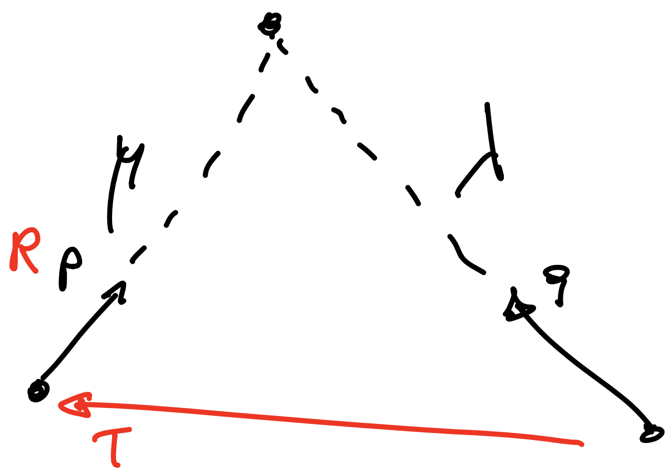
Structure from Motion II



$$\lambda q = R \mu p + T$$

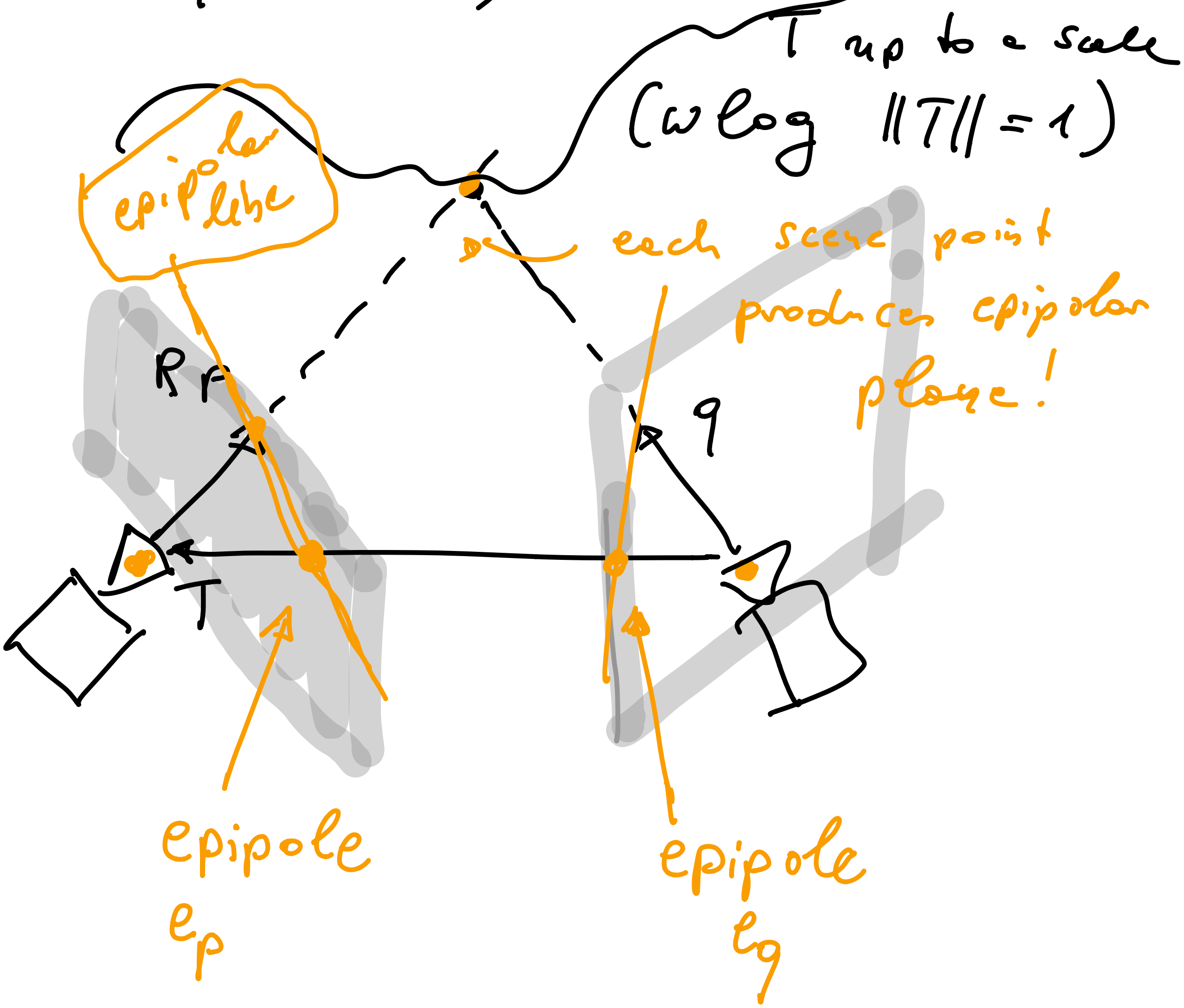
Given: (p_i, q_i)

