# CIS 580, Machine Perception, Spring 2021
# Homework 3 Solutions
# Due: Thursday Feb 25 2021, 5:59pm

## Instructions

- This is an individual homework and worth 100 points
- You must submit your solutions on Gradescope, the entry code is X3GJR8. We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Start early! If you get stuck, please post your questions on Piazza or come to office hours!

## Homework

1. (10 pts.) The image of the rectangle-shaped facade of a building has two vanishing points, one at $(-b, 0)$ corresponding to horizontal lines and one at $(0, h)$ corresponding to the vertical lines. Find the transformation that will map the facade to a rectangle. Assume that the origin (0,0) and the point $(1, 1)$ remain fixed.

   **Answer:**
   We call $H$ the transformation that maps a rectangle to the facade and $P$ the transformation that maps the facade to the rectangle. Using the 2 vanishing points $(1, 0, 0)^T$ and $(0, 1, 0)^T$ along with the origin $(0, 0, 1)^T$ we have:

$$\begin{pmatrix} -\alpha b & 0 & 0 \\ 0 & \beta h & 0 \\ \alpha & \beta & \gamma \end{pmatrix} = H \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   then

$$\begin{pmatrix} -\alpha b & 0 & 0 \\ 0 & \beta h & 0 \\ \alpha & \beta & \gamma \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

   we can assume $\lambda = 1$. So $\alpha = -\frac{1}{b}, \beta = \frac{1}{h}$ and

$$\alpha + \beta + \gamma = 1 \implies \gamma = 1 + \frac{1}{b} - \frac{1}{h} = \frac{bh + h - b}{hb}$$

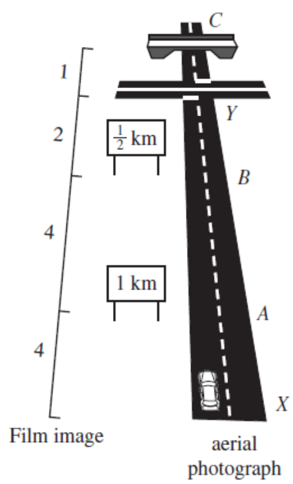
So

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-1}{b} & \frac{1}{h} & \frac{bh+h-b}{hb} \end{pmatrix}$$

And the transformation $P$ we are looking for is

$$P = H^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{h}{bh+h-b} & \frac{-b}{bh+h-b} & \frac{bh}{bh+h-b} \end{pmatrix}$$

2. (20 pts.) The following diagram represents an aerial photograph of a straight road on flat ground. At A there is a sign 'Junction 1 km', at B asign 'Junction $\frac{1}{2}$ km', and C is the road junction. Also, a police patrol car is at X,and a bridge is at Y .The distances marked on the left of the diagram are measured in cm from the photograph. Calculate the actual distances (in km) of the patrol car and the bridge from the junction.




**Answer:**

$$\frac{AY}{AC} : \frac{BY}{BC} = \frac{A_wY_w}{A_wC_w} : \frac{B_wY_w}{B_wC_w} \implies \frac{6}{7} : \frac{2}{3} = \frac{(1 - C_wY_w)}{1} : \frac{(0.5 - C_wY_w)}{0.5}$$
$$\implies \frac{9}{7} = \frac{(1 - C_wY_w)}{(1 - 2C_wY_w)}$$
$$\implies C_wY_w = 2/11$$

for $X_wC_w$ we have

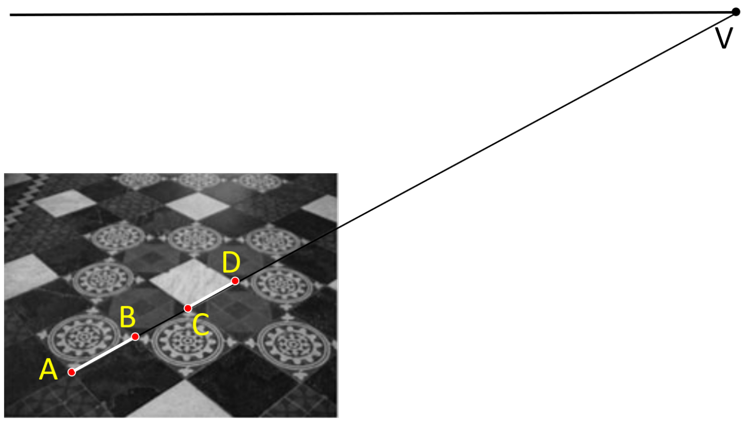
$$\frac{X_wB_w}{X_wC_w} : \frac{A_wB_w}{A_wC_w} = \frac{XB}{XC} : \frac{AB}{AC} = \frac{8}{11} : \frac{4}{7} = \frac{14}{11}$$
$$\implies \frac{(X_wC_w - 0.5)}{X_wC_w} : \frac{0.5}{1} = \frac{14}{11}$$
$$\implies \frac{(2X_wC_w - 1)}{X_wC_w} = \frac{14}{11}$$
$$\implies X_wC_w = \frac{11}{8}$$

