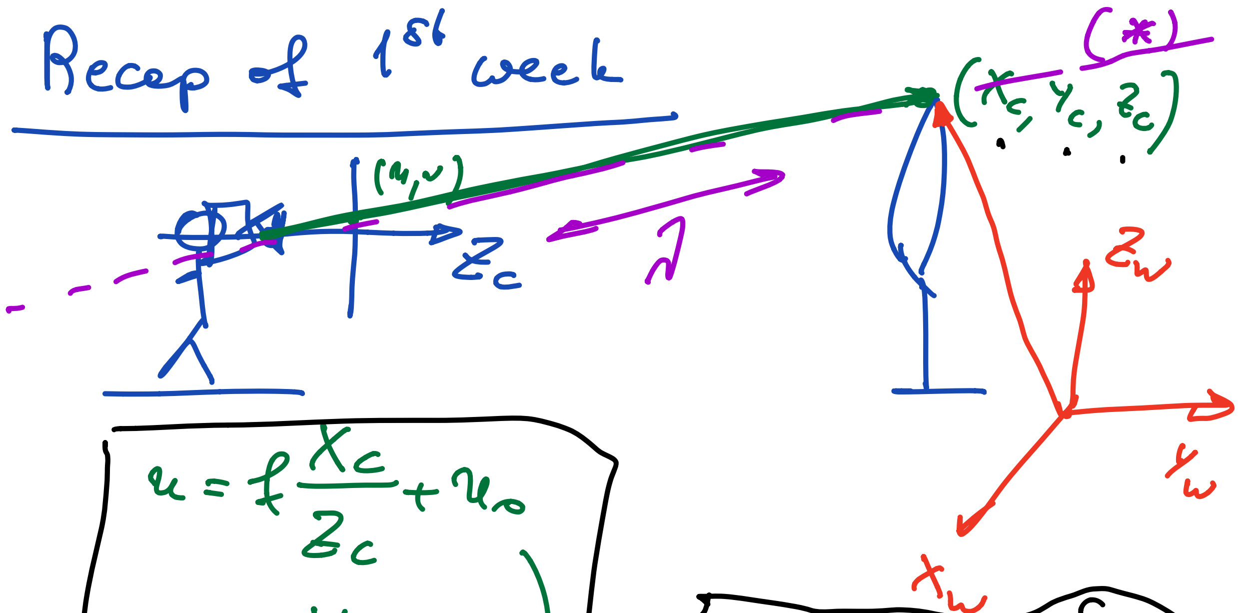
Recap of 1st week



$$u = f\frac{X_c}{Z_c} + u_0$$

$$v = f\frac{Y_c}{Z_c} + v_0$$

[pixels]

$${}^{c}P = {}^{c}R_w\,{}^{w}P + {}^{c}T_w$$

where is camera

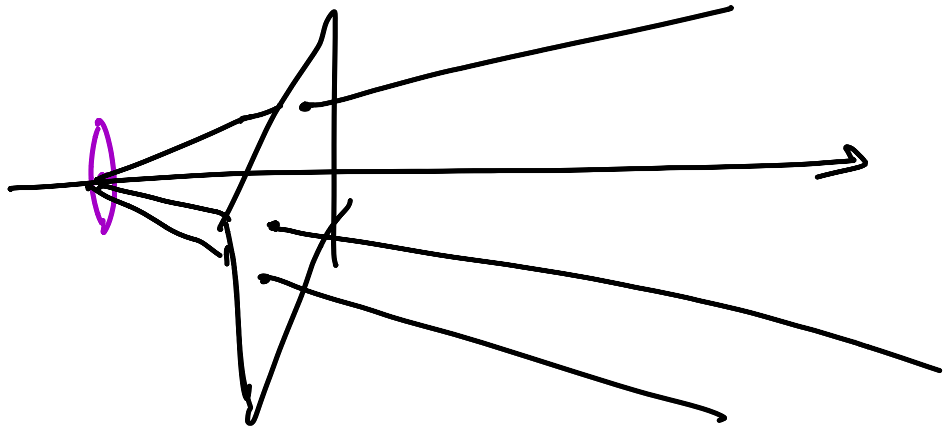
$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\underset{3\times 3}{\begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}}}_{K \text{ intrinsic}} \underbrace{\underset{3\times 4}{\overbrace{\begin{pmatrix} {}^{c}R_w & {}^{c}T_w \end{pmatrix}}^{\text{where is camera}}}}_{M \text{ extrinsic}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

given (u,v)
where is $\begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix}$?

$$\begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} = -{}^{c}R_w^{-1}\,{}^{c}T_w + \lambda\,{}^{c}R_w^{-1} K^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

(*) line equation

given $\begin{pmatrix} u \\ v \end{pmatrix}$ we know only the <u>ray</u> through $\begin{pmatrix} u \\ v \end{pmatrix}$ and the projection center (origin of cam. c.s.).

