CIS580 Problem Set 6

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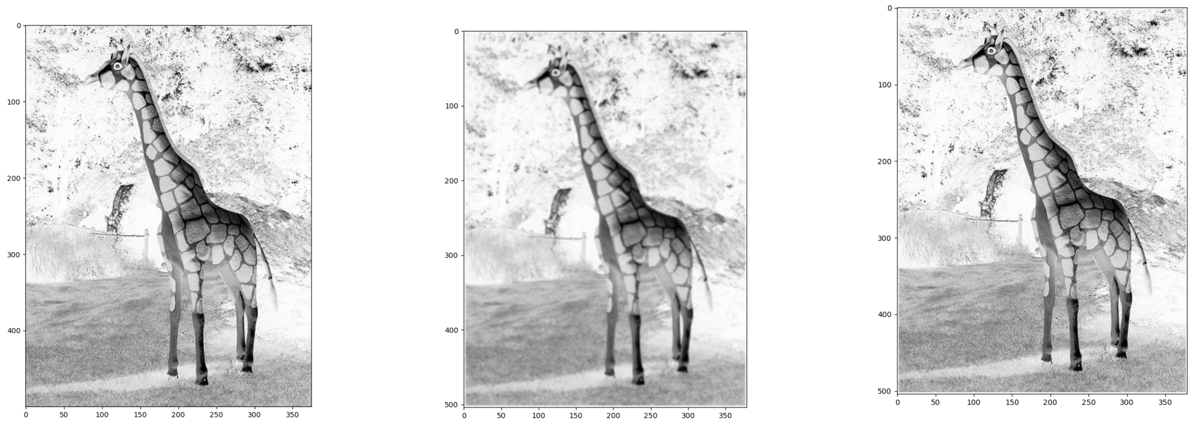
Contents

1 Convolution of image with a Gaussian



(a) Original Image (b) $\sigma = 1$ (c) $\sigma = 0.1$

Figure 1: Three simple graphs

The difference between the three images included above is that by increasing σ from $\sigma = 0$ (original image) to 0.1 and 1, we increase the Gaussian blur applied to the image. This blurring reduces the image noise and detail.

2 Convolution of Gaussians

We know that the 1-D Gaussian $g(t)$ is:

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$

Convolving two Gaussian kernels with σ_1 and σ_2 standard deviations:

$$g_1(t) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$$
$$g_2(t) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_2^2}}$$

$$(g_1 * g_t)(t) = \int_{-\infty}^{\infty} \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}} \cdot \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(t-x)^2}{2\sigma_2^2}} \, dx$$

$$= \int_{-\infty}^{\infty} \frac{e^{-\frac{t^2}{2\sigma_1^2}} \cdot e^{-\frac{(t-x)^2}{2\sigma_2^2}}}{\sigma_1\sigma_2(\sqrt{2\pi})^2}$$

$$= \frac{1}{\sigma_1\sigma_2\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2\sigma_2^2}}$$

The above result shows that convolving two Gaussian kernels g_1 and g_2 with standard deviations σ_1 and σ_2 respectively results in a Gaussian kernel with standard deviation $\sigma_1 \cdot \sigma_2$.

3 Convolution of Step Edge with Gaussian derivative

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$

$$\implies g'(t) = \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3}$$

$$(g' * h)(t) = \int_{-\infty}^{0} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot \frac{-H}{2} \cdot dx + \int_{0}^{\infty} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot \frac{H}{2} \cdot dx$$

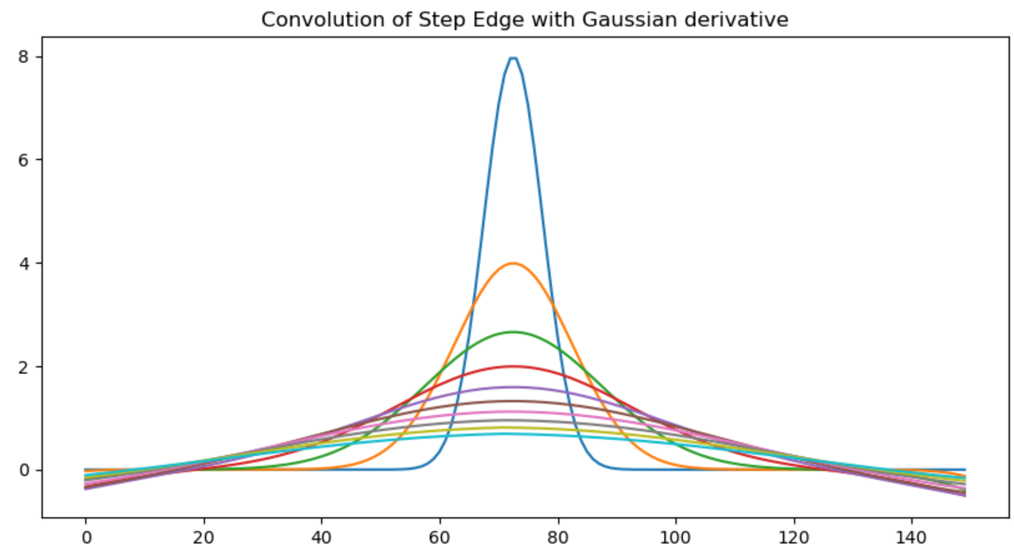


Figure 2

4 Box Function

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$

$$\implies g'(t) = \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3}$$

$$(g' * h)(t) = \int_{-\infty}^{\frac{-1}{2}} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot 0 \cdot dx + \int_{\frac{-1}{2}}^{\frac{1}{2}} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot dx + \int_{\frac{1}{2}}^{\infty} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot 0 \cdot dx$$

