CIS580 Problem Set 3

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Contents

1 Transformation to map facade to rectangle

$$W' \sim P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \alpha W' = P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = T_1$$

$$X' \sim P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \implies \beta X' = P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = T_2$$

$$Y' \sim P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \implies \gamma Y' = P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = T_3$$

$$Z' \sim P \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \implies \delta Z' = P \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = T_1 + T_2 + T_3$$

Combining the above equations:

$$\delta \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} -b \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ h \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can infer that $\delta = 1$, and simplify the system of equations to:

$$\delta \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} -b \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ h \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix} = \left(\begin{bmatrix} -b & 0 & 0 \\ 0 & h & 0 \\ 1 & 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Solving for the inverse of the coefficient matrix, we obtain:

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} -1/b \\ 1/h \\ 1/b - 1/h + 1 \end{bmatrix}$$

From this, we obtain the transformation P by multiplying α, β and γ into the above equation:

$$T^{-1} = \alpha \begin{bmatrix} -b \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ h \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{-1}{b} \begin{bmatrix} -b \\ 0 \\ 1 \end{bmatrix} + \frac{1}{h} \begin{bmatrix} 0 \\ h \\ 1 \end{bmatrix} + \left(\frac{1}{b} - \frac{1}{h} + 1 \right) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-1}{b} & \frac{1}{h} & \frac{1}{b} - \frac{1}{h} + 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Taking the inverse of the above transformation matrix, we obtain the matrix T:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{h}{h-b+bh} & \frac{-b}{h-b+bh} & \frac{bh}{h-b+bh} \end{bmatrix}$$

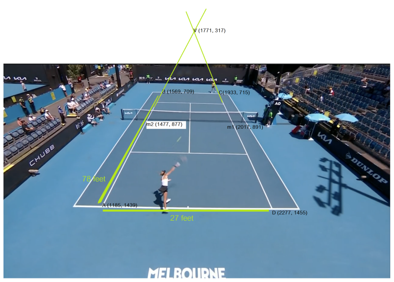
2 Compute distances of patrol car and bridge

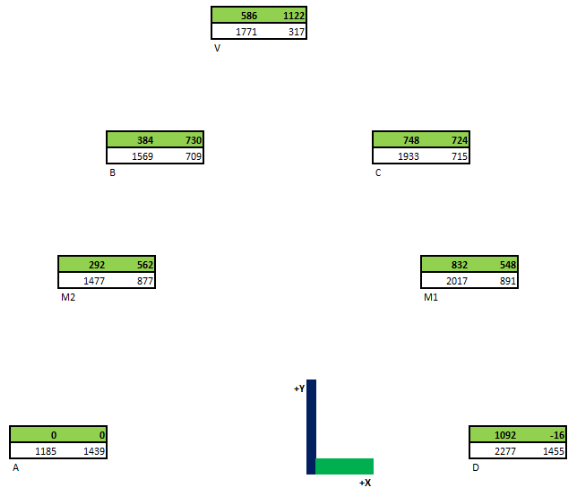
3 Compute distances from the image

4 Different perspectives in a tennis match

- You should get the coordinates of the points in the image. To do so you can use any unit you want. Using pixels is the most convenient.
- The cross ratio should be applied in one of the baselines, and you can use the net in the middle of the court as an extra point.
- Since you have measured coordinates of A and B, you can use the similar triangle to obtain the coordinate of V.
- I had taken average of the vanishing points obtained by using cross ratio on AB and CD due to the minor differences between them.
- For each pair of sidelines you should find 1 vanishing point (so 2 in total for the two images). In the same image using the cross ratio in the two sidelines, you should get the same vanishing points (they will not be exactly the same but they will be close enough, you can just use one of them)
- For the intersection of baselines you can use intersection of lines.
- You cannot define a coordinate system where (0,0,1) is one corner and (1,1,1) is the other because you will mess the distances. In the tennis case we don't have a square. You should modify the method so that it uses [27,78,1] instead of [1,1,1]
- Oh, I see. We should use two images of the same plane, tennis court, to construct two vanishing lines and find the principle point.

