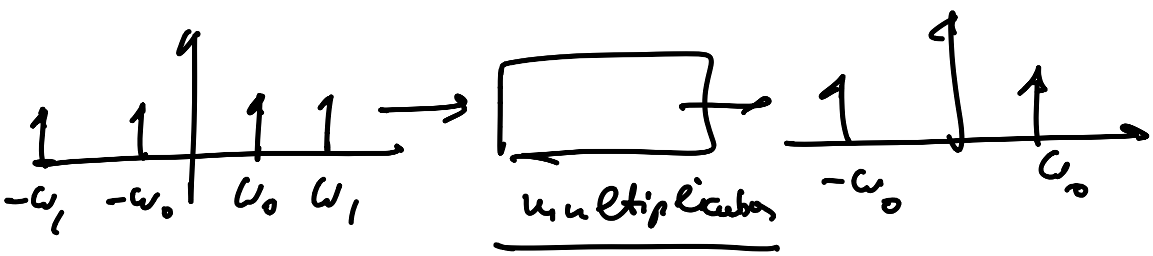
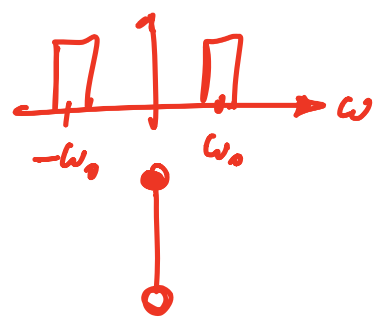
"frequency selective neuron"

$$\cos\omega_0 x + \cos\omega_1 x \longrightarrow \boxed{?} \longrightarrow \cos\omega_0 x$$

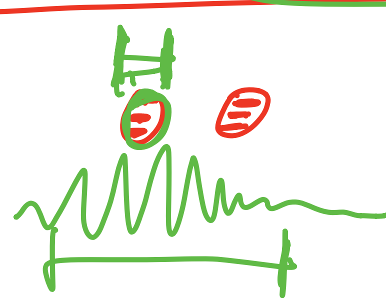
convolution



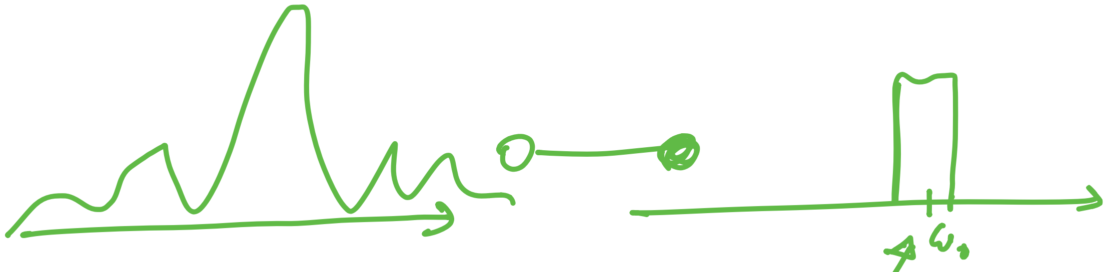


very wide !