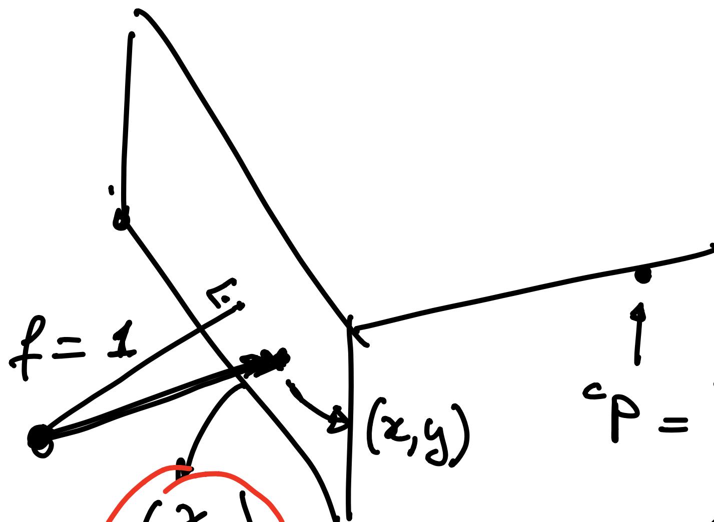


we need a new geometry to deal with a set of rays through one point (pencil of rays)

(old geometry : euclidean geometry)

Axiom : Parallel lines never
"Def" intersect.

Picture taking (Perspective projection) can make parallel lines not parallel (intersect) ?



$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad {}^{c}P = \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix} = \begin{pmatrix} sx \\ sy \\ s \end{pmatrix}$$

homogeneous coordinates

not always one ?!

We will define a new set where each ray is one element !

= set of rays

"all points along one ray are equivalent" relation (x, y)

- $x \sim x$
- $x \sim y \Rightarrow y \sim x$
- $x \sim y$ and $y \sim z$ then $x \sim z$

