23/06/2025

Chapter 03

Boolean Algebra

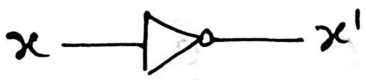
Binary number system এ

set of values, $B = \{0, 1\}$

set of operation $= \{NOT, AND, OR\}$

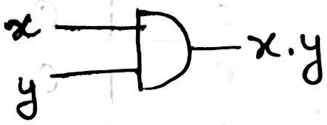
$NOT \rightarrow x'$, $AND \rightarrow x \cdot y$, $OR \rightarrow x+y$

NOT



x	x'
0	1
1	0

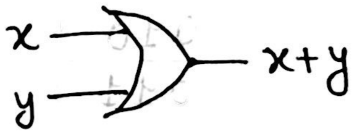
AND



x	y	$x.y$
0	0	0
0	1	0
1	0	0
1	1	1

$x.y$ কে xy লিখা যাবে (without dot)

OR



x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

Postulate 2: (OR, AND)

(a) $x+0=x$

(b) $x \cdot 1 = x$

Postulate 3: (Commutative law)

(a) $x+y=y+x$

(b) $xy=yx$

Postulate 4: (Distributive Law)

(a) $x(y+z) = xy+xz$

(b) $x+yz = (x+y).(x+z)$

Prove (a): (not important for exam) # exam এ prove আসবে না

xyz	(y+z)	x(y+z)	xy	xz	xy+xz
000	0	0	0	0	0
001	1	0	0	0	0
010	1	0	0	0	0
011	1	0	0	0	0
100	0	0	0	0	0
101	1	1	0	1	1
110	1	1	1	0	1
111	1	1	1	1	1

Postulate 5:

(a) $x+x' = 1$

(b) $x.x' = 0$

- Theorem 1: (a) $x+x = x$
  (b) $x.x = x$
- Theorem 2: (a) $x+1 = 1$
  (b) $x.0 = 0$
- Theorem 3: Involution law $(x')' = x$
- Theorem 4: (Associative law) (a) $x+(y+z) = (x+y)+z$
  (b) $x(yz) = (xy)z$

Theorem 5: (De Morgan's Law)

(a) $(x+y)' = x'.y'$

(b) $(x+y+z)' = x'.y'.z'$

(c) $(xy)' = x'+y'$

(d) $(ABC)' = A'+B'+C'$

Operator Precedence:

1. Parentheses
2. NOT
3. AND
4. OR

- $xy' + z(x+y)$ for $x=0$, $y=1$ and $z=0$

$xy' + z(x+y)$

$= 0.1' + 0(0+1)$

$= 0.0 + 0.1$

$= 0+0$

$= 0$

- $F_1 = xyz'$, $F_2 = x+y'z$, $F_3 = x'y'z + x'yz + xy'$, $F_4 = xy' + x'z$

Truth table for F_1,

x y z	z'	xy	xyz'
0 0 0	1	0	0
0 0 1	0	0	0
0 1 0	1	0	0
0 1 1	0	0	0
1 0 0	1	0	0
1 0 1	0	0	0
1 1 0	1	1	1
1 1 1	0	1	0

Another easy method to solve truth table of $F1$ is :-

$\therefore F_1 = xyz'$

$= 110$ [prime থাকলে 0 ধরবো]

xyz	F_1	F_2	F_3	F_4
000	0	0	0	0
001	0	1	1	1
010	0	0	0	0
011	0	0	1	1
100	0	1	1	1
101	0	1	1	1
110	1	1	0	0
111	0	1	0	0

Other functions,

$\therefore F_2 = x + y'z$

1_ _ + _01

100 001
101 101
110
111

$\therefore F_3 = x'y'z + x'yz + xy'$

001 011 10_
100
101

$\therefore F_4 = xy' + x'z$

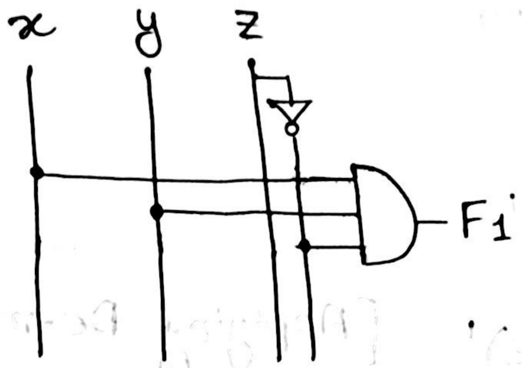
10_ 0_1
100 001
101 011

Here, F_3 and F_4 functions are same but their expressions are different

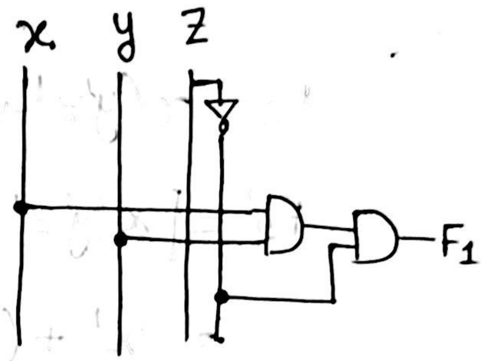
$$\begin{aligned}
F_3 &= x'y'z + x'yz + xy' \\
&= x'z(y'+y) + xy' \\
&= x'z.1 + xy' \\
&= x'z + xy' \\
&= xy' + x'z \\
&= F_4
\end{aligned}$$

