What do I have to know for the midterm?

- All material starting with Structure from Motion. Pyramids and Sampling Theorem are excluded (they are in the slides).
- Know everything about SfM as outlined in the questions below. Bundle adjustment is excluded.
- How to compute the derivatives of the Gaussian.
- How to compute the convolution of a step function and a box function with Gaussian derivatives.
- How to normalize a Gaussian derivative so that the intrinsic scale of a signal can be selected.
- How to rotate a gaussian derivative.
- You have to know the convolution theorem because it simplifies the computation of convolutions. If a Fourier transform is needed we will give it to you (but feel free to look it up during the exam). It is good to know that we can use the Fourier Transform when we encounter cos, sin, or Gaussians.
- You do not need to know how to compute a Fourier transform or any other Fourier theorem. We will not have a Fourier quiz.
- Study slides of optical flow and 3D velocities.

$$
\begin{aligned}
R_x(\theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \\
R_y(\theta) &= \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \\
R_z(\theta) &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

Geometry Review Questions including 3D velocities

1. Explain why the translation magnitude cannot be recovered from two calibrated views?

★ **SOLUTION:**

$$\lambda q = \mu R p + T \tag{1}$$
$$k\lambda q = k\mu R p + kT \tag{2}$$

for any k we have the above equation, therefore we cannot solve the magnitude of T.

2. Explain what happens if you try to compute the essential matrix from two calibrated views but one view is a pure rotation of the other.

★ **SOLUTION:** for the pure rotation, we have

$$q^T E p = q^T E R^T q = 0 \tag{3}$$

let $ER^T = A$, then for any q we have $q^T A q = 0$, which means A can be any skew-symmetric matrix. Therefore $E = \hat{T} R$ where T can be any vector.

If it is not pure rotation, we have

$$q^T E p = q^T E(R^T q - R^T T) = 0 \tag{4}$$

Let $ER^T = A$, then for any q we have $q^T A q - q^T A T = 0$, which implies $q^T A q = 0$ and $AT = 0$ for any q, therefore $A = \hat{T}$

3. How can you recognize from the essential matrix of two calibrated views without computing the SVD that the motion is a pure translation?

★ **SOLUTION:** Then $E = \hat{T}$, it is a skew-symmetric matrix.

There are four solutions: $(T, \boldsymbol{I})$, $(-T, \boldsymbol{I})$ (because -E also is a solution), $(T, \boldsymbol{R}_{T,\pi})$ and $(-T, \boldsymbol{R}_{T,\pi})$

4. Explain triangulation: How can we find depth from two views given intrinsics, R, and T.

★ **SOLUTION:**

$$\begin{pmatrix} q & -Rp \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = T \tag{5}$$

we have $\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} q & -Rp \end{pmatrix}^+ T$

5. Assume that two views differ only by a translation $T = (b, 0, 0)$:

$$Q = P + T$$

Given one calibrate correspondence $(p_x, 0)$ and $(q_x, 0)$ and b compute the depth Q_z of this point.

★ **SOLUTION:** we have

$$Q_z \begin{pmatrix} q_x \\ 0 \\ 1 \end{pmatrix} = P_z \begin{pmatrix} p_x \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Therefore we have $Q_z = P_z$ and $Q_z = \frac{b}{q_x - p_x}$

6. State the necessary and sufficient condition for a matrix to be essential (decomposable into the product of an antisymmetric matrix and an orthogonal matrix).

★ **SOLUTION:** It's singular values are $\sigma_1 = \sigma_2 > 0$ and $\sigma_3 = 0$

7. Suppose we know that the camera always moves (rotation and translation) in a plane parallel to the image plane.

$$R = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T = \begin{pmatrix} t\cos\alpha & t\sin\alpha & 0 \end{pmatrix}^T, \tag{7}$$

with t the magnitude of the translation. Show that the essential matrix $E = \widehat{T}R$ is of special form

$$E = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{pmatrix}$$

Without using the SVD-based decomposition, find a solution to α and β in terms of (a, b, c, d).

★ **SOLUTION:** we can caculate $E = \hat{T}R$,

$$\hat{T}R = \begin{pmatrix} 0 & 0 & tcos(\alpha) \\ 0 & 0 & -tsin(\alpha) \\ -tcos(\alpha+\beta) & tsin(\alpha+\beta) & 0 \end{pmatrix} \tag{8}$$

t cannot be solved since the magnitude cannot be recovered. $\alpha = \text{atan2}(-b, a)$ and $\alpha + \beta = \text{atan2}(d, -c)$

8. (10pts) A camera translates only along the optical axis and rotates only around the optical axis. Write the essential matrix between two views. What are the necessary and sufficient condition (in terms of E_{ij}) for a 3x3 matrix to be an essential corresponding to this case (translation in z and rotation around Z-axis).

