Ingenuity flight:

$$\text{VIO} + \underbrace{\text{1P Distance}}_{\text{Altimeter}}$$

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exam ≈ 50% + 50%

SfM

3D velocity

image processing

Gaussian

# Geometry Review

1) "two calibrated views"

$\parallel$

SfM

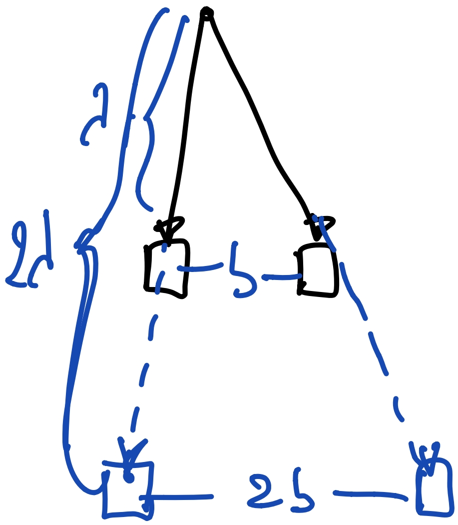
<u>given</u>: correspondences in calibrated coordinates $(p, q)$

<u>find</u>: $(R, T)$, $\lambda, \mu$

$$\lambda q = R \mu p + T \qquad R^T R = I$$

$$2\lambda q = R(2\mu)p + 2T$$

If $T, \lambda, \mu$ are solutions then $2T, 2\lambda, 2\mu$ are solutions, too.




another explanation:

if $T$ satisfies

$$q^T(T \times Rp) = 0$$

the $2T$ satisfies it, too.

2) what if the camera undergoes a pure rotation but we do not know it?

$$q^T E p = 0 \Rightarrow E = ?$$

but $p \sim Rq$ in reality!

$$ERq = 0$$

Solution at the end!

v like

quadratic for $q^T A q = 0$

$q^T(A+A^T)q = 0 \Leftarrow$ $q \quad - \quad 0$

$$\begin{pmatrix} q_x p^T & q_y p^T & q_z p^T \end{pmatrix} \left( \boxed{E} \right) = 0$$

$p \sim Rq$

$9 \times 1$

$$q_x q^T R^T \quad q_y q^T R^T \quad \boxed{q_z q^T R^T}$$

$1 \times 9$

prove that the three <u>columns</u> of $q_z q^T R^T$ are linear combination of the first six columns.

cont. Off

in practice: we check if $\exists H$: positive for all correspondences!

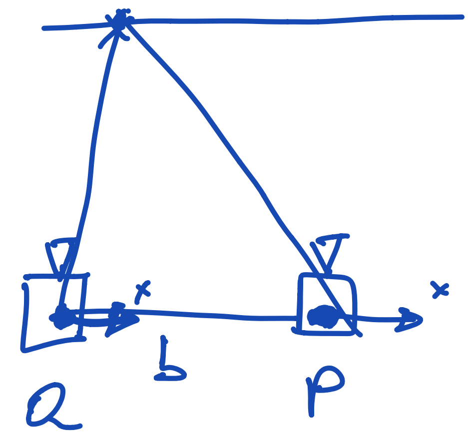
(3) if $E = \hat{T}$ then $E$ is skew-symmetric.

$$E^T T = 0$$

The question is "ill-posed" because if $(T, R = I)$ is a solution then $(T, R_T(180°))$ is also a solution.

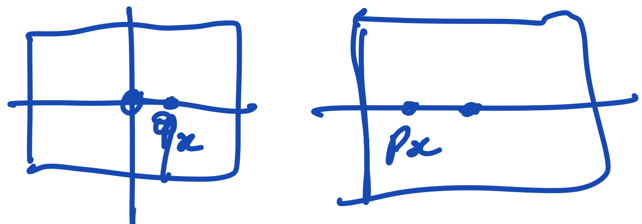
$$\underset{E}{\text{skew symm}} = \underbrace{\text{skew symm} \cdot R_T(180°)}_{\text{this still counts a pure rotation}}$$

4




$$Q = P + T, \quad P = 0$$

$$Q = T$$




$$Q_z \begin{pmatrix} q_x \\ 0 \\ 1 \end{pmatrix} = P_z \begin{pmatrix} p_x \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

