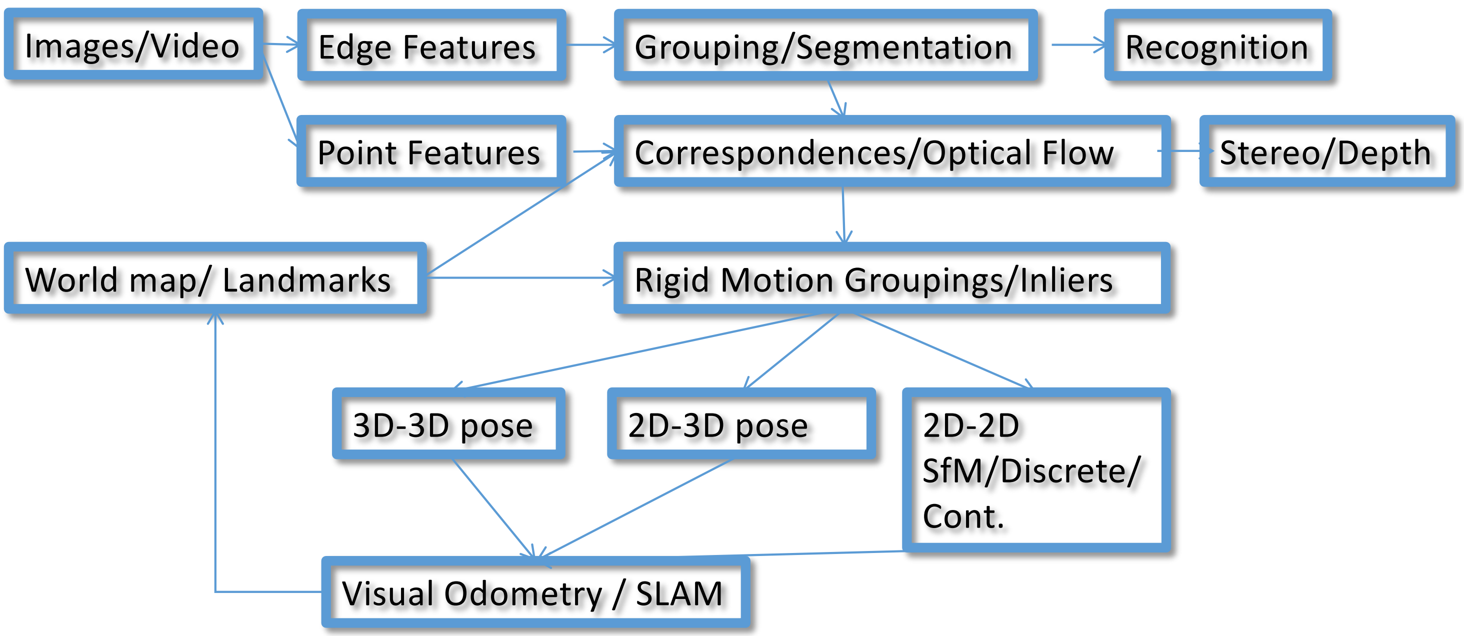
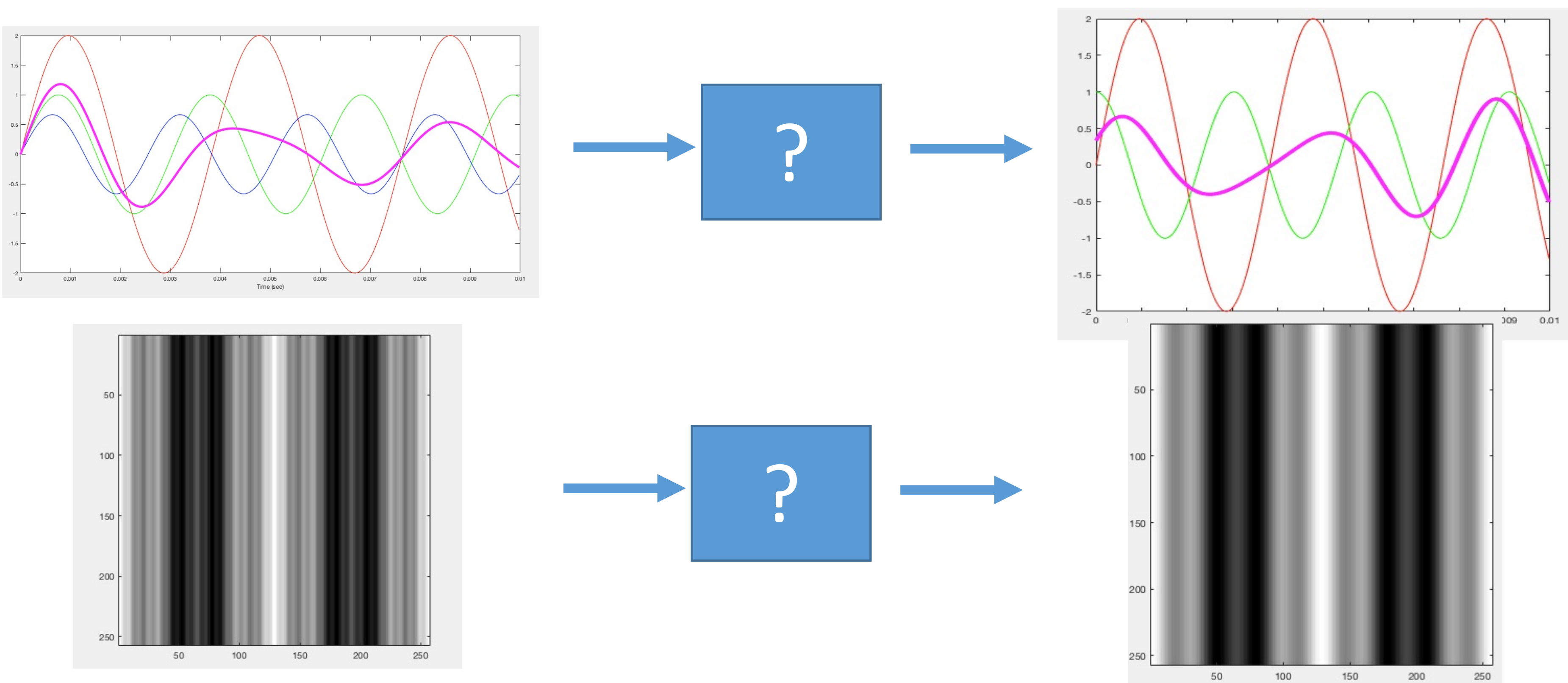
Big Picture: Perception Processing



