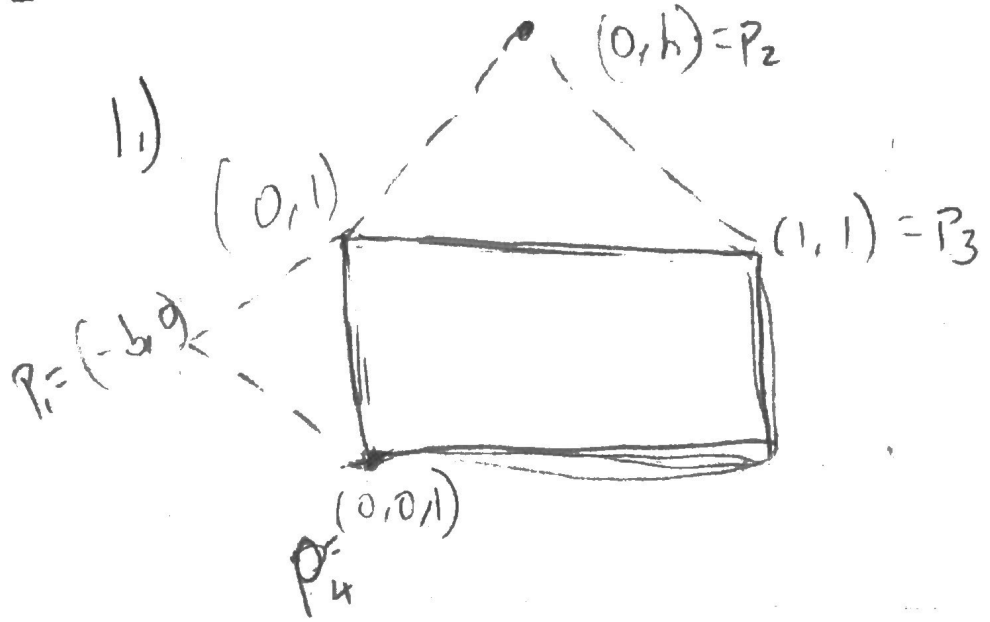
1.)



$$\alpha P_1 = T\begin{pmatrix}1\\0\\0\end{pmatrix} = t_1$$

$$\beta P_2 = T\begin{pmatrix}0\\1\\0\end{pmatrix} = t_2$$

$$\gamma P_3 = T\begin{pmatrix}1\\1\\1\end{pmatrix} = t_1 + t_2 + t_3$$

$$\delta P_4 = T\begin{pmatrix}0\\0\\1\end{pmatrix} = t_3$$

$$\gamma P_3 = \alpha P_1 + \beta P_2 + \delta P_4$$

$$\gamma\begin{pmatrix}1\\1\\1\end{pmatrix} = \alpha\begin{pmatrix}-b\\0\\1\end{pmatrix} + \beta\begin{pmatrix}0\\h\\1\end{pmatrix} + \delta\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$\gamma = 1$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -b & 0 & 0 \\ 0 & h & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \delta \end{pmatrix}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \delta \end{pmatrix} = \begin{pmatrix} -b & 0 & 0 \\ 0 & h & 0 \\ 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1/b & 0 & 0 \\ 0 & 1/h & 0 \\ 1/b & -1/h & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \delta \end{pmatrix} = \begin{pmatrix} -1/b \\ 1/h \\ 1/b - 1/h + 1 \end{pmatrix}$$

$$T = (\alpha P_1 \ \beta P_2 \ \delta P_4)$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{b} & 1/h & 1/b - 1/h + 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{h}{h-b+bh} & \frac{-b}{h-b+bh} & \frac{bh}{h-b+bh} \end{bmatrix}$$

2.1 $X'A' = 4$ $\quad BY = 2$

$A'B' = 4$ $\quad Y'C' = 1$

$$\frac{A'Y'}{A'C'} : \frac{B'Y'}{B'C'} = \frac{6}{7} : \frac{2}{3}$$

$$= 1.286$$

$$\frac{AY}{AC} : \frac{BY}{BC} = \frac{1 - CY}{1} : \frac{0.5 - CY}{0.5} = 1.286$$

$$\frac{0.5(1 - CY)}{0.5 - CY} = 1.286$$

$$(1 - CY) = 1.286(1 - 2CY)$$

$$1.571(CY) = 0.286$$

$$\boxed{CY = 0.1818} \rightarrow \text{Actual distance between bridge and junction}$$

$$CY + YB = 0.5$$

so $YB = 0.5 - 0.1818 = 0.3182$

$$\frac{X'B'}{X'Y'} \cdot \frac{A'B'}{A'Y'} = \frac{8}{10} \cdot \frac{4}{6} = 1.2$$

$$\frac{XB}{XY} \cdot \frac{AB}{AY} = \frac{0.5+XA}{0.3182+0.5+XA} \cdot \frac{0.5}{0.3182+0.5} = 1.2$$

$$\frac{0.8182(0.5+XA)}{0.5(0.8182+XA)} = 1.2$$

$$0.3182(0.5+XA) = 0.6(0.8182+XA)$$

$$0.1591 + 0.3182XA = 0.0955 + 0.6XA$$

$$XA = 0.375$$

So distance between X and C is

1.375 km

3.)

A) $\frac{A_W C_W}{A_W D_W} : \frac{B_W C_W}{B_W D_W}$

$\frac{8}{12} : \frac{4}{8} = \frac{4}{3}$

$\frac{AC}{AD} : \frac{BC}{BD} = \frac{3+BC}{5+BC} : \frac{BC}{2+BC} = \frac{4}{3}$

$\frac{(3+BC)(2+BC)}{BC(5+BC)} = \frac{4}{3}$

$\boxed{BC \approx 2.424}$

B) $\frac{BD}{BV} : \frac{CD}{CV} = \frac{B_W D_W}{C_W D_W} = \frac{8}{4} = 2$

$\frac{BD}{BV} : \frac{CD}{CV} = \frac{4.424}{4.424+DV} : \frac{2}{2+DV} = 2$

$\frac{4.424(2+DV)}{2(4.424+DV)} = 2$

$\boxed{DV \approx 20,87}$