1a

$$Q_z q_x = P_z p_x + b$$

$$0 = 0$$

$$Q_z = P_z$$

$$Q_z (q_x - p_x) = b$$

$$Q_z = \frac{b}{q_x - p_x}$$

$q_x - p_x$ = disparity

6. $$\sigma_1 = \sigma_2 > 0$$
$$\sigma_3 = 0$$

7. $$\hat{T} R =$$

$$\begin{pmatrix} 0 & 0 & t\cos\alpha \\ 0 & 0 & -t\sin\alpha \\ -t\cos\alpha & t\sin\alpha & 0 \end{pmatrix} \begin{pmatrix} \overset{\cos(\beta)}{c} & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & t\cos\alpha \\ 0 & 0 & -t\sin\alpha \\ -t\cos\alpha\cos\beta + t\sin\alpha\sin\beta & t\cos\alpha\sin\beta + t\sin\alpha\cos\beta & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & t\cos\alpha \\ 0 & 0 & -t\sin\alpha \\ -t\cos(\alpha+\beta) & t\sin(\alpha+\beta) & 0 \end{pmatrix}$$

<u>given:</u> $\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{pmatrix}$

<u>find:</u> $\alpha$, $\beta$ (you cannot find $t$ because it is the magnitude of the translation.

$$a = t\cos\alpha$$
$$b = -t\sin\alpha \quad \Rightarrow \quad \tan\alpha = -\frac{b}{a}$$
$$c = -t\cos(\alpha+\beta)$$
$$d = t\sin(\alpha+\beta) \quad \tan(\alpha+\beta) = -\frac{d}{c}$$

8. Similar to 7.

9. $E = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

$T = \begin{pmatrix} 0 \\ +1 \\ +1 \end{pmatrix} \quad E^T T = 0$

$R = I$ or $R = R_T(\pi)$

$T = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$ 4 solution

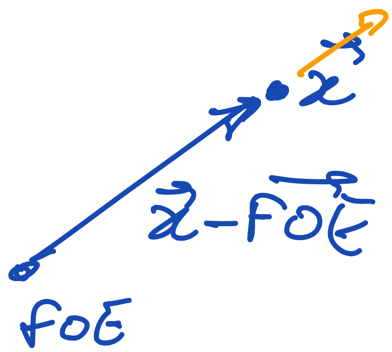
10. $\dot{x} = \frac{1}{Z}(-V_x + x V_z)$

$V_z \neq 0 \quad \dot{y} = \frac{1}{Z}(-V_y + y V_z)$

$\dot{x} = \frac{V_z}{Z}\left(x - \frac{V_x}{V_z}\right)$

$\dot{y} = \frac{V_z}{Z}\left(y - \frac{V_y}{\ldots}\right)$

$$\dot{\vec{x}} = \lambda \left( \vec{x} - \vec{FOE} \right)$$




linear

radial!

11. $TTC = \dfrac{Z}{V_z} =$ [sec]

$$\dot{\vec{x}} = \frac{V_z}{Z} \left( \vec{x} - \vec{FOE} \right)$$

$$\frac{Z}{V_z} = \frac{\| \vec{x} - \vec{FOE} \|}{\| \dot{\vec{x}} \|}$$

$= \dfrac{\text{distance from FOE}}{\text{magnitude of flow}}$

does not hold for $\vec{x} = FOE$

12. $$\begin{pmatrix}\dot x\\ \dot y\end{pmatrix} = \underbrace{\frac{1}{Z}}\overset{2\times3}{A}\,\overset{3\times1}{V} + \overset{2\times3}{B}\,\overset{3\times1}{\Omega}$$

IMU

$$\underbrace{\begin{pmatrix}\dot x\\ \dot y\end{pmatrix} - B\Omega} = \frac{1}{Z} A V$$

$$\dot{\vec{x}}_{TRANS} = \frac{1}{Z} A V$$

find where all flow vectors intersect $\overset{!}{=} V$

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Owe you Nr. 2

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Triangulation

4. $$\lambda q = R\gamma p + T$$

$$\underset{3\times 2}{\begin{pmatrix} q & -Rp \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}} = \underset{3\times 1}{T}$$

$$\begin{pmatrix} \lambda \\ \gamma \end{pmatrix} = \begin{pmatrix} q & -Rp \end{pmatrix}^{+} T$$

2. If $q \sim Rp$

then any $E = \hat{a}R$

for arbitrary $a \in \mathbb{R}^3$

satisfies $q^T E p = 0$

proof:

$$q^T \hat{a} R p$$

$$= q^T \hat{a} R R^T q$$

$$= q^T \hat{a} q$$

$$= q^T (a \times q)$$

$$= 0 \quad \forall a.$$

Hence there exists a three parametric family of solutions $E = \hat{a} R$

where $a = (a_x, a_y, a_z)$

are the 3 parameters