B1 (Basic Projective Geometry)

Take two vanishing points

$H\begin{pmatrix}1\\0\\0\end{pmatrix} \sim h_1$ (first column of H)

$H\begin{pmatrix}0\\1\\0\end{pmatrix} \sim h_2$ (second column of H)

Horizon is the line passing through h_1, h_2

So

$$\underline{l_h = h_1 \times h_2} = \begin{pmatrix}1\\0\\1\end{pmatrix} \times \begin{pmatrix}0\\1\\1\end{pmatrix} \sim \begin{pmatrix}-1\\-1\\1\end{pmatrix}$$

B2

~~If~~ For the parallel lines to remain parallel we need the horizon to be the line at infinity $\begin{pmatrix}0\\0\\1\end{pmatrix}$

So:

$$H\begin{pmatrix}1\\0\\0\end{pmatrix} \sim h_1 = \begin{pmatrix}h_{11}\\h_{21}\\h_{31}\end{pmatrix}$$

$$H\begin{pmatrix}0\\1\\0\end{pmatrix} \sim h_2 = \begin{pmatrix}h_{12}\\h_{22}\\h_{32}\end{pmatrix}$$

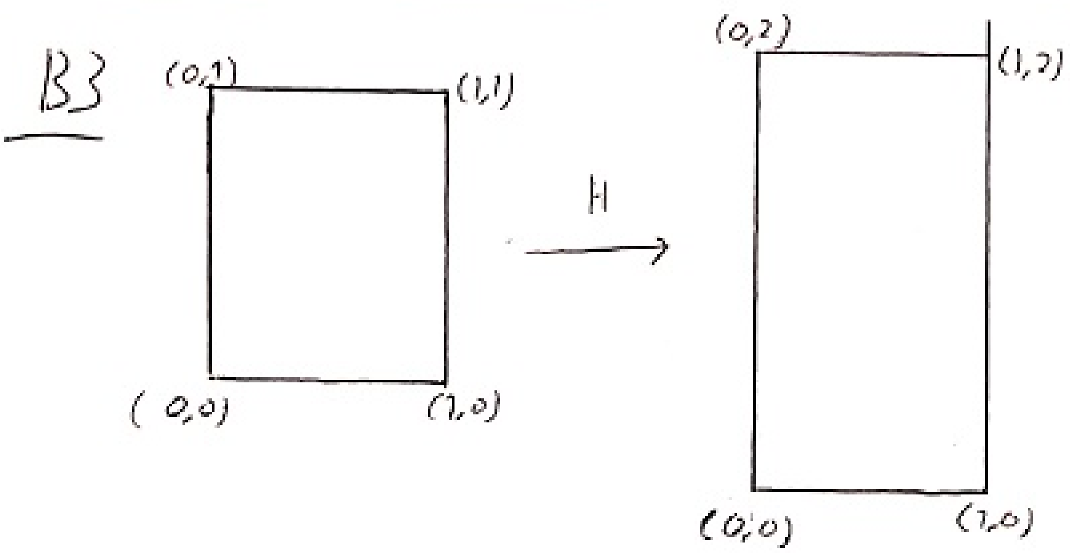
and

$$l_h = h_1 \times h_2 = \begin{pmatrix}h_{21}h_{32} - h_{31}h_{22}\\ -h_{11}h_{32} + h_{12}h_{31}\\ h_{11}h_{22} - h_{12}h_{21}\end{pmatrix} \sim \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow$$

$$\begin{pmatrix}h_{21} & -h_{22}\\ -h_{11} & h_{12}\end{pmatrix}\begin{pmatrix}h_{32}\\h_{31}\end{pmatrix} = 0 \Longrightarrow \underline{h_{32} = h_{31} = 0}$$

$$h_{11}h_{22} - h_{12}h_{21} \neq 0 \xRightarrow{\text{this means}} \det\begin{pmatrix}h_{21} & -h_{22}\\ -h_{11} & h_{12}\end{pmatrix} \neq 0$$

B3



$$\begin{pmatrix}1\\0\\0\end{pmatrix} \sim H\begin{pmatrix}1\\0\\0\end{pmatrix} = h_1 \Rightarrow h_1 = \lambda_1\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \sim H\begin{pmatrix}0\\1\\0\end{pmatrix} = h_2 \Rightarrow h_2 = \lambda_2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix} \sim H\begin{pmatrix}0\\0\\1\end{pmatrix} = h_3 \Rightarrow h_3 = \lambda_3\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$H\begin{pmatrix}1\\1\\1\end{pmatrix} = \lambda_4\begin{pmatrix}1\\2\\1\end{pmatrix} \xRightarrow{\lambda_4 = 1} \lambda_1\begin{pmatrix}1\\0\\0\end{pmatrix} + \lambda_2\begin{pmatrix}0\\1\\0\end{pmatrix} + \lambda_3\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{pmatrix} = \begin{pmatrix}1\\2\\1\end{pmatrix} \Rightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 1$$

and

$$H = \begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

B4

$\begin{pmatrix}1\\0\\0\end{pmatrix} \sim H\begin{pmatrix}1\\0\\0\end{pmatrix} = h_1 \Rightarrow h_1 = \lambda_1\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad$ $H\begin{pmatrix}1\\1\\1\end{pmatrix} \sim \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow H\begin{pmatrix}1\\1\\1\end{pmatrix} = \lambda_4\begin{pmatrix}1\\1\\1\end{pmatrix} \xrightarrow{\lambda_4 = 1}$

