CIS 580: Machine Perception, Spring 2021
Homework 4
Due: Mar 4, 2021 at 11:59pm

Instructions

- This is an individual homework.
- You must submit your solutions on Gradescope, the entry code is MB8ZJP. We recommend that you use LaTeX, but we will accept scanned solutions as well. Please box your answers if you submit scanned versions.
- Start early! If you get stuck, please post your questions on Piazza or come to office hours!

1 Problem 1, 30pts

There are four $2D$ points $(0,1)$, $(1,0)$, $(1,1)$ and $(3,4)$. After same rotation and translation they become $(1.15, 1.28)$, $(0.28, 0.17)$, $(1.27, 0.29)$, $(4.5, -1.32)$.

(i) Could you use the solution for orthogonal Procrustes problem to obtain the rotation matrix?

(ii) Find a solution that solves directly for the rotation angle θ and a translation t_x, t_y by minimizing

$$\sum_{i=1}^{N} \{(x_i' - x_i \cos\theta + y_i \sin\theta - t_x)^2 + (y_i' - x_i \sin\theta - y_i \cos\theta - t_y)^2\}.$$

2 Problem 2, 40pts

You are holding your phone vertically with the optical axis Z_c parallel to the ground. Assume that your camera coordinates are calibrated $K = I$. You see two points on the ground whose world coordinates we assume to be (0,0,0) and (a,0,0). For example, they can be end points of a rod of length a. Their projections are (x_1, y_1) and (x_2, y_2), respectively.

1. Assume that the relative angle between the x-axis of the world and the x-axis of your phone camera is θ, also called the yaw angle. Assume that the translation vector from the projection center to the world origin (0,0,0) is $T = (T_x, T_y, T_z)$. Write the projection equations for (x_1, y_1) and (x_2, y_2) (4 equations total, left hand side must be (x_1, y_1) and (x_2, y_2), right hand side should include only a, θ, T).
2. Solve these 4 equations for the yaw angle θ and (T_x, T_y, T_z).
3. What are the conditions on the camera position in order to obtain a unique or finite number of solutions.

3 Problem 3 (30pts)

Assume that we have two cameras looking at the same ground plane π. Camera 1 is at height $h = 2$ above the ground plane. We also know the gravity vector $g_1 = \left[0, \sqrt{3}/2, 1/2\right]^T$, in the camera's 1 coordinate system and the gravity vector $g_2 = \left[0, 1/2, \sqrt{3}/2\right]^T$ in the camera's 2 coordinate system. The gravity vector is vertical to the ground plane π. Both cameras have camera matrix $K = I$.

Your are given homography H that maps points from camera's 1 image plane to camera's 2 image plane.

$$H = \frac{1}{8}\begin{bmatrix} -4 & 0 & 8 \\ 6 & \sqrt{3}-12 & 5-4\sqrt{3} \\ -2\sqrt{3} & 7+4\sqrt{3} & \sqrt{3}+4 \end{bmatrix}$$

Decompose H so that you find the rotation R and the translation T from camera 1 to camera 2.