CIS 580 Spring 2021: Midterm 2

- **Once you begin the exam you will have 120min to finish and submit to Gradescope. All SDS accommodations will still apply accordingly.**
- **You are not allowed to post the exam to anyone/anywhere.**
- **You are not allowed to collaborate with other students.**
- **During the exam we will post clarification in this Google doc. If you have a question please first check the document and if your question is not there please send a PRIVATE question in piazza. We will copy the question and answer it in the doc.**
- **If you use anything verbatim from the Internet you should cite it properly (like URL).**
- **Use your own paper if you want and submit the same way you submit a 580 math homework.**

1. Problem Structure from Motion

Two views are separated by a rotation around the y-axis and a translation in the xz-plane reflecting a motion of the camera in the xz-plane, a situation called in-plane motion. For example a vacuum robot would satisfy this equation if the camera's y axis is perpendicular to the ground.

$$Q = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} P + \begin{pmatrix} T_x \\ 0 \\ T_z \end{pmatrix}$$

1. **[5 Points]** Compute the essential matrix, given the above rotation and translation. We call this matrix the "vacuum cleaner essential" matrix.

2. **[15 Points]**
   - Which elements of the essential matrix are always zero?
   - How can you tell if a matrix E is "vacuum cleaner essential" ?
   - Suppose that you have found a "vacuum cleaner essential" matrix. Show how to extract θ and (T_x, T_z). The latter up to a scale factor.

① $E = \hat{T}_x R =$ where $R = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

Cross product w/ T $\Leftarrow \hat{T}_x = \begin{bmatrix} 0 & -T_z & 0 \\ T_z & 0 & -T_x \\ 0 & T_x & 0 \end{bmatrix}$

$$\Rightarrow E = \begin{bmatrix} 0, & -T_z, & 0 \\ T_x\sin\theta + T_z\cos\theta, & 0, & -T_x\cos\theta + T_z\sin\theta \\ 0, & T_x, & 0 \end{bmatrix}$$

a)

2 You can tell if E is vacum cleaner essential if its decomposition into R & T ~~s~~satisfies:

- Rotation only about Y-axis
- Translation only in xz plane

b) Main diagonal & off diagonal are zero in vacum cleaner matrix

c) extracting θ, T_x & T_z:

$\Rightarrow -E_{0,1} = T_z$

row col

(assuming rows & cols are zero-indexed)

$\Rightarrow E_{2,1} = T_x$

$$T_x \sin\theta + T_2 \cos\theta = A \quad \leftarrow \quad E_{1,0}$$

$$-T_x \cos\theta + T_2 \sin\theta = B \quad \leftarrow \quad E_{1,2}$$

$+$

$$\cos\theta(T_2 - T_x) + \sin\theta(T_2 + T_x) = A + B$$

This equation can lead to multiple valid solutions for θ

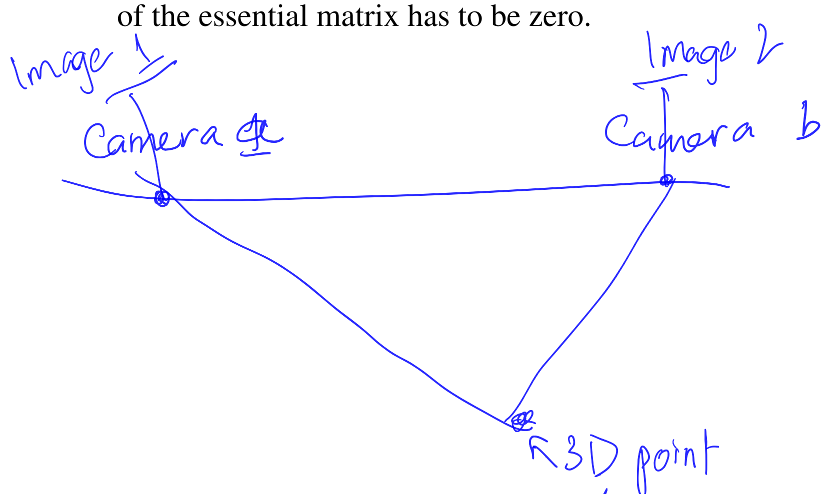
2. Problem Structure from Motion

1. **[5 Points]** In the structure from motion problem, give an example of a pair of translation and rotation, that result in the two epipoles being at exactly the same pixel position in both images. A sketch would help.

