AR practical example

- 2D AR (virtual blending on a plane) - define a square on the ground where the "logo" will be blended

  [recognize where in this square]
  ↳ 4 corners in my image

  only proj. transformation needed

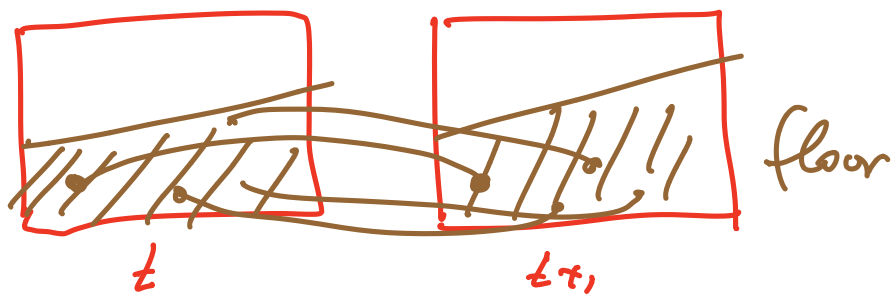
- 3D AR : CAD inserted and kept at a desired position

  ① identify the dominant horizontal* plane (*gravity $N \parallel g$)

  (horizon is in the middle of the image ? this only says sth about the relative orientation to the camera)

  (gravity from where ? accelerometer or recognize vertical edges $\Rightarrow V_v$ $\Rightarrow K^{-1} V_v \sim g$)

(1e) recognize a plane:
semantic segmentation but there is also a geometric way:



correspondences from feature matching (late March)

if correspondences $(x_1, y_1) \leftrightarrow (x_2, y_2)$ satisfy

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim H \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

then with high* probability I can say that this a plane.

* two view collineations:

1) pure rotation (waiting for the phone to translate)
2) all points are very far away. Why?

$$1^{st}_{image} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \sim K \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{wrt camera at pose 1.}}$$

$$2^{nd}_{image} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim K \left(R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + T \right)$$

Z very large $Z \to \infty$

$$\sim \begin{pmatrix} K & R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 0 \end{pmatrix} \quad *$$

∞ in $\mathbb{P}^3$

$* \ \mathbb{P}^2 \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

$\mathbb{P}^3 \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$

$$\sim K R^{*} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

same as pure rotation

look at stationary clouds from two positions : $\begin{matrix} x_2 \\ y_2 \\ 1 \end{matrix} \sim K R K^{-1} \begin{matrix} x_1 \\ y_1 \\ 1 \end{matrix}$

$* \quad K = I \quad \lambda_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + T$

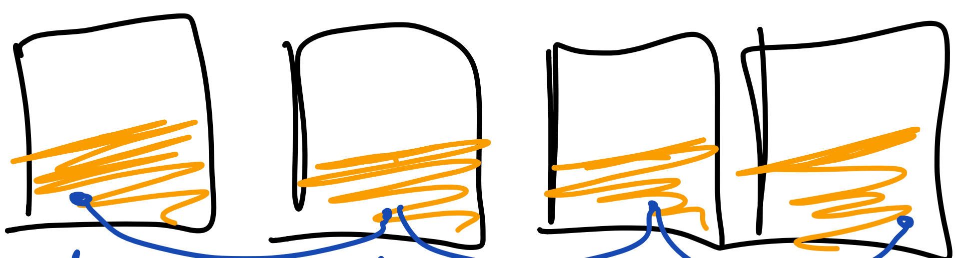
$$\lambda_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = R \underset{\substack{\parallel \\ Z}}{\lambda_1} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} + T$$

$$x_2 = \frac{d_1 (r_{11} x_1 + r_{12} y_1 + r_{13}) + T_x}{d_1 (r_{31} x_1 + r_{32} y_1 + r_{33}) + T_z}$$

$$= \frac{r_{11} x_1 + r_{12} x_2 + r_{13} + T_z / d_1}{r_{31} x_1 + r_{32} x_2 + r_{33} + T_z / d_1}$$

$$\lim_{d_1 \to \infty} = \frac{r_{11} x_1 + r_{12} y_1 + r_{13}}{r_{31} x_1 + r_{32} y_2 + r_{33}}$$

(2) track points on the plane in time



$\Downarrow$ H

$\Downarrow$

$\boxed{\text{reference}}$ = $\boxed{\text{world}}$

$$t_i \quad \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \overset{*}{\sim} K (R_i \; T_i) \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

If I know R_i T_i we can draw the CAD model

$R_i T_i$ → proj. eq *

$\begin{pmatrix} x_r \\ y_r \\ 1 \end{pmatrix} \rightarrow \begin{matrix} X_r \\ Y_r \\ Z_r \end{matrix} \rightarrow$ origin of world

3 between the point we need to know by which R, T camera moved!

