Correspondence → model like checkerboard or April tag and image

image - image

(collineation)

panorama

H

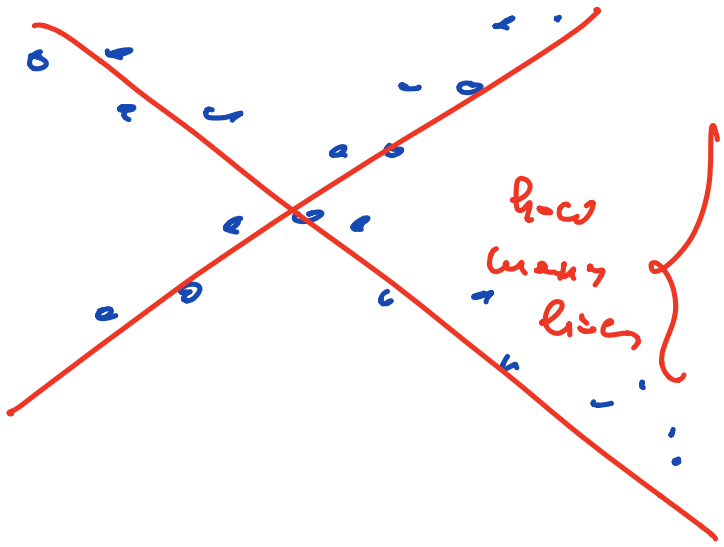
R, T

world - image

P4P

Abstraction : line fitting

(1) multiple line detection



? PCA

? k-means

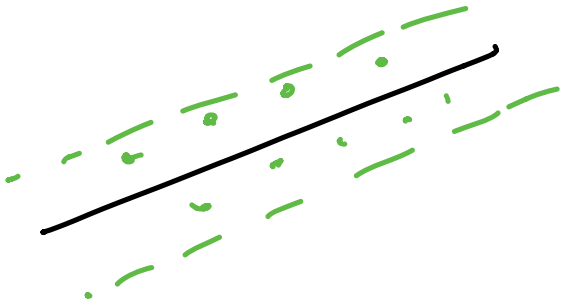
? EM

k-c
means
lines

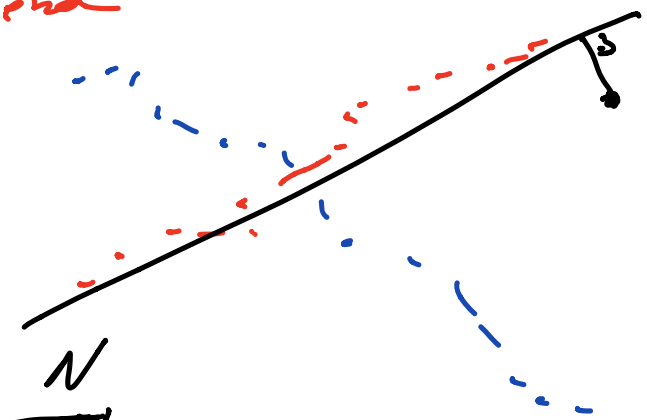
expectation-
maximization

# Chicken and egg

If we know the line equation then we can decide which point corresponds to which line



To find the line equation we need to know which points correspond to the line




$$\frac{1}{N}\sum_{i=1}^{N}(x_i\cos\theta + y_i\sin\theta - d)^2 \Rightarrow \min_{\theta,\,d}$$

$$\frac{\partial}{\partial d} \sum (\quad)^2$$

$$= \frac{1}{N} \cancel{2} \sum (x_i \cos\theta + y_i \sin\theta - d)\cancel{(-1)} = 0$$

