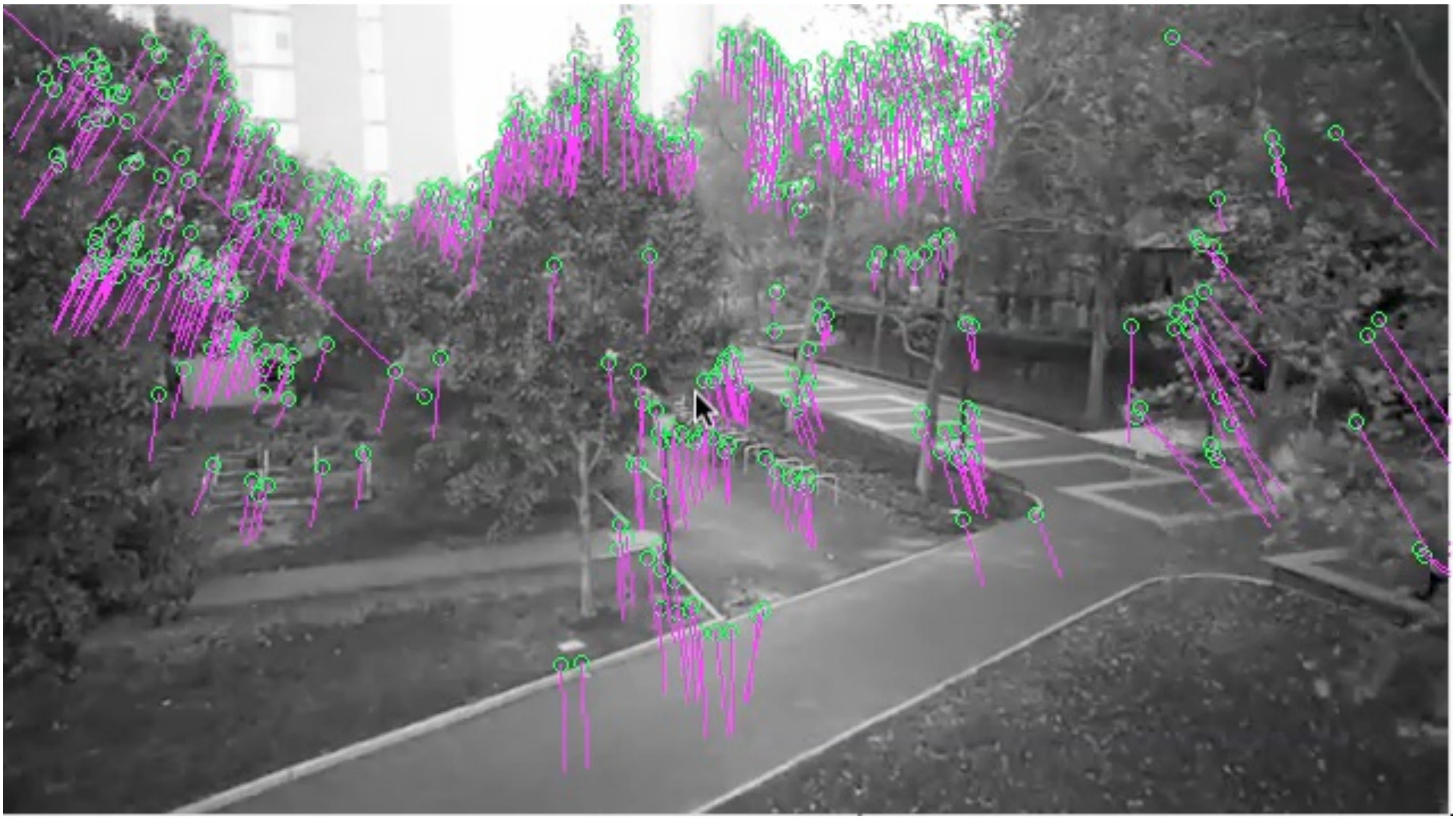
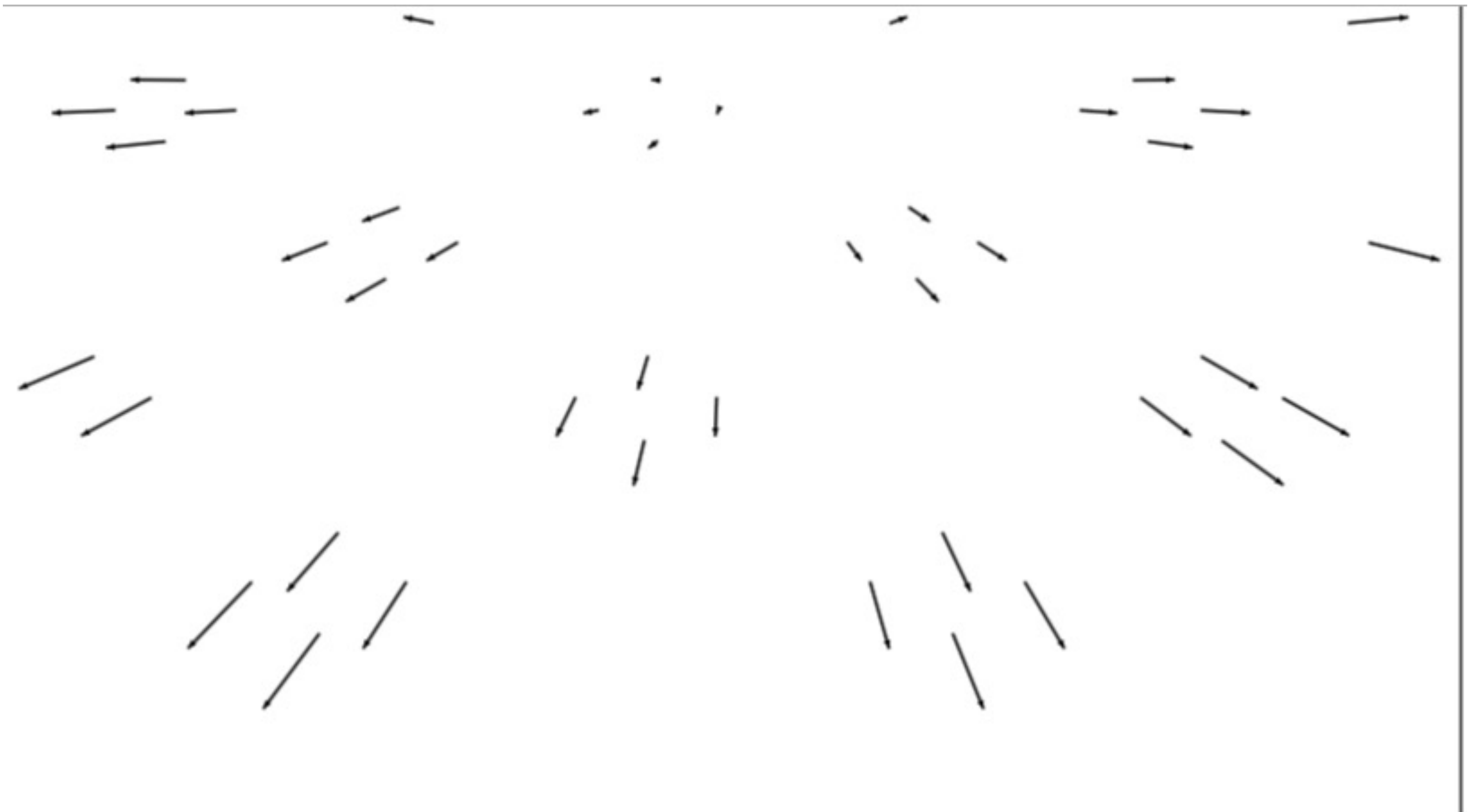
3D Velocities from Optical Flow