$\begin{pmatrix}0\\5\\1\end{pmatrix} \sim H\begin{pmatrix}0\\1\\0\end{pmatrix} = h_2 \Rightarrow h_2 = \lambda_2\begin{pmatrix}0\\5\\1\end{pmatrix}$ $\quad$ $\begin{pmatrix}\lambda_1\\5\lambda_2\\\lambda_2+\lambda_3\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{matrix}\lambda_1 = 1\\ \lambda_2 = 1/5\\ \lambda_3 = 4/5\end{matrix}$

$\begin{pmatrix}0\\0\\1\end{pmatrix} \sim H\begin{pmatrix}0\\0\\1\end{pmatrix} = h_3 \Rightarrow h_3 = \lambda_3\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\quad$ and $\underline{H = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 1/5 & 4/5\end{pmatrix}}$

B5) We assume that when we double the focal length we also double u_0, v_0

so the camera matrix becomes

$$K' = \begin{pmatrix}2f & 0 & 2u_0\\0 & 2f & 2v_0\\0 & 0 & 1\end{pmatrix}$$

We search for H where

$\begin{pmatrix}x'\\y'\\1\end{pmatrix} = H\begin{pmatrix}x\\y\\1\end{pmatrix} = \underbrace{HK}_{K'}(R\,T)\begin{pmatrix}X\\Y\\W\end{pmatrix}$ $\quad|\quad$ $H\begin{pmatrix}f & 0 & u_0\\0 & f & v_0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}2f & 0 & 2u_0\\0 & 2f & 2v_0\\0 & 0 & 1\end{pmatrix} \Rightarrow$

$$H = \begin{pmatrix}2f & 0 & 2u_0\\0 & 2f & 2v_0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}1/f & 0 & -u_0/f\\0 & 1/f & -v_0/f\\0 & 0 & 1\end{pmatrix} = \underline{\begin{pmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{pmatrix}}$$

B6

a) $\begin{pmatrix}u\\v\\w\end{pmatrix} \sim K(r_1\ r_2\ r_3\ t)\begin{pmatrix}X_w\\Y_w \to 0\\Z_w\\W_w\end{pmatrix} \Rightarrow \begin{pmatrix}u\\v\\w\end{pmatrix} \sim \underbrace{K(r_1\ r_3\ t)}_{H}\begin{pmatrix}X_w\\Z_w\\W_w\end{pmatrix}$

So the projective transformation is $H = K(r_1\ r_3\ t)$

B) Similar with B1 we find the horizon

$v_1 = H\begin{pmatrix}1\\0\\0\end{pmatrix} = K(r_1 r_3 t)\begin{pmatrix}1\\0\\0\end{pmatrix} = Kr_1$ $\quad$ and $\underline{l_h = v_1 \times v_2 = (Kr_1 \times Kr_3)}$

$v_2 = H\begin{pmatrix}0\\1\\0\end{pmatrix} = K(r_1 r_3 t)\begin{pmatrix}0\\1\\0\end{pmatrix} = Kr_3$

B7)

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim (K\ 0) \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \quad Z = AX+BY+C \Rightarrow \begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim (K_1\ K_2\ K_3\ 0) \begin{pmatrix} X \\ Y \\ AX+BY+C \\ 1 \end{pmatrix}$$

$$= K_1X + K_2Y + K_3AX + K_3BY + K_3C$$

$$= (K_1 + AK_3,\ K_2 + BK_3,\ CK_3) \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} \Rightarrow$$

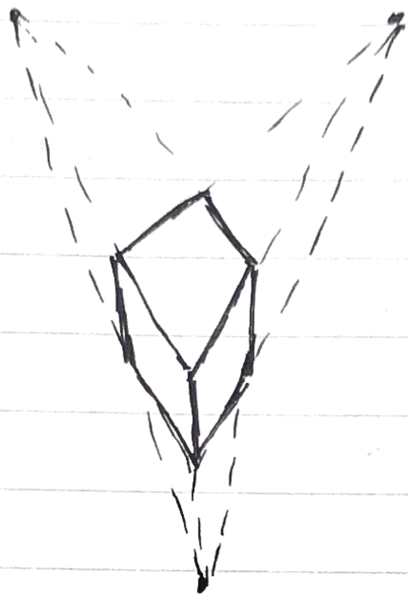
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim (K_1 + AK_3,\ K_2 + BK_3,\ CK_3) \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

B8

- When we approach the scene instead of zooming the projective distortion of parallel lines is larger.
- We cannot tell the difference when the plane we are looking is vertical to the optical axis

See discussion on lecture 1

V ①



(http://mathworld.wolfram.com/Perspective.html)