$$= \int_{\frac{-1}{2}}^{\frac{1}{2}} \frac{-te^{\frac{-(t^2)}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3} \cdot dx$$

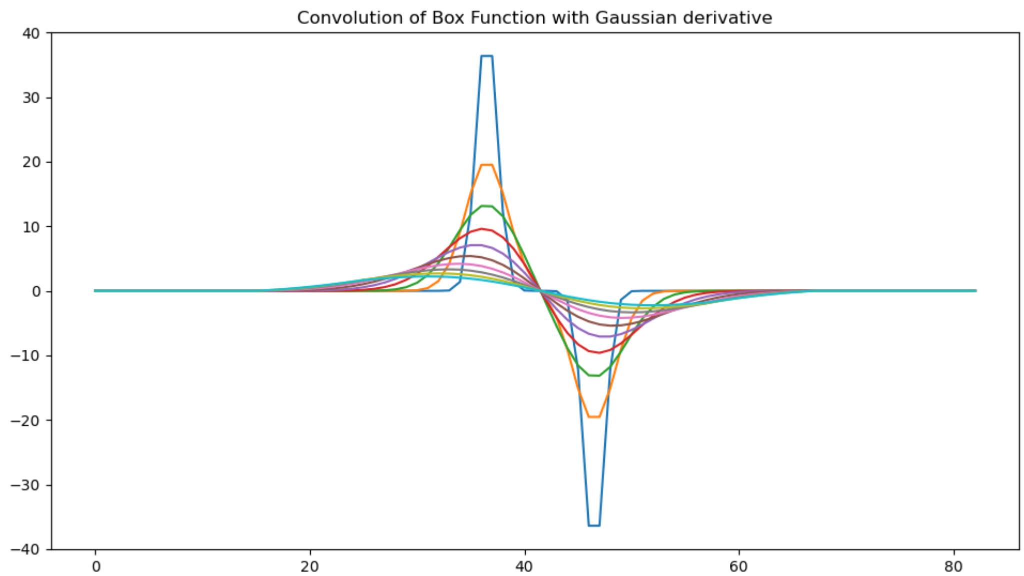

Convolution of Box Function with Gaussian derivative
40
30
20
10
0
−10
−20
−30
−40
0
20
40
60
80