$$\frac{1}{N}\cos\theta \sum x_i + \sin\theta \frac{1}{N} \sum y_i = d$$

$$\cos\theta\, \bar{x} + \sin\theta\, \bar{y} = d$$

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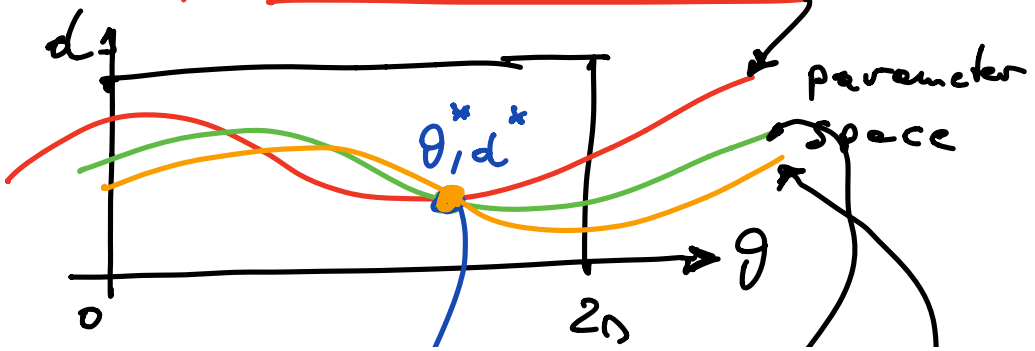
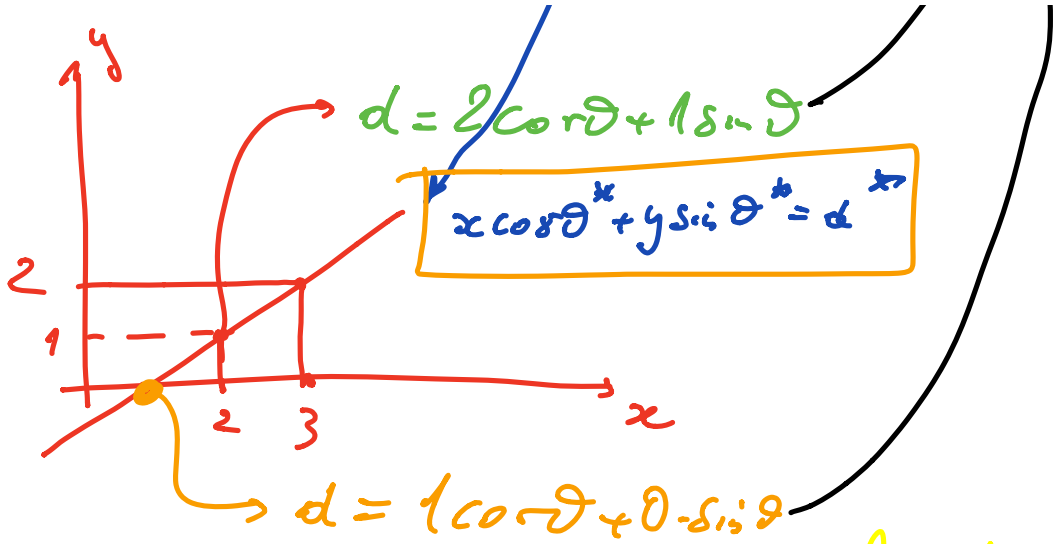
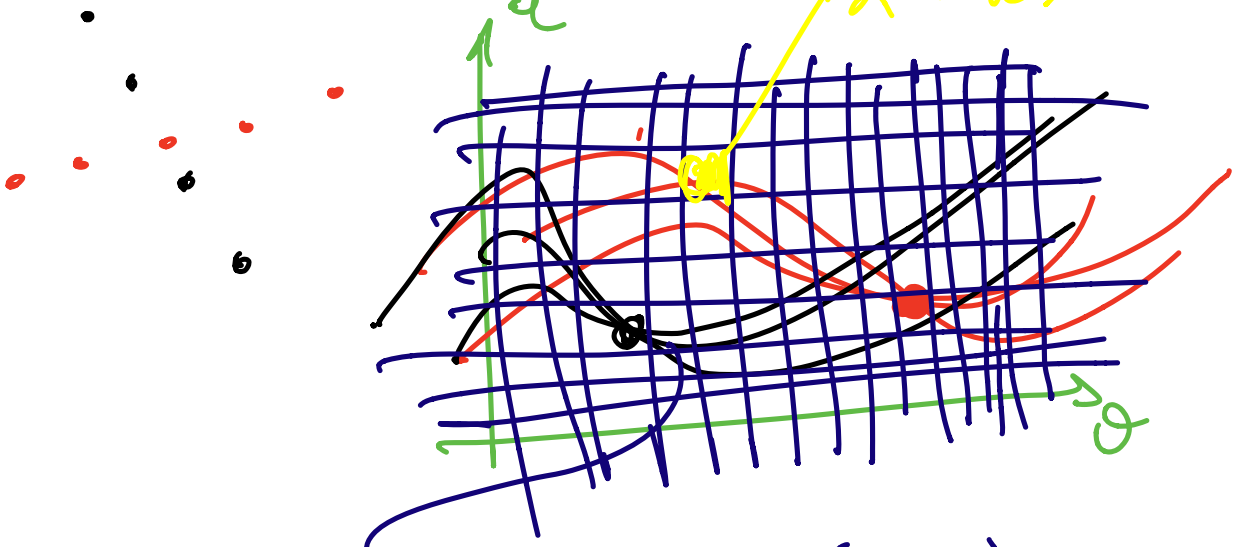
Alternative: Solve by voting in parameter space

