

# Two view metrology

Perception: Kostas Daniilidis

We have learnt that the transformation from a world plane (like the road plane) to the image is a projective transformation !



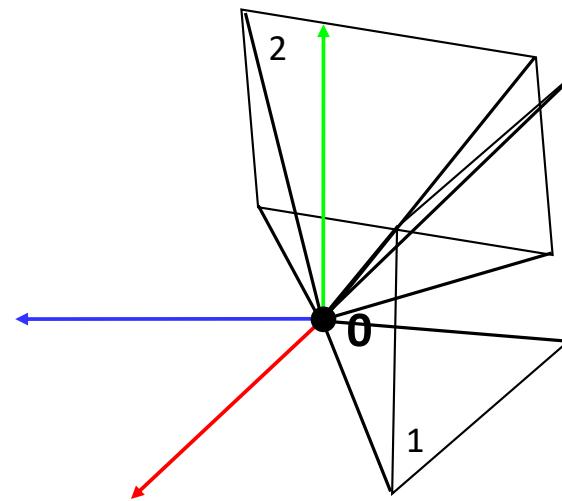
# Two-view collineations: panoramas

- All slides by Richard Szeliski (author of the book).

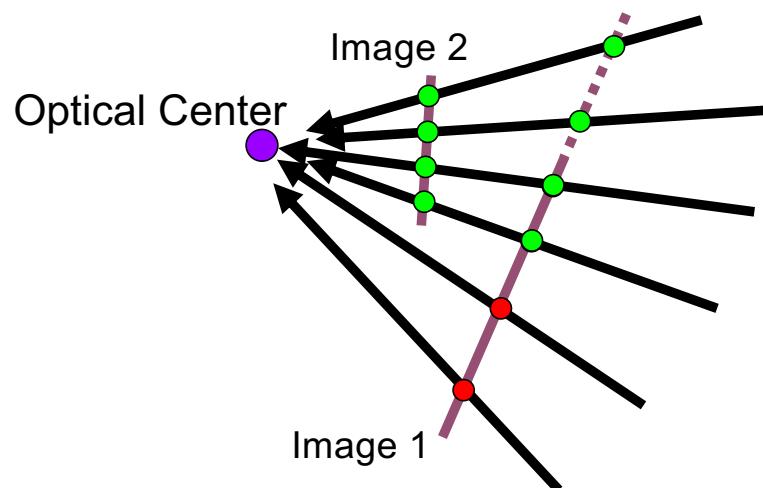
# Creating a panorama

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - If there are more images, repeat

# Geometric interpretation of mosaics



# What is the transformation?



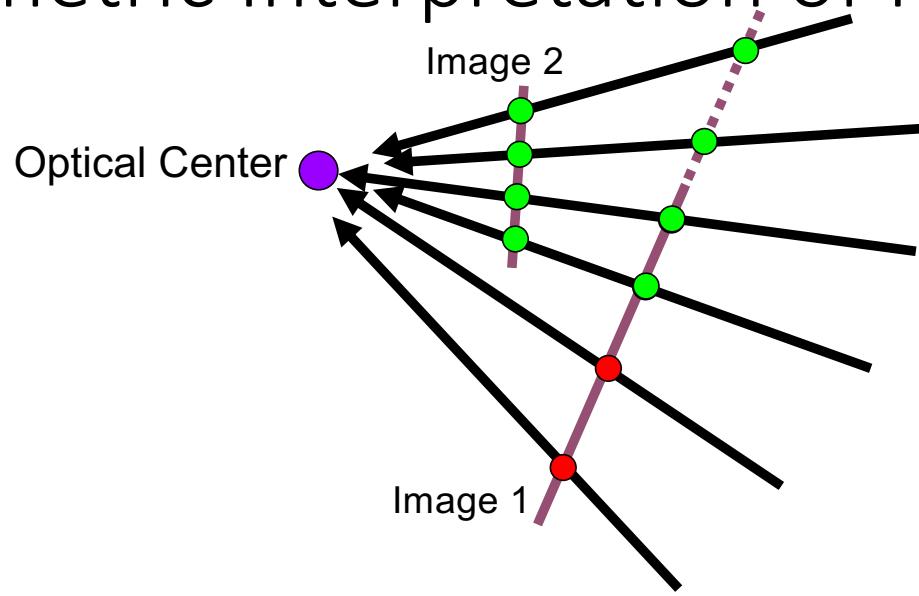
How do we transform image 2 onto image 1's projection plane?

$$\begin{array}{ll} \text{image 1} & \text{image 2} \\ \mathbf{K}_1 & \mathbf{K}_2 \\ \mathbf{R}_1 = \mathbf{I}_{3 \times 3} & \mathbf{R}_2 \end{array}$$

$$\begin{aligned} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} &= \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} & \text{3D ray coords} & \text{image coords} \\ && \downarrow & \\ \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} &= \mathbf{R}_1 \mathbf{R}_2^T \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} & \text{3D ray coords} & \text{image coords} \\ && \downarrow & \\ \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} &\sim \mathbf{K}_1 \mathbf{R}_1 \mathbf{R}_2^T \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} & \text{image coords} & \text{image coords} \end{aligned}$$

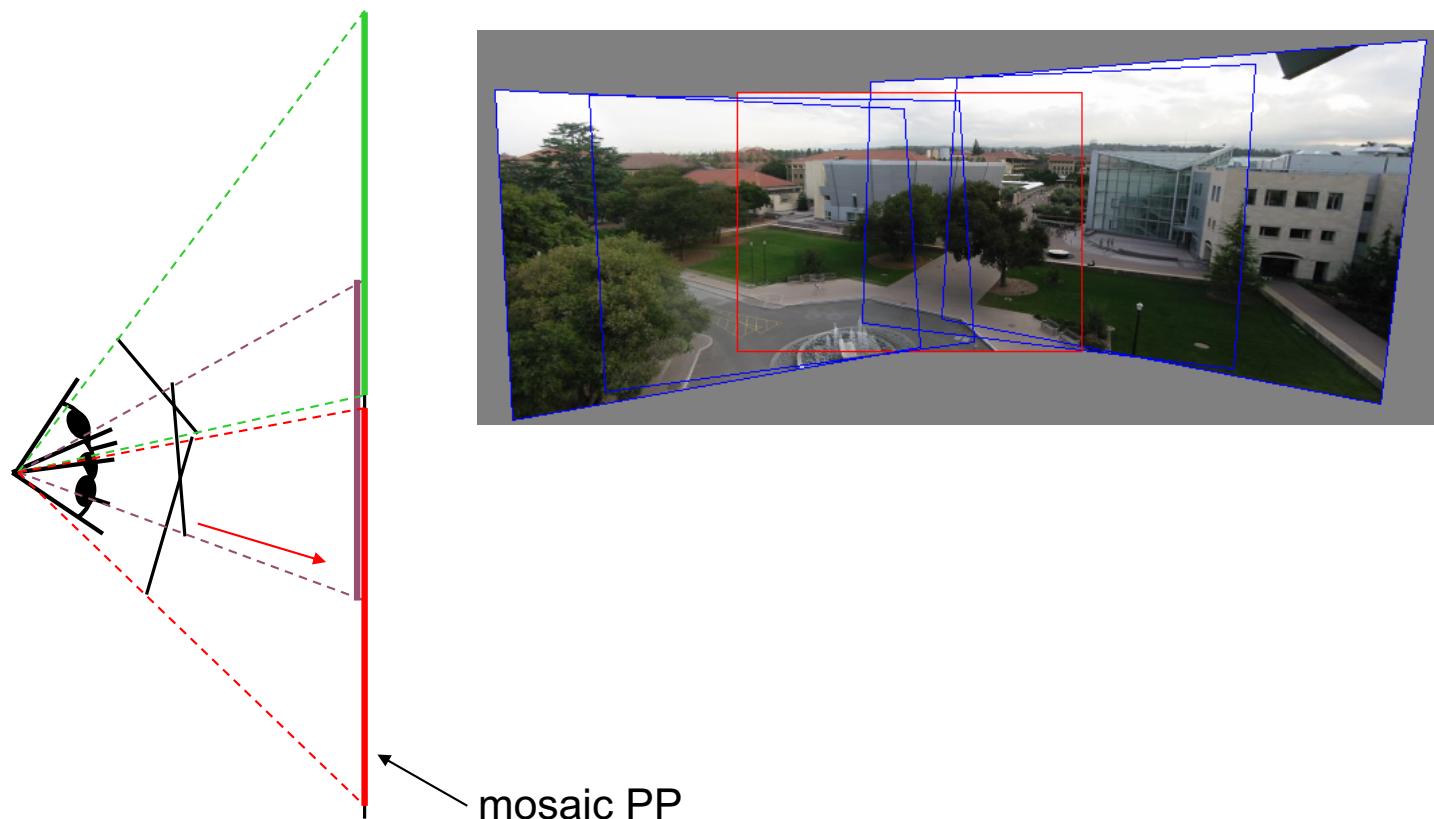
**3x3 homography**

# Geometric Interpretation of Mosaics



- If we capture all  $360^\circ$  of rays, we can create a  $360^\circ$  panorama
- The basic operation is *projecting* an image from one plane to another
- The projective transformation is scene-INDEPENDENT
  - This depends on all the images having the same optical center

# Projecting images onto a common plane

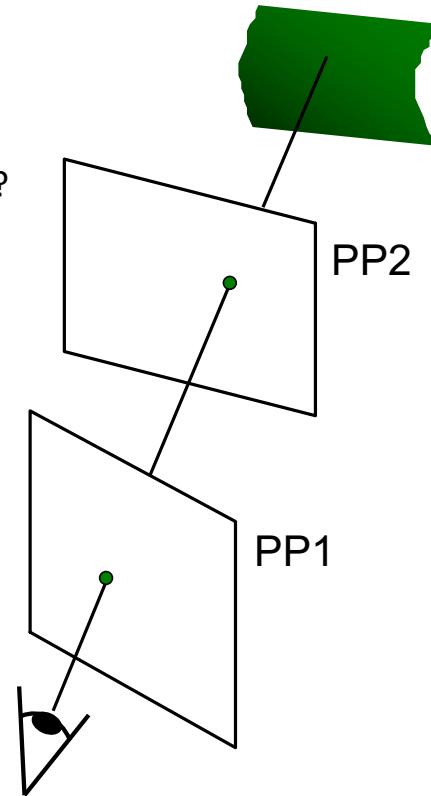


# Image reprojection

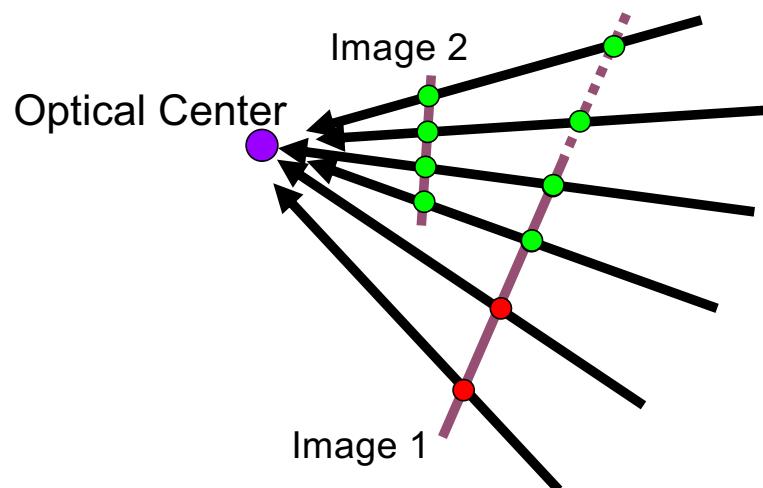
- Basic question
  - How to relate two images from the same camera center?
    - how to map a pixel from PP1 to PP2

Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2



# What is the transformation?



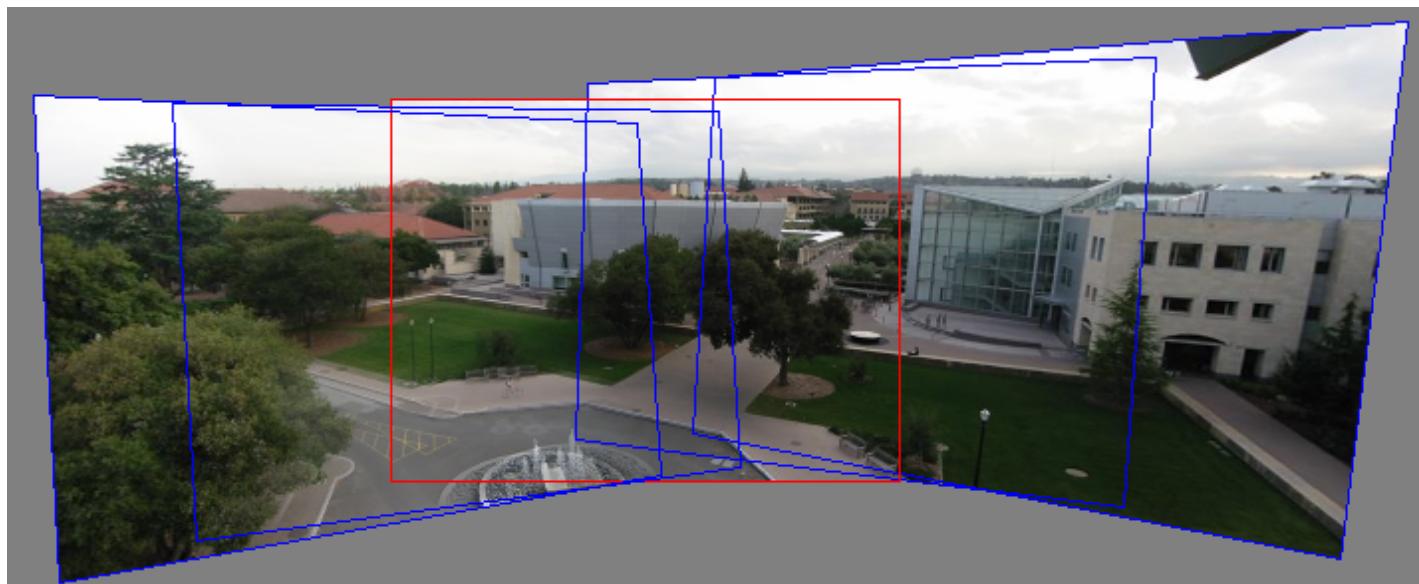
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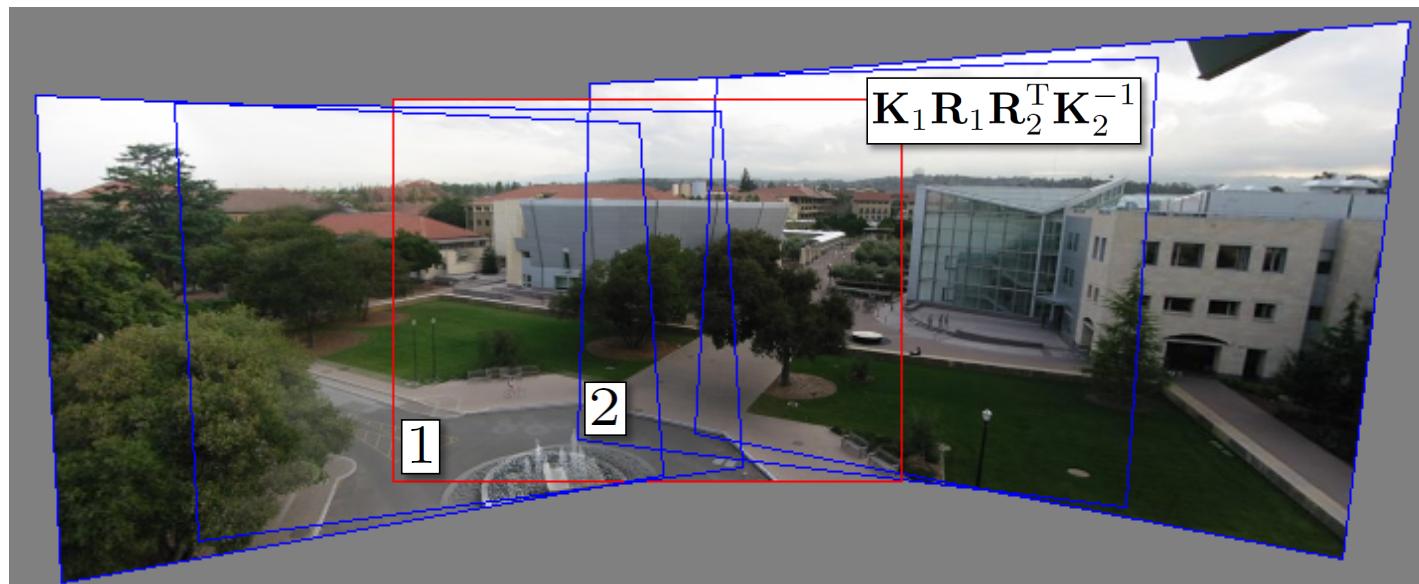
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**3x3 homography**

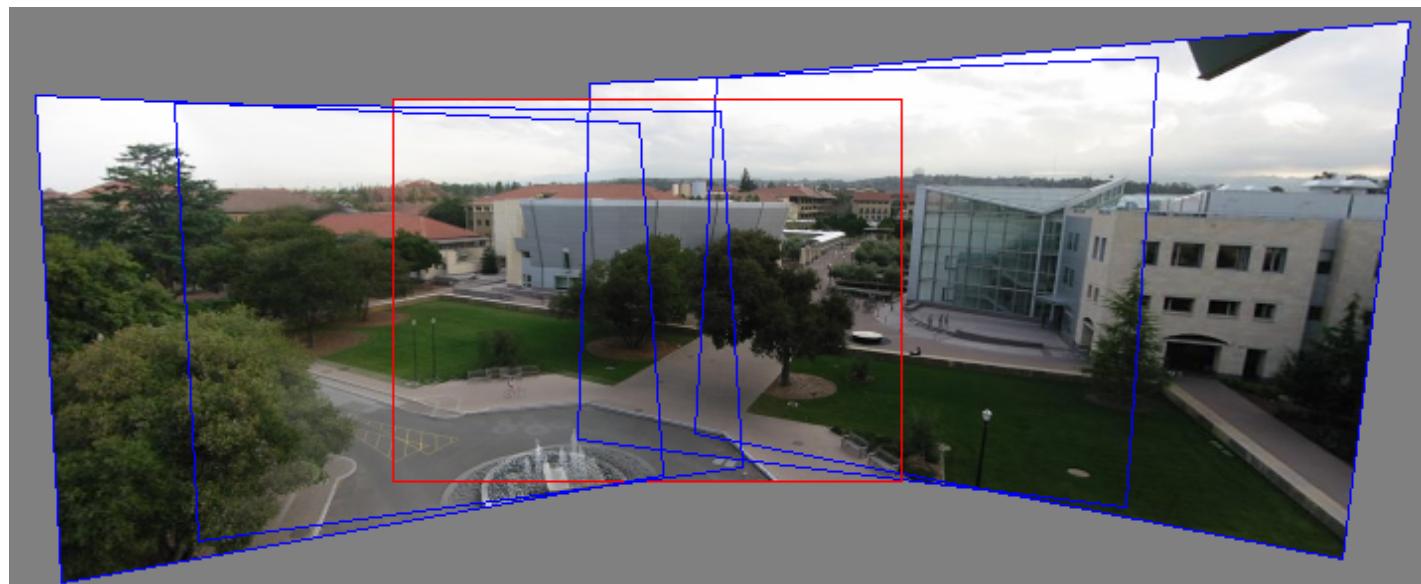
# Image alignment



# Image alignment

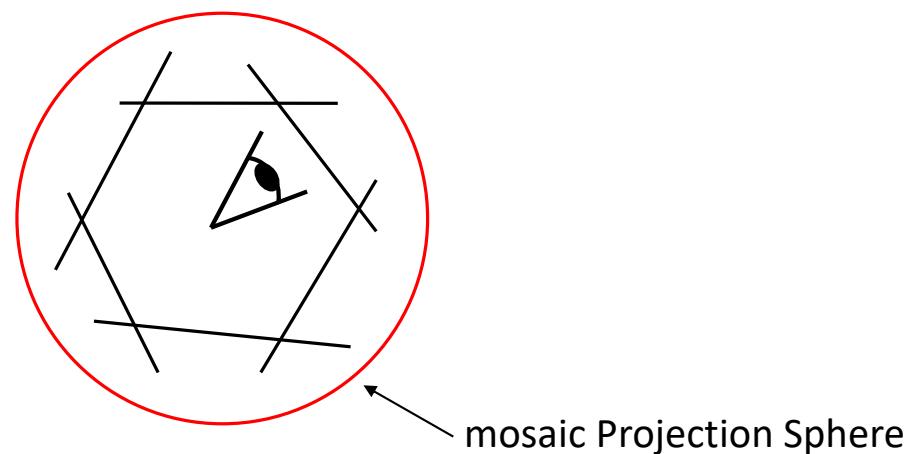


Can we use homography to create a 360 panorama?