Kostas Daniilidis

Which direction is the vehicle moving?

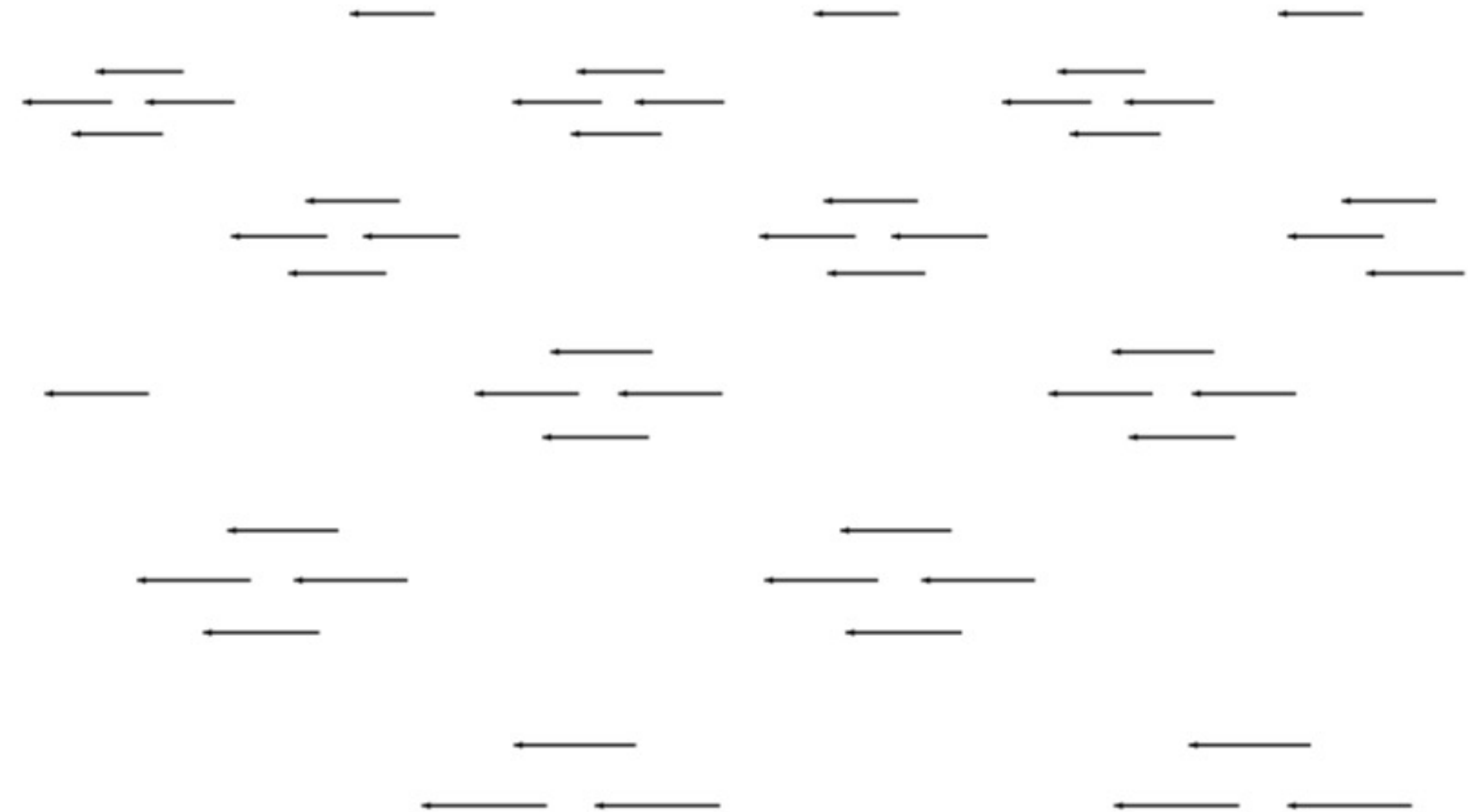


Pure translation

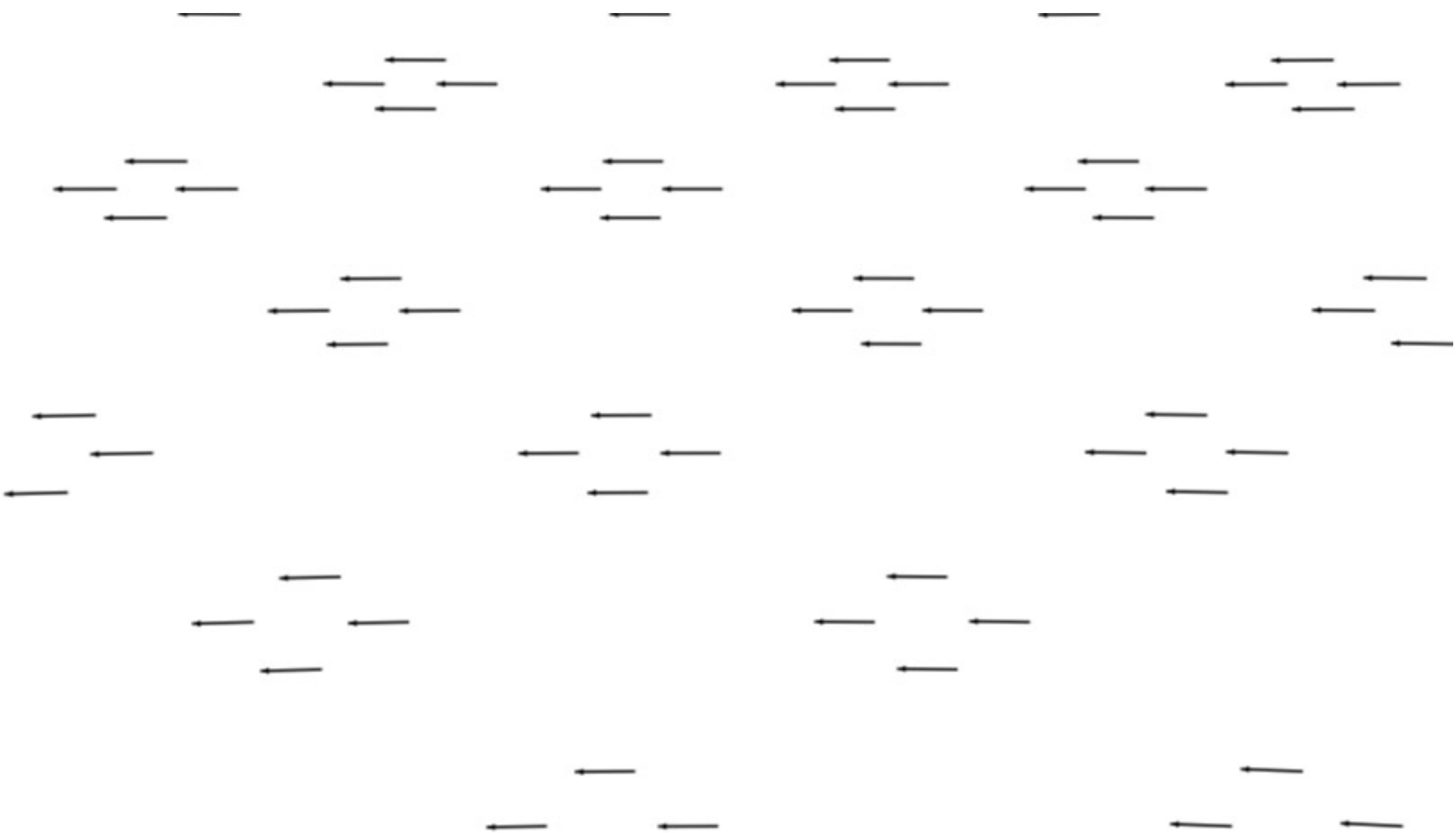


Pure translation to the right

Pure horizontal translation



Pure rotation around vertical axis