★ **SOLUTION:** We have $T = [0, 0, z]$ and $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$. So $\hat{T} = \begin{bmatrix} 0 & -z & 0 \\ z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and

$$E = \hat{T}R = \begin{bmatrix} -z\sin(\theta) & -z\cos(\theta) & 0 \\ z\cos(\theta) & -z\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In order for E to be an essential matrix it needs to have 1 zero singular value and 2 non-zero equal singular values. We first compute the eigenvalues of $E^T E$.Where

$$E^T E = \begin{bmatrix} -z\sin(\theta) & z\cos(\theta) & 0 \\ -z\cos(\theta) & -z\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -z\sin(\theta) & -z\cos(\theta) & 0 \\ z\cos(\theta) & -z\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} z^2 & 0 & 0 \\ 0 & z^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So $E^T E$ has eigenvalues $z^2, z^2, 0$. As a results the singular values of E are $|z|, |z|, 0$ and in order for E to be an essential matrix we need $z \neq 0$ (because we need 2 non-zero equal singular values)

9. If $E = \hat{T}R$ and

$$E = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

find T (up to a scale) and R.

★ **SOLUTION:** It is easy to see that for $T = [0, 1, 1]^T$ we get $\hat{T} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. So

$$E = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \hat{T}I$$

where I is the identity matrix. The fact the we can decompose E as shown above shows that $T = [0, 1, 1]^T$ and $R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Due to the twisted pair and mirror ambiguity we can also have

$$T = [0, 1, 1], \quad R = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T = [0, -1, -1], \quad R = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T = [0, -1, -1], \quad R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

10. Explain why a pure translational optical flow field is radial.

★ **SOLUTION:** When we have a pure translation the equation for the optical flow becomes

$$\dot{p} = \frac{V_z}{z}\begin{bmatrix} x - \frac{V_x}{V_z} \\ y - \frac{V_y}{V_z} \end{bmatrix}$$

This shows that at each point (x, y) the direction of the optical flow is parallel to the line that connects the point (x, y) and the point $(\frac{V_x}{V_z}, \frac{V_y}{V_z})$. So it is a radial field with center $(\frac{V_x}{V_z}, \frac{V_y}{V_z})$ which is the Focus of Expansion

11. Explain how we find the Time to Collision.

★ **SOLUTION:** Page 14 Slide 3D Optical Flow 4/21

12. An IMU gives us an estimate of the angular velocity in the camera coordinate system. Explain how we can compute the translation direction.

★ **SOLUTION:** The equation that connect the velocity with the optical flow is

$$\dot{p} = \frac{1}{Z}\begin{bmatrix} xV_z - V_x \\ yV_z - V_y \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\Omega$$

If we know Ω from an IMU we can compute $\dot{p}_{\text{trans}}$ as

$$\dot{p}_{\text{trans}} = \dot{p} - \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\Omega$$

Then using $\dot{p}_{\text{trans}}$ we can compute the translation direction following the method from Pages 18-20 in Slides 3D Optical Flow (4/21)

Image Processing Review Questions

1. **Convolution of two Gaussians** The 1D Gaussian function reads

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}. \tag{9}$$

We give the Fourier transform of a Gaussian as

$$g(t) \circ\!\!-\!\!\bullet\; e^{-\frac{\sigma^2\omega^2}{2}} \tag{10}$$

Prove that the convolution of two Gaussians with σ_1 and σ_2 respectively is a Gaussian. Find its σ in terms of σ_1 and σ_2.

★ **SOLUTION:** In this question, using Fourier is simpler than directly computing the convolution.

Simple solution using Fourier

$$g_{\sigma_1}(t) \circ\!\!-\!\!\bullet\ e^{-\omega^2\sigma_1^2/2}, \qquad g_{\sigma_2}(t) \circ\!\!-\!\!\bullet\ e^{-\omega^2\sigma_2^2/2}.$$

Therefore:

$$(g_{\sigma_1} \star g_{\sigma_2})(t) \circ\!\!-\!\!\bullet\ e^{-\omega^2\sigma_1^2/2}e^{-\omega^2\sigma_3^2/2} = e^{-\omega^2(\sigma_1^2+\sigma_2^2)/2},$$

and we recognize the Fourier Transform of a Gaussian of variance $\sqrt{\sigma_1^2 + \sigma_2^2}$.

