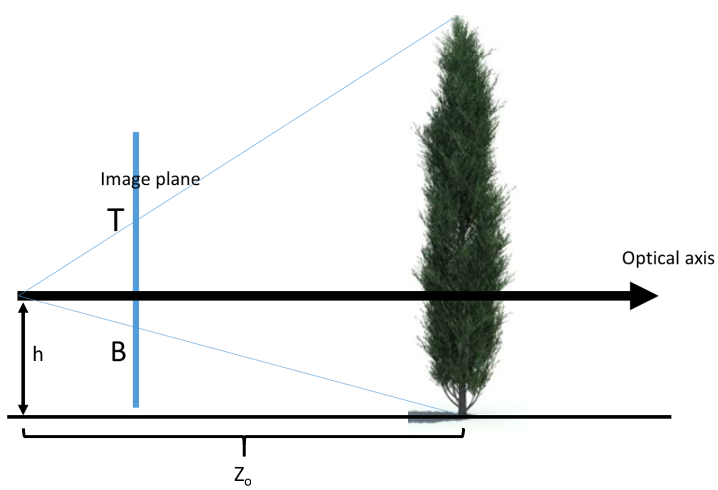
CIS 580, Machine Perception, Spring 2021
Homework 1 Solutions

Instructions

- This is a homework you have to solve by yourself.
- You must submit your solutions on Gradescope. We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Start early! If you get stuck, please post your questions on Piazza or come to office hours!

Homework

1. Assume that you see the bottom and the top of a vertical tree in front of you. The optical axis is parallel to the ground and the height of the projection center with respect to the ground is h. The image plane is vertical as well and you see the bottom and the top of the tree at calibrated ($K = I$) coordinates $B = (0, y_1)$ and $T = (0, y_2)$, respectively.
   Compute the horizontal distance Z_0 between the projection center and the tree.
   (Compute the result with respect to h, y_1, y_2. It is not required that all of them appear in the result)



Answer: Because we have $K = I$ we can infer that $f = 1$, so:

$$|y_1| = |f\frac{h}{Z_0}| \implies Z_0 = \frac{h}{|y_1|}$$

2. Assume the same configuration as Question 1, but while we keep the projection center at the same position we double the distance between the image plane and projection center.

   (a) What are the new coordinates B', T' for the projections of the bottom and the top of the tree at the image plane?

   (b) How much would we have to move the projection center so that the bottom and the top of the tree appear at the original coordinates $B = (0, y_1)$, $T = (0, y_2)$ in the image plane? (We assume that while we move the projection center we keep the distance between the projection center and the image plane constant)

   Answer:

   (a) when we double the distance between the projection center and the image plane we get $f' = 2$. So

   $$y_1' = f'\frac{h}{Z_0} = 2y_1$$
   $$y_2 = f'\frac{h}{Z_0} = 2y_2$$

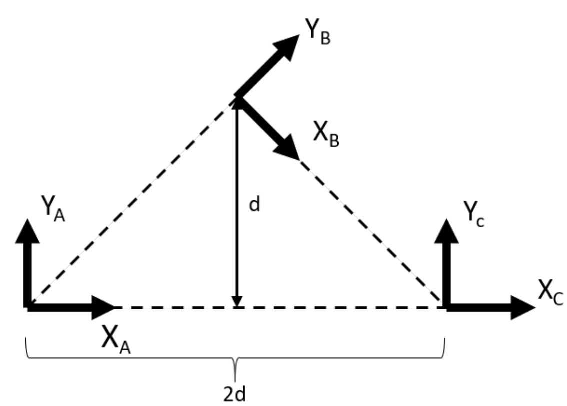
   which means $B' = (0, 2y_1)$, $T' = (0, 2y_2)$

   (b) We have to move the projection center away from the tree by distance l where:

   $$y_1 = f'\frac{h}{Z_0 + l} \implies \frac{h}{Z_0} = \frac{2h}{Z_0 + l} \implies l = Z_0$$

3. Find the transformations T_1, T_2 given the configuration from the figure below:

$$\begin{pmatrix} X_A \\ Y_A \\ 1 \end{pmatrix} = T_1 \begin{pmatrix} X_B \\ Y_B \\ 1 \end{pmatrix}, \quad \begin{pmatrix} X_B \\ Y_B \\ 1 \end{pmatrix} = T_2 \begin{pmatrix} X_C \\ Y_C \\ 1 \end{pmatrix}$$



Note: $\cos(\pi/4), \sin(\pi/4)$ can appear in the solution. You should not replace them with their numerical value. You have to multiply out the matrices.
(Clarifications about figure: the triangle is an isosceles triangle with height d and base $2d$*.)*
Answer: Because the triangle is isosceles we have that $C\hat{A}B = A\hat{C}B = \pi/4$. We will use the columns interpretation to find the rotation matrix

