

CIS 580 Spring 2021: Midterm 2

- Once you begin the exam you will have 120min to finish and submit to Gradescope. All SDS accommodations will still apply accordingly.
- You are not allowed to post the exam to anyone/anywhere.
- You are not allowed to collaborate with other students.
- During the exam we will post clarification in this **Google doc**. If you have a question please first check the document and if your question is not there please send a PRIVATE question in piazza. We will copy the question and answer it in the doc.
- If you use anything verbatim from the Internet you should cite it properly (like URL).
- Use your own paper if you want and submit the same way you submit a 580 math homework.

1. Problem Structure from Motion

Two views are separated by a rotation around the y -axis and a translation in the xz -plane reflecting a motion of the camera in the xz -plane, a situation called in-plane motion. For example a vacuum robot would satisfy this equation if the camera's y axis is perpendicular to the ground.

$$Q = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} P + \begin{pmatrix} T_x \\ 0 \\ T_z \end{pmatrix}$$

1. [5 Points] Compute the essential matrix, given the above rotation and translation. We call this matrix the "vacuum cleaner essential" matrix.

2. [15 Points]

- Which elements of the essential matrix are always zero?
- How can you tell if a matrix E is "vacuum cleaner essential"?
- Suppose that you have found a "vacuum cleaner essential" matrix. Show how to extract θ and (T_x, T_z) . The latter up to a scale factor.

$$\textcircled{1} \quad E = T_x R = \quad \text{where} \quad R = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$\text{Cross product} \Rightarrow T_x = \begin{bmatrix} 0 & -T_z & 0 \\ T_z & 0 & -T_x \\ 0 & T_x & 0 \end{bmatrix}$$

$$\Rightarrow E = \begin{bmatrix} 0, & -T_z, & 0 \\ T_x \sin \theta + T_z \cos \theta, & 0, & -T_x \cos \theta + T_z \sin \theta \\ 0, & T_x, & 0 \end{bmatrix}$$

a)

② You can tell if E is vacuum cleaner essential if its decomposition into R & T satisfies:

- Rotation only about y -axis
- Translation only in xz plane

b) Main diagonal & off diagonal are zero in vacuum cleaner matrix

c) Extracting $\theta, T_x \text{ & } T_z$:

$$\Rightarrow -E_{0,1} = T_z$$

↑
row ↑
col

(assuming
rows &
cols are
zero-indexed)

