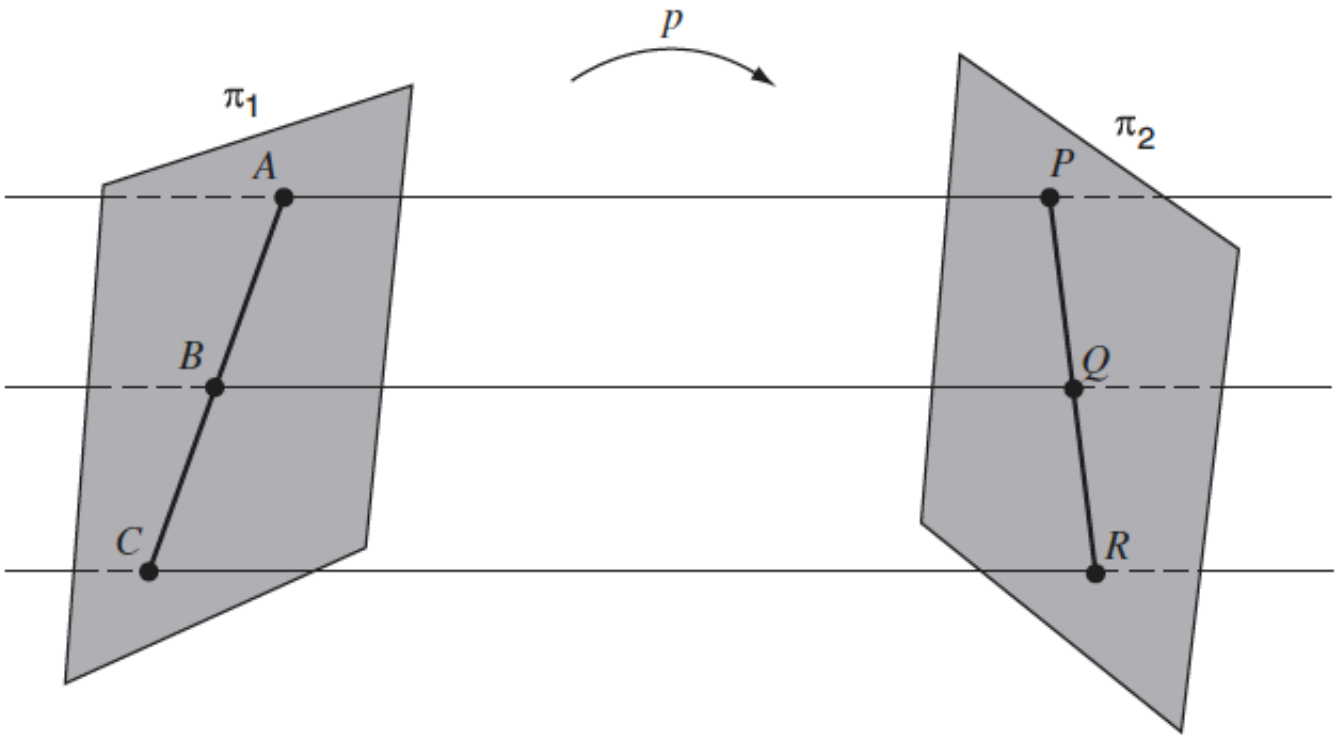
Cross Ratios
and
Single View Metrology!

Perception

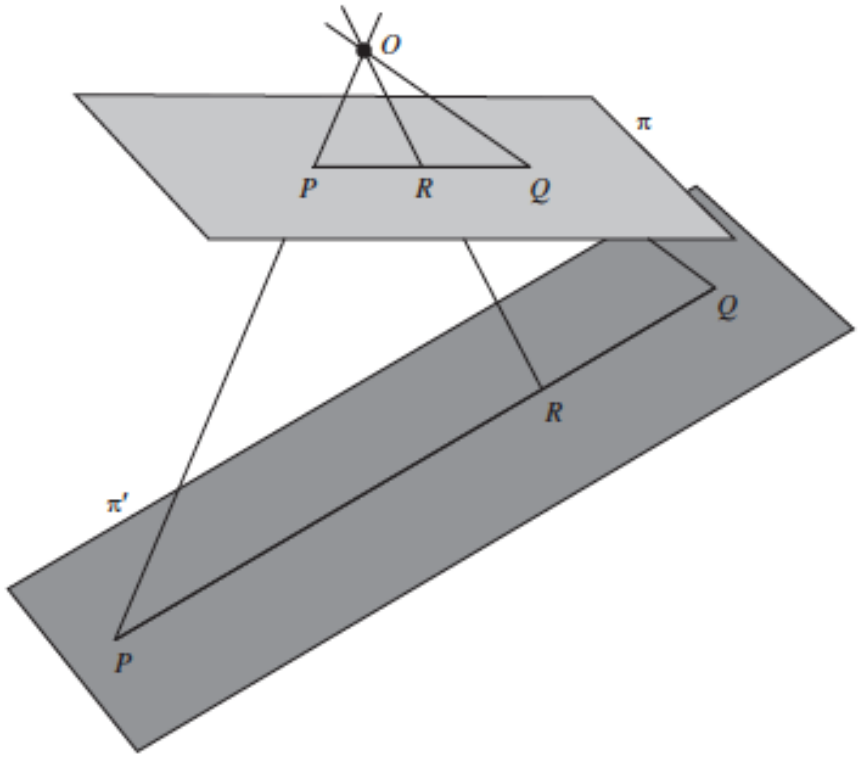
Kostas Daniilidis

Is the middle point of a segment preserved under parallel projection? Yes !



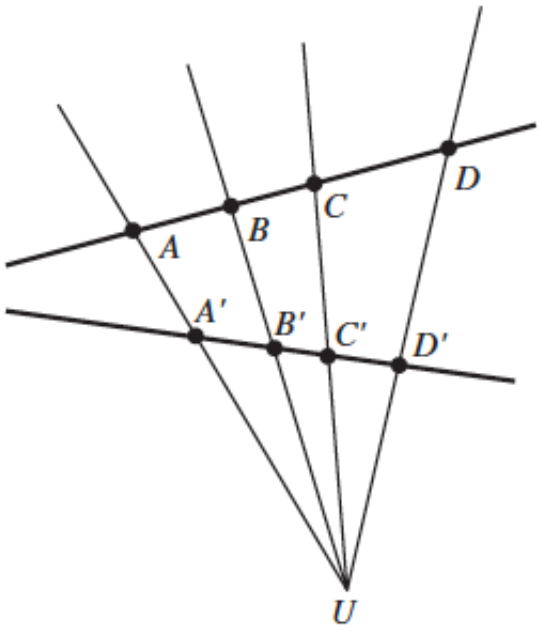
Brannan et al. Geometry

Is the middle point of a segment preserved under perspective projection? No!



