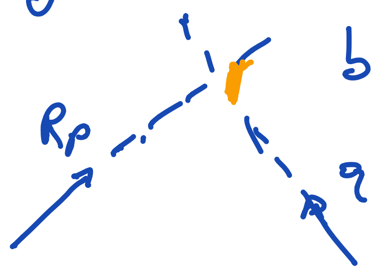
Triangulation

$$\begin{pmatrix} q & -Rp \end{pmatrix}\begin{pmatrix}\lambda\\ \mu\end{pmatrix} = T$$

$$3\times 2 \qquad 2\times 1$$

Why is it overdetermined?



because the two rays do not exactly intersect!

Overdetermined $\Rightarrow$ least squares

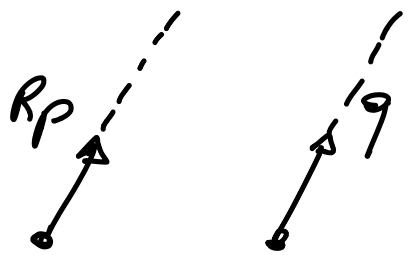
$$\min_{\lambda,\mu} \|\lambda q - Rp\mu - T\|^2$$

$$\underset{3\times 2}{B}\ \overset{2\times 1}{x} = \underset{3\times 1}{b} \qquad x = B^+ b$$

if B full rank
$B^+ = (B^TB)^{-1}B^T$

$\exists$ even if B is not full rank!

When is B not full rank?



$q \parallel Rp$ when? point very far away (sky, clouds)..

Pseudoinverse for matrices of defficient rank!

$$A = USV^T \qquad A^+ = V S^+ U^T$$

A: $n \times n$

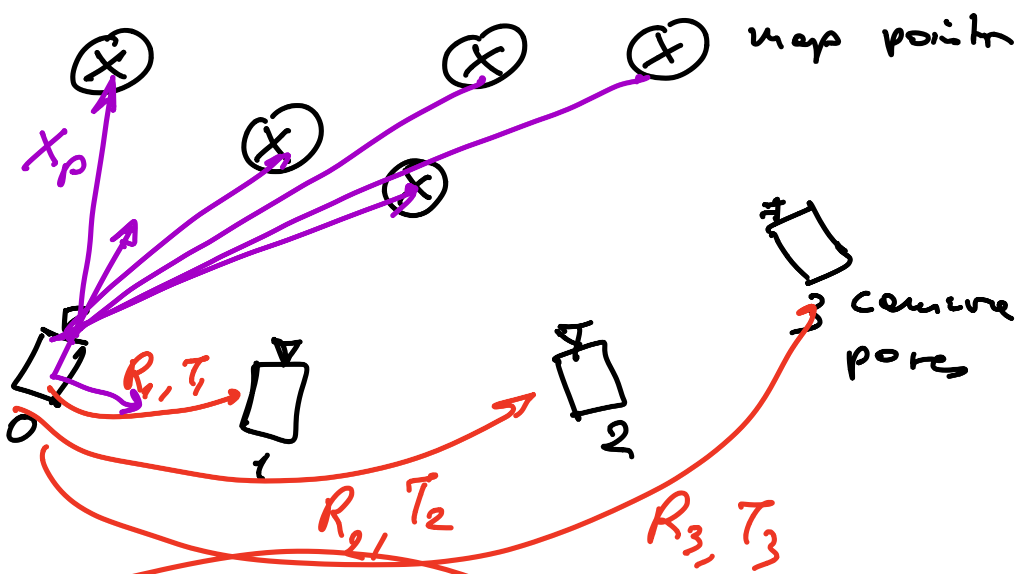
$$S = \begin{pmatrix} \sigma_1 & & & & & & \\ & \sigma_2 & & & & & \\ & & \ddots & & & & \\ & & & \sigma_r & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix} \quad \nexists S^{-1}$$

