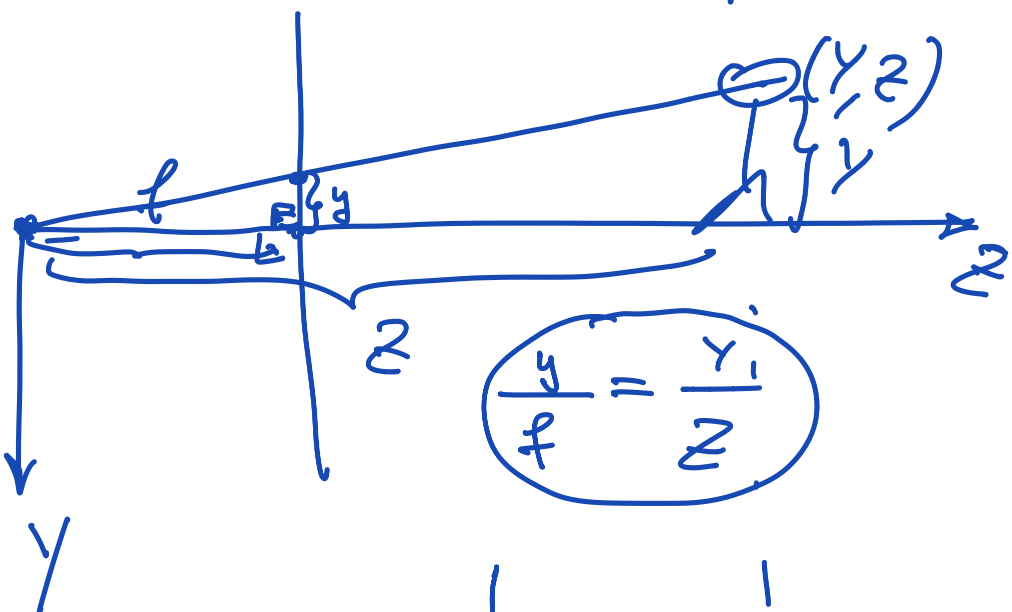
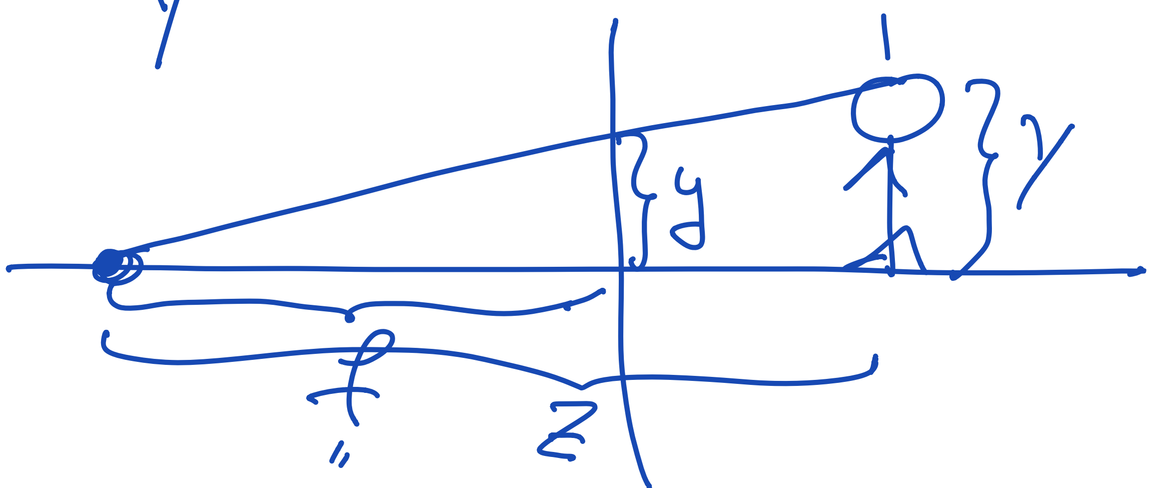
Pinhole model : everything is sharp.