$$d = x\cos\theta + y\sin\theta$$

$(x,y) = (3,2)$ $\quad d = 3\cos\theta + 2\sin\theta$






every cell $(\vartheta, d)$ we count the votes (how many curves go through this cell)

$$V(\vartheta, d) = \sum_{i=0}^{N} \mathbb{1}\left(|x_i \cos\vartheta + y_i \sin\vartheta - d| < \varepsilon\right)$$

# Hough Transform

- circle detection (3 parameters)
- pure rotation estimation (3 parameters)
  ↳ $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = R \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$

#parameter > 3 : prohibitive

(ellipse detection, proj. transformation)

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# RANSAC P3P

not about multiple models (lines)

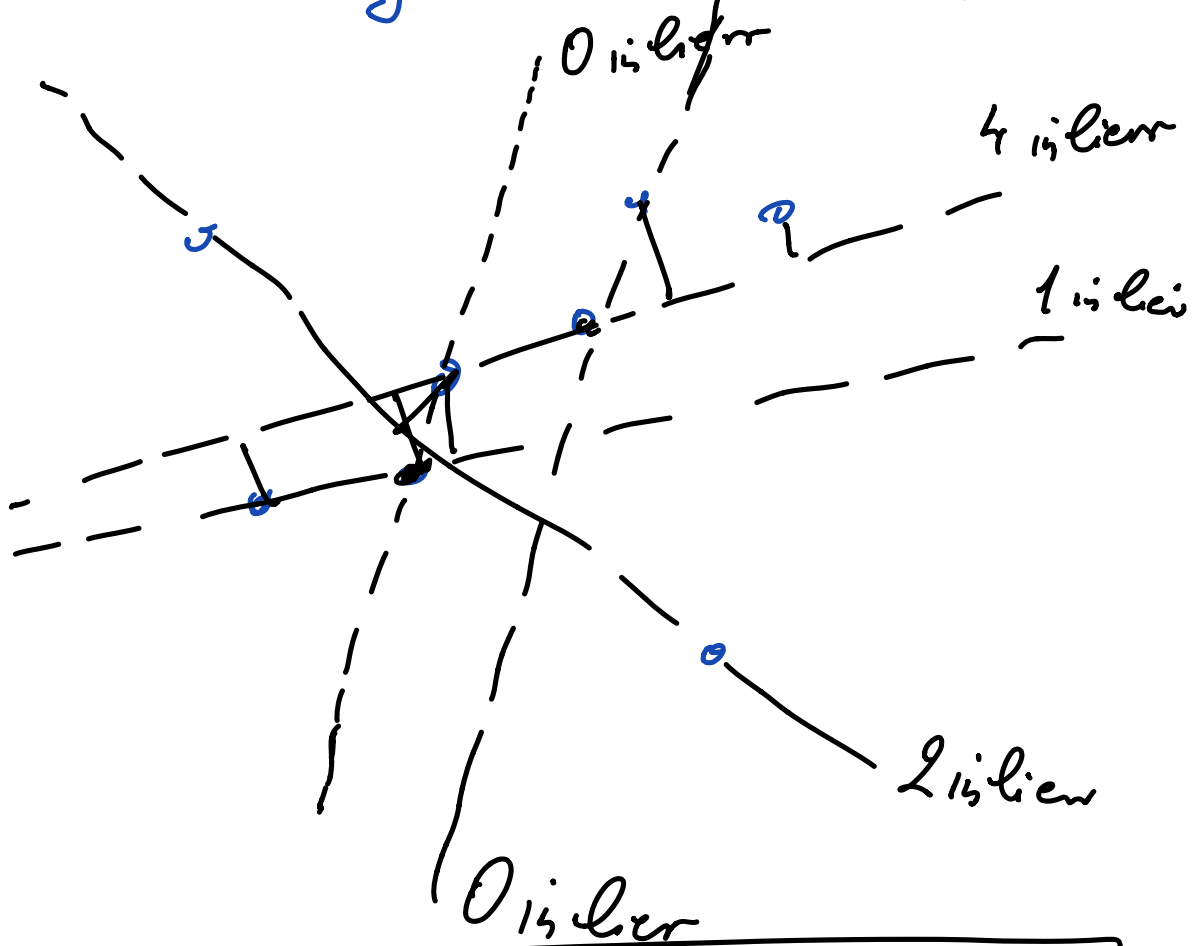
but one model (line, proj. transform)

