CompSci 206 Homework 5

June 16, 2017

```
In [4]: import numpy as np
        %matplotlib inline
        import matplotlib.pyplot as plt

        # set output format
        np.set_printoptions(suppress=True)
```

1 9.30

```
In [707]: # 9.30
          alpha = 0.01
          beta = 0.5
          maxiters = 1000
          gradtol = 1e-3
          nttol = 1e-6

          m = 100
          n = 30
```

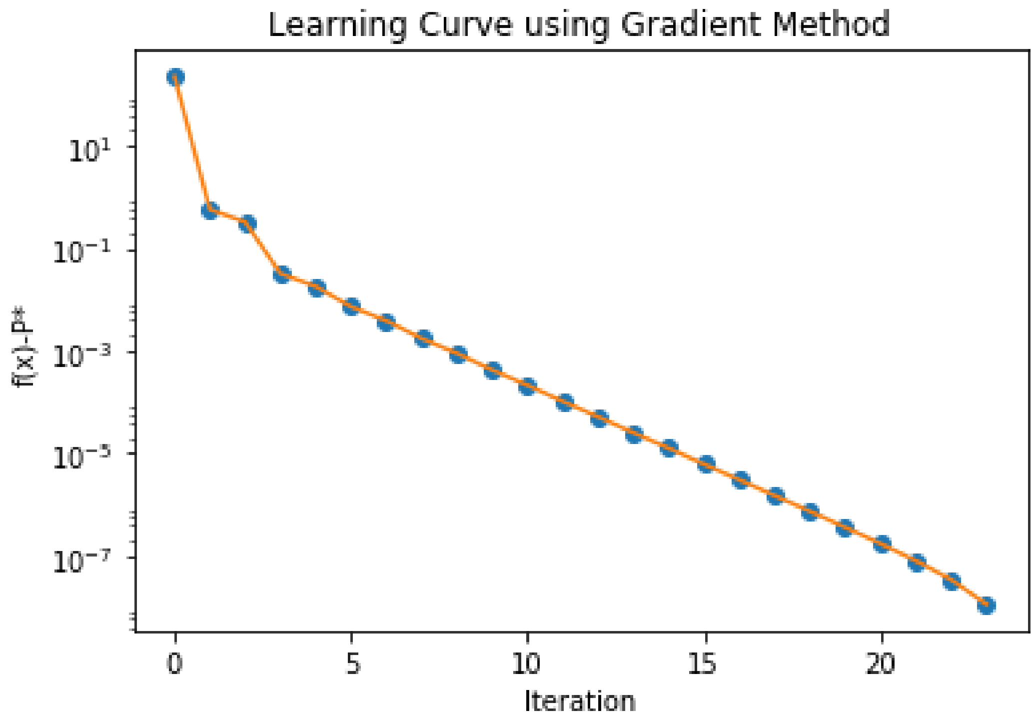
```
In [710]: x = np.zeros(n)
          A = np.random.rand(n,m)
```