↳ we destroy localization



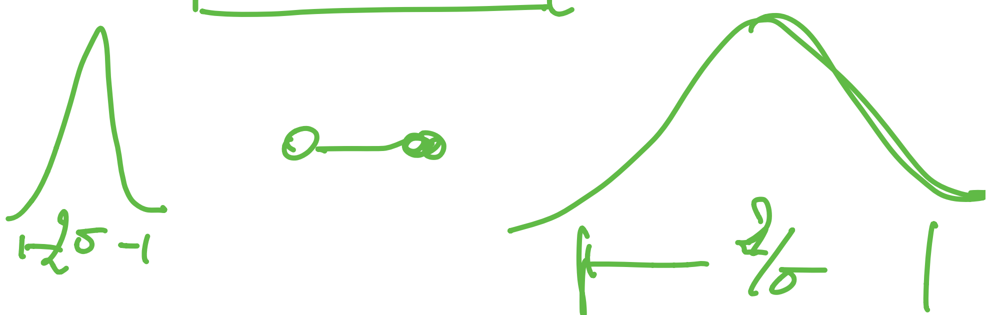
uncertainty principle



long support
= bad localization

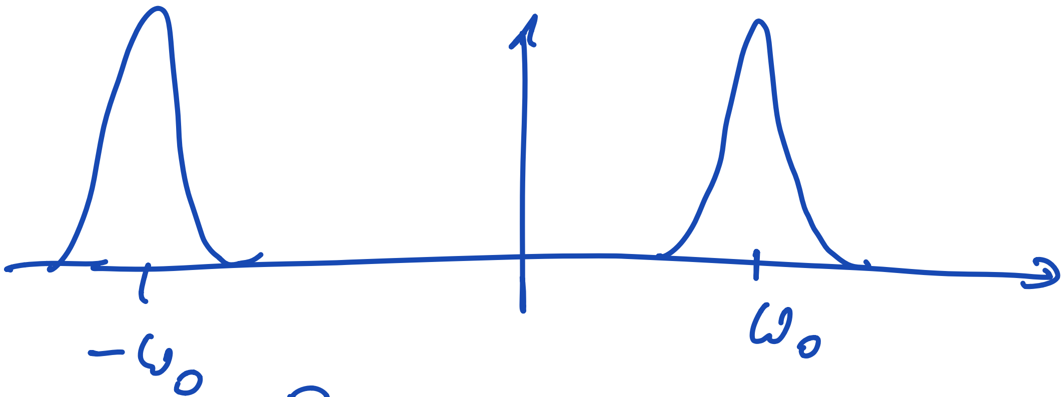
extreme
frequency
selectivity

Gaussian



best function
to deal with this trade off
is the Gaussian

$$\underbrace{\Delta x}_{\text{uncertainty in localization}} \cdot \underbrace{\Delta \omega}_{\text{uncertainty in frequency selectivity}} \geq C$$

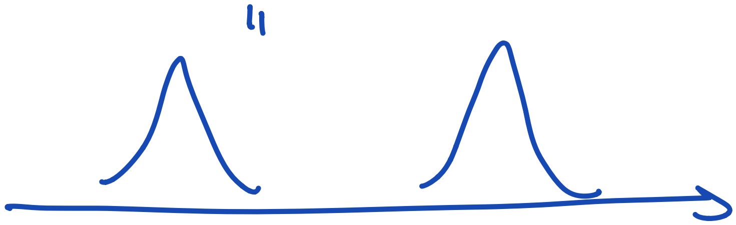


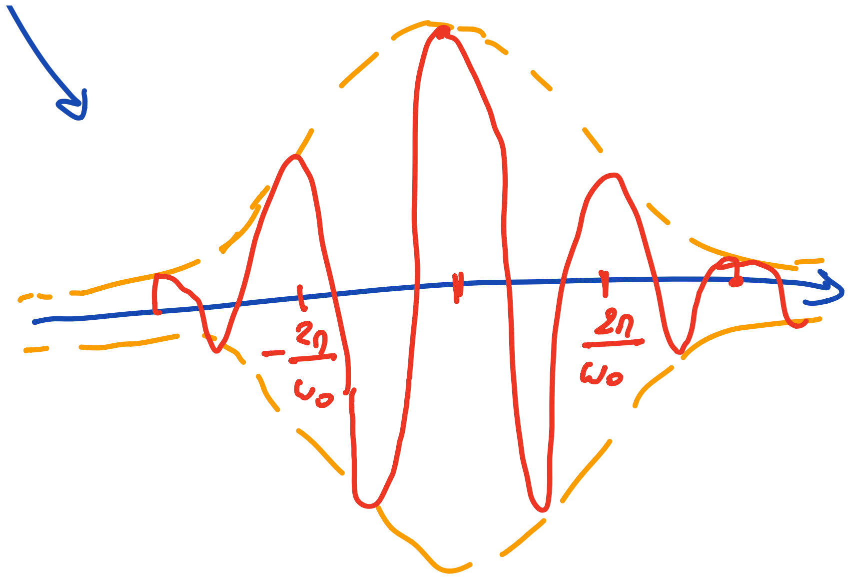
Bandpass

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \cos \omega_0 x$$

$$e^{-\frac{\omega^2\sigma^2}{2}} \ast$$

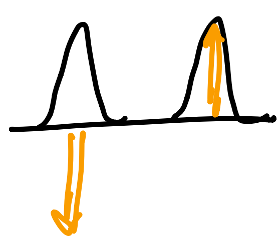
=

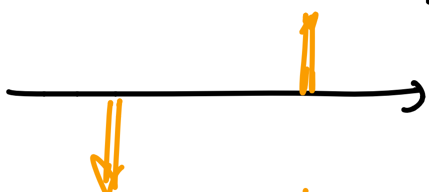




$$\sin\omega_0 t * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \cos\omega_0 t =$$

