CIS580 Problem Set 2

Contents

1 Line that passes through the points

1.1 $[0, a, 0], [0, 0, a]$

$$l = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} = \begin{bmatrix} a^2 - 0 \\ 0 - 0 \\ 0 - 0 \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

1.2 $[a, a, 1], [a, a, 2]$

$$l = \begin{bmatrix} a \\ a \\ 1 \end{bmatrix} \times \begin{bmatrix} a \\ a \\ 2 \end{bmatrix} = \begin{bmatrix} 2a - a \\ a - 2a \\ a^2 - a^2 \end{bmatrix} = \begin{bmatrix} a \\ -a \\ 0 \end{bmatrix}$$

1.3 $[a, b, 0], [c, d, 0]$

$$l = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \times \begin{bmatrix} c \\ d \\ 0 \end{bmatrix} = \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ ad - bc \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ ad - bc \end{bmatrix}$$

2 Point of Intersection $\in \mathbb{P}^2$

2.1 $x - y + w = 0, w = 0$

$$P = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 - 0 \\ 0 - 1 \\ 0 - 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

2.2 $3x - w = 0, 4y - w = 0$

$$P = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 + 4 \\ 0 + 3 \\ 12 - 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 12 \end{bmatrix}$$

2.3 $x - y + 5w = 0, x - y + 2w$

$$P = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 + 5 \\ 5 - 2 \\ -1 + 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

3 Find λ such that three lines intersect

Write the lines in a matrix system of equations:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & \lambda & \lambda \\ \lambda & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Given that $w = 0$, we reduce the system of equations to the following:

$$\begin{bmatrix} 1 & \lambda \\ \lambda & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Set the determinant equal to zero:

$$\begin{aligned} 1 - \lambda &= 0 \\ (1 - \lambda)(1 + \lambda) &= 0 \\ \lambda &= -1, 1 \end{aligned}$$

We choose $\lambda = -1$ since in the other case, we do not have three distinct lines. Using this, we compute the point of intersection as follows:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0+1 \\ 1+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

4 Find projective transformation A

We wish to preserve:

$$\begin{aligned} P_1 &= (1, 0, 0) \\ P_2 &= (0, 1, 0) \\ O &= (0, 0, 1) \end{aligned}$$

We wish to map: $P_3 = (1, 1, 1) \to P_3{}' = (3, 2, 1)$

$$(P_1{}', P_2{}', P_3{}', O) = M(P_1, P_2, P_3, O)$$

$$\begin{bmatrix} \lambda_1 & 0 & 3\lambda_3 & 0 \\ 0 & \lambda_2 & 2\lambda_3 & 0 \\ 0 & 0 & \lambda_3 & \lambda_4 \end{bmatrix} = M \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$