Brannan et al. Geometry

What is preserved under a projective transformation? Cross-Ratio!



Given four points A, B, C, D, we define the cross-ratio of their distances as

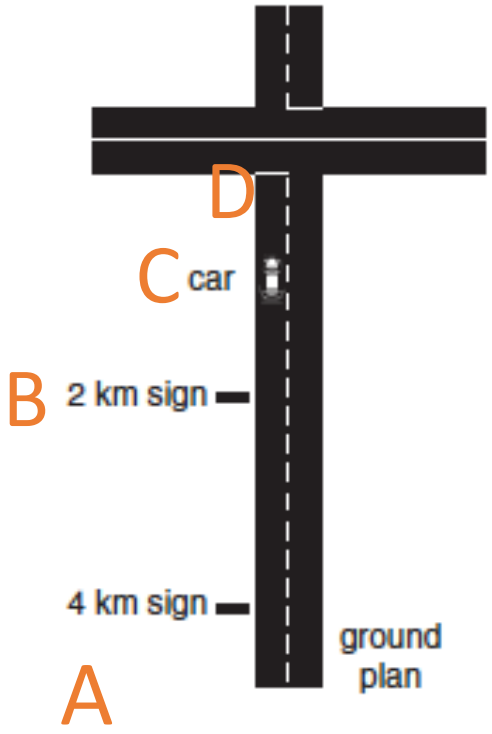
$$CR(A,B,C,D) = \frac{AC}{AD} : \frac{BC}{BD}.$$

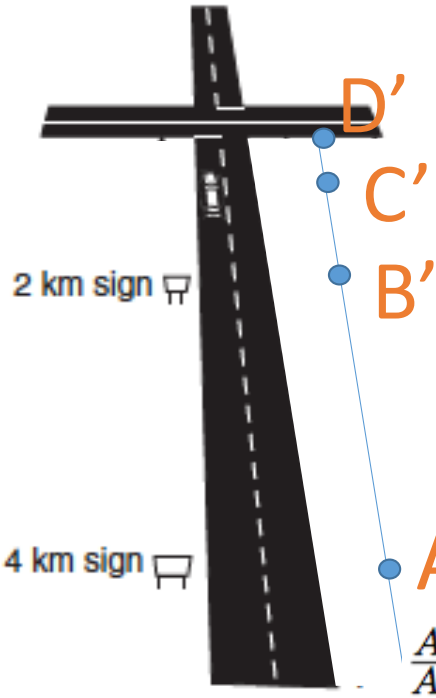
$CR(A,B,C,D)$ remains invariant under projective transformations

$$\frac{AC}{AD} : \frac{BC}{BD} = \frac{A'C'}{A'D'} : \frac{B'C'}{B'D'}$$

Brannan et al. Geometry

How can it be used for metrology?





How far is car from intersection:

CD=?

A'D'=300 pixels

A'C'=275pixels

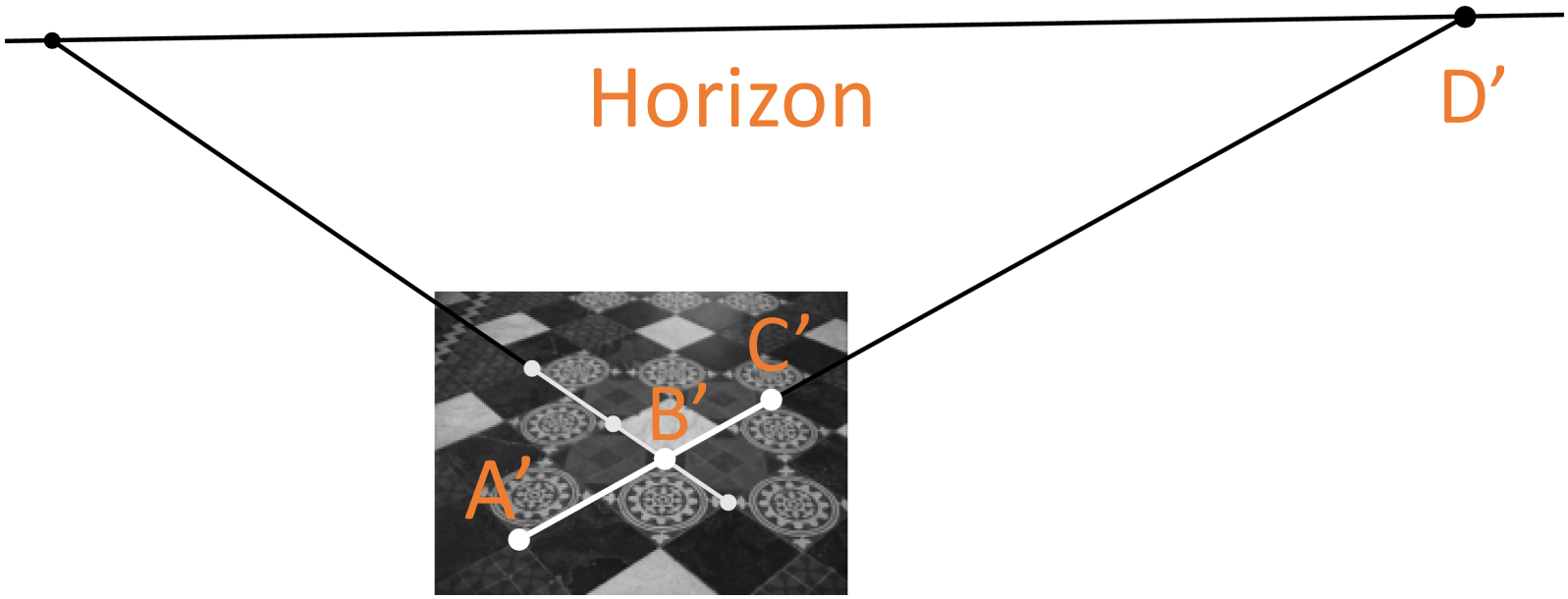
B'C'=50 pixels

$$\frac{A'C'}{A'D'} : \frac{B'C'}{B'D'} = \frac{275}{300} : \frac{50}{75} = 1.375$$

