CIS 580 Spring 2021: Midterm 2

- **Once you begin the exam you will have 120min to finish and submit to Gradescope. All SDS accommodations will still apply accordingly.**
- **You are not allowed to post the exam to anyone/anywhere.**
- **You are not allowed to collaborate with other students.**
- **During the exam we will post clarification in this Google doc. If you have a question please first check the document and if your question is not there please send a PRIVATE question in piazza. We will copy the question and answer it in the doc.**
- **If you use anything verbatim from the Internet you should cite it properly (like URL).**
- **Use your own paper if you want and submit the same way you submit a 580 math homework.**

1. Problem Structure from Motion

Two views are separated by a rotation around the y-axis and a translation in the xz-plane reflecting a motion of the camera in the xz-plane, a situation called in-plane motion. For example a vacuum robot would satisfy this equation if the camera's y axis is perpendicular to the ground.

$$Q = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} P + \begin{pmatrix} T_x \\ 0 \\ T_z \end{pmatrix}$$

1. **[5 Points]** Compute the essential matrix, given the above rotation and translation. We call this matrix the "vacuum cleaner essential" matrix.

2. **[15 Points]**

   - Which elements of the essential matrix are always zero?
   - How can you tell if a matrix E is "vacuum cleaner essential" ?
   - Suppose that you have found a "vacuum cleaner essential" matrix. Show how to extract θ and (T_x, T_z). The latter up to a scale factor.

2. Problem Structure from Motion

1. **[5 Points]** In the structure from motion problem, give an example of a pair of translation and rotation, that result in the two epipoles being at exactly the same pixel position in both images. A sketch would help.

2. **[5 Points]** Assume two cameras with cooordinate systems Q and P and $Q = RP + T$. Give an example of a pair of translation T and rotation R, that causes epipole in image plane q being at infinity and epipole in image plane p being in the center of the image. A sketch would help.

3. **[5 Points]** Suppose that two views have intersecting optical axes. Show that the element E_{33} of the essential matrix has to be zero.

3. Problem 3D Velocities

1. **[10 Points]** Assume that we are moving with a pure translational velocity $(0, 0, V_z)$. Show how can we compute the time to collision to a point in the scene that is projected on the calibrated point (x, y).
2. **[10 Points]** Assume that we drive towards a wall parallel to the image plane and we observe a circle. Explain how we can find the time to collision (assuming constant velocity) from the area of the circle A and the rate of change of the area $\dot{A}$.

4. Problem

1. **[10 Points]** Your system allows you to perform only convolutions with a Gaussian of $\sigma = 1/\sqrt{2}$. How many convolutions would you need to approximate a Gaussian (0th derivative) of $\sigma = 4$. Subsampling is not allowed.

2. **[10 Points]** Assume that you want to approximate the 2nd derivative for $\sigma = 4$. Your system still allows you only to convolve with a Gaussian of $\sigma = 1/\sqrt{2}$. Establish a procedure so that you can compute the equivalent of a convolution with the 2nd derivative of $\sigma = 4$. Subsampling is not allowed.

5. Problem

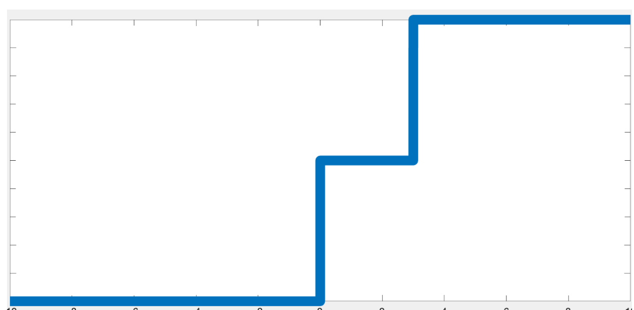
1. **[5 Points]** Compute the convolution of the 1D edge

$$h(x) = \begin{cases} H & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases}$$

with the first derivative of the Gaussian function with standard deviation σ. Plot or draw the edge, the first derivative of the Gaussian and the result of the convolution.

2. **[10 Points]** Assume the double-step function of the figure defined as

$$h(x) = \begin{cases} 2H & \text{if } x > a, \\ H & \text{if } 0 \leq x \leq a\ . \\ 0 & \text{if } x < 0 \end{cases}$$



Compute its convolution with the first derivative of a Gaussian with standard deviation σ and call the response $d(x, \sigma, a)$.

3. **[5 Points]** Show that $d(x, \sigma, a)$ has always an extremum at $x = a/2$.
Show that $d(x, \sigma, a)$ does hot have extrema at $x = 0$ and $x = a$ as one would anticipate.

4. **[5 Points]** Plot $d(x, \sigma, a)$ for $\sigma = 1$ and $a = 5$.
Plot $d(x, \sigma, a)$ for $\sigma = 4$ and $a = 5$.
You can use python, matlab, wolfram alpha, or even a Ti84 and you are allowed to draw the curves by hand.
Explain the curves. What do you observe.