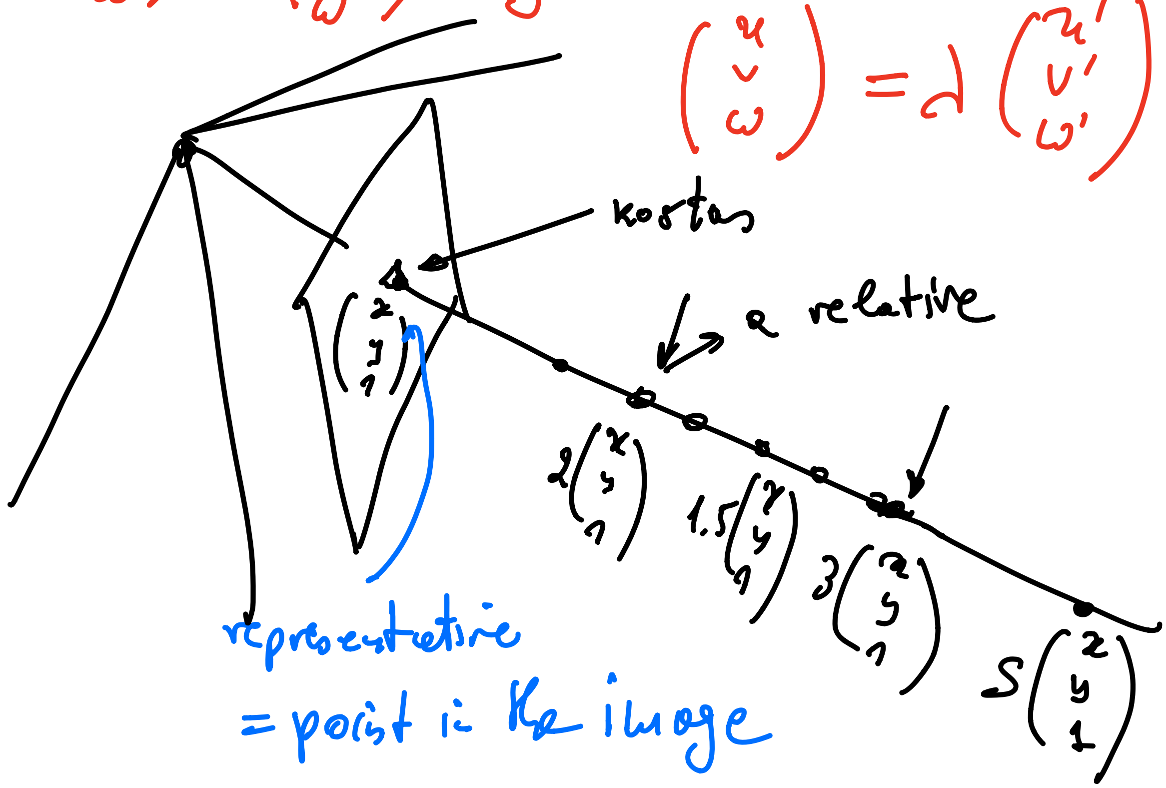
equivalence relation

equivalence class $= \{ x \in X : x \sim a$ for an $a \in X \}$

representative

$\boxed{\text{projective equivalence}}$

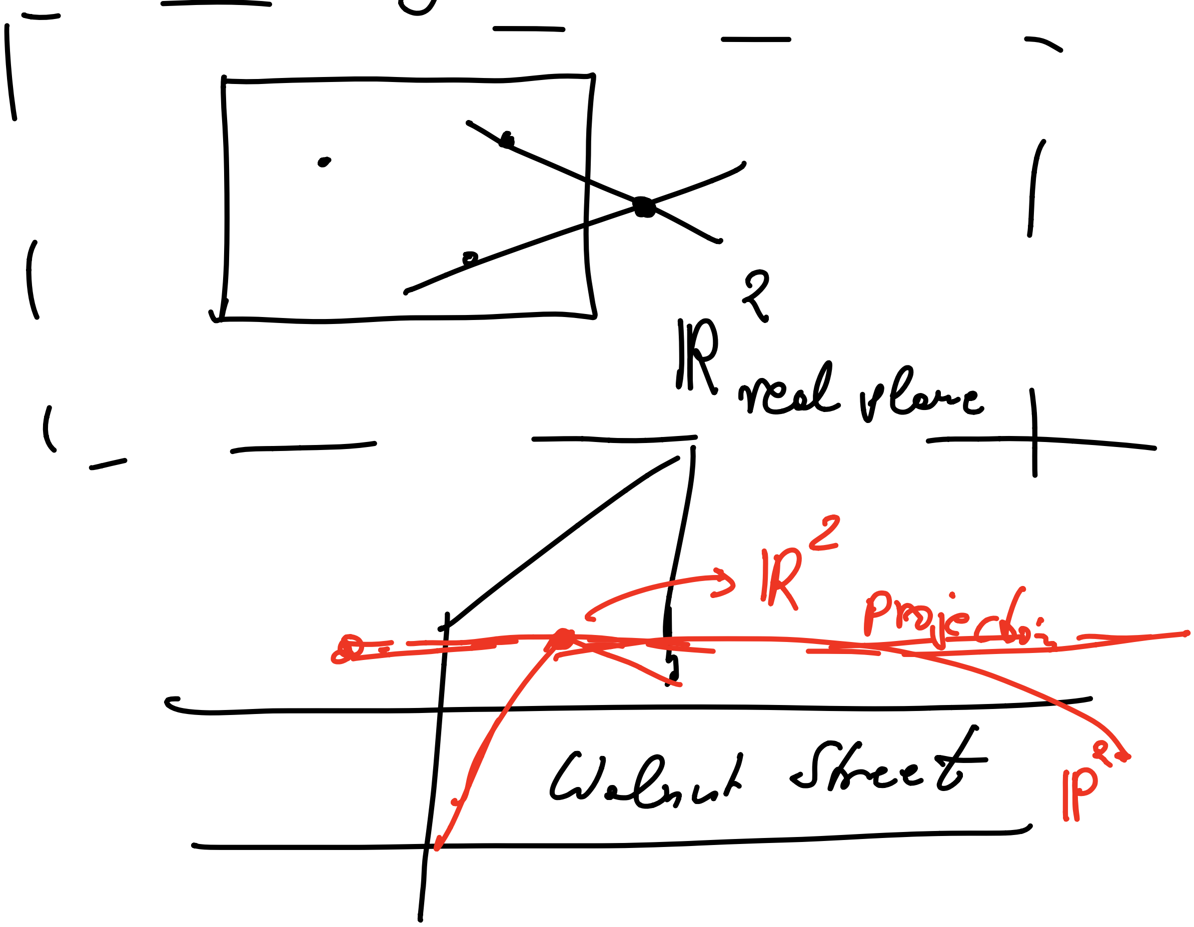
$$0 \neq \begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} \neq 0 \quad \text{iff} \quad \exists \lambda \neq 0 : \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \lambda \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix}$$



projective plane $\mathbb{P}^2$ = set of all rays $s\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ when $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \neq 0$

$s \in \mathbb{R} \setminus \{0\}$

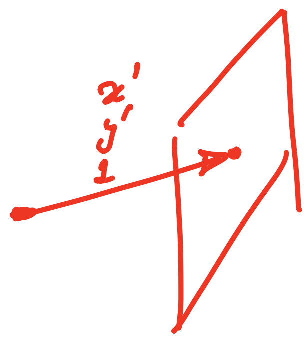
Why was the image plane not enough? $\mathbb{R}^2$



Main reason for introducing $\mathbb{P}^2$ is the projection of points at infinity!