$$\Rightarrow E_{2,1} = T_x$$

$$T_n \sin\theta + T_2 \cos\theta = A \quad \leftarrow E_{1,0}$$

$$-T_n \cos\theta + T_2 \sin\theta = B \quad \leftarrow E_{1,2}$$

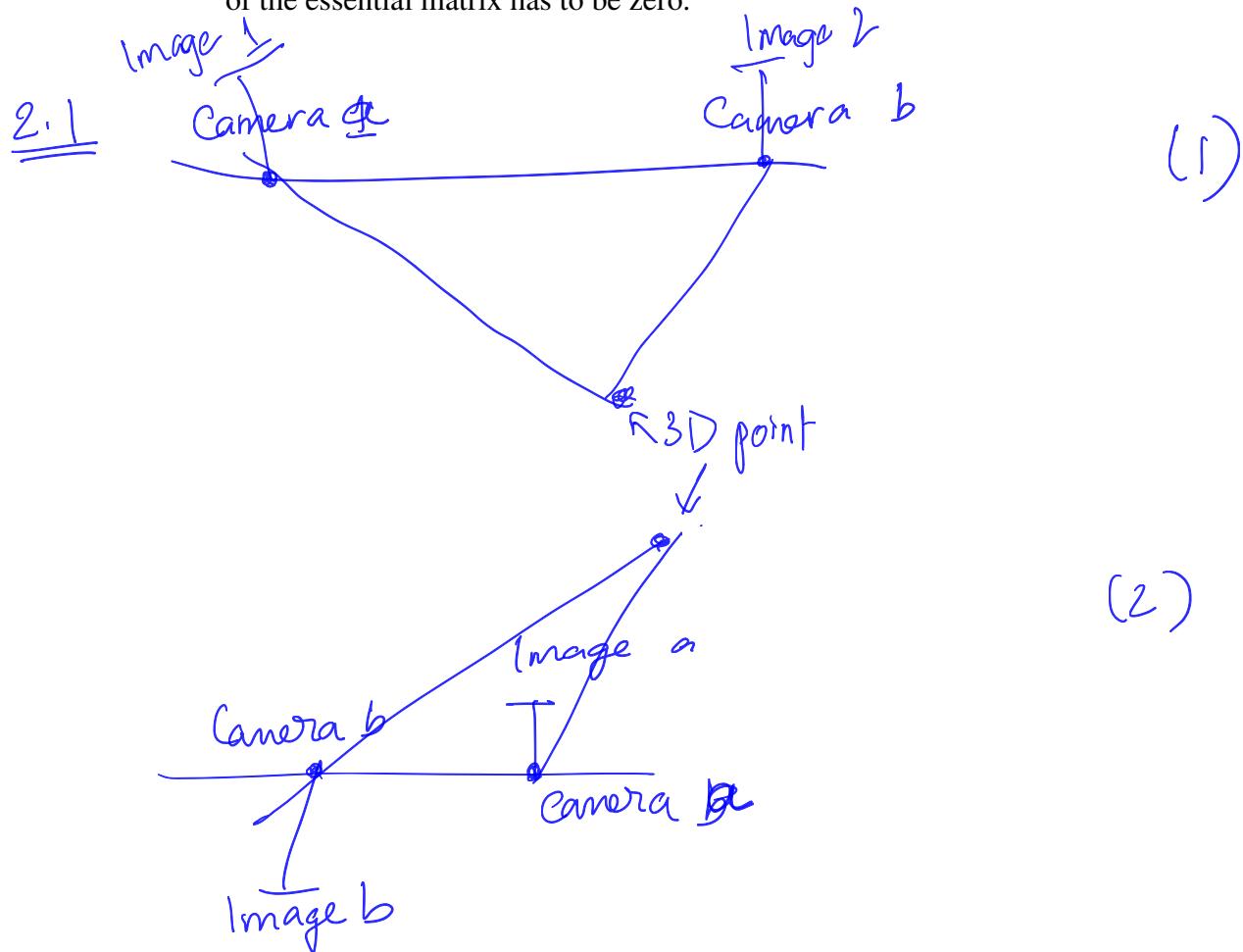
+

$$\cos\theta(T_2 - T_n) + \sin\theta(T_2 + T_n) = A + B$$

↓ This equation can lead to multiple valid solutions for θ

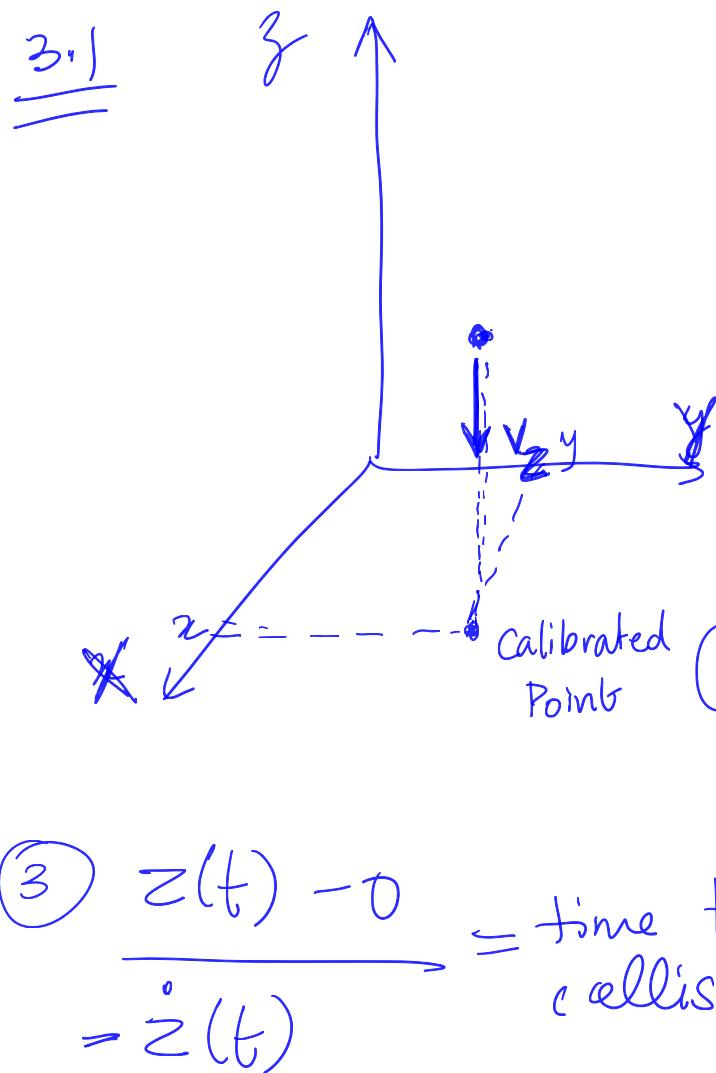
2. Problem Structure from Motion

1. [5 Points] In the structure from motion problem, give an example of a pair of translation and rotation, that result in the two epipoles being at exactly the same pixel position in both images. A sketch would help.
2. [5 Points] Assume two cameras with coordinate systems Q and P and $Q = RP + T$. Give an example of a pair of translation T and rotation R , that causes epipole in image plane q being at infinity and epipole in image plane p being in the center of the image. A sketch would help.
3. [5 Points] Suppose that two views have intersecting optical axes. Show that the element E_{33} of the essential matrix has to be zero.

2.2

3. Problem 3D Velocities

1. [10 Points] Assume that we are moving with a pure translational velocity $(0, 0, V_z)$. Show how can we compute the time to collision to a point in the scene that is projected on the calibrated point (x, y) .
2. [10 Points] Assume that we drive towards a wall parallel to the image plane and we observe a circle. Explain how we can find the time to collision (assuming constant velocity) from the area of the circle A and the rate of change of the area \dot{A} .



Algorithm to compute
time to collision (assuming
point masses)

① If moving body's x &
y coordinates do not
match calibrated point,
no collision

② If particle is moving
away from $z=0$, no
collision (i.e.,
 $\dot{z} > 0$)

