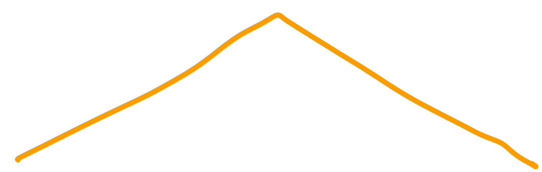
Few examples :

2 0 0 4 1 1 1 0 0 * 1 1 1

$h[-1]$ $h[0]$ $h[1]$

= 0 4 2 3 2 1 0



$$\sum_{k=-\infty}^{\infty} f[k]\, h[n-k]$$

covid 7-day average

1/7	1/7	1/7				1/7

$h[0]$ $h[1]$ $h[6]$

$h[-k]$ 1/7 . . . 1/7

$h[-6]$ $h[0]$

. . . 3 5 . . . 4 0 10

value of inner product

Convolution in probabilities

$x: \; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 6 \qquad p(x) = 1/6$

$y: \; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 6 \qquad p(y) = 1/6$

$z = x + y = 2, 3, \ldots, 12$

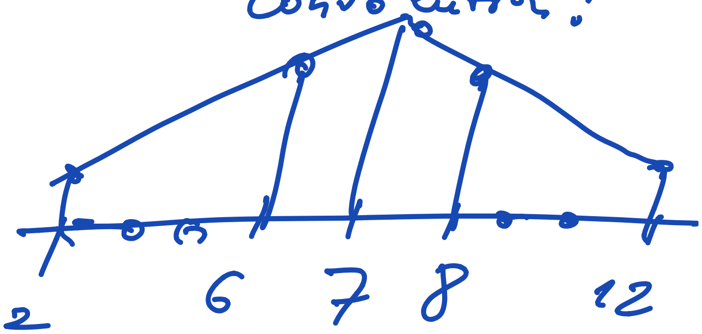
$$p(z) = p(x)\,p(y) = \underbrace{\int_{-\infty}^{\infty} p(x)\,p(z-x)\,dx}_{\text{convolution!}}$$