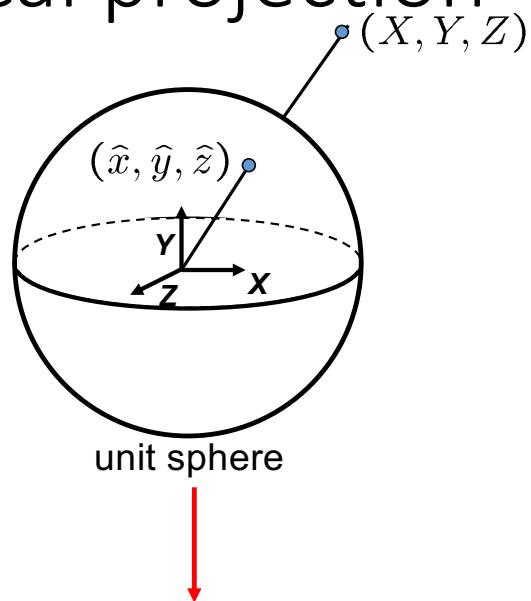


# Panoramas

- What if you want a 360° field of view?



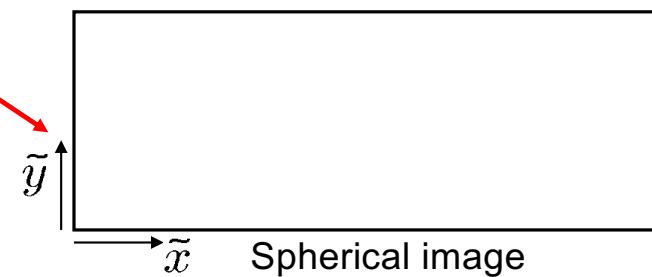
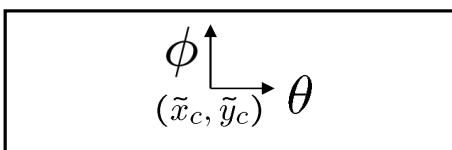
# Spherical projection



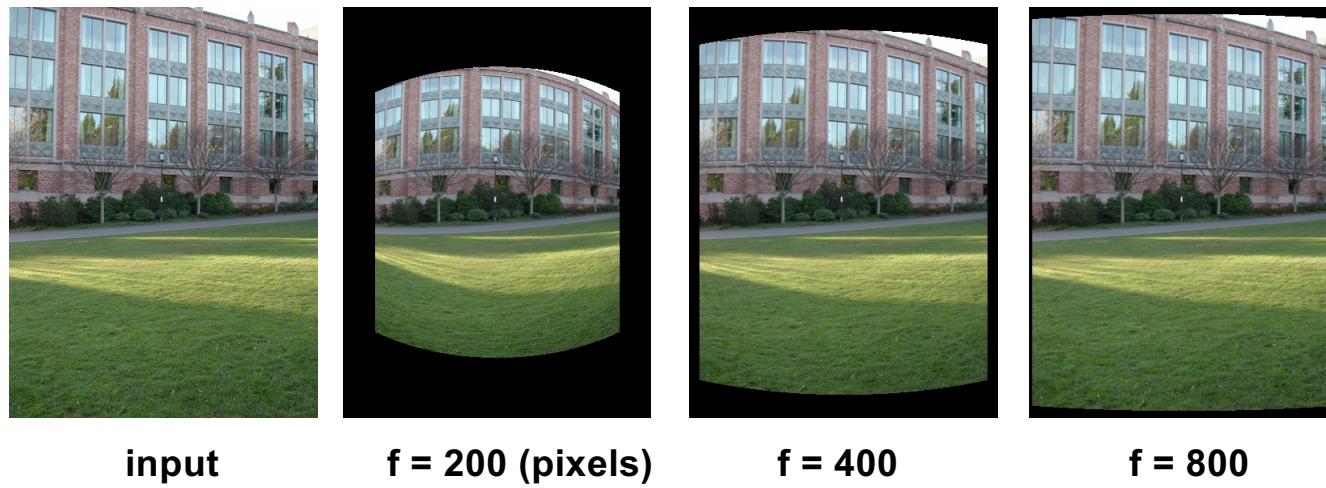
- Map 3D point  $(X, Y, Z)$  onto sphere

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2+Y^2+Z^2}}(X, Y, Z)$$

- Convert to spherical coordinates  
 $(\sin\theta\cos\phi, \sin\phi, \cos\theta\cos\phi) = (\hat{x}, \hat{y}, \hat{z})$
- Convert to spherical image coordinates  
 $(\tilde{x}, \tilde{y}) = (s\theta, s\phi) + (\tilde{x}_c, \tilde{y}_c)$ 
  - $s$  defines size of the final image
    - » often convenient to set  $s = \text{camera focal length}$



# Spherical reprojection



- Map image to spherical coordinates
  - need to know the focal length

# Aligning spherical images

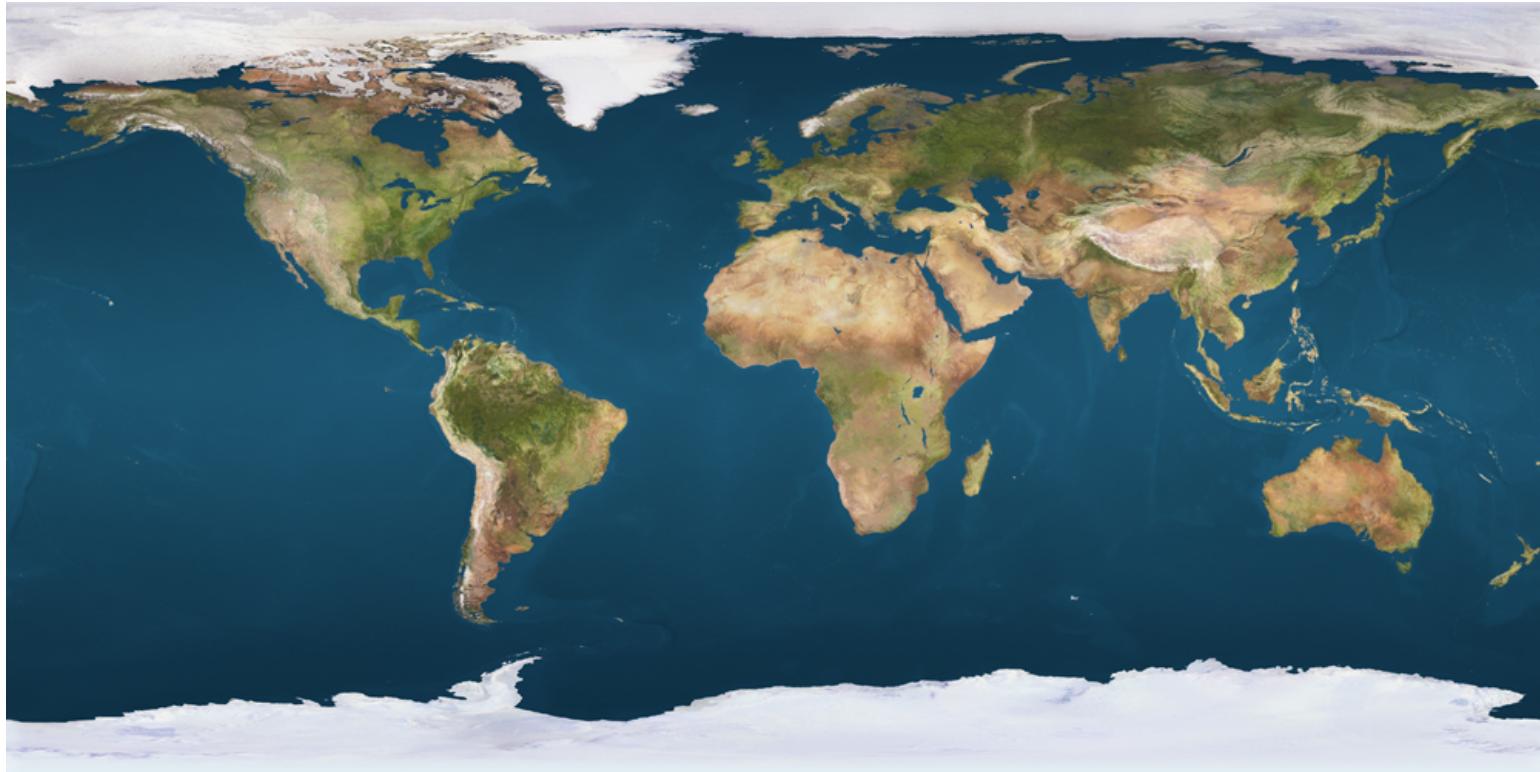


- Suppose we rotate the camera by  $\theta$  about the vertical axis
  - How does this change the spherical image?



# Mapping a sphere

Credit: JHT's Planetary Pixel Emporium

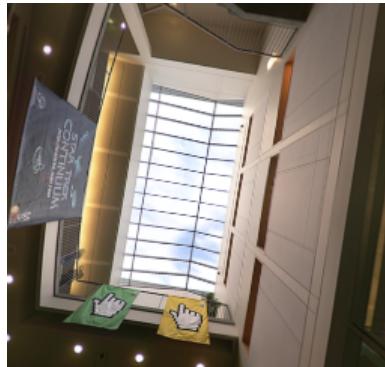


# Spherical panoramas



Microsoft Lobby: <http://www.acm.org/pubs/citations/proceedings/graph/258734/p251-szeliski>