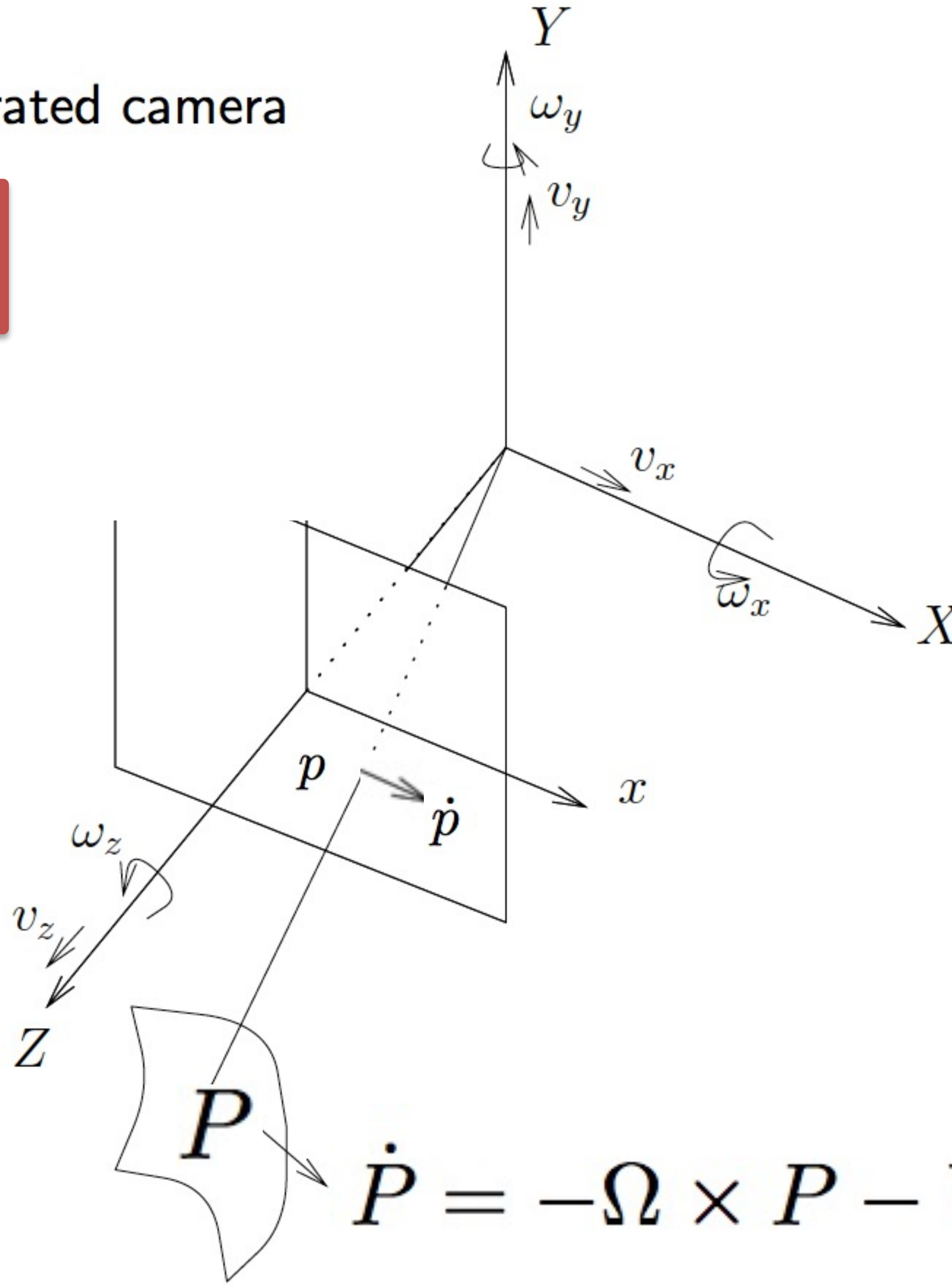
Projection equations for calibrated camera

$x = \frac{X}{Z}$, $y = \frac{Y}{Z}$

or in vector notation $\boxed{p = \frac{1}{Z}P}$

Differentiating w.r.t. time yields:

$$\dot{p} = \frac{\dot{P}}{Z} - \frac{\dot{Z}}{Z}p$$



$$\dot{P} = -\Omega \times P - V$$

