CIS580 Spring 2021: Midterm

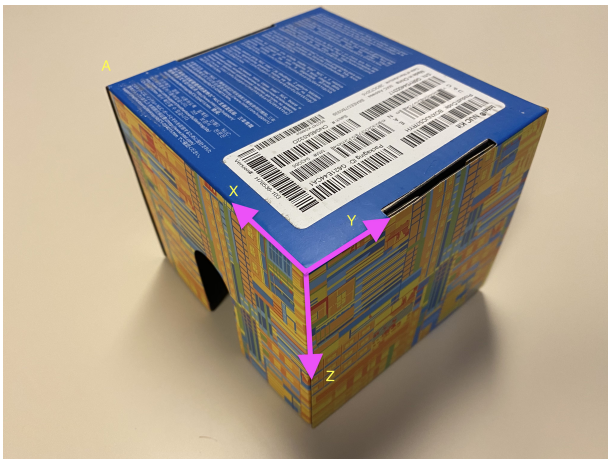
- Once you begin the exam, you will have 100min to finish. All SDS accommodations will still apply accordingly.
- The exam will be administered over Gradescope. You will have the option of uploading written answers as pictures.
- The exam does not require any ruler or any interactive clicking on the pictures, so the original images will not be given.
- The exam does not require any SVD either. There is a question with a matrix-vector multiplication with numbers where you have to use a calculator or excel or google sheets or matlab or wolfram alpha or etc. A trigonometric calculation is also necessary.
- During the exam, we will post clarifications in the google doc https://docs.google.com/document/d/1BoJIOiwfwGtc437-Wzlhyg01OHgs_tDocXxdpjN6Psc/edit?usp=sharing

  If you have a question please first check the document and if your question is not there please send a PRIVATE question in piazza. We will copy the question and answer it in the doc.
- There will be no zoom during the exam.
- The exam is open book/notes/Internet. If by any chance you want to use a resource from the internet or a book that is not in our class material please cite this source.
- If you have an Internet interruption please try with other means like your cell phone to send an email to kostas@cis.upenn.edu with your answers and upload the answers later when Internet is back.
- For any emergency please send a private post in Piazza and email kostas@cis.upenn.edu.
- **The distribution of the exam itself is strictly prohibited.**
- **This includes postings of individual exam questions in public forums and social media.**
- **Collaboration is strictly prohibited.**
- **There will be severe consequences for anyone caught in violation of this policy, including for anyone who helps others cheat.**
- **Even if we discover a violation in just one of the questions we will zero the exam and all homeworks submitted til today.**
- **If you want to appeal our decision we will send your case to the Office of Student Conduct.**

1. **Problem 40 points**

A cube is projected on an image plane. A world (object) coordinate system is attached to the cube as in the picture. The intrinsic camera matrix reads

$$K = \begin{pmatrix} 3275 & 0 & 2016 \\ 0 & 3275 & 1512 \\ 0 & 0 & 1 \end{pmatrix}$$



a. (10pts) The origin of the world is projected to pixel (2021,1778). If the distance from the projection center to the origin is 111mm find T in the projection equation

$$\lambda \begin{pmatrix} u_{pix} \\ v_{pix} \\ 1 \end{pmatrix} = K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X_{mm} \\ Y_{mm} \\ Z_{mm} \\ 1 \end{pmatrix}$$

b. (10pts) In the above projection equation the rotation R reads

$$R = \begin{pmatrix} -0.6356 & 0.7708 & 0.0434 \\ -0.5383 & -0.4828 & 0.6907 \\ 0.5534 & 0.4156 & 0.7218 \end{pmatrix}.$$

Find the position of the camera (projection center) with respect to the world coordinate system (make sure that the signs of this vector make sense and that your answer is in mm).

c. (10pts) Find the pixel positions of the three orthogonal vanishing points corresponding to x,y, and z directions. You should find them by using the equation, NOT by using a ruler or clicking on a hypothetical image.

d. (10pts) Assume now that you know the image center (2016,1512) but you do not know the focal length or any of the extrinsic parameters. Compute the focal length using the vanishing points in x and y direction from the last question. Please show all your work. You are supposed to find an f value close to the one above.