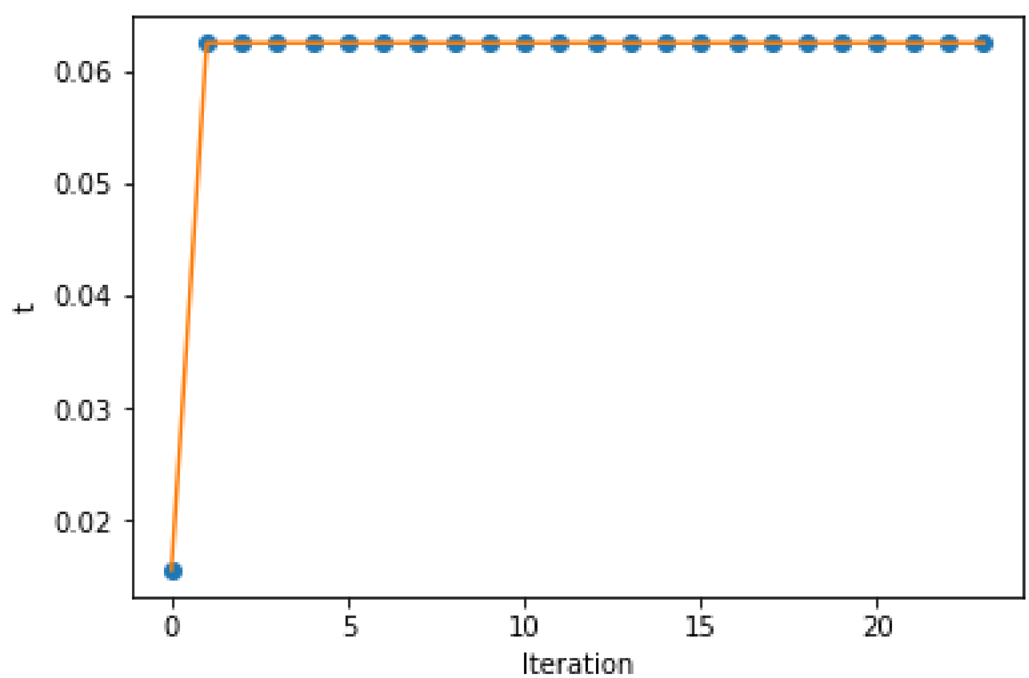
1.1 Gradient Descent Method

```
In [712]: ## gradient descent method
          fs = []
          steps = []

          for iter in range(maxiters):
              f = - np.sum(np.log(1-np.dot(A.T,x))) - \
              np.sum(np.log(1+x)) - np.sum(np.log(1-x))
              fs.append(f)
              grad = np.dot(A, (1/(1-np.dot(A.T,x)))) - 1/(1+x) + 1/(1-x)
              v = -grad
              fprime = np.dot(grad.T, v)

              if np.linalg.norm(grad) < gradtol:
```

```
            break

        t = 1
        while np.amax(np.dot(A.T, (x+t*v))) >= 1 or \
        np.amax(np.absolute(x+t*v)) >= 1:
            t = beta*t

        while - np.sum(np.log(1-np.dot(A.T, (x+t*v)))) \
        - np.sum(np.log(1+x+t*v)) \
        - np.sum(np.log(1-x-t*v)) > f + alpha*t*fprime:
            t = beta*t
        x = x+t*v
        steps.append(t)

    pstar = fs[len(fs)-1]
    print "P*:", pstar
    print

    plt.semilogy(range(len(fs)-1), (fs[0:len(fs)-1]-pstar), 'o',
                 range(len(fs)-1), (fs[0:len(fs)-1]-pstar), '-')
    plt.title('Learning Curve using Gradient Method')
    plt.xlabel("Iteration"); plt.ylabel("f(x)-P*")
    plt.show()

    plt.plot(range(len(steps)), steps, "o",
             range(len(steps)), steps, "-")
    plt.xlabel("Iteration"); plt.ylabel("t")
    plt.show()
```

```
P*: -224.497255892
```


Learning Curve using Gradient Method
f(x)-p*
Iteration
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
0
5
10
15
20

