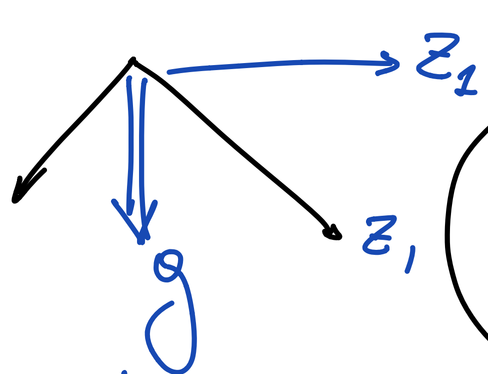
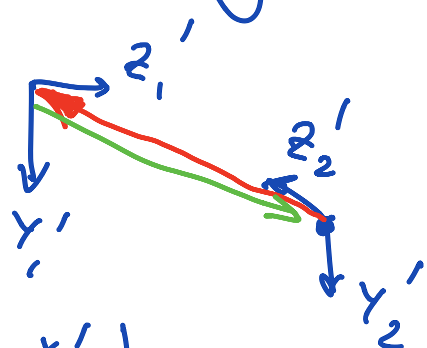
Mar 6 OH

Transformations: Always write the equation with points not just transformations.



$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_1 \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix}$$



test: $g = R_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 1/2 \\ \sqrt{3}/2 \end{pmatrix} = \text{2nd col of } R_1$$

$$\begin{pmatrix} x_2' \\ y_2' \\ z_2' \end{pmatrix} = R' \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} + T'$$

$$\begin{pmatrix} x_2' \\ y_2' \\ z_2' \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + T'$$

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} = R' \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} + T'$$

? $R_1^T R' R_2$

$R_2 R' R_1^T$

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = R_2 \begin{pmatrix} x_2' \\ y_2' \\ z_2' \end{pmatrix}$$

$$= R_2 \left(R' \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} + T' \right)$$

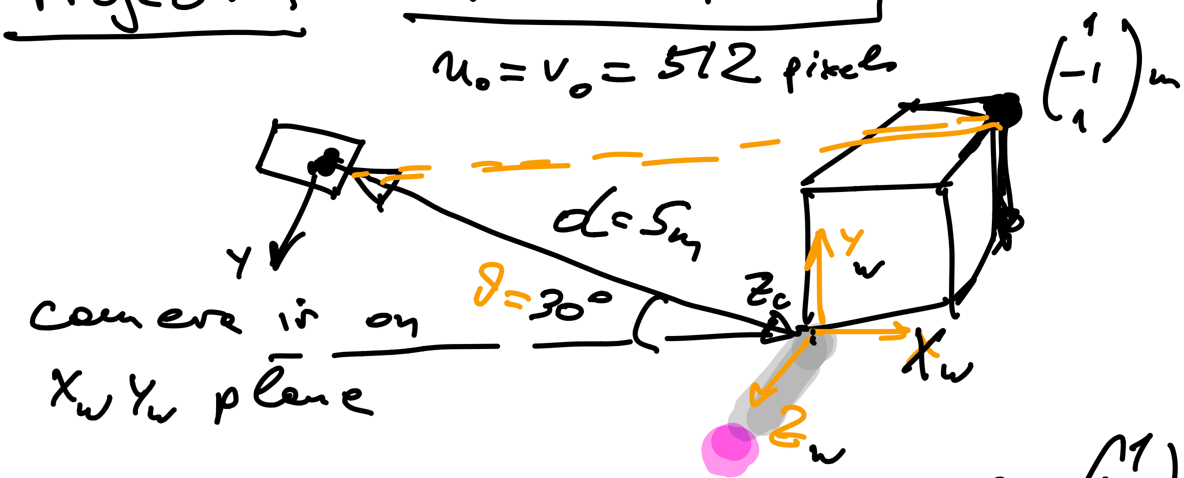
$$= R_2 R' R_1^{-1} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + R_2 T'$$