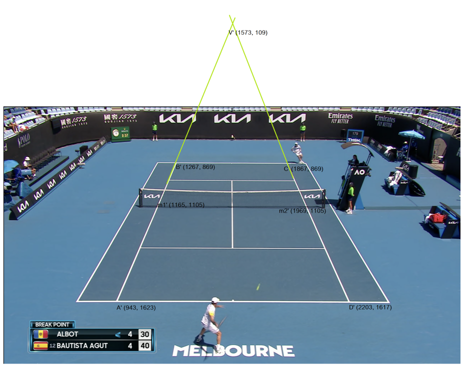
4.1 Why is the perspective different

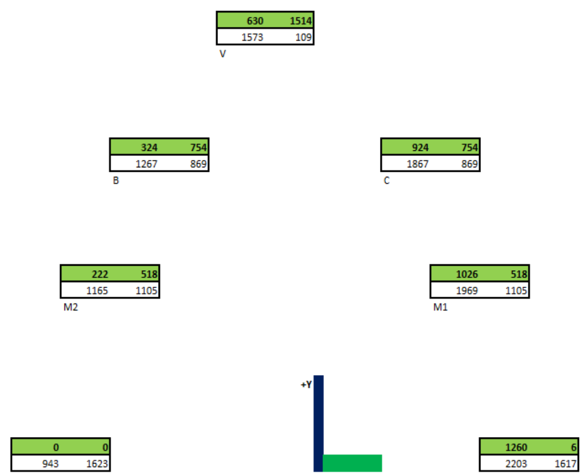




Distances

AB	824.8
CD	816.0
M1C	195.0
DV	1245.4
CV	429.7





Distances

AB	820.7
CD	820.0
M1C	257.1
DV	1634.3
CV	814.9

4.2 Find vanishing points using cross-ratios

Known quantities:

Units	Segment	Value
Image	AD	$\sqrt{1092^2+16^2} = 1092$ pixels
Image	BC	$\sqrt{364^2+6^2} = 364$ pixels
Image	AB	$\sqrt{730^2+384^2} = 825$ pixels
Image	CD	$\sqrt{344^2+740^2} = 816$ pixels
Image	M_1M_2	$\sqrt{540^2+14^2} = 540$ pixels
Image	$A'D'$	$\sqrt{1260^2+6^2} = 1260$ pixels
Image	$B'C'$	$\sqrt{600^2+0^2} = 600$ pixels
Image	$A'B'$	$\sqrt{324^2+754^2} = 821$ pixels
Image	$C'D'$	$\sqrt{336^2+748^2} = 820$ pixels
Image	$M_1'M_2'$	$\sqrt{804^2+0^2} = 804$ pixels
World	$AD, A'D'$	27 feet
World	$BC, B'C$	27 feet
World	$AB, A'B'$	78 feet
World	$CD, C'D'$	78 feet

Vanishing Point for Image 1

$$\begin{aligned}
\frac{DC/DV}{M_1C/M_1V} &= \frac{C_wD_w/D_wV_w}{M_{1w}C_w/M_{1w}V_w} \\
\frac{816/DV}{195/M_1V} &= 2 \cdot M_{1w}V_w/D_wV_w \\
\frac{816}{195} \cdot \frac{M_1V}{DV} &= 2 \cdot M_{1w}V_w/D_wV_w \\
\frac{195+CV}{816+CV} &= \frac{195}{816} \cdot 2 \cdot 1 \\
(195+CV) \cdot 816 &= 390 \cdot (816+CV) \\
CV \cdot (816-390) &= 816 \cdot (390-195) \\
CV &= \frac{816 \cdot 195}{426} \\
CV &= 374
\end{aligned}$$

Given the slope of the line and the distance CV, we can compute V as follows:

$$m = \frac{548-724}{832-748} = \frac{724-V_y}{748-V_x} \tag{1}$$

$$374 = \sqrt{(724-V_y)^2 + (748-V_x)^2} \tag{2}$$

Solving the above system of equations using Mathematica, we obtain the possible vanishing points as:

$$V_x \to 909.093, V_y \to 386.472$$
$$V_x \to 586.907, V_y \to 1061.53$$

Double-checking the answer using cross-products:
Image 1 Vertical Vanishing Point

$$L_1 = \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1092 \\ -16 \\ 1 \end{bmatrix} = \begin{bmatrix} 724 + 16 \\ 1092 - 748 \\ -16 * 748 - 724 * 1092 \end{bmatrix} = \begin{bmatrix} 740 \\ 344 \\ -802576 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 384 \\ 730 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 730 \\ -384 \\ 0 \end{bmatrix}$$