V ②

$$K(r_1\ r_2\ T)\begin{pmatrix}1\\0\\0\end{pmatrix} \sim K r_1 \sim \begin{pmatrix}a_1\\b_1\\c_1\end{pmatrix}$$

$$K(r_1 r_2 T)\begin{pmatrix}0\\1\\0\end{pmatrix} \sim K r_2 \sim \begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix}$$

$$r_1^T r_2 = 0$$

$$K = \begin{pmatrix}f & 0 & 0\\0 & f & 0\\0 & 0 & 1\end{pmatrix}$$

$$\therefore r_1 \sim \begin{pmatrix}\frac{a_1}{f}\\\frac{b_1}{f}\\c_1\end{pmatrix} \qquad r_2 \sim \begin{pmatrix}\frac{a_2}{f}\\\frac{b_2}{f}\\c_2\end{pmatrix}$$

$$\therefore \frac{a_1a_2+b_1b_2}{f^2} + c_1c_2 = 0$$

$$f^2 = -\frac{a_1a_2+b_1b_2}{c_1c_2}$$

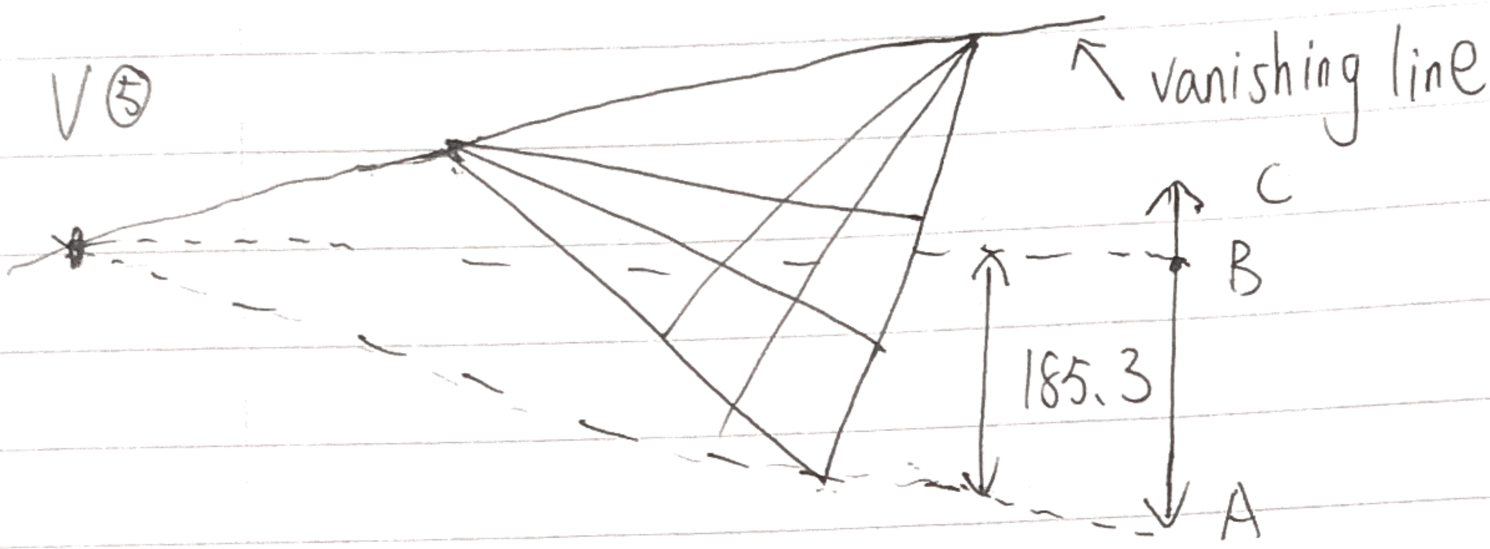
V ③

$$\frac{A_wC_w}{A_wD_w} : \frac{B_wC_w}{B_wD_w} = \frac{A_wC_w}{B_wC_w} = 2 = \frac{AC}{AC+CD} : \frac{BC}{BC+CD}$$

find CD

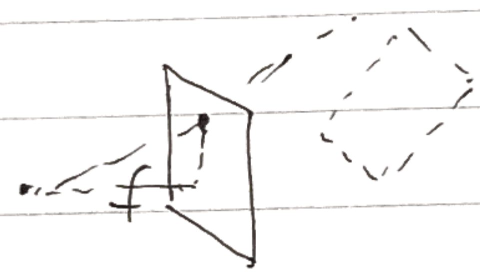
V④ $\frac{A_w C_w}{B_w C_w} = H \frac{A_w B_w}{B_w C_w} = \frac{AC}{AD} : \frac{BC}{BD}$

V⑤



$$\frac{AC}{x} = \frac{AB}{185.3}$$

V⑥ Yes, it will chang



L①: 4 positions → calculate H

since K known

$K^{-1}H \simeq \lambda(r_1 \; r_2 \; T)$

solution see the slide of

pose from collineations Addendum

L② see the slide of pnp

page 14 – 17

∠③ impossible to localize from 2P

Camera orientation is Unkown

∠④ see the slide of procrustes
page 7–8