3. (15 pts.) In the following image, the points $A, B, C, D$ are collinear. Point $V$ is the vanishing point for line $AB$. For the world points we know that $A_w B_w = B_w C_w = C_w D_w = 4$.

   Given the distances $AB = 3$, $CD = 2$, compute

   (a) The distance $BC$

   (b) The distance $DV$




**Answer:**

(a)

$$\frac{AC}{AD} : \frac{BC}{BD} = \frac{A_w C_w}{A_w D_w} : \frac{B_w C_w}{B_w D_w} = \frac{8}{12} : \frac{4}{8} = \frac{4}{3}$$

$$\implies \frac{(3+BC)}{(5+BC)} : \frac{BC}{(2+BC)} = \frac{(3+BC)(2+BC)}{BC(5+BC)} = \frac{4}{3}$$

$$\implies BC = \frac{\sqrt{97}}{2} - \frac{5}{2} \approx 2.4244$$

b)

$$\frac{AD}{AV} : \frac{BD}{BV} = \frac{A_w D_w}{A_w V_w} : \frac{B_w D_w}{B_w V_w} = \frac{A_w D_w}{B_w D_w} = \frac{3}{2}$$

$$\implies \frac{7.4244}{(7.4244 + DV)} : \frac{4.4244}{(4.4244 + DV)} = \frac{3}{2}$$

$$\implies DV \approx 20.848$$

4. (40 pts.) For this question you will need to read the paper

   You are given the following images from 2 different tennis matches captured by the same camera. It is known that the single court tennis width (inner lines) is 27 ft., and the court length is 78 ft. This means that for the world points we know the distances $A_w D_w = B_w C_w = A'_w D'_w = B'_w C'_w = 27ft$. and $A_w B_w = C_w D_w = A'_w C'_w = C'_w D'_w = 78ft$. Also the net in both images is in the middle of the court.

   (a) Explain why is the perspective different in the two images.

   (b) Using the known cross-ratios find the vanishing points of the sidelines ($AB$, $CD$, $A'B'$, $C'D'$) in both images.

(c) Find the vanishing points for the court baselines ($AD$, $BC$, $A'D'$, $B'C'$) using the intersection of the corresponding lines. For each image, modify the method from class and solve for 3 unknowns to compute the homography that maps the tennis court to the image plane. (the method with the 3 unknowns is presented in the slides from the lecture on 2/3, in pages 8-20).

[for half the points, you can use the method that solves the problem with 8 unknowns using the SVD]

(d) Using the homographies that you found, apply the method presented in the paper "Camera Calibration using Vanishing Points" to compute the focal length of each image.

(e) Compute again the vanishing points of the sidelines (similar to question 4(b)). Now instead of using the cross-ratio use the intersection of the parallel lines. Compare the vanishing points found with this method and the vanishing points from question 4(b).

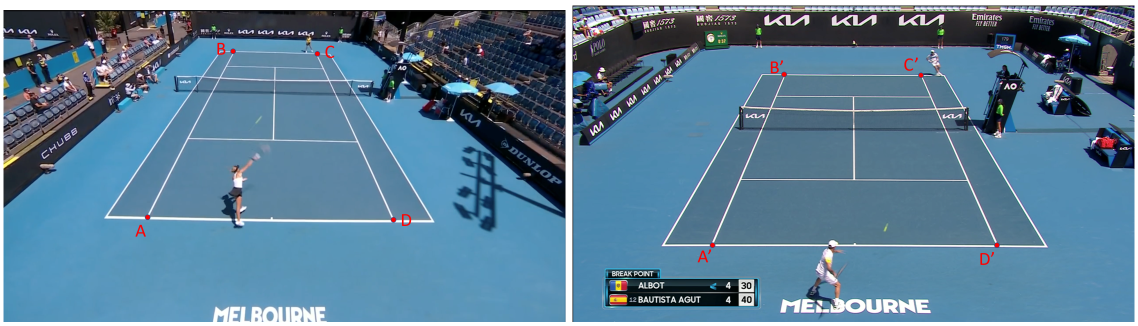



Figure 1: The two original images can be found here

*To solve the above problem you should take measurements on the given images. Make sure to clearly define the coordinate system that you use both in the image plane and the real world plane.*

