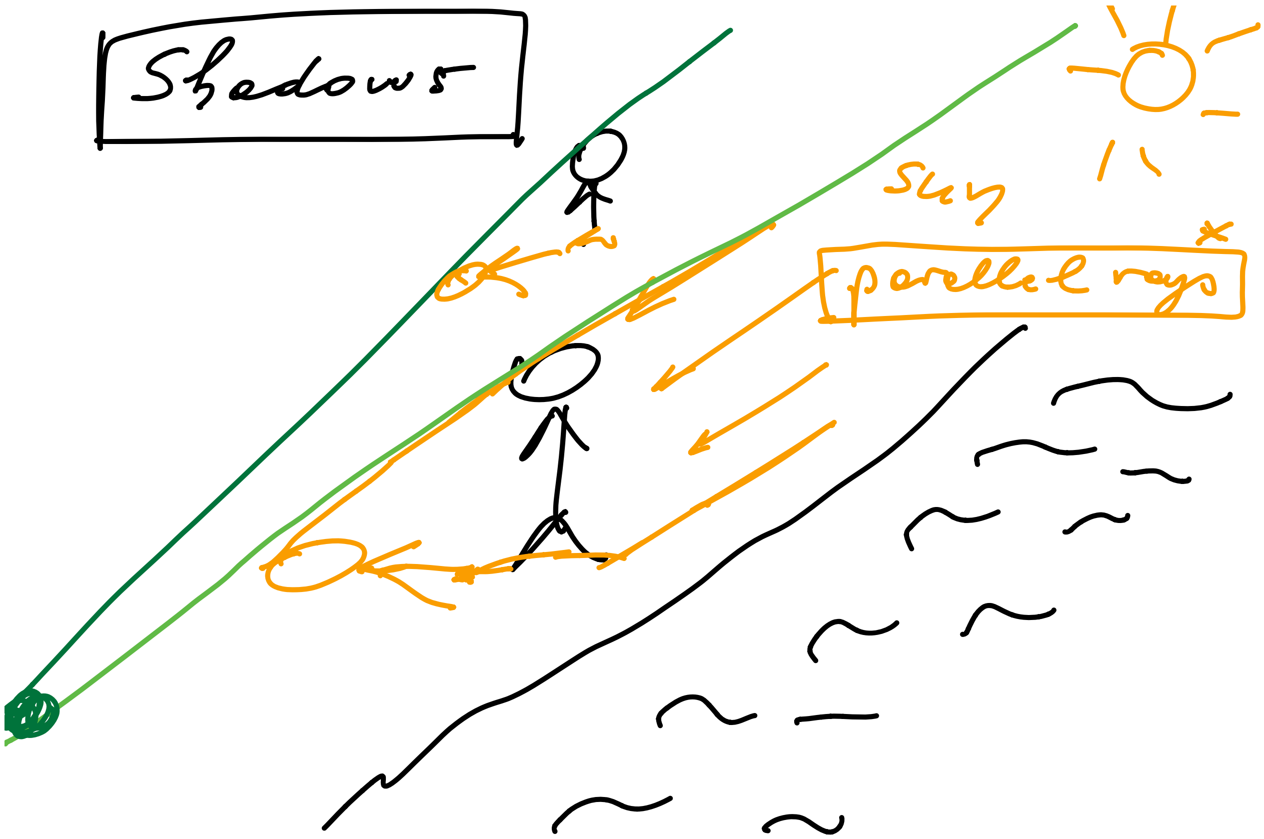