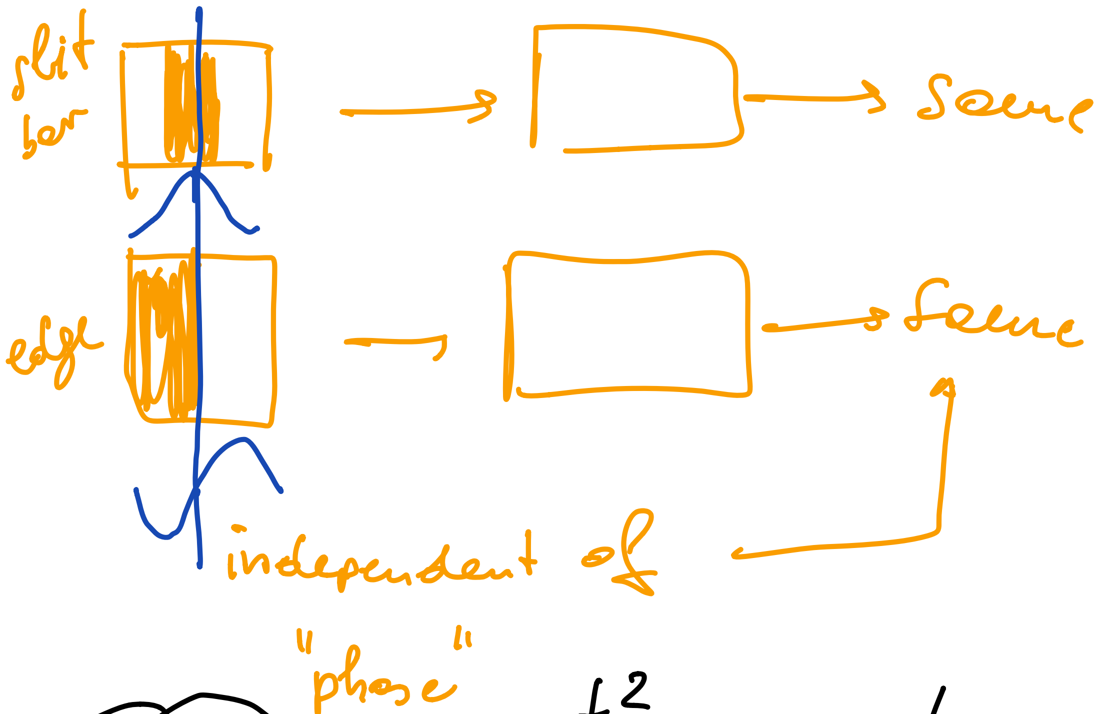
$$\frac{1}{2j}\left(-\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right)\cdot$$





$$\left(-\frac{1}{\omega_0}\frac{d\cos}{dt}\right) * \left(\quad\right)$$

$$= -\frac{1}{\omega_0}\frac{d}{dt}\left(\overbrace{\cos * (\quad)}^{\cos}\right) = -\frac{1}{\omega_0}\sin\omega_0 t$$



Answer $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \underbrace{e^{j\omega_0 t}}_{\cos\omega_0 t + j\sin\omega_0 t}$