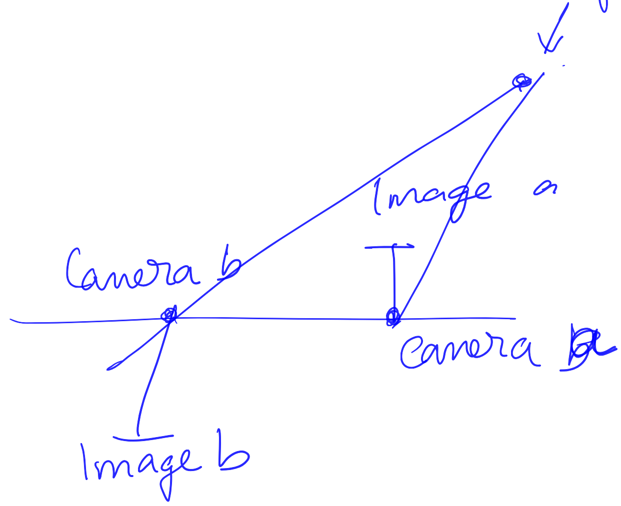
2. **[5 Points]** Assume two cameras with cooordinate systems Q and P and $Q = RP + T$. Give an example of a pair of translation T and rotation R, that causes epipole in image plane q being at infinity and epipole in image plane p being in the center of the image. A sketch would help.

3. **[5 Points]** Suppose that two views have intersecting optical axes. Show that the element E_{33} of the essential matrix has to be zero.

2.1



(1)



(2)

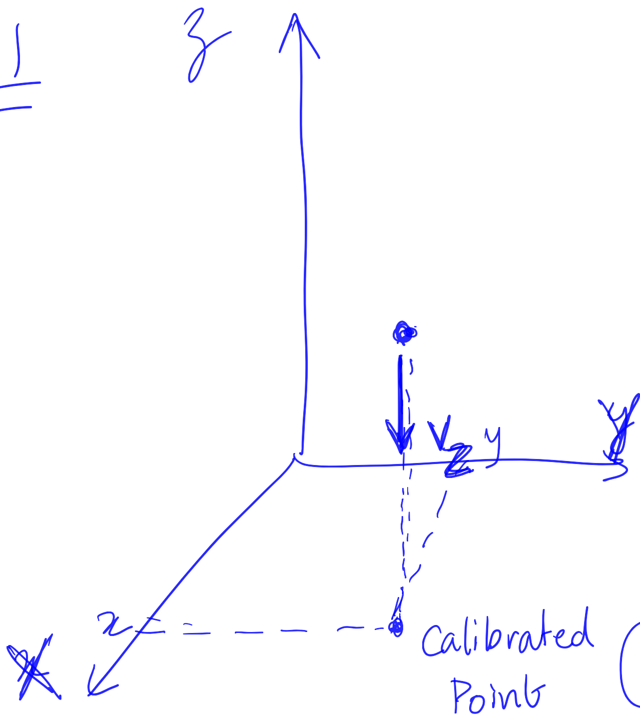
2.2

3. Problem 3D Velocities

1. **[10 Points]** Assume that we are moving with a pure translational velocity $(0,0,V_z)$. Show how can we compute the time to collision to a point in the scene that is projected on the calibrated point (x,y).

2. **[10 Points]** Assume that we drive towards a wall parallel to the image plane and we observe a circle. Explain how we can find the time to collision (assuming constant velocity) from the area of the circle A and the rate of change of the area $\dot{A}$.

3.1



Algorithm to compute time to collision (assuming point masses)

① If moving body's x & y coordinates do not match calibrated point, no collision

② If particle is moving away from $z=0$, no collision (i.e., $\dot{z} > 0$)