# Different projections are possible

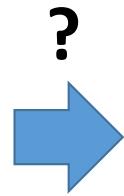


# Computing transformations

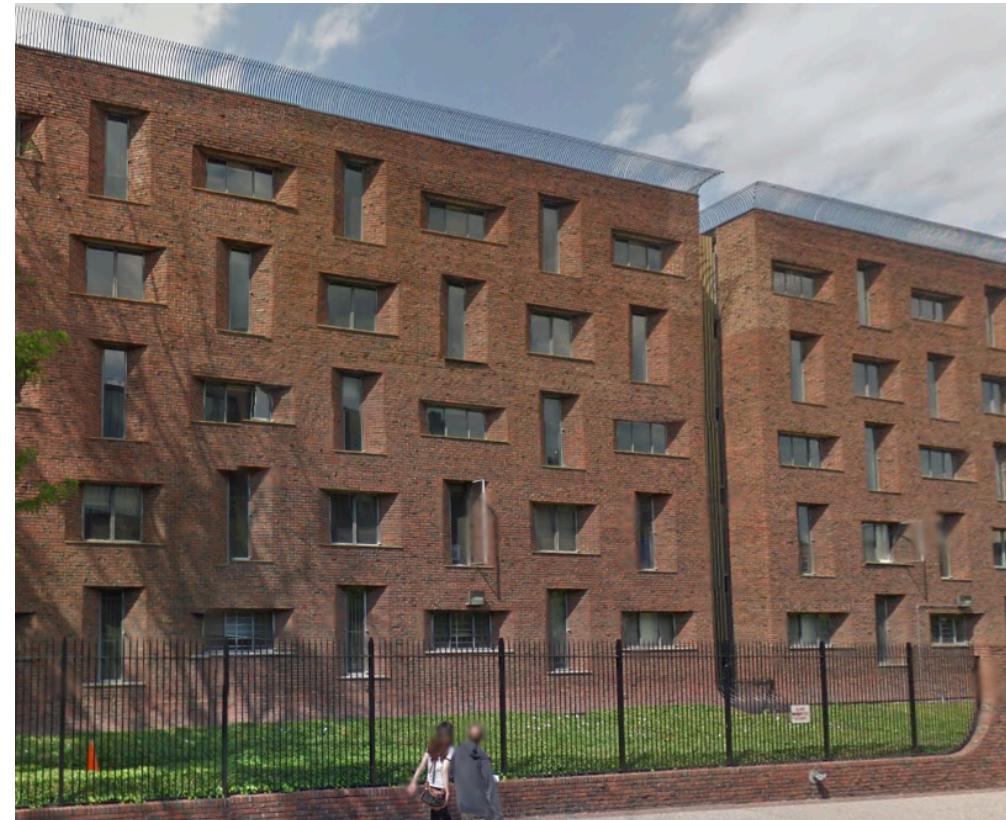
- Given a set of matches between images A and B
  - How can we compute the transform  $T$  from A to B?



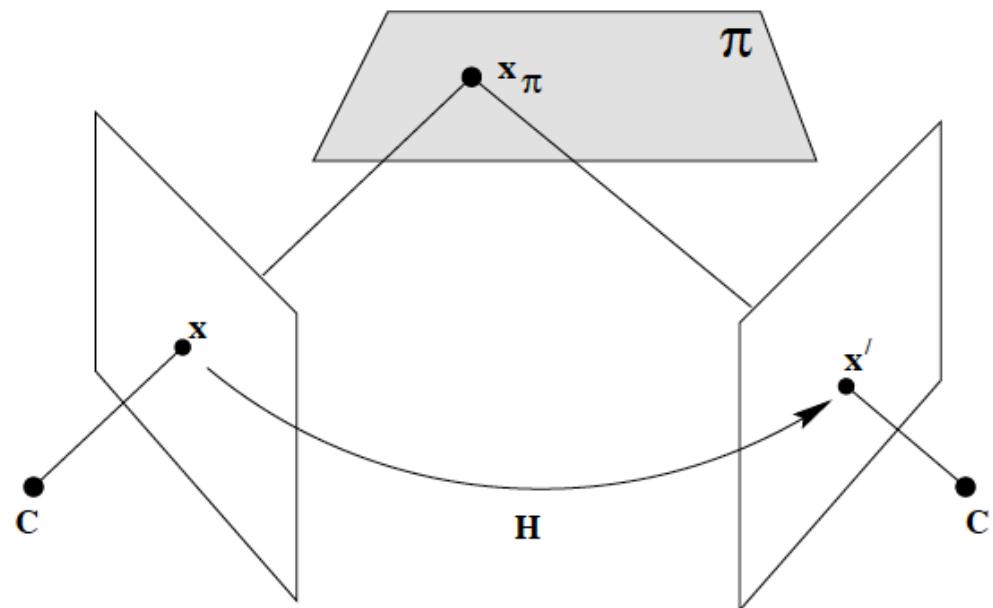
# Computing transformations



What about two views of the same plane (façade)?



Projective transformation between two images of the same plane!



# 1966 World Cup: England-Germany



Was it a goal?



[https://en.wikipedia.org/wiki/1966\\_FIFA\\_World\\_Cup\\_Final](https://en.wikipedia.org/wiki/1966_FIFA_World_Cup_Final)

England's third goal has remained controversial ever since the match. According to the Laws of the Game the definition of a goal is when "the whole of the ball passes over the goal line". [9] English supporters cited the good position of the linesman and the statement of Roger Hunt, the nearest England player to the ball, who claimed it was a goal and that was why he wheeled away in celebration rather than attempting to tap the rebounding ball in. **Modern studies using film analysis and computer simulation have shown that the whole ball never crossed the line – only 97% did.** Both Duncan Gillies of the Visual Information Processing Group at Imperial College London and Ian Reid and Andrew Zisserman of the Department of Engineering Science at University of Oxford have stated that the ball would have needed to travel a further 2.5–6.0 cm to fully cross the line.[10]

## Goal-directed Video Metrology

Ian Reid and Andrew Zisserman

Dept of Engineering Science, University of Oxford, Oxford, OX1 3

**Abstract.** We investigate the general problem of accurate metro from uncalibrated video sequences where only partial information available. We show, via a specific example – plotting the position of a goal-bound soccer ball – that accurate measurements can be obtained and that both qualitative and quantitative questions about the data can be answered.

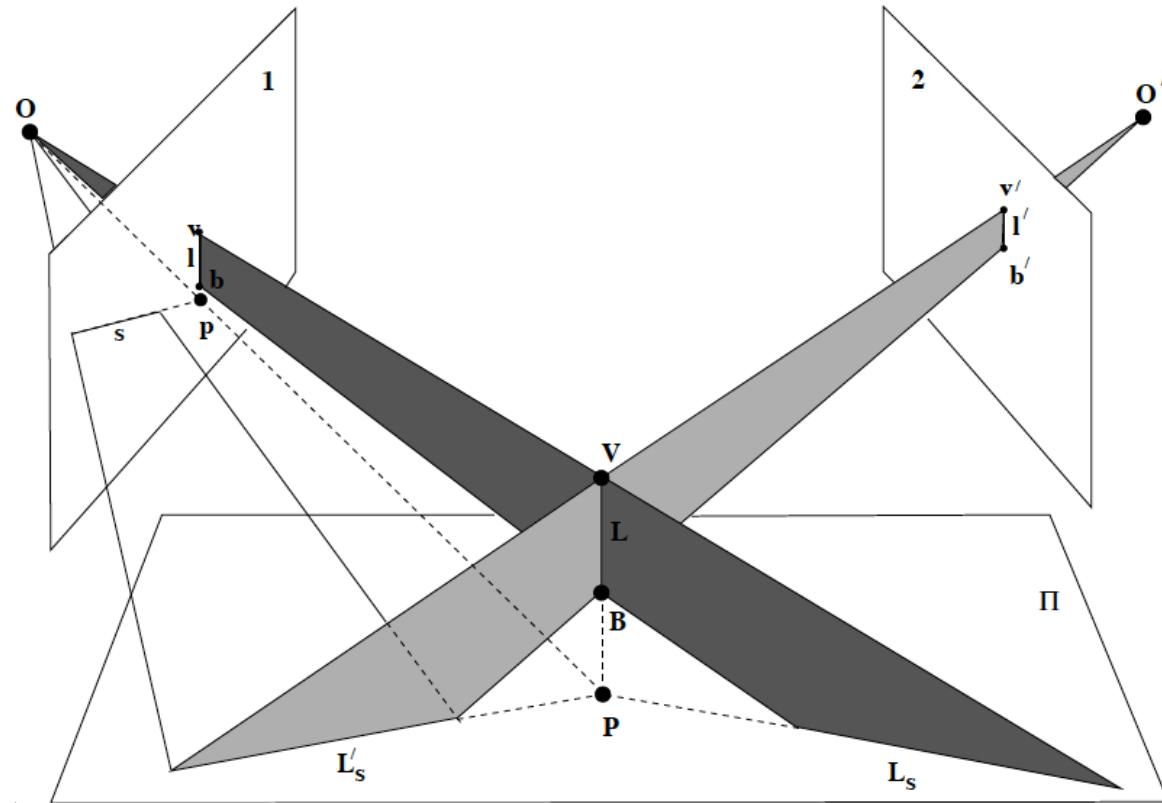
From two video sequences of an incident captured from different viewpoints, we compute a novel (overhead) view using pairs of corresponding images. Using projective constructs we determine the point at which a vertical line through the ball pierces the ground plane in each frame. Throughout we take care to consider possible sources of error and see how these may be eliminated, neglected, or we derive appropriate uncertainty measures which are propagated via a first-order analysis.

Can we infer from  
two different  
viewpoints  
whether the ball  
was inside the  
goal ?

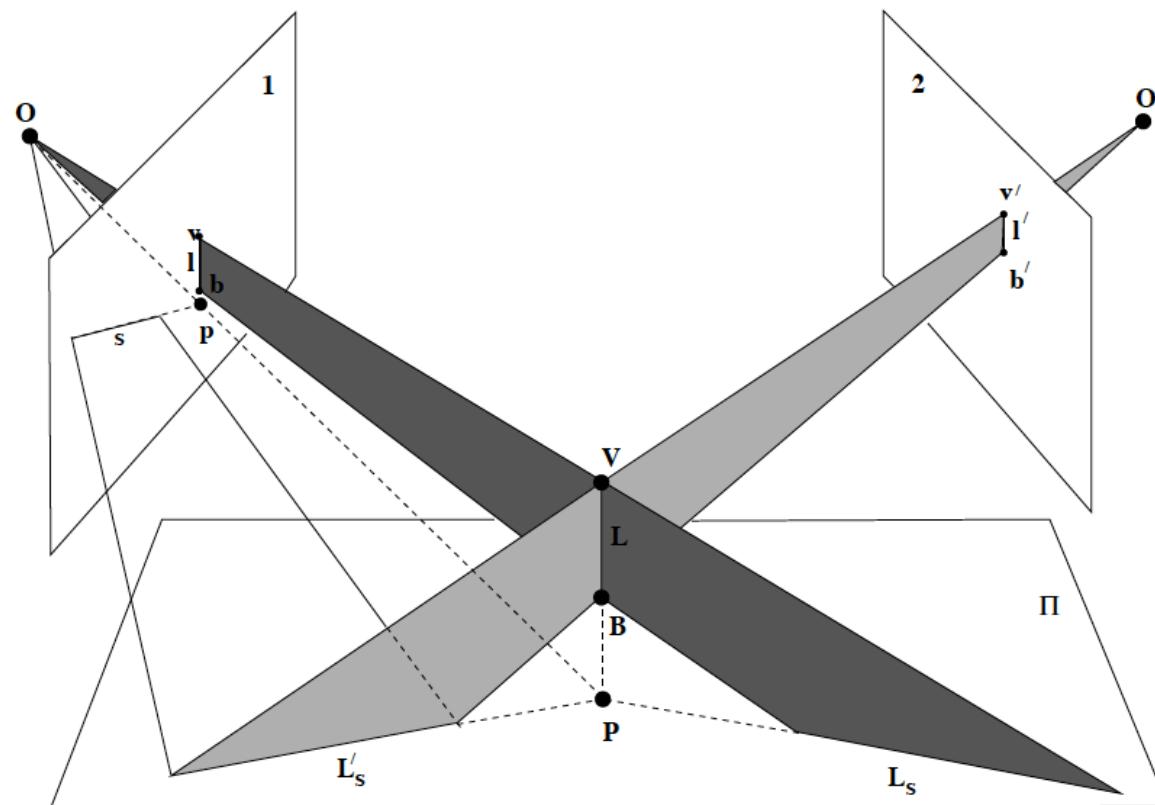
Today, this is done  
with what is called  
Goal Line  
Technology!