$\boxed{\text{Gabor function}}$

$\circ\!-\!\bullet$

$$e^{-\frac{\sigma^2(\omega-\omega_0)^2}{2}}$$

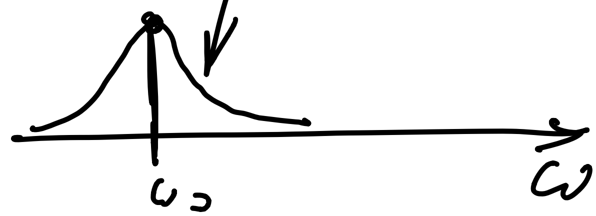
$\parallel$

$$f(t) * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} e^{j\omega_0 t} = Re + j\,Im$$

$$\sqrt{Re^2 + Im^2}$$

 $*$ Gabor

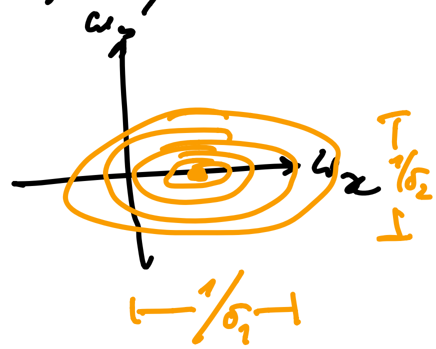
$\frac{\omega_0}{2\pi}$

$=$ 

2D Gabor

$$\frac{1}{\sigma_1 \sigma_2 2\pi} e^{-\frac{x^2}{2\sigma_1^2} - \frac{y^2}{2\sigma_2^2}} e^{j\omega_1 x}$$

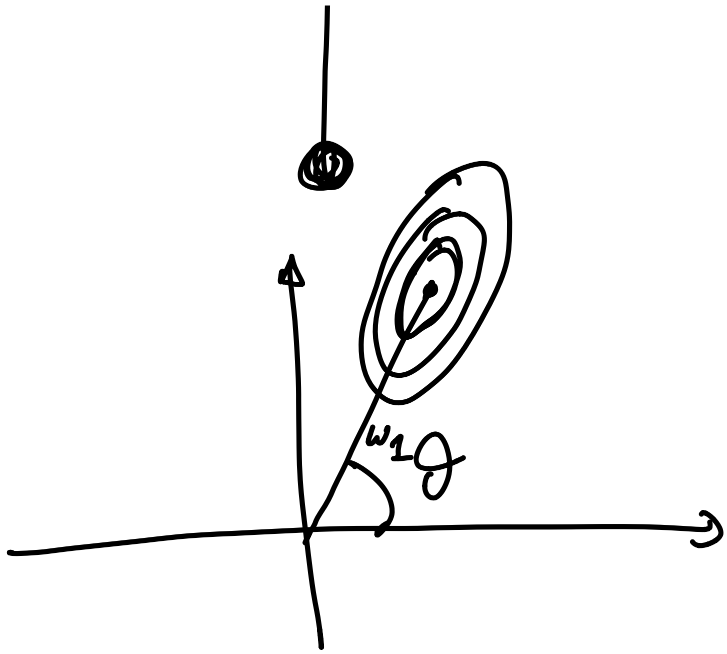
frequency domain



rotate

minus

$$e^{-\frac{(x\cos\theta + y\sin\theta)^2}{2\sigma_1^2} - \frac{(x\sin\theta + y\cos\theta)^2}{2\sigma_2^2}} \quad e^{j\omega_1 (x\cos\theta + y\sin\theta)}$$