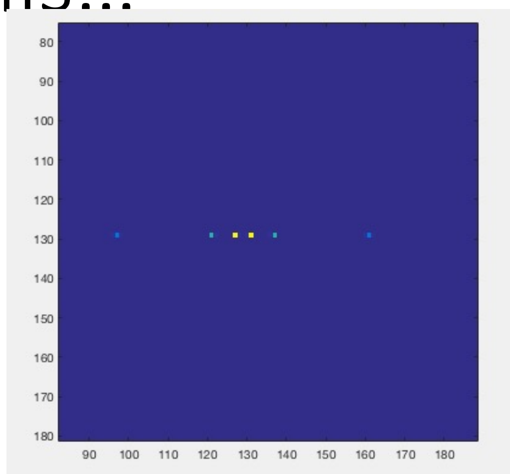
Frequency selectivity of filters

Frequency-selective filters

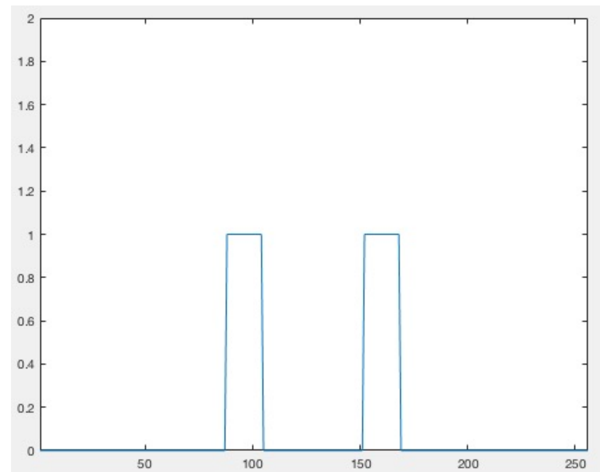
How can we filter out a frequency out of an image?