2. **Problem 10 points**

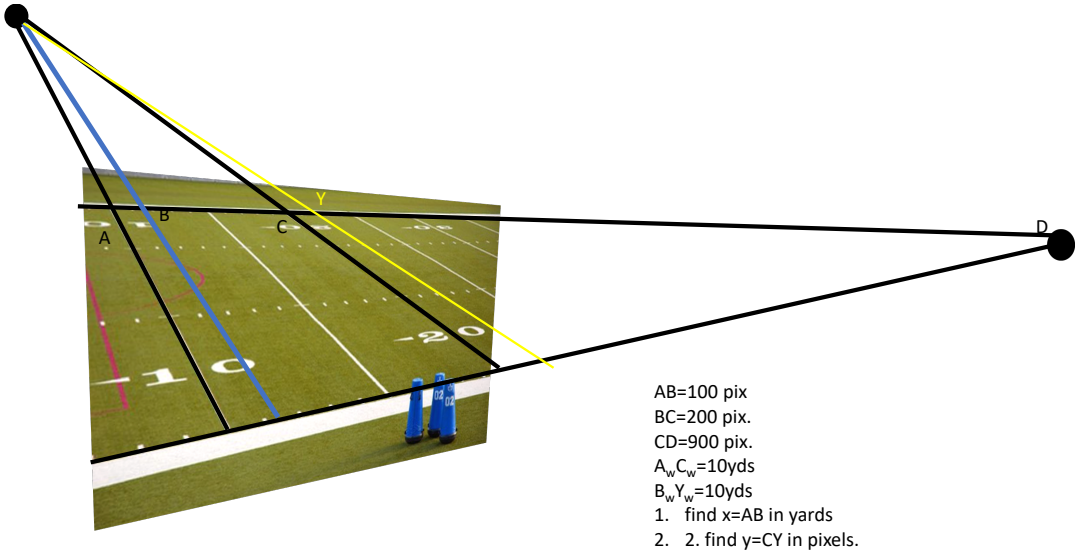
Assume a camera with image center $(u_0, v_0) = (0, 0)$ and focal length $f = 600$. Consider a target point at position $(u_T, v_T) = (0, 200\sqrt{3})$. Write a pure rotation (as a matrix) that the camera has to undergo so that the target is at $(0, 0)$ after the rotation. There are infinite solutions to this problem. Any answer that will bring the point (u_T, v_T) to $(0, 0)$ is correct. Your eye does such kind of fixation all the time by applying saccadic movements.

3. **Problem 20 points**

In this problem we will solve the yellow line problem. In the picture below you are given the distances between points A,B,C and D in pixels.

1. (10pts) Find the distance $x = AB$ in yards.

2. (10pts) Find where to put the yellow line, namely, the point Y or equivalently the distance CY we will call y in pixels.



4. **Problem 20 points** Let a camera rotate about the y-axis with angle θ.

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \sim \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}.$$

a. (5pts) Write the equation of rotation for the projections (u_0, v_0, w_0), (u, v, w) of the points (X_0, Y_0, Z_0), (X, Y, Z), respectively. Assume that the intrinsic camera matrix is $K \neq I$.

b. (10pts) Assume $K = I$. Consider a point $(u_0, v_0, w_0) = (0, 1, 1)$ on the $y - axis$. If we have a continuous pure rotation with changing θ this point will traverse a curve in the image plane. By eliminating θ and embedding in a real plane $x = u/w, y = v/w$, show that the trajectory of the point (x, y) is a hyperbola.

c. (5pts) (Assume again that $K = I$) Suppose that the camera is not able mechanically to undergo a pure rotation and a translation is always present. Assume that the camera is seeing a checkerboard. What image point would you choose so that the resulting trajectory is still a hyperbola.

5. **Problem 10 points** The drone sees a Siemens star at position $P_1 = (p, q, r)$ with respect to camera coordinates. Computes a translation T such that when the drone moves by T following the equation

$$\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} + T$$

the lines on the star will appear parallel in the **image** of the camera at coordinate system (X_2, Y_2, Z_2).