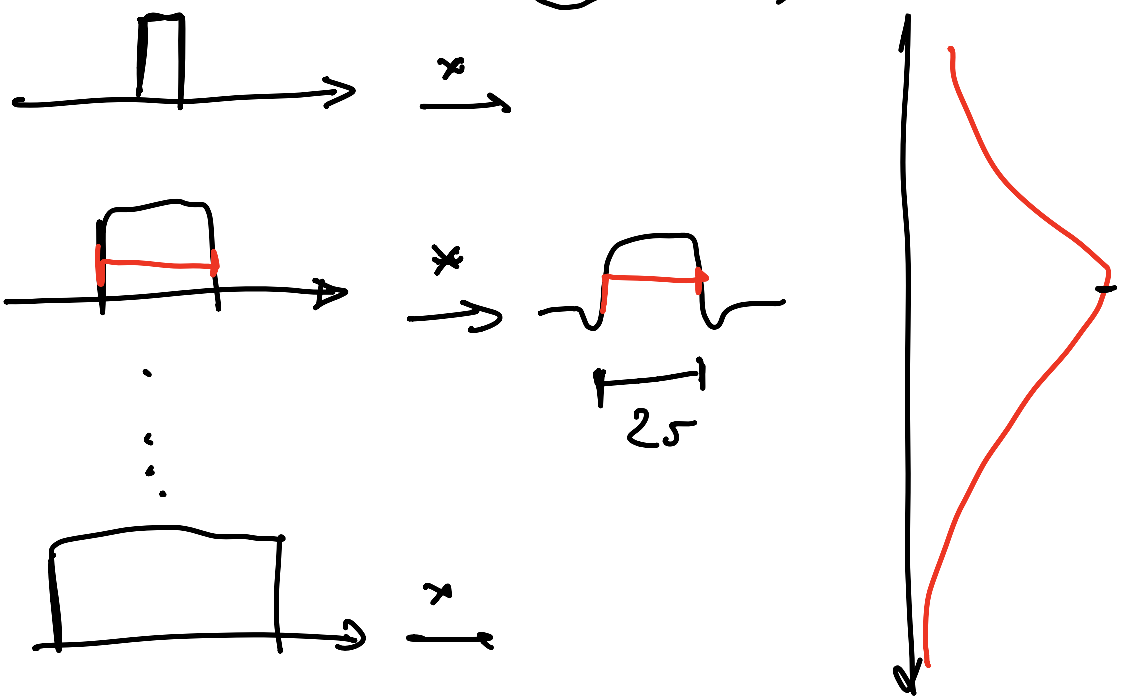


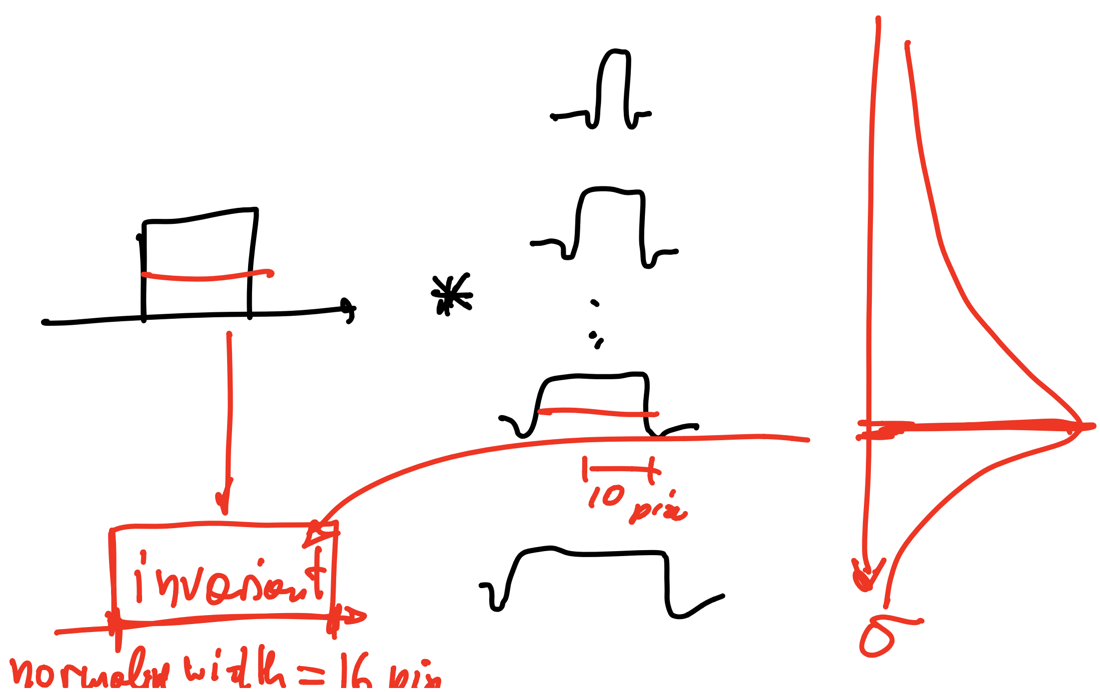
$$f(x, y) \longrightarrow f\left(R(\theta)^T \begin{pmatrix} x \\ y \end{pmatrix}\right)$$

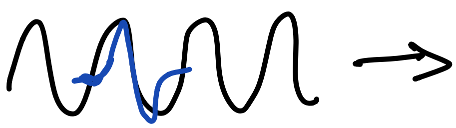
$$f(x) \longrightarrow f(x - T)$$

Scale ~~invariance~~ selection

(SIFT)