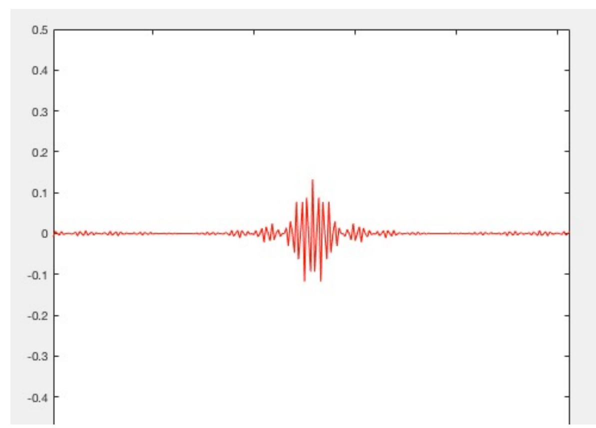
One might thing of a bandpass filter in frequency like this…



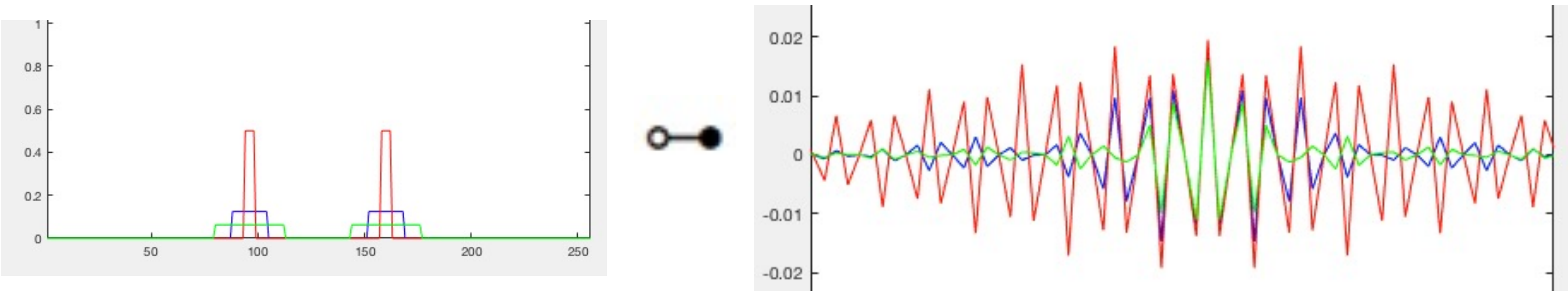
times



Would mean convolving with a huge mask:



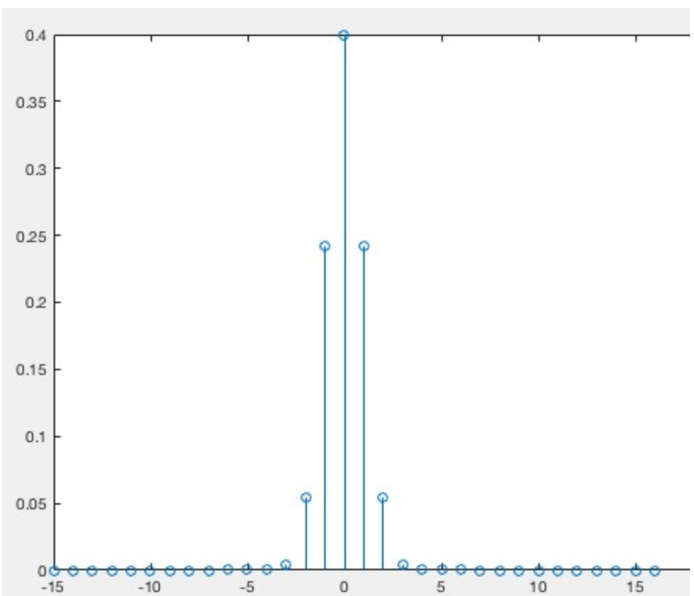
Frequency selectivity inversely proportional to spatial support (and hence location selectivity)