$$\mathbb{P}^2 = \mathbb{R}^2 \cup \{\text{points at infinity}\}$$

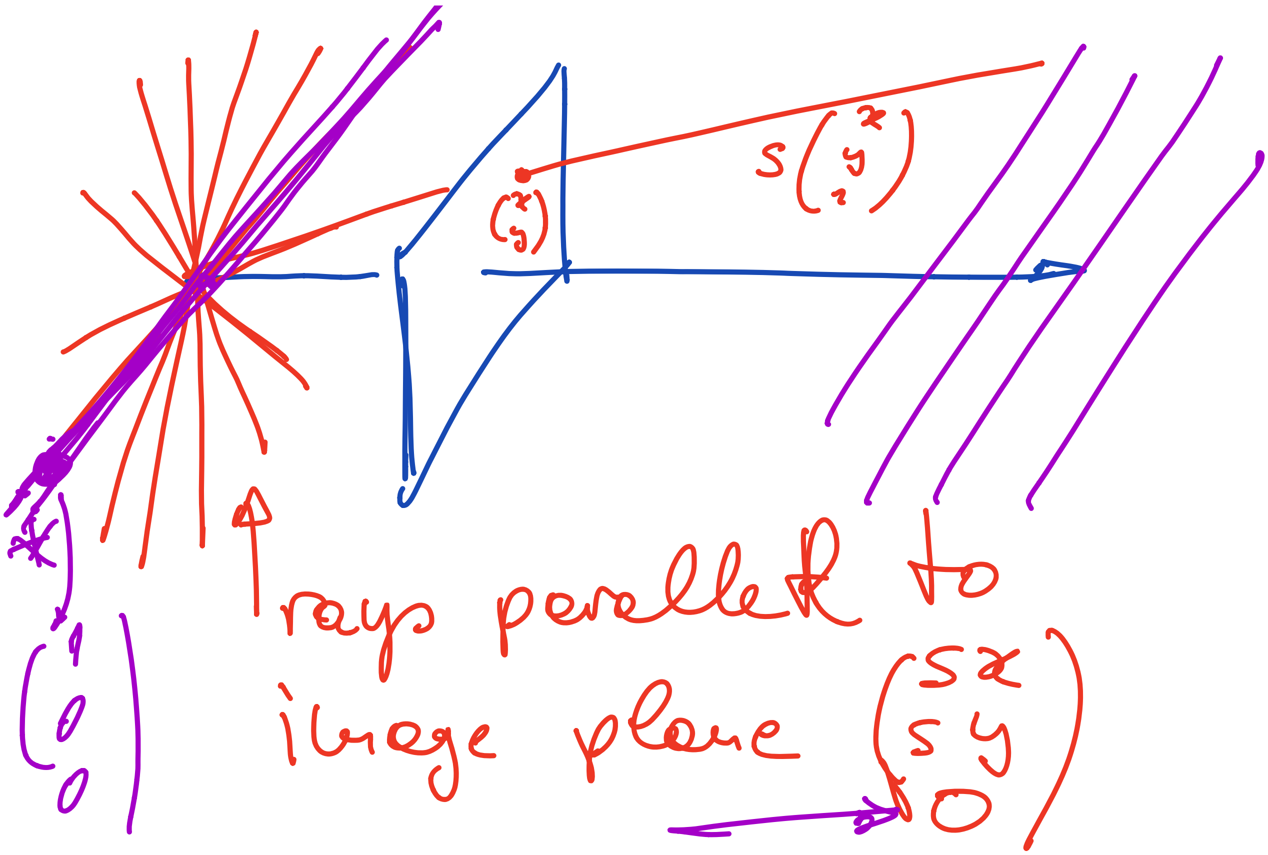
$$\begin{matrix} x' \\ y' \end{matrix} \in \mathbb{R}^2 \longrightarrow \mathbb{P}^2 \ni \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$



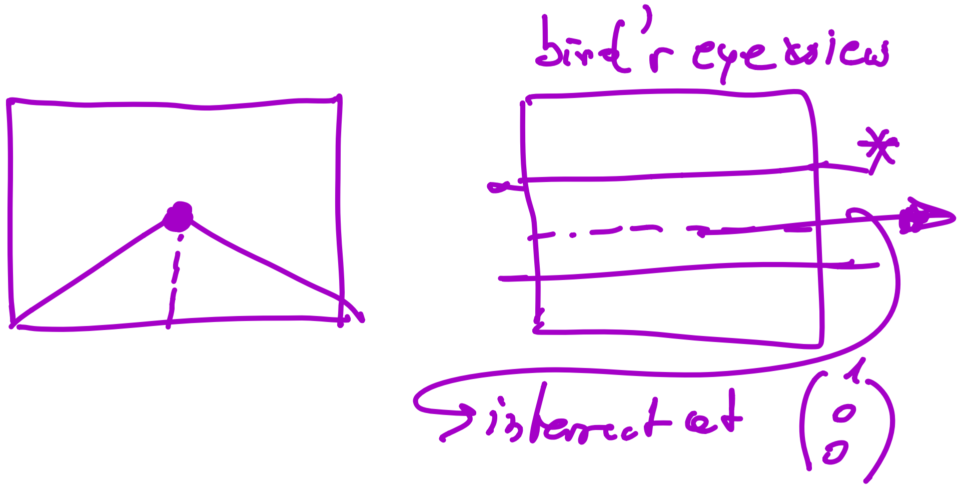
$$\begin{matrix} u \\ v \\ w \end{matrix} \in \mathbb{P}^2 \Longrightarrow \mathbb{R}^2 \ni \begin{pmatrix} u/w \\ v/w \end{pmatrix}$$

if $w \neq 0$.

