Rotation and Translation from Two Views of a Plane

All slides by the authors of the Ma, Soatto, Kosecka, Sastry book

Epipolar Geometry – Planar Case

- Plane in first camera coordinate frame

$$aX + bY + cZ + d = 0$$

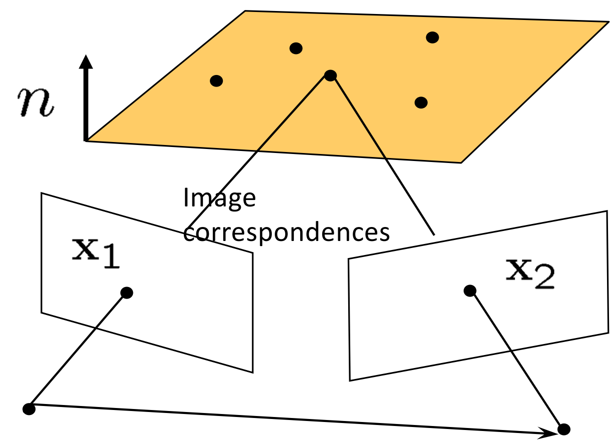
$$\frac{1}{d}N^T\mathbf{X} = 1$$

$$\lambda_2\mathbf{x}_2 = R\lambda_1\mathbf{x}_1 + T$$

$$\lambda_2\mathbf{x}_2 = (R + \frac{1}{d}TN^T)\lambda_1\mathbf{x}_1$$

$$\mathbf{x}_2 \sim H\mathbf{x}_1$$

$$\boxed{H = (R + \frac{1}{d}TN^T)}$$



Planar homography

Linear mapping relating two corresponding planar points in two views

Decomposition of H

- Algebraic elimination of depth $\widehat{\mathbf{x}_2} H \mathbf{x}_1 = 0$
- H_L can be estimated linearly $H_L = \lambda H$
- Normalization of $H = H_L / \sigma_3$
- Decomposition of H into 4 solutions $H = (R + \frac{1}{d} T N^T)$

$R_1 = W_1 U_1^T$ $N_1 = \widehat{v_2} u_1$ $\frac{1}{d} T_1 = (H - R_1) N_1$	$R_3 = R_1$ $N_3 = -N_1$ $\frac{1}{d} T_3 = -\frac{1}{d} T_1$	$R_2 = W_2 U_2^T$ $N_2 = \widehat{v_2} u_2$ $\frac{1}{d} T_2 = (H - R_2) N_2$	$R_4 = R_2$ $N_4 = -N_2$ $\frac{1}{d} T_4 = -\frac{1}{d} T_2$

- $$H^T H = V \Sigma V^T \quad V = [v_1, v_2, v_3] \quad \Sigma = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2)$$
  $$u_1 \doteq \frac{\sqrt{1-\sigma_3^2}\, v_1 + \sqrt{\sigma_1^2 - 1}\, v_3}{\sqrt{\sigma_1^2 - \sigma_3^2}} \quad u_2 \doteq \frac{\sqrt{1-\sigma_3^2}\, v_1 - \sqrt{\sigma_1^2 - 1}\, v_3}{\sqrt{\sigma_1^2 - \sigma_3^2}}$$
  $$U_1 = [v_2, u_1, \widehat{v_2} u_1], \quad W_1 = [H v_2, H u_1, \widehat{H v_2} H u_1];$$
  $$U_2 = [v_2, u_2, \widehat{v_2} u_2], \quad W_2 = [H v_2, H u_2, \widehat{H v_2} H u_2].$$

Motion and pose recovery for planar scene

- Given at least 4 point correspondences $\widehat{\mathbf{x}_2^j} H \mathbf{x}_1^j = 0$
- Compute an approximation of the homography matrix H_l^s
- Compute Homography
- Normalize the homography matrix χ $H = H_L / \sigma_3$

- Decompose the homography matrix

$$H^T H = V \Sigma V^T$$

- Select two physically possible solutions imposing positive depth constraint

Example