Let B be the ball ! The question is whether the vertical projection of the ball P is behind the goal-line or not!



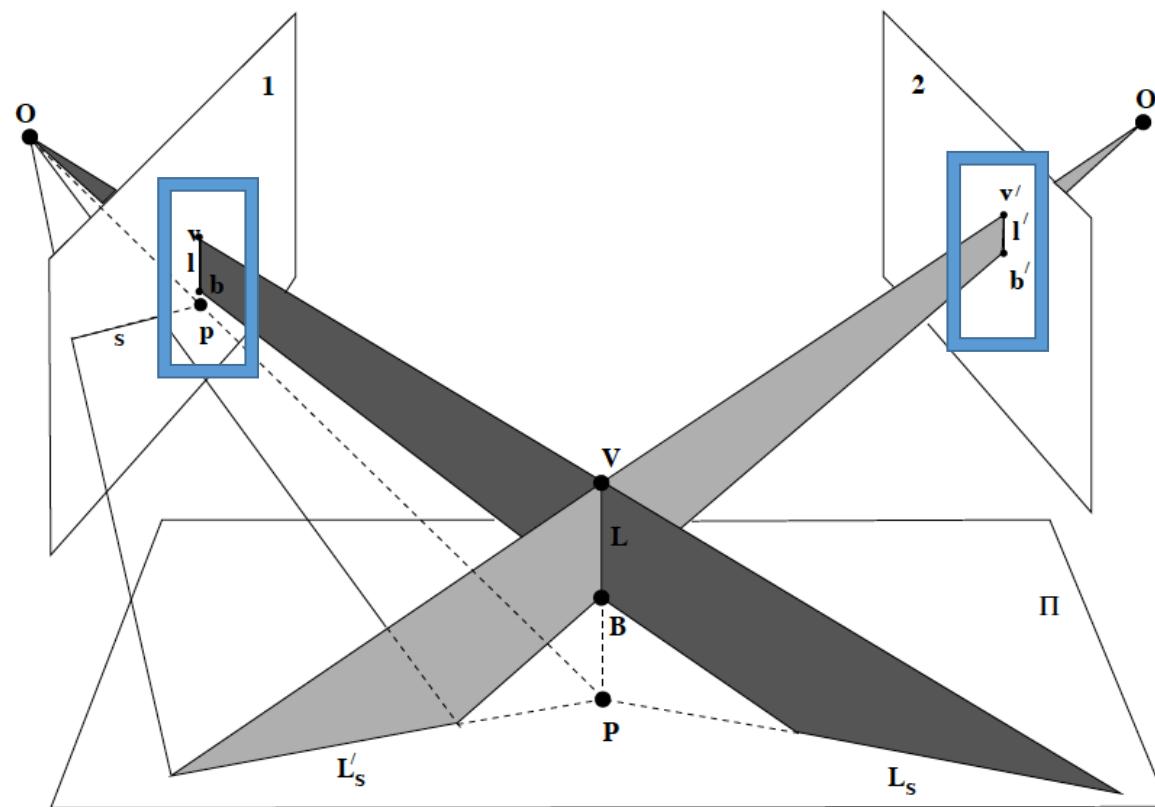
Let us also assume that we can find the vertical vanishing points  $v$  and  $v'$  in both images !



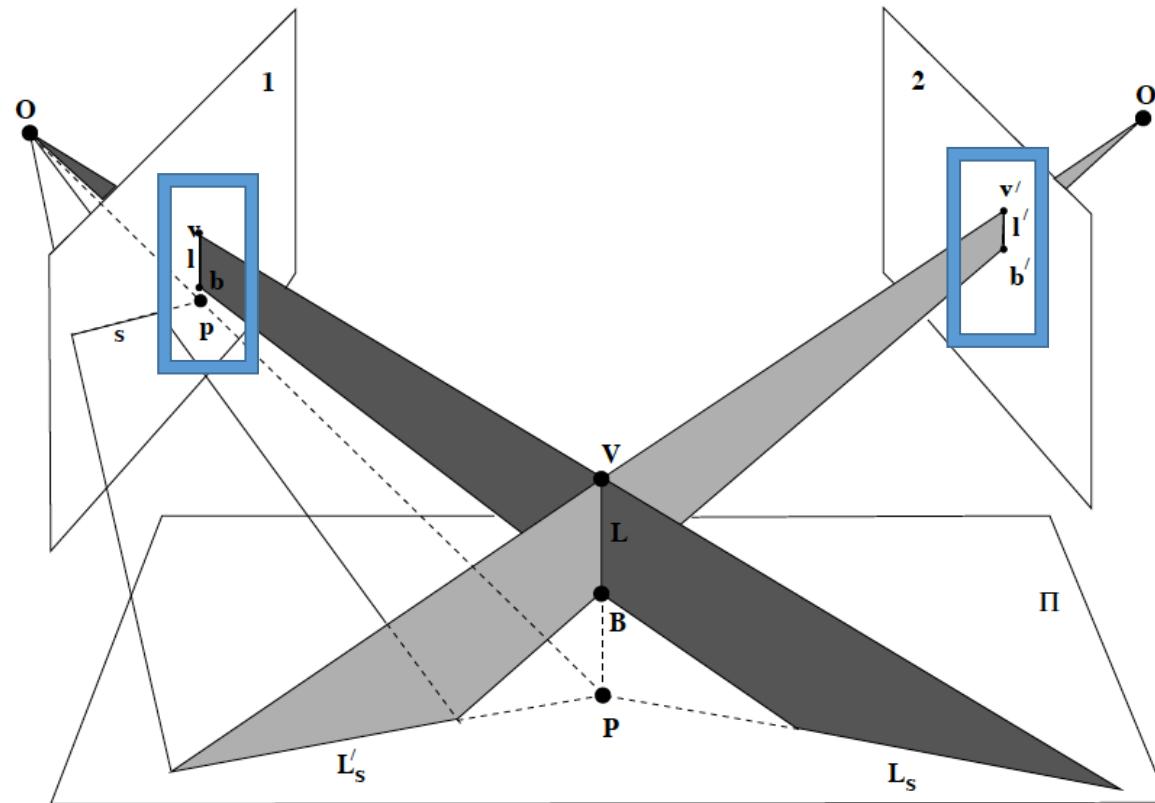
# Vertical vanishing points



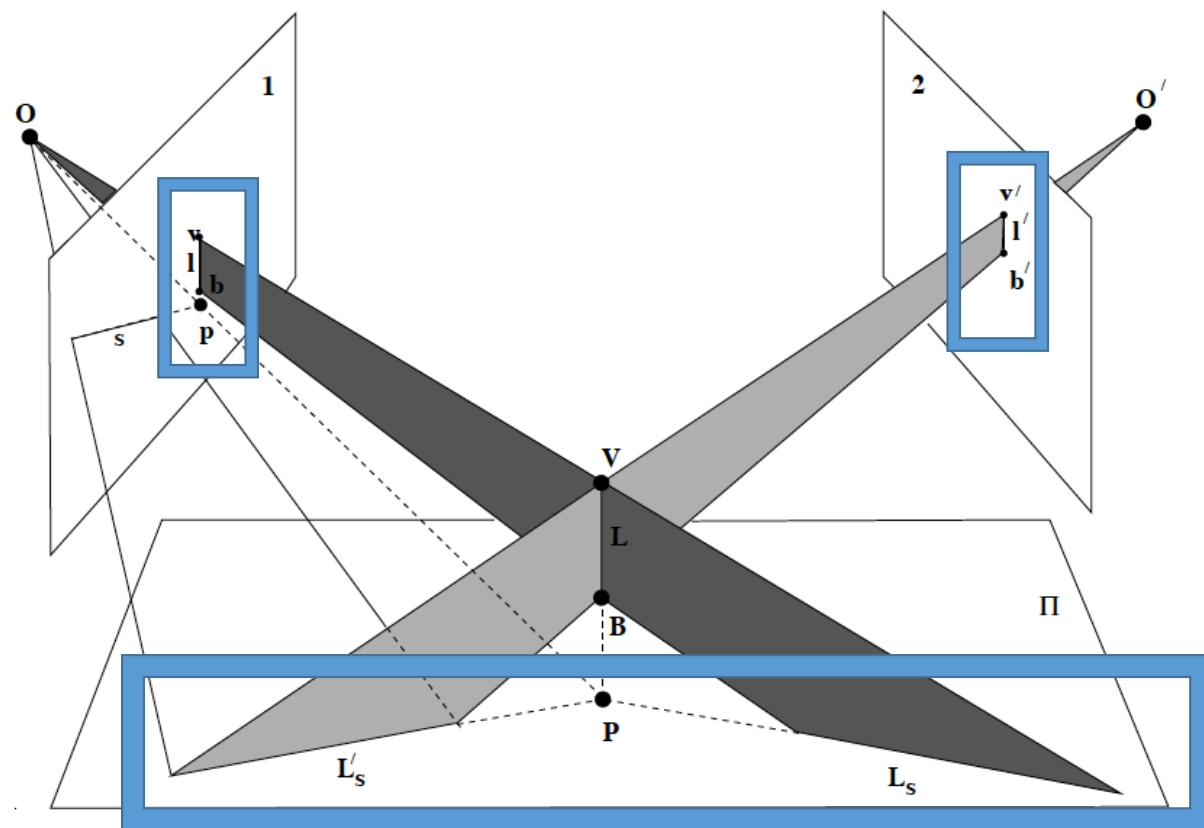
Then we know in both images the projections of vertical lines  $l$  and  $l'$ .



