

# CIS580 Problem Set 3

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# 1 Transformation to map facade to rectangle

## **2 Compute distances of patrol car and bridge**

### **3 Compute distances from the image**

## 4 Different perspectives in a tennis match

- You should get the coordinates of the points in the image. To do so you can use any unit you want. Using pixels is the most convenient.
- The cross ratio should be applied in one of the baselines, and you can use the net in the middle of the court as an extra point.
- Since you have measured coordinates of A and B, you can use the similar triangle to obtain the coordinate of V.
- I had taken average of the vanishing points obtained by using cross ratio on AB and CD due to the minor differences between them.
- For each pair of sidelines you should find 1 vanishing point (so 2 in total for the two images). In the same image using the cross ratio in the two sidelines, you should get the same vanishing points (they will not be exactly the same but they will be close enough, you can just use one of them)
- For the intersection of baselines you can use intersection of lines.
- You cannot define a coordinate system where  $(0,0,1)$  is one corner and  $(1,1,1)$  is the other because you will mess the distances. In the tennis case we don't have a square. You should modify the method so that it uses  $[27,78,1]$  instead of  $[1,1,1]$
- Oh, I see. We should use two images of the same plane, tennis court, to construct two vanishing lines and find the principle point.

## 4.1 Why is the perspective different

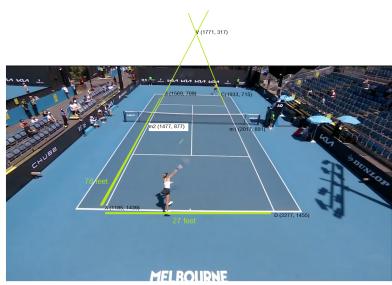
586	1122
1771	317
V	

384	730
1569	709
8	

748	724
1933	715
C	

292	562
1477	877
M2	

832	548
2017	891
M1	



0	0
1185	1439
A	



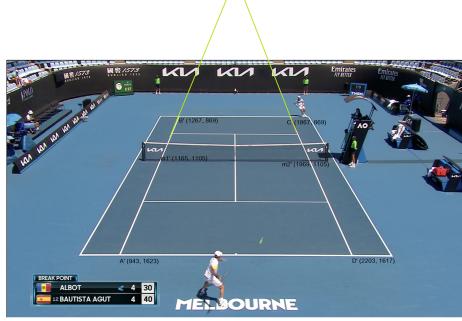
1092	-16
2277	1455
D	

Distances	
AB	824.8
CD	816.0
M1C	195.0
DV	1245.4
CV	429.7

630	1514
1573	109
V	

324	754
1257	869
B	

924	754
1867	869
C	



0	0
943	1623
A	



1260	6
2203	1617
D	

Distances	
AB	820.7
CD	820.0
M1C	257.1
DV	1634.3
CV	814.9

## 4.2 Find vanishing points using cross-ratios

Known quantities:

Units	Segment	Value
Image	$AD$	$\sqrt{1092^2 + 16^2} = 1092$ pixels
Image	$BC$	$\sqrt{364^2 + 6^2} = 364$ pixels
Image	$AB$	$\sqrt{730^2 + 384^2} = 825$ pixels
Image	$CD$	$\sqrt{344^2 + 740^2} = 816$ pixels
Image	$M_1M_2$	$\sqrt{540^2 + 14^2} = 540$ pixels
Image	$A'D'$	$\sqrt{1260^2 + 6^2} = 1260$ pixels
Image	$B'C'$	$\sqrt{600^2 + 0^2} = 600$ pixels
Image	$A'B'$	$\sqrt{324^2 + 754^2} = 821$ pixels
Image	$C'D'$	$\sqrt{336^2 + 748^2} = 820$ pixels
Image	$M'_1M'_2$	$\sqrt{804^2 + 0^2} = 804$ pixels
World	$AD, A'D'$	27 feet
World	$BC, B'C$	27 feet
World	$AB, A'B'$	78 feet
World	$CD, C'D'$	78 feet