($\mathrm{rank}(A) = r$)

$$S^+ = \begin{pmatrix} 1/\sigma_1 & & & & & \\ & \ddots & & & & \\ & & 1/\sigma_r & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

Bundle Adjustment

(graph pose optimisation)



$$\sum_{f,p} \left\| \begin{pmatrix} x_p^f \\ y_p^f \end{pmatrix} - \text{proj.}\left(R^f X_p + T^f\right) \right\|^2 \Rightarrow \min_{\substack{R^f, T^f \\ X_p}}$$

$$= \varepsilon^*$$

minimization is Bundle Adjustment

nonlinear least squares

$$\boxed{F \geq 3 \text{ frames} \qquad N \geq 4 \text{ points}}$$

Solved with iterative approaches (example: gradient descent)

Gradient descent is very slow.

Gauss-Newton / Levenberg-Marquardt iteration $u_{k+1} = u_k + \Delta u$

$$(J^T J)\,\Delta u = -J^T \varepsilon$$

unknown

why?

bottleneck

$$(6F + 3N - 7) \times (6F + 3N - 7)$$

frame

points

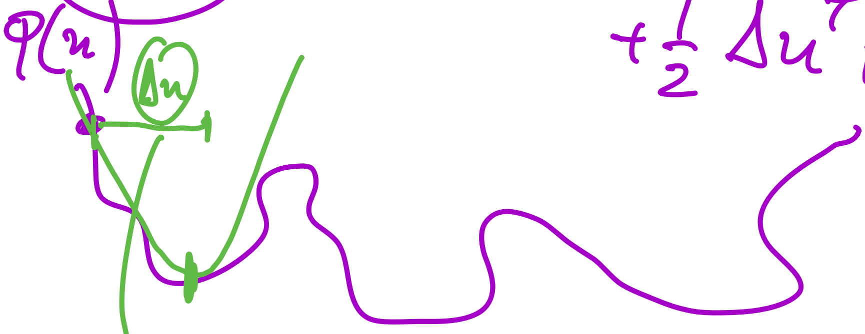
$R_0 = I$

6 0^{th} frame fixed $T_0 = 0$

+ 1 $Z_{p=0} = 5m$ for example!

why?

$$\varphi(u+\Delta u) = \varphi(u) + \Delta u^T \nabla\varphi(u) + \frac{1}{2}\Delta u^T H \Delta u + \dots$$



$$\nabla\varphi + H\Delta u = 0$$
$$\downarrow J^TJ$$
$$(J^TJ)\Delta u = -\nabla\varphi$$

$$J = \frac{\partial \varepsilon}{\partial \begin{pmatrix}\text{motion}\\ \text{structure}\end{pmatrix}}$$

$$J^TJ = \begin{pmatrix} \frac{\partial^T}{\partial\,\text{motion}}\frac{\partial}{\partial\,\text{motion}} : \ \blacksquare_{6\times6}\ \ \blacksquare_{6\times6} & \text{mixed} \\ \text{mixed}^T & \frac{\partial^T}{\partial\,\text{structure}}\frac{\partial}{\partial\,\text{struct}} : \ \blacksquare_{3\times3}\ \blacksquare_{3\times3}\ \blacksquare_{3\times3}\ \dots \end{pmatrix}$$