Figure 3

5 1D FFT Quiz

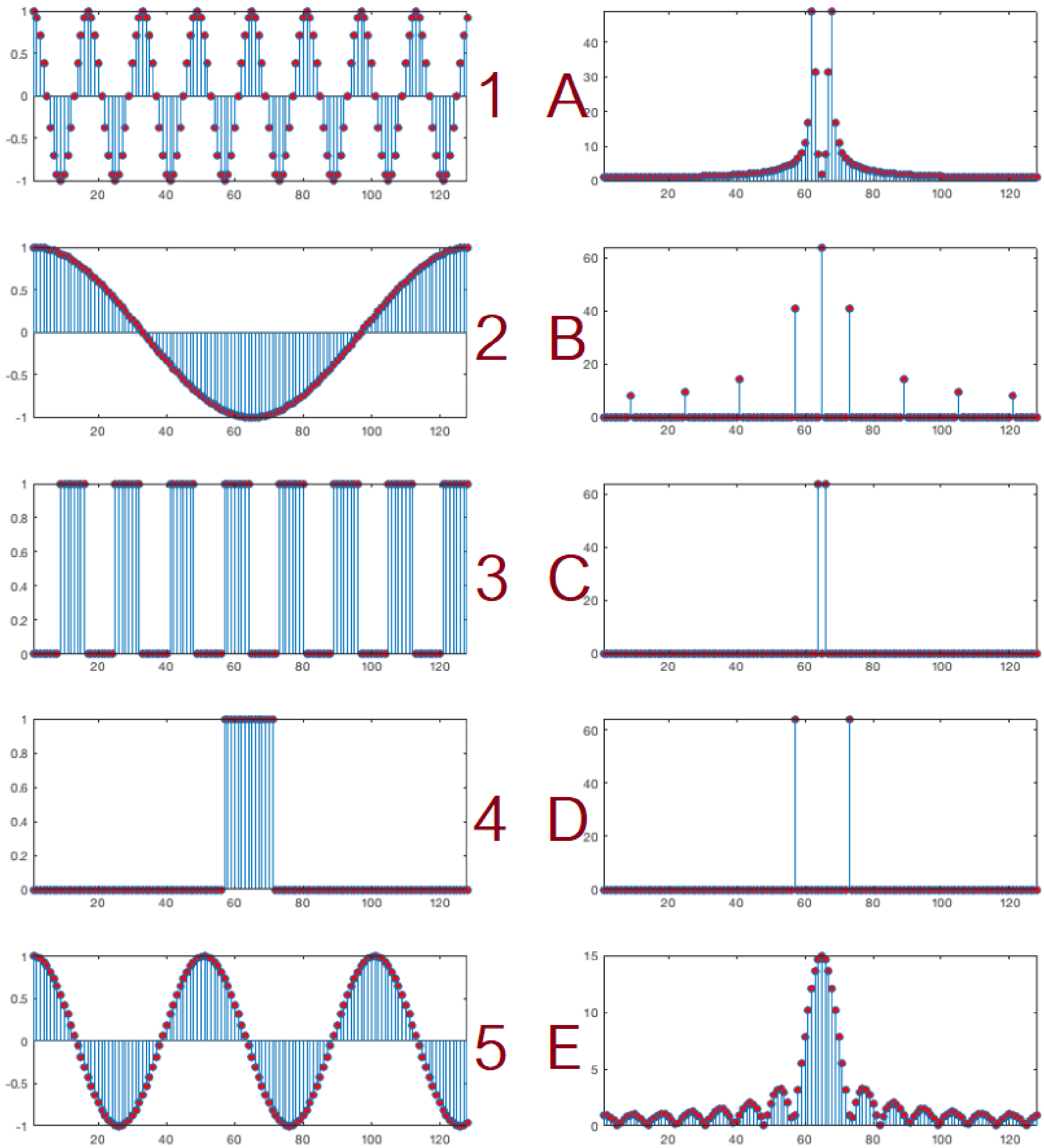


Figure 4

Function	FFT Plot
1	D
2	C
3	B
4	E
5	A

My reasoning for the above matching is that in frequency domain, the distance between the two peaks is proportional to the frequency of the curves.

6 2D Fourier Transform

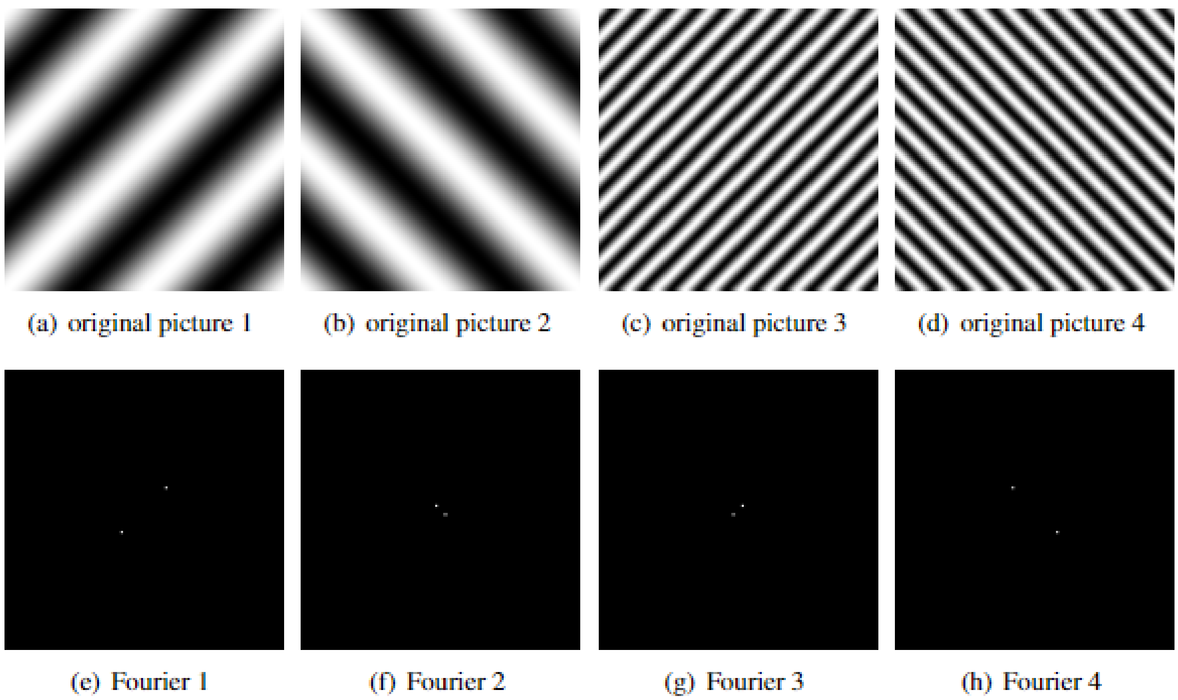


Figure 5: Functions and corresponding FFTs

Function	FFT Plot
A	F
B	G
C	H
D	E

My reasoning for the above matching is that the separation of the white dots in the frequency domain is proportional to the frequency of the black and white signals. This reasoning is similar to the 1-dimensional case where the distance between the two peaks in the frequency domain was proportional to the frequency of the curves.

7 Filter Design