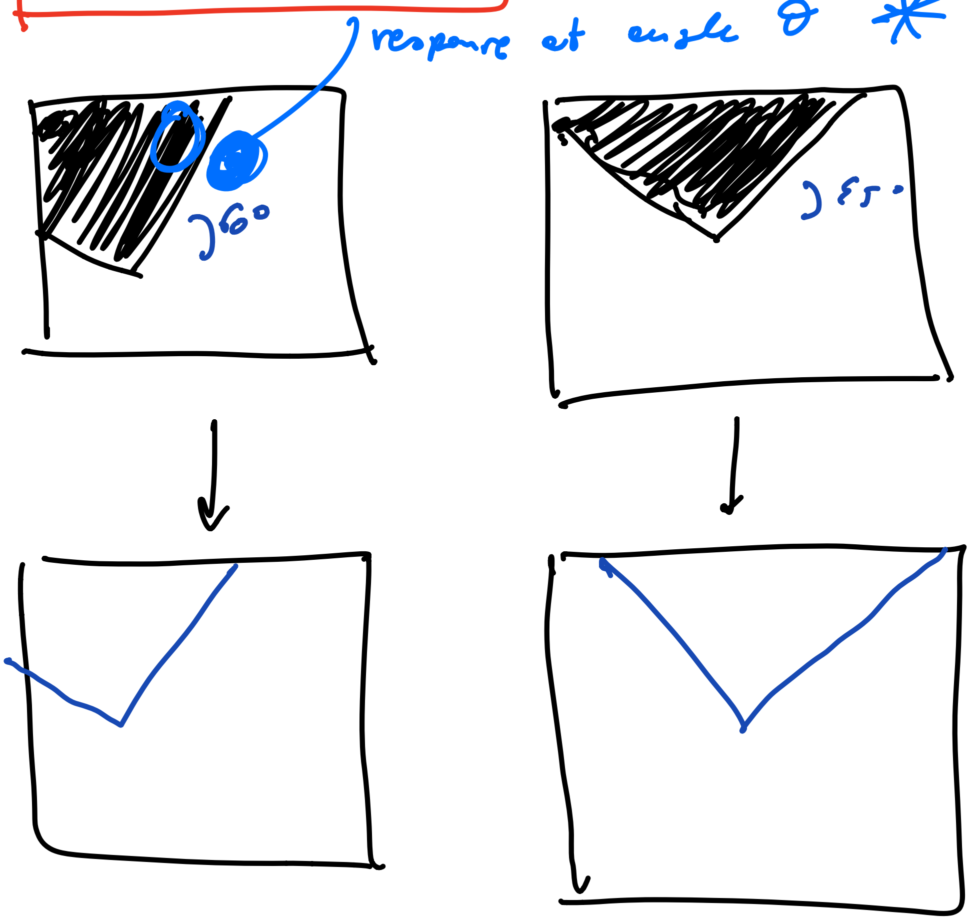
Why do we want edges or keypoints to be rotation and scale invariant?

- detected from different viewpoints (angles and distances)

  - geometry (matching)
  - image retrieval (matching, histogram of features)

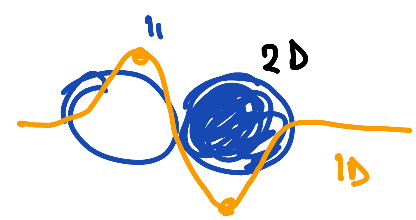
- a keypoint has a descriptor of its neighborhood (e.g. histogram of gradient orientations inside a neighborhood)

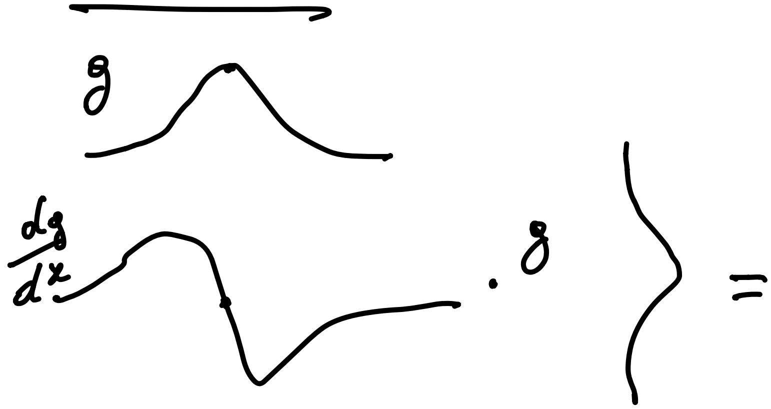
Orientation

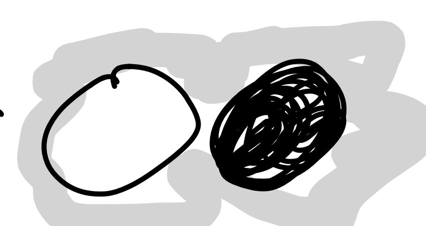


edge detection : $I(x,y) * \frac{dg}{dx}$

recall:







$$I(x,y) * \underbrace{\frac{dg}{dx}\left(\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^T\begin{pmatrix}x\\y\end{pmatrix}\right)}_{\text{|| proved}}$$

has an edge at θ

$$\cos\theta\,\frac{dg}{dx} + \sin\theta\,\frac{dg}{dy}$$

$$\boxed{\cos\theta\left(I * \frac{dg}{dx}\right) + \sin\theta\left(I * \frac{dg}{dy}\right)}$$

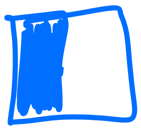
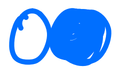
to compute the response at angle θ we do not need to rotate our filter!

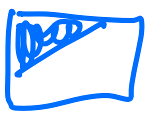
We do not need: $\theta = 1^\circ : 1^\circ : 360^\circ$ compute $I * \frac{dg}{dx}(\theta)$

We need only TWO convolutions

and interpolate for θ $\square$.

Strength : $\sqrt{I_x^2 + I_y^2}$ invariant to θ!

 $*$  $= I * \frac{dg}{dx}$

 $*$ 

$$I_\theta(x,y) * \frac{dg_\theta}{dx} = \left(I * \frac{dg}{dx}\right)_\theta$$

$$\begin{tikzcd}
I \arrow[r, "\theta"] \arrow[d, "\frac{dg}{dx}"'] & I_\theta \arrow[d, "* \frac{dg}{dx}_\theta"] \\
{} \arrow[r, "\theta"'] & I_\theta * g_\theta
\end{tikzcd}$$

What about $\frac{d^2 g}{dx^2}$?

$$g_{xx}(\theta) = \cos^2\theta \frac{\partial^2 g}{\partial x^2} + \sin^2\theta \frac{\partial^2 g}{\partial y^2} + 2\cos\theta\sin\theta \frac{\partial^2 g}{\partial x \partial y}$$

Property : <u>steerability</u>

$\Downarrow$

we can always compute an $\boxed{\text{invariance}}$

$$\frac{\partial^2 g}{\partial x \partial y}(\theta) = \dots\dots$$

$$\theta$$

$$j^2 \omega_x \omega_y G(\omega_x, \omega_y)$$

rotated

$$\underbrace{(\omega_x \cos\theta - \omega_y \sin\theta)(\omega_x \sin\theta + \omega_y \cos\theta)}\, G(\omega_x, \omega_y)$$

$$\cos\theta \sin\theta \underbrace{\omega_x^2 G}_{\frac{\partial^2 g_\theta}{\partial x^2}}$$

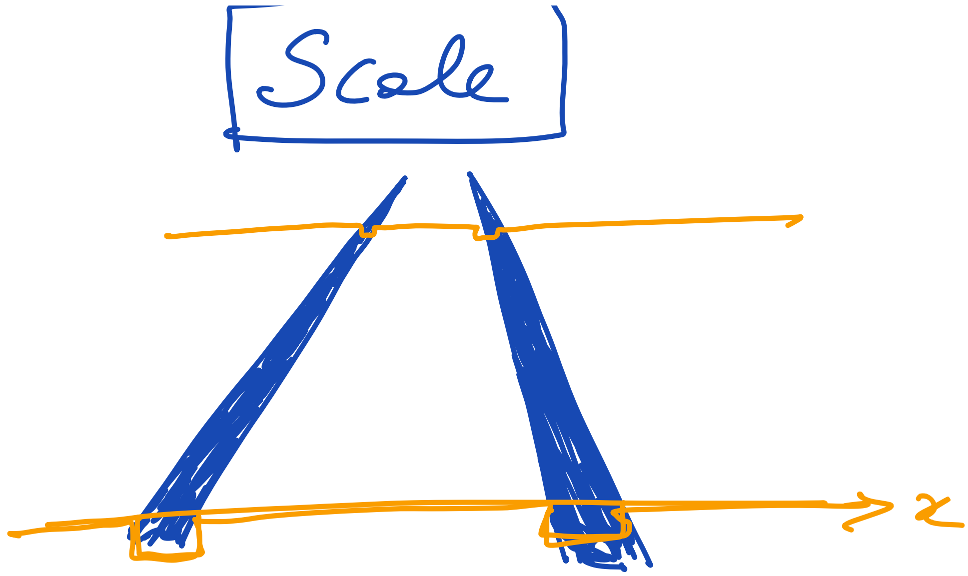
Example of non-steerable function: $\sqrt{x}$, $\cos x$

filter

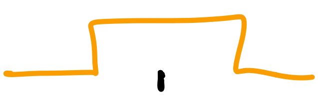
$$\sqrt{x\cos\theta + y\sin\theta} \neq A(\theta) f(x,y) + B(\theta) g(x,y)$$