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = R \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + T$$

Projection

$\boxed{f = 1000 \text{ pixels}}$

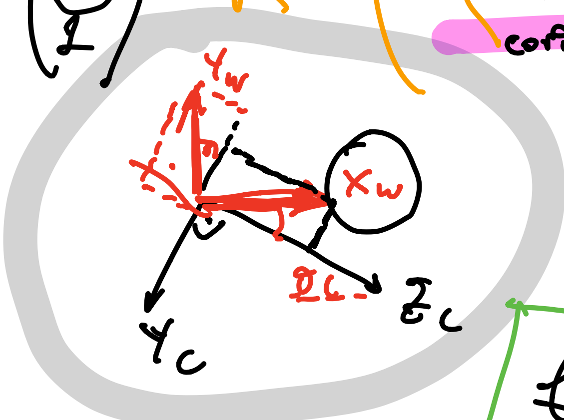
$u_0 = v_0 = 512$ pixels



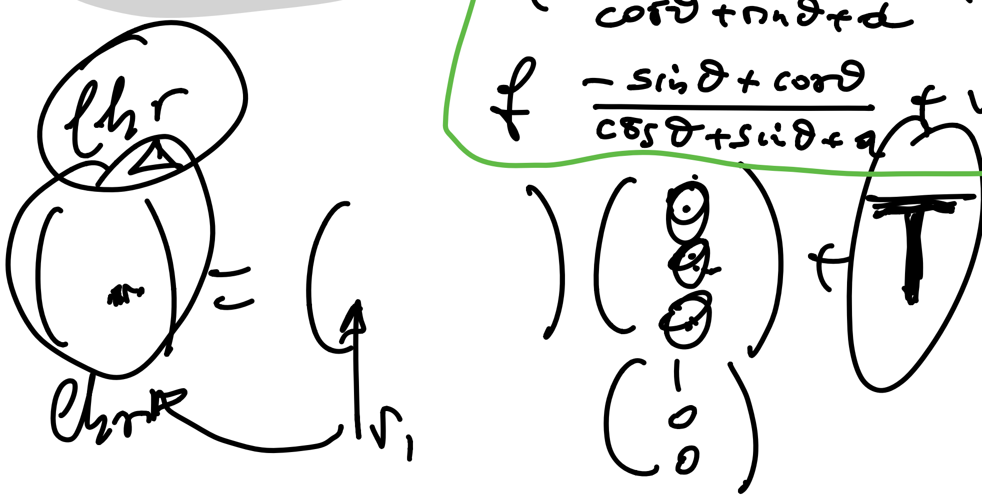
→ pixel coordinates of projection of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$

$$\lambda \begin{pmatrix}u\\v\\1\end{pmatrix} = K\left(\begin{pmatrix}0&0&1\\-\sin\theta&-\cos\theta&0\\\cos\theta&-\sin\theta&0\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix}+\begin{pmatrix}0\\0\\d\end{pmatrix}\right)$$

$$\begin{pmatrix}1+0\\-\sin\theta+\cos\theta+0\\\cos\theta+\sin\theta+d\end{pmatrix}$$



$$\boxed{\begin{array}{l} f\,\dfrac{1}{\cos\theta+\sin\theta+d}+u_0 \\[2ex] f\,\dfrac{-\sin\theta+\cos\theta}{\cos\theta+\sin\theta+d}+v_0 \end{array}}$$



In the figure above :

collineation from $\boxed{Y_w=0}$ to image H

$$\lambda \begin{pmatrix}u\\v\\1\end{pmatrix} \sim K \begin{pmatrix}r_1 & r_3 & T\end{pmatrix}\begin{pmatrix}X_w\\Z_w\\W_w\end{pmatrix}$$

