CIS 580 Midterm, Spring 2020

Name: Max time: 80 minutes

Problem	Points	Score
1	15	
2	15	
3	15	
4	20	
5	15	
6	20	
Sum	100	

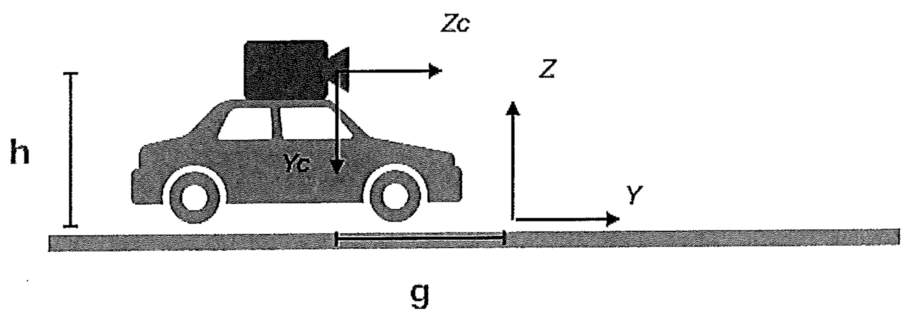
$$
\begin{aligned}
\text{cross ratio} \quad \{A, B; C, D\} &= \frac{AC}{AD} : \frac{BC}{BD} \\
\cos(\alpha + \beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\sin(\alpha + \beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
Ap \times Aq &= \frac{1}{\det(A)} A^{-T}(p \times q) \quad \text{for} \quad p, q \in \mathbb{R}^3, A \in \mathbb{R}^{3\times 3} \\
a \times b = \hat{a}b \qquad \text{where} \qquad \hat{a} &= \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix} \\
\text{rotation about x} \qquad R_x &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \\
\text{rotation about y} \qquad R_y &= \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}
\end{aligned}
$$

Q.1 15 points Assume that : (i) there is no radial distortion in the picture although it is visible close to the border. (ii) the horizon is horizontal (iii) the vertical lines are parallel. Do not assume that the center of the picture is the image center.

1. Draw the horizon of the ground plane.
2. Find the height of the person in the picture (including heels) as designated by the line segment if the height of the end of the toilet booth panel is 180cm as shown in the picture.

Show your work with the ruler, clearly name the points and lines you will use. No need to carry out the calculations. Answers like 16780/365*24 get full credit including approximate answers.

Extra credit: Who is the person in the picture. From which movie is this shot.



Q.2 15 points

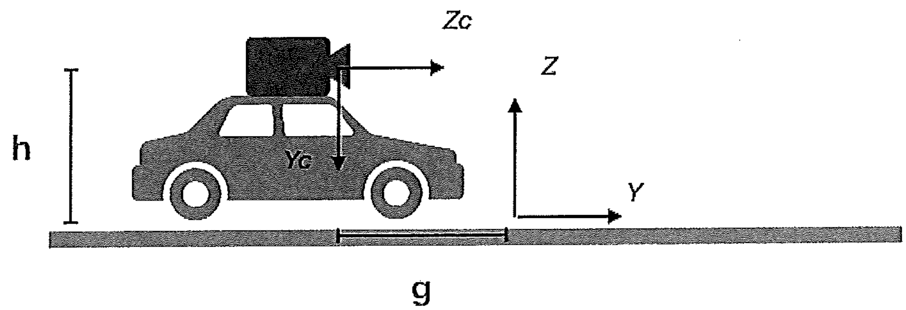
1. What are the R and T in the camera's projection equation from world to image plane:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$$

2. Assume $K = I$. Write the projective transformation from the plane $Z = 0$ to the image plane.

3. Find the 3x1 vector of coefficients of the horizon (projection of the line at infinity in the image plane)

4. Imagine there is a line $X = m$ in the world plane (in homogeneous coordinates it reads $X - mW = 0$). Find its projection in the image plane.



Q.3 15 points All is same as in Q.2.

1. Imagine that the camera moves forward by ΔZ along its Z_c axis. Prove that the transformation between the new image plane (u', v', w') and the old image plane (u, v, w) is a projective transformation.

