Review : 12 Geometry (G)
8 Image Processing (IP)

(IP1) 1. convolution theorem

$$\frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_1^2}} = g_1(t) \circ\!\!-\!\!\bullet\, G_1(\omega)$$

$$\ldots = g_2(t) \circ\!\!-\!\!\bullet\, G_2(\omega)$$

Fourier pair

$$g_1 * g_2 \circ\!\!-\!\!\bullet\, G_1 G_2$$

$$\parallel$$

$$e^{-\frac{\sigma_1^2\omega^2}{2} - \frac{\sigma_2^2\omega^2}{2}}$$

$$\parallel$$

$$\frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2+\sigma_2^2}} e^{-\frac{x^2}{2(\sigma_1^2+\sigma_2^2)}} \circ\!\!-\!\!\bullet\, e^{-\frac{(\sigma_1^2+\sigma_2^2)\omega^2}{2}}$$

$$\sigma = \sqrt{\sigma_1^2+\sigma_2^2}$$

$$\left.\begin{array}{l}\sigma_1 = 1\\ \sigma_2 = 1\end{array}\right\} \sigma = \sqrt{2}$$

Additional question: You are allowed to use only Gaussian with $\sigma = 1$. How can you realize a convolution with $\sigma = 8$?

$$\sqrt{\underbrace{1^2 + 1^2 \quad \cdots \quad + 1^2}_{64 \text{ convolutions with } \sigma = 1}} = 8$$

Why?

$\sigma = 1$: 5×1

$\sigma = 8$: 41×1

IP 2: $h(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$\int_{-\infty}^{\infty} h(t)\, g''(x-t)\, dt$$

$$= \int_{-\infty}^{0} 0 \cdot g''\, dt + \int_{0}^{\infty} 1 \cdot g''(x-t)\, dt$$

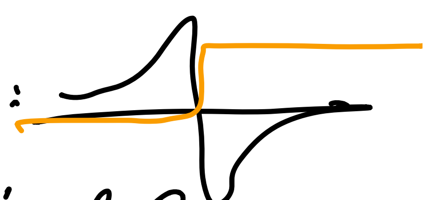
$\xi = x - t \quad d\xi = -dt$

$\xi : x \to -\infty$

$$= \int_{x}^{-\infty} g''(\xi)\,(-d\xi)$$

$$= \int_{-\infty}^{x} g''(\xi)\, d\xi$$

$$= g'(x)$$

Answer: 

1st deriv of Gaussian

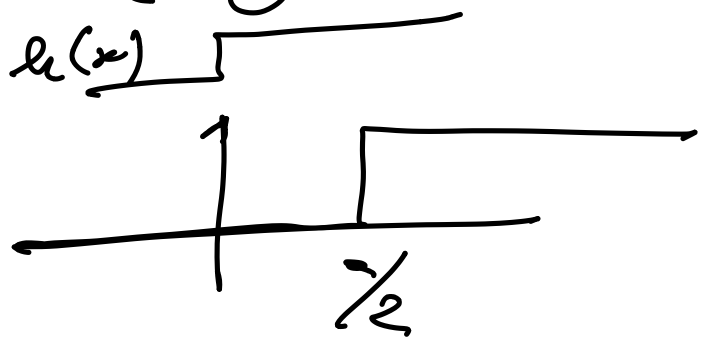
IP3

$$\mathrm{box}(x) = \begin{cases} \frac{1}{a} & -\frac{a}{2} \le x \le \frac{a}{2} \\ 0 & \end{cases}$$

