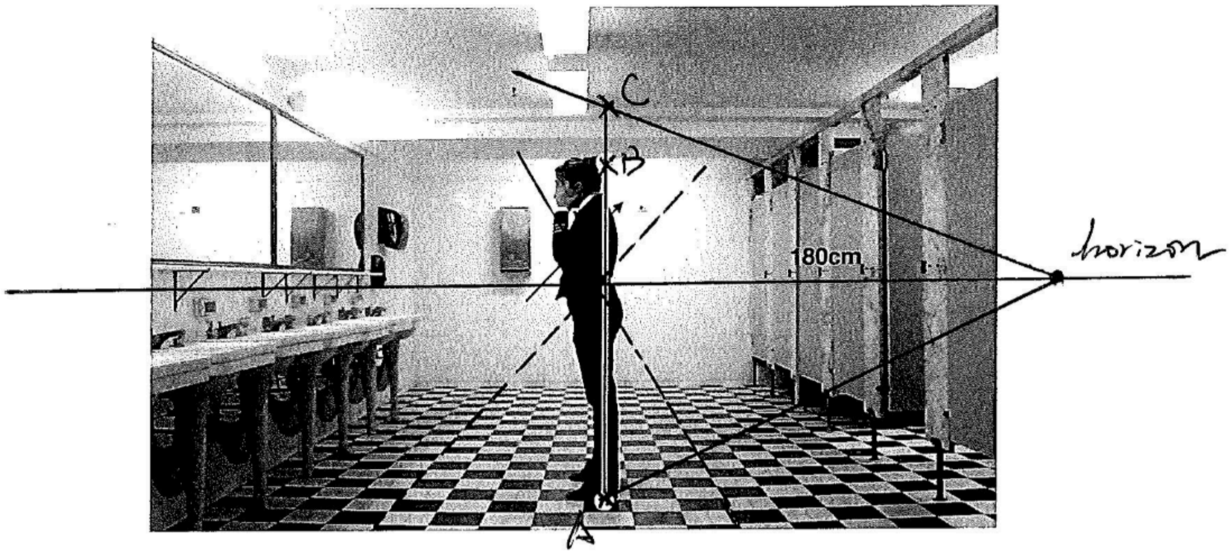
Q1

1. find horizon:



2. Vertical lines parallel -> intersection of vertical parallel lines at infinity

From measurement, $AB : 4.1cm, AC : 4.8cm$

$\frac{AC}{AB} = \frac{180}{H}, H = 180 \times \frac{4.1}{4.8} = 153.75cm$

Q2

1. Rotation: $R_x(90) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$, Translation: $T = \begin{bmatrix} 0 \\ h \\ g \end{bmatrix}$
2. Since Z=0 and K=I, $H = I \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & h \\ 0 & 1 & g \end{bmatrix}$
3. Horizon passes through two vanishing points $(1, 0, 0)$ and $(0, 1, 0)$, they get projected to points $(1, 0, 0)$ and $(0, 0, 1)$. Taking the cross product of the two gives $l'_\infty = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$
4. line equation $l = \begin{bmatrix} 1 \\ 0 \\ -m \end{bmatrix}$ Take two points on the line $(m, 0, 1)$ and $(m, 1, 1)$, they get projected to points (m, h, g) and $(m, h, 1 + g)$. Taking the cross product of the two mapped points gives us the projected line

$$l' = \begin{bmatrix} -h \\ m \\ 0 \end{bmatrix}$$

Q3

1. New transformation $H' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & h \\ 0 & 1 & g - \Delta_z \end{bmatrix}$, $det(H') = -h \neq 0$ H' is projective transformation
2. horizon stays the same $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$
3. projection of line $l' = A^{-T} l, H^{-T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{z-g}{h} & \frac{1}{h} \\ 0 & 1 & 0 \end{bmatrix}$

$$l'' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{z-g}{h} & \frac{1}{h} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -m \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{m}{h} \\ 0 \end{bmatrix}$$

4.