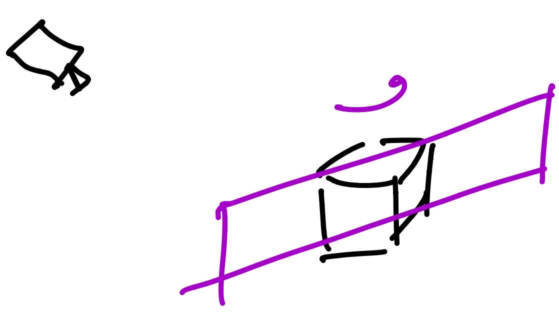
$$\begin{pmatrix}r_1 & r_2 & r_3\end{pmatrix}\begin{pmatrix}X_w\\0\\Z_w\end{pmatrix}+T$$

from $X_w=0$ to image :

$$\begin{pmatrix}r_1 & r_2 & r_3\end{pmatrix}\begin{pmatrix}0\\Y_w\\Z_w\end{pmatrix}+T$$

$$N_w^T\begin{pmatrix}X_w\\Y_w\\Z_w\end{pmatrix}=D \qquad \boxed{AX_w+BY_w+CZ_w=D}$$

collineation from this to image!



$$\begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T$$

$$= \begin{pmatrix} r_1 & r_2 & r_3 & T \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} r_1 & r_2 & r_3 & T \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ \frac{AX_w + BY_w + CZ_w}{D} \end{pmatrix}$$

$$\overset{\text{"}}{1}$$

$$= X_w r_1 + Y_w r_2 + Z_w r_3 + \frac{T}{D} A X_w + \frac{T}{D} B Y_w + \frac{T}{D} C Z_w$$

$$= \underbrace{\begin{pmatrix} r_1 + \frac{T}{D} A & r_2 + \frac{T}{D} B & r_3 + \frac{T}{D} C \end{pmatrix}}_{H} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} \in \mathbb{P}^2$$

dirty

cannot say which coordinate corresponds

forget it for exam

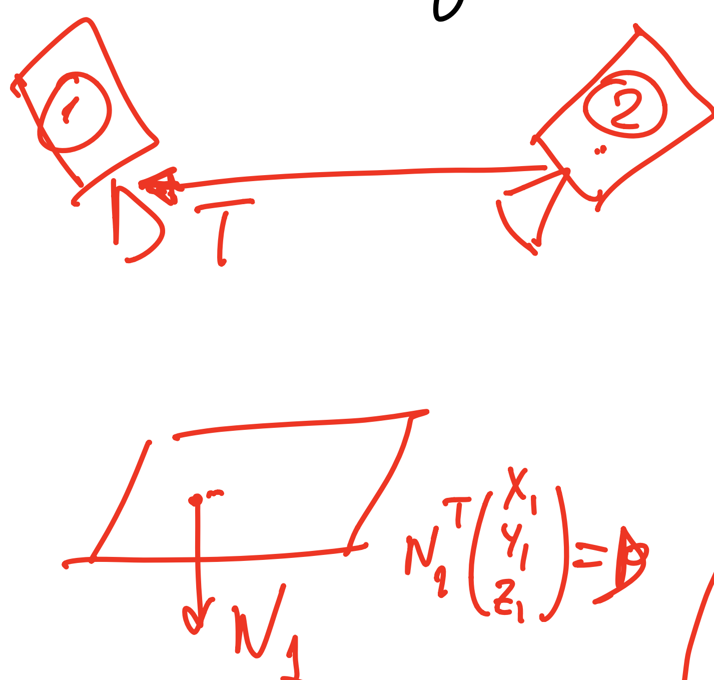
$$\begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix} + \frac{T}{D}\begin{pmatrix} A & B & C \end{pmatrix}$$

N^T

similar to two views

$$\lambda \begin{pmatrix} u_2 \\ v_2 \\ w_2 \end{pmatrix} \sim K\left(R + \frac{T N_1^T}{D}\right) K^{-1} \begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix}$$

image plane



$$N_1^T \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = D$$

— projection equation

— projection of point at infinity

— projection equation ⇒ collineation

— f, u_0, v_0 from H or orthogonal V.P.

