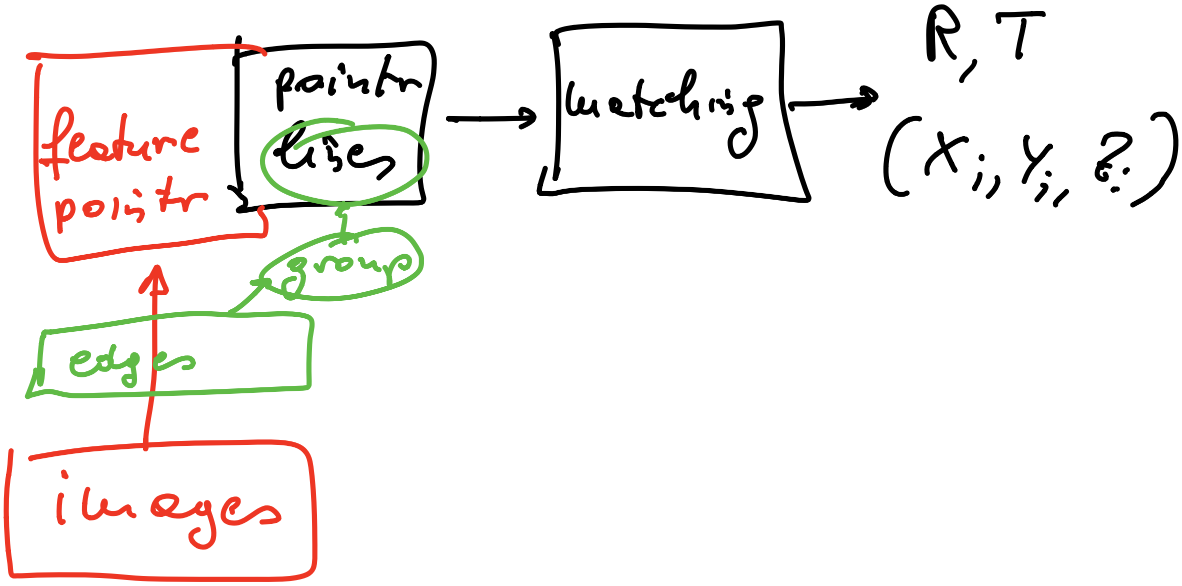
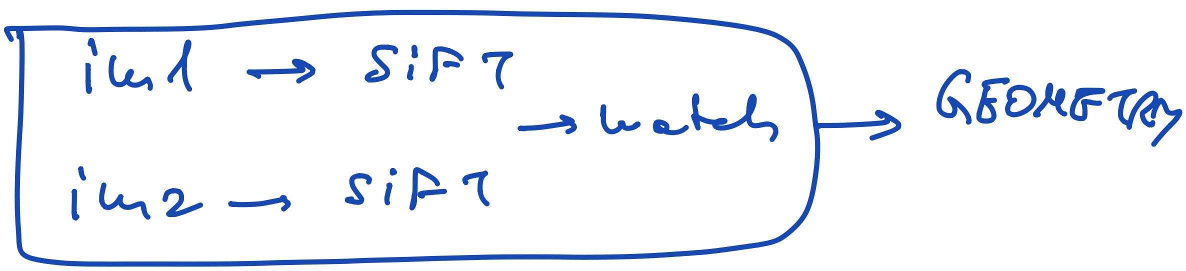


HW5



next 3 weeks

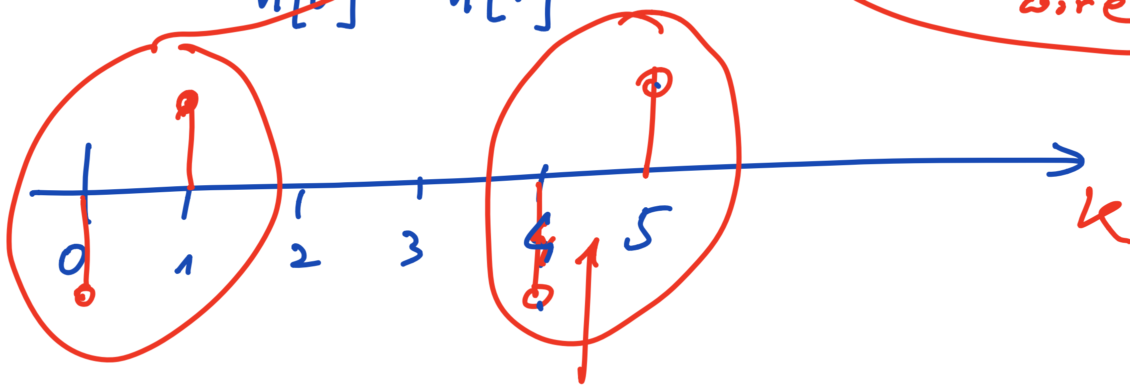
- feature extraction
- matching / correspondence

pixels ⇒ ?