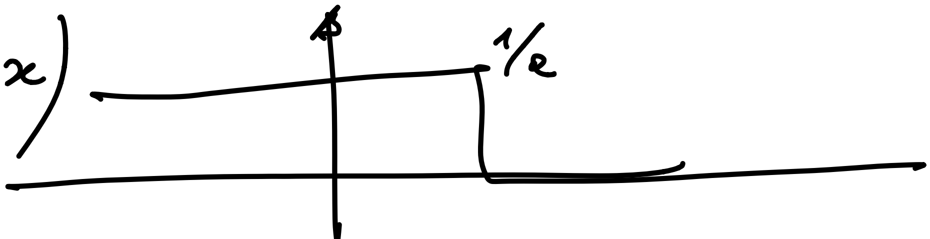
step

$h\left(x - \frac{a}{2}\right)$

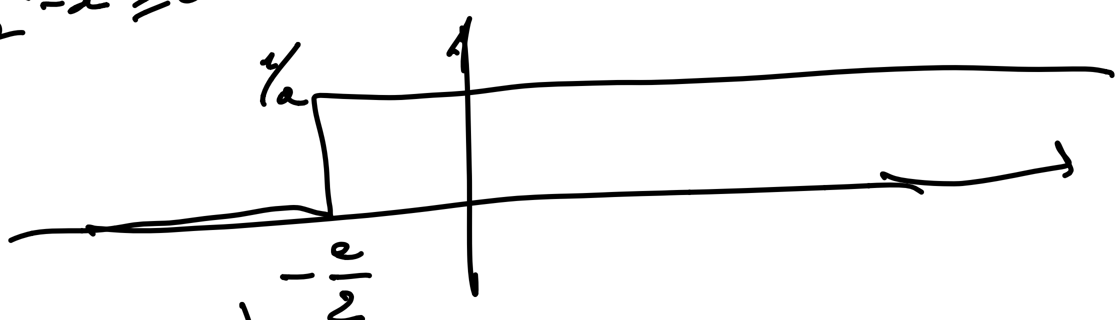
$\mapsto$



$h\left(\frac{a}{2} - x\right)$



$\frac{a}{2} - x \ge 0$

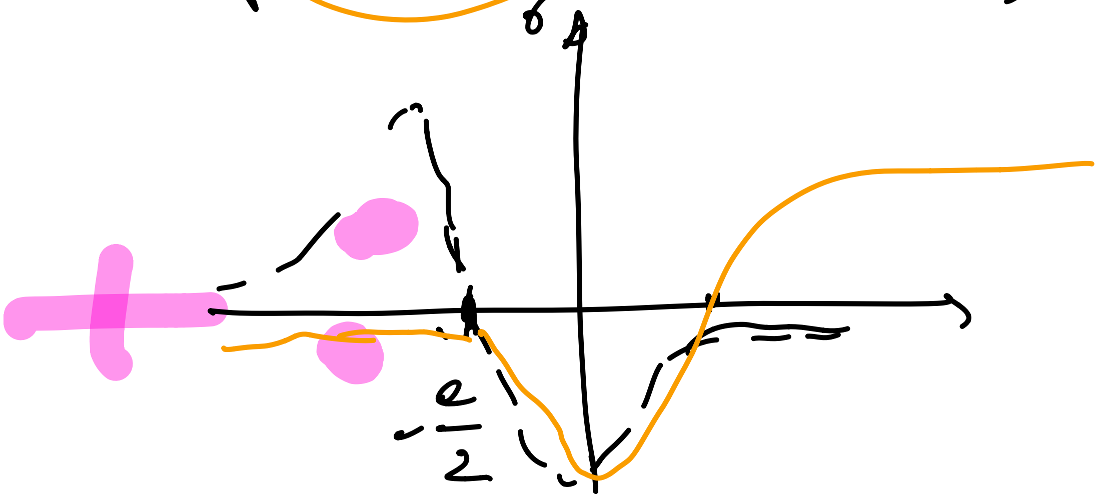


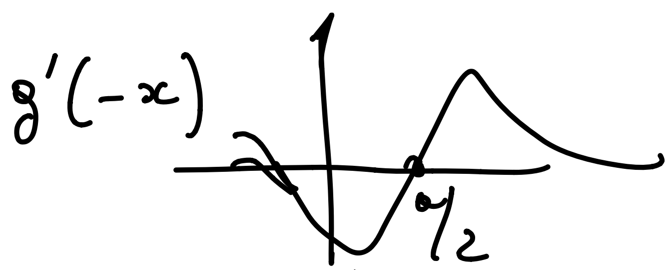
$h\left(x + \frac{a}{2}\right)$

$$\mathrm{box}(x) = \frac{1}{2}\left(h\left(\frac{a}{2} - x\right) + h\left(x + \frac{a}{2}\right)\right)$$

$$\operatorname{box}(x) * g''(x)$$

$$= \frac{1}{2}\left(g'\left(\frac{e}{2} - x\right) + g'\left(x + \frac{e}{2}\right)\right)$$