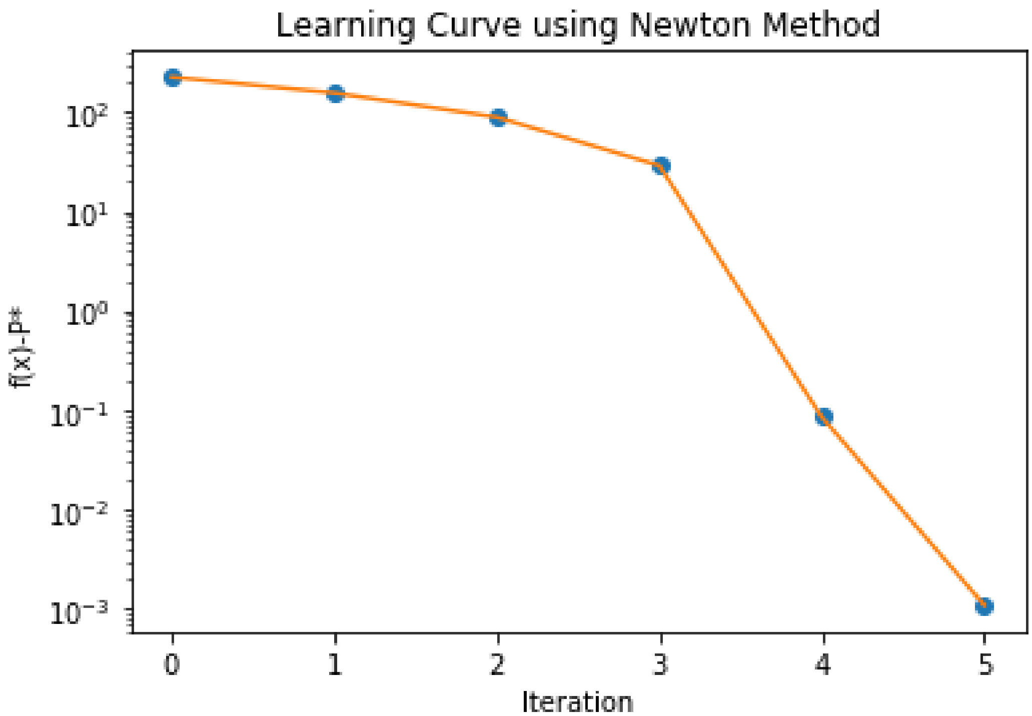

t
Iteration
0.06
0.05
0.04
0.03
0.02
0
5
10
15
20

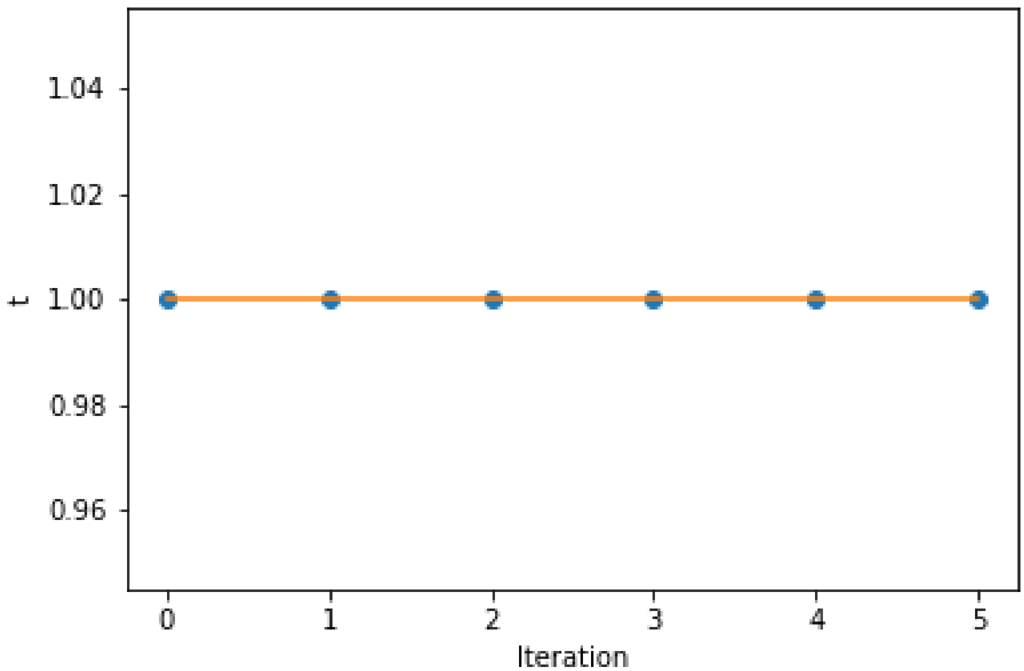
1.2 Newton's Method

In [713]:
```
## Newton's Method
fs = []
steps = []
x = np.zeros(n)

for iter in range(maxiters):
    f = - np.sum(np.log(1-np.dot(A.T,x))) - \
    np.sum(np.log(1+x)) - np.sum(np.log(1-x))
    fs.append(f)
    d = 1/(1-np.dot(A.T,x))
    grad = np.dot(A, d) - 1/(1+x) + 1/(1-x)
    hess = np.dot(A.dot(np.diag(np.diag(np.outer(d,d.T)))), A.T) \
    + np.diag(np.diag(1/np.outer((1+x),(1+x).T) \
                      + 1/np.outer((1-x),(1-x).T)))
    v = - np.linalg.solve(hess, grad)
    fprime = np.dot(grad.T, v)

    if np.absolute(fprime) < nttol:
        break

    t = 1
    while (np.amax(np.dot(A.T, (x+t*v))) >= 1) \
    or (np.amax(np.absolute(x+t*v)) >= 1):
        t = beta*t

    while - np.sum(np.log(1-np.dot(A.T, (x+t*v)))) - \
    np.sum(np.log(1+x+t*v)) - \
    np.sum(np.log(1-x-t*v)) > f + alpha*t*fprime:
        t = beta*t

    x = x+t*v
    steps.append(t)

pstar = fs[len(fs)-1]
print "P*:", pstar
print

#print "steps:", steps

plt.semilogy(range(len(fs)-1), (fs[0:len(fs)-1]-pstar), 'o',
             range(len(fs)-1), (fs[0:len(fs)-1]-pstar), '-')
plt.title('Learning Curve using Newton Method')
plt.xlabel("Iteration"); plt.ylabel("f(x)-P*")
plt.show()

plt.plot(range(len(steps)), steps, "o",
```

```
            range(len(steps)), steps, "-")
    plt.xlabel("Iteration"); plt.ylabel("t")
    plt.show()
```

```
P*: -224.497255475
```