Find: R, T, λ, μ

$$q^T (T \times R p) = 0$$ epipolar constraint

1. fact T, λ, μ can be computed up to a factor!

$q^T(T \times Rp) = 0$ R



2. fact epipolar line
= intersection of epipolar
plane with image plane

epipolar line is q-plane:

$\boxed{q^T l = 0}$ $q^T(T \times Rp) = 0$

$$\boxed{q = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad \ell = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \quad Ax + By + C = 0}$$

coefficients of line

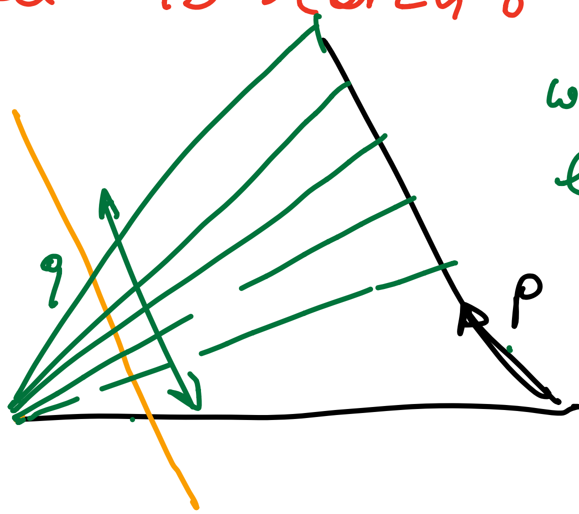
epipolar line in p-plane:

$$p^T \underbrace{R^T (q \times T)}_{\text{coefficients of line}} = 0$$

Significance:

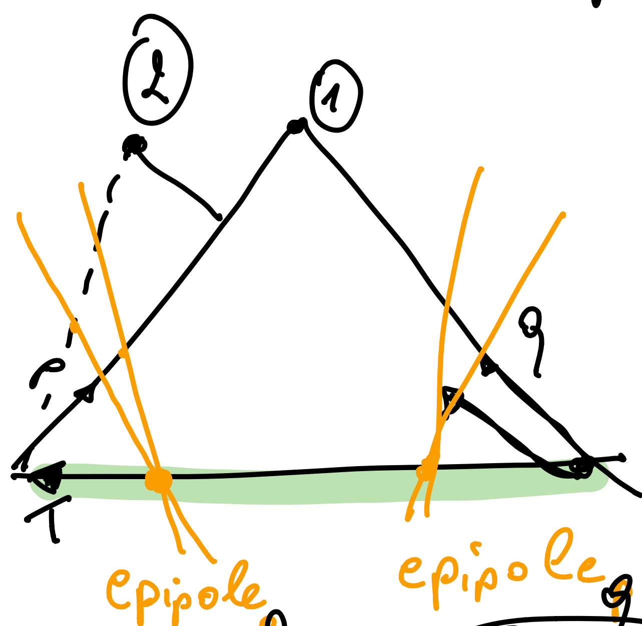
given p, T, R we know that q will lie in $q^T(T \times Rp) = 0$

Correspondence: (which q corresponds to p) will be an 1D-search!



where on the line $\Leftrightarrow$ depth of q

$\boxed{\text{3. fact}}$ For different scene points epipolar lines intersect at epipole!



$$e_p \sim -R^T T \qquad e_q \sim T$$

4. fact If we recover epipole e_q then we know Translation!