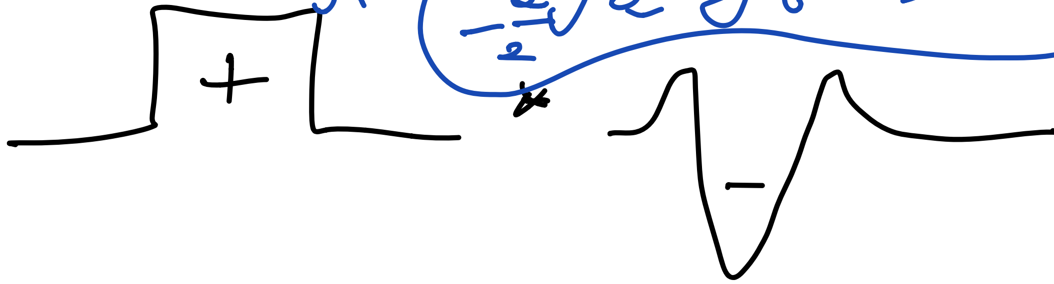




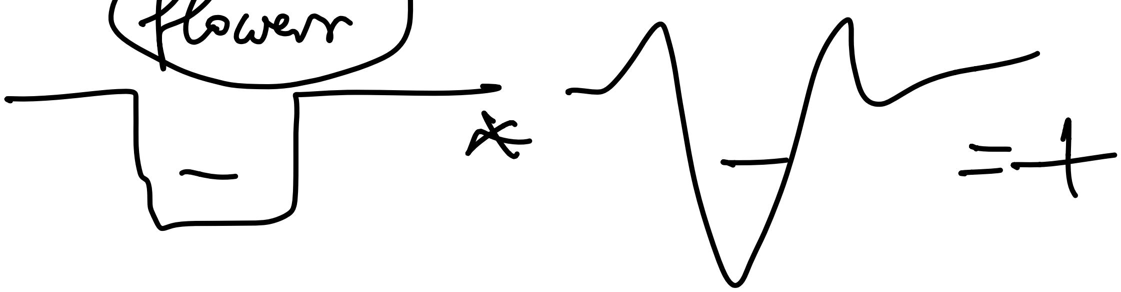
$$g'\left(-\left(x - \frac{e}{2}\right)\right) = g'\left(\frac{e}{2} - x\right)$$

$=$

Alternatively $\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{a}\, g(t-x)\, dx$