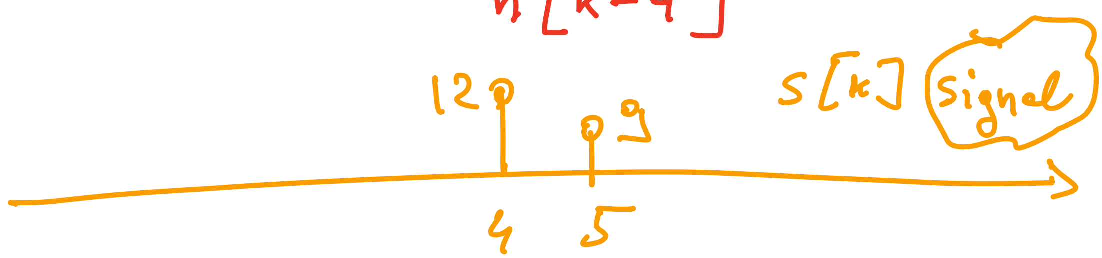
$h : \quad -1 \quad 1$

$h[0] \quad h[1]$

$$h[k] = \begin{cases} -1 & k=0 \\ 1 & k=1 \\ 0 & \text{otherwise} \end{cases}$$



$h[k-4]$



$12 \cdot (-1) + 9 \cdot 1 = -3$

$y[4] = s[4]\,h[0] + s[5]\,h[1]$

length of signal ← N

$$y[n] = \sum_{k=1}^{N} s[k]\,h[k-n]$$

$\boxed{\text{correlation}}$

output

input signal

filter/mask

$$s \to \boxed{h} \to y$$

$$y[4] = \sum_{k=-\infty}^{\infty} s[k]\, h[k-4] \qquad \neq 0$$

$k=4$

$k=5$

$$= s[4]h[0] + s[5]h[1]$$

How does convolution differ from correlation?

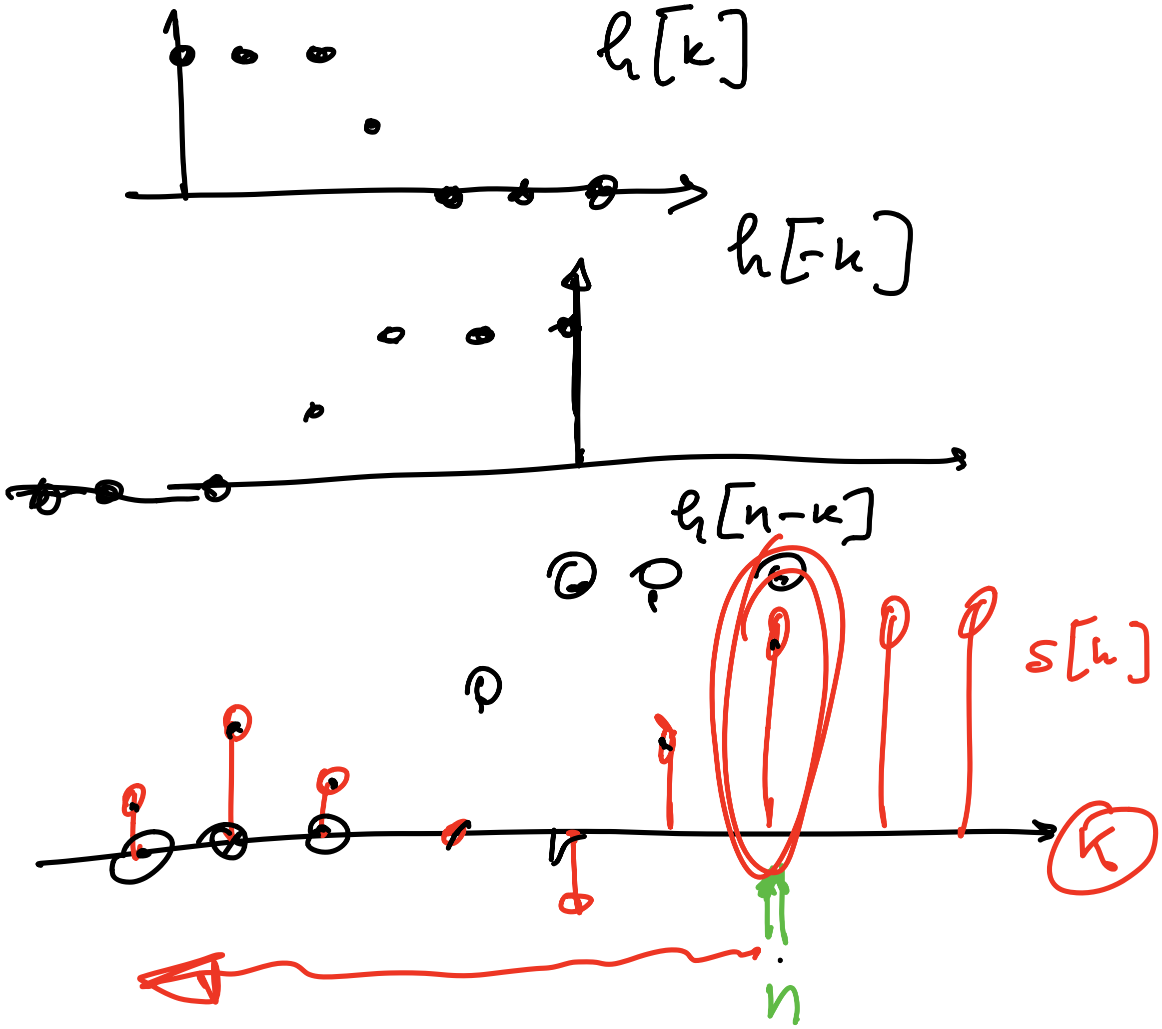
$\boxed{\text{Mask is first reflected!}}$