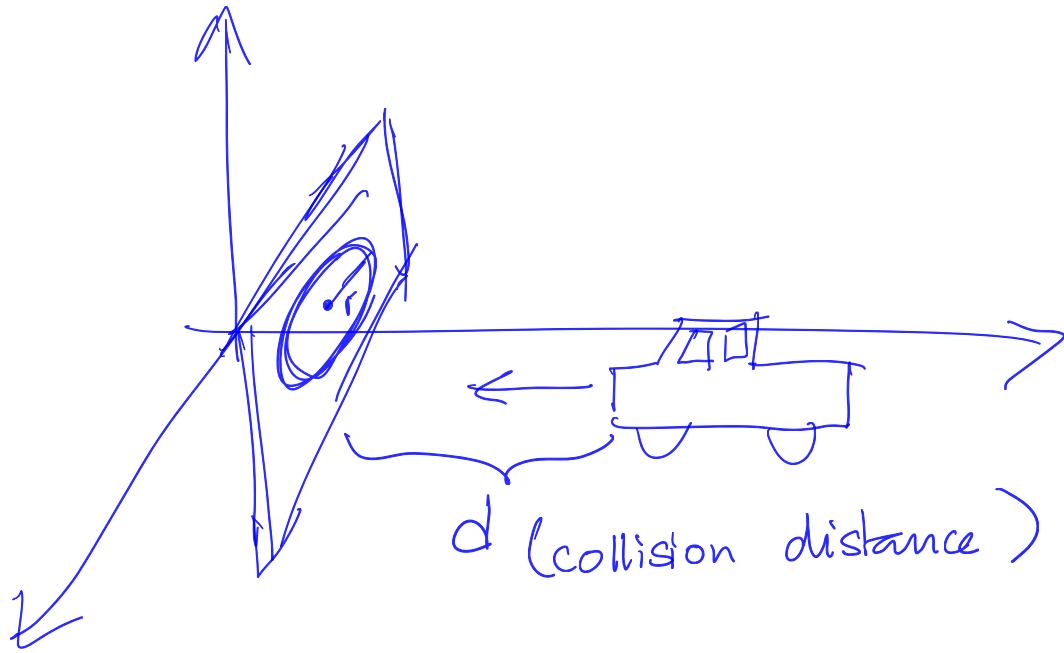
$$\textcircled{3} \quad \frac{z(t) - 0}{\dot{z}(t)} = \text{time to collision}$$

where $z(t)$ is z -coord of
moving particle

3.02 Area of circle $A = \pi r^2$

⇒ We know r is proportional to distance from object ($r \propto d$)

$$\Rightarrow \frac{dA}{dr} = 2\pi r \quad (\propto d)$$



We can use the fact that \dot{A} is directly proportional to collision distance to compute time to collision

4. Problem

- [10 Points] Your system allows you to perform only convolutions with a Gaussian of $\sigma = 1/\sqrt{2}$. How many convolutions would you need to approximate a Gaussian (0th derivative) of $\sigma = 4$. Subsampling is not allowed.
- [10 Points] Assume that you want to approximate the 2nd derivative for $\sigma = 4$. Your system still allows you only to convolve with a Gaussian of $\sigma = 1/\sqrt{2}$. Establish a procedure so that you can compute the equivalent of a convolution with the 2nd derivative of $\sigma = 4$. Subsampling is not allowed.

4.1

$$\sigma = 4 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 \times 8 \times 4} = \sqrt{\frac{32}{2}} = 4$$

~~32 convolutions~~

4.2 Step 1: compute $\sigma=4$ Gaussian
using technique from 4.1

Step 2: Use LoG method twice:

$$F_\alpha = F * \{1 \quad 0 \quad -1\}$$

$$F_{\alpha\alpha} = F_\alpha * \{1 \quad 0 \quad -1\}$$

5. Problem

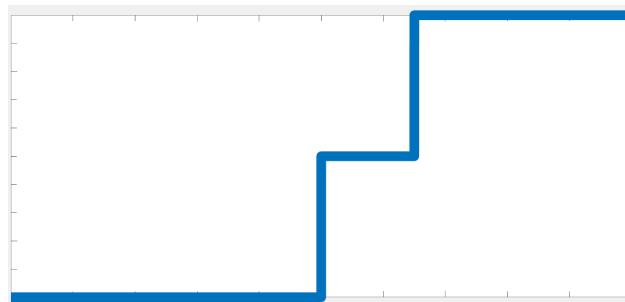
1. [5 Points] Compute the convolution of the 1D edge

$$h(x) = \begin{cases} H & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases}$$

with the first derivative of the Gaussian function with standard deviation σ . Plot or draw the edge, the first derivative of the Gaussian and the result of the convolution.

2. [10 Points] Assume the double-step function of the figure defined as

$$h(x) = \begin{cases} 2H & \text{if } x > a, \\ H & \text{if } 0 \leq x \leq a . \\ 0 & \text{if } x < 0 \end{cases}$$



Compute its convolution with the first derivative of a Gaussian with standard deviation σ and call the response $d(x, \sigma, a)$.

