$\boxed{\text{Extract pose } (R,T) \text{ from a projective transformation } H}$

- 3 orthogonal V.P. $\Rightarrow K\ (f, u_0, v_0)$
- Mapping between two planes where no intrinsics (K) or extrinsics (R, T) were needed.

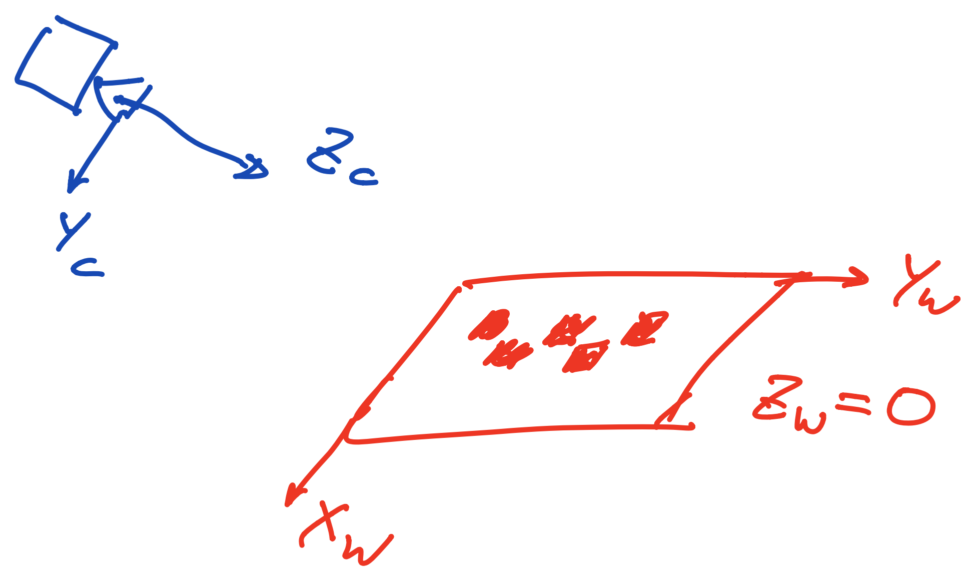
STEPS:

1) Given $N \geq 4$ correspondences between world plane and image plane $\Rightarrow H$

2) $H \xrightarrow{?} R, T$

Where is the camera?

- foundation of AR
- localization of robots (cars etc)



$\boxed{\text{1st lecture}}$

$$\lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{K \\ 3\times 3}} \underbrace{\begin{pmatrix} r_1 & r_2 & r_3 & T \end{pmatrix}}_{3\times 4} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ W_w \end{pmatrix}$$

0

proj. plane

proj. space

We do not assume that $w \neq 0$, that's why we cannot set it to 1.

$Z_w = 0$

$$\lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix} = K \left(r_1 X_w + r_2 Y_w + r_3 \cdot \underset{z_w = 0}{0} + T \cdot W_w \right)$$

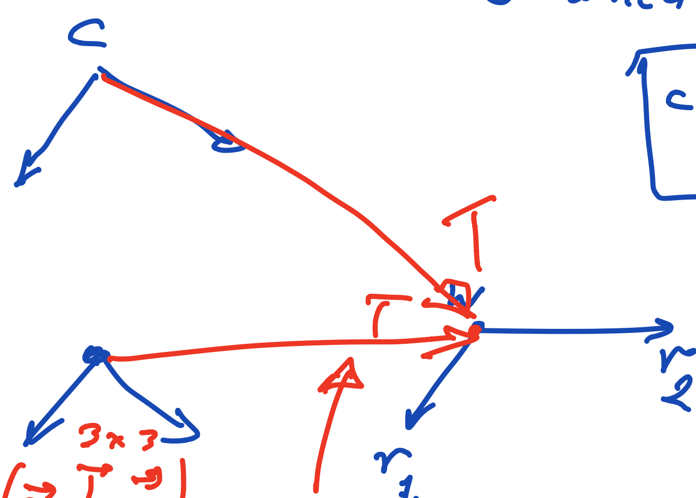
drop subscript w for "world"
and keep world as upper case!

$$\lambda \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underbrace{\underset{3\times3}{K}\ \underset{3\times3}{(r_1\ \ r_2\ \ T)}}_{H} \begin{pmatrix} X \\ Y \\ W \end{pmatrix}$$

proj. transformation is any <u>invertible</u>
3x3 matrix! Is H invertible

$$\det(H) = \det(K)\det(r_1\ r_2\ T)$$

$$= f^2 \underbrace{(r_1 \times r_2)^T T}_{=0 \text{ when } r_1, r_2, T \text{ coplanar}}$$



$$\det(\vec{a}\ \vec{b}\ \vec{c}) = \vec{a}^T(\vec{b} \times \vec{c})$$

$$T^T(r_1 \times r_2) = 0$$

$$\boxed{{}^cP = R\,{}^wP + T}$$

$$\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{r_1} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$${}^cP = T$$

$$H \sim K\underbrace{(r_1\ r_2\ T)}_{\text{find}}$$

given

$$\lambda \overset{3\times 3}{H} = \overset{3\times 3}{K}\overset{3\times 3}{(r_1\ r_2\ T)}$$

9 equations

1 unknown   3   3   3

f, u_0, v_0

first two columns contain the entire information.