2 10.15

In [36]:
```
# 10.15
n = 100
p = 30
A = np.random.rand(p,n)
## Check A is full rank
print "Rank(A)=", np.linalg.matrix_rank(A)

xhat = np.random.uniform(0,1,n)
x0 = np.ones(n)
b = np.dot(A,xhat)
```

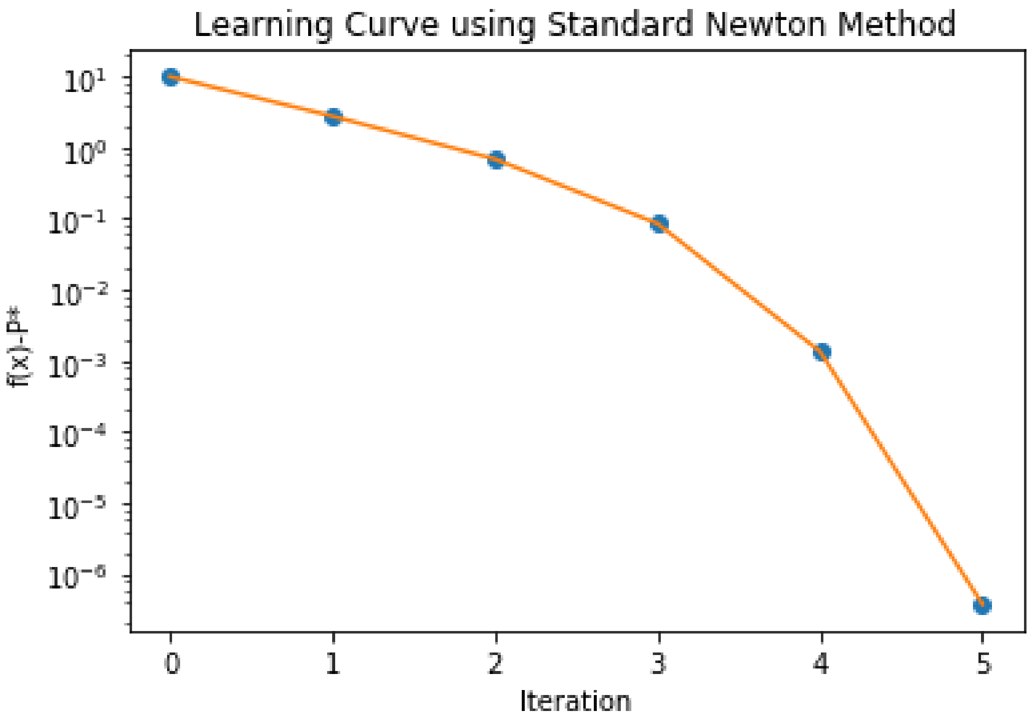
```
Rank(A)= 30
```

2.1 (a) Standard Newton Method

In [37]:
```
## (a) Standard Newton Method
alpha = 0.01
beta = 0.5
maxiters = 100
nttol = 1e-7
```

```python
        fs = []
        x = xhat

        for iter in range(maxiters):
            f = np.dot(x.T, np.log(x))
            fs.append(f)
            grad = 1+np.log(x)
            hess = np.diag(1/x)
            kkt1 = np.concatenate((hess, A.T), axis=1)
            kkt2 = np.concatenate((A, np.zeros((p,p))), axis=1)
            kkt = np.concatenate((kkt1, kkt2), axis=0)
            sol = - np.linalg.solve(kkt,(np.concatenate(\
                                                (grad, np.zeros(p)), \
                                                axis=0)))
            v = sol[0:n]
            fprime = np.dot(grad.T, v)
            if np.absolute(fprime) < nttol:
                break

            t=1
            while np.amin(x+t*v) <= 0:
                t = beta*t
            while np.dot((x+t*v).T, np.log(x+t*v)) \
            >= f + t*alpha*fprime:
                t = beta*t

            x = x + t*v

        pstar = fs[len(fs)-1]
        print "P*:", pstar

        plt.semilogy(range(len(fs)-1),
                     (fs[0:len(fs)-1]-pstar), 'o',
                     range(len(fs)-1),
                     (fs[0:len(fs)-1]-pstar), '-')
        plt.title('Learning Curve using Standard Newton Method')
        plt.xlabel("Iteration"); plt.ylabel("f(x)-P*")
        plt.show()
```

```
P*: -33.450731312
```



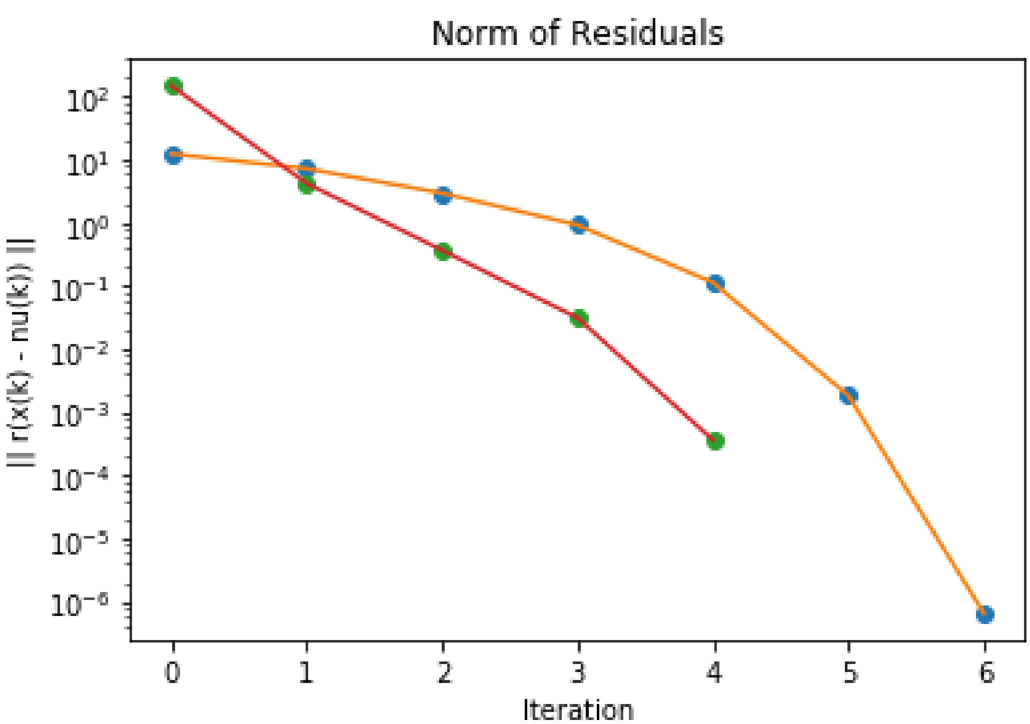
2.2 (b) Infeasible Start Newton Method

```
In [42]: ## (b) Infeasible start Newton method
         restol = 1e-7

         x = xhat ## feasible start
         nu = np.zeros(p)
         resdls = []

         for iter in range(maxiters):
             r = np.concatenate(((1+np.log(x)+np.dot(A.T, nu)), \
                                 (np.dot(A,x)-b)), axis=0)
             resdls.append(np.linalg.norm(r))
             kkt1 = np.concatenate((np.diag(1/x), A.T), axis=1)