3. [5 Points] Show that $d(x, \sigma, a)$ has always an extremum at $x = a/2$.

Show that $d(x, \sigma, a)$ does not have extrema at $x = 0$ and $x = a$ as one would anticipate.

4. [5 Points] Plot $d(x, \sigma, a)$ for $\sigma = 1$ and $a = 5$.

Plot $d(x, \sigma, a)$ for $\sigma = 4$ and $a = 5$.

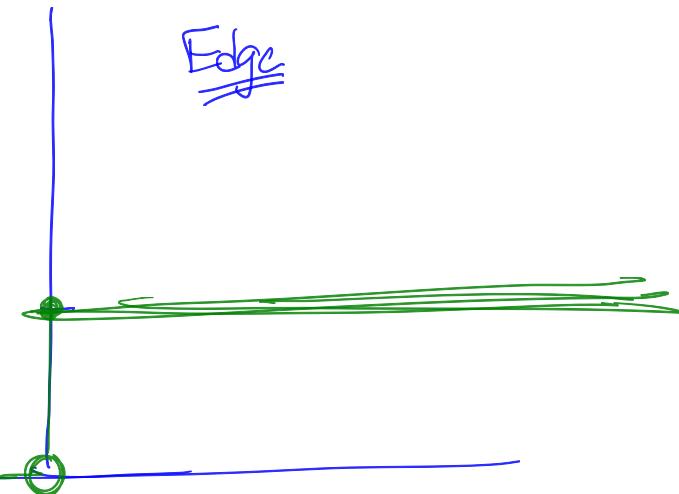
You can use python, matlab, wolfram alpha, or even a Ti84 and you are allowed to draw the curves by hand.

Explain the curves. What do you observe.

D.L

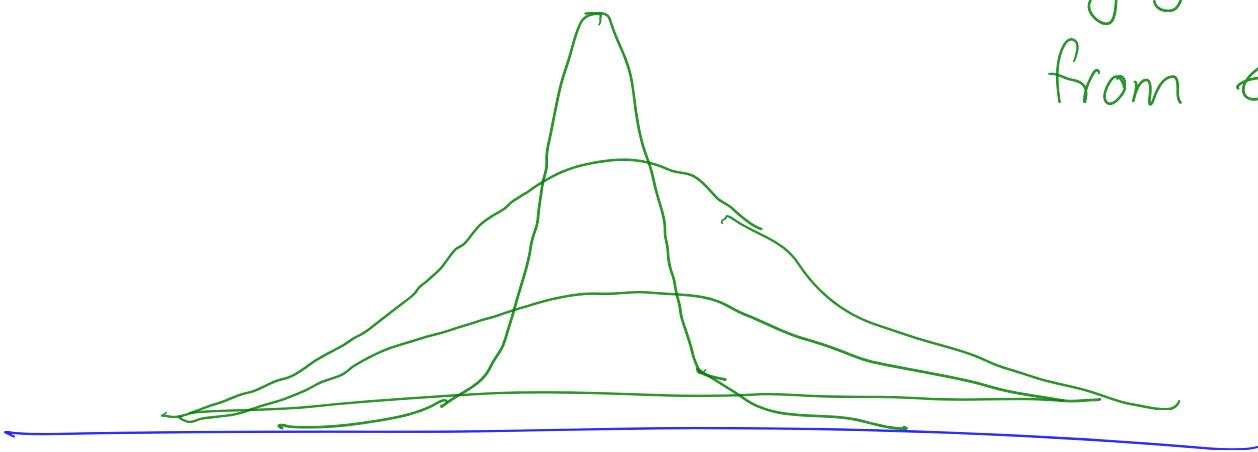
Edge

H



Result of Convolution

(varying sigma
from $\sigma \in [0, 5]$)



