Q1 a) Given : ① O (2021, 1778)

② distance from projection center to origin is 111 mm

③ $K = \begin{pmatrix} 3275 & 0 & 2016 \\ 0 & 3275 & 1512 \\ 0 & 0 & 1 \end{pmatrix}$

To find : T where $\lambda \begin{pmatrix} u_{pix} \\ v_{pix} \\ 1 \end{pmatrix} = K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$

$\Rightarrow \lambda \begin{pmatrix} 2021 \\ 1778 \\ 1 \end{pmatrix} = K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2021 \\ 1778 \\ 1 \end{pmatrix} \sim K \begin{pmatrix} r_1 & r_2 & r_3 & t \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2021 \\ 1778 \\ 1 \end{pmatrix} \sim K (t)$

Given K, we can solve for T as

$T = \begin{pmatrix} 2021 \\ 1778 \\ 1 \end{pmatrix} \cdot K^{-1} \cdot 111$ lambda

$\Rightarrow T = \begin{bmatrix} 68.5 \\ 60.3 \\ 0 \end{bmatrix}$

b) Given R, find position of camera w.r.t

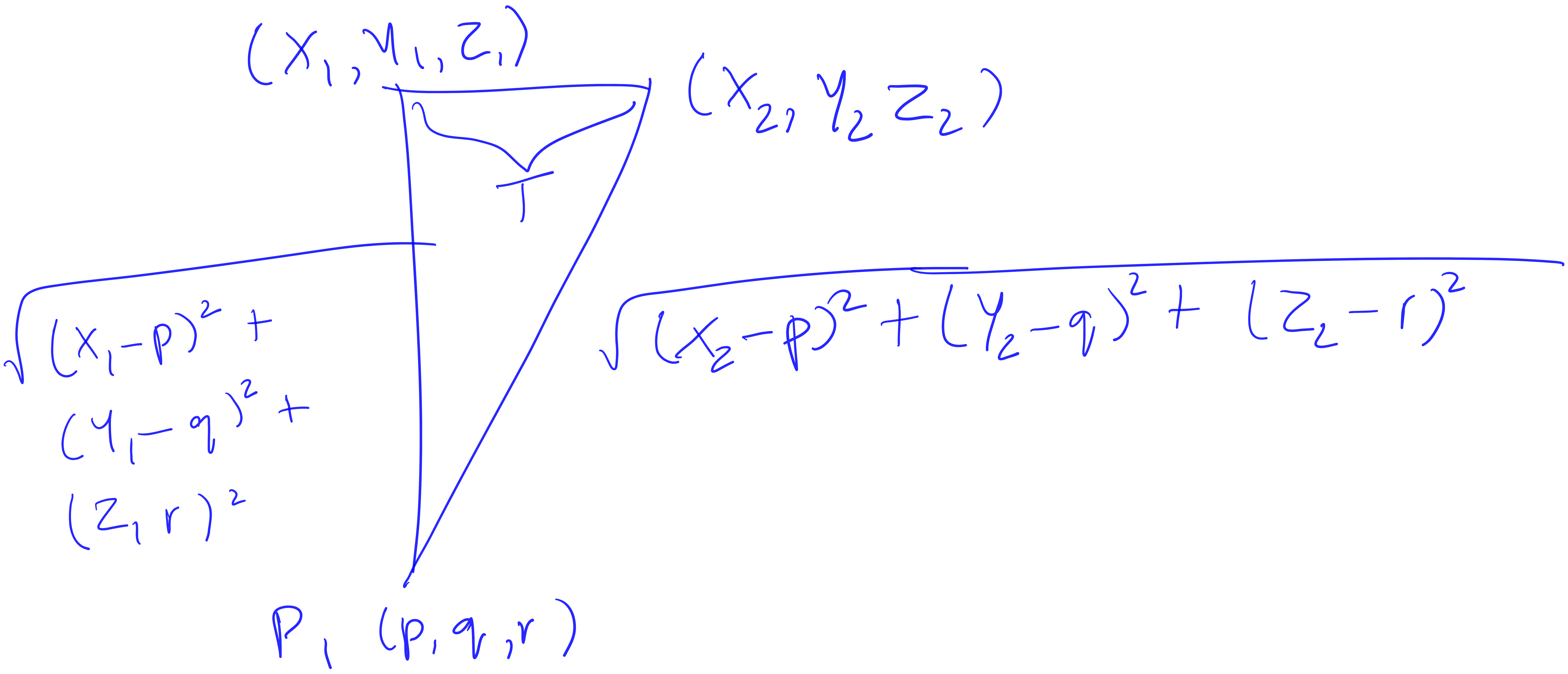
Q1 world coordinate system

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

We ~~know~~ assume pixel coordinates of camera are $(0, 0, 0)$ and use the projection equation to find X_{mm}, Y_{mm}, Z_{mm}

$$\Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = K \begin{pmatrix} R & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

(5)



T should be equal to $\left[\begin{matrix} \sqrt{(x_2-x_1)^2}, \\ \sqrt{y_2-y_1{}^2}, \\ \sqrt{z_2-z_1{}^2} \end{matrix} \right]$

Since when the drone is on top of the origin of the Siemens ~~star~~ star, the lines will appear parallel

2 (a) $K \neq I$

TODO : Rotation for projections of the points

$$\boxed{\begin{array}{c} (X_0, Y_0, Z_0) \longrightarrow (u_0, v_0, w_0) \\ \hline (X, Y, Z) \longrightarrow (u, v, w) \end{array}}$$

We know $\lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix} = K \left(R \; \underline{I} \right) \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$

$= \underline{I}$dentity

$\because$ no translation

$\Rightarrow$ Rotation equation for (u_0, v_0, w_0):

$$\lambda \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix} = K R \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$ where R is specified in the problem

Likewise for $\begin{pmatrix} u \\ v \\ w \end{pmatrix}$:

$$\lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix} = K R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