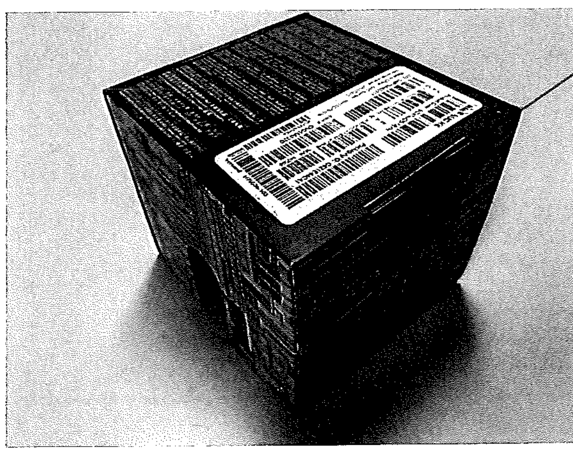
2. What is the new equation of the horizon in (u', v', w') ?

3. What is the new equation of the projection of $X = m$ in (u', v', w') ?

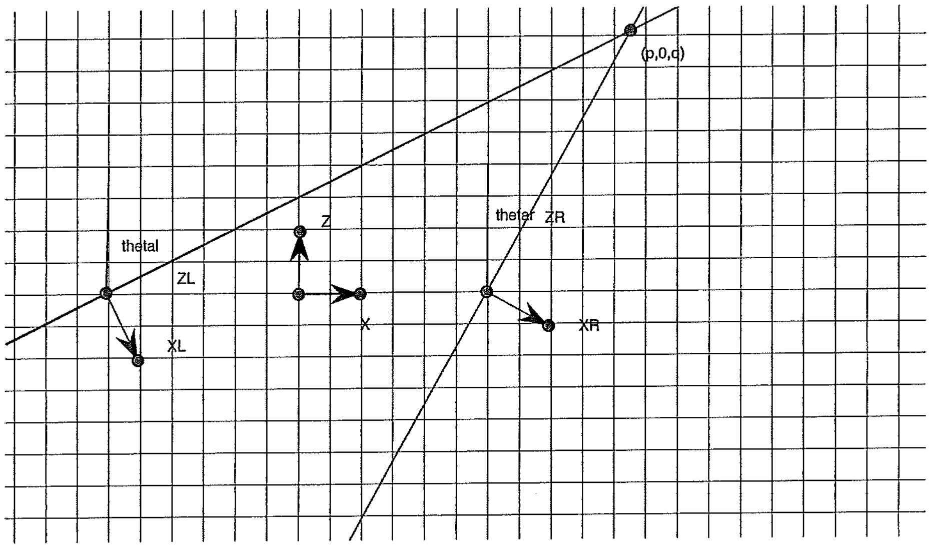
4. Imagine that the camera tilts down (rotates -30deg about X_c axis) instead of moving forward. Consider now the projection of a point above the ground at coordinates that projects on a point (u, v, w) before the rotation and (u'', v'', w''). Prove that the transformation between the new image plane (u'', v'', w'') and the old image plane (u, v, w) is a projective transformation.

Q.4 20 pts 1) Using the ruler find the image center of the picture.

2) Find the focal length in pixels. Assume that the width of the picture is 4000 pixels and the height 3000 pixels.



Q.5 15 pts Will the vanishing points change pixel position in the following cases:
(i) camera translates
(ii) camera zooms (meaning f changes)
(iii) camera rotates around an axis through its projection center
Explain your answer, best with equations.



Q.6 20 pts Ignore the coordinate system in the center.

1. Find R and T in the rigid transformation

$$\begin{pmatrix} X_L \\ Y_L \\ Z_L \\ 1 \end{pmatrix} = \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X_R \\ Y_R \\ Z_r \\ 1 \end{pmatrix}$$

2. Following the above transformation, write the essential matrix for this two-view system in terms of θ_L, θ_R and the distance between the two coordinate systems b (baseline).

3. Given θ_L, θ_R and b find the intersection of the two optical axes $(p, 0, q)$.