$F = 10$ frames

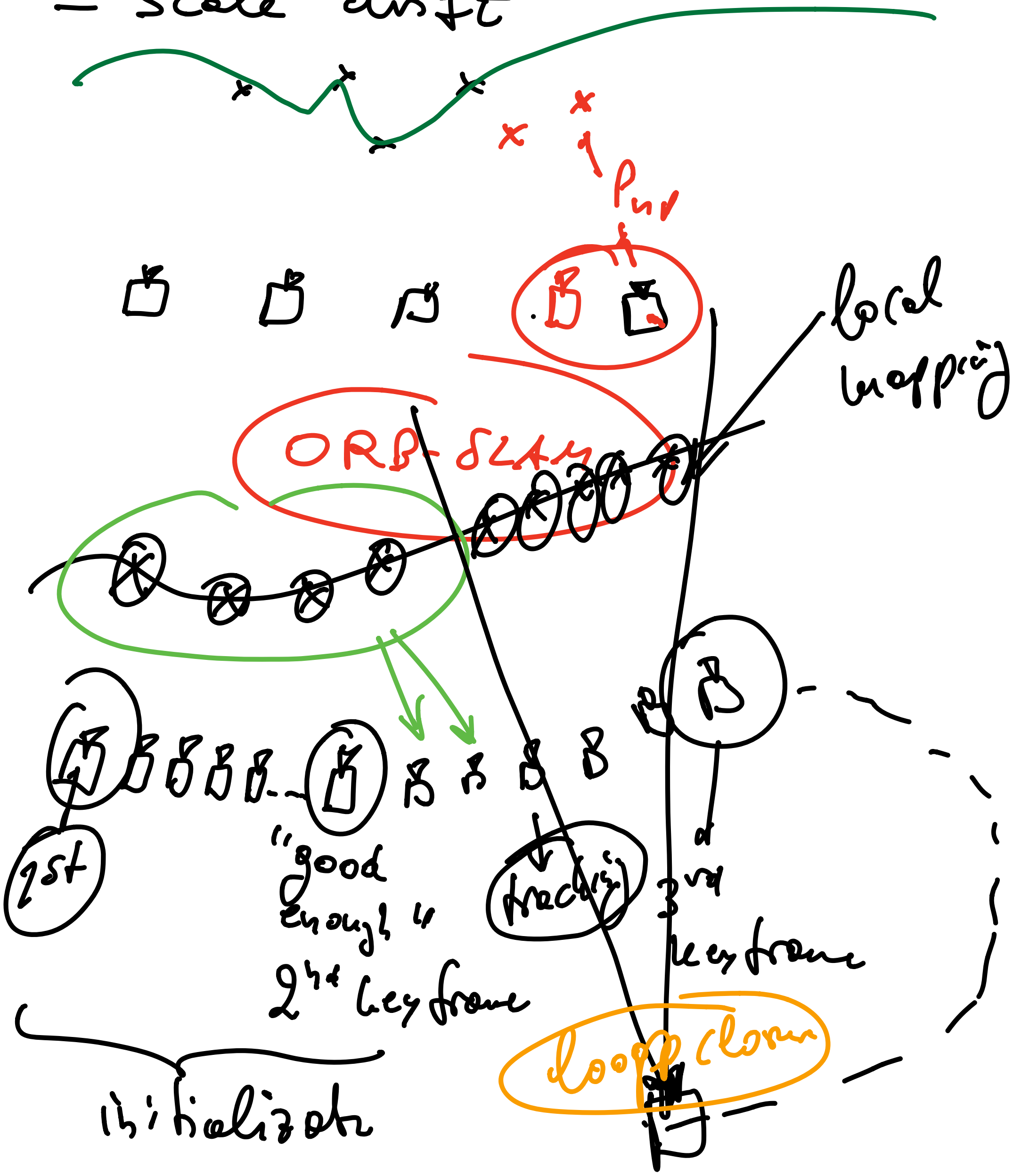
$N = 100$ points

invert $6F \times 6F$ $\quad 60 \times 60 \Rightarrow$ motion

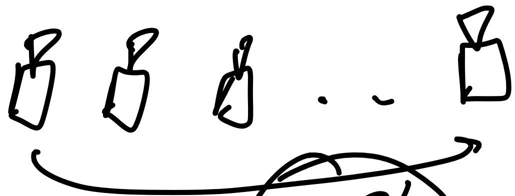
invert 3×3 for the points.