$$\frac{AC}{AD} : \frac{BC}{BD} = \frac{4-CD}{4} : \frac{2-CD}{2} = 1.375$$

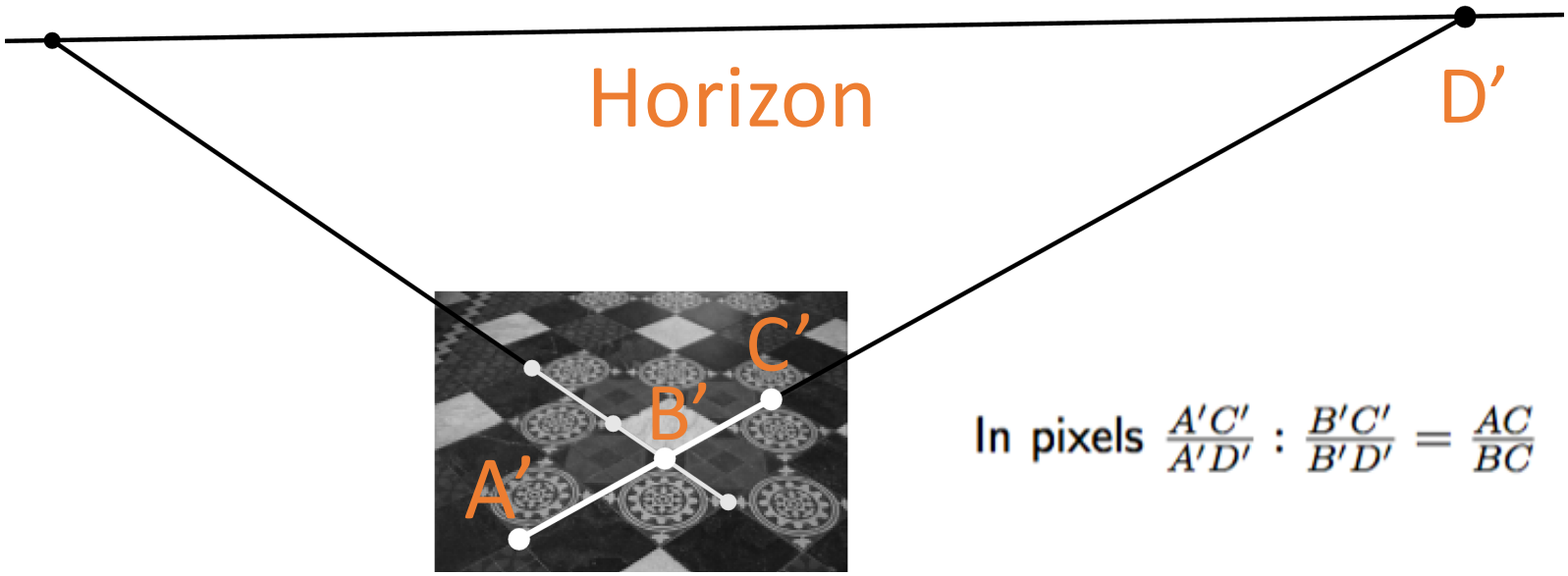
$$CD = 0.857km$$

What happens when one of the points is at infinity?



While D’ is a finite point, D on the original plane is at infinity !

What happens when one of the points is at infinity?

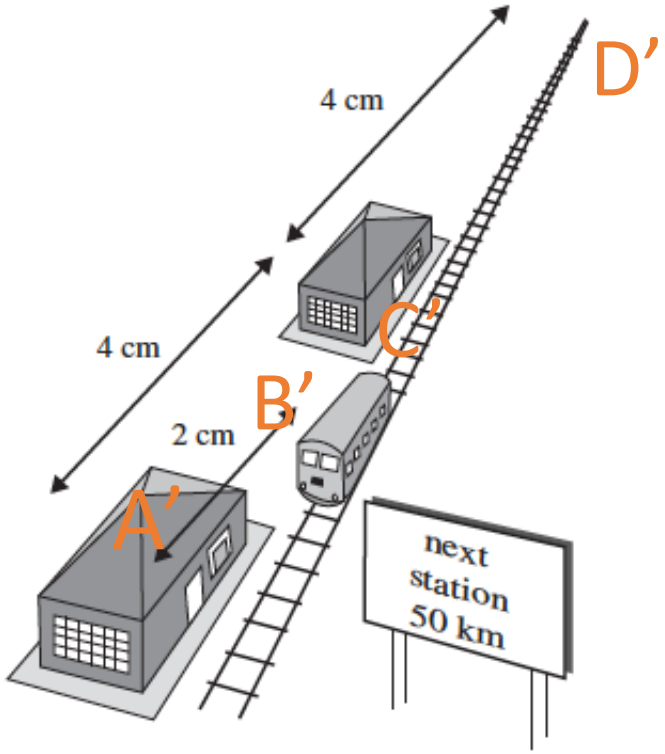


In pixels $\frac{A'C'}{A'D'} : \frac{B'C'}{B'D'} = \frac{AC}{BC}$

When a point D is at infinity, the cross-ratio becomes a ratio !

$$\frac{AC}{AD} : \frac{BC}{BD} = \frac{AC}{BC}$$

Knowledge of a vanishing point allows us to measure ratios in the original plane !



How far away is the train from the next station? Or BC=?

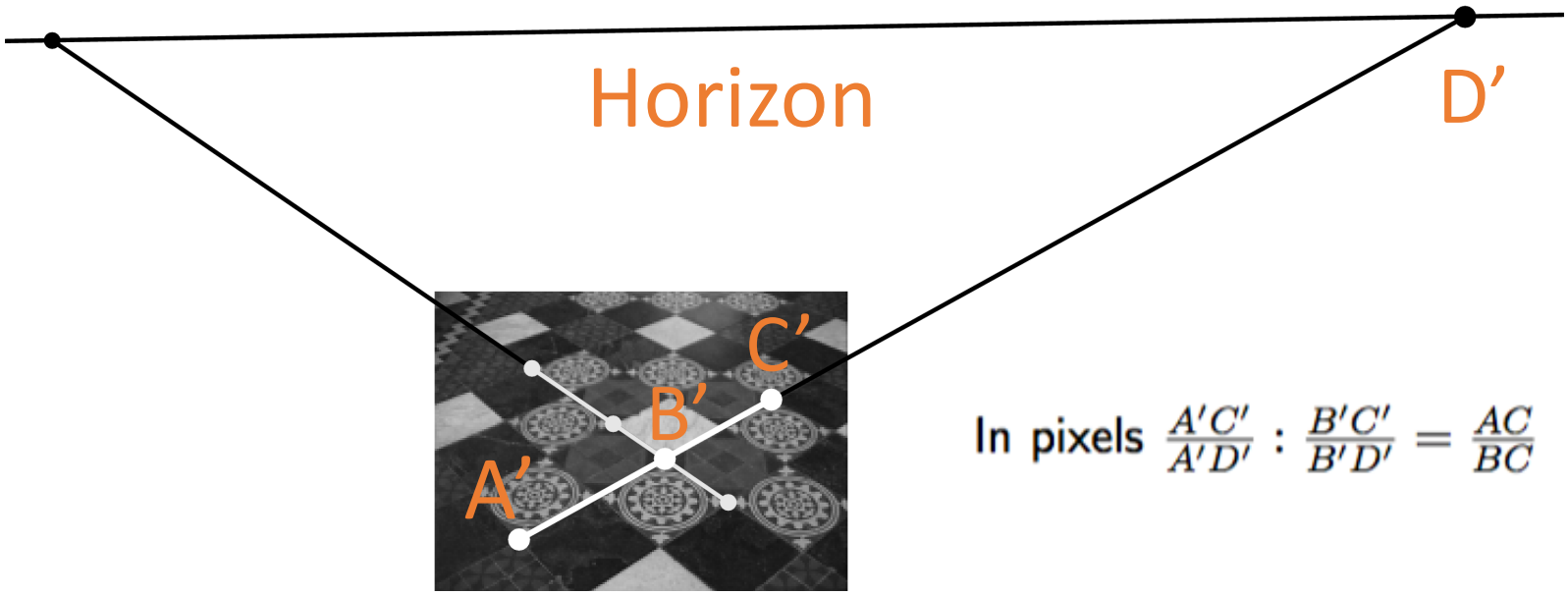
$$\frac{A'C'}{A'D'} : \frac{B'C'}{B'D'} = \frac{AC}{BC}$$

