CIS580 Spring 2021: Midterm Review Questions
Reading: all lectures, resources and HW until and including the lecture of Feb 24, 2021

Compiled on March 3, 2021 at 17:02:57GMT

Basic Projective Geometry

1. Points in a world plane p are mapped to points in the image plane q via the projective transformation $q \sim Hp$ with

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

   Find the equation of the horizon (projection of the line at infinity).

2. Find a condition on the elements of a projective transformation H such that parallel lines remain parallel?

3. A projective transformation H leaves horizontal lines horizontal and vertical lines vertical. It maps the origin to the origin and the point $(1, 1)$ to the point $(1, 2)$. Make a sketch of the two figures and compute H.

4. A projective transformation leaves the horizontal lines horizontal but maps all vertical lines to lines going through the point $(0, 5)$. It leaves the points $(0, 0)$ and $(1, 1)$ at the same place. Compute H.

5. Write the projective transformation that corresponds to doubling the focal length.

6. The projection from world to the image plane reads:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim K \begin{pmatrix} r_1 & r_2 & r_3 & t \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ W_w \end{pmatrix},$$

   where K an invertible 3x3 matrix of intrinsic parameters and r_1, r_2, r_3 the three columns of a rotation matrix.

   (a) Find the projective transformation from the plabe $Y_W = 0$ to the image plane

   (b) There is a line at infinity $W = 0$ in the plane $Y_W = 0$. Find the line coefficients of its projection in the image plane.

7. If the projection equation reads

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim \begin{pmatrix} K & \mathbf{0} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

with K an invertible 3x3 matrix of intrinsic parameters, write the equation of the projection of the plane $Z = AX + BY + C$ to the image plane.

8. What is the difference between zooming in and approaching a 3D scene? If there are parallel lines in the scene can you tell the difference? How does the scene have to look like so that you cannot tell the difference?

Vanishing points and Single View Geometry

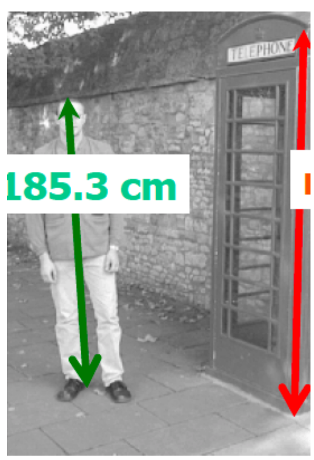
1. Draw a perspective projection of a cube in the case when the image plane is not parallel to any of the faces of the cube. To prove correctness you have to extend edges to vanishing points.

2. Prove that we can compute the focal length from two **orthogonal** vanishing points if the image center is at $(0, 0)$.

3. Given the projection A, B and C of 3 collinear points equidistant in the world find the the distance CD to the vanishing point D along this line.

4. If we have the projections of two lines parallel in the world intersecting in the image at a known vanishing point D and we know the pixel locations A, B, C of 3 points along one of those lines, explain how we can find the ratio $\frac{A_w B_w}{B_w C_w}$.

5. Explain how you can find the height of the phone booth in the picture below. Use a sketch but no need to enter numbers.



6. Does zooming (changing the focal length) change the position of the vanishing points ?

Localization

1. Explain how you can find the pose R, T of a camera given the projection of four coplanar points whose coordinates are known in the world.
2. Explain how you can find the pose R, T of the camera from the projection of a known triangle (P3P). Explain the steps without solving the equations.
3. Why is it impossible to find the location of the camera from the projection of two points of known coordinates in the world?
4. In the 3D-3D registration problem $P_i = RQ_i + T$ we are looking for (R, T) given correspondences of 3D points (P_i, Q_i). Explain how we can eliminate T from these equations.