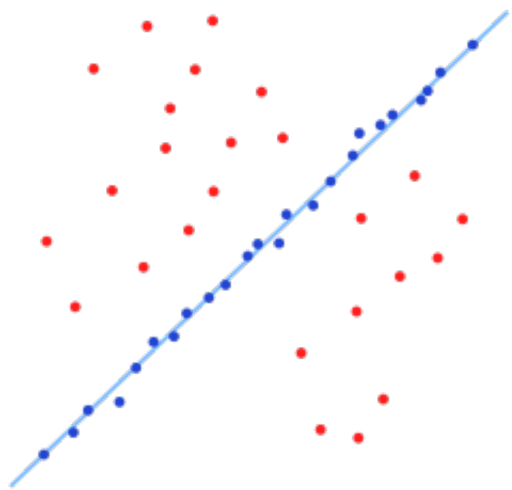
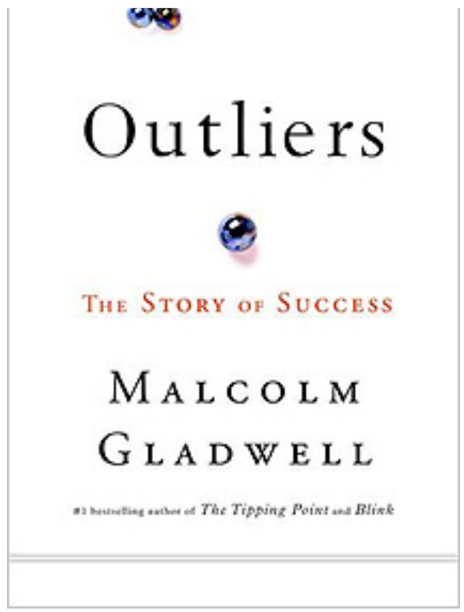
# RANSAC

Let us suppose that our only problem is to detect outliers
A point is an *outlier* if it does not fit the underlying probability likelihood model.

$$e^{-\frac{1}{2\sigma_i^2}(\cos\theta x_i + \sin\theta y_i - d)^2}$$



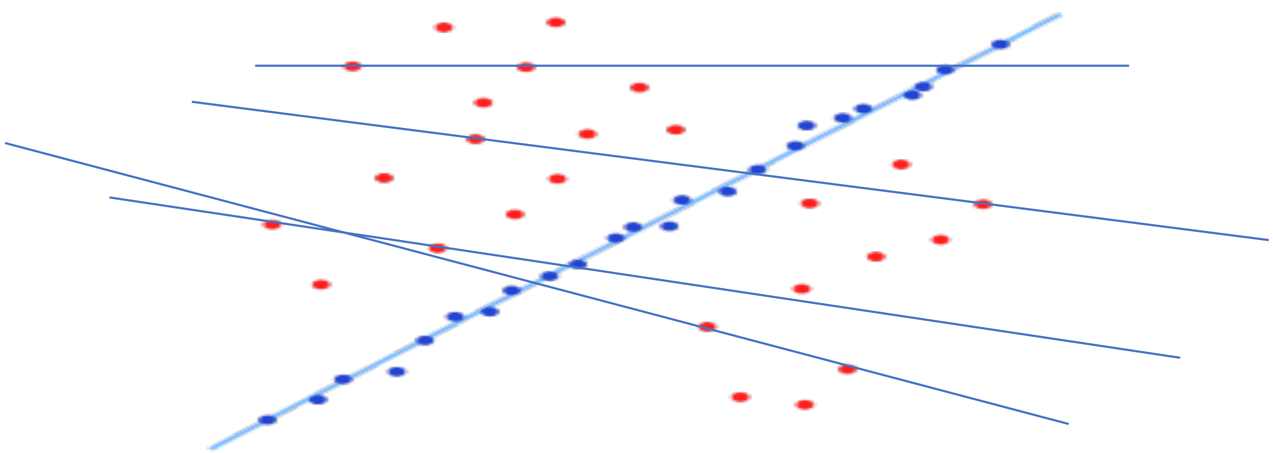


# Sample Consensus

1. **Hypothesis:** Each minimal sample set (two points) defines a line.
2. **Test:** Which points of the dataset satisfy the hypothesis (no. of inliers)

Exhaustive Search:

1. Choose all $\binom{n}{2}$ pairs.
2. Keep the one with the maximum number of inliers (above a threshold).



# RANdom SAmple Consensus or RANSAC

Graphics and Image Processing
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## Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

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**and analysis conditions. Implementation details and computational examples are also presented.**

**Key Words and Phrases: model fitting, scene analysis, camera calibration, image matching, location determination, automated cartography.**

**CR Categories: 3.60, 3.61, 3.71, 5.0, 8.1, 8.2**

### I. Introduction

We introduce a new paradigm, Random Sample Consensus (RANSAC), for fitting a model to experimental data; and illustrate its use in scene analysis and automated cartography. The application discussed, the location determination problem (LDP), is treated at a level beyond that of a mere example of the use of the RANSAC paradigm; new basic findings concerning the conditions under which the LDP can be solved are presented and a comprehensive approach to the solution of this problem that we anticipate will have near-term practical applications is described.