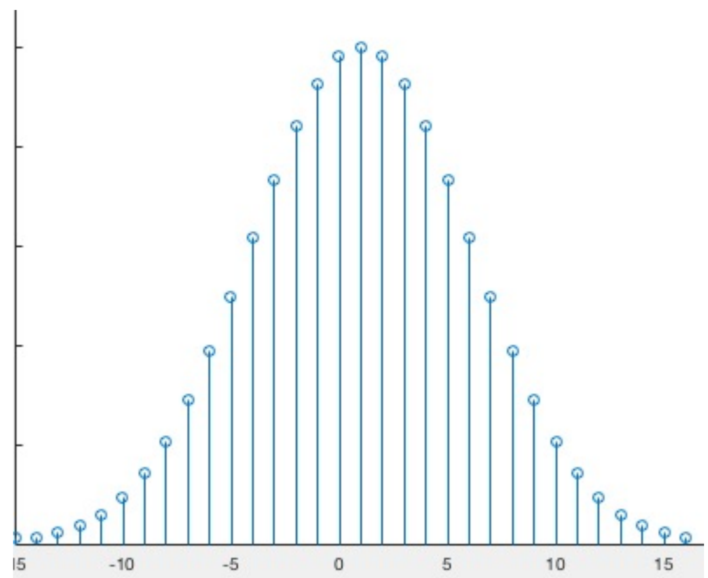


The uncertainty principle of signal processing

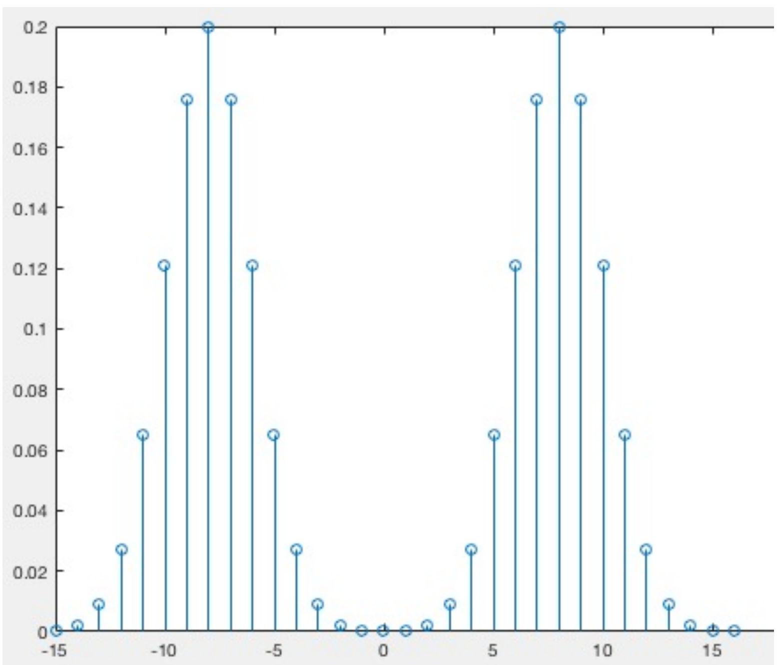
Best way to alleviate uncertainty principle….

Replace the rectangle with a Gaussian!!





How to make a bandpass out of a smoother (lowpass) ?



How to make a bandpass out of a smoothing filter (lowpass) ?

By creating two copies!

How?

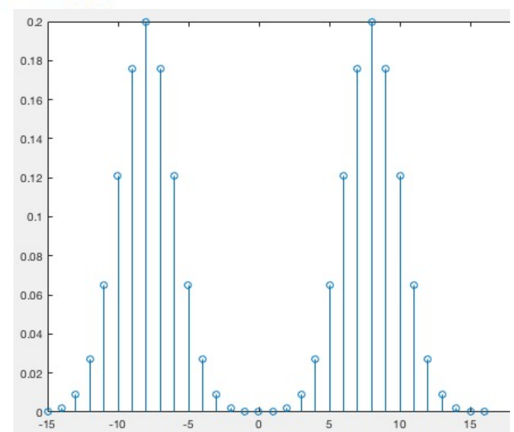
Modulating with a cosine!