### Vanishing Point for Image 1

$$\begin{aligned}
 \frac{DC/DV}{M_1C/M_1V} &= \frac{C_wD_w/D_wV_w}{M_{1w}C_w/M_{1w}V_w} \\
 \frac{816/DV}{195/M_1V} &= 2 \cdot M_{1w}V_w/D_wV_w \\
 \frac{816}{195} \cdot \frac{M_1V}{DV} &= 2 \cdot M_{1w}V_w/D_wV_w \\
 \frac{195 + CV}{816 + CV} &= \frac{195}{816} \cdot 2 \cdot 1 \\
 (195 + CV) \cdot 816 &= 390 \cdot (816 + CV) \\
 CV \cdot (816 - 390) &= 816 \cdot (390 - 195) \\
 CV &= \frac{816 \cdot 195}{426} \\
 CV &= 374
 \end{aligned}$$

Given the slope of the line and the distance  $CV$ , we can compute  $V$  as follows:

$$m = \frac{548 - 724}{832 - 748} = \frac{724 - V_y}{748 - V_x} \quad (1)$$

$$374 = \sqrt{(724 - V_y)^2 + (748 - V_x)^2} \quad (2)$$

Solving the above system of equations using Mathematica, we obtain the possible vanishing points as:

$$\begin{aligned} V_x &\rightarrow 909.093, V_y \rightarrow 386.472 \\ V_x &\rightarrow 586.907, V_y \rightarrow 1061.53 \end{aligned}$$

Double-checking the answer using cross-products:

### **Image 1 Vertical Vanishing Point**

$$L_1 = \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1092 \\ -16 \\ 1 \end{bmatrix} = \begin{bmatrix} 724 + 16 \\ 1092 - 748 \\ -16 * 748 - 724 * 1092 \end{bmatrix} = \begin{bmatrix} 740 \\ 344 \\ -802576 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 384 \\ 730 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 730 \\ -384 \\ 0 \end{bmatrix}$$

Now, computing  $L_1 \times L_2$  to obtain the vanishing point:

$$\begin{bmatrix} 740 \\ 344 \\ -802576 \end{bmatrix} \times \begin{bmatrix} 730 \\ -384 \\ 0 \end{bmatrix} = \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix}$$

### **Vanishing Point for Image 2**

$$\begin{aligned} \frac{D' C' / D' V'}{M_1' C' / M_1' V'} &= \frac{C_w' D_w' / D_w' V_w'}{M_{1w}' C_w' / M_{1w}' V_w'} \\ \frac{820 / DV}{257 / M_1 V} &= 2 \cdot M_{1w} V_w / D_w V_w \\ \frac{820}{257} \cdot \frac{M_1 V}{DV} &= 2 \cdot M_{1w} V_w / D_w V_w \\ \frac{257 + CV}{820 + CV} &= \frac{257}{820} \cdot 2 \cdot 1 \\ (257 + CV) \cdot 820 &= 514 \cdot (820 + CV) \\ CV \cdot (820 - 514) &= 820 \cdot (514 - 257) \\ CV = \frac{820 \cdot 257}{306} \\ CV &= 689 \end{aligned}$$

Given the slope of the line and the distance  $CV$ , we can compute  $V$  as follows:

$$m = \frac{518 - 754}{1026 - 924} = \frac{754 - V_y}{924 - V_x} \quad (3)$$

$$689 = \sqrt{(754 - V_y)^2 + (924 - V_x)^2} \quad (4)$$

Solving the above system of equations using Mathematica, we obtain the possible vanishing points as:

$$V_x \rightarrow 1197.35, V_y \rightarrow 121.544$$

$$V_x \rightarrow 650.65, V_y \rightarrow 1386.46$$

### 4.3 Find vanishing points for court baselines

#### Image 1 Horizontal Vanishing Point

$$L_1 = \begin{bmatrix} 384 \\ 730 \\ 1 \end{bmatrix} \times \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 364 \\ -268024 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1092 \\ -16 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 1092 \\ 0 \end{bmatrix}$$