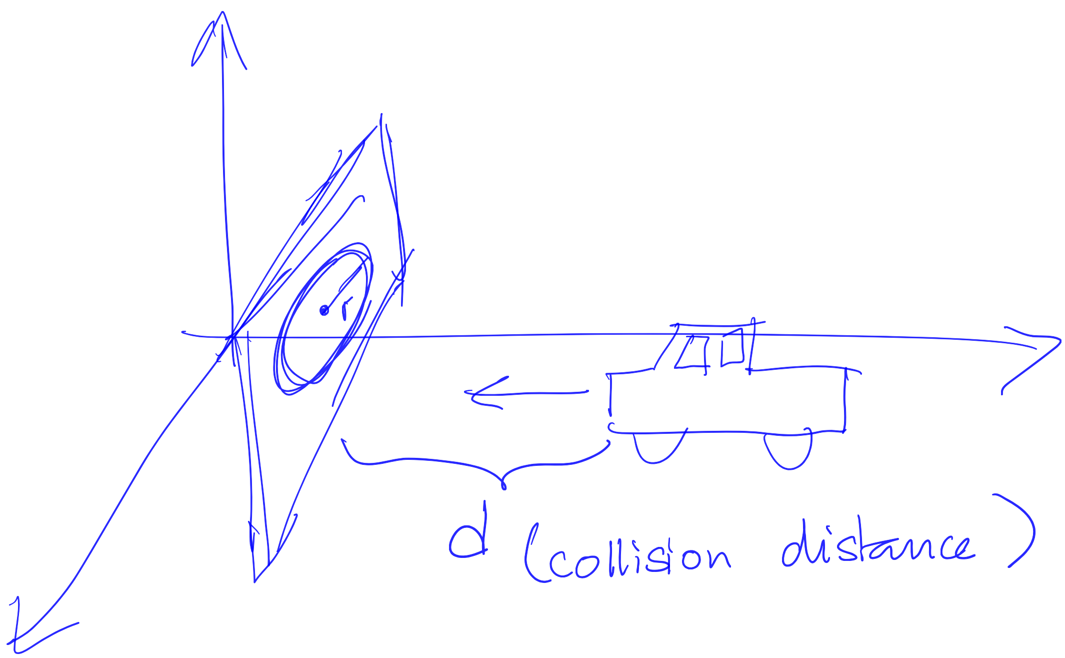
③ $$\frac{z(t) - 0}{-\dot{z}(t)} = \text{time to collision}$$

where $z(t)$ is z-coord of moving particle

3.2 Area of circle $A = \pi r^2$

$\Rightarrow$ We know r is proportional to distance from object ($r \propto d$)

$\Rightarrow \frac{\partial \dot{A}}{\partial r} = 2\pi r \quad (\propto d)$



We can use the fact that $\dot{A}$ is directly proportional to collision distance to compute time to collision

4. Problem

1. **[10 Points]** Your system allows you to perform only convolutions with a Gaussian of $\sigma = 1/\sqrt{2}$. How many convolutions would you need to approximate a Gaussian (0th derivative) of $\sigma = 4$. Subsampling is not allowed.

2. **[10 Points]** Assume that you want to approximate the 2nd derivative for $\sigma = 4$. Your system still allows you only to convolve with a Gaussian of $\sigma = 1/\sqrt{2}$. Establish a procedure so that you can compute the equivalent of a convolution with the 2nd derivative of $\sigma = 4$. Subsampling is not allowed.

4.1

$$\sigma = 4 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 \times 8 \times 4} = \sqrt{\frac{32}{2}} = 4$$

$\boxed{\text{32 convolutions}}$

4.2

Step 1: compute $\sigma = 4$ Gaussian using technique from 4-1

Step 2: Use LoG method twice:

$$F_x = F * \{1 \quad 0 \quad -1\}$$

$$F_{xx} = F_x * \{1 \quad 0 \quad -1\}$$

5. Problem

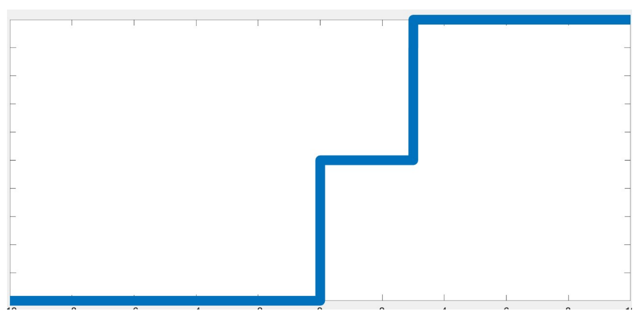
1. **[5 Points]** Compute the convolution of the 1D edge

$$h(x) = \begin{cases} H & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases}$$

with the first derivative of the Gaussian function with standard deviation σ. Plot or draw the edge, the first derivative of the Gaussian and the result of the convolution.