Modulation with a cosine means

$$g(t) = \frac{1}{\sigma\sqrt{(2\pi)}} e^{-t^2/2\sigma^2} \circ\!\!-\!\!\bullet\ G(\omega) = e^{-\sigma^2\omega^2/2}$$

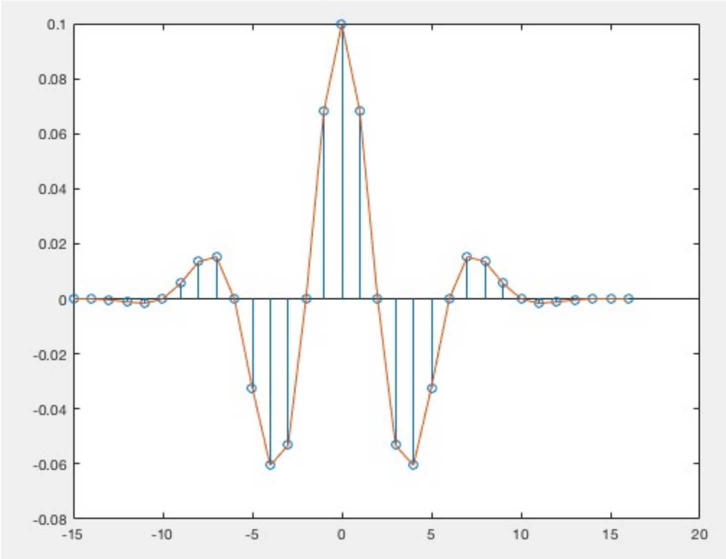
$$f(t) = \cos(\omega_0 t) \circ\!\!-\!\!\bullet\ \frac{1}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$F(\omega)G(\omega) = e^{-\sigma^2\omega_0^2/2}(\frac{1}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)))$$



Modulated Gaussian

$$\frac{1}{\sigma\sqrt{(2\pi)}}e^{-t^2/2\sigma^2}\cos\omega_0 t$$



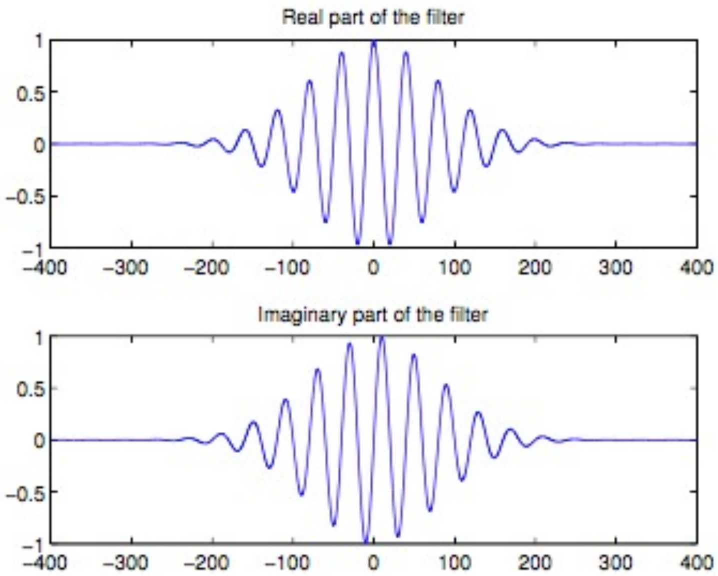
What happened with the phase?

$$\sin(\omega_0 t) \star \quad \frac{1}{\sigma\sqrt{(2\pi)}} e^{-t^2/2\sigma^2} \cos\omega_0 t \qquad =? \qquad \sin(\omega_0 t)$$

We need a phase-independent result!