More complex solution by computing the convolution We first derive the Gauss by representing I in terms of 2 variables:

$$I^2 = \left(\int_\infty^\infty e^{(-\frac{t^2}{2\sigma^2})}dt\right)\left(\int_\infty^\infty e^{(-\frac{s^2}{2\sigma^2})}ds\right) = \int_\infty^\infty \int_\infty^\infty e^{(-\frac{t^2+s^2}{2\sigma^2})}dtds$$

The point (s,t) in Cartesian coordinates can be transformed to polar coordinates by $s = r\cos(\varphi)$ and $t = r\sin(\varphi)$. This coordinate transformation is reflected in the above integration as follows:

$$\begin{aligned}
\int_\infty^\infty \int_\infty^\infty e^{(-\frac{t^2+s^2}{2\sigma^2})}dtds &= \int_0^\infty \int_0^{2\pi} e^{(-\frac{r^2}{2\sigma^2})}rdrd\varphi \\
&= -2\pi\sigma^2 e^{-\frac{r^2}{2\sigma^2}}|_0^\infty \\
&= 2\pi\sigma^2
\end{aligned}$$

Hence $I = \sigma\sqrt{2\pi}$.

$$\begin{aligned}
(g_1 \star g_2)(t) &= \int_{-\infty}^\infty g_1(t-s)g_2(s)ds \\
&= \frac{1}{2\pi\sigma_1\sigma_2}\int_{-\infty}^\infty \exp - \left(\frac{(t-s)^2}{2\sigma_1^2} + \frac{t^2}{2\sigma_2^2}\right)ds \\
&= \frac{1}{2\pi\sigma_1\sigma_2}\int_{-\infty}^\infty \exp - \left(\left(\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{\sigma_1\sigma_2\sqrt{2}}s - \frac{\sigma_2}{\sigma_1\sqrt{2}\sqrt{\sigma_1^2+\sigma_2^2}}\right)^2 + \frac{t^2}{2(\sigma_1^2+\sigma_2^2)}\right)ds = \\
&= \frac{1}{2\pi\sigma_1\sigma_2}\exp\left(-\frac{t^2}{2(\sigma_1^2+\sigma_2^2)}\right)\int_{-\infty}^\infty \exp - \left(\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{\sigma_1\sigma_2\sqrt{2}}s - \frac{\sigma_2 t}{\sigma_1\sqrt{2}\sqrt{\sigma_1^2+\sigma_2^2}}\right)^2 ds \\
&= \frac{1}{(\sigma_1^2+\sigma_2^2)\sqrt{2}}\exp\left(-\frac{t^2}{\sigma_1^2+\sigma_2^2}\right)
\end{aligned}$$

In the last step of the above derivation we make use of the Gauss integral. Hence, $(g_1 \star g_2)$ is also a Gaussian with variance $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$.

2. Compute and plot the convolution of a step edge with the second derivative of a Gaussian.

★ **SOLUTION:**

$$h(t) = \begin{cases} 1, \text{if } t \geq 0 \\ 0, \text{otherwise.} \end{cases} .$$

We want to calculate the convolution of h(t) with the second derivative of a Gaussian, g”(t), where g(t) is the Gaussian.

$$\begin{aligned}
(h \star g'')(x) &= \int_{-\infty}^{\infty} h(t) g''(x-t) dt \\
&= \int_{-\infty}^{0} 0 g''(x-t) dt + \int_{0}^{\infty} 1 g''(x-t) dt \\
u &= x - t \\
du &= -dt \\
&= \int_{x}^{-\infty} g''(u)(-du) \\
&= \int_{-\infty}^{x} g''(u) du \\
&= g'(x)
\end{aligned}$$

3. Compute the convolution of the 1D box function

$$box(t) = \begin{cases} 1/a, \text{if } |t| \leq a/2 \\ 0, \text{otherwise.} \end{cases} .$$

with the 2nd derivative of the Gaussian.

★ **SOLUTION:**

$$box(t) = \frac{1}{a}\left(h\left(t - \frac{a}{2}\right) - h\left(t + \frac{a}{2}\right)\right)$$

We can rewrite the box function in terms of the step function used in the previous problem.

$$\begin{aligned}
(box(x) \star g'') &= \frac{1}{a}\left(h\left(x - \frac{a}{2}\right) - h\left(x + \frac{a}{2}\right)\right) \star g'' \\
&= \frac{1}{a}\left(g'\left(x - \frac{a}{2}\right) - g'\left(x + \frac{a}{2}\right)\right)
\end{aligned}$$

4. Suppose that you are sampling the 1D first derivative of a Gaussian with standard deviation σ in order to produce a filter of length 2M+1 so there are M taps for positive values and M taps for negative values. As a practitioner you want to have the minimum M possible so that your mask is not unnecessarily long. But this depends on σ. To capture well the support we are asking that the area under the first derivative from -M to 0 captures a fraction α (for example 0.9) of the total area from $-\infty$ to 0.