Actions

If you have two cameras with intrinsics K_1 and K_2, then you can write down the projection equation for the second camera as $\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = K_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = K_2^{-1} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$. Now we have projection equation for the first camera as $\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = K_1 R \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, substituting (x,y,z) from above we have $\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = K_1 R K_2^{-1} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$. Since $det(K_1 R K_2^{-1}) \neq 0$, this is a homography.

In this case, $det(K R_x(90) K^{-1}) = det(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}) = 1 \neq 0$

Q4

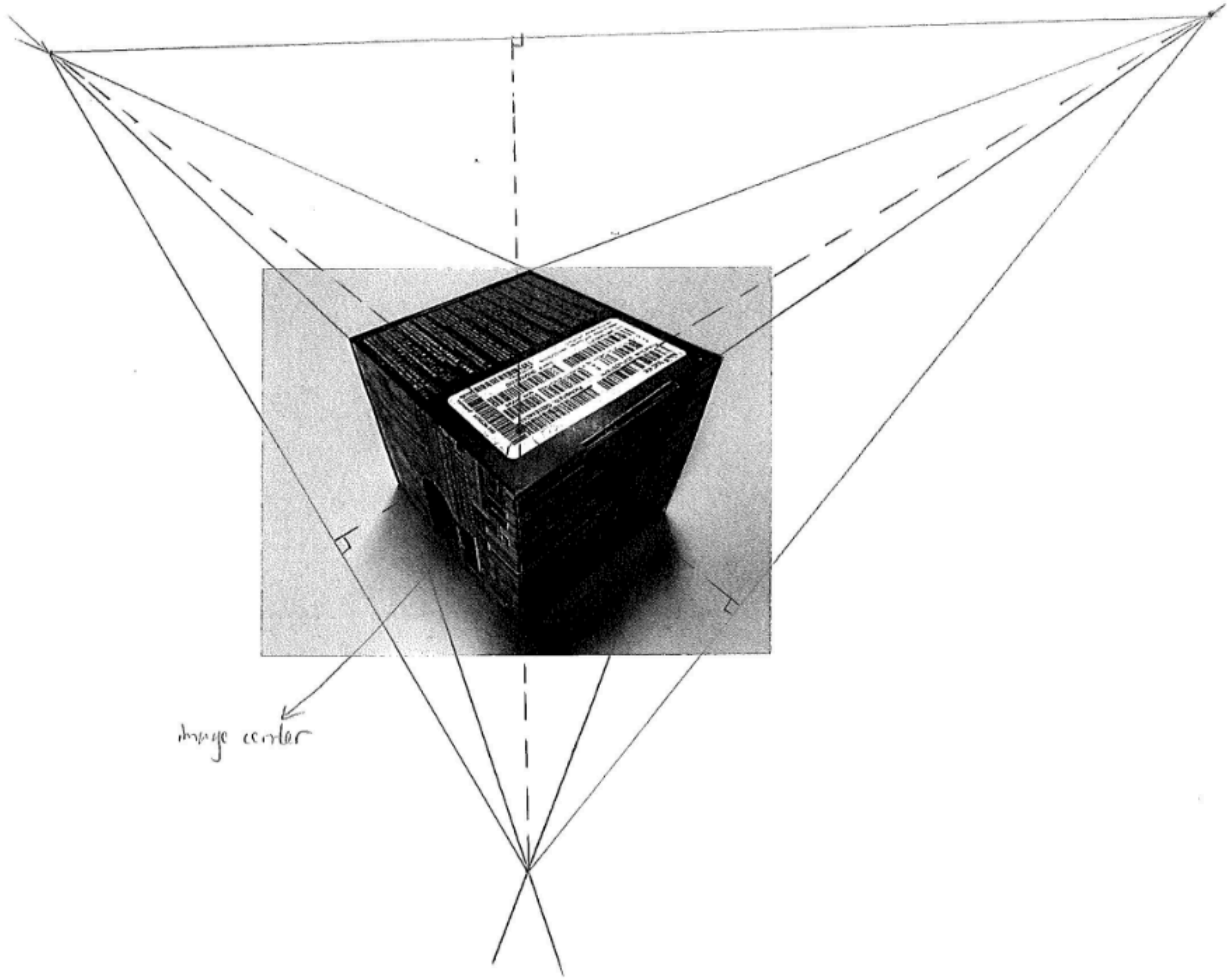
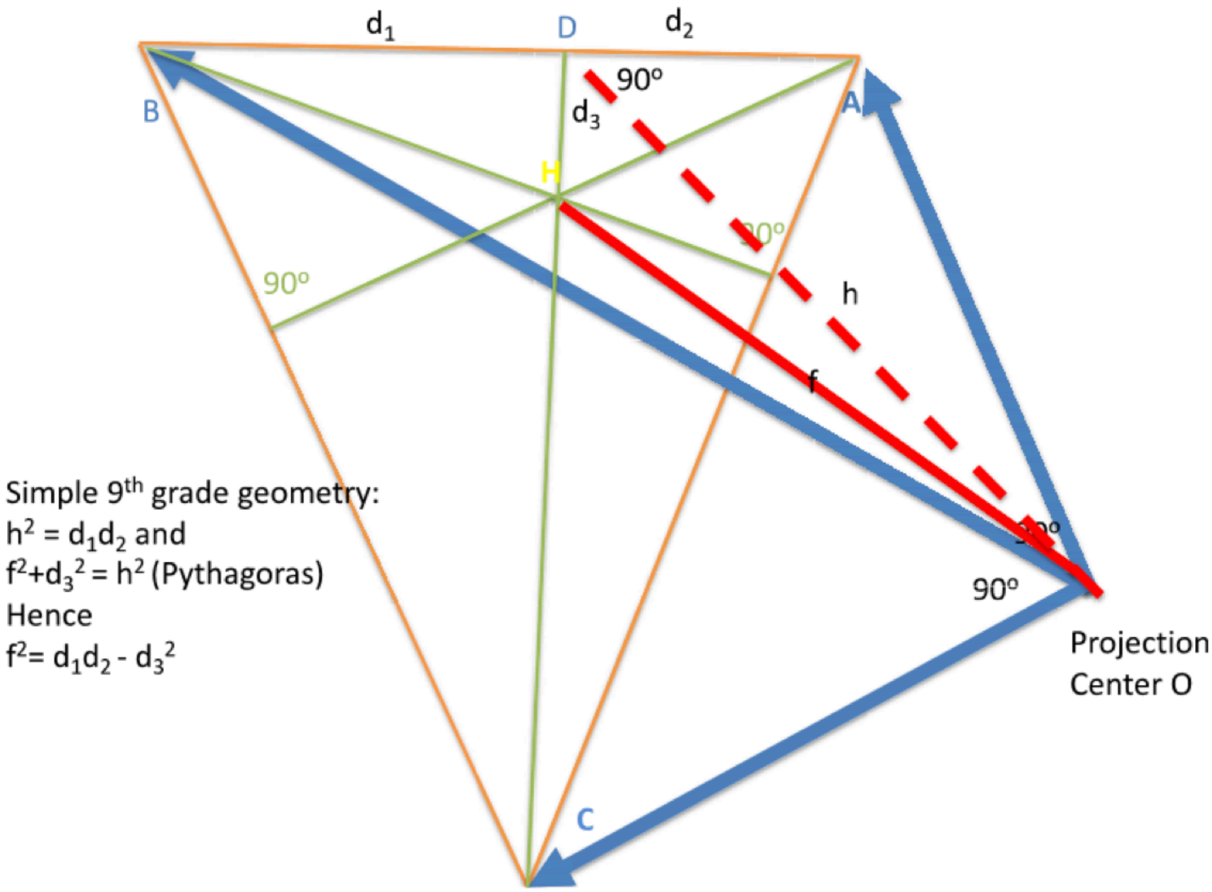


image center



$$f = \sqrt{d_1 d_2 - d_3^2} \times \frac{\text{pixel width}}{\text{measured width}}$$