$$\frac{4}{8} : \frac{2}{6} = \frac{50}{BC}$$

$$\frac{4}{8} : \frac{2}{6} = \frac{50}{BC} \text{ and } BC = 33.33 \text{ km}.$$

Brannan et al. Geometry

And vice versa, we can find where is the vanishing point if we know ratios in the ground plane!

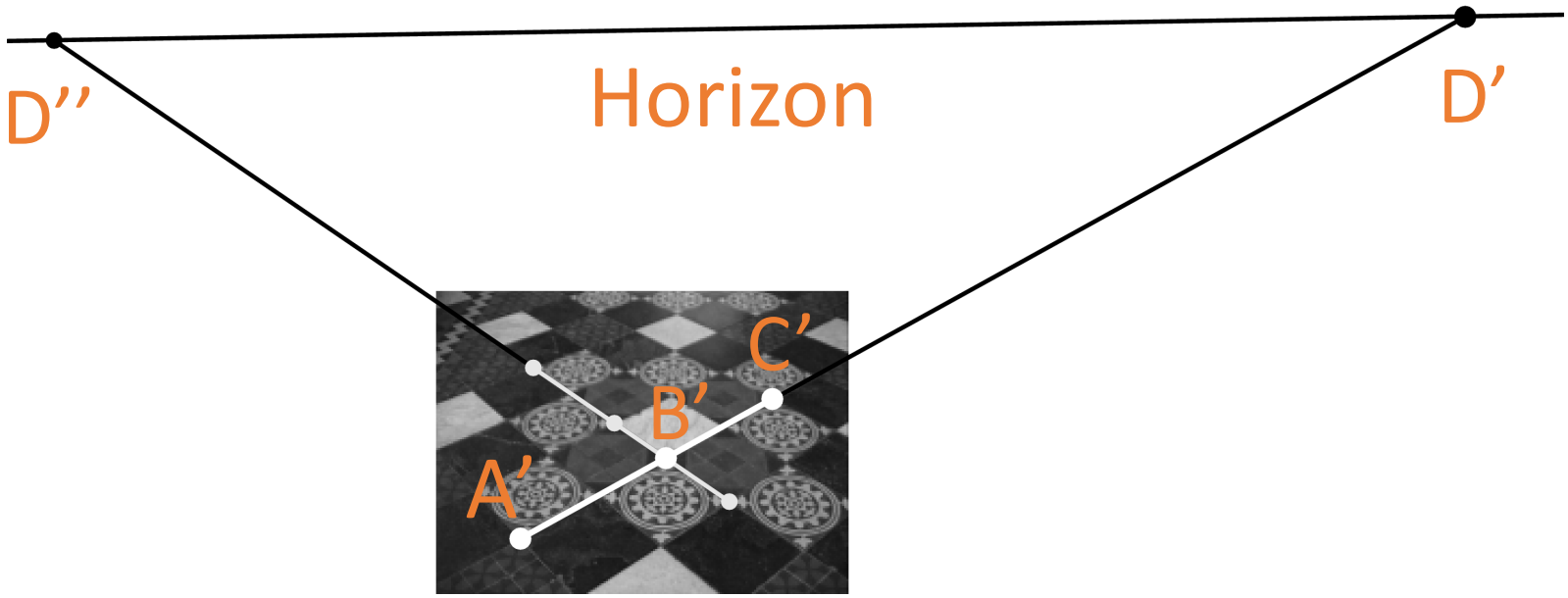


In pixels $\frac{A'C'}{A'D'} : \frac{B'C'}{B'D'} = \frac{AC}{BC}$