where $z = x + y$

$p(8) = 5/36$

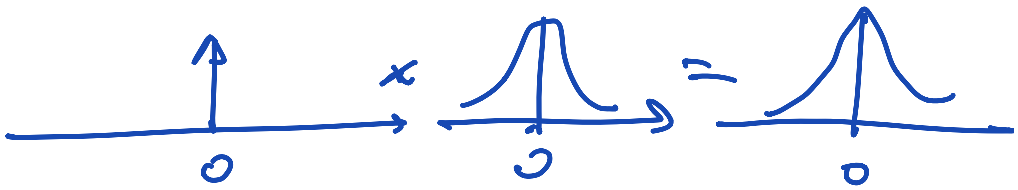
$p(\cancel{7}) = \dfrac{6}{36}$

$p(6) = \dfrac{5}{36}$

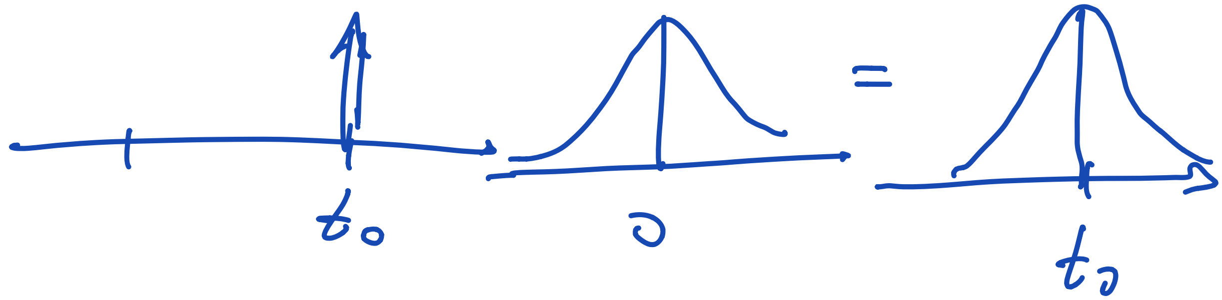


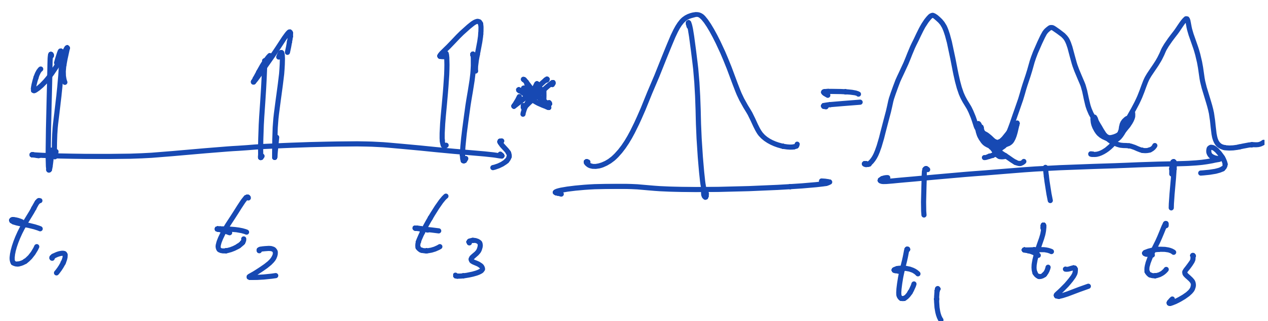
$p(2) = p(12) = \dfrac{1}{36}$

Dirac: $\quad \delta(t) * h(t) = h(t)$



$$\delta(t-t_0) * h(t) = h(t-t_0)$$