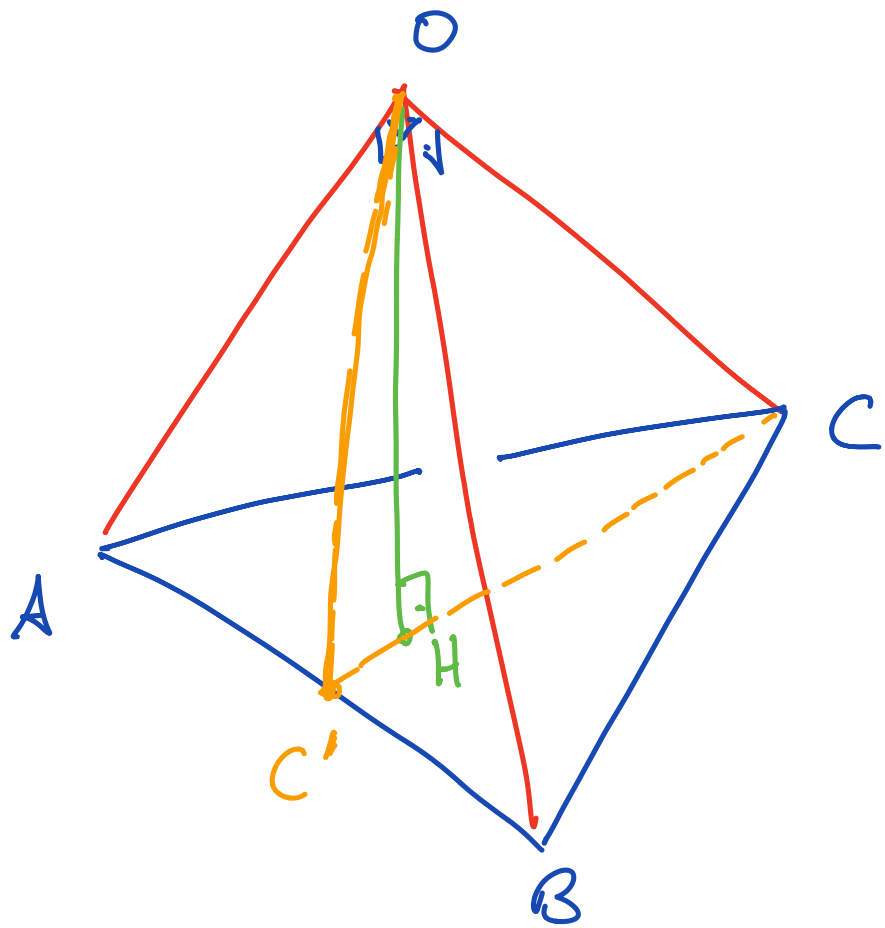
Shadows



