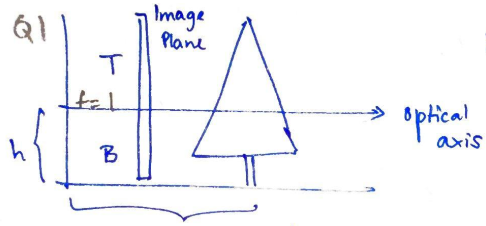
Q1



$K = I$

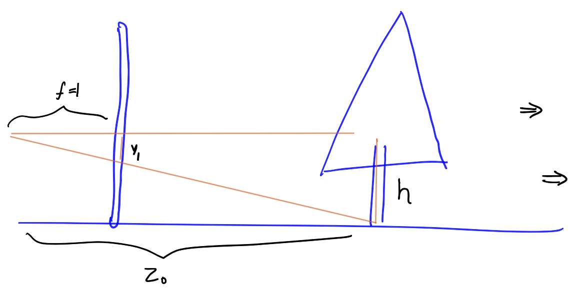
$B = (0, y_1)$ Bottom

$T = (0, y_2)$ Top

$Z_0 = ?$ (w.r.t. h, y_1 & y_2)

K (camera matrix)
$\Rightarrow f = 1$
$\Rightarrow (u_0, v_0) = (0,0)$

$$\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\Rightarrow \frac{h}{Z_0} = \frac{y_1}{f}$$

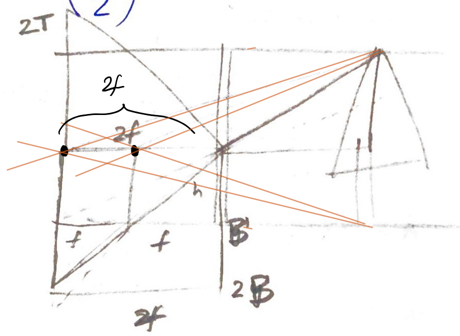
$$\Rightarrow Z_0 = \frac{h \cdot f}{y_1} = \frac{h}{y_1} \quad (\because f=1)$$

$$\boxed{Z_0 = \frac{h}{y_1}}$$

Q2 Double the distance between image plane & projection center $\left(f' = 2f\right)$
$= 2$

(a) $B' = ?$ new coordinates for top & bottom
$T' = ?$ of tree projected on the image plane

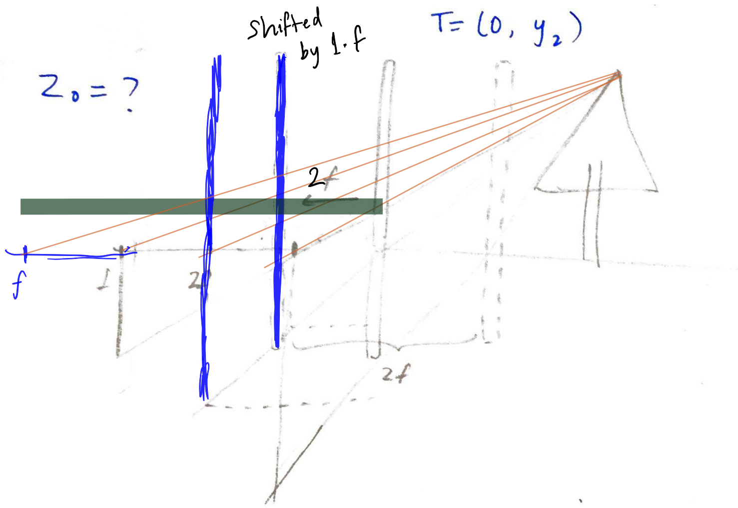
$$\begin{pmatrix} x' \\ y' \\ f \end{pmatrix} = \frac{f}{Z} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



$$\boxed{\begin{aligned} B &= 2y_1 \\ T &= 2y_2 \end{aligned}}$$

(b) How much to move projection center so that bottom & top of the tree appear at original coordinates $B = (0, y_1)$
$T = (0, y_2)$

** Keeping distance between the projection center & image plane constant **



Ans Move the projection center by $2f$ $(=2)$ to the left (i.e. away from the tree)

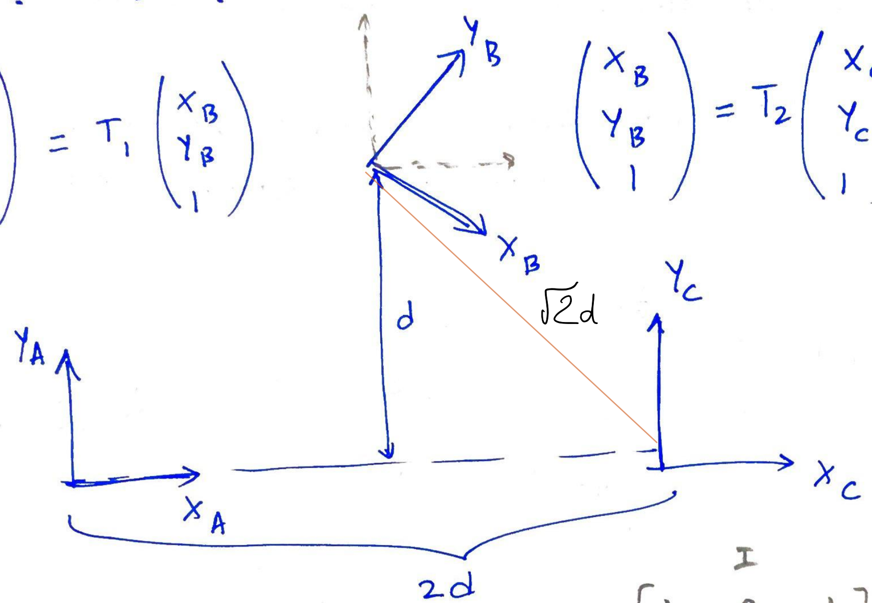
Due to similar triangles, this will reduce the image projection height by a factor of 2