Quadrature: complex filter with Re^2+Im^2=1

$$e^{-t^2/(2\sigma^2)}(\cos(\omega_0 t) + j\sin(\omega_0 t))$$



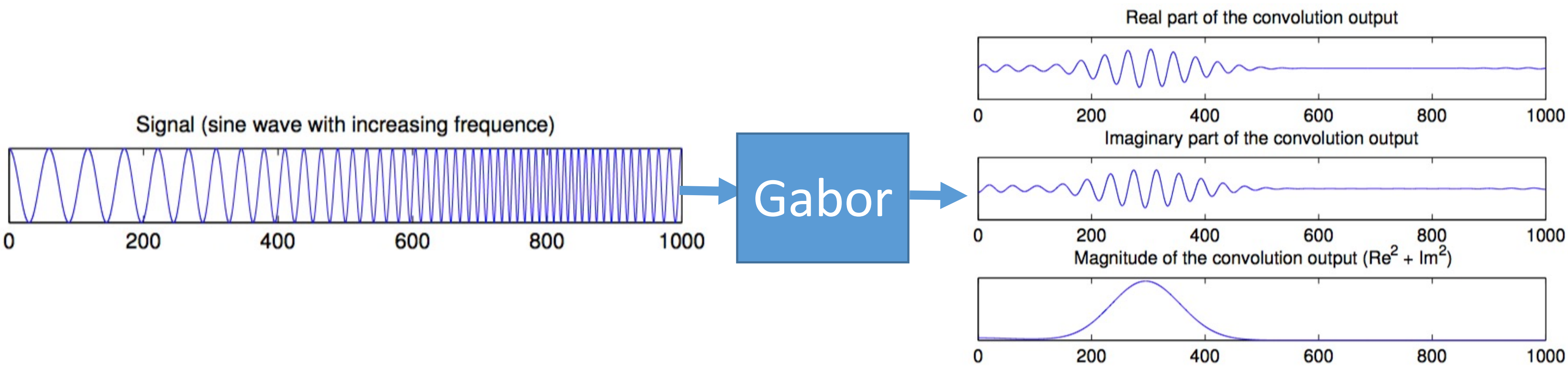
The Gabor Function

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/(2\sigma^2)} e^{j\omega_0 t}$$

$$\circ\!\!-\!\!\bullet$$

$$e^{-\sigma^2(\omega-\omega_o)^2/2}$$

Frequency selectivity

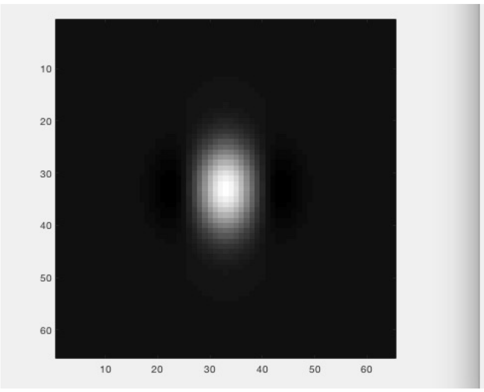


2D Gabor Function: selectivity in frequency and orientation

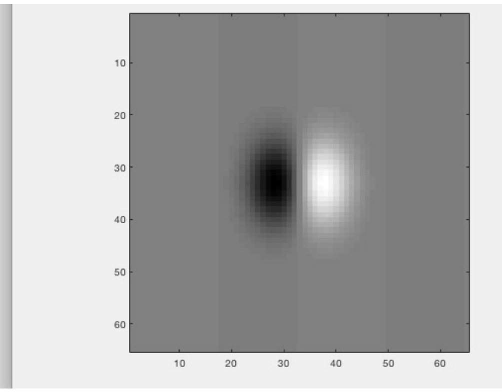
$$h(x, y, \sigma_1, \sigma_2, \omega_1) = \frac{1}{\sigma_1 \sigma_2 2\pi} e^{-(x^2/2\sigma_1^2 + y^2/2\sigma_2^2)} e^{j\omega_1 x}$$

$$e^{-(x^2/2\sigma_1^2 + y^2/2\sigma_2^2)} e^{j\omega_1 x} \circ\!\!-\!\!\bullet\; e^{-(\sigma_1^2(\omega_1 - \omega_x)^2 + \sigma_2^2 \omega_y^2)/2}$$

Real part of Gabor

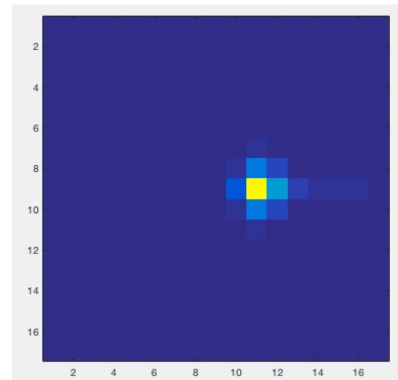


Imaginary part of Gabor



$\circ\!\!-\!\!\bullet$

Fourier Transform of Gabor is real



$$e^{-(x^2/2\sigma_1^2+y^2/2\sigma_2^2)}\, e^{j\omega_1 x} \;\circ\!\!-\!\!\bullet\; e^{-(\sigma_1^2(\omega_1-\omega_x)^2+\sigma_2^2\omega_y^2)/2}$$

Rotated 2D Gabor Filter