$$\frac{y}{f} = \frac{Y}{Z}$$



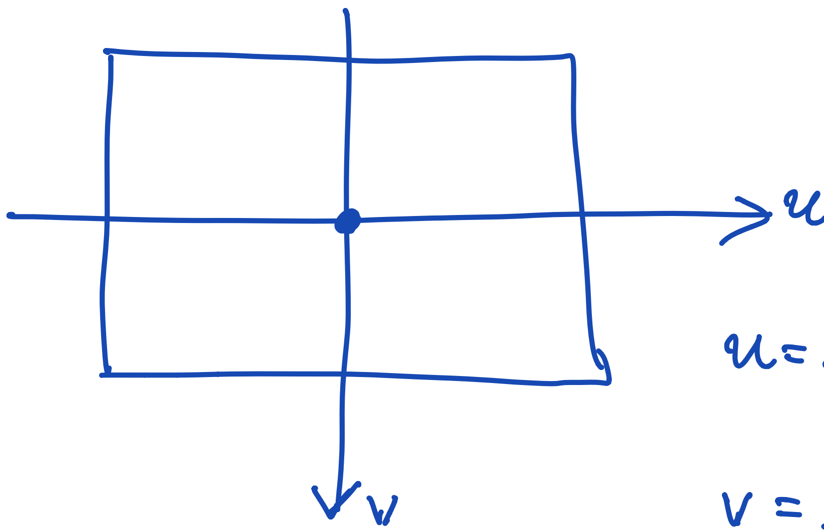
zooming

$$f = b\,\frac{1}{1-\frac{a}{f}} \approx 1$$

Z

f is cropping and resizing.

f is just a scaling factor of the camera!