□ Logic design diagram:-

• $F_1 = xyz'$

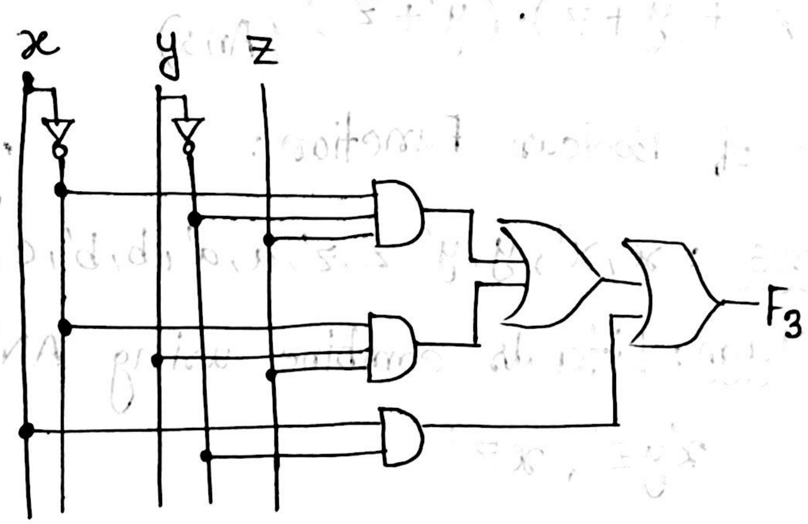


৩ ইনপুটের জন্য diagram



২ ইনপুটের জন্য diagram

• $F_3 = x'y'z + x'yz + xy'$



□ Complement of Boolean Function:

$F_1 = x'yz' + x'y'z$

$F_1' = (x'yz' + x'y'z)'$

Applying De-morgan's law, $F_1 = (x'yz' + x'y'z)'$

$= (x'yz')' . (x'y'z)'$

[P.T.O]

$$= [(x')' + y' + (z')'] . [(x')' + (y')' + z]$$

$$= (x + y' + z) . (x + y + z')$$

(Ans)

$$F_2 = x(y'z' + yz)$$

$$F_2' = [x(y'z' + yz)]'$$

$$= x' + (y'z' + yz)' \quad \text{[Applying De-morgan's Law]}$$

$$= x' + [(y'z')' . (yz)']$$

$$= x' + \{(y')' + (z')'\} . (y' + z')$$

$$F_2' = x' + (y + z) . (y' + z') \quad \text{(Ans)}$$

□ Forms of Boolean Function:

- Literals : $x, x', y, y', z, z', a, a', b, b', c, c'$
- Product Term: Literals combine using AND operation.

$x'yz', \; xz'$

- Minterm: Product terms where all the variables are present. (no missing variable)

$x'yz', \; xyz'$

- SumTerm: Literals combined using OR operation

$(x+y), \; (x+y+z)$

• **Max Term** : Sum Term where all the variables are present

$(x'+y'+z)$, $(x'+y+z)$

Variables

xyz	Minterm	Maxterm
000	$x'.y'.z'$	$x+y+z$
001	$x'.y'.z$	$x+y+z'$
010	$x'.y.z'$	$x+y'+z$
011	$x'.y.z$	$x+y'+z'$
100	$x.y'.z'$	$x'+y+z$
101	$x.y'.z$	$x'+y+z'$
110	$x.y.z'$	$x'+y'+z$
111	$x.y.z$	$x'+y'+z'$

* Minterm এর ক্ষেত্রে,

0 হলে x', y', z'

1 হলে x, y, z

* Maxterm এর ক্ষেত্রে,

0 হলে x, y, z

1 হলে x', y', z'

Minterm এর complement Maxterm. (Viceversa)

H.W Practice $(x'yz)' = x+y'+z'$

$(x'+y+z')' = xy'z$

26/06/25

Canonical Forms

Sum of Product (SOP)

$$f = xy' + x'yz + xyz$$

no missing variable → **Canonical sum of product (CSOP) / Sum of minterm**

$$f_1 = \underset{001}{x'y'z} + \underset{100}{xy'z'} + \underset{111}{xyz}$$

$$f_2 = \underset{011}{x'yz} + \underset{101}{xy'z} + \underset{110}{xyz'} + \underset{111}{xyz}$$

xyz	f_1	f_2
000	0	0
001	1	0
010	0	0
011	0	1
100	1	0
101	0	1
110	0	1
111	1	1

Short notation of Sum of Minterm

$$f_1(x,y,z) = \Sigma(1,4,7)$$

$$f_2(x,y,z) = \Sigma(\underset{011}{3}, \underset{101}{5}, \underset{110}{6}, \underset{111}{7})$$