Projection equations for calibrated camera

$x = \frac{X}{Z}$, $y = \frac{Y}{Z}$

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Differentiating w.r.t. time yields:

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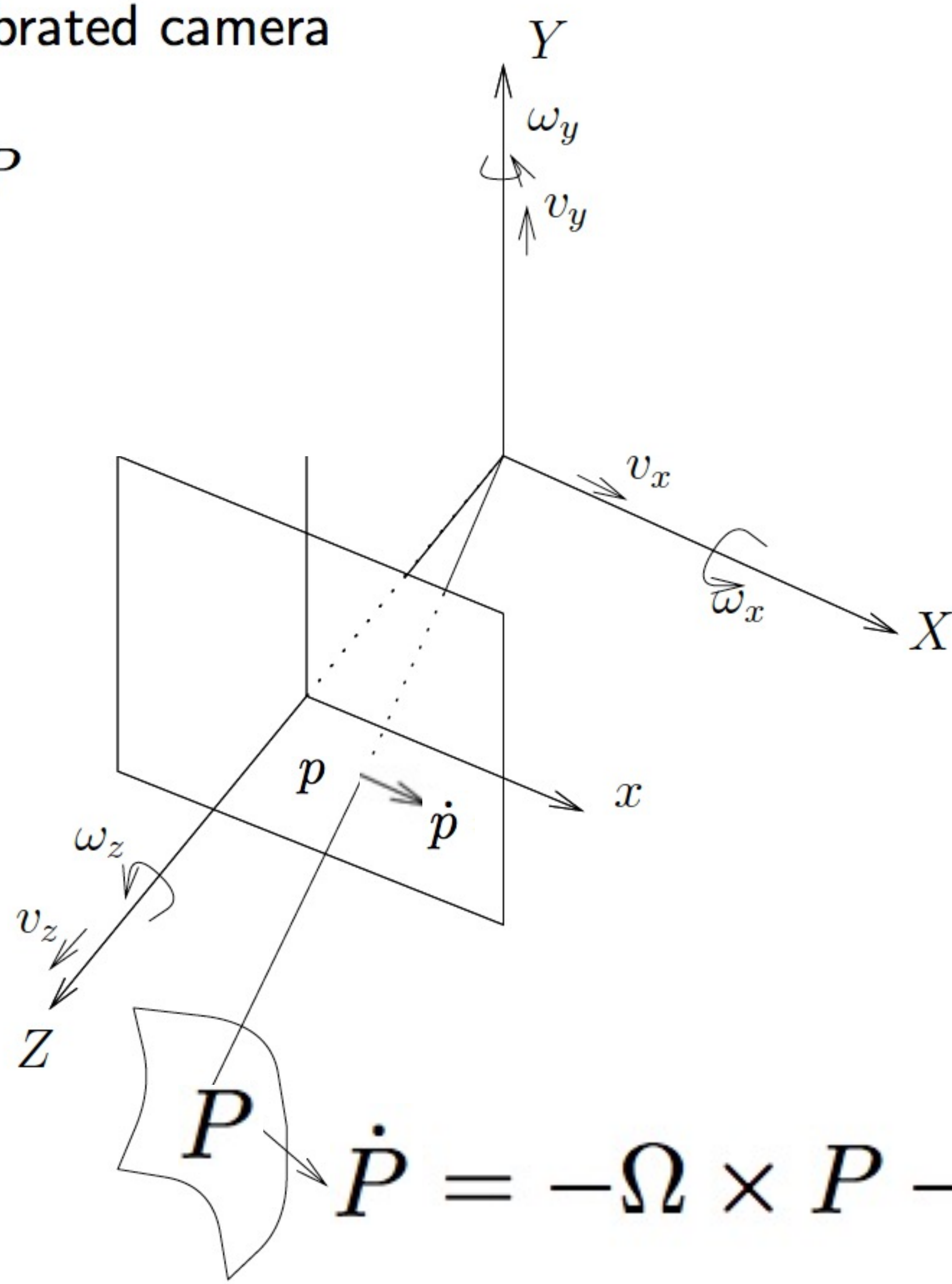