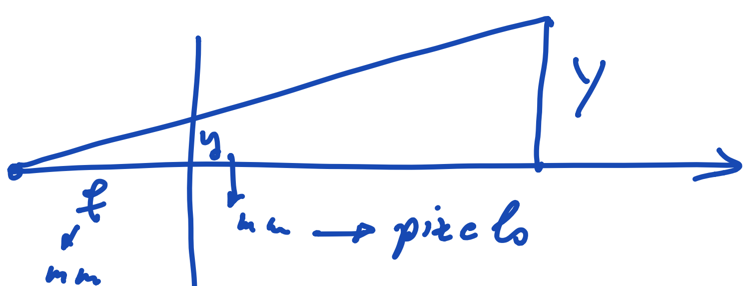


$$u = f\frac{X}{Z}$$

$$v = f\frac{Y}{Z}$$

u, v are in pixels

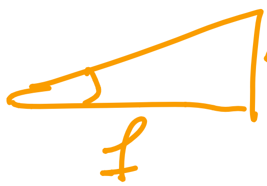
$\Rightarrow f$ is in pixels



CMOS
chip

$$f[\text{pixels}] = f[\text{mm}] \cdot \#\frac{\text{pixel}}{\text{mm}}$$

Field of View Interpretation

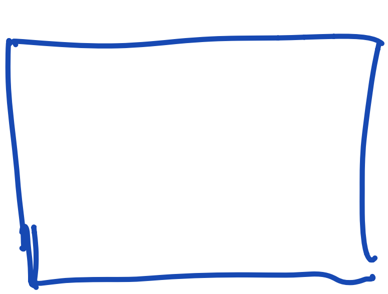


$$\frac{\text{half of width}}{f} = \tan\frac{FOV}{2}$$

$$f = \frac{W/2}{\tan\frac{FOV}{2}} = \boxed{2000 \text{ pixels}}$$

for example: $FOV = 90^\circ$ horiz

$W = 4K$



During zooming

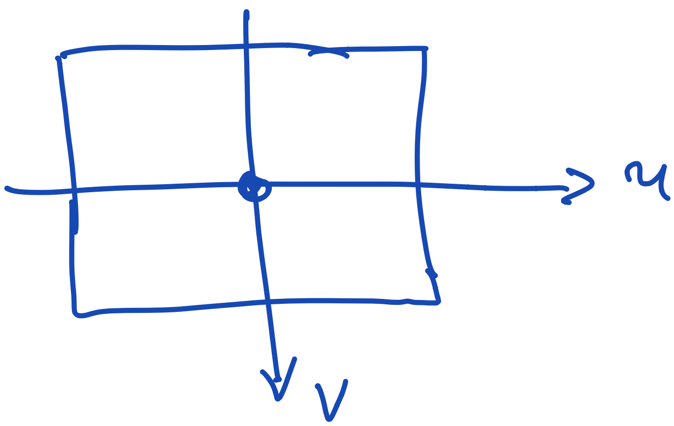
$FOV \downarrow \Rightarrow f \uparrow$

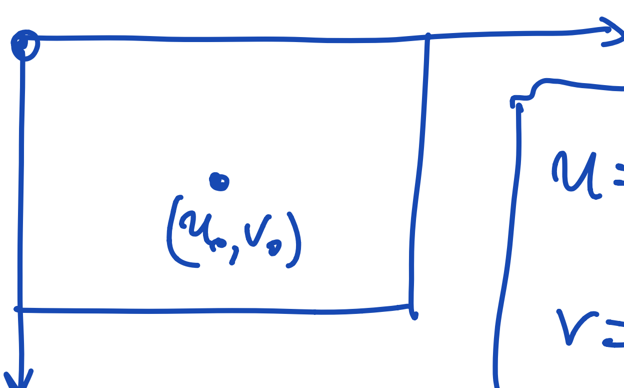
Where is the origin of the image plane?

image center = intersection of the optical axis with the image plane

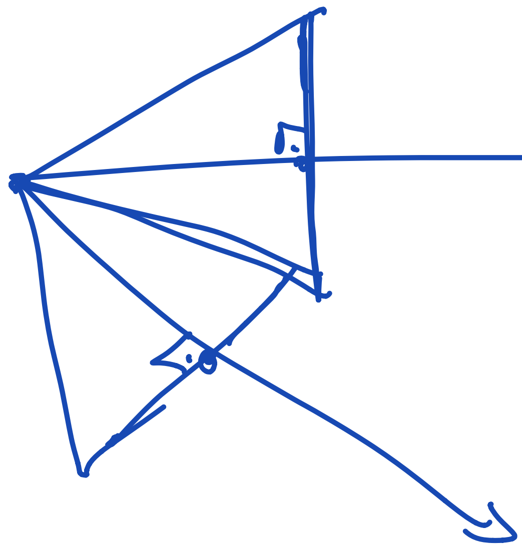
$\neq$

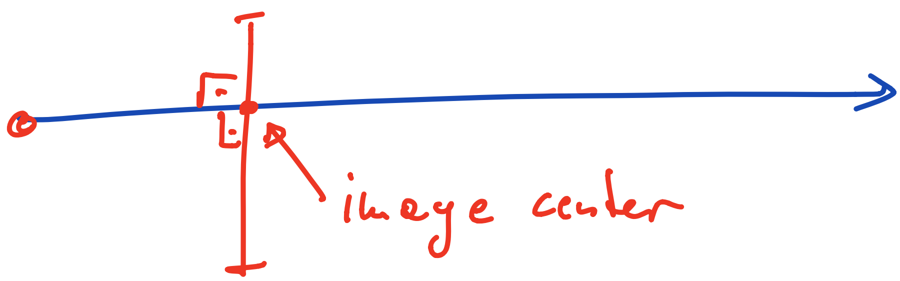
projection center





$$\boxed{\begin{aligned} u &= f\frac{X}{Z} + u_0 \\ v &= f\frac{Y}{Z} + v_0 \end{aligned}}$$

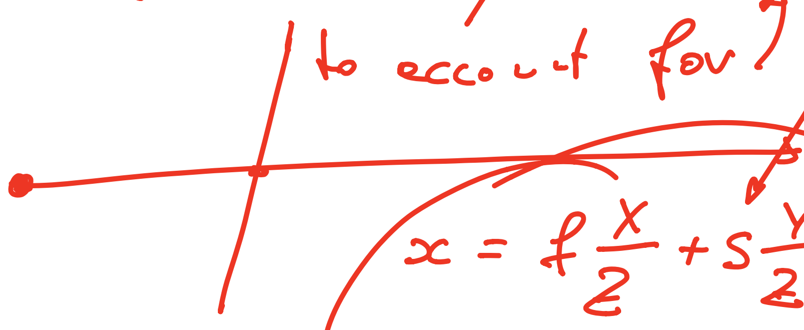




what happens if

image plane $\not\perp$ optical axis

to account for skew factor



$$x = f\frac{X}{Z} + s\frac{Y}{Z}$$

$$y = f\frac{Y}{Z}$$

From now on we assume

$$\boxed{\begin{aligned} u &= f\frac{X_c}{Z_c} + u_0 \\ v &= f\frac{Y_c}{Z_c} + v_0 \end{aligned}}$$