**Answer:**

b)For the first image, we have the mapping $A_w = [0, 0, 1]$ to $A = [638, 916, 1]$, $B_w = [0, 78, 1]$ to $B = [1017, 182, 1]$, $C_w = [27, 78, 1]$ to $C = [1382, 193, 1]$, $D_w = [27, 0, 1]$ to $D = [1720, 926, 1]$. Between A and B we have the point $E_w = [0, 78/2, 1]$ that we use to find the vanishing point with the cross ratio

$$\frac{AB}{AB + BV_v} : \frac{EB}{EB + BV_v} = \frac{A_w B_w}{E_w B_w} = 2 \implies BV_v = 403.084$$

.

With similar triangles we find vertical vanishing point distance $V_v = [1197, -178.63, 1]$. For the second image we have $A'_w = [0, 0, 1]$ to $A' = [619, 1055, 1]$, $B'_w = [0, 78, 1]$ to $B' = [936, 305, 1]$, $C'_w = [27, 78, 1]$ to $C' = [1540, 309, 1]$, $D'_w = [27, 0, 1]$ to $D' = [1874, 1061, 1]$ and with cross ratio we find $B'V'_v = 732.629$

With similar triangles we get the vertical vanishing point is $V'_v = [1220.9, -369.96, 1]$

c) With cross products for the first image we find the lines $AD = [-10, 1082, -984732]$, $BC = [11, -365, 55243]$ that intersect in the horizontal vanishing point $V_h = [36312, 1245, 1]$.

With cross product for the second image we find the lines $A'D' = [-6, 1255, -1320311]$, $B'C' = [4, -604, 180476]$, that intersect in the horizontal vanishing point $V'_h = [409004, 3007, 1]$.

For the first image we have that

$$H = \left(\alpha V_h^T, \beta V_v^T, \gamma A^T\right)$$

and also

$$\left(\alpha V_h^T, \beta V_v^T, \gamma A^T\right) C_w^T = C^T \implies$$

$$\left(C_w[1] V_h^T, C_w[2] V_v^T, C_w[3] A^T\right) \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = C^T \implies$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \left(C_w[1] V_h^T, C_w[2] V_v^T, C_w[3] A^T\right)^{-1} C^T = \begin{pmatrix} 0.00038715 \\ 0.00850827 \\ 0.32590182 \end{pmatrix}$$

So by substituting $\alpha, \beta, \gamma$ back at the equation for $H$ we get

$$H = \begin{pmatrix} 14.0584484 & 10.1858529 & 207.925364 \\ 0.482274266 & -1.51983945 & 298.526072 \\ 0.000387 & 0.00850827 & 0.32590 \end{pmatrix}$$

For the second image we follow the same process to find

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0.000054838 \\ 0.00673782 \\ 0.47296872 \end{pmatrix}$$

and

$$H' = \begin{pmatrix} 22.429161 & 8.2262177 & 292.767643 \\ 0.16492328 & -2.4927556 & 498.982008 \\ 0.00005483840 & 0.006737828 & 0.47296872 \end{pmatrix}$$

(d) We follow the paper's method where $a^{-1} = H$. So for the first image we get

$$L = a^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.11391408e-04 \\ 2.74624275e-03 \\ 6.23918076e-01 \end{pmatrix}$$

$$V_a = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times L = \begin{pmatrix} -0.00274624 \\ -0.00011139 \\ 0 \end{pmatrix}$$

$$I_a = aV_a = \begin{pmatrix} -2.02004369e-04 \\ 9.19169427e-06 \\ 0 \end{pmatrix}$$

$$I_b = \begin{pmatrix} -I_a[2] \\ I_a[1] \\ 0 \end{pmatrix} = \begin{pmatrix} -9.19169427e-06 \\ -2.02004369e-04 \\ 0 \end{pmatrix}$$

$$Vb = a^{-1} I_b = \begin{pmatrix} 1.26972661e+03 \\ -1.75687834e+02 \\ 1 \end{pmatrix}$$

$$I_c = I_b + I_a = \begin{pmatrix} -0.0002112 \\ -0.00019281 \\ 0 \end{pmatrix}$$

$$V_c = a^{-1} I_c = \begin{pmatrix} 2.86427808e+03 \\ -1.11010619e+02 \\ 1 \end{pmatrix}$$

also $M$ is vertical to $L$ and passes from $V_b$ so we have that $M^T = [-L[2], L[1], M[3]]$ and