Points at infinity $w = 0$

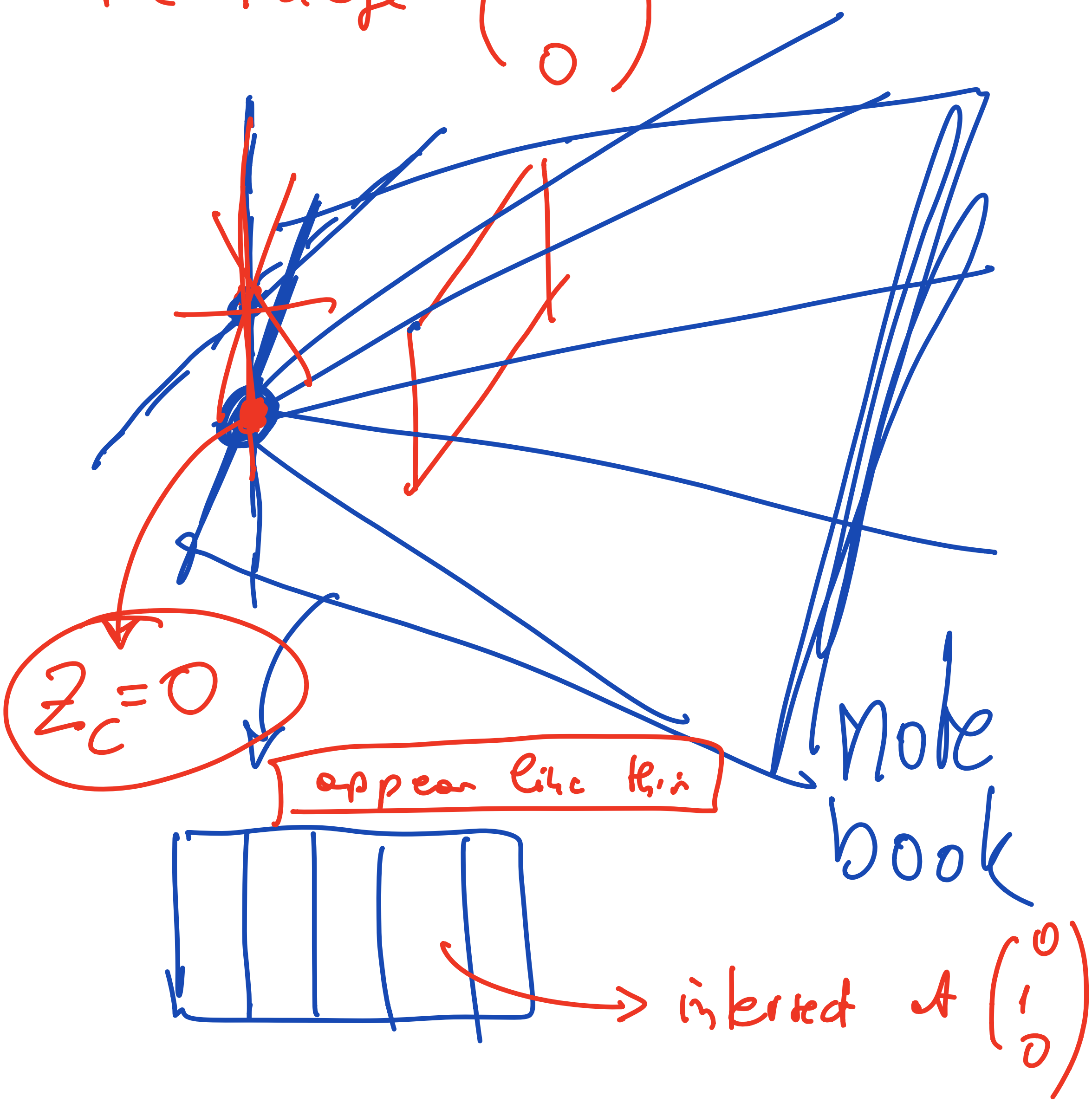


$\begin{pmatrix} x \\ y \end{pmatrix}$
$s\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
rays parallel to image plane
$\begin{pmatrix} sx \\ sy \\ 0 \end{pmatrix}$



bird's eye view
intersect at $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Can intersecting lines in the world appear parallel in the image? → intersection in the image $\begin{pmatrix} \cdot \\ \cdot \\ 0 \end{pmatrix}$