* images of all parallel rays intersect at one point (true even if parallel rays are not on the same plane)

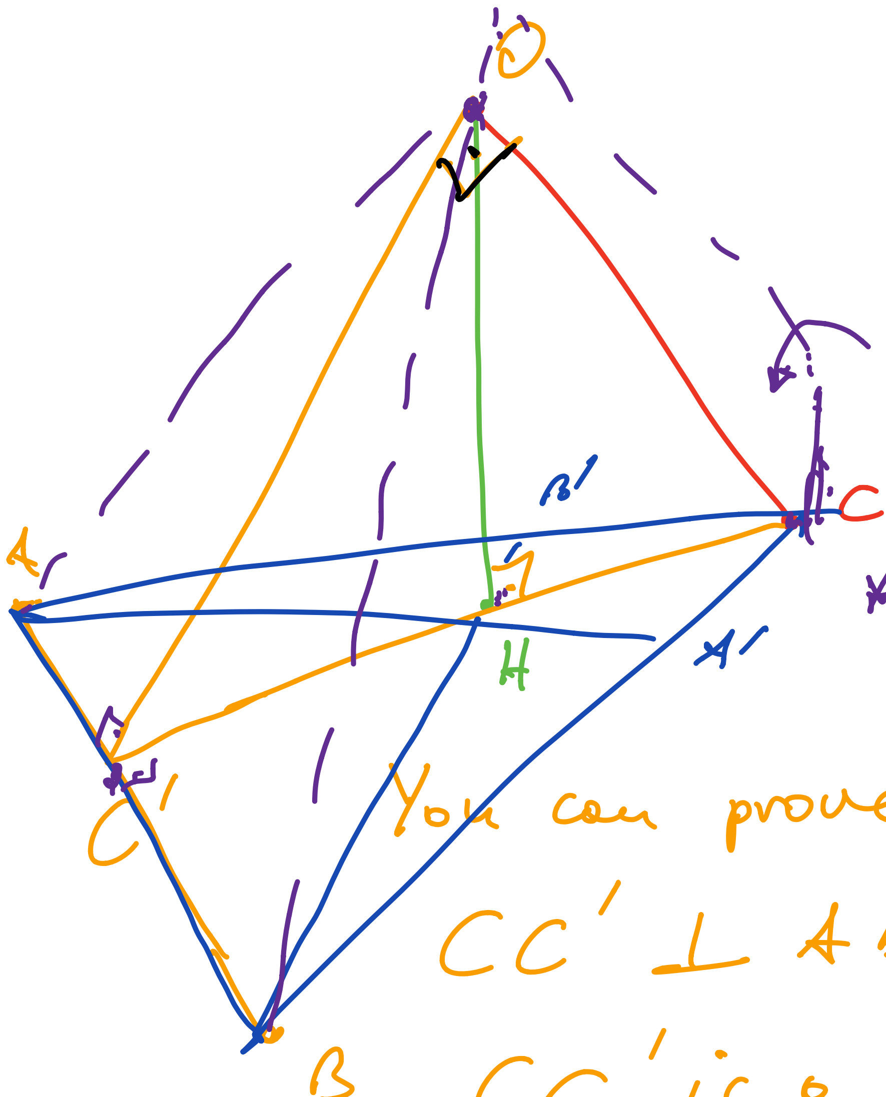
What about if shadows are induced from a lamp? they still intersect in the image?