2. **[10 Points]** Assume the double-step function of the figure defined as

$$h(x) = \begin{cases} 2H & \text{if } x > a, \\ H & \text{if } 0 \leq x \leq a \ . \\ 0 & \text{if } x < 0 \end{cases}$$



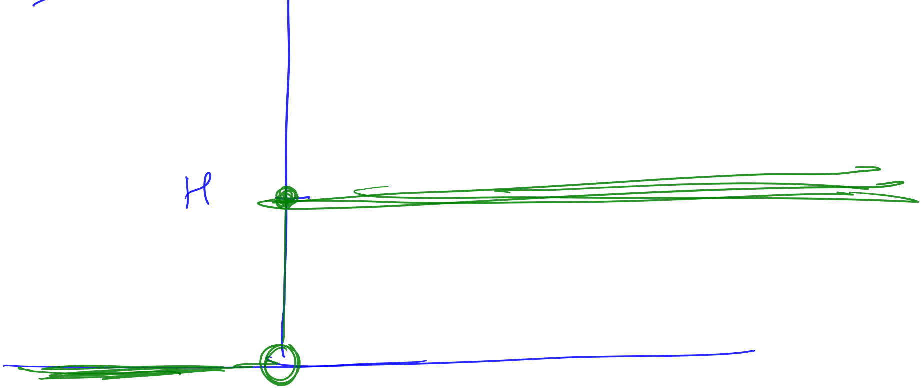
Compute its convolution with the first derivative of a Gaussian with standard deviation σ and call the response $d(x, \sigma, a)$.

3. **[5 Points]** Show that $d(x, \sigma, a)$ has always an extremum at $x = a/2$.
Show that $d(x, \sigma, a)$ does hot have extrema at $x = 0$ and $x = a$ as one would anticipate.

4. **[5 Points]** Plot $d(x, \sigma, a)$ for $\sigma = 1$ and $a = 5$.
Plot $d(x, \sigma, a)$ for $\sigma = 4$ and $a = 5$.
You can use python, matlab, wolfram alpha, or even a Ti84 and you are allowed to draw the curves by hand.
Explain the curves. What do you observe.

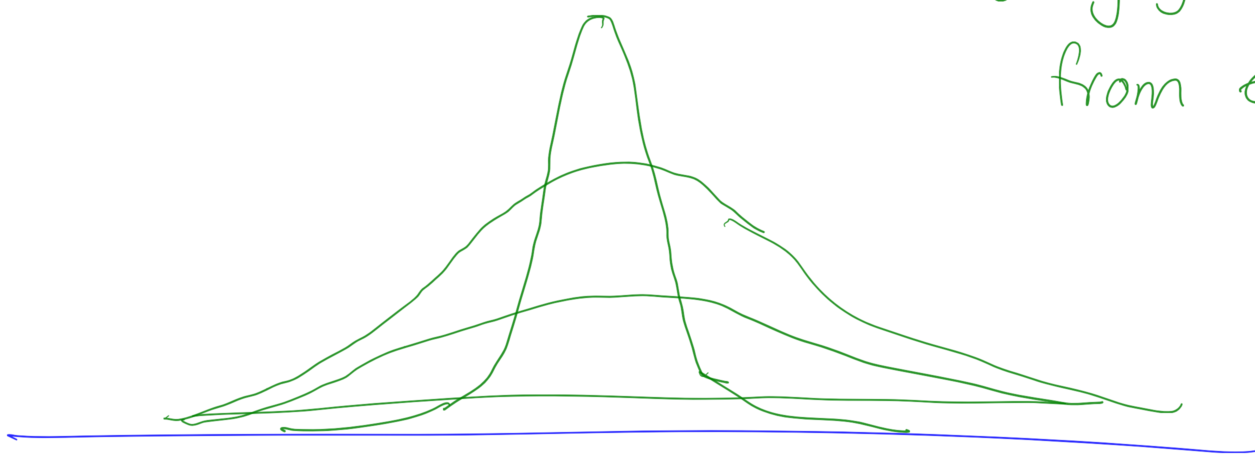
5.1

Edge



Result of Convolution

(varying sigma from $\sigma = [0,5]$



Convolution by hand

Derivative of Gaussian

$$f(x) = \frac{-t \times e^{\frac{-t^2}{2\sigma^2}}}{\sqrt{2\pi \cdot \sigma^3}}$$