$$h[k] \longrightarrow h[-k]$$

correlation

$$\sum s[k]\, h[-(k-n)]$$

$$= \sum s[k]\, h[n-k] \quad \text{convolution}$$



$$\sum{}' s[k]\, h[n-k]$$

Why is the reflection needed? Because we want the response at time n to depend only on time $k \le n$

not on the future : causal property !

$$f[n] \longrightarrow \boxed{\;T\;} \longrightarrow g[n]$$

<u>linear</u>

$$a f_1 + b f_2 \longrightarrow \boxed{\quad} \longrightarrow a g_1 + b g_2$$

$$f(t) \longrightarrow \boxed{\;T\;} \longrightarrow f(t) - f(t-1)$$

<u>nonlinear</u>

$$f(t) \longrightarrow \boxed{\;T\;} \longrightarrow \max(f(t), f(t-1))$$

$$\text{median}(f(t), \dots f(t-5))$$

NN $\quad f(t) \longrightarrow \boxed{\quad} \longrightarrow \text{ReLu}(f(t))$

$$= \begin{cases} f(t) & f(t) \geq 0 \\ 0 & f(t) < 0 \end{cases}$$

$\boxed{\text{shift-invariant}}$

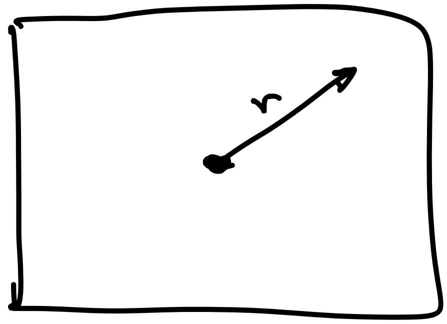
$$f(t-t_0) \to \boxed{T} \to g(t-t_0)$$

non shift-invariant

$$f(t) \to \boxed{T} \to t\,f(t)$$

image

$$f(x,y) \to \boxed{T} \to \sqrt{x^2+y^2}\,f(x,y)$$



LSI or LTI

shift

time

Theorem: A linear shift-invariant system can always be implemented by a <u>convolution</u>.