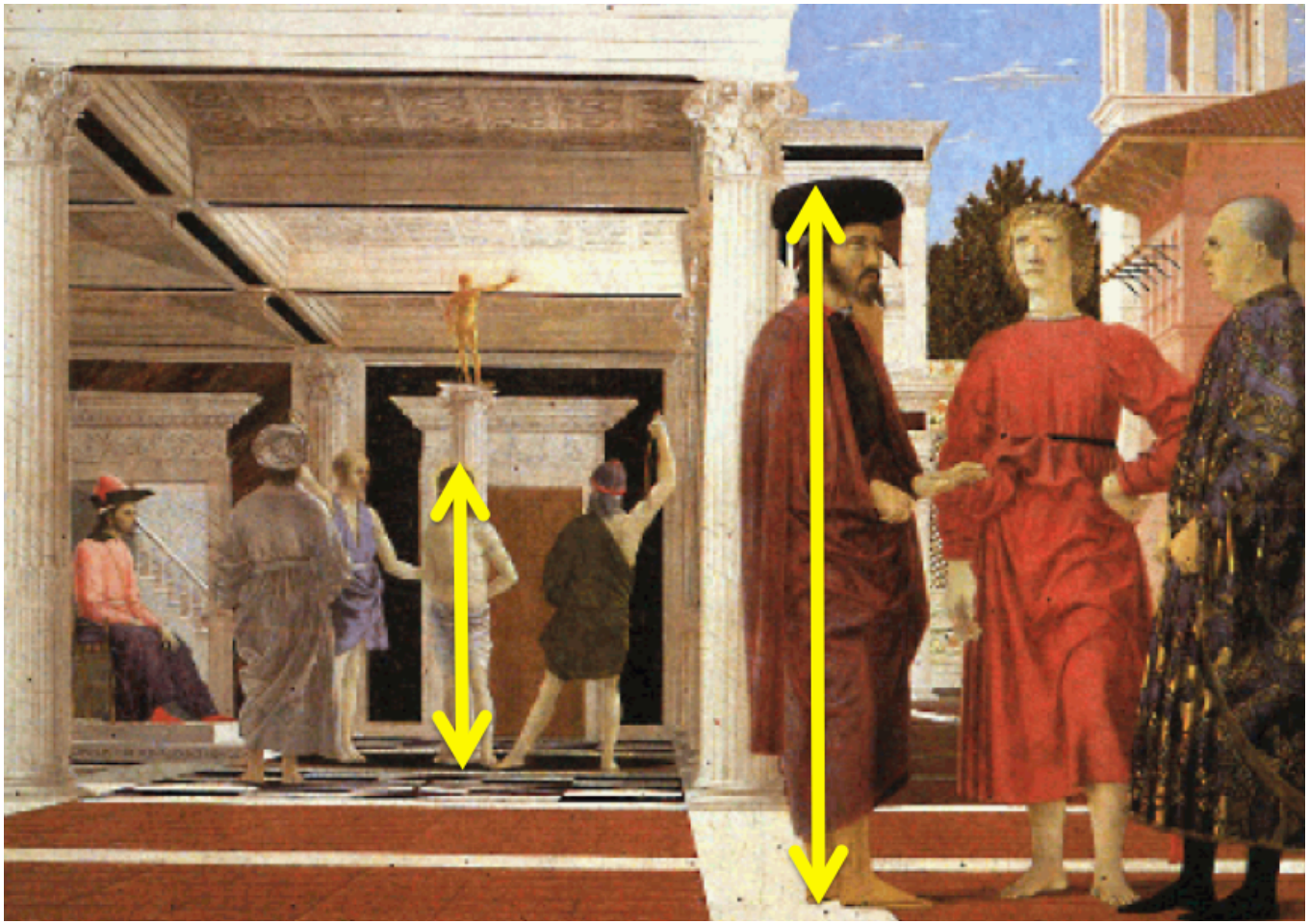
$$\frac{A'C'}{A'D'} : \frac{B'C'}{B'D'} = \frac{AC}{BC} = 2$$

If we know A', B', C; in pixels we can find D'.

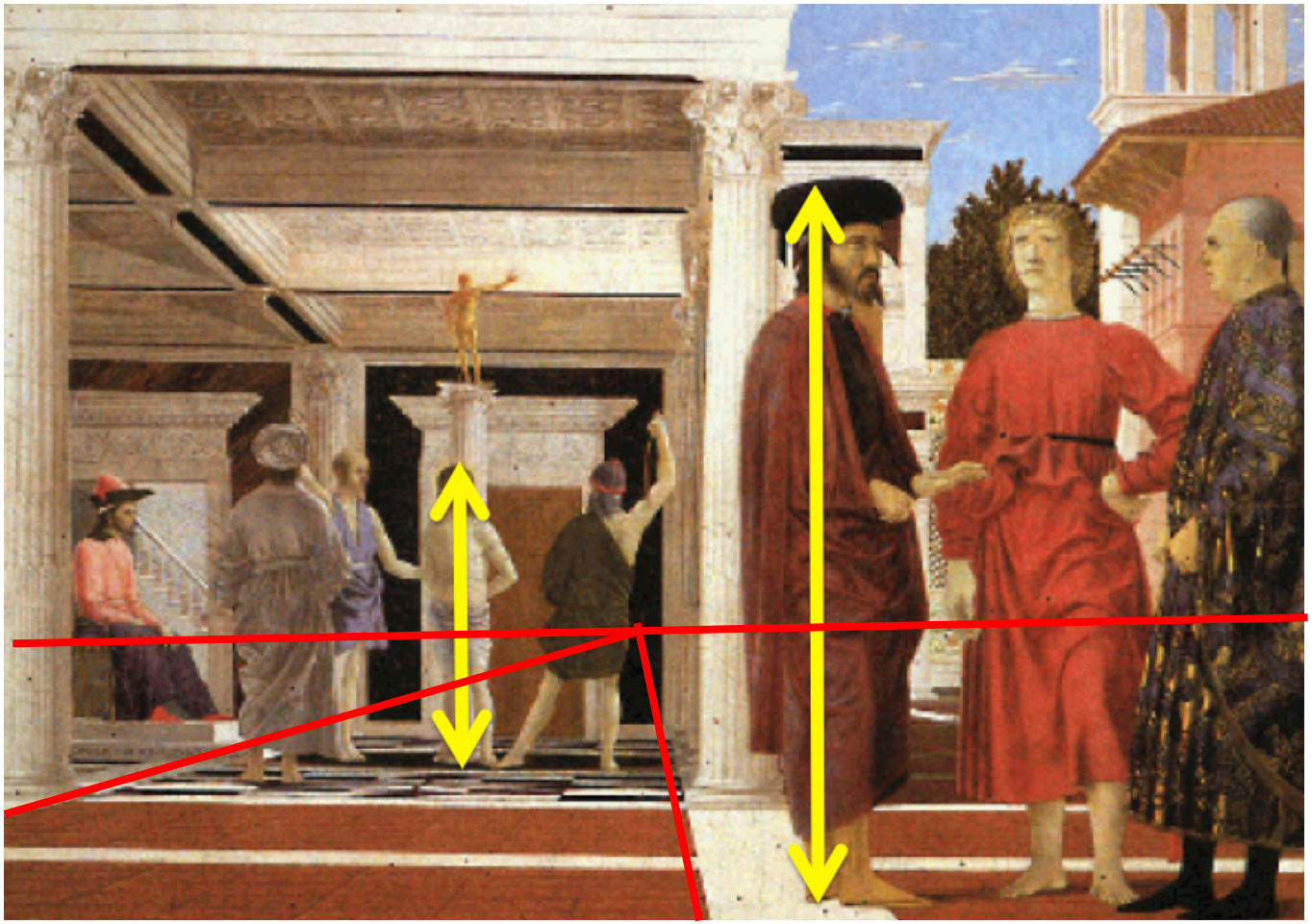
This means that if we have two ratios we can find two vanishing points and the horizon without needing parallel lines!