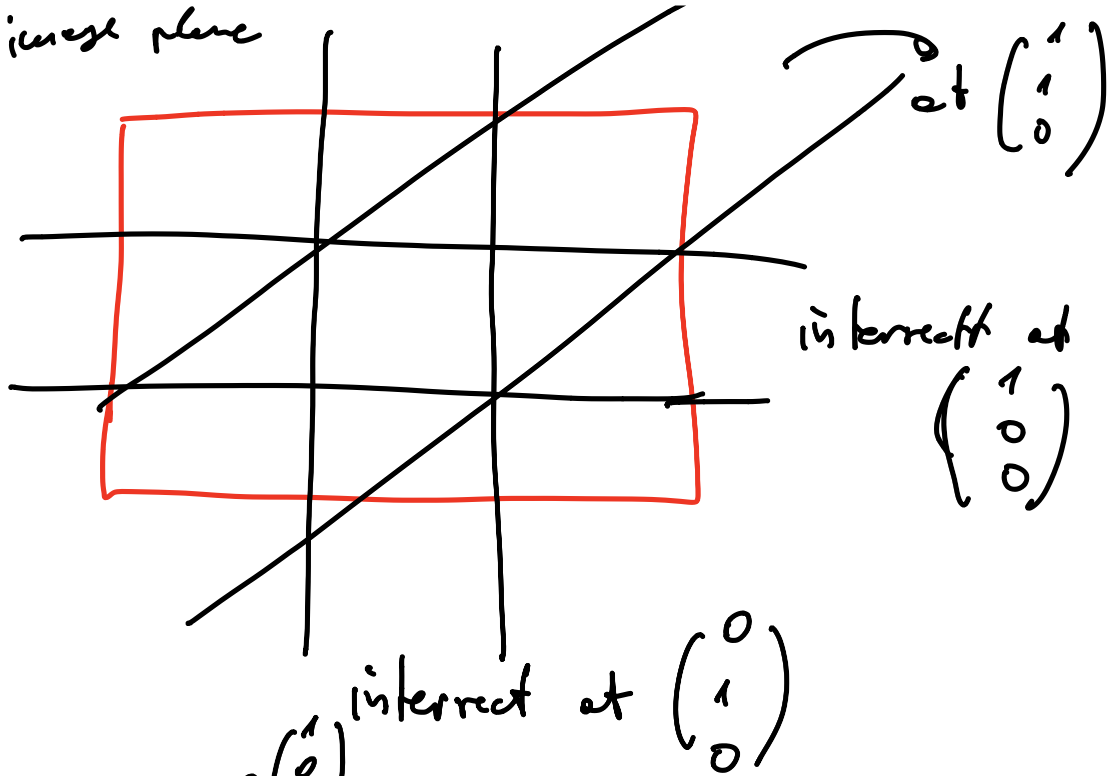
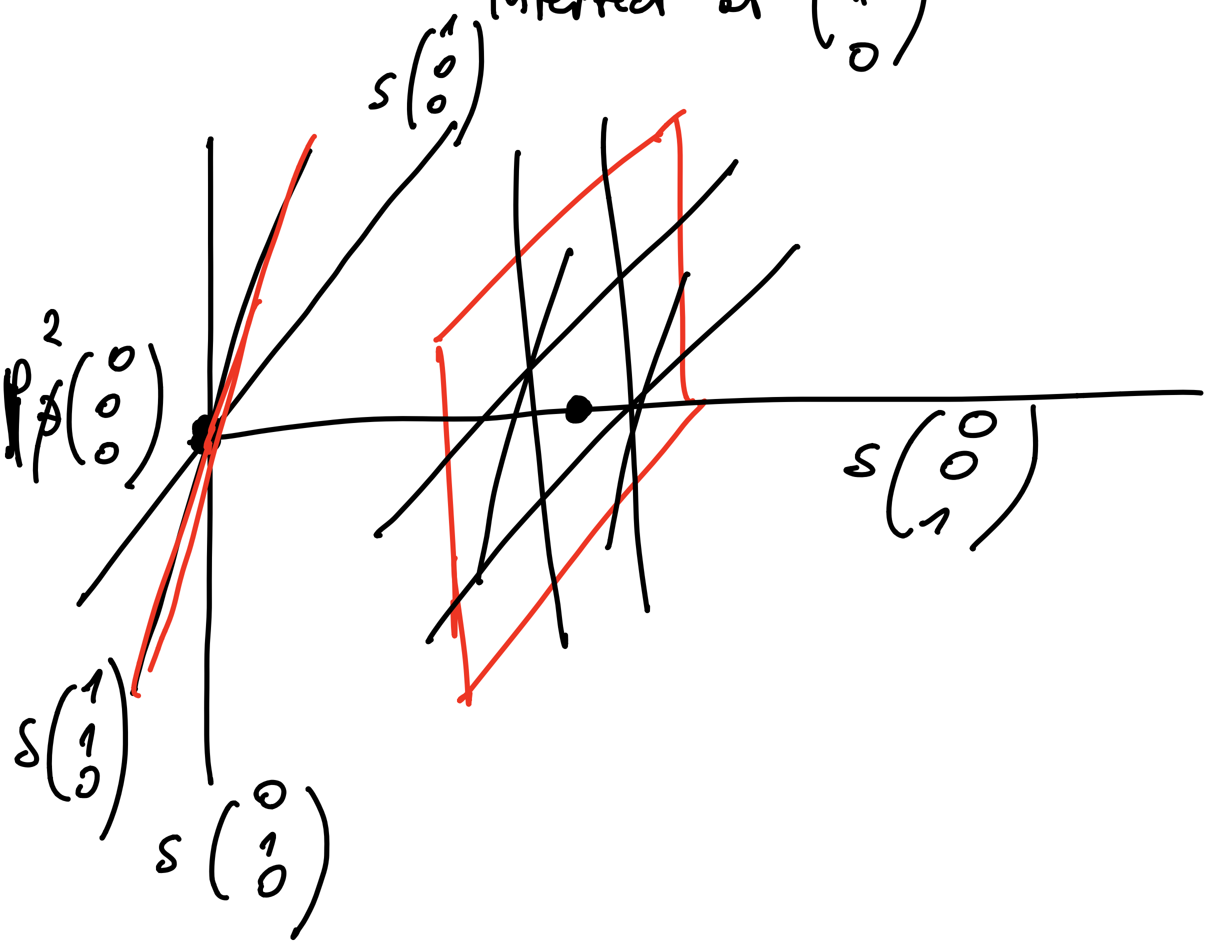


