Problem: Given correspondences between a planar pattern and an image find where is the camera (R, T).

Assume K is given (f, u_0, v_0).

1. $N \geq 4$ point pairs $\Rightarrow H$

2. $$H = \lambda K (r_1 \; r_2 \; T)$$

$$\underbrace{K^{-1} H}_{} = \lambda (r_1 \; r_2 \; T)$$

$$H' = \lambda (r_1 \; r_2 \; T)$$

solve for λ, r_1, r_2, T : $\begin{array}{l} r_1^T r_2 = 0 \\ \|r_1\| = \|r_2\| = 1 \end{array}$

call $H' = (\underset{3\times1}{a} \; \underset{3\times1}{b} \; \underset{3\times1}{c})$

3. $$\| (a \; b \; c) - \lambda (r_1 \; r_2 \; T) \|_F \Rightarrow \min$$

$$\underset{3\times2}{(a \; b)} = U_{3\times2} \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} V^T_{2\times2}$$

solution : $(r_1 \;\; r_2) = U_{3\times 2} V_{2\times 2}^T$

$$\lambda = \frac{s_1 + s_2}{2} \Rightarrow T = \frac{c}{\lambda}$$

$$R = (r_1 \;\; r_2 \;\; r_1 \times r_2)$$

$$R^T R = I \qquad \det R = 1$$

A more general problem

$$K = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ with f unknown!

(common situation if somebody gives you a photo)

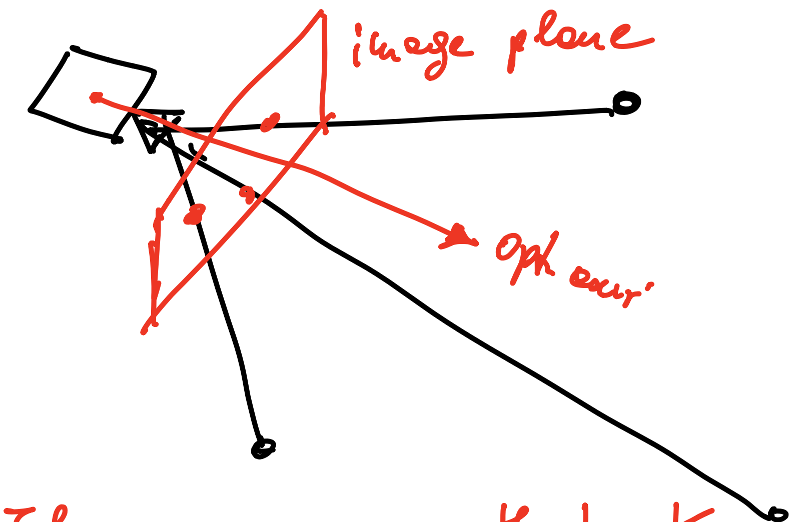
$$H = \lambda \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} (r_1 \;\; r_2 \;\; T)$$

find λ, f, r_1, r_2, T

$\boxed{\text{Continue with pose estimation}}$

"localization given a map"

Perspective N Point (PnP)

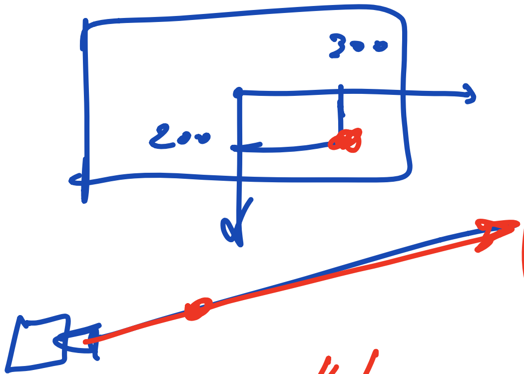


If we assume that K is known then we know the "vector" of the ray to the point. $K^{-1}\begin{pmatrix}u\\v\\1\end{pmatrix}$.