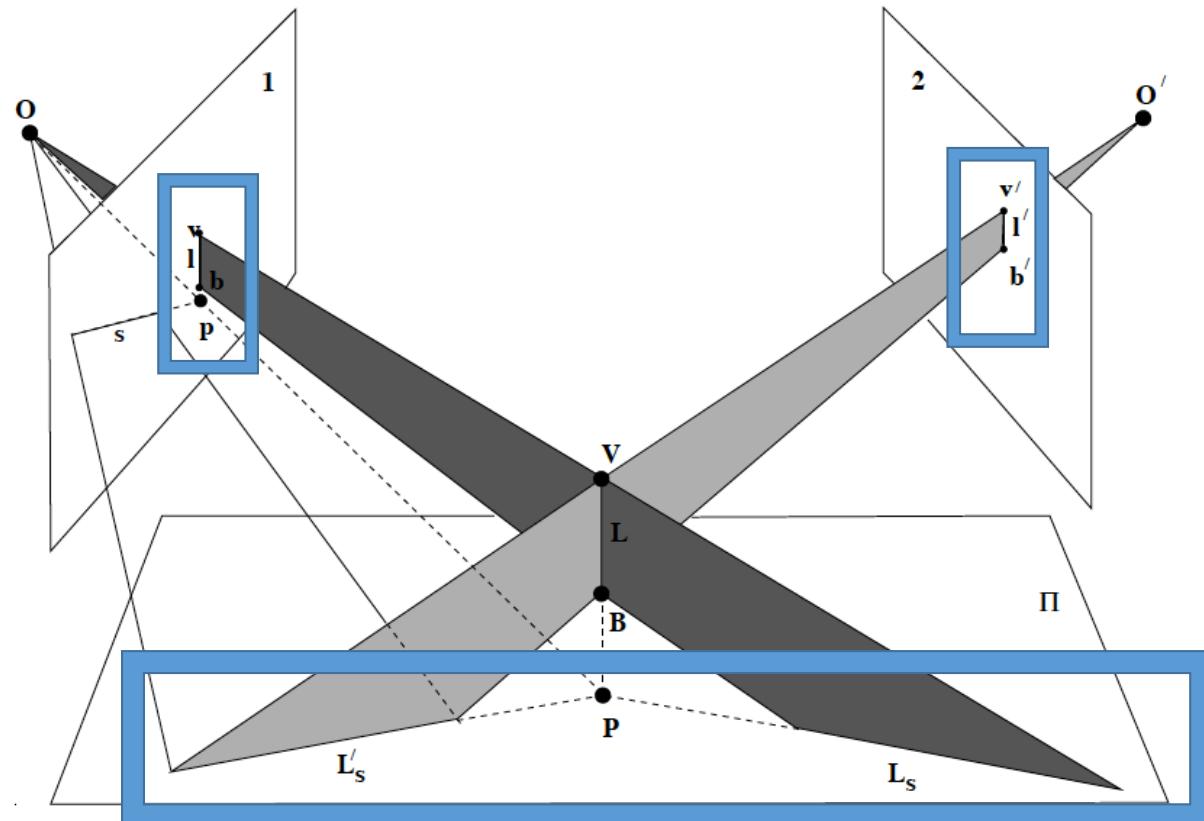
If we project them back on the soccer field we will find the shadows of the vertical through the ball.



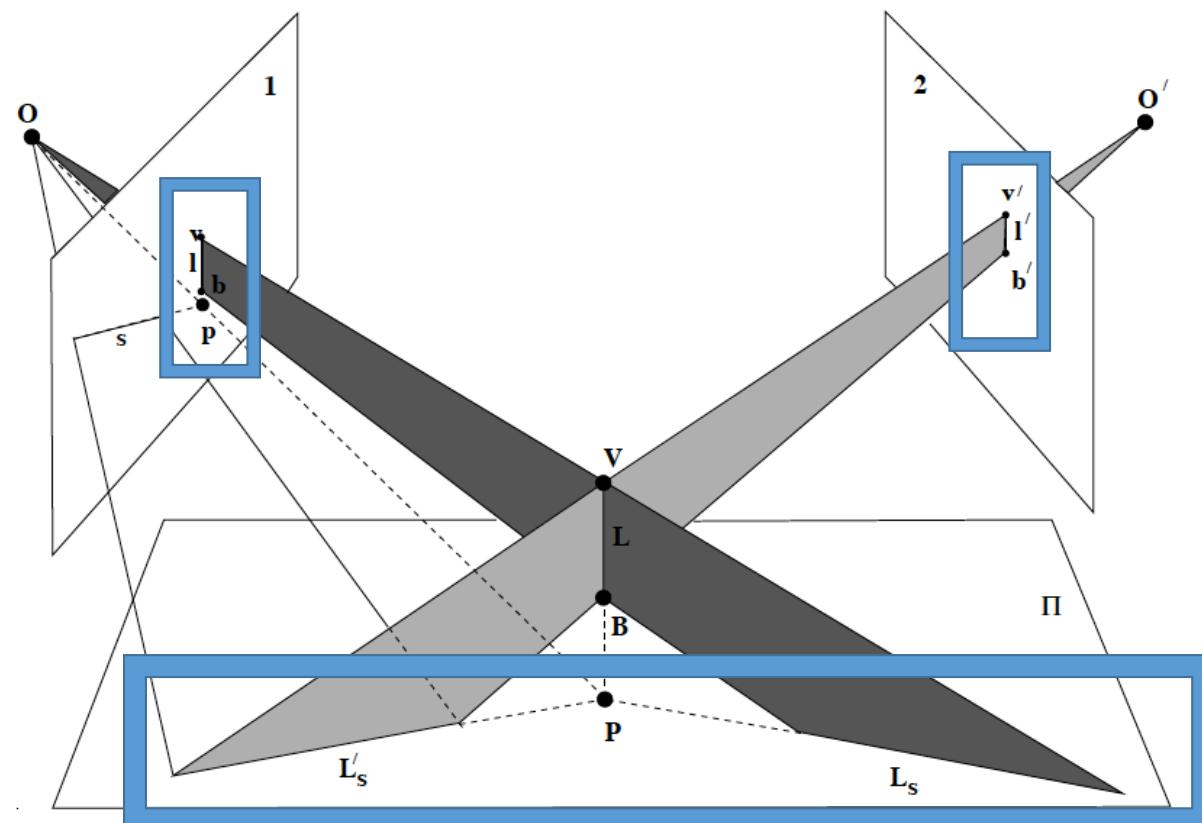
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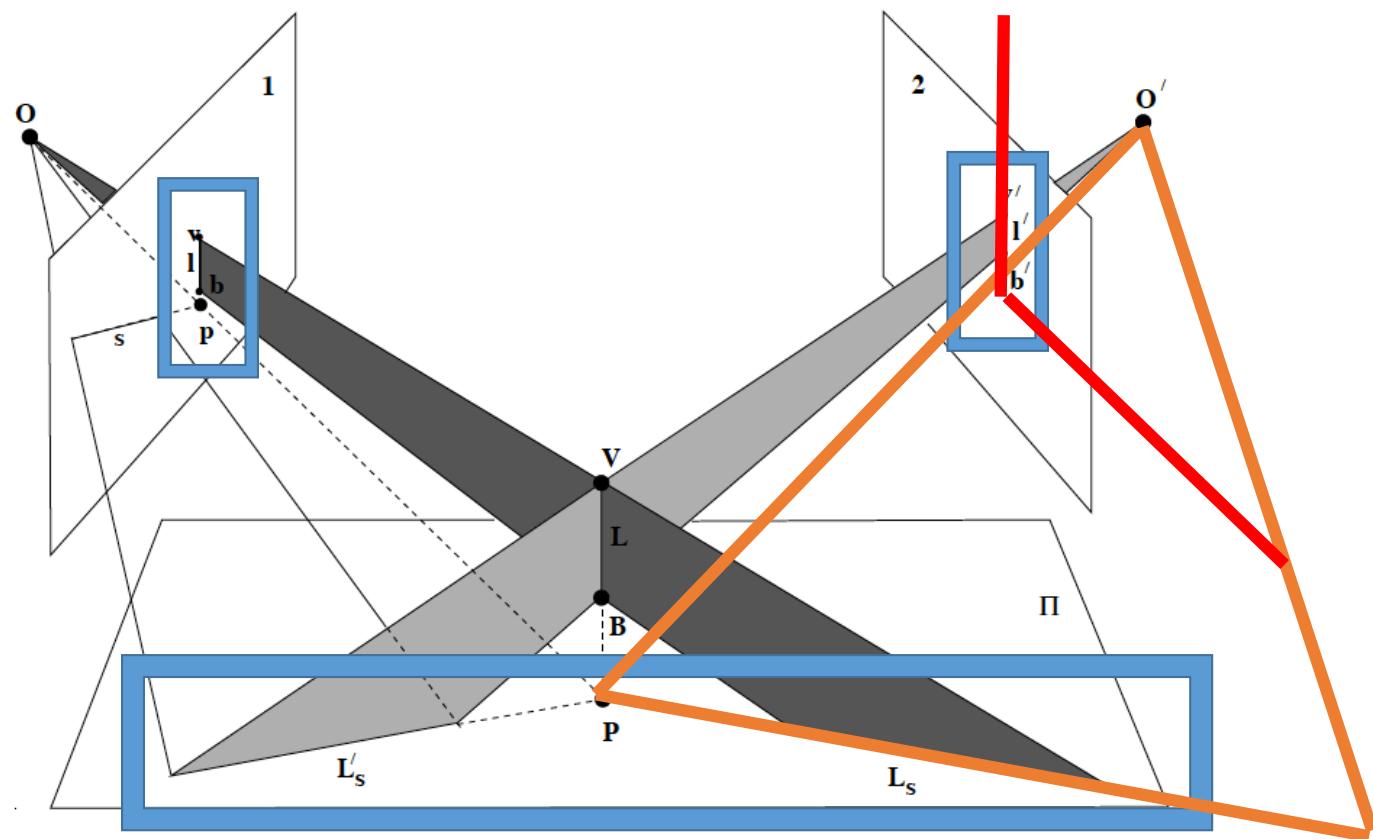
Shadows intersect at the desired point P !