image plane
at $\begin{pmatrix}1\\1\\0\end{pmatrix}$
intersect at $\begin{pmatrix}1\\0\\0\end{pmatrix}$
intersect at $\begin{pmatrix}0\\1\\0\end{pmatrix}$

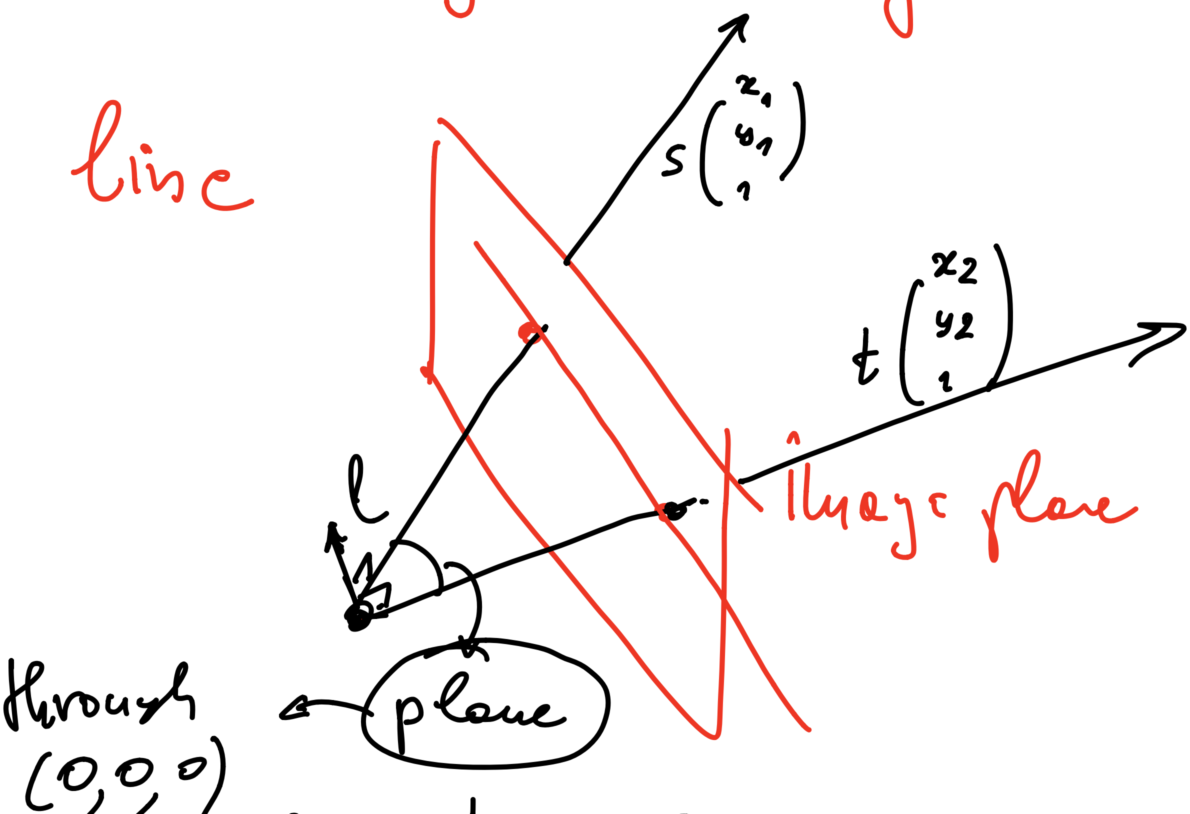


$s\begin{pmatrix}1\\0\\0\end{pmatrix}$
$\mathbb{P}^2 \ni \begin{pmatrix}0\\0\\0\end{pmatrix}$
$s\begin{pmatrix}0\\0\\1\end{pmatrix}$
$s\begin{pmatrix}1\\1\\0\end{pmatrix}$
$s\begin{pmatrix}0\\1\\0\end{pmatrix}$

$$\mathbb{P}^2 = \underbrace{\left\{ s\begin{pmatrix}x\\y\\1\end{pmatrix} \right\}_{s\neq 0}}_{\text{ray}} \cup \underbrace{\left\{ t\begin{pmatrix}x\\y\\0\end{pmatrix}\right\}}_{\text{ray}}$$