$$u = \frac{fX_c + u_0 Z_c}{Z_c}$$

$$\Updownarrow$$

$$\lambda u = fX_c + u_0 Z_c$$

$$\lambda = Z_c$$

$$\Updownarrow$$

$$\lambda \begin{pmatrix} u \\ 1 \end{pmatrix} = \begin{pmatrix} f & u_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_c \\ Z_c \end{pmatrix}$$

$$\boxed{\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}} \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}}$$

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + t$$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \left(\underbrace{R}_{3\times 3} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + \underbrace{t}_{3\times 1} \right)$$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{K}_{3\times 3} \underbrace{\underbrace{\begin{pmatrix} R & t \end{pmatrix}}_{3\times 4}}_{M} \underbrace{\begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}}_{4\times 1}$$

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{K M}_{P_{3\times 4}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

[mm]

projection matrix

unknown [pixels]

f, u_0, v_0 intrinsic

extrinsic

world to camera projection

$$\begin{pmatrix} X_w \\ Y_w \\ W_w \end{pmatrix} \longrightarrow \underset{P = KM}{\text{projection}} \longrightarrow \begin{pmatrix} u \\ v \end{pmatrix}$$

world or object

pixels

rendering (without accounting for visibility or color)

$$\begin{pmatrix} u \\ v \end{pmatrix} \longrightarrow \text{inverse projection} \longrightarrow ?$$

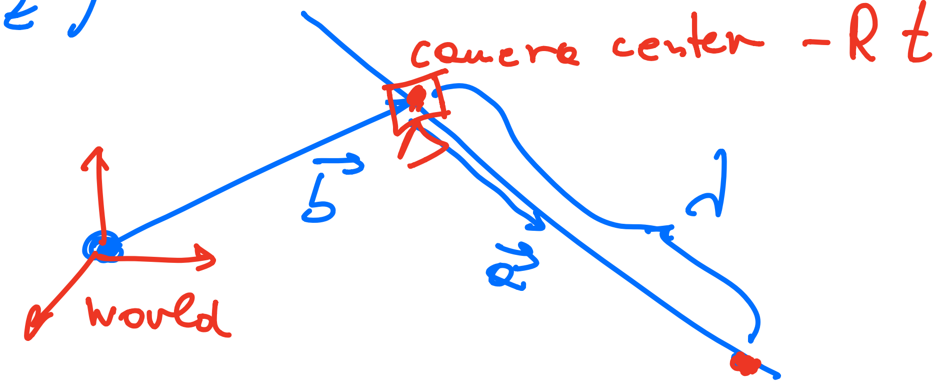
ray through this pixel

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = KM \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\lambda K^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = R \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + t$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \lambda R^{-1} K^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} - R^{-1} t$$

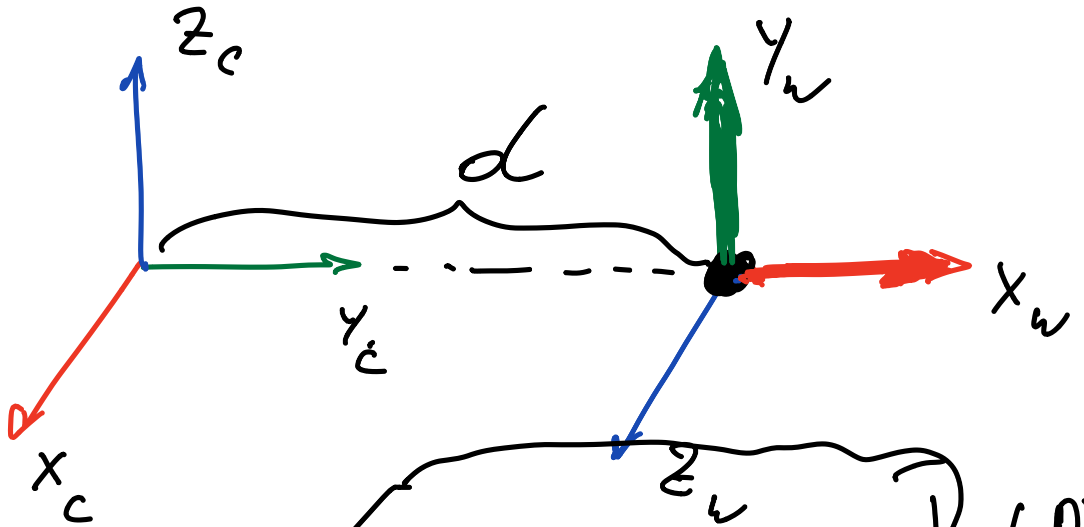
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \vec{a} + \vec{b}$$