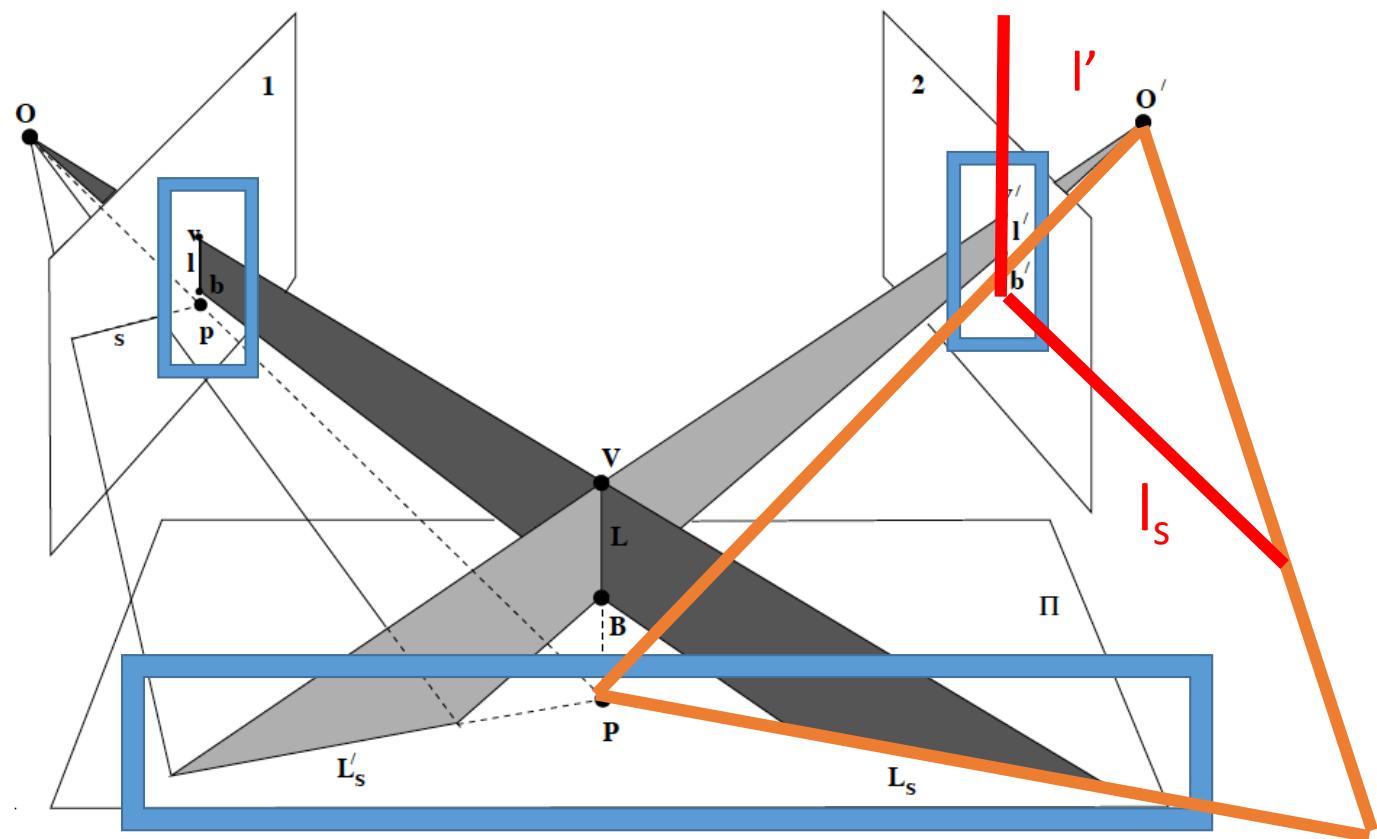
How can we find the projection of P in one of the image planes?



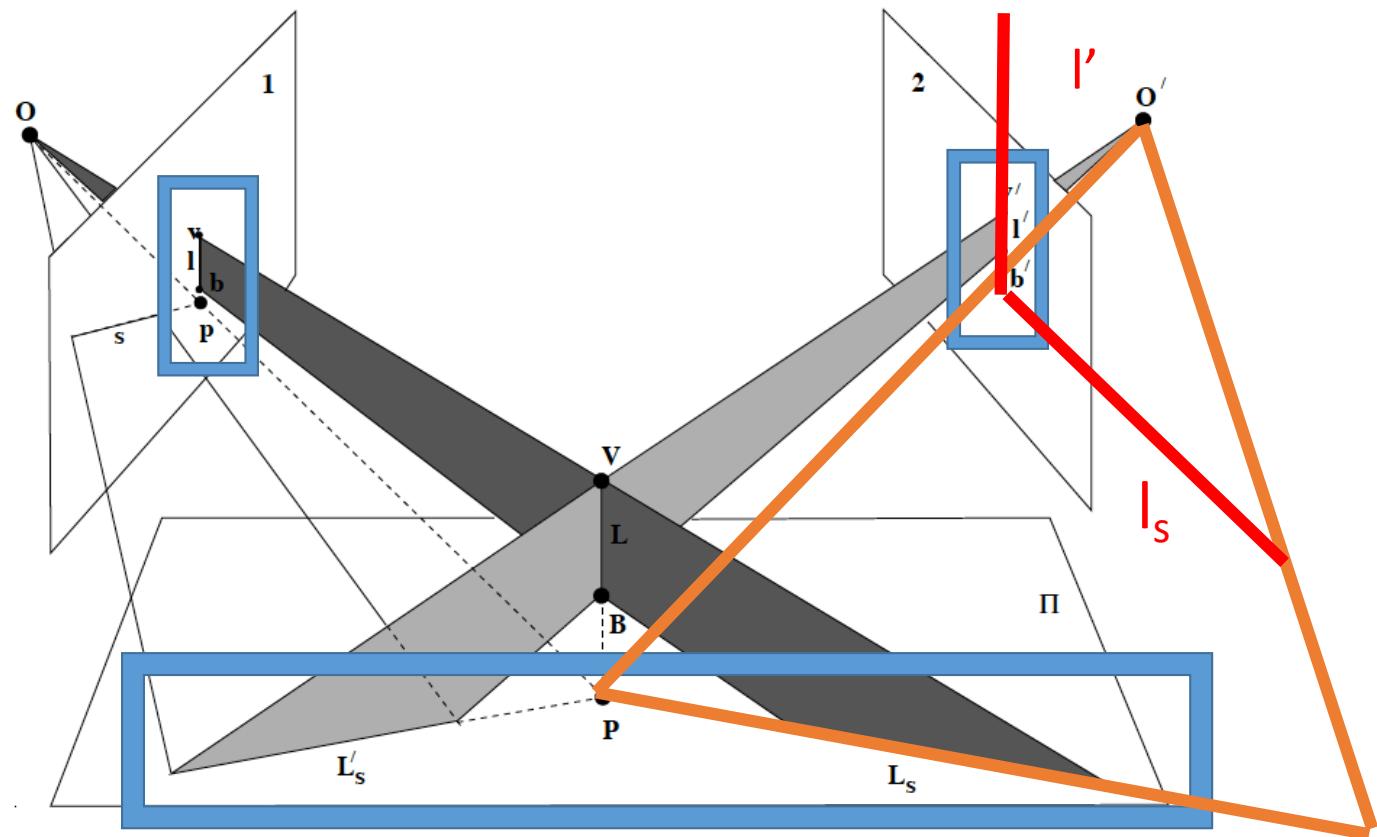
By computing the projections of the shadows!  
The red lines in the right image plane!



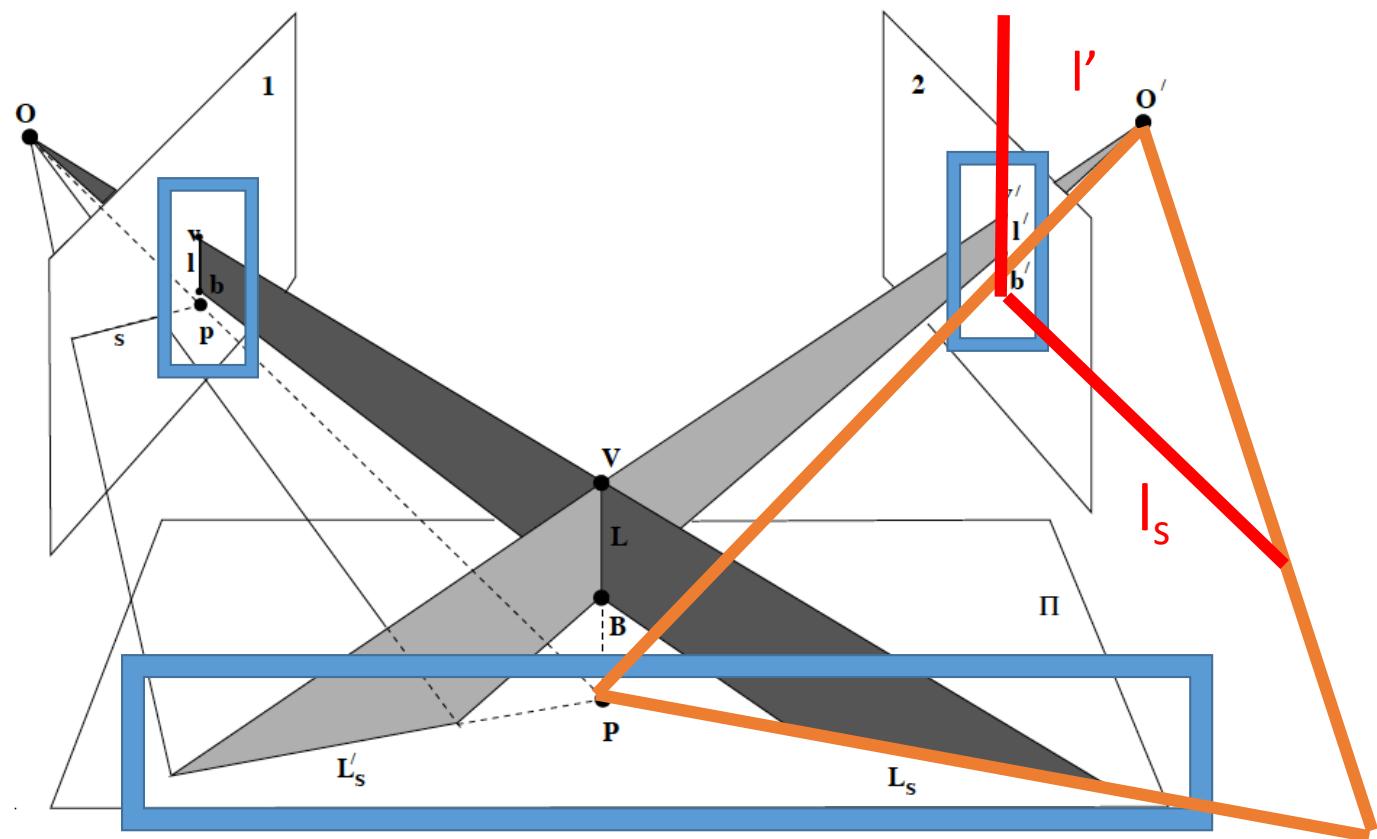
By computing the projections of the shadows  $l'$  and  $l_s$ ,  
the red lines in the right image plane!