A, B, C 3 VP of
3 orthogonal direction

Optical axis $\perp$ ABC image plane

OH $\perp$ ABC



$* \; AB \perp OCC'$

You can prove that

$CC' \perp AB$ *

CC' is a height in the triangle ABC

similarly that $AA' \perp BC$

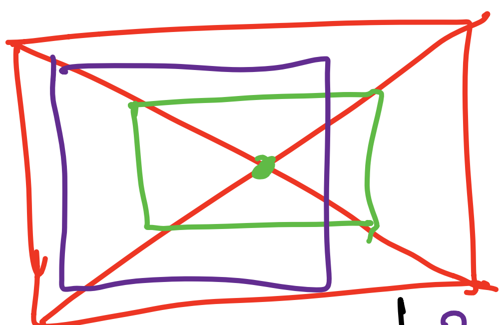
$BB' \perp AC$

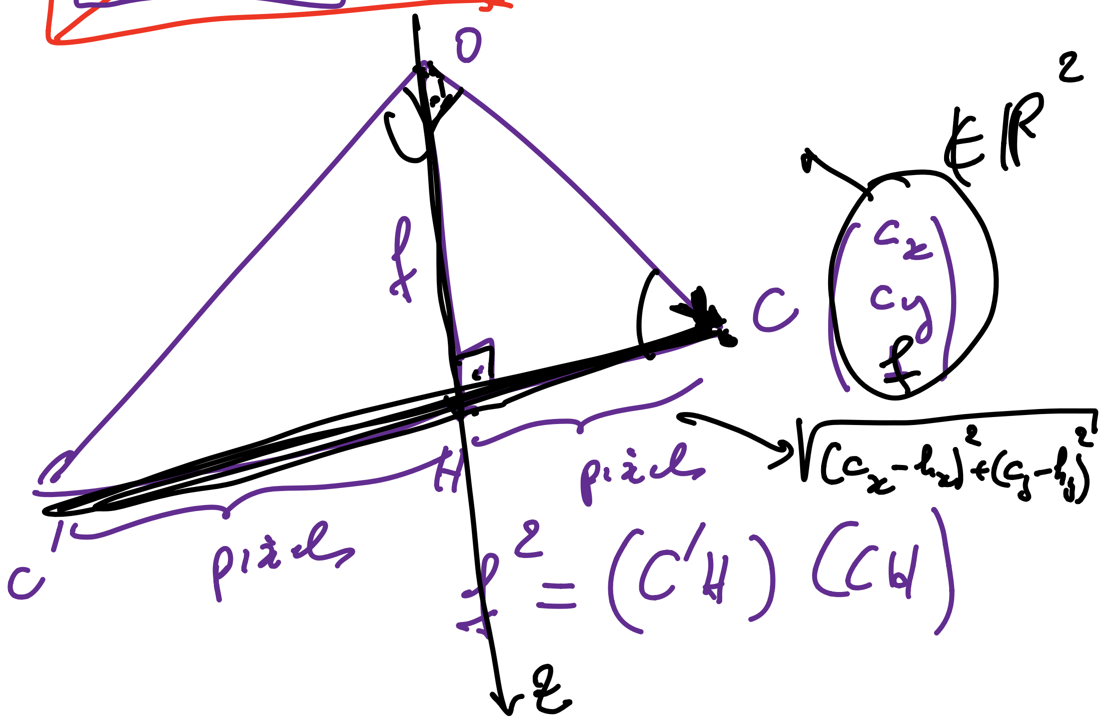
$\Rightarrow$ H orthocenter

iff H is the image center.

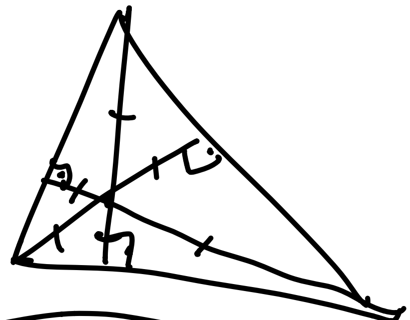
"Cropped photo" detection :

if ortho center is not in the middle of the image then image must have been cropped asymmetrically.





what about $(B'H)\,(BH)$ or
$(A'H)\,(AH)$.



Alternatively

$$\lambda \begin{pmatrix} c_x \\ c_y \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{=K} \overrightarrow{OC}$$

$$\lambda' \begin{pmatrix} c'_x \\ c'_y \\ 1 \end{pmatrix} = \begin{pmatrix} K \end{pmatrix} \overrightarrow{OC'}$$

$OC \perp OC'$

$$\boxed{\left(K^{-1} \begin{pmatrix} c'_x \\ c'_y \\ 1 \end{pmatrix} \right)^T \left(K^{-1} \begin{pmatrix} c_x \\ c_y \\ 1 \end{pmatrix} \right) = 0}$$

$$\begin{pmatrix} (c'_x - u_0)/f \\ (c'_y - v_0)/f \\ 1 \end{pmatrix}^T \begin{pmatrix} \frac{c_x - u_0}{f} \\ \frac{c_y - v_0}{f} \\ 1 \end{pmatrix} = 0$$

$$OC^T OC' = 0$$

$$\boxed{(c'_x - u_0)(c_x - u_0) + (c'_y - v_0)(c_y - v_0) + f^2 = 0}$$

this must be the same as

$$f^2 = \underbrace{\sqrt{(c'_x - u_0)^2 + (c'_y - v_0)^2}}_{c'H} \; \underbrace{\sqrt{(c_x - u_0)^2 + (c_y - v_0)^2}}_{CH}$$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$u = f \frac{X}{Z} + u_0$$