             kkt2 = np.concatenate((A, np.zeros((p,p))), axis=1)
             kkt = np.concatenate((kkt1, kkt2), axis=0)
             sol = - np.linalg.solve(kkt, r)
             Dx = sol[0:n]
             Dnu = sol[n:(n+p)]

             if np.linalg.norm(r) < restol:
                 break
```

```
    t = 1
    while np.amin(x+t*Dx) <= 0:
        t = beta*t
    while np.linalg.norm(np.concatenate \
                         (((1+np.log(x+t*Dx) \
                            +np.dot(A.T, (nu+Dnu))),\
                          (np.dot(A,x+Dx)-b)), axis=0)
                        ) \
          > (1 - alpha*t)*np.linalg.norm(r):
        t = beta*t

    x = x + t*Dx
    nu = nu + t*Dnu

print "p* using feasible start:", np.dot(x.T, np.log(x))
###########################################
x = x0 ## infeasible start
nu = np.zeros(p)
resdls2 = []

for iter in range(maxiters):
    r = np.concatenate(((1+np.log(x)+np.dot(A.T, nu)), \
                       (np.dot(A,x)-b)), axis=0)
    resdls2.append(np.linalg.norm(r))
    kkt1 = np.concatenate((np.diag(1/x), A.T), axis=1)
    kkt2 = np.concatenate((A, np.zeros((p,p))), axis=1)
    kkt = np.concatenate((kkt1, kkt2), axis=0)
    sol = - np.linalg.solve(kkt, r)
    Dx = sol[0:n]
    Dnu = sol[n:(n+p)]

    if np.linalg.norm(r) < restol:
        break

    t = 1
    while np.amin(x+t*Dx) <= 0:
        t = beta*t
    while np.linalg.norm(np.concatenate \
                         (((1+np.log(x+t*Dx) \
                            +np.dot(A.T, (nu+Dnu))),\
                          (np.dot(A,x+Dx)-b)), axis=0)
                        ) \
          > (1 - alpha*t)*np.linalg.norm(r):
        t = beta*t

    x = x + t*Dx
    nu = nu + t*Dnu
```

```
print "p* using infeasible start:", np.dot(x.T, np.log(x))

plt.semilogy(range(len(resdls)-1),
             (resdls[0:len(resdls)-1]), 'o',
             range(len(resdls)-1),
             (resdls[0:len(resdls)-1]), '-')
plt.plot(range(len(resdls2)-1),
             (resdls2[0:len(resdls2)-1]), 'o',
             range(len(resdls2)-1),
             (resdls2[0:len(resdls2)-1]), '-')

plt.title('Norm of Residuals')
plt.xlabel("Iteration"); plt.ylabel("|| r(x(k) - nu(k)) ||")
plt.show()
```

```
p* using feasible start: -33.450731312
p* using infeasible start: -33.450731312
```