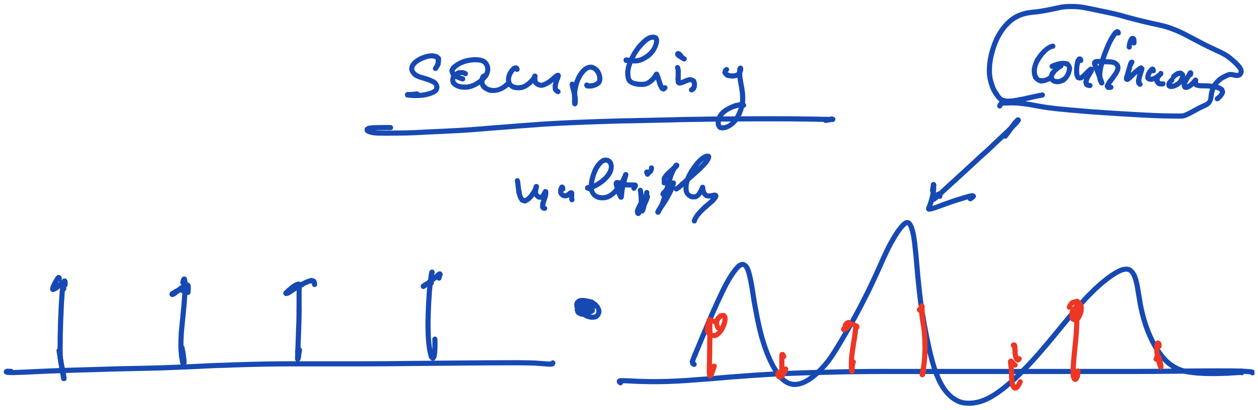


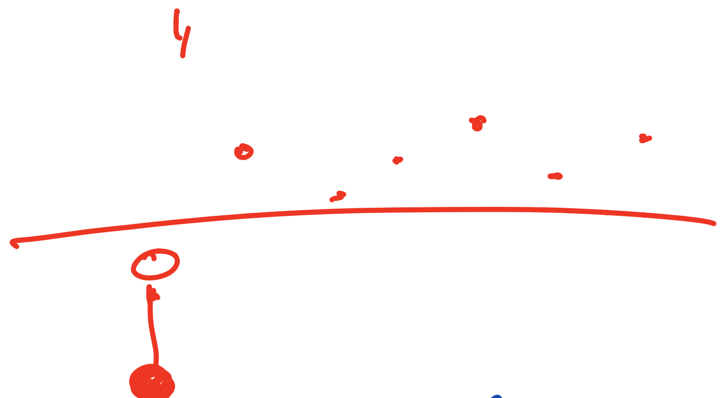
Dirac series
"comb"
impulse train

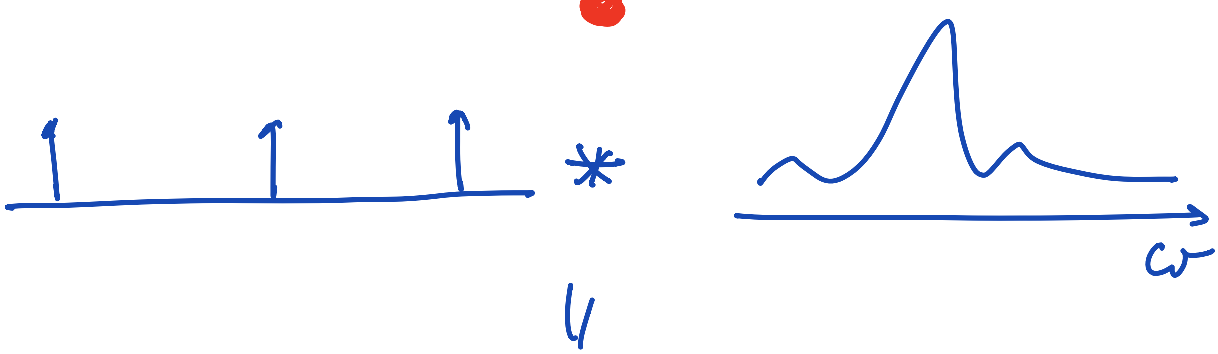
"xeroxing"

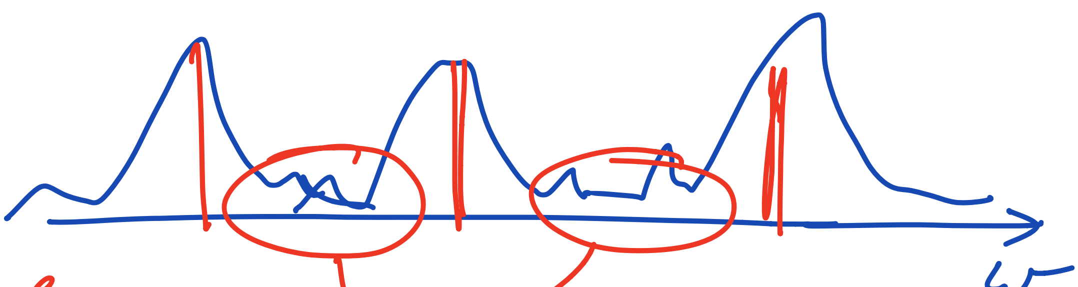
sampling

multiply







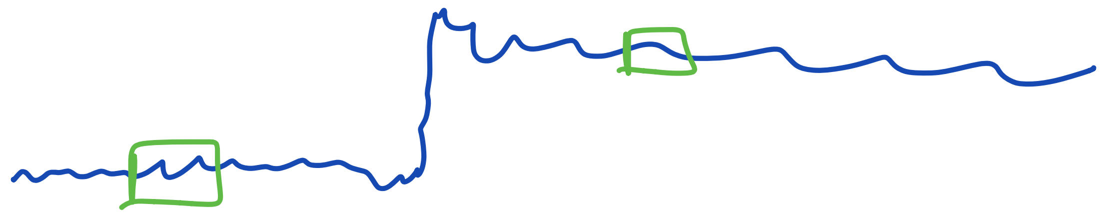


sampling freq ≥ 40 kHz — aliasing

≥ 2 max freq.