→ looking only at the plane → (R,T) from H

→ anywhere

next Monday

SfM

Structure from Motion

4 not in this class

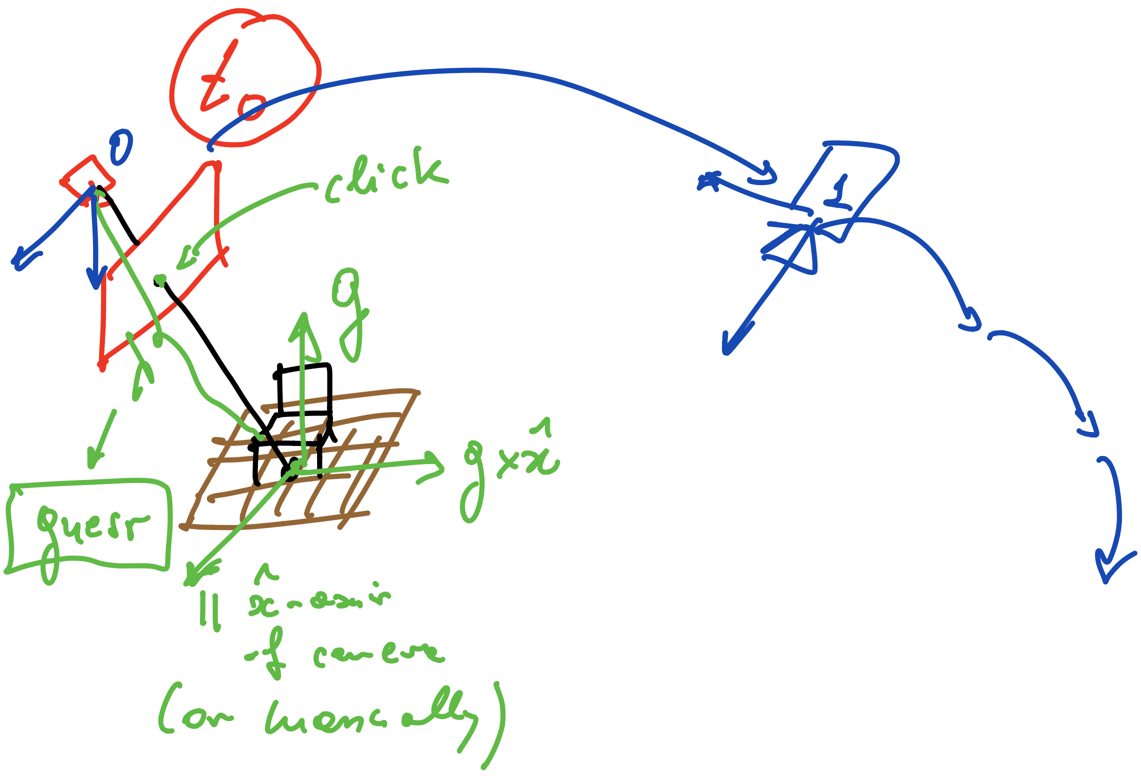
$(R,T)_{0,1}$ $(R,T)_{1,2}$ $(R,T)_{2,3}$

"

$\boxed{R_{0,t}, T_{0,t}}$ absolute

We need a "Kalman Filter"

ARkit, ARcore fuse $\boxed{\text{IMU}}$



slides $\quad H \Rightarrow R, T$

if points lie on a plane

$$\boxed{\lambda_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \lambda_1 R \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} + T}$$

calibrated

$$K^{-1}\begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} \qquad K^{-1}\begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} \qquad \lambda_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = K \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$$

1st image

equation of plane in image 1. 3D 2D

$$a X_1 + b Y_1 + c Z_1 = d \qquad \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \lambda_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

$$\lambda_1 \left(a x_1 + b y_1 + c \right) = d$$

normal of plane $\|N\| = 1 \quad N = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

distance from plane d

$$\lambda_1 N^T \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = d$$

↳ height of camera

$$\lambda_2 \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \left(R + \frac{T N^T}{d} \right) \lambda_1 \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim \underbrace{\left(R + \frac{T N^T}{d} \right)}_{H} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}}$$

two views of a plane

Assume N unknown (not necessarily g)

1. $N \geq 4$ correspondences $\Longrightarrow H$ $\boxed{\text{SVD method}}$

2. $\underset{3\times 3}{H} = \lambda \left(R + \frac{1}{d} T N^T \right)$

1 | 3 unknown | 1 | 3 | 2

10 unknowns

We cannot disambiguate between (T, d) or $(2T, 2d)$

Set $\|T\| = 1$ 9 unknowns

$$R^T R = I$$

3. $$H = USV^T \quad S = \begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix}$$

$$\lambda = \sigma_2 \quad (\text{Ch } 5.2)$$

$$\frac{H}{\sigma_2} = R + \frac{T}{d} N^T$$

4 solutions (slide)

using U, V $\boxed{\hat{u}_2 u_1 = u_2 \times u_1}$

select correct solution

that yields positive λ_1, λ_2

4. remaining problem

$$\|T\| = ?$$