representative $\to \mathbb{R}^2$

representative

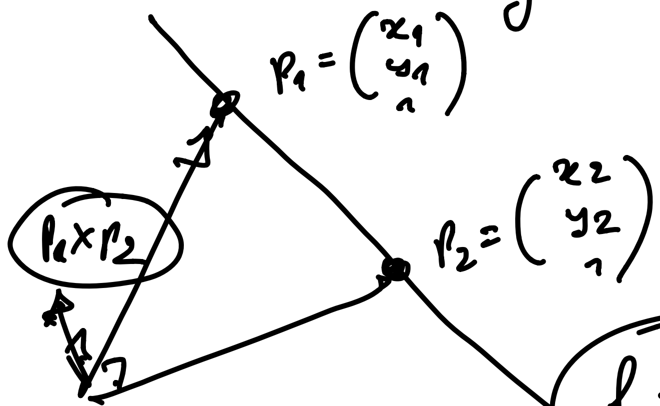


$$ax + by + cz = 0$$

$$\begin{pmatrix}a & b & c\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = 0$$

homogeneous coordinates $\to$ $l^T p = 0$

line is not that different then real lines $ax + by + c = 0$



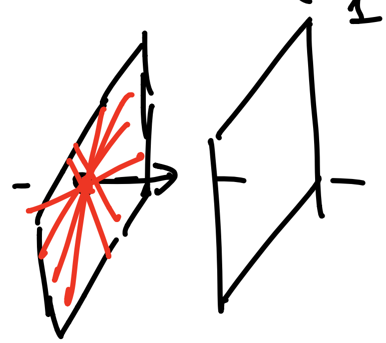
$$l = p_1 \times p_2$$

$$ax + by = 0$$

passing through image origin (not projection center)

which line has the form $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$? $0x + 0y + 1 \cdot z = 0$

this is the line at infinity: l_∞

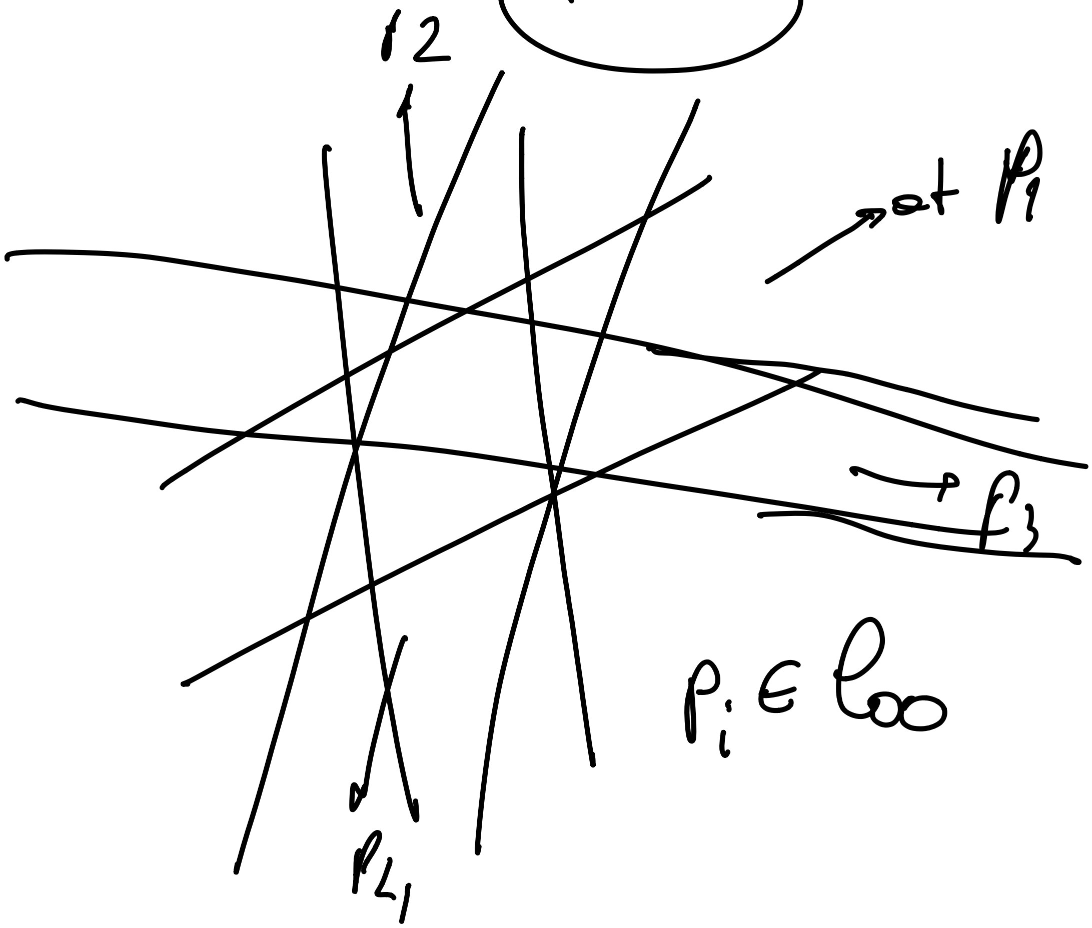


$$\ell_\infty = \left\{ \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

ℓ_∞ ir $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \notin \mathbb{R}^2$

$$0x + 0y + 1 = 0$$

$$1 = 0$$



$p_i \in \ell_\infty$