$$\dot{P} = -\Omega \times P -$$

If we combine the equations for $\dot{P}$ and the optical flow $\dot{p}$ we obtain

$$\dot{p} = \frac{1}{Z} e_3 \times (p \times V) + e_3 \times (p \times (p \times \Omega))$$

where $e_3 = (0, 0, 1)^T$.

Written out in coordinates

$$\dot{p} = \underbrace{\frac{1}{Z} \begin{bmatrix} xV_z - V_x \\ yV_z - V_y \end{bmatrix}}_{\text{translational flow}} + \underbrace{\begin{bmatrix} xy & -(1 + x^2) & y \\ (1 + y^2) & -xy & -x \end{bmatrix} \Omega}_{\text{rotational flow independent of depth}}$$

Optical flow has two additive components: translational and rotational.

$$\dot{p} = \underbrace{\frac{1}{Z}\begin{bmatrix} xV_z - V_x \\ yV_z - V_y \end{bmatrix}}_{\text{translational flow}} + \underbrace{\begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\Omega}_{\text{rotational flow independent of depth}}$$

If Z is known, $\dot{p}$ is linear in V and Ω.

Having at least 3 optical flow vectors not on collinear points and corresponding depths we can solve for the 3D velocities from 6 equations.

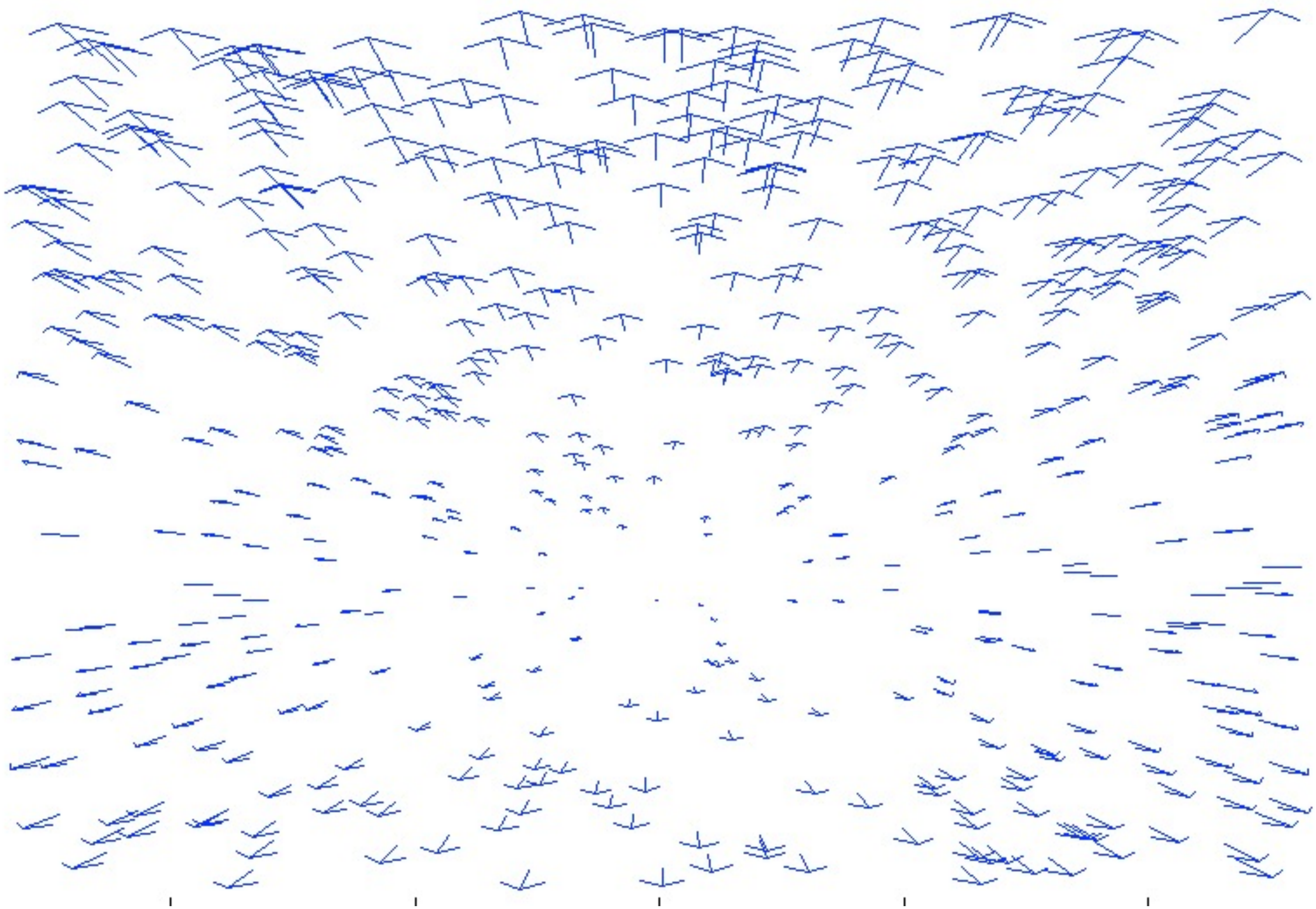
If the field is purely rotational then we have no information about depth.