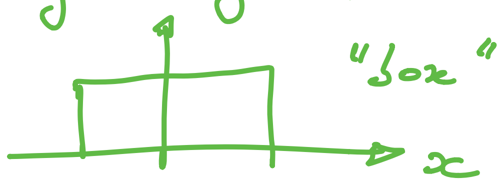
↳ 20 kHz

edge detection



smooth first!

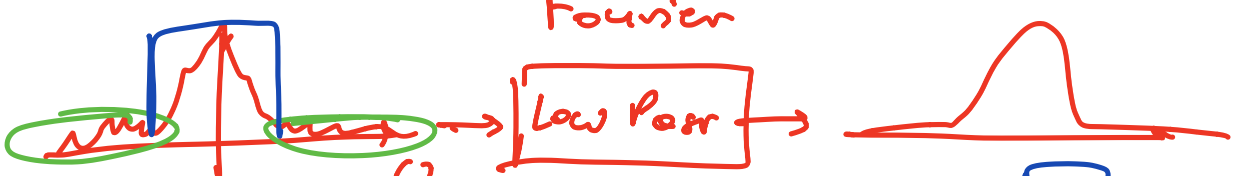
is averaging a good smoothing filter?

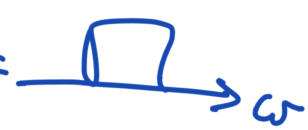


creates additional edges

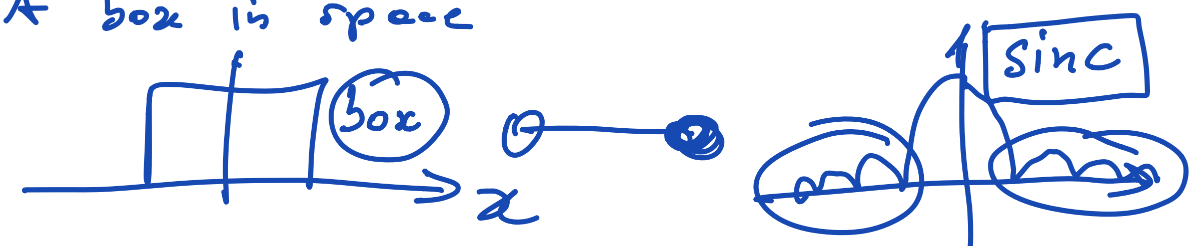
smoothing is low-pass filtering

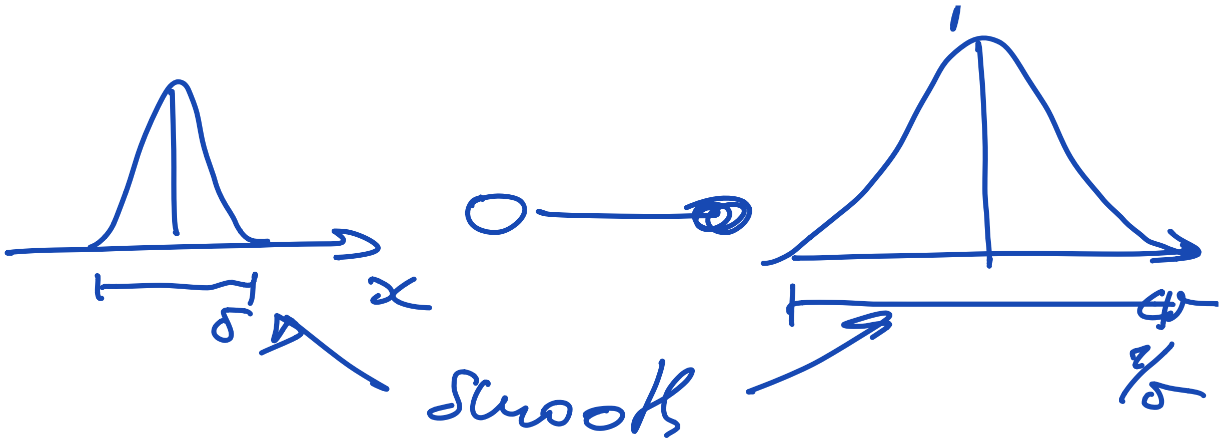
Fourier



ideal smoother = 

A box in space





GAUSSIAN

edge detection

1. smooth with g
2. first-derivative

$$f \longrightarrow \boxed{} \longrightarrow e$$

$$\frac{d(f*g)}{dx}$$