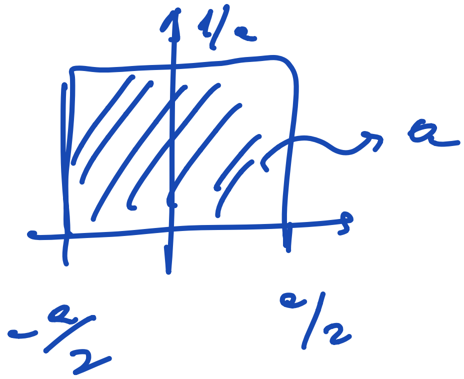
continuous: $\exists\, h$

$$f(t) \rightarrow \boxed{\text{LSI}} \rightarrow \int_{-\infty}^{\infty} f(t')\,h(t-t')\,dt' = g(t)$$

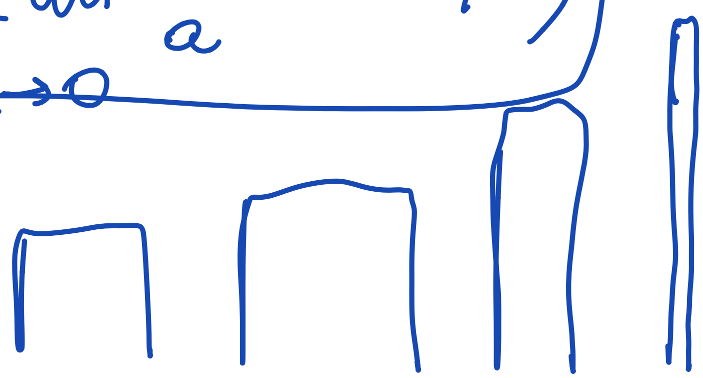
$$(f*h)(t)$$

Impulse function

box $\quad \mathrm{rect}(t/a) = \begin{cases} 1/a & |t| \le \frac{a}{2} \\ 0 & \text{else} \end{cases}$



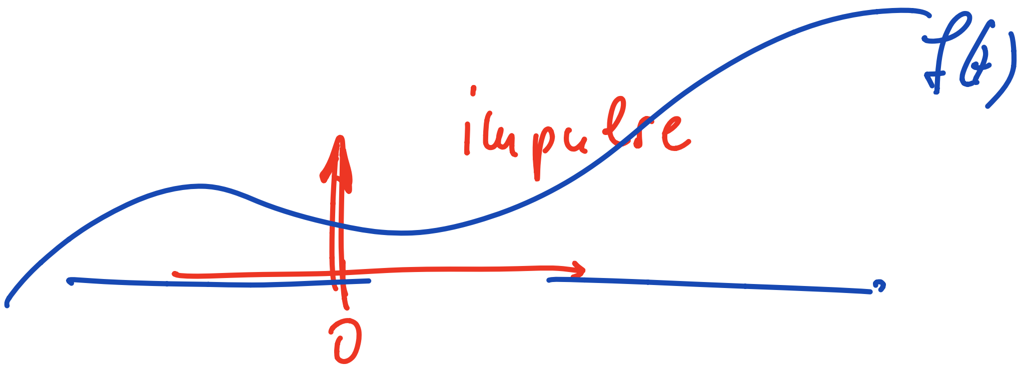
$$\boxed{\delta(t) = \lim_{a \to 0} \frac{1}{a} \, \mathrm{rect}(t/a)}$$



$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) = 1$$

in continuous t we never say $\delta(t)=1$



$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

$$f(t)\,\delta(t) \neq f(0)$$

Absorption

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

$$\delta(t) \longrightarrow \boxed{LSI} \longrightarrow \int \delta(t')\,h(t-t')\,dt'$$

pistol

* proof at the end

Absorption property $= h(t)$

$$\delta(t) \longrightarrow \boxed{\text{LSI}} \longrightarrow h(t)$$

Dirac impulse   filter

Example: "echo". We do not know the formula for "echoing". Instead we imitate the Dirac with a pistol shot.

$$f(t) \longrightarrow \boxed{\text{echo}} \longrightarrow (f * h)(t)$$

Definition of a filter:

The response of a system to the Dirac on input.

$h(t)$ = impulse response

Prove: If $\int f(t)\,\delta(t-t_0)\,dt = f(t_0)$ (*)

then $\int \delta(t')\,h(t-t')\,dt' = h(t)$

$$\int_{-\infty}^{\infty} \delta(t')\,h(t-t')\,dt' \qquad \tau = t-t'$$

$$d\tau = -dt'$$

$$\tau: \infty \to -\infty$$

$$= \int_{\infty}^{-\infty} \delta(t-\tau)\,h(\tau)\,(-d\tau)$$

$$= \int_{-\infty}^{\infty} h(\tau)\,\delta(t-\tau)\,d\tau$$

$\overset{\shortparallel}{\delta(\tau-t)}$ because $\delta(t) = \delta(-t)$

$$= \int_{-\infty}^{\infty} h(\tau)\,\delta(\tau - t)\,d\tau \overset{*}{=} h(t)$$