$$\dot{p} = \underbrace{\begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}}_{\text{rotational flow independent of depth}} \Omega$$

This also means that if we know Ω from other sources we can *derogate* he flow field without knowing the depth.

Translational Flow:

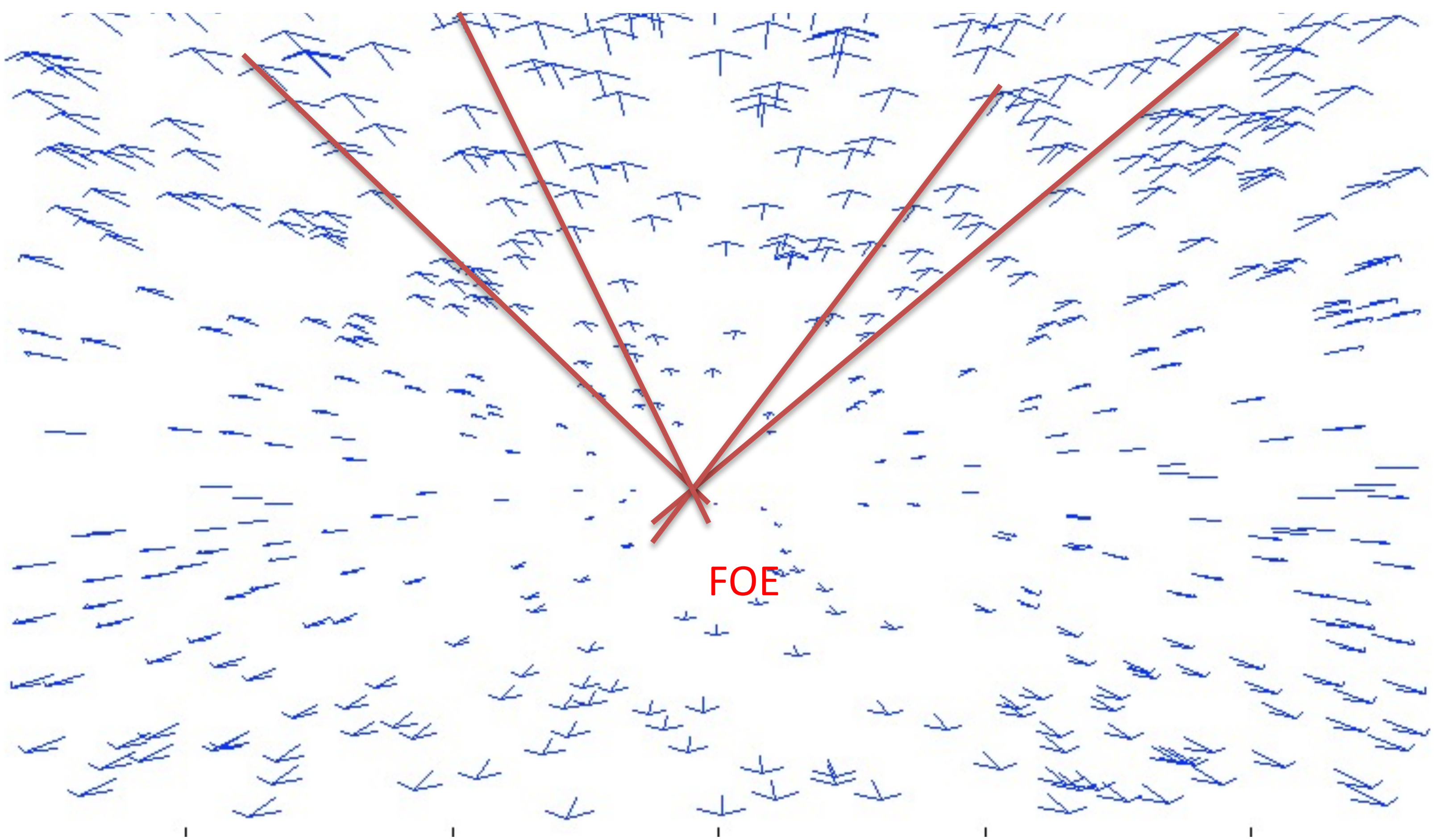
$$\dot{p}_{\mathsf{trans}} = \frac{V_z}{Z} \begin{bmatrix} x - \frac{V_x}{V_z} \\ y - \frac{V_y}{V_z} \end{bmatrix}$$



By intersecting the lines spanned by $\dot{p}_{\text{trans}}$, we can obtain the Focus of Expansion (FOE) also called Epipole

$$FOE = (V_x/V_z, V_y/V_z)$$

FOE can also be at infinity if $V_z = 0$.