④ ⓑ Assume $K = I$

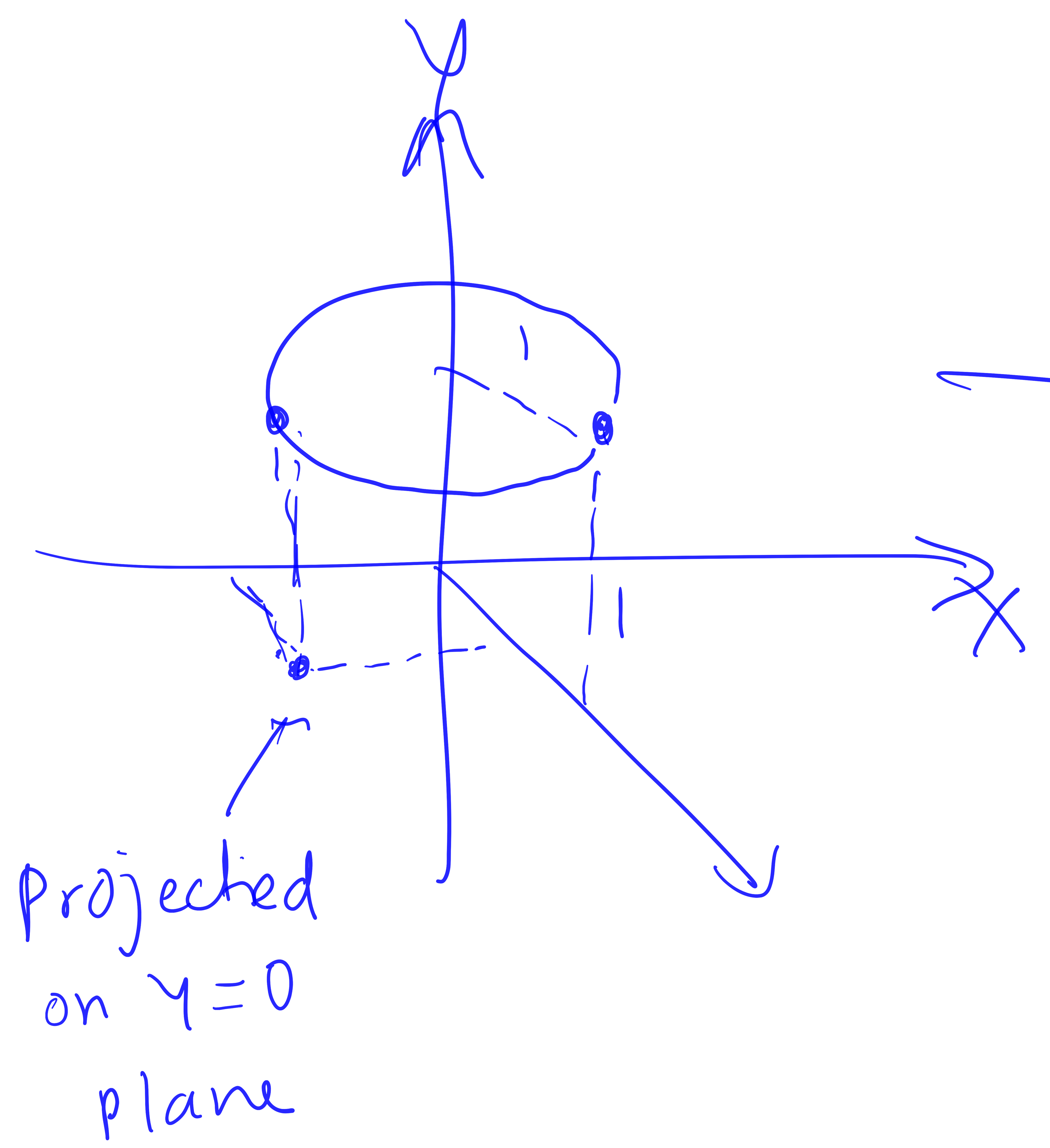
$(u_0, v_0, w_0) = (0, 1, 1)$ on y-axis

