4)$



(active_high output 3x8 decoder)

Conversion

$F_2 = (A,B,C)$

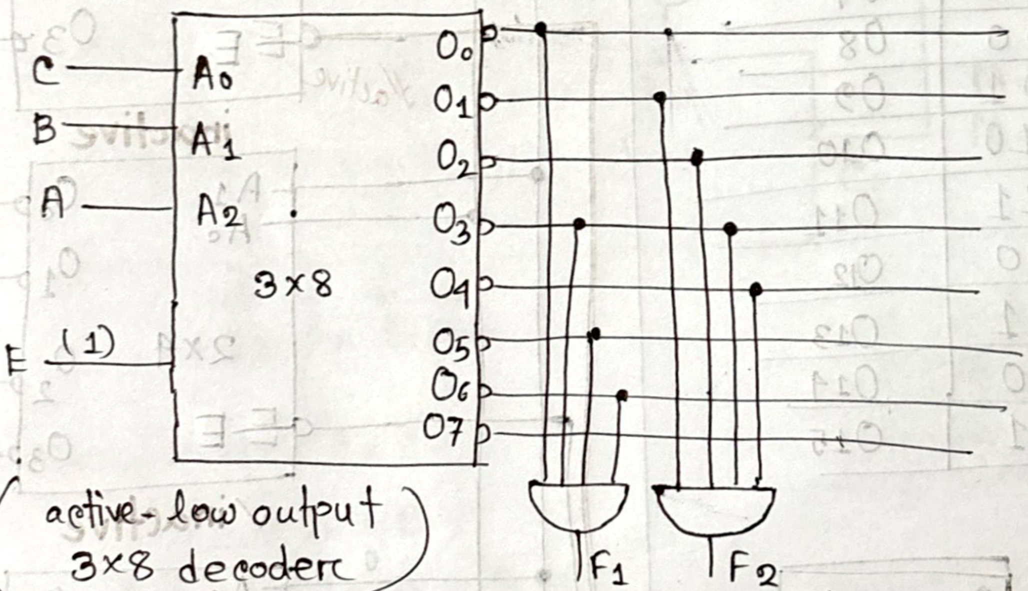
$= \pi(1,2,3,4)$

$= M_1 \cdot M_2 \cdot M_3 \cdot M_4$

$= ((M_1 \cdot M_2 \cdot M_3 \cdot M_4)')'$

$= (M_1' + M_2' + M_3' + M_4')'$

$= (m_1 + m_2 + m_3 + m_4)'$



(active-low output 3x8 decoder)

Expansion of Decoder:-

Implementation of 4x16 Decoder using 2x4 Decoders (active-LOW Enable Input and active_LOW outputs)

Answer:

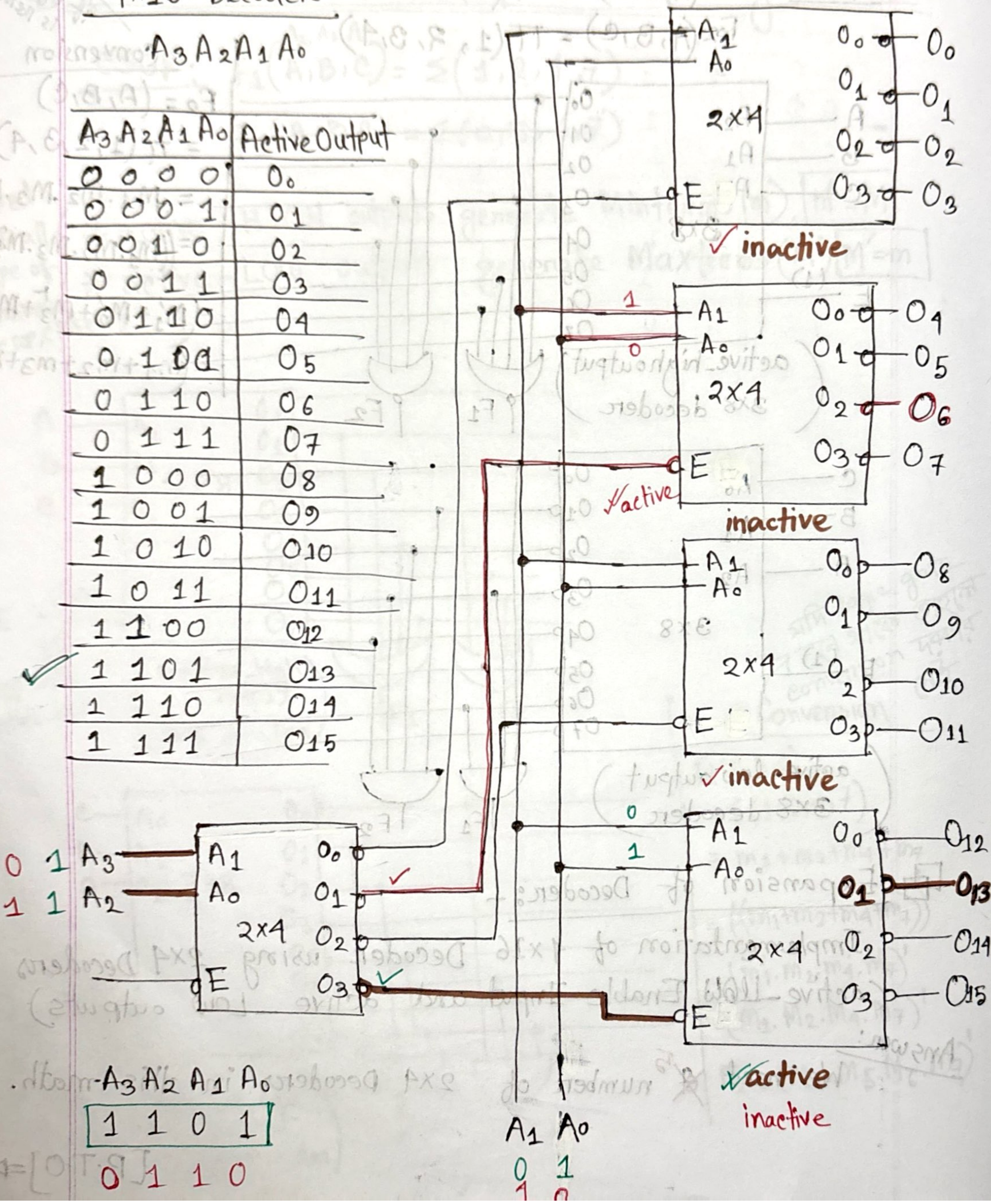
We need ~~4~~ 5 number of 2x4 Decoders in this math.