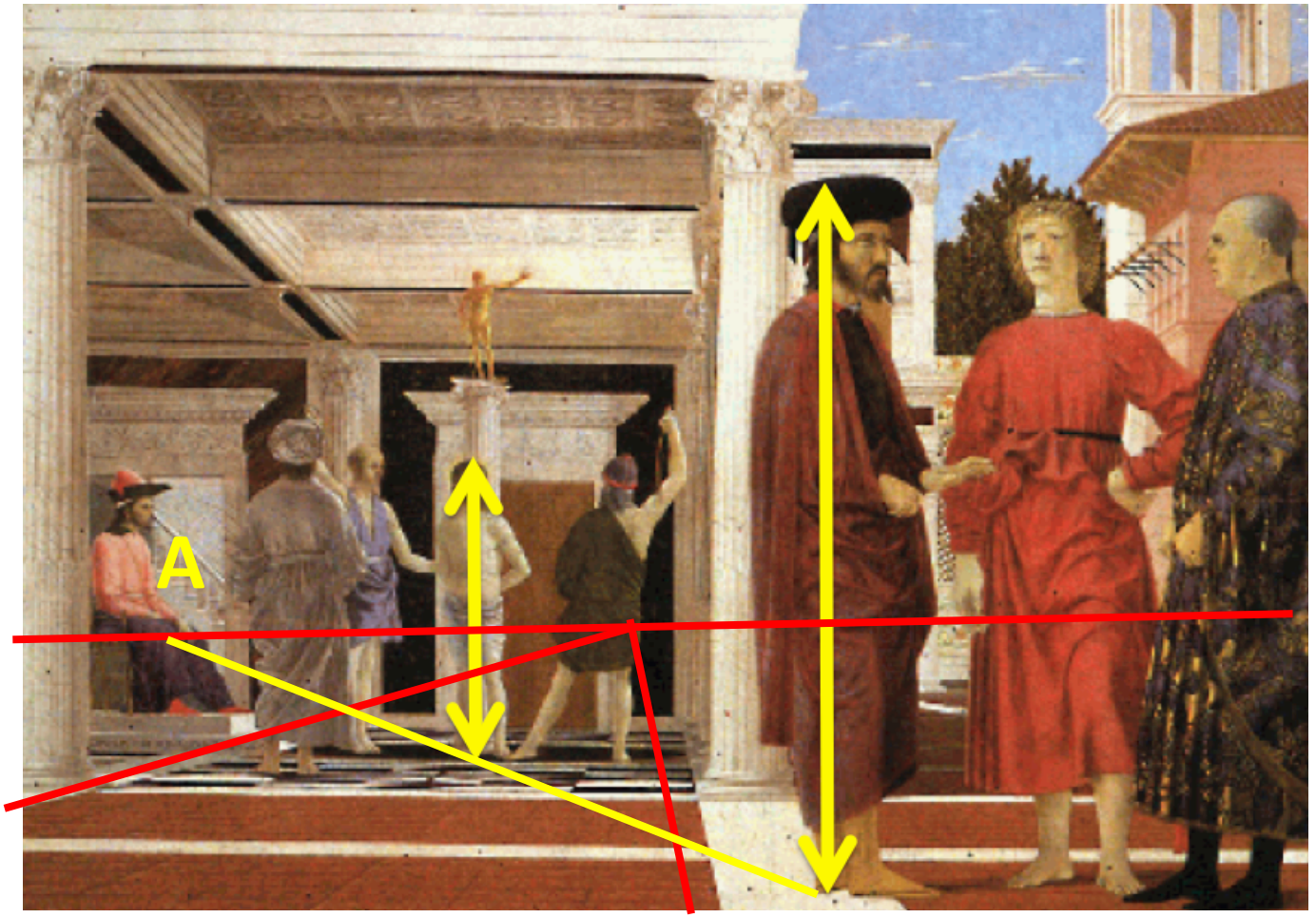
Distance Transfer: How tall is the man if the statue is 180cm ?



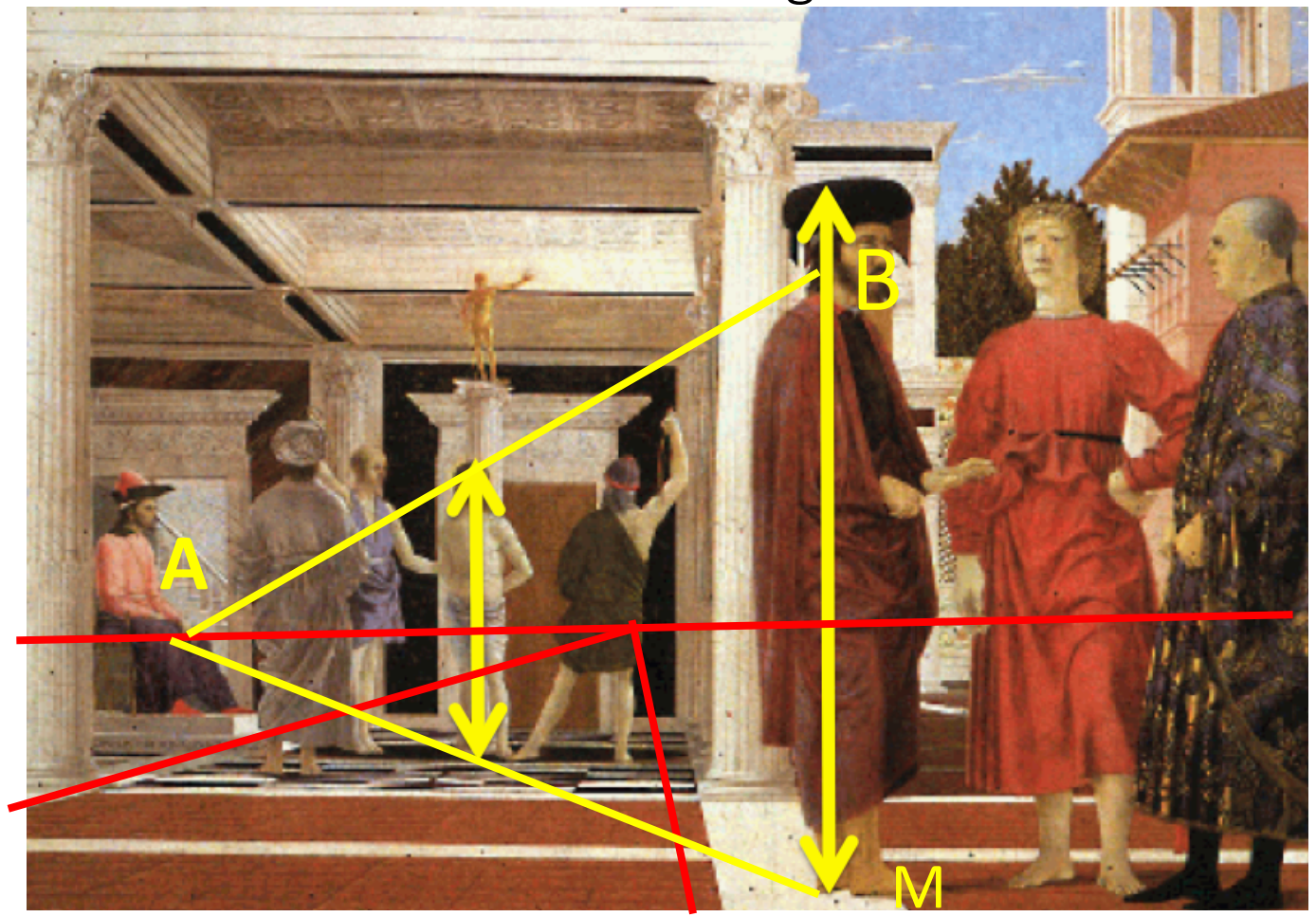
1. Assume that the horizon is horizontal and find a vanishing point!