To a large extent, scene analysis (and, in fact, science in general) is concerned with the interpretation of sensed data in terms of a set of predefined models. Conceptually, interpretation involves two distinct activities: First, there is the problem of finding the best match between the data and one of the available models (the classification

# RANdom SAmple Consensus or RANSAC

Sample minimal sample sets instead of exhaustively traverse them over $k$ iterations.

```
Repeat for k iterations
1.  Choose a minimal sample set
2.  Count the inliers for this set
3.  Keep maximum, if it exceeds a desired number of inliers
stop.
```

Assume that the minimal sample set has $M$ points ($M = 2$ for the case of line fitting).

What is the probability that your minimal sample set is a set of inliers ?

If the probability of a point to be an inlier is $\epsilon$ then the probability of choosing an inlier pair is $\epsilon^M$.

In $k$ iterations the probability of NON hitting a single inlier pair is

$$(1-\epsilon^M)^k$$

Thus the probability of choosing at least one inlier pair in $k$ iterations is

$$p = 1-(1-\epsilon^M)^k.$$

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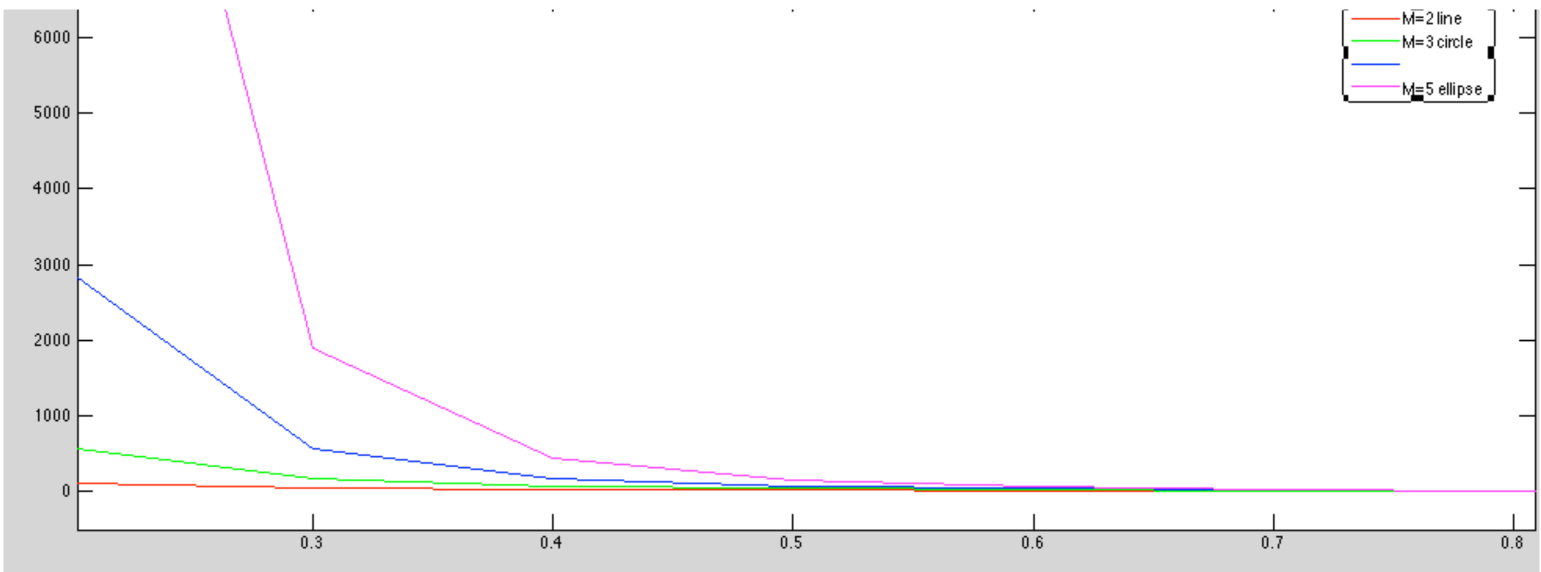
How tractable is RANSAC ?

$$k = \frac{\log(1-p)}{\log(1-\epsilon^M)}$$

Number of iterations needed to hit an inlier with probability 0.99 for the cases of line, circle, homograph, and ellipse fitting




Fraction of inliers

# RANSAC vs Hough

- RANSAC can deal only with one model (inliers vs outliers) while Hough detects multiple models
- RANSAC is more efficient when fraction of outliers is low
- RANSAC requires the solution of a minimal set problem,
  - For example, solve of a system of 5 polynomial equations for 5 unknowns
- Hough needs a bounded parameter space
- Hough is intractable for large number of unknowns