Convolution by hand

Derivative of Gaussian

$$f(x) = -t \times e^{\frac{-t^2}{2\sigma^2}}$$

$$\sqrt{2\pi} \cdot \sigma^3$$

$$g(x) = \text{Stepwise: } \begin{cases} 0 & \text{if } t < 0 \\ H & \text{otherwise} \end{cases}$$

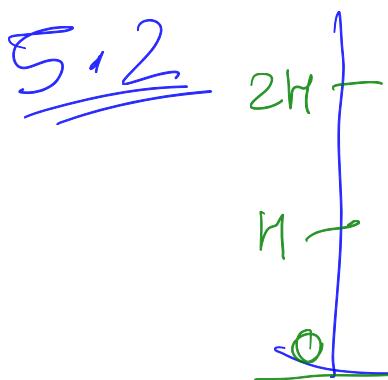
$$(f * g)(t) = \int_{-\infty}^0 f(t-u)g(u)du \Rightarrow 0$$

$- \infty$

$$+ \int_0^\infty f(t-u)g(u)du$$

\uparrow

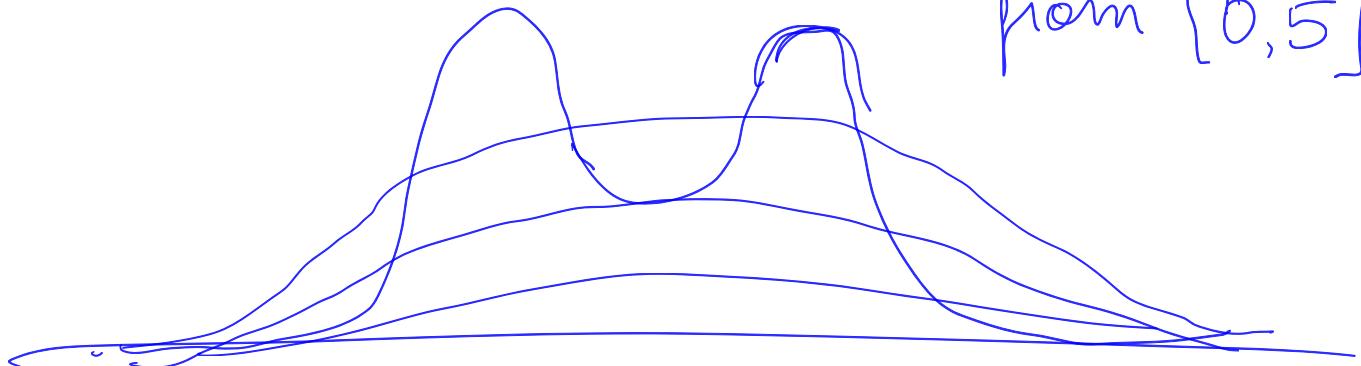
evaluated numerically using matlab



Edge function

Convolution Result

(varying Sigma
from [0, 5])