If we recover e_p
we know some part* of
the rotation $(R^T T)$

* If we know $\vec{a}$ and $R\vec{a}$
can we recover the whole R?

$$R\vec{a} = RR_{\vec{a}}\vec{a} \qquad R_{\vec{a}}\vec{a} = \vec{a}$$

axis

5. Two epipoles contain
T-direction and R
upto a rotation around T.

Solve for R, T in two steps.

$$\underbrace{q^T (T \times R}_{E} p) = 0$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \underset{3\times 3}{\hat{a}}\, b$$

cross product = antisymm. matrix x vector

or skew matrix

$$\hat{a} b = [a]_\times b$$

$$\boxed{\hat{a}^T = -\hat{a}}$$

$$\underset{3\times 3}{\hat{a}}\ \underset{3\times 1}{a} = a \times a = 0$$

$$\Downarrow$$

$$\det \hat{a} = 0 \Longleftarrow \operatorname{rank} \hat{a} = 2$$

$$q^T \underbrace{\hat{T} R}_{E} p = 0 = q^T E p \; *$$

$\boxed{\text{essential matrix}}$

$$q_{pix}^T \underbrace{K^{-T} E K^{-1}}_{F} p_{pix} = 0$$

fundamental matrix

song

$$\boxed{q_{pix}^T F p_{pix} = 0}$$

? $q_i^T E p_i = 0 \Rightarrow E$

$i = 1..N$

coefficients

* $q^T (Ep) = 0$ epipolar line in q-plane

$p^T (E^T q) = 0$ in p-plane

$$\underset{3\times3}{E} = \begin{pmatrix} \underset{3\times1}{e_1} & \underset{3\times1}{e_2} & \underset{3\times1}{e_3} \end{pmatrix}$$

$$q^T \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} p_x q^T & p_y q^T & p_z q^T \end{pmatrix}}_{a} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

1×9 $\qquad$ 9×1

for $\boxed{\text{one point}}$ correspondence

$$\begin{array}{l} \\ 1^{st}\text{ point} \rightarrow \\ 2^{nd}\text{ point} \rightarrow \\ \\ N^{th} \rightarrow \end{array} \underbrace{\overset{A}{\begin{pmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_N^T \end{pmatrix}}}_{N \times 9} \underset{9 \times 1}{E_{stacked}} = 0$$

$$a^T = (p_x q_x \;\; p_x q_y \;\; p_x q_z \;\; p_y q_x \;\; p_y q_y \;\; p_y q_z \;\; \ldots \;\; p_z q_x \;\; p_z q_y \;\; p_z q_z)$$

$$A = U S \underset{9 \times 9}{V^T}$$

solution $E_{stacked} = V(:, 9)$

$$N \geq 8$$ → 8 independent equations

understanding linear homogeneous system : 3 unknowns

$$x + y - z = 0$$

$$\boxed{\begin{aligned} 2x + 2y - 2z &= 0 \\ x - y + z &= 0 \end{aligned}}$$

two independent equations suffice

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$2x + 2y = 2z$$
$$x - y = -z$$

$$\boxed{\begin{aligned} x &= 0 \\ y &= z \\ z &= z \end{aligned}}$$

Why not 5 points ?

$$T = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \quad R = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

$$q^T (T \times Rp) = \begin{pmatrix} q_x & q_y & q_z \end{pmatrix} \begin{pmatrix} t_z (R_{21} p_x + \dots) \end{pmatrix}$$

P3P $\Rightarrow$ 4th degree polynomial

SfM-8p $\Rightarrow$ SVD $\Rightarrow$ E $\overset{?}{\Rightarrow}$ R, T

SfM-5p $\Rightarrow$ highly non linear

(Nister 2005 5p algorithm)

dirty but correct trick

(3+1p) $\boxed{\text{Naroditsky 2012}}$

sampling