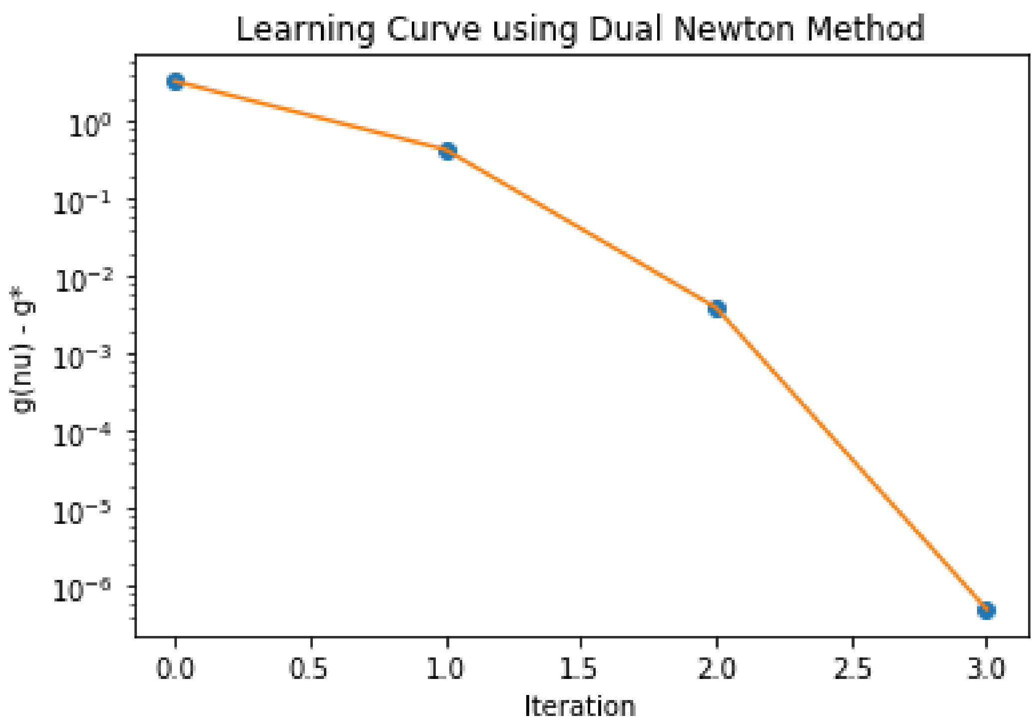


2.3 (c) Dual Newton Method

```
In [43]: ## (c) Dual Newton Method
         gs = []
```

```python
nu = np.zeros(p)

for iter in range(maxiters):
    g = np.dot(b.T, nu) + np.sum(np.exp(-np.dot(A.T, nu)-1))
    gs.append(g)
    grad = b - np.dot(A, np.exp(-np.dot(A.T, nu)-1))
    hess = np.dot(A.dot(np.diag(np.exp(-np.dot(A.T, nu)-1))), A.T)
    v = - np.linalg.solve(hess, grad)
    fprime = np.dot(grad.T, v)

    if np.absolute(fprime) < nttol:
        break
    t = 1
    while np.dot(b.T, (nu+t*v)) + \
    np.sum(np.exp(-np.dot(A.T, (nu+t*v))-1)) \
          > g + t*alpha*fprime:
        t = beta*t
    nu = nu+t*v

gstar = -gs[len(gs)-1]
print "g*:", gstar

plt.semilogy(range(len(gs)-1), (gs[0:len(gs)-1]+gstar), 'o',
             range(len(gs)-1), (gs[0:len(gs)-1]+gstar), '-')
plt.title('Learning Curve using Dual Newton Method')
plt.xlabel("Iteration"); plt.ylabel("g(nu) - g*")
plt.show()
```

```
g*: -33.450731312
```



3 11.22

In [44]:
```
# 11.22
maxiters = 200
alpha = 0.01
beta = 0.5
nttol = 1e-8
mu = 20
tol = 1e-4

A = np.matrix([
    [0,-1],[2,-4],[2,1],[-4,4],[-4,0]
])
n = np.shape(A)[1]
m = np.shape(A)[0]

b = np.array([np.ones(5)]).T

Ap = np.fmax(A,0)
Am = np.fmax(-A,0)

r = np.amax(np.dot(Ap, np.ones(n)) + np.dot(Am, np.ones(n)))
```