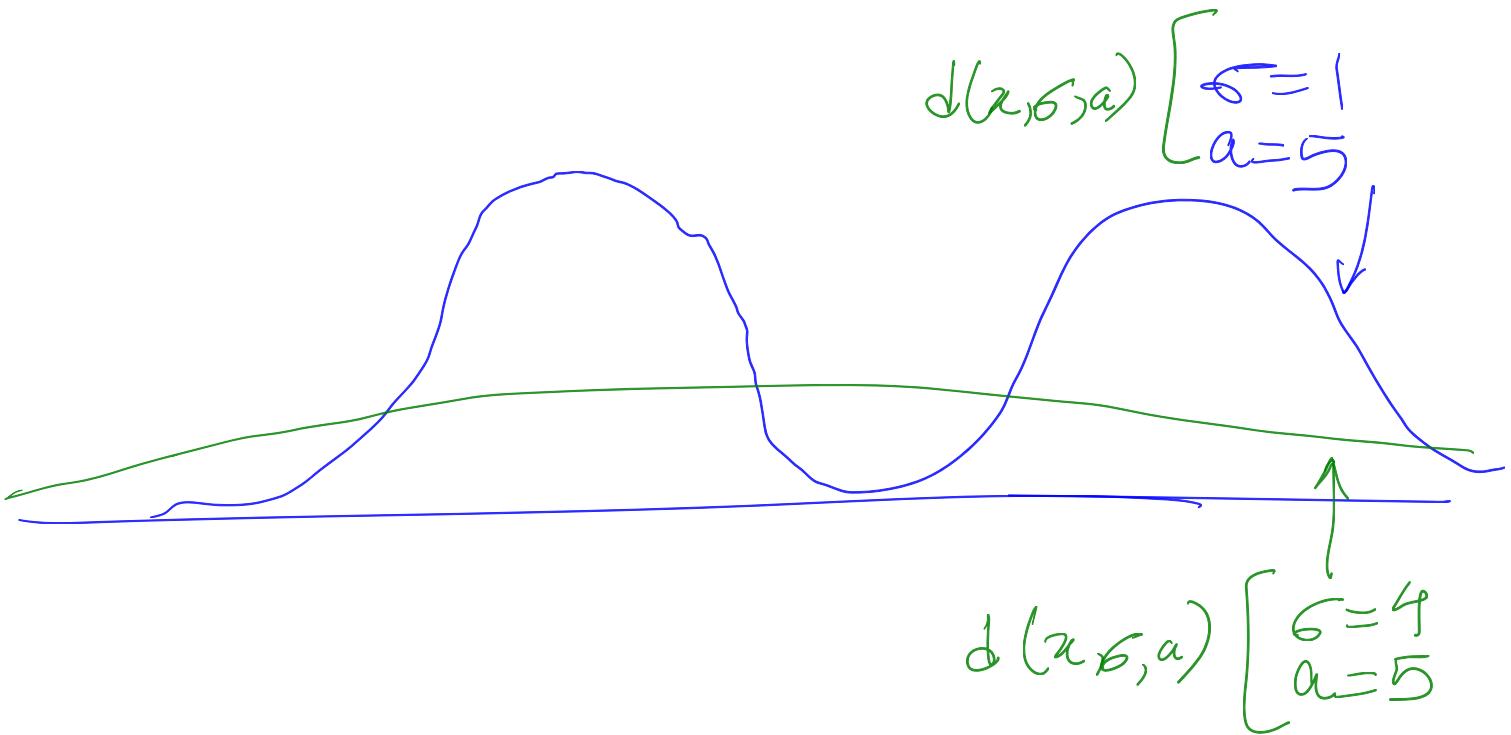
$f(x) = \text{Derivative of Gaussian}$

$g(n) = \text{stepwise: } 0 \quad \text{if } t < 0$
 $\quad \quad \quad H \quad \underline{0 \leq t \leq a}$
 $\quad \quad \quad 2H \quad \text{otherwise}$

$$(f * g)(t) = \int_{-\infty}^0 f(t-u) \cancel{g(u)} du + \int_0^a f(t-u) \cdot H dt + \int_a^\infty f(t-u) \cdot 2H dt$$

(Numerically evaluated to make plot)

5.4



I observe that as s increases, the twin peaks converge to a singular global maxima (distance inversely proportional to s)

5.3 We can prove that $d(x, s, a)$ always has extrema @ $x=a/2$ by altering the expression used to compute the convolution

$$\begin{aligned}
 \frac{\partial}{\partial t} ((f * g)(t)) &= (f(t-a))H - \\
 &\quad f(t)H + \\
 0 - 2H(f(t-a)) \\
 &= -H f(t-a) - f(t) H \\
 &= -H(f(t-a) - f(t)) = 0
 \end{aligned}$$

δ solve for $t \Rightarrow t = \frac{a}{2}$
 is local
 minimum