$g(x)$ = Stepwise: 0 if $t < 0$

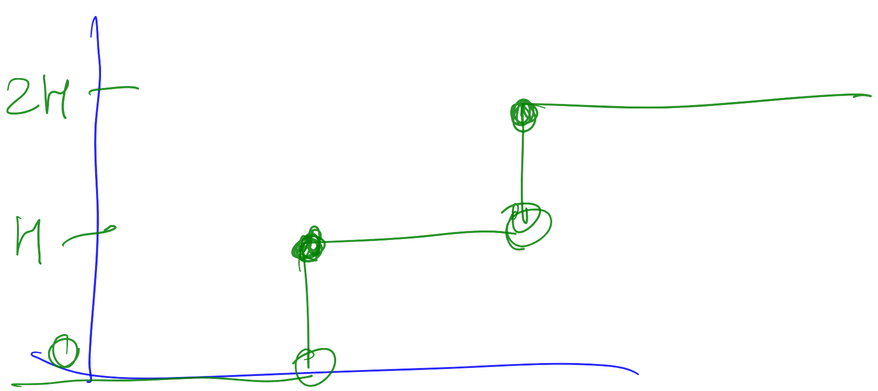
H otherwise

$$(f * g)(t) = \int_{-\infty}^{0} f(t-u)\, \cancel{g(u)}\, du \Rightarrow 0$$

$$+ \int_{0}^{\infty} f(t-u)\, g(u)\, du$$

evaluated numerically using matlab

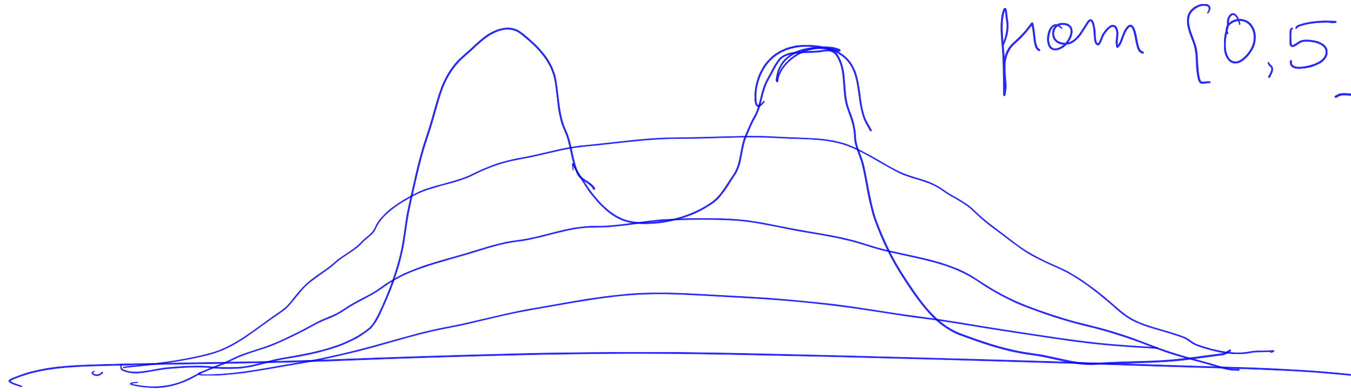
5.2



Edge function

Convolution Result

(varying sigma from [0,5]



$f(x) =$ Derivative of Gaussian

$$g(x) = \text{stepwise:} \quad \begin{cases} 0 & \text{if } t < 0 \\ H & 0 \leq t \leq a \\ 2H & \text{otherwise} \end{cases}$$