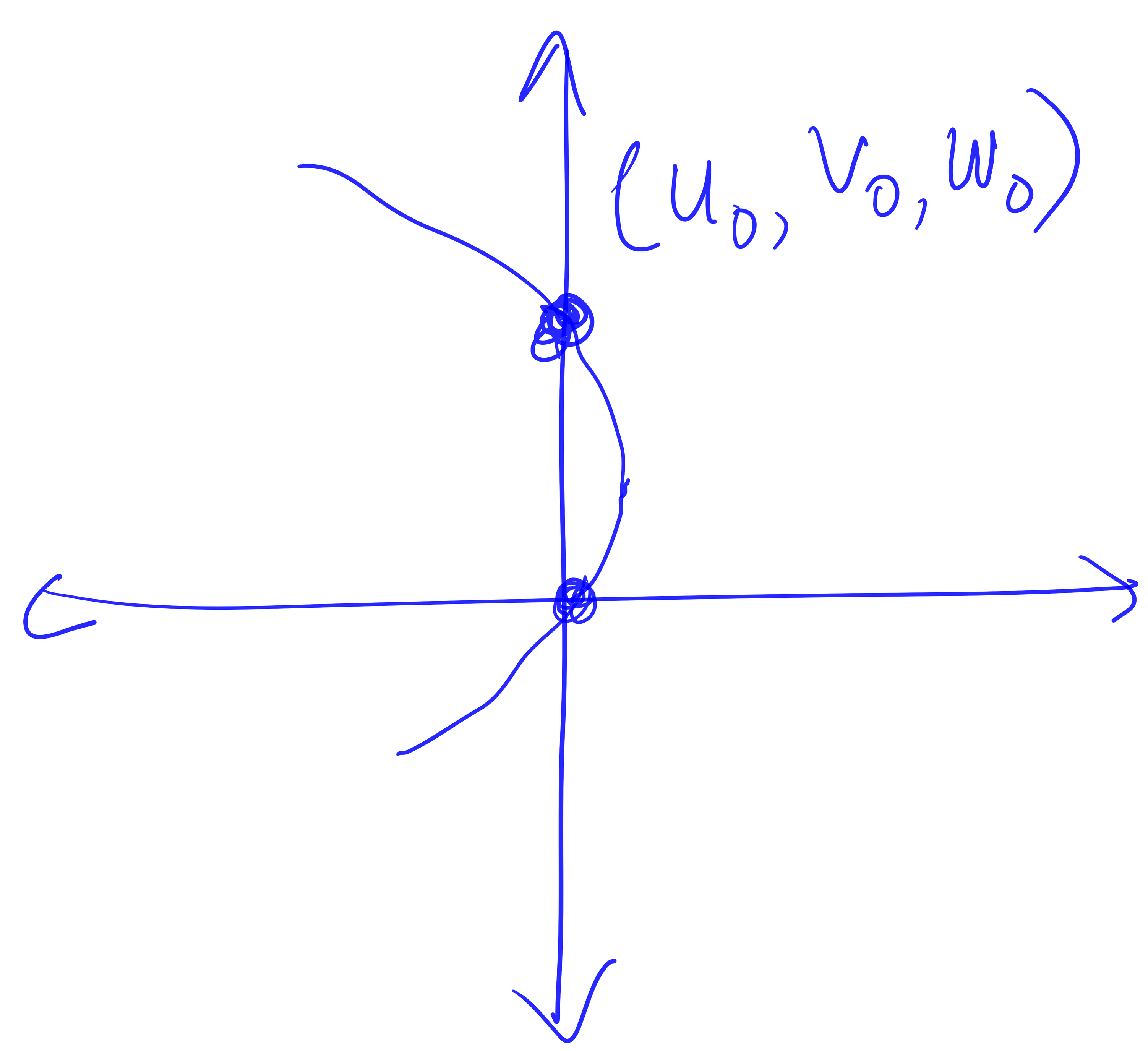
Show that the trajectory of this point in the real ~~world~~ plane is a hyperbola