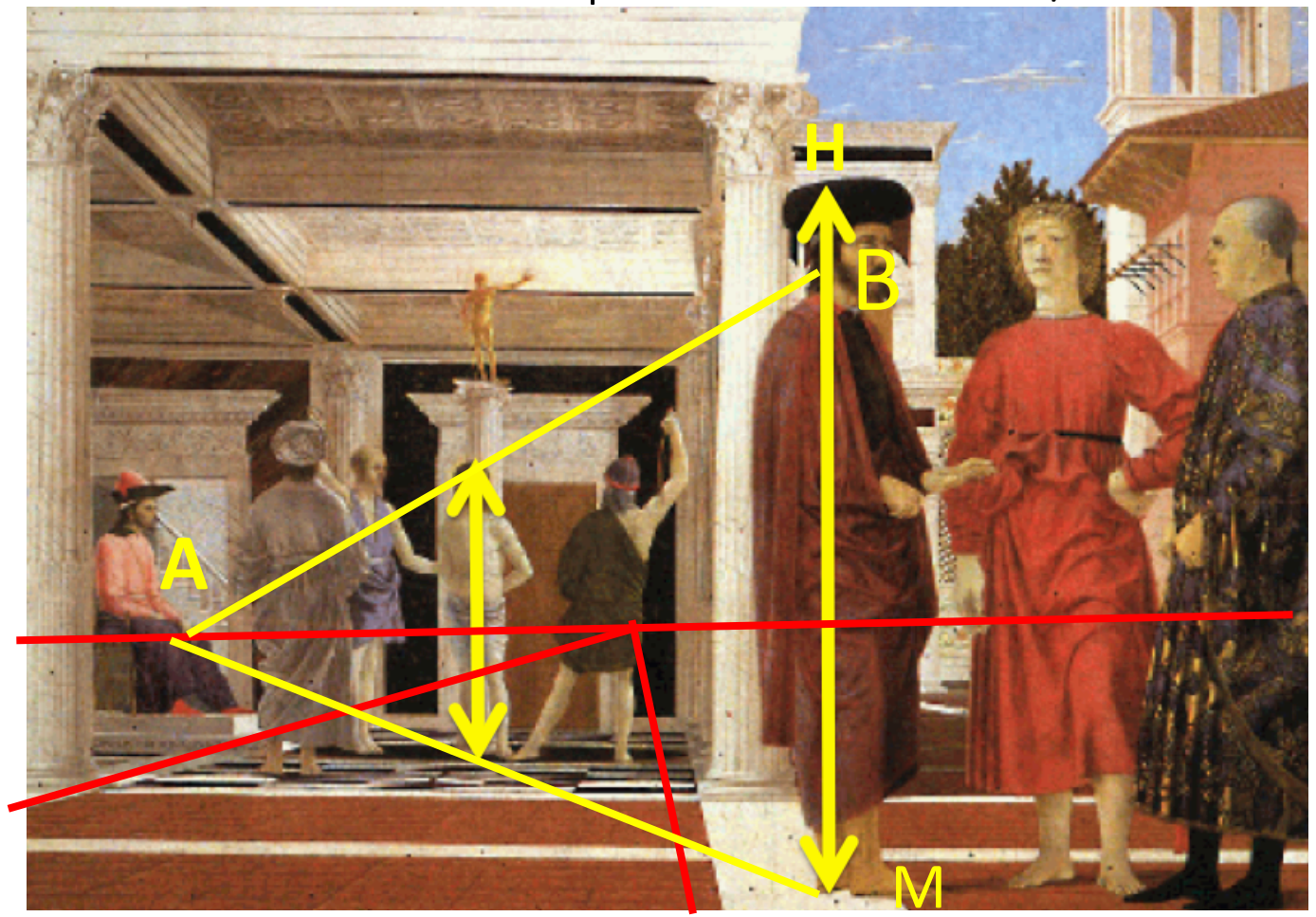
2. Connect the feet of the man and the statue and find intersection A with horizon!