$$v = f \frac{Y}{Z} + v_0 .$$

$$\boxed{3^{rd} \text{ way}}$$

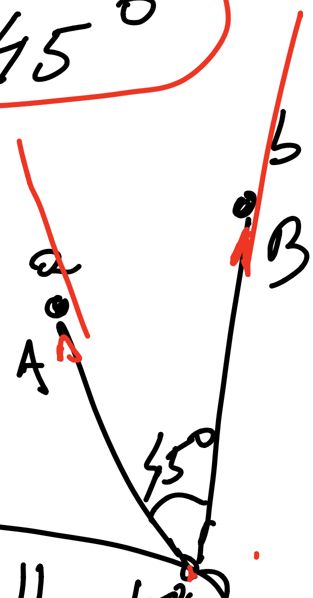
$$\left.\begin{aligned}
\alpha \begin{pmatrix} a_x \\ a_y \\ 1 \end{pmatrix} &= K \, OA \\
\beta \begin{pmatrix} b_x \\ b_y \\ 1 \end{pmatrix} &= K \, OB \\
\gamma \begin{pmatrix} c_x \\ c_y \\ 1 \end{pmatrix} &= K \, OC
\end{aligned}\right\} \Rightarrow K \, ?$$

$$OA^T OB = 0 \quad OB^T OC = 0$$

$$OC^T OA = 0$$

$$\boxed{3 \text{ eq. with unknown } f, u_0, v_0}$$

Let's $\measuredangle(OA, OB) = 45^\circ$



$$\alpha a = K\, OA$$

$$\beta b = K\, OB$$

$$OA^T OB = \|OA\|\,\|OB\| \cos 45^\circ$$

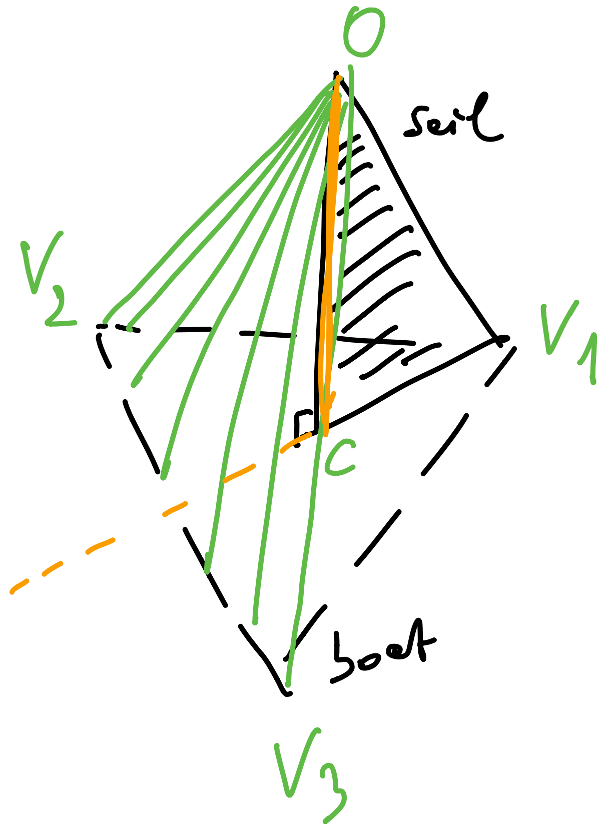
if $u_0 = v_0 = 0$ then we can find f!

$$K^{-1}a = \frac{1}{\alpha} OA$$

$$K^{-1}b = \frac{1}{\beta} OB$$

$$\measuredangle(K^{-1}a, K^{-1}b) = 45^\circ$$

$$(K^{-1}a)^T(K^{-1}b) = \cos 45^\circ\, \|K^{-1}a\|\, \|K^{-1}b\|$$



$OC \perp V_1V_2V_3$

$\cancel{OV_1 \perp OV_2}$

$\Downarrow$

$OCV_1 \perp V_1V_2V_3$

$\Downarrow$

$V_2V_3 \perp OCV_1$

$\Downarrow$

$V_2V_3 \perp CV_1$