$$\begin{bmatrix} u/w \\ v/w \\ 0 \end{bmatrix} \qquad \begin{bmatrix} u \\ v \\ w \end{bmatrix} \lambda = KR \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \lambda = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\mathbb{R}^3$ $\qquad$ $\mathbb{R}^2$



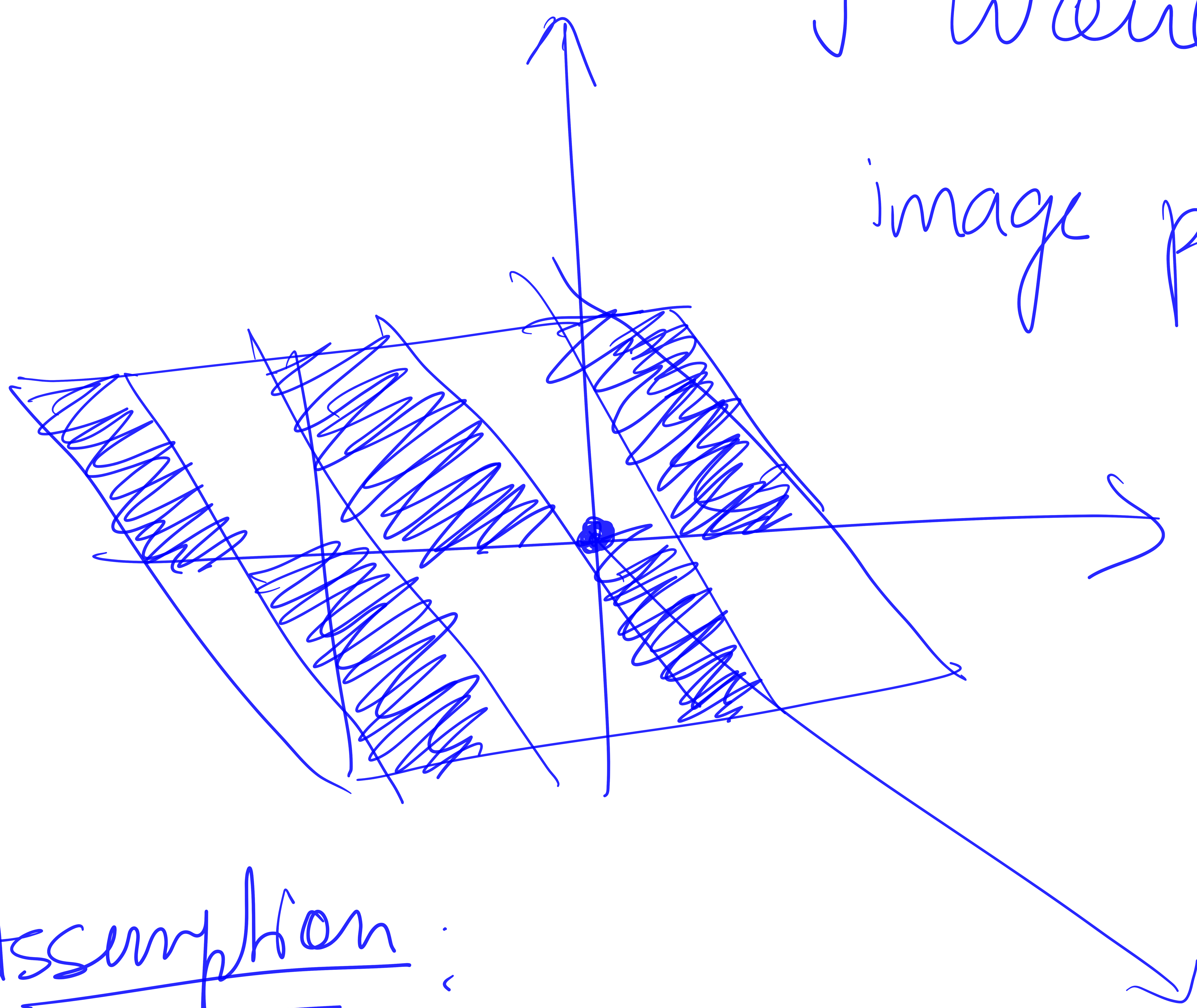


By plotting a few points, we see it is a hyperbola

4 c) $K = I$

What image point on a checkerboard to choose such that resulting trajectory is still a hyperbola

(cannot undergo rotation & translation simultaneously)



I would select an image point on y-axis of (0, 1, 0) so pure rotation and no translation

Assumption:

our camera cannot undergo both rotation & translation at the same timestep, but can rotate alone or translate alone.

③ ⓐ

$$\frac{\frac{AB}{AC}}{\frac{BC}{BA}} = \frac{\boxed{\frac{A_wB_w}{A_wC_w}}}{\frac{B_wC_w}{B_wA_w}} \ ?$$