collineation : $1p \Rightarrow H$

- relation between v.p. and H

horizon and H

cross ratio and cross ratio with one point at infinity

R, T localization :

$$P_c \sim K \underbrace{\overbrace{(r_1 \; r_2}^{H'} \; T)}_{H} P_w$$

1) $H \Rightarrow R, T$

$$\boxed{\lambda H' = (r_1 \; r_2 \; T)}$$

$$(h_1' \; h_2') = USV^T$$

$$R = UV^T \text{ (det. correction)}$$

$$H' = K^{-1} H$$

$$\lambda = \frac{\sigma_1 + \sigma_2}{2} \qquad T = \lambda h_3'$$

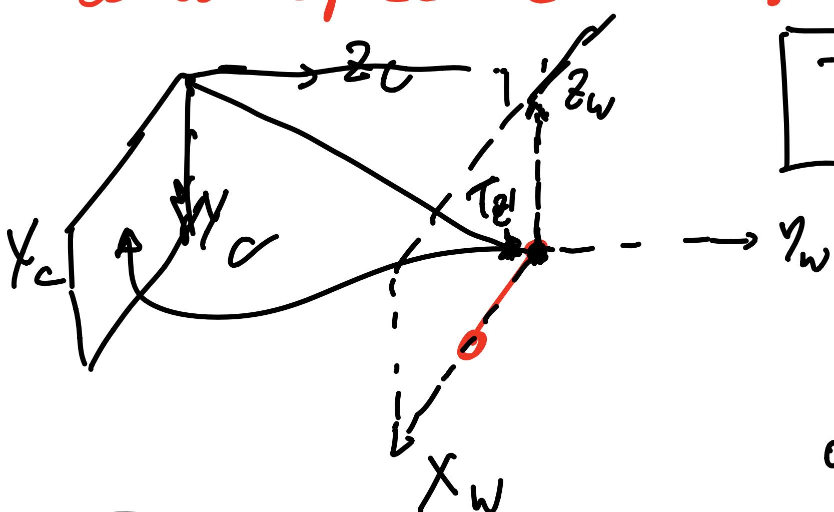
2) $3p \Rightarrow R, T$ involves

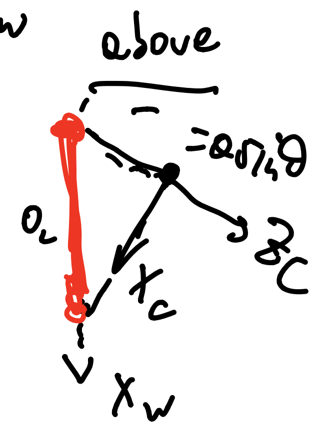
3) Procrustes $\boxed{3D-3D} \Rightarrow R, T$

4) two views: $\boxed{H \sim R + \frac{1}{d} T N^T}$

and special cases.

