open OH Sat Mar 6 12 noon

default time : for midterm / gradescope $\boxed{90}$ min + Δt scanning

12^{noon} March 8

Exam will not assume prob/ler!

Open everything

(if you use any theorem or anything that is not in our webpage cite it!)

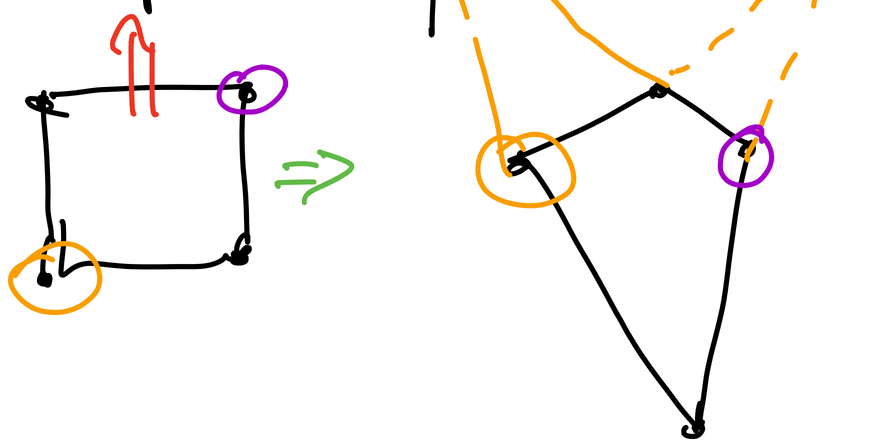
Review

Basic Projective Geometry

line $\ell \sim p \times q$

point $p \sim \ell \times m$

$$p \sim Hq$$



$H \Rightarrow$ horizon

$H\begin{pmatrix}1\\0\\0\end{pmatrix} \sim h_1$

$H\begin{pmatrix}0\\1\\0\end{pmatrix} \sim h_2$

$h_1 \times h_2$

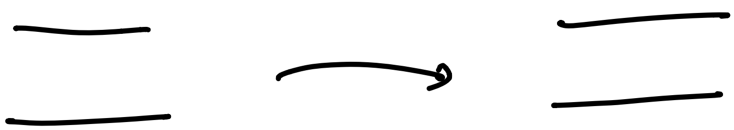
$\|$

horizon

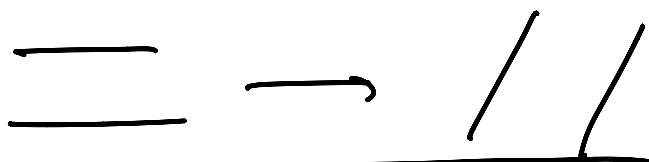
also $H\begin{pmatrix}1\\0\\0\end{pmatrix} \times H\begin{pmatrix}0\\1\\0\end{pmatrix}$

Remark 2: <u>check if H leaves parallel lines parallel</u>

$$H\begin{pmatrix}1\\0\\0\end{pmatrix} \sim \begin{pmatrix}1\\0\\0\end{pmatrix} \leftarrow \text{too restrictive}$$



ok



$$\boxed{H\begin{pmatrix}u\\v\\0\end{pmatrix} \sim \begin{pmatrix}u'\\v'\\0\end{pmatrix}} \quad \forall u,v$$

$$\Updownarrow$$

$$h_{31}u + h_{32}v = 0 \quad \forall u,v$$

$$\Updownarrow$$

$$h_{31} = h_{32} = 0$$

$$\begin{pmatrix}\cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\\ 0 & 0 & 1\end{pmatrix}$$

alternatively: horizon line at infinity → line at infinity

$$\begin{array}{ccc} i & j & k \\ h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \end{array}$$

$$h_1 \times h_2 \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad w = 0$$

$$\begin{pmatrix} h_{21} h_{32} - h_{31} h_{22} \\ -h_{11} h_{32} + h_{12} h_{31} \\ \cdot \end{pmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$$h_{31} = h_{32} = 0$$

Q5 relation between H and K

Assume $u_0 = v_0 = 0$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim H \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} \sim \underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} H}_{= H'} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