Now, computing $L_1 \times L_2$ to obtain the vanishing point:

$$\begin{bmatrix} 740 \\ 344 \\ -802576 \end{bmatrix} \times \begin{bmatrix} 730 \\ -384 \\ 0 \end{bmatrix} = \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix}$$

Vanishing Point for Image 2

$$\begin{aligned}
\frac{D'C'/D'V'}{M_1{}'C'/M_1{}'V'} &= \frac{C_w{}'D_w{}'/D_w{}'V_w{}'}{M_{1w}{}'C_w{}'/M_{1w}{}'V_w{}'} \\
\frac{820/DV}{257/M_1V} &= 2 \cdot M_{1w}V_w/D_wV_w \\
\frac{820}{257} \cdot \frac{M_1V}{DV} &= 2 \cdot M_{1w}V_w/D_wV_w \\
\frac{257 + CV}{820 + CV} &= \frac{257}{820} \cdot 2 \cdot 1 \\
(257 + CV) \cdot 820 &= 514 \cdot (820 + CV) \\
CV \cdot (820 - 514) &= 820 \cdot (514 - 257) \\
CV &= \frac{820 \cdot 257}{306} \\
CV &= 689
\end{aligned}$$

Given the slope of the line and the distance CV, we can compute V as follows:

$$m = \frac{518 - 754}{1026 - 924} = \frac{754 - V_y}{924 - V_x} \tag{3}$$

$$689 = \sqrt{(754 - V_y)^2 + (924 - V_x)^2} \tag{4}$$

Solving the above system of equations using Mathematica, we obtain the possible vanishing points as:

$$V_x \to 1197.35, V_y \to 121.544$$
$$V_x \to 650.65, V_y \to 1386.46$$

4.3 Find vanishing points for court baselines

Image 1 Horizontal Vanishing Point

$$L_1 = \begin{bmatrix} 384 \\ 730 \\ 1 \end{bmatrix} \times \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 364 \\ -268024 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1092 \\ -16 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 1092 \\ 0 \end{bmatrix}$$

Now, computing $L_1 \times L_2$ to obtain the vanishing point:

$$\begin{bmatrix} 6 \\ 364 \\ -268024 \end{bmatrix} \times \begin{bmatrix} 16 \\ 1092 \\ 0 \end{bmatrix} = \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix}$$

Image 2 Horizontal Vanishing Point

$$L_1 = \begin{bmatrix} 324 \\ 754 \\ 1 \end{bmatrix} \times \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 600 \\ -452400 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1260 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 1260 \\ 0 \end{bmatrix}$$

Now, computing $L_1 \times L_2$ to obtain the vanishing point:

$$\begin{bmatrix} 0 \\ 600 \\ -452400 \end{bmatrix} \times \begin{bmatrix} -6 \\ 1260 \\ 0 \end{bmatrix} = \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix}$$

Finding the homography that maps tennis court to image plane

Image 1

$$W' \sim P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \implies \alpha W' = P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = P_3$$

$$X' \sim P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \implies \beta X' = P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = P_2$$

$$Y' \sim P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \gamma Y' = P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = P_1$$

$$Z' \sim P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \implies \delta Z' = P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} = 27P_1 + 78P_2 + P_3$$

Combining the above equations:

$$\delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} = 27\gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix} + 78\beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can infer that $\alpha = 1$, and simplify the system of equations to:

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} - 78\beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} - 27\gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -44928 \\ -85410 \\ -78 \end{bmatrix} + \gamma \begin{bmatrix} -10854972 \\ 159057 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} \delta & \beta & \gamma \end{bmatrix} = \left(\begin{bmatrix} 748 & -44928 & -10854972 \\ 724 & -85410 & 159057 \\ 1 & -78 & -27 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solving for the inverse of the coefficient matrix, we obtain:

$$\begin{bmatrix} \delta \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 3.0027 \\ 0.0256 \\ 0.0001 \end{bmatrix}$$