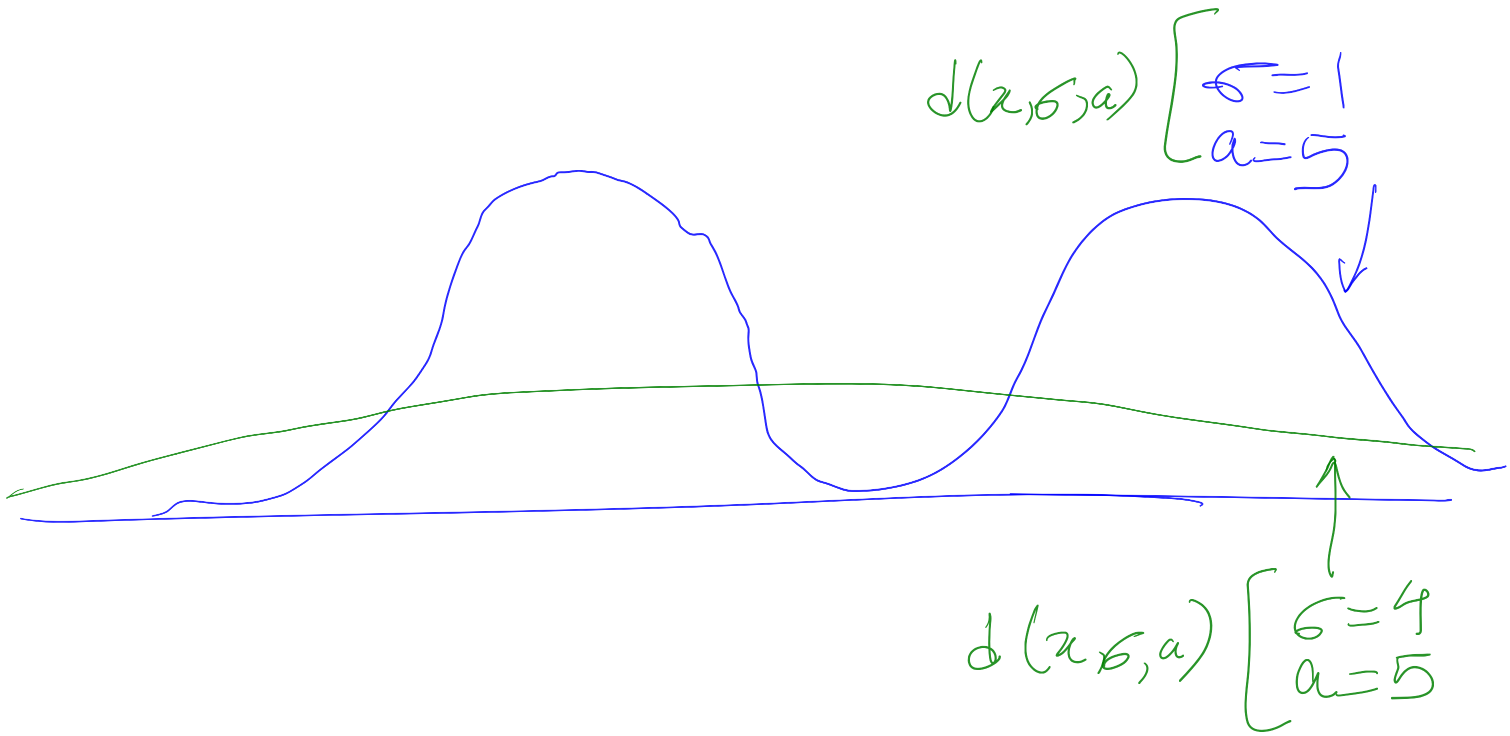
$$(f * g)(t) = \int_{-\infty}^{0} f(t-u)\, \cancel{g(u)}_{\,0}\, du \Longrightarrow 0$$

$$+ \int_{0}^{a} f(t-u) \cdot H \, du$$

$$+ \int_{a}^{\infty} f(t-u) \cdot 2H \, du$$

(Numerically evaluated to make plot)

5.4



I observe that as σ increases, the twin peaks converge to a singular global maxima (distance inversely proportional to σ)

5.3 We can prove that $d(x,\sigma,a)$ always has extrema @ $x = a/2$ by observing the expression used to compute the convolution

$$\frac{\partial\left((f*g)(t)\right)}{\partial t} = \left(f(t-a)\right)H - f(t)H + 0 - 2H\left(f(t-a)\right)$$

$$= -H\,f(t-a) - f(t)\,H$$

$$= -H\left(f(t-a) - f(t)\right) = 0$$

& solve for $t \Rightarrow t = \frac{a}{2}$

is local minima