HW7 flowers

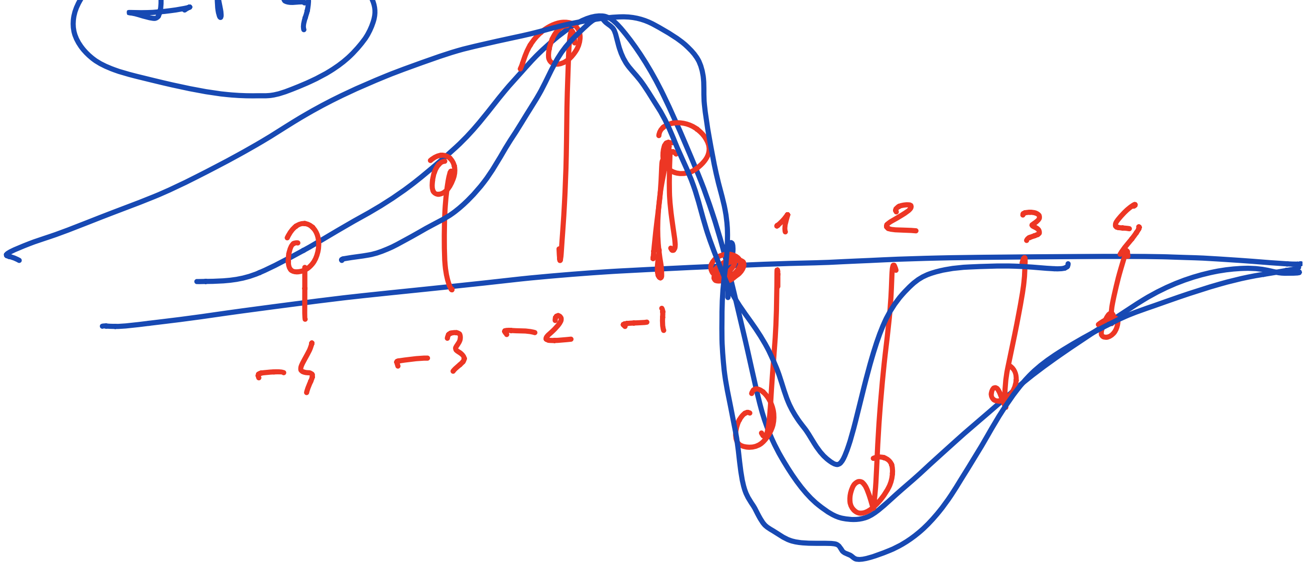


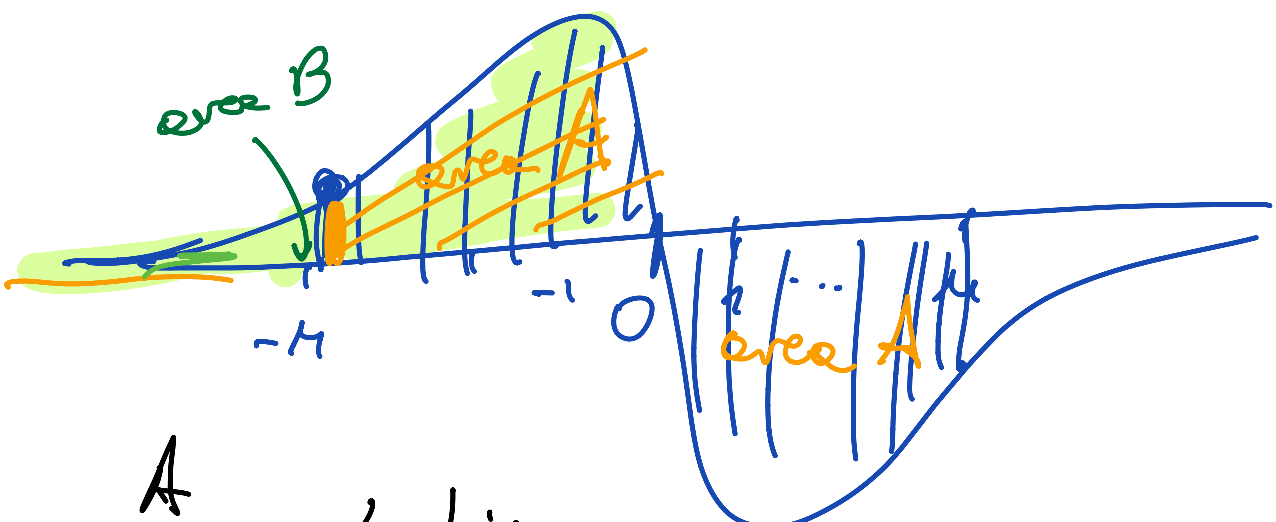
What is the best normalization for g'' so that $g''(\sigma)$ has a maximum proportional to a.

$$\frac{\partial}{\partial \sigma}\left(g'_\sigma\left(\frac{a}{2}-x\right)+g'_\sigma\left(x+\frac{a}{2}\right)\right)=0$$

solve for $\sigma \longrightarrow$ proportional to a!

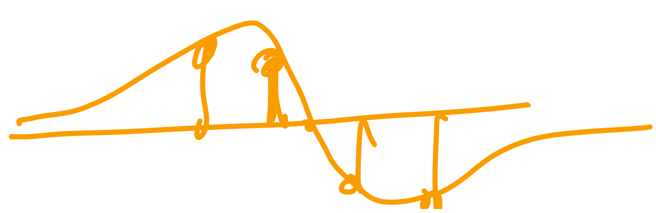
IP4





$$\frac{A}{B} = \alpha$$ big enough to capture the variation.

avoid



$$B = \int_{-\infty}^{0} g'(x)\,dx = g(0)$$

$$= \frac{1}{\sigma\sqrt{2\pi}}$$

$$A = \int_{-\mu}^{0} g'(x)\,dx = g(0) - g(-\mu)$$

$$\frac{g(0) - g(-\mu)}{g(0)} = \alpha = 0.9$$

$$1 - \frac{\cancel{\frac{1}{\sigma\sqrt{2\pi}}}\, e^{-\frac{\mu^2}{2\sigma^2}}}{\cancel{\frac{1}{\sigma\sqrt{2\pi}}}} = \alpha$$