   (a) Find a formula for the M as a function of σ and α.

   ★ **SOLUTION:**

$$\frac{A}{B} = \alpha$$

$$\begin{aligned} B &= \int_{-\infty}^{0} g'(x)dx = g(0) = \frac{1}{\sigma\sqrt{2\pi}} \\ A &= \int_{-M}^{0} g'(x)dx = g(0) - g(-M) \end{aligned}$$

$$\begin{aligned} \alpha &= \frac{A}{B}\frac{g(0) - g(-M)}{g(0)} \\ &= 1 - \frac{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{M^2}{\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}}} \\ 1 - \alpha &= e^{-\frac{M^2}{\sigma^2}} \\ log_e(1-\alpha) &= -\frac{M^2}{\sigma^2} \\ M^2 &= 2\sigma^2 log_e\left(\frac{1}{1-\alpha}\right) \\ M &= \sigma\sqrt{2log_e\left(\frac{1}{1-\alpha}\right)} \end{aligned}$$

   (b) Assume that you can use only masks with 3 taps ($M = 1$). Compute which σ corresponds to $M = 1$ if we want $\alpha = 0.9$. How many times would we need to convolve with the $M = 1$ mask to achieve the equivalent of a convolution with $\sigma = 2$.

   ★ **SOLUTION:**

$$\begin{aligned} M &= \sigma\sqrt{2log_e\left(\frac{1}{1-\alpha}\right)} \\ 1 &= \sigma\sqrt{2log_e\left(\frac{1}{1-0.9}\right)} \\ \sigma &= 0.46599 \end{aligned}$$

In order to obtain an equivalent convolution with $\sigma = 2$,

$$\begin{aligned} \sqrt{n\sigma_1^2} &= \sigma_2 \\ n &= \frac{\sigma_2^2}{\sigma_1^2} \\ n &= \frac{2^2}{0.46599^2} = 18.42 \end{aligned}$$

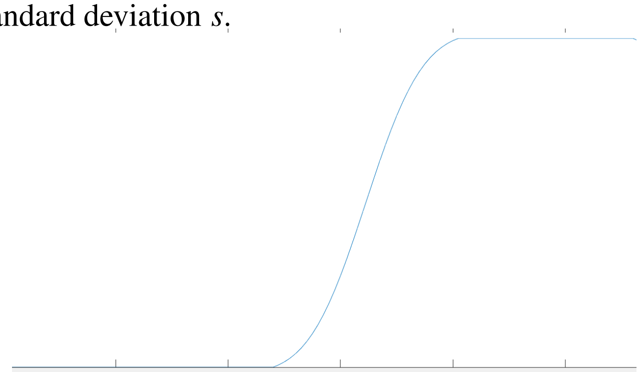
5. Compute the convolution of a step edge

$$h(t) = \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{if } t < 0 \end{cases}.$$

with the second derivative of a Gaussian with standard deviation σ. All computations should be done in the continuous domain. Plot the result.

★ **SOLUTION:** See question 2.

6. Assume that an image was blurred and that the step appears as already convolved with a Gaussian with standard deviation s.



a. Show that its convolution with the first derivative of a Gaussian with standard deviation σ is a Gaussian. Find the standard deviation of that Gaussian.

★ **SOLUTION:**

$$
\begin{aligned}
(h \star g'_\sigma)(x) &= \int_{-\infty}^{\infty} h(t) g'_\sigma(x-t)dt \\
&= \int_{-\infty}^{0} 0 g'_\sigma(x-t)dt + \int_{0}^{\infty} 1 g'_\sigma(x-t)dt \\
u &= x-t \\
du &= -dt \\
&= \int_{x}^{-\infty} g'_\sigma(u)(-du) \\
&= \int_{-\infty}^{x} g'_\sigma(u)du \\
&= g_\sigma(x)
\end{aligned}
$$

b. Compute the convolution of the blurred step edge with the second derivative. The response at $t = 0$ is a function of σ, let us denote it as $f(\sigma)$. Does $f(\sigma)$ decrease or increase with the the blurriness s of the input edge?