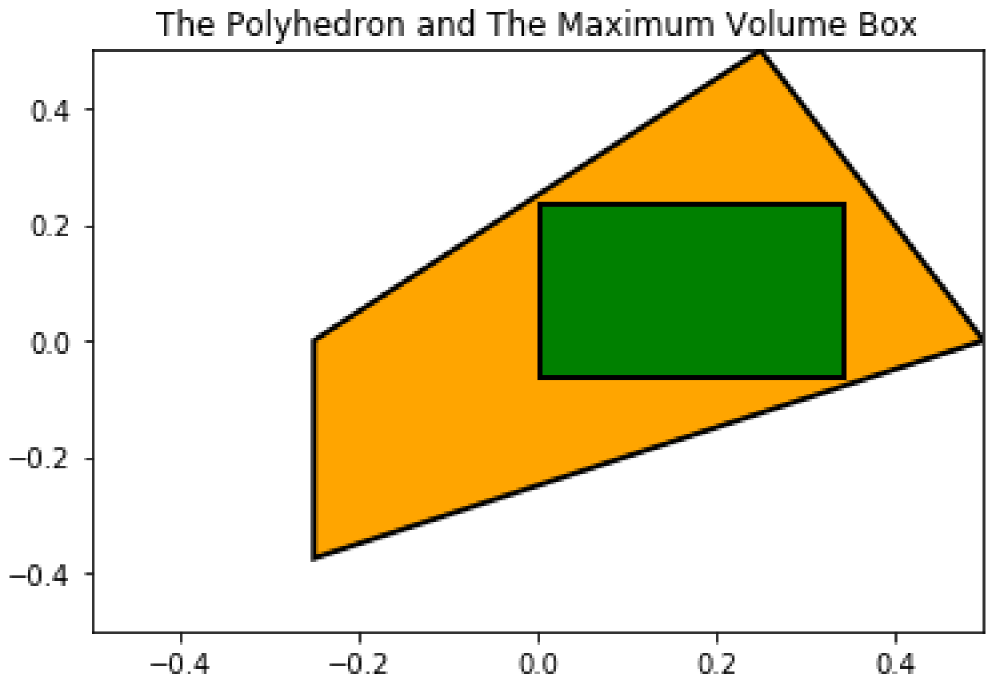
```
        u = np.array([(0.5/r)*np.ones(n)]).T
        l = np.array([-(0.5/r)*np.ones(n)]).T
```

```
In [45]: t = 1
         for iter in range(maxiters):
             y = b + np.dot(Am,l)-np.dot(Ap,u)
             val = -t*np.sum(np.log(u-l)) - np.sum(np.log(y))
             grad = (t*np.concatenate((1/(u-l), -1/(u-l)), axis=0)
                     + np.dot(np.concatenate((-Am.T, Ap.T), axis=0),(1/y)))

             kkt1 = np.concatenate((np.diagflat(np.square(1/(u-l))),
                                    np.diagflat(-np.square(1/(u-l)))), axis = 1)
             kkt2 = np.concatenate((np.diagflat(-np.square(1/(u-l))),
                                    np.diagflat(np.square(1/(u-l)))), axis = 1)
             kkt = np.concatenate((kkt1, kkt2), axis = 0)

             hess = t*kkt + np.dot(np.concatenate((-Am.T, Ap.T), axis=0).dot
                    (np.diagflat(np.square(1/y))),
                    np.concatenate((-Am, Ap), axis=1))

             step = - np.linalg.solve(hess, grad)
             fprime = np.dot(grad.T, step)

             if np.absolute(fprime) < nttol:
                 gap = (2*m)/t
                 print "iter:", str(iter), "; gap:", str(gap)
                 if gap < tol:
                     break
                 t = mu*t

             else:
                 dl = step[0:n]
                 du = step[n:(2*n)]
                 dy = np.dot(Am, dl) - np.dot(Ap, du)
                 tls = 1
                 while (np.amin(np.concatenate(
                       (u-l+tls*(du-dl),y+tls*dy), axis = 0)) <= 0):
                     tls = beta*tls

                 while (-t*np.sum(np.log(u-l+tls*(du-dl))) -
                         np.sum(np.log(y+tls*dy)) >= val+tls*alpha*fprime):
                     tls = beta*tls

                 l = l+tls*dl
                 u = u+tls*du


         print "l:", l
```

```
        print "u:", u
```

```
iter: 3 ; gap: 10
iter: 9 ; gap: 0
l: [[ 0.00228159]
 [-0.06305297]]
u: [[ 0.34371329]
 [ 0.23397095]]
```

```
In [27]: import matplotlib.pyplot as plt
         from matplotlib.path import Path
         import matplotlib.patches as patches

         verts = [
            (-1/4., 0.), (-1/4., -3/8.), (1/2., 0.),(1/4., 1/2.), (-1/4., 0)
             ]
         codes = [Path.MOVETO,
                  Path.LINETO,
                  Path.LINETO,
                  Path.LINETO,
                  Path.CLOSEPOLY,
                  ]

         verts1 = [
            (0.00228159, -0.06305297), (0.00228159, 0.23397095),
             (0.34371329, 0.23397095),
             (0.34371329, -0.06305297), (0.34371329, 0.23397095)
             ]
         codes1 = [Path.MOVETO,
                  Path.LINETO,
                  Path.LINETO,
                  Path.LINETO,
                  Path.CLOSEPOLY,
                  ]
         path = Path(verts, codes)
         path1 = Path(verts1, codes1)
         fig = plt.figure()
         ax = fig.add_subplot(111)
         patch = patches.PathPatch(path, facecolor='orange', lw=2)
         ax.add_patch(patch)

         patch1 = patches.PathPatch(path1, facecolor='green', lw=2)
         ax.add_patch(patch1)

         ax.set_xlim(-0.5,0.5)
         ax.set_ylim(-0.5,0.5)
         ax.set_title("The Polyhedron and The Maximum Volume Box")
         plt.show()
```


The Polyhedron and The Maximum Volume Box
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4