$$e^{-\frac{\mu^2}{2\sigma^2}} = 1 - \alpha$$

Natural log

$$-\frac{\mu^2}{2\sigma^2} = \log(1-\alpha)$$

$$\mu^2 = 2\sigma^2 \log\frac{1}{1-\alpha}$$

$$\boxed{\mu = \sigma\sqrt{2\log\frac{1}{1-\alpha}}}$$

$$\alpha = 0.9 \qquad \sqrt{2\log 10} = 2.146$$

$$\mu = \sigma\, 2.146$$

$\sigma = 1 \qquad \mu = 2 \qquad 2\mu+1 = 5$ mesh

$\sigma = 2 \qquad \mu = 4.2 \qquad 9$ mesh

$$\mu = 1 \qquad \sigma = \frac{1}{\sqrt{2\log\frac{1}{1-\alpha}}} = 0.46$$

To realize a convolution

with $\sigma = 2$

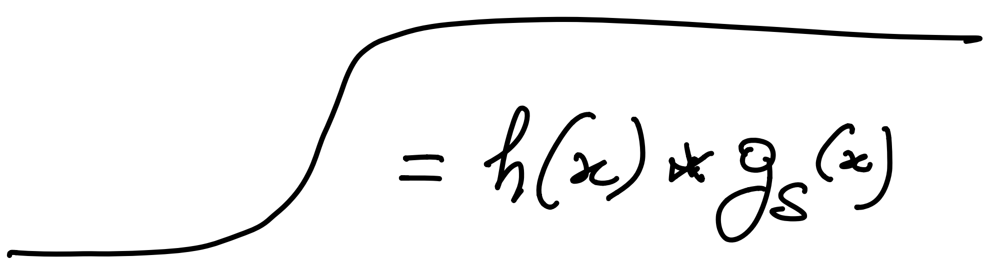
$$\sqrt{n \, 0.46^2} = 2$$

$$n = \frac{4}{0.46^2} = 18.9$$

19 convolutions

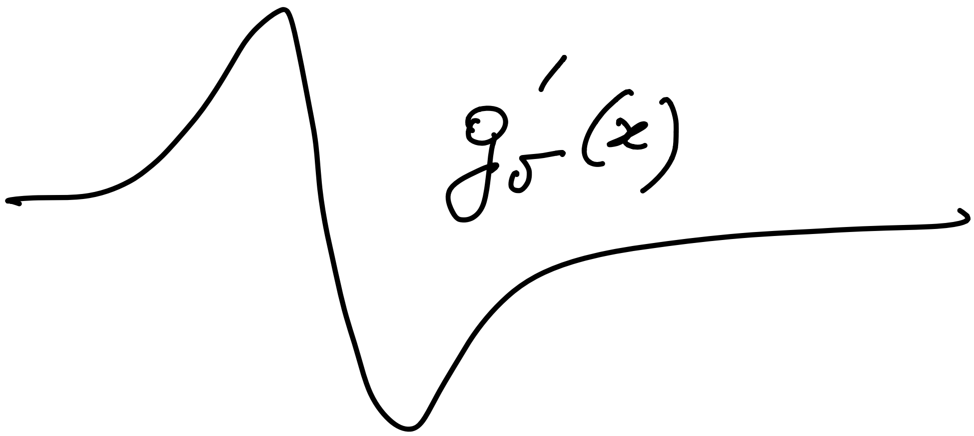
with a 3x1 mask

5b. blurred step edge



$$= h(x) * g_s(x)$$

$*$



$$\left(h(x) * g_s(x)\right) * g_\sigma'(x)$$

we have to show $= g_{?^2}(x)$

we know that $\boxed{h(x) * g'(x) = g(x)}$

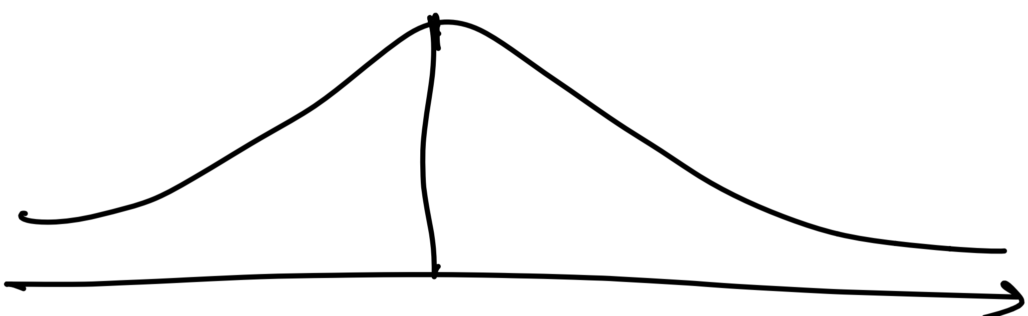
$$h(x) * \left(g_s(x) * g_\sigma'(x)\right)$$