Now, computing  $L_1 \times L_2$  to obtain the vanishing point:

$$\begin{bmatrix} 6 \\ 364 \\ -268024 \end{bmatrix} \times \begin{bmatrix} 16 \\ 1092 \\ 0 \end{bmatrix} = \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix}$$

#### Image 2 Horizontal Vanishing Point

$$L_1 = \begin{bmatrix} 324 \\ 754 \\ 1 \end{bmatrix} \times \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 600 \\ -452400 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1260 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 1260 \\ 0 \end{bmatrix}$$

Now, computing  $L_1 \times L_2$  to obtain the vanishing point:

$$\begin{bmatrix} 0 \\ 600 \\ -452400 \end{bmatrix} \times \begin{bmatrix} -6 \\ 1260 \\ 0 \end{bmatrix} = \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix}$$

## Finding the homography that maps tennis court to image plane

### Image 1

$$\begin{aligned}
 W' &\sim P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \implies \alpha W' = P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = P_3 \\
 X' &\sim P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \implies \beta X' = P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = P_2 \\
 Y' &\sim P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \gamma Y' = P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = P_1 \\
 Z' &\sim P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \implies \delta Z' = P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} = 27P_1 + 78P_2 + P_3
 \end{aligned}$$

Combining the above equations:

$$\delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} = 27\gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix} + 78\beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can infer that  $\alpha = 1$ , and simplify the system of equations to:

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} - 78\beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} - 27\gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -44928 \\ -85410 \\ -78 \end{bmatrix} + \gamma \begin{bmatrix} -10854972 \\ 159057 \\ -27 \end{bmatrix}$$

$$[\delta \quad \beta \quad \gamma] = \left( \begin{bmatrix} 748 & -44928 & -10854972 \\ 724 & -85410 & 159057 \\ 1 & -78 & -27 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solving for the inverse of the coefficient matrix, we obtain:

$$\begin{bmatrix} \delta \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 3.0027 \\ 0.0256 \\ 0.0001 \end{bmatrix}$$

From this, we obtain the transformation  $P$  by multiplying  $\alpha, \beta$  and  $\gamma$  into the above equation:

$$\begin{aligned} \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} &= \gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 13.4944 & 4.9186 & 0 \\ -0.1977 & 9.3505 & 0 \\ 0 & 0.0085 & 0.3300 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \end{aligned}$$

## Image 2

$$\begin{aligned} W' \sim P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} &\implies \alpha W' = P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = P_3 \\ X' \sim P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} &\implies \beta X' = P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = P_2 \\ Y' \sim P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} &\implies \gamma Y' = P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = P_1 \\ Z' \sim P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} &\implies \delta Z' = P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} = 27P_1 + 78P_2 + P_3 \end{aligned}$$

Combining the above equations:

$$\delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} = 27\gamma \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix} + 78\beta \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can infer that  $\alpha = 1$ , and simplify the system of equations to:

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} - 78\beta \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} - 27\gamma \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -50778 \\ -108108 \\ -78 \end{bmatrix} + \gamma \begin{bmatrix} -4275180 \\ -20358 \\ -27 \end{bmatrix}$$

$$[\delta \quad \beta \quad \gamma] = \left( \begin{bmatrix} 924 & -50778 & -4275180 \\ 754 & -108108 & -20358 \\ 1 & -78 & -27 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solving for the inverse of the coefficient matrix, we obtain:

$$\begin{bmatrix} \delta \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 2.2010 \\ 0.0153 \\ 0.0003 \end{bmatrix}$$

From this, we obtain the transformation  $P$  by multiplying  $\alpha, \beta$  and  $\gamma$  into the above equation:

$$\begin{aligned} \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} &= \gamma \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 21.1528 & 4.5240 & 0 \\ 0.1007 & 9.6318 & 0 \\ 0.0001 & 0.0069 & 0.4543 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \end{aligned}$$

#### 4.4 Compute the focal length of each image

#### 4.5 Compute the vanishing points using intersection of parallel lines