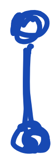




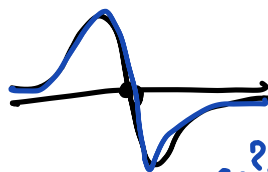


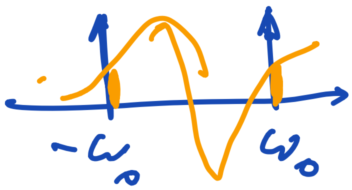
$$\boxed{\frac{dg_\sigma}{dx}}$$

$$-\omega_0 e^{-\frac{\omega_0^2\sigma^2}{2}} \sin \omega_0 t$$



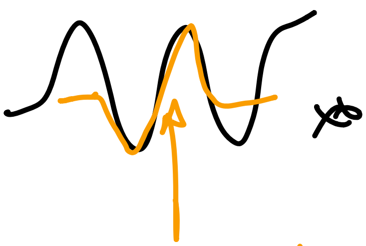
$*$





$$\cdot \; (j\omega)\, e^{-\frac{\omega^2\sigma^2}{2}} =$$

derivative
Fourier

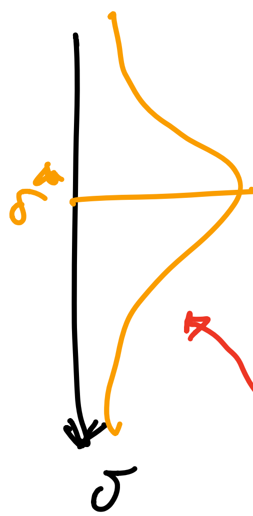


ω_0 matches σ^*

$\frac{dg_{\sigma=1}}{dx}$

$\sigma=2$

$\sigma=3$



unfortunately $e^{-\frac{\omega_0^2\sigma^2}{2}}$

does not follow their pattern

Solution for scale selection (meaning exhibiting a maximum as a function of σ) is to normalize the Gaussian derivative!

$$\boxed{\frac{d^m g_n}{dx^m} = \sigma^m \frac{d^m g}{dx^m}}$$

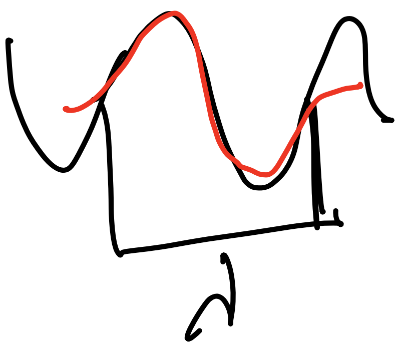
$$\sigma e^{-\frac{\sigma^2 \omega_0^2}{2}}$$

σ^*

$$\cos \omega_0 x * \left(\sigma \frac{dg}{dx}\right) =$$

$$\frac{\partial}{\partial \sigma} \sigma e^{-\frac{\sigma^2 \omega_0^2}{2}} = e^{-\frac{\sigma^2 \omega_0^2}{2}} + \sigma\left(-\frac{2\sigma\omega_0^2}{2}\right) e^{-\frac{\sigma^2 \omega_0^2}{2}} = 0$$

$$1 - \sigma^2 \omega_0^2 = 0$$



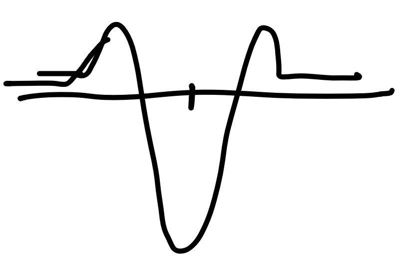
$$\sigma^* = \frac{1}{\omega_0} = \frac{\lambda}{2\pi}$$

wave length

To detect blobs

we convolve with

the Laplacian of Gaussian



$$\frac{\partial^2 g_\sigma}{\partial x^2} + \frac{\partial^2 g_\sigma}{\partial y^2}$$

and take the maximum

in (x, y) and then in σ!

On can show that

$$\frac{\partial^2 g}{\partial x^2} = \sigma \frac{\partial g}{\partial \sigma}$$

Diff. of Gaussians

LoG DoG