★ **SOLUTION:**

$$
\begin{aligned}
(h(x) \star g_\sigma(x)) \star g'_\sigma(x) \\
h(x) \star (g_\sigma(x) \star g'_\sigma(x)) \\
h(x) \star \frac{d}{dx}(g_\sigma(x) \star g_\sigma(x)) \\
h(x) \star \frac{d}{dx}(g_{\sqrt{\sigma^2+\sigma^2}}(x)) \\
h(x) \star g'_{\sqrt{\sigma^2+\sigma^2}}(x) \\
g_{\sqrt{\sigma^2+\sigma^2}}(x)
\end{aligned}
$$

c. Compute the derivative $\frac{df(\sigma)}{d\sigma}$. For which σ does this derivative vanish?

d. The normalized 1st derivative of Gaussian is the original 1st derivative multiplied by σ. If we replace the old 1st derivative with the normalized one the result will be also multiplied by σ so it will be $\sigma f(\sigma)$. Show that $\sigma f(\sigma)$ will have a maximum at $\sigma = s$, i.e., we can recover the width of the box by finding the maximum over σ's.

7. Given the signal $s(t) = cos(\omega_0 t+\phi)$ and the Gabor Filter $g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}(\cos(\omega_0 t) + j\sin(\omega_0 t))$: Compute the convolution $(s * g)(t)$. Show that the magnitude of the convolution $|(s * g)(t)|$ does not depend on ϕ (phase invariant).

★ **SOLUTION:** (To show that we need to assume that $e^{-\omega_0^2} \simeq 0$)
The Fourier transform of the Gabor filter is $G(\omega) = 2\pi * e^{-(\omega-\omega_0)^2/2}$
The Fourier transform for the cosine is $S(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega - \omega_0)]e^{j\omega\phi/\omega_0}$. So

$$S(\omega)G(\omega) = 2\pi^2 e^{j\phi}\delta(\omega - \omega_0) + 2\pi^2 e^{-\omega_0^2}\delta(\omega + \omega_0)e^{-j\phi} \simeq 2\pi^2 e^{j\phi}\delta(\omega - \omega_0)$$

This gives us that

$$(s * g)(t) = \pi e^{j\omega_0 t} e^{j\phi} = e^{j(\omega_0 t+\phi)} = \pi[\cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi)]$$

So

$$|(s * g)(t)| = \pi\sqrt{\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi)} = \pi$$

8. Consider a wave $f(x, y, t) = \cos(\omega(x - ut)) + \cos(\omega(y - vt))$ moving with constant optical flow (u, v). Verify that the brightness change constraint equation

$$\frac{\partial f}{\partial x}u + \frac{\partial f}{\partial y}v + \frac{\partial f}{\partial t} = 0$$

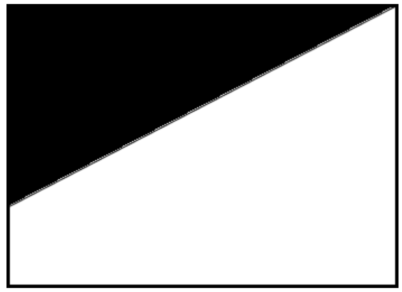
holds for $f(x, t)$.

★ **SOLUTION:**

$$\begin{aligned}
\frac{\partial f}{\partial x} &= -\omega\sin(\omega(x - ut)) \\
\frac{\partial f}{\partial y} &= -\omega\sin(\omega(y - vt)) \\
\frac{\partial f}{\partial t} &= -\omega u[-\sin(\omega(x - ut))] - \omega v[-\sin(\omega(y - vt))] = -\frac{\partial f}{\partial x}u - \frac{\partial f}{\partial y}v \\
\implies \frac{\partial f}{\partial x}u &+ \frac{\partial f}{\partial y}v + \frac{\partial f}{\partial t} = 0
\end{aligned}$$

9. Explain the aperture problem with a sketch as well as an equation.

★ **SOLUTION:** The aperture problem happens when we try to estimate the optical flow of a scene that we are looking with a small receptive field (for example through a whole). If we can only observe a the motion of a one dimensional structure like a line there is ambiguity in the optical flow. An example of such a structure is shown in the picture below



When all gradients in all the points in the image are parallel then $\sum \nabla I \nabla I^T = \sum_{x,y \in N} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right) \end{bmatrix}$ is singular so we can not find a unique solution for the optical flow equation

$$\left(\sum_{x,y \in N} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right) \end{bmatrix}\right) \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum \Delta I \frac{\partial I}{\partial x} \\ \sum \Delta I \frac{\partial I}{\partial y} \end{bmatrix}$$