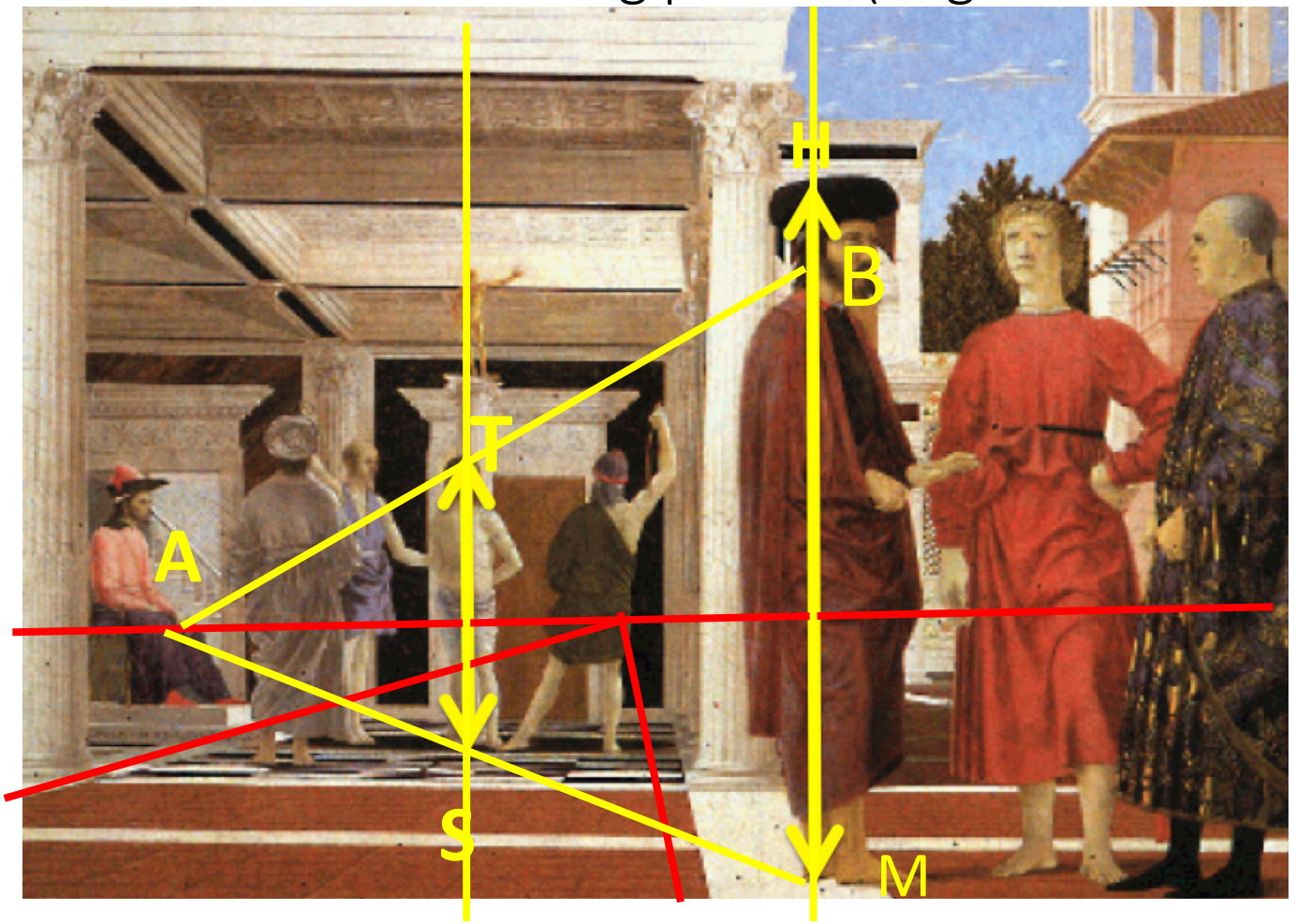
3. Connect A with top of statue. Then AB is parallel to the ground, hence MB is the same height as the statue.



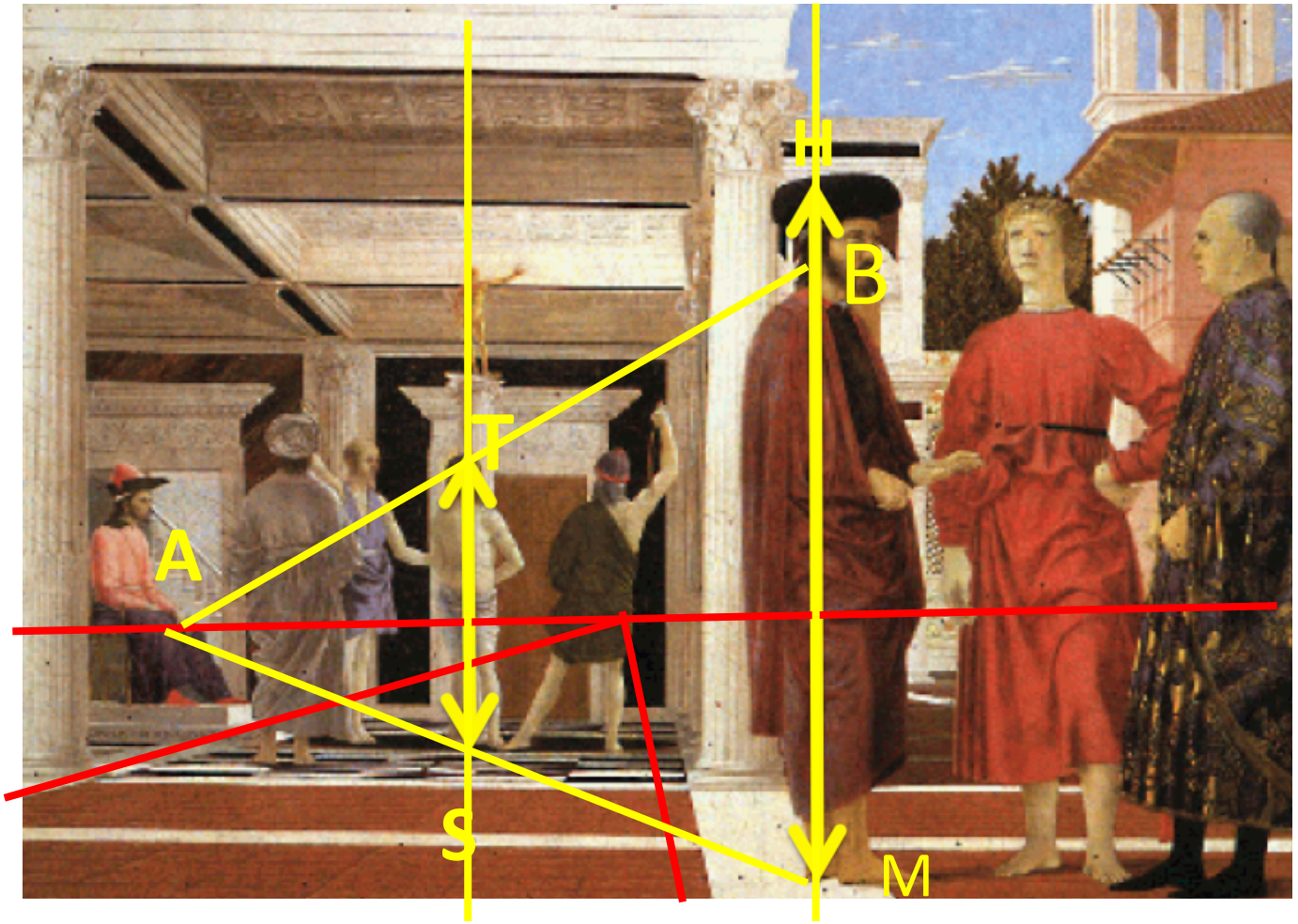
4. But we want MH in the world !
How can we compute the ratio MH/MB ?



4. Only if we know a vanishing point in the vertical direction. Let ST and MB intersect at a vertical vanishing point V (might be at infinity or not) .



5. Then we can compute the crossratio {V,H,B,M} in pixels and hence the ratio of MH/MB in the world since V in the world lies at infinity!



Single View Metrology via Cross Ratios

- If we know a vanishing point we can compute any ratio along this direction!
- We can transfer distances among parallel lines in the world if we know two vanishing points.
- In none of these steps we used focal length or any other intrinsics.
- We can do some image forensics on paintings or old photos!