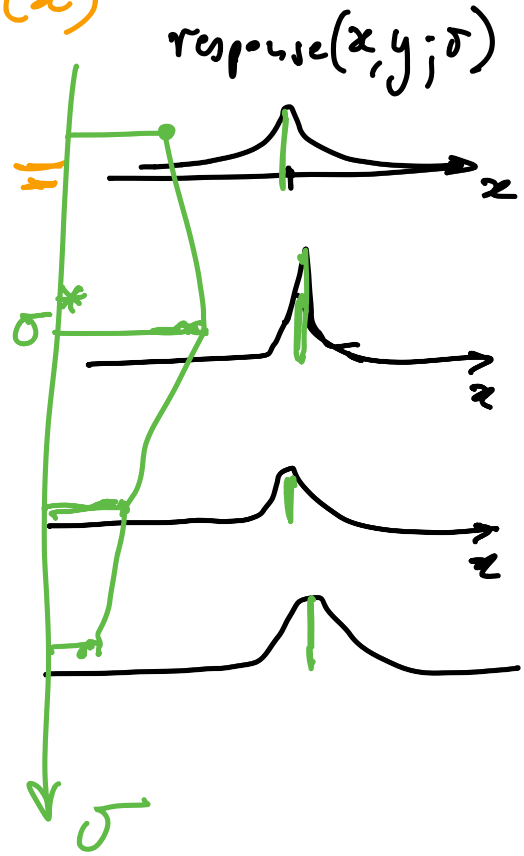
$$\cos(\cos\theta\, x + \sin\theta\, y) \neq \text{this way}$$

Scale



rectangle detection $h(x)$

 $* \; h_{\sigma=1}(x) =$



$h_{\sigma=1.7}$

$h_{\sigma=2}$

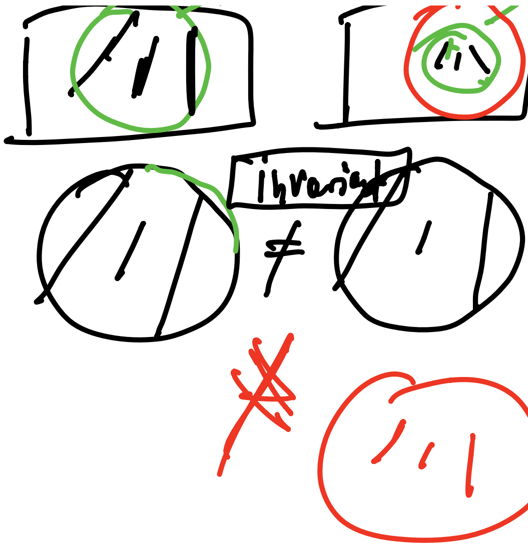
$h_{\sigma=3}$

we want

σ* ~ width of the rectangle

$\downarrow$

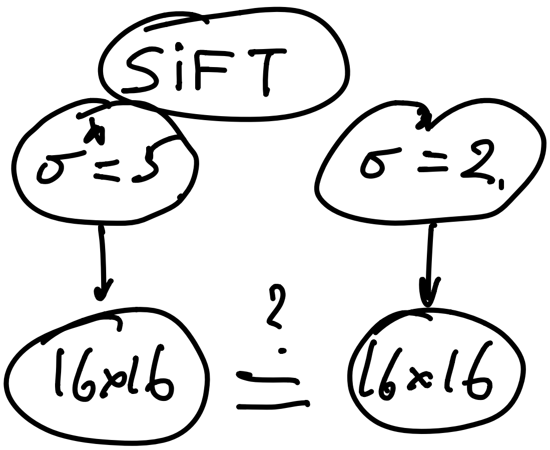
intrinsic scale selection

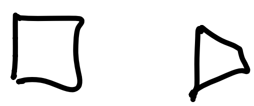


response $(x, y; \sigma)$

$\max_{x,y} \rightarrow$ where is the line?

$\max_{\sigma} \rightarrow$ what is the line's width!





beyond scale and rotation:

"affine" invariant