From this, we obtain the transformation P by multiplying α, β and γ into the above equation:

$$\begin{bmatrix} 748 \\ 724 \\ 1 \end{bmatrix} = \gamma \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13.4944 & 4.9186 & 0 \\ -0.1977 & 9.3505 & 0 \\ 0 & 0.0085 & 0.3300 \end{bmatrix} \times \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix}$$

Image 2

$$W' \sim P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \implies \alpha W' = P \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = P_3$$

$$X' \sim P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \implies \beta X' = P \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = P_2$$

$$Y' \sim P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \gamma Y' = P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = P_1$$

$$Z' \sim P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} \implies \delta Z' = P \begin{bmatrix} 27 \\ 78 \\ 1 \end{bmatrix} = 27P_1 + 78P_2 + P_3$$

Combining the above equations:

$$\delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} = 27\gamma \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix} + 78\beta \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can infer that $\alpha = 1$, and simplify the system of equations to:

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} - 78\beta \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} - 27\gamma \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \delta \begin{bmatrix} 924 \\ 754 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -50778 \\ -108108 \\ -78 \end{bmatrix} + \gamma \begin{bmatrix} -4275180 \\ -20358 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix}\delta & \beta & \gamma\end{bmatrix} = \left(\begin{bmatrix}924 & -50778 & -4275180\\ 754 & -108108 & -20358\\ 1 & -78 & -27\end{bmatrix}\right)^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Solving for the inverse of the coefficient matrix, we obtain:

$$\begin{bmatrix}\delta\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}2.2010\\ 0.0153\\ 0.0003\end{bmatrix}$$

From this, we obtain the transformation P by multiplying α, β and γ into the above equation:

$$\begin{bmatrix}924\\754\\1\end{bmatrix} = \gamma\begin{bmatrix}158340\\754\\1\end{bmatrix} + \beta\begin{bmatrix}651\\1386\\1\end{bmatrix} + \alpha\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}27\\78\\1\end{bmatrix}$$

$$= \begin{bmatrix}21.1528 & 4.5240 & 0\\ 0.1007 & 9.6318 & 0\\ 0.0001 & 0.0069 & 0.4543\end{bmatrix} \times \begin{bmatrix}27\\78\\1\end{bmatrix}$$

4.4 Compute the focal length of each image

Horizon of Image 1

We can compute the horizon by taking the cross-product of the horizontal and Vertical vanishing points of this image, as computed in the previous parts.

$$h = \begin{bmatrix} 402036 \\ -5891 \\ 1 \end{bmatrix} \times \begin{bmatrix} 576 \\ 1095 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6986 \\ -401460 \\ 443622636 \end{bmatrix}$$

$$\implies V_A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -6986 \\ -401460 \\ 443622636 \end{bmatrix}$$

$$= \begin{bmatrix} 401460 \\ -6986 \\ 0 \end{bmatrix}$$

From the previous part, we also know that the homography H is as follows:

$$\begin{bmatrix} 13.4944 & 4.9186 & 0 \\ -0.1977 & 9.3505 & 0 \\ 0 & 0.0085 & 0.3300 \end{bmatrix}$$

Horizon of Image 2

We can compute the horizon by taking the cross-product of the horizontal and Vertical vanishing points of this image, as computed in the previous parts.

$$h = \begin{bmatrix} 158340 \\ 754 \\ 1 \end{bmatrix} \times \begin{bmatrix} 651 \\ 1386 \\ 1 \end{bmatrix} = \begin{bmatrix} -632 \\ -157689 \\ 218968386 \end{bmatrix}$$

$$\implies V_A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -632 \\ -157689 \\ 218968386 \end{bmatrix} = \begin{bmatrix} 157689 \\ -632 \\ 0 \end{bmatrix}$$

From the previous part, we also know that the homography H is as follows:

$$\begin{bmatrix} 21.1528 & 4.5240 & 0 \\ 0.1007 & 9.6318 & 0 \\ 0.0001 & 0.0069 & 0.4543 \end{bmatrix}$$

4.5 Compute the vanishing points using intersection of parallel lines