If K and R, t are known

pixel position $\Rightarrow$ ray

Rigid Transformations



$$
\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ r_1^{1} & r_2^{0} & r_3^{0} \\ \underset{3\times1}{}^{0} & \underset{3\times1}{}^{1} & \underset{3\times1}{}^{0} \end{pmatrix}}_{3\times3} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix}}_{3\times1}
$$

translation: set $X_w, Y_w, Z_w = 0$ origin of RHS c.s. w.r.t. LHS

rotation: set $t = 0$

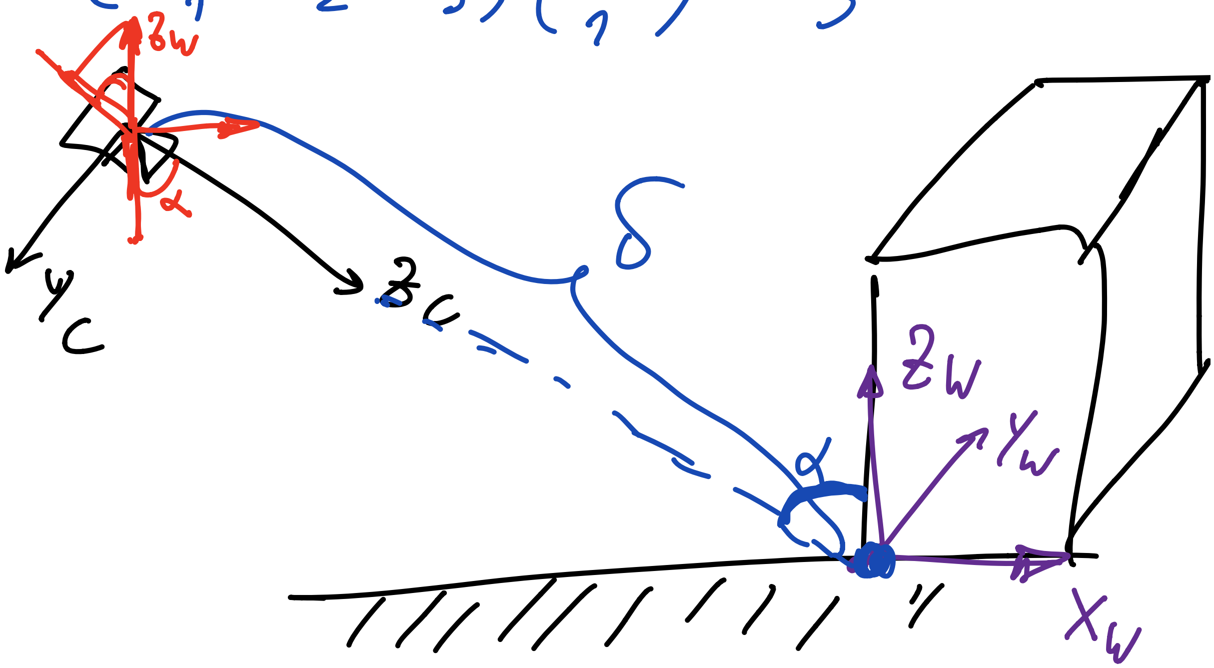
1st col is X_w-axis wrt camera, 2nd col is Y_w-axis, 3rd col is the Z_w.

$$\begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = r_1$$

X_w-axis

$$\begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = r_2$$

$$\begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = r_3$$



$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ -\cos\alpha & 0 & -\sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \end{pmatrix} \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \delta \end{pmatrix}$$