$$\Rightarrow \frac{\frac{100\,px}{100\,\cancel{AB} + 200\,px}}{\frac{200}{100}} \Rightarrow \frac{1\cancel{00}}{3\cancel{00}} \times \frac{1\cancel{00}}{2\cancel{00}} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{6} = \frac{(AB)_w^2}{(AC)_w(BC)_w} = \frac{(AB)_w^2}{(10)(\underset{10}{\cancel{AC}} - AB)_w}$$

$$\Rightarrow \frac{10}{6} = \frac{AB^2}{\underset{10}{\cancel{AC}} - AB}$$

$$\Rightarrow \underset{10}{\cancel{AC}} - AB = \frac{6}{10} AB^2$$

$$\Rightarrow \cancel{x =} \ AB = -5, \boxed{\frac{10}{3} \text{ yards}}$$

(b) $\cancel{AC}$

$$\frac{\left(\frac{AY}{CY}\right)}{\left(\frac{BC}{BY}\right)} = \frac{\left(\frac{AY}{CY}\right)_w}{\left(\frac{BC}{BY}\right)_w}$$

$$\Rightarrow \frac{\left(\frac{\overset{100}{\cancel{AB}} + \overset{200}{\cancel{BC}} + CY}{CY}\right)}{\left(\frac{200}{\underset{200}{\cancel{BC}} + CY}\right)} = \frac{\left(\frac{BY + AB}{BY - BC}\right)}{\left(\frac{AC - AB}{10}\right)}$$

$$\Rightarrow \frac{300 + CY}{CY} \times \frac{200 + CY}{200}$$

continued...