$$= \frac{d}{dx}\left(g_s * g_\sigma\right)$$

$$\overbrace{h(x) * g'_{\sqrt{s^2+\sigma^2}}}$$

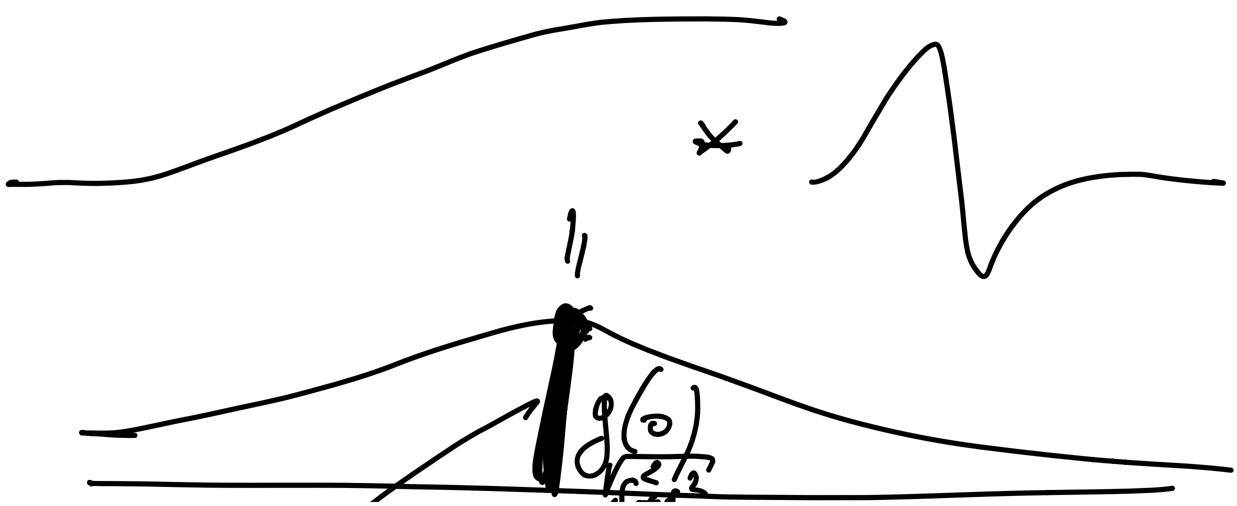
$$= g_{\sqrt{s^2+\sigma^2}}(x)$$

$$\parallel$$



max $\Rightarrow$ edge

What abou'



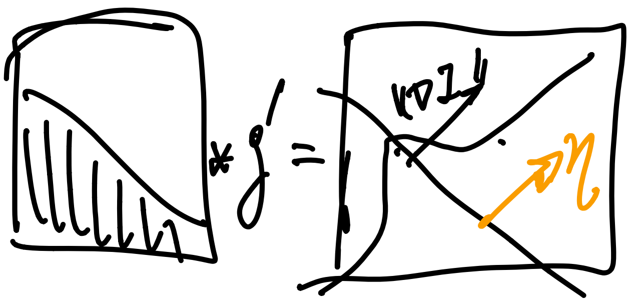
↳ can the magnitude of the convolution with $g'(x)$ reveal how blurred the original image is?

$$g_{\sqrt{s^2+\sigma^2}}(0) = \frac{1}{\sqrt{2\pi}\sqrt{s^2+\sigma^2}} \downarrow \quad s \text{ increasi}$$

IP6 same as IP3

IP7 : 2D

$$\|\nabla I\| = \sqrt{I_x^2 + I_y^2}$$



$$\eta = \frac{\nabla I}{\|\nabla I\|} \qquad \frac{\partial \|\nabla I\|}{\partial \eta} = 0$$

look up $\frac{\partial \|v\|}{\partial v} = \frac{v}{\|v\|}$

unit vector

$$\frac{\partial f}{\partial \vec{\eta}} \quad \text{directional derivative}$$

$$= \vec{\eta}^{T} \nabla f$$

$$f = \|\nabla I\|$$

$$\frac{\partial \|\nabla I\|}{\partial \vec{\eta}} = \vec{\eta}^{T} \underbrace{\nabla \|\nabla I\|}$$

$$= \vec{\eta} \frac{\nabla I}{\|\nabla I\|}$$

$$= \frac{\nabla I}{\|\nabla I\|}^{T} \frac{\nabla I}{\|\nabla I\|} = 1$$

sth is wrong here!!!

to be continued!