$\boxed{T_z = a \sin\theta}$



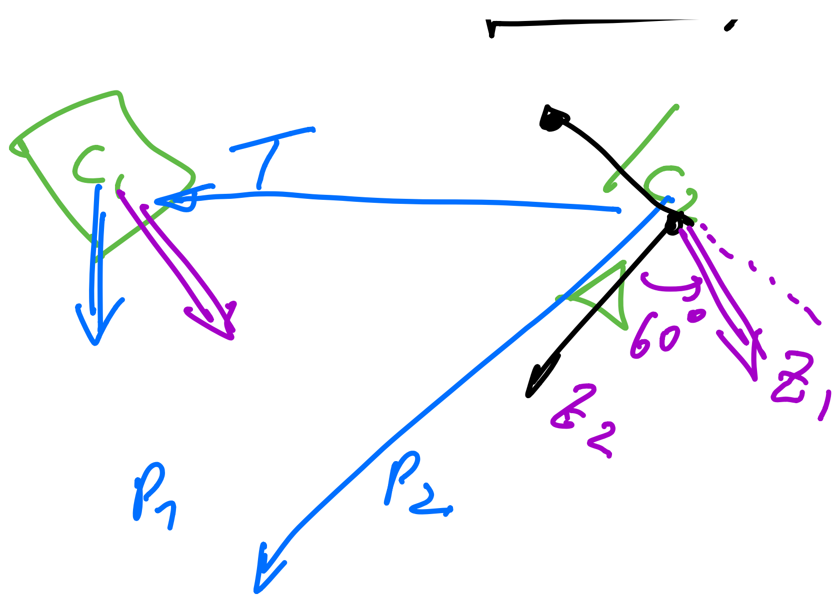


division by zero mean that the image point goes to infinity

$\downarrow$

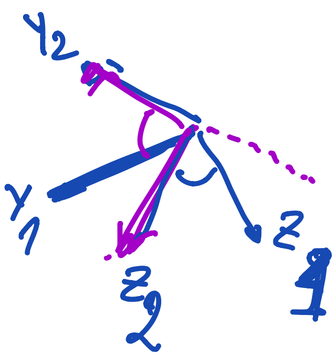
on $Z_c = 0$

impossible → exists not / not unique



$$P_2 = R_x P_1 + T$$

$$R_x(+60°)$$



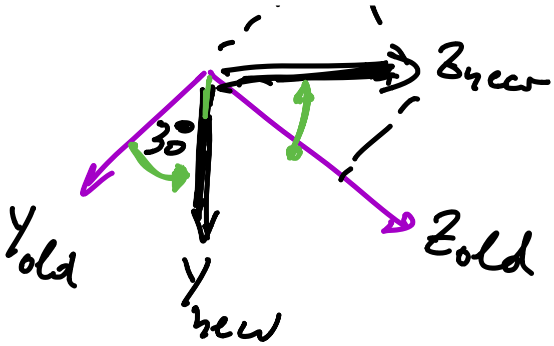
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 \\ 0 & \sin 60 & \cos 60 \end{pmatrix}$$

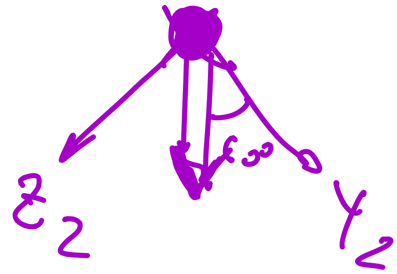
y_1 axis w.r.t 2nd

LH r

z_1 axis wrt 2nd

LEJS





$$old = R_x(+30°)\, new$$

$$\text{Chg} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30° \\ 0 & \sin 30 & \cos 30° \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Y_{new}

$$\text{blue} = A\ \text{green}$$

Procrustes

$$\|A - RB\|_F^2 = \mathrm{tr}\left((A-RB)^T(A-RB)\right)$$

$$\|X\|_F^2 = \mathrm{tr}(X^T X)$$

$$\| \qquad \qquad \qquad \qquad \|$$

$$x_{11}^2 + x_{12}^2 + x_{21}^2 + x_{22}^2$$

$$\min \quad -\mathrm{tr}(A^T R B)$$

$$-\mathrm{tr}(B^T R^T A)$$

$$\max \quad \mathrm{tr}(A^T R B) + \mathrm{tr}(B^T R^T A)$$

$$\mathrm{tr}(X) = \mathrm{tr}(X^T) \qquad \overset{\text{''}}{\mathrm{tr}(A^T R B)}$$

$$2\,\mathrm{tr}(A^T R B)$$

$$\operatorname{tr}(R \underbrace{BA^T}_{\text{SVD}})$$

$$A = \begin{pmatrix} x & \cdots \\ y & \cdots \end{pmatrix}$$

$$B = \begin{pmatrix} x & \cdots \\ y & \cdots \end{pmatrix}$$