Time to Collision (TTC)

5. Spiess, F. N. *et al. Science* **207,** 1421–1433 (1980).
6. Corliss, J. B. *Science* **203,** 1073–1083 (1979).
7. Edmond, J. M. *et al. Earth planet. Sci. Lett.* **46,** 19–30 (1979).
8. Ruby, E. G., Wirsen, C. O. & Jannasch, H. W. *Appl. envir. Microbiol.* (in the press).
9. Cavanaugh, C. M., Gardiner, S. L., Jones, M. L., Jannasch, H. W. & Waterbury, J. B. *Science* **213,** 340–342 (1981).
10. Felbeck, H. *Science* **213,** 336–338 (1981).
11. Jones, M. L. *Science* **213,** 333–336 (1981).
12. Peck, H. D. Jr *Enzymes* **10,** 651–669 (1974).
13. Latzko, E. & Gibbs, M. *Pl. Physiol.* **44,** 295–300 (1969).
14. Reid, R. G. B. *Can. J. Zool.* **58,** 386–393 (1980).
15. Los Angeles County (California) Sanitation District files.
16. Rau, G. H. *Science* **213,** 338–340 (1981).
17. Rau, G. H. *Nature* **289,** 484–485 (1981).
18. Rau, G. H. & Hedges, J. I. *Science* **203,** 648–649 (1979).
19. Emery, K. O. & Hulsemann, J. *Deep-Sea Res.* **8,** 165–180 (1962).
20. Hartman, O. & Barnard, J. L. *Allan Hancock Pacific Exped.* **22,** (1958).
21. Nicholas, D. J. D., Ferrante, J. V. & Clarke, G. R. *Analyt. Biochem.* **95,** 24–31 (1979).
22. Lonsdale, P. *Nature* **281,** 531–534 (1979).

Plummeting gannets: a paradigm of ecological optics

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Getting around the environment and doing things requires precise timing of body movements. Moreover, there is often little time available to pick up the visual information to organize the action. Consider, for instance, a batsman hitting a fast bowler or a bird alighting on a twig swaying in the wind. The visual and motor systems evidently work in close harmony, vision rapidly and directly providing the information for controlling the action. Here, we present evidence in support of a theory to explain how actions are visually timed. The evidence derives from a film analysis of the spectacular plunge dive of one of Britain's largest seabirds, the gannet (*Sula bassana*).

The theory is based on an analysis of the visual input considered as an optic flow field[1,2]. When the organism is moving

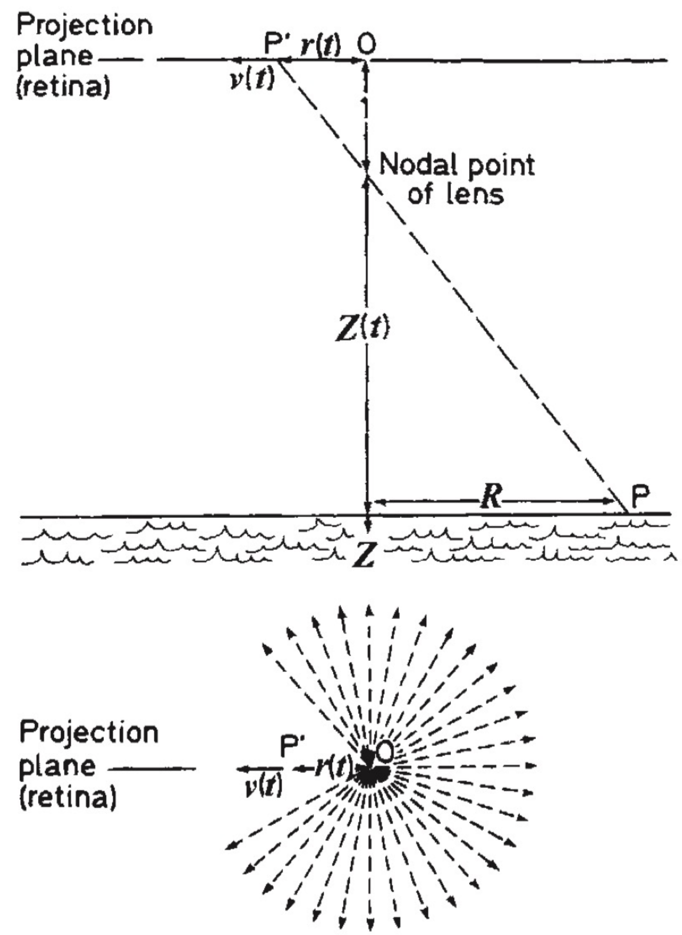

Fig. 1 How time-to-contact is specified in the optic flow field. The schematic eye is, at time t, at height $Z(t)$ and moving vertically downward with velocity $V(t)$ towards the water surface. Light reflected from the surface texture elements (for example, ripples) passes through the nodal point of the lens and projects an expanding optic flow pattern on to the retina. Considering an arbitrary texture element P and its moving image P′, then from similar triangles: $Z(t)/R = 1/r(t)$. Differentiating with respect to time: $V(t)/R = v(t)/r(t)^2$. Finally, eliminating R, $Z(t)/V(t) = r(t)/v(t) = \tau(t)$; that is, the time-to-contact under constant closing velocity is specified by the optical parameter $\tau(t)$. The optical geometry is similar for a slanting dive.