$\Rightarrow$ $B = (0, y_1)$ & $T = (0, y_2)$ again

3) $T_1 = ?$ $T_2 = ?$

$$\begin{pmatrix} x_A \\ y_A \\ 1 \end{pmatrix} = T_1 \begin{pmatrix} x_B \\ y_B \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_B \\ y_B \\ 1 \end{pmatrix} = T_2 \begin{pmatrix} x_C \\ y_C \\ 1 \end{pmatrix}$$



$T_1 \Rightarrow$ - shifted right by d
- shifted up ~~by~~ by d
- rotated clockwise by $45°$ (about Z axis)

$$T_1 = \underline{T}\,\underline{R} \quad \overset{I}{\begin{bmatrix} 1 & 0 & d \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix}} \overset{R}{\begin{bmatrix} \cos(-\frac{\pi}{4}) & \sin(\frac{\pi}{4}) & 0 \\ -\sin(-\frac{\pi}{4}) & \cos(-\frac{\pi}{4}) & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

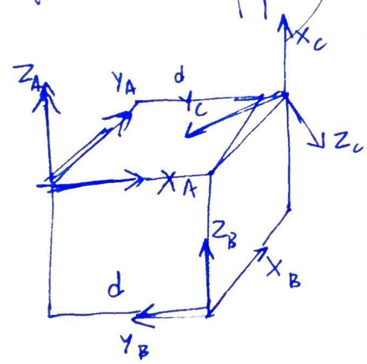
$(-45°)$ about Z

$T_2 = TR$

$$\overset{I}{\begin{bmatrix} 1 & 0 & \sqrt{2}d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos(\frac{\pi}{4}) & \sin(\frac{\pi}{4}) & 0 \\ -\sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4)

$$\begin{pmatrix} x_A \\ y_A \\ z_A \\ 1 \end{pmatrix} = A \begin{pmatrix} x_B \\ y_B \\ z_B \\ 1 \end{pmatrix} \qquad \begin{pmatrix} x_C \\ y_C \\ z_C \\ 1 \end{pmatrix} = B \begin{pmatrix} x_A \\ y_A \\ z_A \\ 1 \end{pmatrix} \qquad \begin{pmatrix} x_C \\ y_C \\ z_C \\ 1 \end{pmatrix} = C \begin{pmatrix} x_B \\ y_B \\ z_B \\ 1 \end{pmatrix}$$



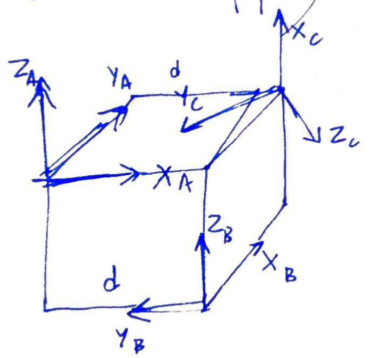
$A =$

$$\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) & 0 & 0 \\ -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A = T \cdot R_{z,90°}$

4)

$$\begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix} = A \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix} \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = B \begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix} \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = C \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix}$$



$A = T \cdot R_{z,90°}$

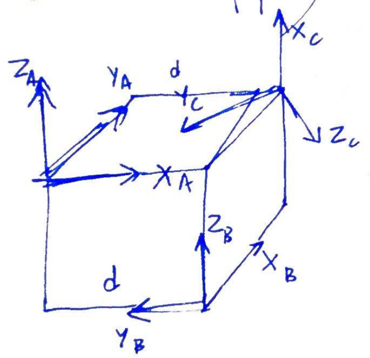
$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \cancel{d\sin(\frac{\pi}{4})} \\ 0 & 0 & 1 & \cancel{d\cos(\frac{\pi}{4})}\ 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sqrt{2}d \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(135°) & \sin(135°) & 0 \\ 0 & -\sin(135°) & \cos(135°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/2) & 0 & -\sin(\frac{\pi}{2}) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\pi/2) & 0 & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$B =$

$T R_{x,135} R_{y,90}$

4)

$$\begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix} = A \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix} \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = B \begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix} \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = C \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix}$$



$A = T \cdot R_{z,90^\circ}$

$$C = T R_{x,-45} R_{y,90} \quad \begin{bmatrix} 1 & 0 & 0 & -d \\ 0 & 1 & 0 & d \sin\left(\frac{\pi}{4}\right) \\ 0 & 0 & 1 & d\cos\left(\frac{\pi}{4}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{-\pi}{4}\right) & \sin\left(\frac{-\pi}{4}\right) & 0 \\ 0 & -\sin\left(\frac{-\pi}{4}\right) & \cos\left(\frac{-\pi}{4}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\left(\frac{\pi}{2}\right) & 0 & -\sin\left(\frac{\pi}{2}\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\frac{\pi}{2}\right) & 0 & \cos\left(\frac{\pi}{2}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$