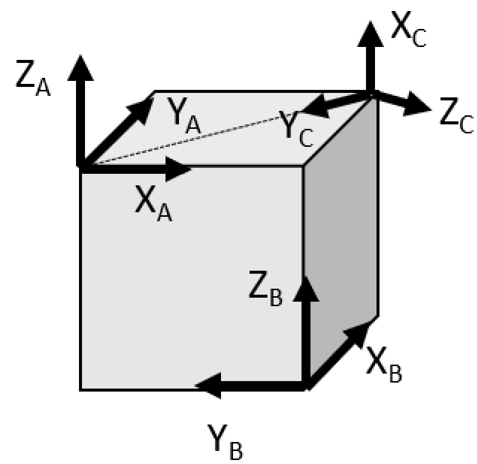
- $$\begin{bmatrix} X_A \\ Y_A \\ 1 \end{bmatrix} = \left[\begin{array}{cc|c} r_{x1} & r_{y1} & t_1 \\ \hline 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} X_B \\ Y_B \\ 1 \end{bmatrix}$$

  the translation is
  $$t_1 = \begin{bmatrix} d \\ d \end{bmatrix}$$
  and for the rotation matrix we have that
  $$r_{x1} = \begin{bmatrix} \cos(-\pi/4) \\ \sin(-\pi/4) \end{bmatrix} = \begin{bmatrix} \cos(\pi/4) \\ -\sin(\pi/4) \end{bmatrix}$$
  $$r_{y1} = \begin{bmatrix} \sin(\pi/4) \\ \cos(\pi/4) \end{bmatrix}$$
  so in total
  $$T_1 = \begin{bmatrix} \cos(\pi/4) & \sin(\pi/4) & d \\ -\sin(\pi/4) & \cos(\pi/4) & d \\ 0 & 0 & 1 \end{bmatrix}$$

- For T_2 we have
  $$\begin{bmatrix} X_B \\ Y_B \\ 1 \end{bmatrix} = \left[\begin{array}{cc|c} r_{x2} & r_{y2} & t_2 \\ \hline 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} X_C \\ Y_C \\ 1 \end{bmatrix}$$
  we find distance $BC = d\sqrt{2}$ so
  $$t_2 = \begin{bmatrix} d\sqrt{2} \\ 0 \end{bmatrix}$$
  and for the rotation matrix we have
  $$r_{x2} = \begin{bmatrix} \cos(\pi/4) \\ \sin(\pi/4) \end{bmatrix}$$
  $$r_{y2} = \begin{bmatrix} -\sin(\pi/4) \\ \cos(\pi/4) \end{bmatrix}$$
  so in total
  $$T_2 = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & d\sqrt{2} \\ \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Suppose that we have the following cube with edges of length d. Write the transformations A, B, C between the coordinate systems :

$$\begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix} = A \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = B \begin{pmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = C \begin{pmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{pmatrix}.$$



You can solve the problem with the rotation column interpretation method or with concatenation of rotations.
(Clarifications about the configuration: X_A, Y_A lie on the cube's edges. Similarly X_B, Y_B lie on the cube's edges. X_C is looking up and vertical to the cube's face and Y_C lies on the cube's face diagonal.)

Answer:

- $$\begin{bmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{bmatrix} = A \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix}$$

  Written int A coordinate we have translation to B frame:

  $$T = \begin{bmatrix} d \\ 0 \\ -d \end{bmatrix}$$

  and we also have rotation matrix

  $$R = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}$$

with

$$r_x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad r_y = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad r_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

so in total

$$A = \begin{bmatrix} 0 & -1 & 0 & d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $$\begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = B \begin{bmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{bmatrix}$$

  The dimension of the diagonal is $l = d\sqrt{2}$, so we have translation written in C coordinates where

  $$T = \begin{bmatrix} 0 \\ d\sqrt{2} \\ 0 \end{bmatrix}$$

  Then for the rotation we have

  $$r_x = \begin{bmatrix} 0 \\ -\cos(\pi/4) \\ \sin(\pi/4) \end{bmatrix} \quad r_y = \begin{bmatrix} 0 \\ -\sin(\pi/4) \\ -\cos(\pi/4) \end{bmatrix} \quad r_z = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

  and in total

  $$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\cos(\pi/4) & -\sin(\pi/4) & 0 & d\sqrt{2} \\ \sin(\pi/4) & -\cos(\pi/4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Finally we have that

  $$\begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = C \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} = B \begin{bmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{bmatrix} = BA \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix}$$

  so we get that

  $$C = BA = \begin{bmatrix} 0 & 0 & 1 & -d \\ -\cos(\pi/4) & \sin(\pi/4) & 0 & \cos(\pi/4)d \\ -\sin(\pi/4) & -\cos(\pi/4) & 0 & \sin(\pi/4)d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$