We know the angles between the rays and the opt. axis.

exemple : $(u_0, v_0) = 0$

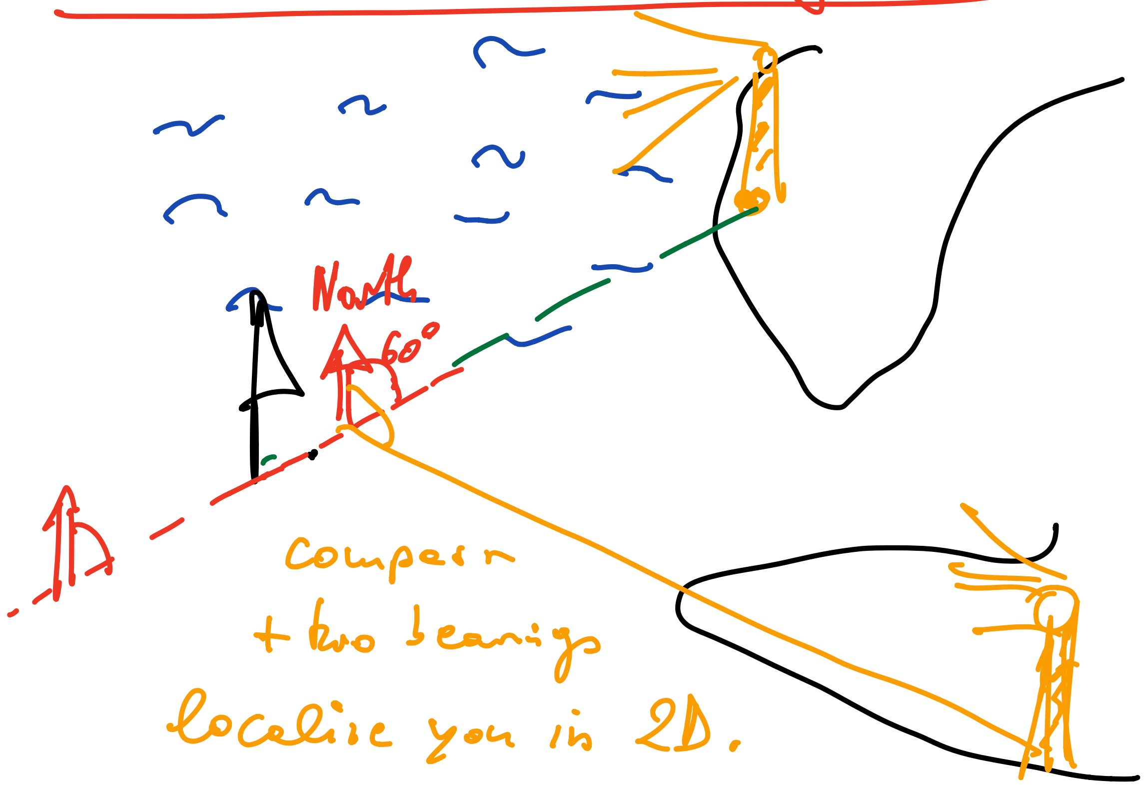
$f = 1000$



$$\begin{pmatrix} 300/1000 \\ 200/1000 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 1 \end{pmatrix}$$

"bearings"