$\boxed{\text{lemma:}}$ Convolution commutes with differentiation

$$\frac{d(f*g)}{dx} = f * \frac{dg}{dx}$$

$$\frac{d}{dx}\int_{-\infty}^{\infty} f(t)\,g(x-t)\,dt$$

proof 1

$$\frac{d}{dx}\int_{-\infty}^{x} f(t)\,g(x-t)\,dt$$

proof 2 via Fourier

f ○—● F

$f*g$ ○—● FG

g ○—● G

$\frac{d}{dx}$ ○—● $j\omega$

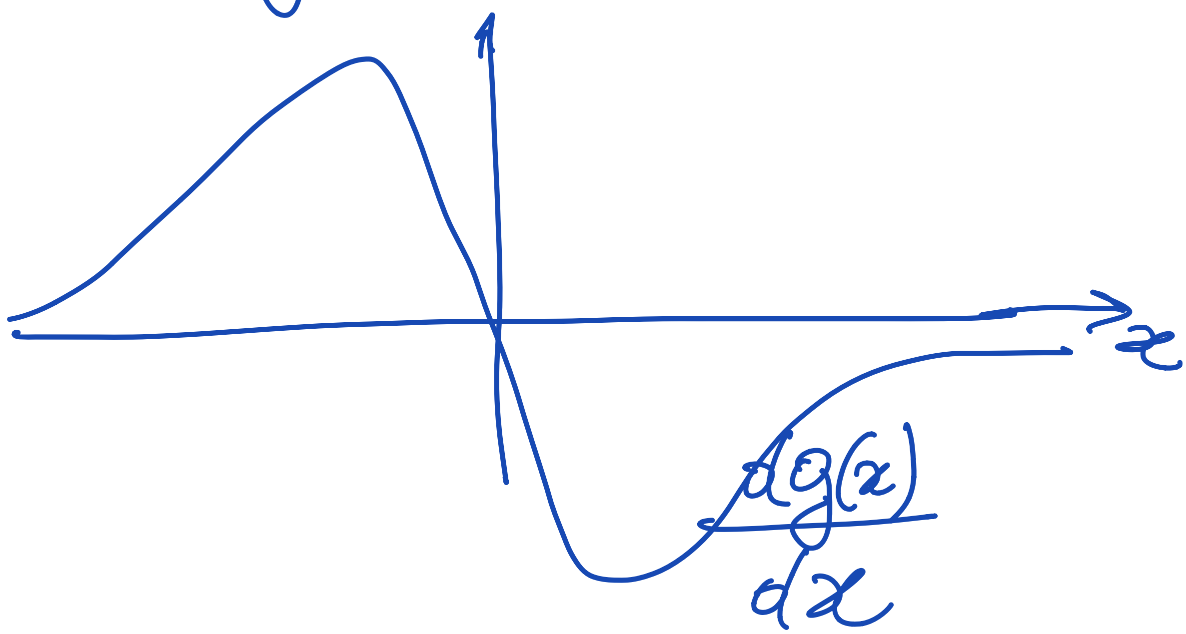
$\frac{d}{dx}(f*g)$ ○—● $j\omega FG$

inverse Fourier

$INV(F) * inv(j\omega G)$

$$= f * \frac{dg}{dx}$$

To detect an edge we convolve with first derivative of a Gaussian! ... and



and we find the maximum!