$MVb = 0 \implies M[1]V_b[1]+M[2]V_b[2]+M[3]V_b[3] = 0 \implies M[3] = (-M[1]V_b[1]-M[2]V_b[2])/V_b[3]$

so

$$M = \begin{pmatrix} -2.74624275e-03 \\ -1.11391408e-04 \\ 3.46740739e+00 \end{pmatrix}$$

For the second image we have $a'^{-1} = H'$ and

$$L' = a'^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.22461682e-05 \\ 1.47858709e-03 \\ 5.61975688e-01 \end{pmatrix}$$

$$V_a' = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times L' = \begin{pmatrix} -1.47858709e-03 \\ -1.22461682e-05 \\ 0 \end{pmatrix}$$

$$I_a' = a' V_a' = \begin{pmatrix} -6.61199011e-05 \\ 5.38142204e-07 \\ 0 \end{pmatrix}$$

$$I_b' = \begin{pmatrix} -I_a'[2] \\ I_a'[1] \\ 0 \end{pmatrix} = \begin{pmatrix} -5.38142204e-07 \\ -6.61199011e-05 \\ 0 \end{pmatrix}$$

$$V_b' = a'^{-1} I_b' = \begin{pmatrix} 1.24791080e+03 \\ -3.69740522e+02 \\ 1 \end{pmatrix}$$

$$I_c' = I_b' + I_a' = \begin{pmatrix} -6.66580433e-05 \\ -6.55817589e-05 \\ 0 \end{pmatrix} V_c' = a'^{-1} I_c' = \begin{pmatrix} 4.56659544e+03 \\ -3.42254031e+02 \\ 1 \end{pmatrix}$$

$$M' = \begin{pmatrix} -1.47858709e-03 \\ -1.22461682e-05 \\ 1.84061690e+00 \end{pmatrix}$$

Then we have that

$$P = M \times M' = \begin{pmatrix} 1240.29357 \\ 549.954048 \\ 1 \end{pmatrix}$$

And we can compute the focal length for the first image with $CV_b = V_c V_b = 1595.86$, $V_b P = 726.23$ and $F = CP = \sqrt{CV_b^2 - V_b P^2} = 1421.04$

For the second image $C'V_b' = V_C' V_b' = 3318.8$, $V_b' P = 919.72$ and $F = C'P = \sqrt{C'V_b'^2 - V_b' P^2} = 3188.8$

(e) For the first image we compute lines $AB = A \times B = [734, 379, -8.15e+05]$, $DC = D \times C = [733, -338, -9.47e+5]$. And the vanishing point is

$$V_v = AB \times DC = [1207, -186, 1]$$

.

For the second image we compute lines $A'B' = A' \times B' = [750, 317, -7.98e+05]$, $D'C = D' \times C' = [752, -334, -10.54e+5]$. And the vanishing point is

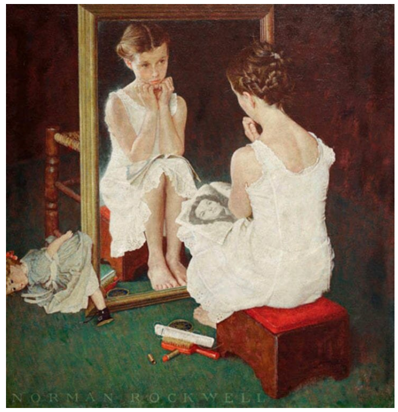
$$V_v' = A'B' \times D'C' = [1229, -389, 1]$$

.

We can see that although small differences the vanishing points are close to the ones computed in (b).

5. (15 pts.) For this question you will need to read the article "How to Detect Faked Photos" (Farid).

   Is the following reflection painting perspectively correct?




   *To answer this question you can draw lines on top of the image and use them for your argument. The original image can be found here*

   **Answer:**

   The perspective is not correct