What do I have to know for the midterm?

- All material starting with Structure from Motion. Pyramids and Sampling Theorem are excluded (they are in the slides).
- Know everything about SfM as outlined in the questions below. Bundle adjustment is excluded.
- How to compute the derivatives of the Gaussian.
- How to compute the convolution of a step function and a box function with Gaussian derivatives.
- How to normalize a Gaussian derivative so that the intrinsic scale of a signal can be selected.
- How to rotate a gaussian derivative.
- You have to know the convolution theorem because it simplifies the computation of convolutions. If a Fourier transform is needed we will give it to you (but feel free to look it up during the exam). It is good to know that we can use the Fourier Transform when we encounter cos, sin, or Gaussians.
- You do not need to know how to compute a Fourier transform or any other Fourier theorem. We will not have a Fourier quiz.
- Study slides of optical flow and 3D velocities.

$$
\begin{aligned}
R_x(\theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \\
R_y(\theta) &= \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \\
R_z(\theta) &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

Geometry Review Questions including 3D velocities

1. Explain why the translation magnitude cannot be recovered from two calibrated views?

2. Explain what happens if you try to compute the essential matrix from two calibrated views but one view is a pure rotation of the other.

3. How can you recognize from the essential matrix of two calibrated views without computing the SVD that the motion is a pure translation?

4. Explain triangulation: How can we find depth from two views given intrinsics, R, and T.

5. Assume that two views differ only by a translation $T = (b, 0, 0)$:

$$Q = P + T$$

   Given one calibrate correspondence $(p_x, 0)$ and $(q_x, 0)$ and b compute the depth Q_z of this point.

6. State the necessary and sufficient condition for a matrix to be essential (decomposable into the product of an antisymmetric matrix and an orthogonal matrix).

7. Suppose we know that the camera always moves (rotation and translation) in a plane parallel to the image plane.

$$R = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T = \begin{pmatrix} t\cos\alpha & t\sin\alpha & 0 \end{pmatrix}^T, \tag{1}$$

   with t the magnitude of the translation. Show that the essential matrix $E = \widehat{T}R$ is of special form

$$E = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{pmatrix}$$

   Without using the SVD-based decomposition, find a solution to α and β in terms of (a, b, c, d).

8. (10pts) A camera translates only along the optical axis and rotates only around the optical axis. Write the essential matrix between two views. What are the necessary and sufficient condition (in terms of E_{ij}) for a 3x3 matrix to be an essential corresponding to this case (translation in z and rotation around Z-axis).

9. If $E = \hat{T}R$ and

$$E = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

   find T (up to a scale) and R.

10. Explain why a pure translational optical flow field is radial.

11. Explain how we find the Time to Collision.

12. An IMU gives us an estimate of the angular velocity in the camera coordinate system. Explain how we can compute the translation direction.

Image Processing Review Questions

1. **Convolution of two Gaussians** The 1D Gaussian function reads

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}. \tag{2}$$

We give the Fourier transform of a Gaussian as

$$g(t) \circ\!\!-\!\!\bullet\; e^{-\frac{\sigma^2\omega^2}{2}} \tag{3}$$

Prove that the convolution of two Gaussians with σ_1 and σ_2 respectively is a Gaussian. Find its σ in terms of σ_1 and σ_2.

2. Compute and plot the convolution of a step edge with the second derivative of a Gaussian.

3. Compute the convolution of the 1D box function

$$h(t) = \begin{cases} 1/a, \text{if } |t| \leq a/2 \\ 0, \text{otherwise.} \end{cases} .$$

with the 2nd derivative of the Gaussian.

4. Suppose that you are sampling the 1D first derivative of a Gaussian with standard deviation σ in order to produce a filter of length 2M+1 so there are M taps for positive values and M taps for negative values. As a practitioner you want to have the minimum M possible so that your mask is not unnecessarily long. But this depends on σ. To capture well the support we are asking that the area under the first derivative from -M to 0 captures a fraction α (for example 0.9) of the total area from $-\infty$ to 0.

   (a) Find a formula for the M as a function of σ and α.

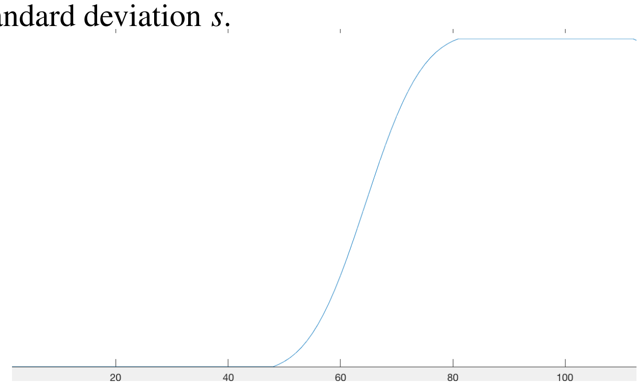
   (b) Assume that you can use only masks with 3 taps ($M = 1$). Compute which σ corresponds to $M = 1$ if we want $\alpha = 0.9$. How many times would we need to convolve with the $M = 1$ mask to achieve the equivalent of a convolution with $\sigma = 2$.

5. Compute the convolution of a step edge

$$h(t) = \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{if } t < 0 \end{cases} .$$

with the second derivative of a Gaussian with standard deviation σ. All computations should be done in the continuous domain. Plot the result.

6. Assume that an image was blurred and that the step appears as already convolved with a Gaussian with standard deviation s.



a. Show that its convolution with the first derivative of a Gaussian with standard deviation σ is a Gaussian. Find the standard deviation of that Gaussian.

b. Compute the convolution of the blurred step edge with the second derivative. The response at $t = 0$ is a function of σ, let us denote it as $f(\sigma)$. Does $f(\sigma)$ decrease or increase with the the blurriness s of the input edge?

c. Compute the derivative $\frac{df(\sigma)}{d\sigma}$. For which σ does this derivative vanish?

d. The normalized 1st derivative of Gaussian is the original 1st derivative multiplied by σ. If we replace the old 1st derivative with the normalized one the result will be also multiplied by σ so it will be $\sigma f(\sigma)$. Show that $\sigma f(\sigma)$ will have a maximum at $\sigma = s$, i.e., we can recover the width of the box by finding the maximum over σ's.

7. Given the signal $s(t) = cos(\omega t + \phi)$ and the Gabor Filter $g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}(\cos(\omega t) + j\sin(\omega t))$: Compute the convolution $(s * g)(t)$. Show that the magnitude of the convolution $|(s * g)(t)|$ does not depend on ϕ (phase invariant).

8. Consider a wave $f(x, y, t) = \cos(\omega(x - t)) + \cos(\omega(y - vt))$ moving with constant optical flow (u, v). Verify that the brightness change constraint equation

$$\frac{\partial f}{\partial x} u + \frac{\partial f}{\partial y} v + \frac{\partial f}{\partial t} = 0$$

holds for $f(x, t)$.

9. Explain the aperture problem with a sketch as well as an equation.