Visual odometry

- scale drift



Map initialization



① $>90\%$

② non-deficient epipolar geometry

$$\underbrace{\begin{pmatrix} & A & \end{pmatrix}}_{N\times 9}\underbrace{e}_{9\times 1}=0 \qquad N\geq 8$$

① good: $\operatorname{rank}(A)=8$

② bad: $\operatorname{rank}(A)=6$?

$$q^T E p = 0$$

$$\exists H : \quad q \sim Hp$$

① pure rotation

② all scene points are on a plane

$$q^T \underbrace{E H^{-1}}_{C} q = 0$$

for what matrix C

$$q^T C q = 0 \quad \forall q \quad ?$$

$$q^T (c \times q) = 0 \quad \forall q$$

$$C = \hat{c}$$

$$E H^{-1} = \hat{c} \Rightarrow E = \hat{c} H$$

If $q \sim Hp$ then there exists a family of solutions for $E = \hat{c} H$: this is a 3-parametric family because c has 3 elements.

This means that the nullspace of A in $Ae=0$ is 3-dimensional hence $\mathrm{rank}(A) = 6$.

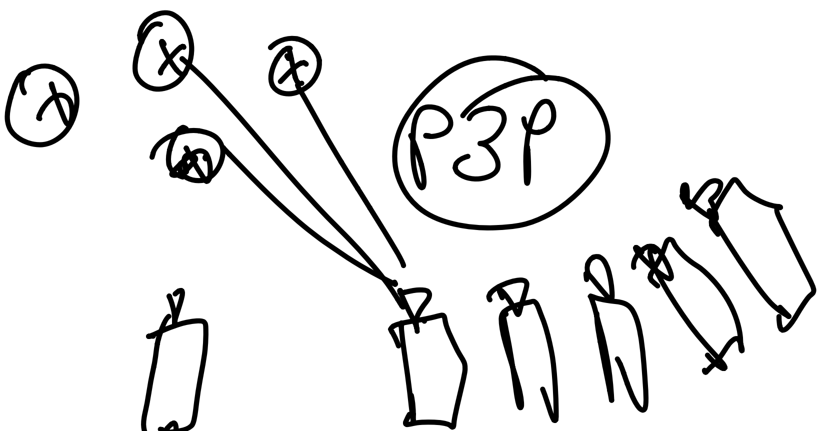
Practical check:

1. $\sigma_8(A) >> \sigma_9(A) \approx 0$

or 2 fit on H

is $q_i \sim H p_i$

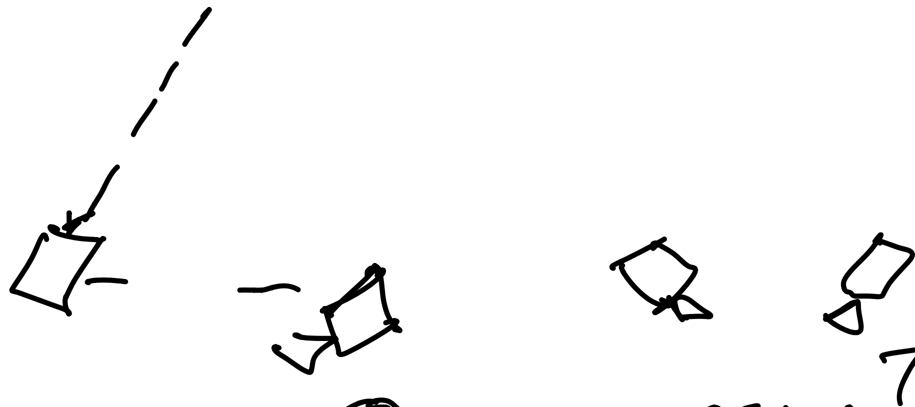
TRACKING



look keyframe

8:30p Tuesday

open OH



$$E = \hat{T} R \to U R_z\left(\frac{\pi}{2}\right) V^T$$

$$E = \hat{T} R_T(\pi) R \to U R_z\left(-\frac{\pi}{2}\right) V^T$$

$$\hat{u}_3 \qquad U \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} U^T$$