$$\frac{300+CY}{CY} \cdot \frac{(200+CY)}{200} = \frac{\dfrac{10+\frac{10}{3}}{\cancel{10}-\left(\cancel{10}-\frac{10}{3}\right)}}{\dfrac{10-\frac{10}{3}}{10}}$$

$$= \frac{\frac{\cancel{40}^{\,4}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{10}}}{\frac{2}{3}} = \frac{4 \cdot 3}{2} = \frac{12}{2} = 6$$

$$\Rightarrow CY = \boxed{100,}\,600$$

PX

② Given

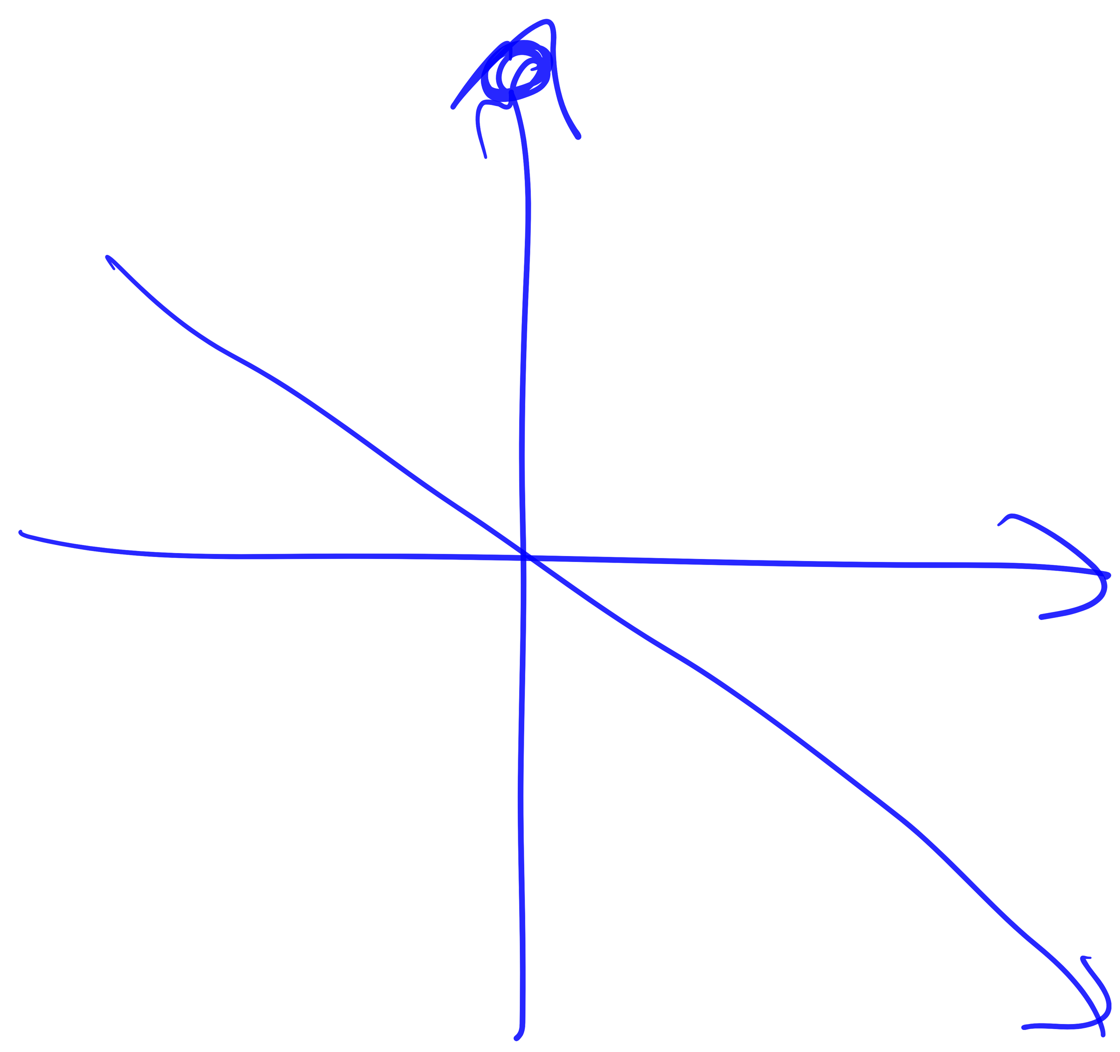
$(u_0, v_0) = (0, 0)$

$f = 600$

Target $(u_T, v_T) = (0, 200\sqrt{3})$

on homogenous coordinates

$$\begin{bmatrix} 0 \\ 200\sqrt{3} \\ 1 \end{bmatrix}$$



Rotation needs to be about Z axis, so general form is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$