$\mathbb{R}^2$

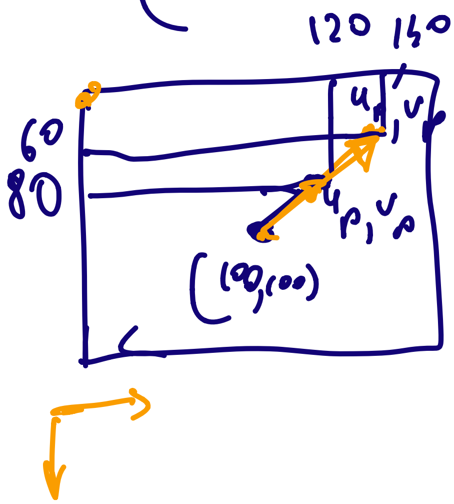
$u'_p = 2 u_p$

$v'_p = 2 v_p$

If $\begin{pmatrix}u_0\\v_0\end{pmatrix} \neq 0$

$$u_p' = 2(u_p - u_0) + u_0$$

$$v_p' = 2(v_p - v_0) + v_0$$



$$\begin{pmatrix}40\\-40\end{pmatrix} \begin{matrix}+100\\+100\end{matrix}$$

$$\begin{pmatrix}140\\-60\end{pmatrix}$$

H'

Image

Q7 $\begin{pmatrix}u\\v\\w\end{pmatrix} \sim \begin{pmatrix}K & 0\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}$

$$Z = AX + BY + C$$

clarification $\begin{pmatrix}u\\v\\w\end{pmatrix} \sim\ ?\ \begin{pmatrix}X\\Y\\1\end{pmatrix}$

$$\begin{pmatrix}K & 0\end{pmatrix}\begin{pmatrix}X\\Y\\AX+BY+C\\1\end{pmatrix} \sim \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix}X\\Y\\1\end{pmatrix}$$

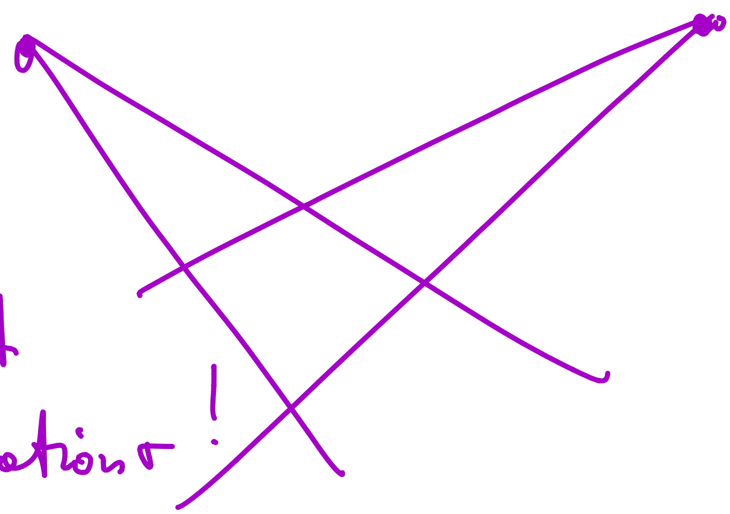
$$\begin{pmatrix} k_1 & k_2 & k_3 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ AX+BY+C \\ 1 \end{pmatrix}$$

$$= X k_1 + Y k_2 + (AX+BY+C) k_3$$

$$= \underbrace{\begin{pmatrix} k_1 + A k_3 & k_2 + B k_3 & C k_3 \end{pmatrix}}_{\text{final answer}} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

Q8

VP invariant to translation!



V2 focal length from two orthogonal vanishing points ?

$u_0 = v_0 = 0$

Assum

$$h_1 \sim K r_1 \sim \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} r_1$$

$$h_2 \sim K r_2 \sim \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} r_2$$

r_2 r_1

$$r_1^T r_2 = 0 \iff h_1^T \begin{pmatrix} 1/f^2 & 0 & 0 \\ 0 & 1/f^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} h_2 = 0$$

$\boxed{\text{is it always possible ?}}$

$$\frac{h_{1x} h_{2x}}{f^2} + \frac{h_{1y} h_{2y}}{f^2} + \underbrace{h_{1w} h_{2w}}_{\neq 0} = 0$$

none of the VP can be at infinity !!!

L1 $H \sim K\begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$

$4p \Rightarrow H \xRightarrow{K} r_1, r_2 \Rightarrow R$

known $\quad T$

Clarify

Problem 1 : Given H'

find r_1, r_2 $\quad r_1^T r_2 = 0 \quad \|r_1\| = \|r_2\|$

s.t $\quad \lambda H' = \begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$

Problem 2 : $\|A - RB\|_F \Rightarrow \min_R$

Procrustes

L3 impossible to localize from 2p