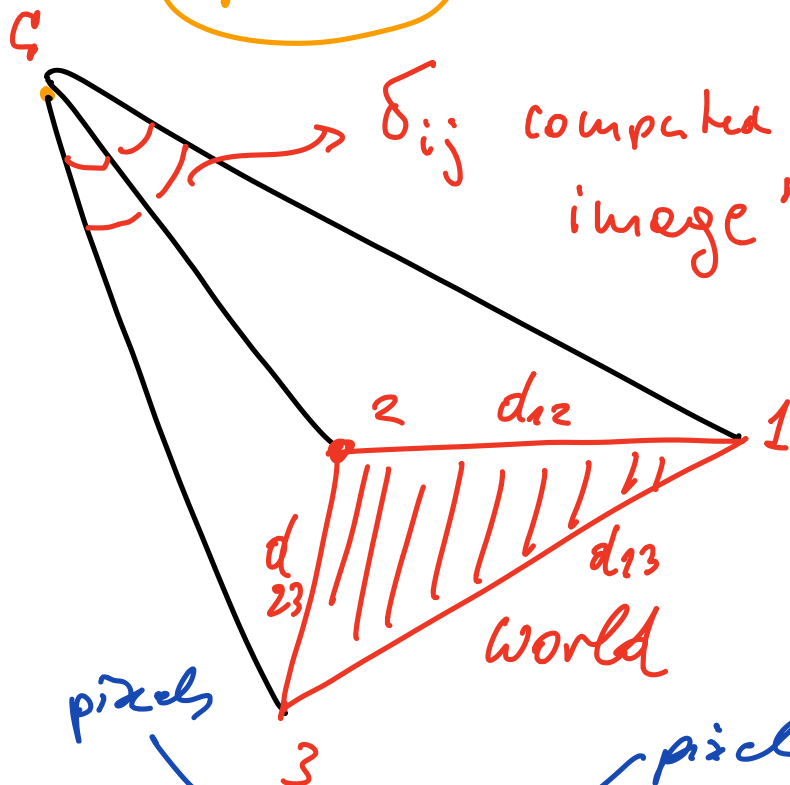
old-time navigation



compass
+ two bearings
localise you in 2D.

slide : $\boxed{\text{no compass}}$

P3P



δ_{ij} computed from image *

$$* \quad \frac{\left(K^{-1}\begin{pmatrix}u_i\\v_i\\1\end{pmatrix}\right)^T\left(K^{-1}\begin{pmatrix}u_j\\v_j\\1\end{pmatrix}\right)}{\left\|K^{-1}\begin{pmatrix}u_i\\v_i\\1\end{pmatrix}\right\|\left\|K^{-1}\begin{pmatrix}u_j\\v_j\\1\end{pmatrix}\right\|} = \cos\delta_{ij}$$

pixels pixels

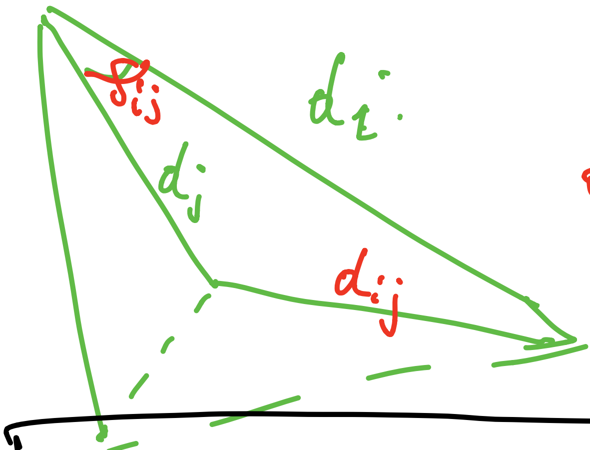
$\boxed{\textbf{given}: \underbrace{d_{12}, d_{23}, d_{13}}_{\text{triangle}}, \delta_{12}, \delta_{23}, \delta_{13}}$

find: where is the camera c?

d_1, d_2, d_3

$\boxed{PnP}$

GPS: d_1, d_2, d_3, satellite 1, 2, 3



if we get d_i

$$d_i \begin{pmatrix} u_i \\ v_i \\ 1 \end{pmatrix} = R P_i + T$$

unknown

$$\boxed{d_i^2 + d_j^2 - 2 d_i d_j \cos \delta_{ij} = d_{ij}^2}$$

3 quadratic equations

3 unknowns

probably 8 solutions

let's call the unknowns

$x = d_1 \quad y = d_2 \quad z = d_3$

$$\boxed{\begin{aligned} x^2 + y^2 - 2xy\cos\alpha &= a^2 \\ y^2 + z^2 - 2yz\cos\beta &= b^2 \\ z^2 + x^2 - 2xz\cos\gamma &= c^2 \end{aligned}}$$

$y = ux \quad z = vx \quad$ call $u = \frac{y}{x} \quad v = \frac{z}{x}$

$$\left.\begin{array}{l} \boxed{\begin{array}{l} x^2 + u^2x^2 - 2x^2u\cos\alpha = a^2 \\ u^2x^2 + v^2x^2 - 2uvx^2\cos\beta = b^2 \end{array}} \\ v^2x^2 + x^2 - 2vx^2\cos\gamma = c^2 \end{array}\right\}$$