[P.T.O] ⇒

4×16 Decoder

$A_3 A_2 A_1 A_0$

$A_3 A_2 A_1 A_0$	Active Output
0 0 0 0	O_0
0 0 0 1	O_1
0 0 1 0	O_2
0 0 1 1	O_3
0 1 1 0	O_4
0 1 0 1	O_5
0 1 1 0	O_6
0 1 1 1	O_7
1 0 0 0	O_8
1 0 0 1	O_9
1 0 1 0	O_{10}
1 0 1 1	O_{11}
1 1 0 0	O_{12}
1 1 0 1	O_{13}
1 1 1 0	O_{14}
1 1 1 1	O_{15}

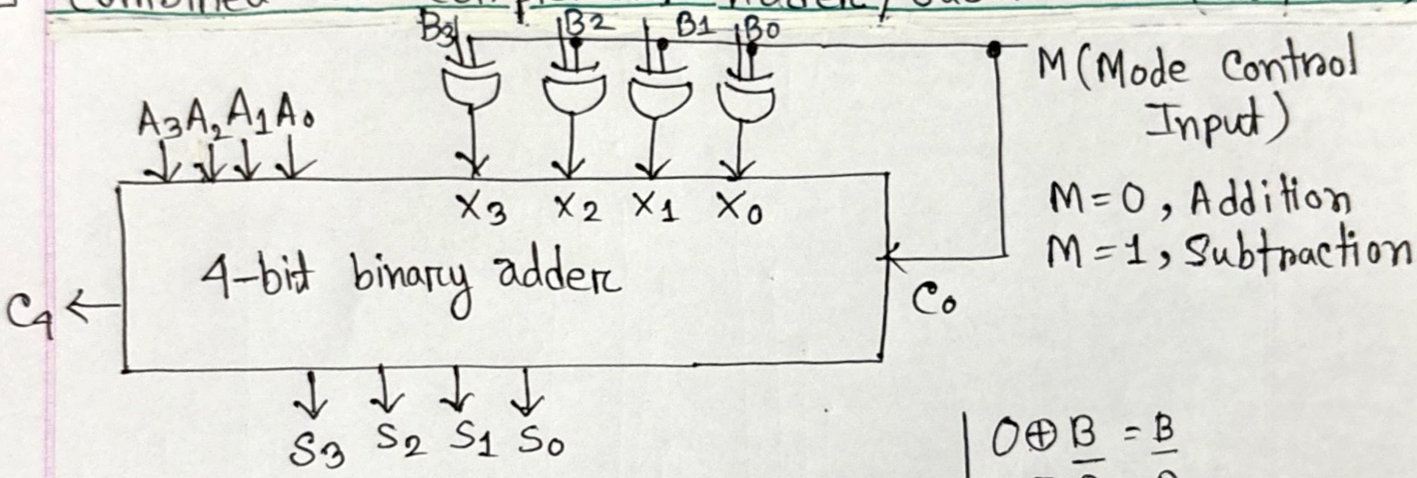


C.W
22/07/2025
Tuesday

Mid Marks Distribution (6 Questions):-

Chapter 2 → 1 Ques − 4 marks
Chapter 3 → " − 6 "
Chapter 5 → " (Kmap) − 6 "
Chapter 6 → Analysis − 6 "
→ Design − 4 "
Chapter 8 → Decoder − 4 "

□ Combined 2's Complement Adder / subtractor (Chapter-7)



$0 \oplus B = B$
$0 \oplus 0 = 0$
$0 \oplus 1 = 1$

When, $M = 0$, $C_0 = 0$

$$X = 0 \oplus B = B$$
$$A + X + C_0 = A + B + 0$$
$$= A + B \quad \text{(Addition)}$$

$1 \oplus B = B'$
$1 \oplus 0 = 1$
$1 \oplus 1 = 0$

When, $M = 1$, $C_0 = 1$

$$X = 1 \oplus B = B'$$
$$A + X + C_0 = A + B' + 1$$
$$= A + (B' + 1) \quad \text{(or বা, addition)}$$
$$= A + \text{2's Complement of } B$$
$$= A + (-B) = A - B \quad \text{(Subtraction)}$$