Alternative method:

Denote two vanishing points coordinates (u_0, v_0, w_0) and (u_1, v_1, w_1) with the orthocenter as origin. Then $\frac{(u_1 u_2 + v_1 v_2)}{f^2} + w_1 w_2 = 0, f = \sqrt{\frac{-w_1 w_2}{u_1 u_2 + v_1 v_2}}$

Q5

1. Translate: No

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = K[RT] \begin{bmatrix} X \\ Y \\ Z \\ 0 \end{bmatrix} = KI \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2. Zoom: Yes. Without loss of generality, assume identity rotation and zero translation. When zooming, f changes, vanishing points move

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

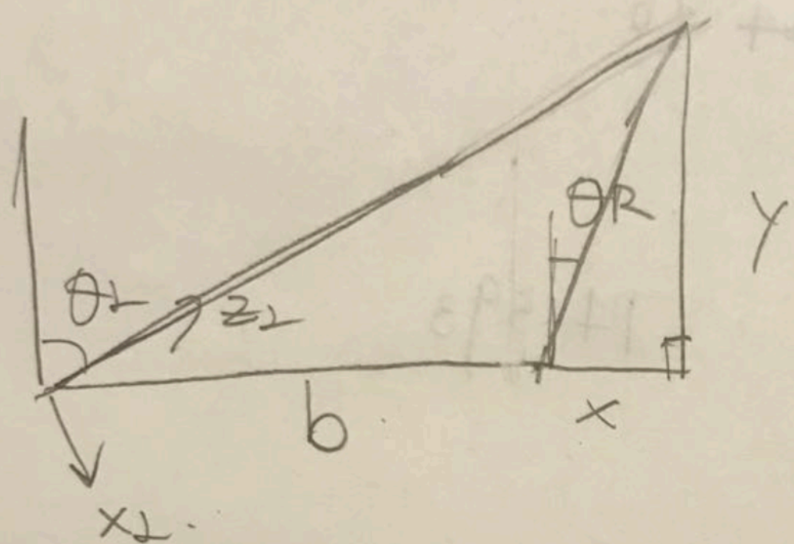
3. Rotate: Yes.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = KR \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Q6

1. Rotation $R_{\theta_R-\theta_L} = \begin{bmatrix} \cos(\theta_R-\theta_L) & 0 & \sin(\theta_R-\theta_L) \\ 0 & 1 & 0 \\ -\sin(\theta_R-\theta_L) & 0 & \cos(\theta_R-\theta_L) \end{bmatrix}$, Translation $T = \begin{bmatrix} b\cos\theta_L \\ 0 \\ b\sin\theta_L \end{bmatrix}$
2. $E = \hat{T}R = \begin{bmatrix} 0 & -b\sin\theta_L & 0 \\ \cos(\theta_R-\theta_L)b\sin\theta_L + \sin(\theta_R-\theta_L)b\cos\theta_L & 0 & \sin(\theta_R-\theta_L)b\sin\theta_L - \cos(\theta_R-\theta_L)b\cos\theta_L \\ 0 & b\cos\theta_L & 0 \end{bmatrix}$

   $= \begin{bmatrix} 0 & -b\sin\theta_L & 0 \\ b\sin\theta_R & 0 & -b\cos\theta_R \\ 0 & b\cos\theta_L & 0 \end{bmatrix}$
3.



$$\begin{cases} \dfrac{b+x}{Y} = \tan\theta_l \\ \dfrac{x}{Y} = \tan\theta_R \end{cases}$$

$$\Rightarrow Y = \frac{b}{\tan\theta_l - \tan\theta_R}$$

in l. coord. $$z = \frac{Y}{\cos\theta_l}$$

$$= \frac{b}{(\tan\theta_l - \tan\theta_R)\cos\theta_l}$$

$$(p, oq) = \left(0, 0, \frac{b}{(\tan\theta_l - \tan\theta_R)\cos\theta_l}\right)$$