$r_3 = r_1 \times r_2$

$\boxed{10 \text{ unknowns}}$

What is solvable?

1) K known   7 unknowns

overconstrained 9 equations

TODAY IN SLIDES

2) $\boxed{u_0, v_0 \text{ known}}$ d, f, 6 DOF $\boxed{8 \text{ equations}}$

1) case K known $\quad H = K(r_1\ r_2\ T)$

call $H' = K^{-1}H$

known

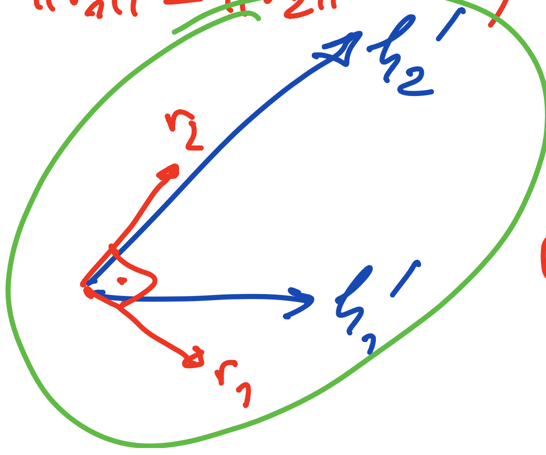
$$\begin{pmatrix} h_1' & h_2' & h_3' \end{pmatrix} = \begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$$

$$\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}$$

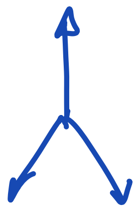
Math problem: we know $r_1^T r_2 = 0$

Given two arbitrary vectors h_1' and h_2' find the closest unit vectors that are perpendicular to each other.

$(\|r_1\| = \|r_2\| = 1)$



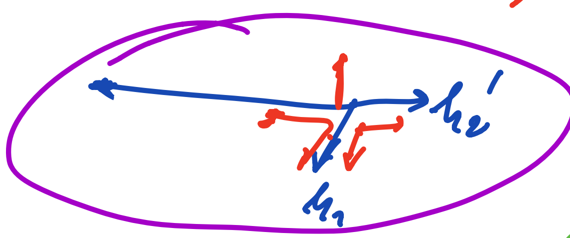
solution is given by the SVD!!!



$$\underbrace{\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix}}_{3\times 3} \overset{\text{SVD}}{=} USV^T$$

alternative $\underbrace{\begin{pmatrix} h_1' & h_2' \end{pmatrix}}_{3\times 2} = USV^T$ same sol.

alternative $\begin{pmatrix} h_1' & h_2' & h_3' \end{pmatrix} = USV^T$ not same sol.

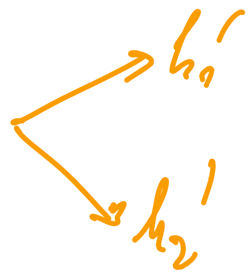


Why do we try to find the closest?

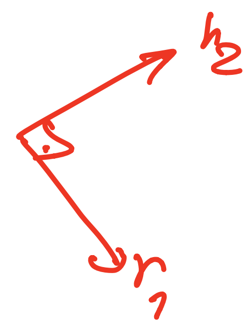
You click, you compute H,
and then $K^{-1}H = \begin{array}{c} \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \\ \;\; h_1' \;\; h_2' \;\;\;\; \end{array}$

if they are perpendicular you are done $r_1 = \dfrac{h_1'}{\|h_1'\|}$ $r_2 = \dfrac{h_2'}{\|h_2'\|}$

Because of clicking and other numerical or sensor errors h_1' and h_2' will not be $\perp$.



closest



$$(h_1' \; h_2' \; h_3') = USV^T$$

orthogonal

$\det = \pm 1$

solution for rotation is

$$R = U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(UV^T) \end{pmatrix} V^T$$

$$\det(R) = \det(U)\det(UV^T)\det(V)$$
$$= \det(U)^2 \det(V)^2 = 1$$

Theorem we used is : $\det(A) \neq 0$

$$\underset{\substack{R \\ R^TR = I}}{\arg\min} \|R - A\| = UV^T$$

when $A = USV^T$.

What about T?

$$(h_1' \; h_2' \; h_3') = (r_1 \; r_2 \; T)$$

closest

What is closest translation?

$$\boxed{T \sim h_3'}$$

$\|T\| = ?$

$$\boxed{T = \frac{h_3'}{\frac{\|h_1'\| + \|h_2'\|}{2}}}$$

instead of $\frac{1}{\|h_1'\|}$ slide!

we localized !!!