1 eq. in u, v

1 eq. u, v

$\Rightarrow$ two quadratic equations in u, v

$$* \quad \boxed{\begin{array}{l} c^2(u^2 + v^2 - 2uv\cos\beta) \\ \qquad = b^2(u^2 + v^2 - 2uv\cos\gamma) \\ a^2(u^2 + v^2 - 2uv\cos\gamma) \\ \qquad = c^2(u^2 + v^2 - 2uv\cos\alpha) \end{array}}$$

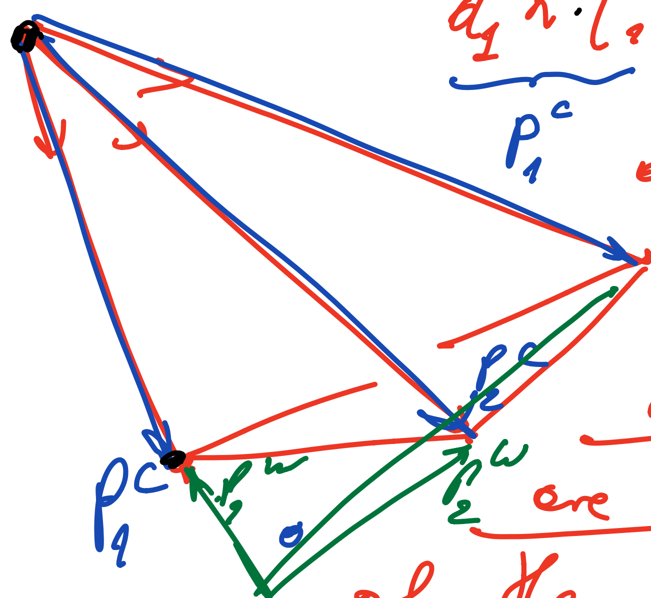
$\Rightarrow$ one eq. in v 4^{th} deg

(not 8^{th} because there were no linear factors in $*$).

$\Rightarrow$ ~~for~~ u: 8 solutions (not all real).

How do you keep one solution given 8 solutions for d_1, d_2, d_3?

$$\underbrace{d_1 K^{-1}\begin{pmatrix}u_1\\ v_1\\ 1\end{pmatrix}}_{P_1^c}, \quad \overbrace{d_2 K^{-1}\begin{pmatrix}u_2\\ v_2\\ 1\end{pmatrix}}^{P_2^c}, \quad \underbrace{d_3 K^{-1}\begin{pmatrix}u_3\\ v_3\\ 1\end{pmatrix}}_{P_3^c}$$



all of them are in front of the camera!

Given P_1^c, P_2^c, P_3^c

$P_1^w \quad P_2^w \quad P_3^w$

Find: $\boxed{P_i^c = R P_i^w + T}$

computer vision ← Procrustes (Statistics, ML)

RGB-D cameras or Lidar (when range is known)

More general : $A_i, B_i \in \mathbb{R}^3$

$$A_i = R B_i + T$$

solve for R (rotation), T

How many points suffice to solve this? $\boxed{\text{Triangle enough}}$

$$(\qquad\qquad) = R\,(\qquad\qquad)$$

$$\underbrace{(\qquad)^{-1}(\qquad) = R}$$

$$\min_{R,T} \sum_{i=1}^{N} \| A_i - R B_i - T \|^2$$

$$T = \bar{A} - R \bar{B}$$

centroid centroid

$$\underbrace{\begin{pmatrix} A_1 - \bar{A} & \cdots & A_N - \bar{A} \end{pmatrix}}_{3 \times N} = \underset{3\times 3}{R} \underset{3 \times N}{\begin{pmatrix} B_1 - \bar{B} & \cdots & B_N - \bar{B} \end{pmatrix}}$$

$$A = R\, B$$

solve $\min_R \| A - RB \|$

$$\underset{3\times 3}{BA^T} = USV^T \Rightarrow R = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(VU^T) \end{pmatrix} U^T$$