Product of Sum (POS)

$$f = (x+y'z)(x+z)(y'+z)$$

Canonical Product of Sum (CPOS) / Product of Maxterm

$$f_1 = \underset{000}{(x+y+z)}\underset{010}{(x+y'+z)}\underset{011}{(x+y'+z')}\underset{101}{(x'+y+z')}\underset{110}{(x'+y'+z)}$$

$$f_2 = \underset{000}{(x+y+z)}\underset{001}{(x+y+z')}\underset{010}{(x+y'+z)}\underset{100}{(x'+y+z)}$$

Short notation of Product of Maxterm

xyz	f1	f2
000	0	0
001	1	0
010	0	0
011	0	1
100	1	0
101	0	1
110	0	1
111	1	1

$$f_1(x,y,z) = \pi(\underset{000}{0}, \underset{010}{2}, \underset{011}{3}, \underset{101}{5}, \underset{110}{6})$$

$$f_2(x,y,z) = \underset{\text{pie sign}}{\pi}(0,1,2,4)$$

Non-canonical Forms

Sum of product (SOP)

Non-canonical form কে canonical form এ নেওয়া যায়।

Algebric way

$f = y' + xy + x'yz'$

$= 1.y'.1 + x.y.1 + x'yz'$

$= (x+x').y'.(z+z') + x.y(z+z') + x'yz'$ [We know, $x + x' = 1$]

$= (xy' + x'y')(z+z') + xyz + xyz' + x'yz'$

$f = xy'z + xy'z' + x'y'z + x'y'z' + xyz + xyz' + x'yz'$ → Canonical form

truth table

$f = y' + xy + x'yz'$

$= \_0\_ + 11\_ + 010$

000 110
001 111
100
101

→ Same answer

x y z	f
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

H.W Practice: $F(A, B, C) = A + B'C$ (Sum of product)

Non-canonical form

Product of sum (POS)

Algebric way

$F = (x'+y)(x+z)(y+z)$

$= (x'+y+0)(x+0+z)(0+y+z)$

$= (x'+y+z.z')(x+y.y'+z)(x.x'+y+z)$ [We know, $x.x' = 0$]

$= (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)(x+y+z)(x'+y+z$

$F = (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)$

(* যদি কোন term repeat হয় তাহলে একবার লিখবো।)

Another Method: From Truth table

$F = (x'+y)(x+z)(y+z)$

$x'+y$	$x+z$	$y+z$
10_	0_0	_00
100	000	000
101	010	100

Same as previous. (Ans).

xyz	F
000	0
001	1
010	0
011	1
100	0
101	0
110	1
111	1

H.W Practice: $F(x,y,z) = x(y'+z)(x'y+z')$

◻ Non-standard Forms:-

$$F(A,B,C,D) = \underbrace{(AB+CD)}_{\text{Sum of product}} \cdot \underbrace{(A'B'+C'D')}_{\text{sum of product}}$$

SOP, CSOP, POS, CPOS } Standard Form

Non-standard form কে standard form এ নেওয়া যায়।

(a) $x(y+z) = \underbrace{xy+xz}_{\text{SOP}}$ → SOP বের করার জন্য এই formula

(b) $x+yz = \underbrace{(x+y)\cdot(x+z)}_{\text{POS}}$ → POS " " " " "

◻ Conversion of Non-standard forms into standard form.

for SOP: $F(A,B,C,D) = \underbrace{(AB+CD)}_{x}(\underbrace{A'B'}_{y}+\underbrace{C'D'}_{z})$

$= (AB+CD)\cdot(A'B') + (AB+CD)\cdot(C'D')$

$= \underbrace{ABA'B'}_{0} + CDA'B' + ABC'D' + \underbrace{CDC'D'}_{0}$ [we know, $x \cdot x' = 0$]

$= CDA'B' + ABC'D'$

$\therefore F = CDA'B' + ABC'D'$ [CSOP]

for POS: $F(A,B,C,D) = (\underbrace{AB}_{x} + \underset{y}{C}\underset{z}{D})(\underbrace{A'B'}_{x} + \underset{y}{C'}\underset{z}{D'})$

$= (\underset{y}{A}\underset{z}{B} + \underset{x}{C})(AB+D) \cdot (A'B'+C')(A'B'+D')$

$F = (A+C)(B+C)(A+D)(B+D)(A'+C')(B'+C')(A'+D')(B'+D')$

[POS]

(* Ques এ SOP/POS থেকে canonical form এ বের করতে বলতে পারে)