Translational Flow:

$$\dot{p}_{\text{trans}} = \frac{V_z}{Z} \begin{bmatrix} x - \frac{V_x}{V_z} \\ y - \frac{V_y}{V_z} \end{bmatrix}$$

The time to collision (which birds and insects estimate) is

$$\frac{Z}{V_z}$$

$$\frac{V_z}{Z} = \frac{||\dot{p}_{\text{trans}}||}{||p - F\vec{O}E||}$$

Points at the same radial distance from FOE have flow vector lengths proportional to inverse depth (or inverse time to collision).

(12) **United States Patent**
Stein et al.

(10) **Patent No.: US 8,879,795 B2**
(45) **Date of Patent: *Nov. 4, 2014**

(54) **COLLISION WARNING SYSTEM**

(71) Applicants: **Gideon Stein**, Jerusalem (IL); **Erez Dagan**, Rehovot (IL); **Ofer Mano**, Modiin (IL); **Amnon Shashua**, Mevasseret Zion (IL)

(72) Inventors: **Gideon Stein**, Jerusalem (IL); **Erez Dagan**, Rehovot (IL); **Ofer Mano**, Modiin (IL); **Amnon Shashua**, Mevasseret Zion (IL)

(73) Assignee: **Mobileye Vision Technologies Ltd.**, Jerusalem (IL)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

This patent is subject to a terminal disclaimer.

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(22) Filed: **Nov. 26, 2013**

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(60) Provisional application No. 60/560,049, filed on Apr. 8, 2004.

(51) **Int. Cl.**

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G08G 1/16	(2006.01)
B60Q 9/00	(2006.01)

(52) **U.S. Cl.**
CPC ***B60Q 9/008*** (2013.01); ***G08G 1/16*** (2013.01); ***G06K 9/00805*** (2013.01); ***G08G 1/166*** (2013.01)
USPC **382/104**; 701/301

(58) **Field of Classification Search**
USPC 382/104; 701/301
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

4,257,703	A	3/1981	Goodrich
4,931,937	A	6/1990	Kakinami et al.

(Continued)

FOREIGN PATENT DOCUMENTS

DE	19926559	12/2000
DE	10258617	4/2004

(Continued)

OTHER PUBLICATIONS

Brauckmann, Michael E., "Toward All Around Automatic Obstacle Sensing for cars," Proceedings of the Intelligent Vehicles '94 Symposium, Oct. 24-26, 1994, pp. 79-84.

(Continued)

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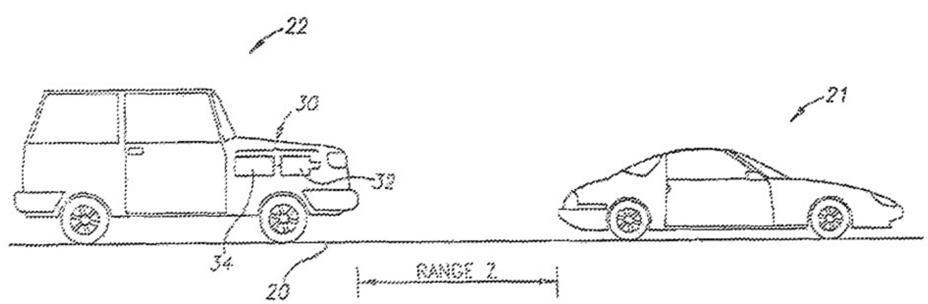
(57) **ABSTRACT**

A method of estimating a time to collision (TTC) of a vehicle with an object comprising: acquiring a plurality of images of the object; and determining a TTC from the images that is responsive to a relative velocity and relative acceleration between the vehicle and the object.

21 Claims, 4 Drawing Sheets



From

$$\dot{p}_{trans}^T(p \times V) = 0$$

we obtain the following coplanarity condition

$$V^T(p \times \dot{p}_{trans}) = 0$$

which says that image point, flow, and linear velocity lie on the same plane.

We can obtain V from two points

$$V \sim (p_1 \times \dot{p_1}) \times (p_2 \times \dot{p_2})$$

and from n points we obtain a homogeneous system

$$\underbrace{\begin{pmatrix} (p_1 \times \dot{p}_1)^T \\ p_2 \times \dot{p}_2)^T \\ \dots \\ p_n \times \dot{p}_n)^T \end{pmatrix}}_{A} V = 0 \tag{1}$$

Then V is the nullspace of A which can be obtained from SVD.

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Then V is the nullspace of A which can be obtained from SVD.

Both V and Ω unknown

Recall that

$$\dot{p} = \frac{1}{Z} F(x,y)V + G(x,y)\Omega$$

This is can be written linearly in inverse depths and Ω:

$$\dot{p} = \left[F(x,y)V \;\; G(x,y)\right] \begin{bmatrix} \frac{1}{Z} \\ \Omega \end{bmatrix}$$

For n points we can write out a system of equations:

$$\begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \ldots \\ \dot{p}_n \end{pmatrix} = \Phi(V) \begin{pmatrix} \frac{1}{Z_1} \\ \frac{1}{Z_1} \\ \ldots \\ \frac{1}{Z_N} \\ \Omega \end{pmatrix}$$

The Φ matrix is a 2N by (N+3) matrix and is a function of V

$$\dot{d} = \Phi(V) \begin{pmatrix} \frac{1}{Z_1} \\ \frac{1}{Z_1} \\ \dots \\ \frac{1}{Z_N} \\ \Omega \end{pmatrix}$$

If we solve for the unknown vector of inverse depths and Ω we obtain

$$\Phi^+(V)\dot{d}$$

which we can insert back in the objective function.

A search on the sphere yields then V:

$$\arg\min_{V \in S^2} \|\dot{d} - \Phi(V)\Phi(V)^+\dot{d}\|^2$$

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