6DoF Pose from Projective Transformations

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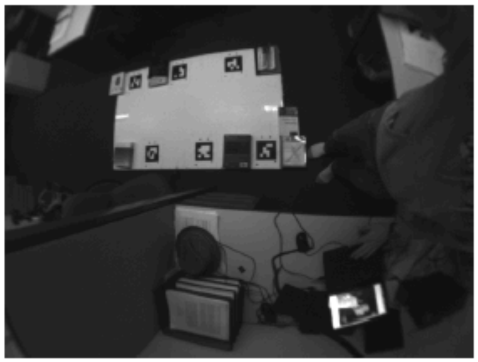
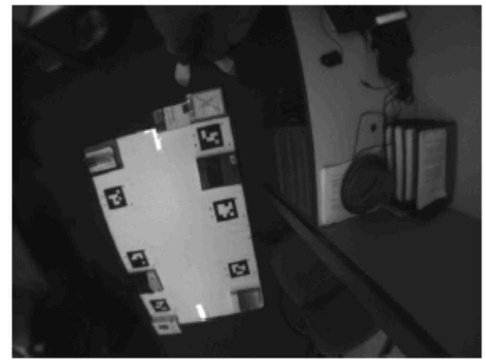
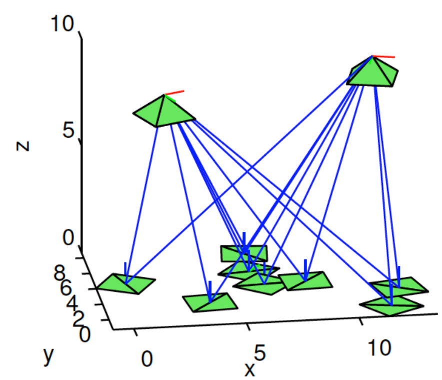
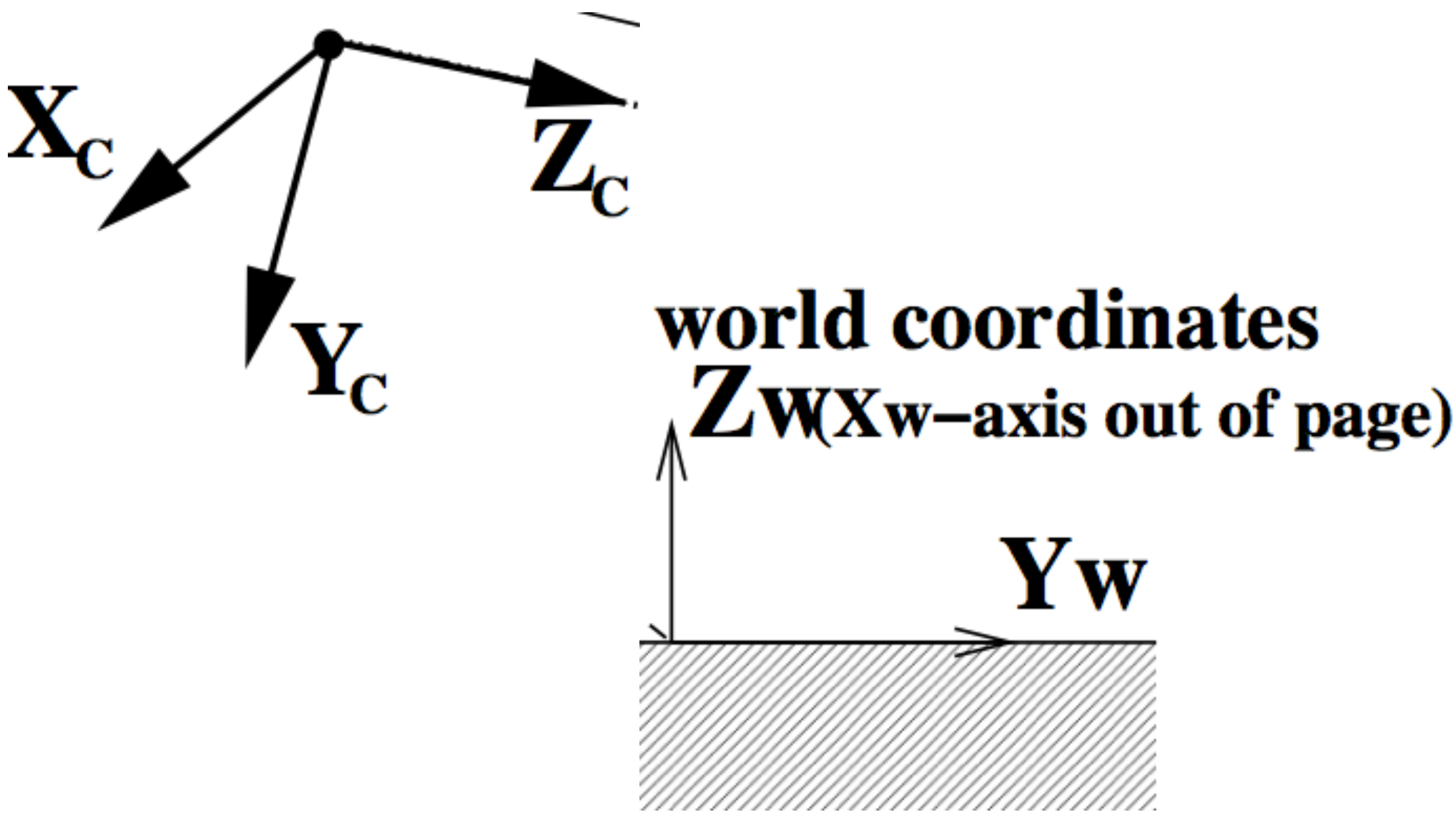
Using the projective transformation the pose of a robot with respect to a planar pattern:







Pose from reference points on plane Z_w=0



Pose from Projective Transformation

Recall the projection from world to camera

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim K \begin{pmatrix} r_1 & r_2 & r_3 & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$$

and assume that all points in the world lie in the ground plane $Z = 0$.

Then the transformation reads

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim K \begin{pmatrix} r_1 & r_2 & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ W \end{pmatrix}$$

H is a transformation from $\mathbb{P}^2$ to $\mathbb{P}^2$:

$$H \sim K \begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$$

Is it a projective transformation? Let us inspect its determinant:

$$\det \begin{pmatrix} r_1 & r_2 & T \end{pmatrix} = T^T (r_1 \times r_2)$$

which vanishes only if the camera lies in the ground plane $Z = 0$. In this case all points would project on a line.

Since $\det(K) = f^2$, H is invertible iff

$$T^T (r_1 \times r_2) \neq 0$$

Suppose we estimate an H from $N \geq 4$ correspondences.

Let us assume that we know the intrinsic parameters K.

Pose estimation means finding R, T given H and intrinsics K.

We observe that

$$K^{-1}H = \begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$$

has specific properties: its first two columns are orthogonal unit vectors.

Nothing guarantees that that the H we computed will satisfy this condition.

Let us name the columns of $K^{-1}H$:

$$K^{-1}H = \begin{pmatrix} h_1' & h_2' & h_3' \end{pmatrix}$$

We seek orthogonal r_1 and r_2 that are the closest to h_1' and h_2'. The solution to this problem is given by the Singular Value Decomposition.

We find the orthogonal matrix R that is the closest to $\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix}$:

$$\underset{R \in SO(3)}{\arg\min} \left\| R - \begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix} \right\|_F^2$$

$$\boxed{\arg\min_{R\in SO(3)} \|R - \begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix}\|_F^2}$$

If the SVD of

$$\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix} = USV^T$$

then the solution is

$$R = U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(UV^T) \end{pmatrix} V^T$$

The diagonal matrix is inserted to guarantee that $\det(R) = 1$.

To find the translation : $T = h_3'/\|h_1'\|$

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