and many outliers.

# Let's do it for the line fitting

Hypothesis: Two points define a line

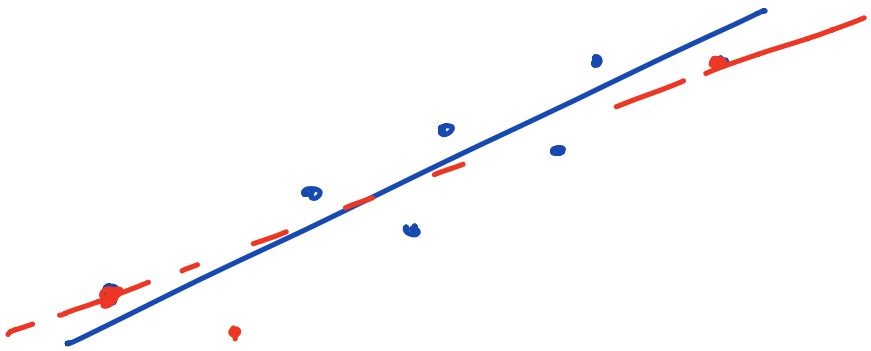
Test: Which points satisfy the hypothesis (inliers)




# threshold on inliers

- choose all $\binom{7}{2}$ pairs
- keep one with max. # inliers (> threshold)

The best line is none of them (does not pass through any pair) This is only an inlier selection $\Rightarrow$ fit a line $\underset{\theta, d}{\operatorname{argmin}} \sum_{i \in \text{inliers}} (x_i \cdot \text{cos}\theta + y_i \cdot \text{sin}\theta - d)^2$ $\leftarrow N$



Instead of exhaustive $\binom{n}{2}$

1. choose a pair
2. count inliers (dist < ε)
3. keep the maximum
4. if max > #inliers STOP

OUTPUT: the pair

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- choose a pair for a line
- in general choose minimal set

minimal set for line : 2 points

- plane in 3D : 3 points
- circle : 3 points
- ellipse : 5 points
- line in 3D: 2 points
- wifi-based localization : 3

- proj. transformation : 4 corresp.
- PnP : 3 correspondences
- Procrustes : 3 correspondences

This is the reason we described all minimal solutions.

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office hours Tue 6:30p →

in class review Wed

office hours noon Saturday