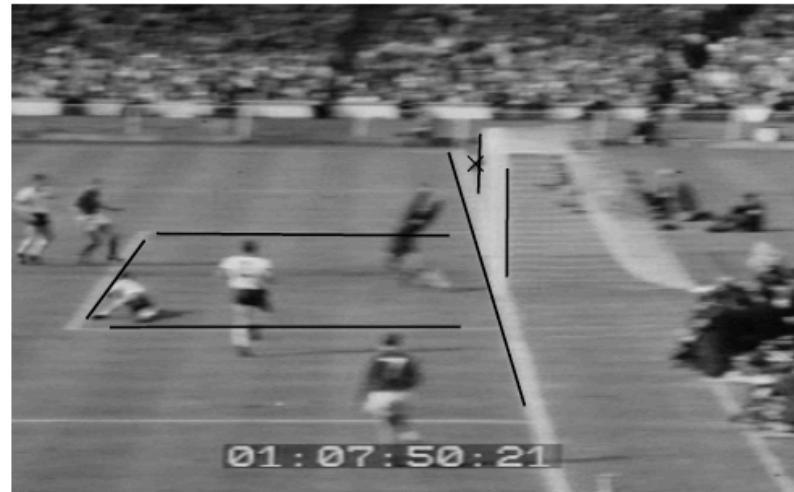
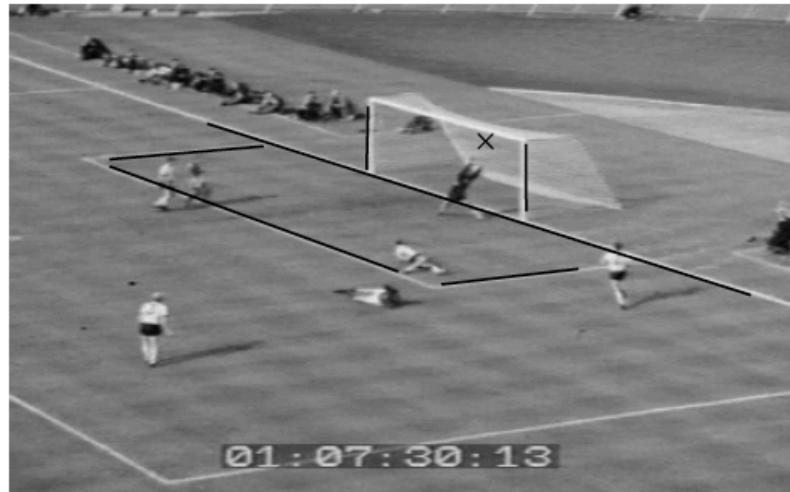
But we only know  $l'$ . But  $l_s$  and  $l$  (left) are the projections from the same shadow line on the soccer field.



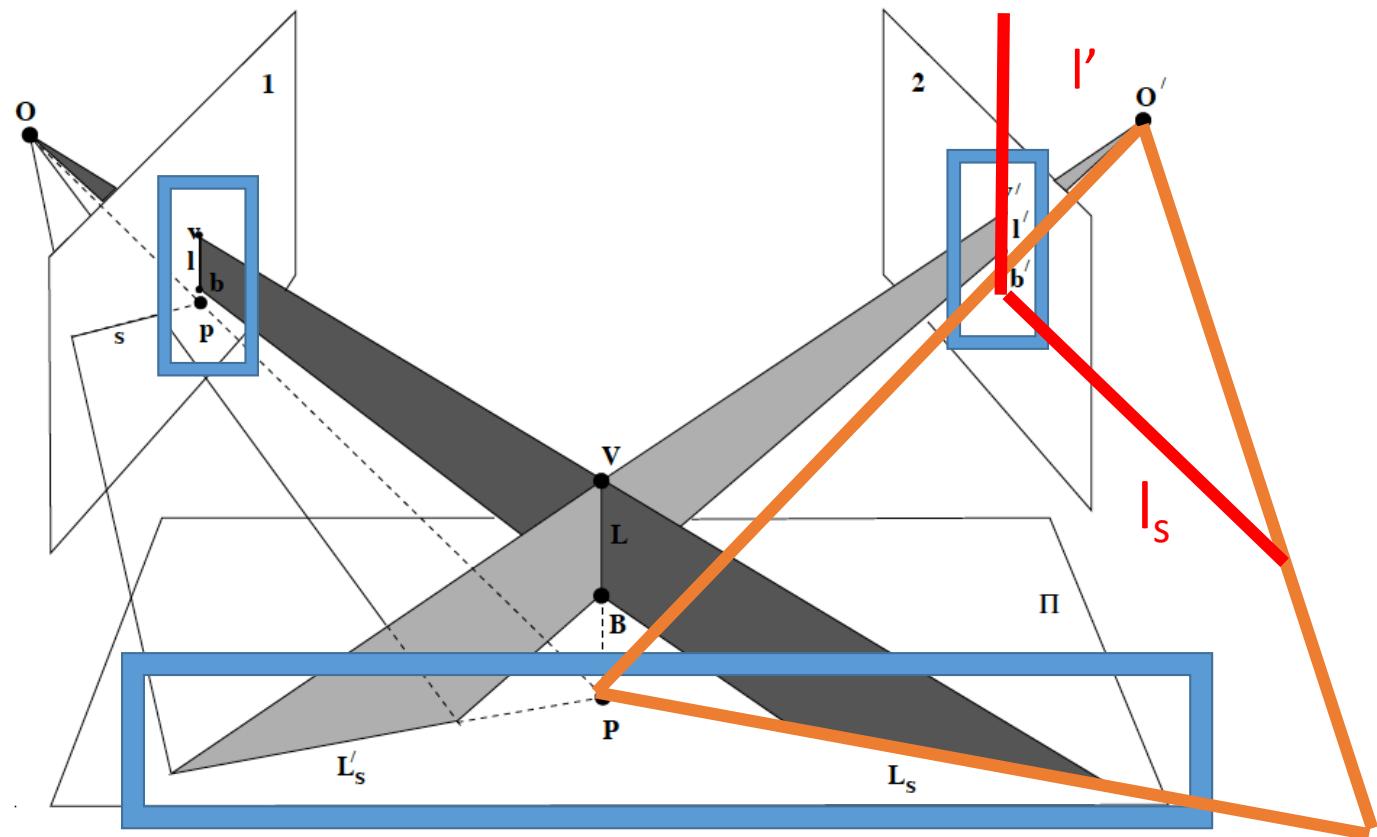
Because the soccer field is planar, they are related by a homography (collineation).



Homography of soccer field can be computed because we see the same features:

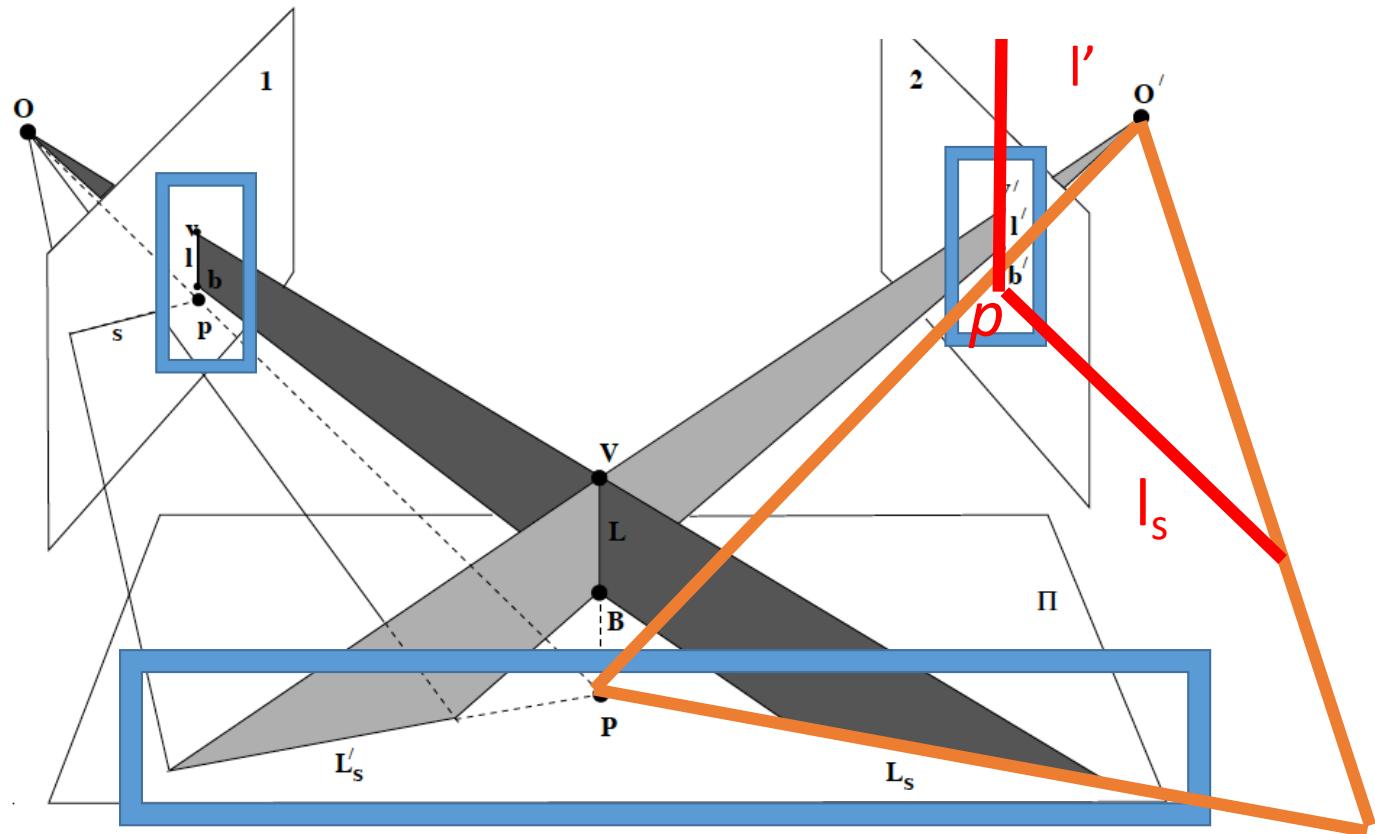


If  $H$  is the homography between points, then  $\mathbf{l}_s \sim H^{-T} \mathbf{l}$



And the projection  $p$  is  
intersection of two lines

$$p \sim H^{-T}l \times l'$$



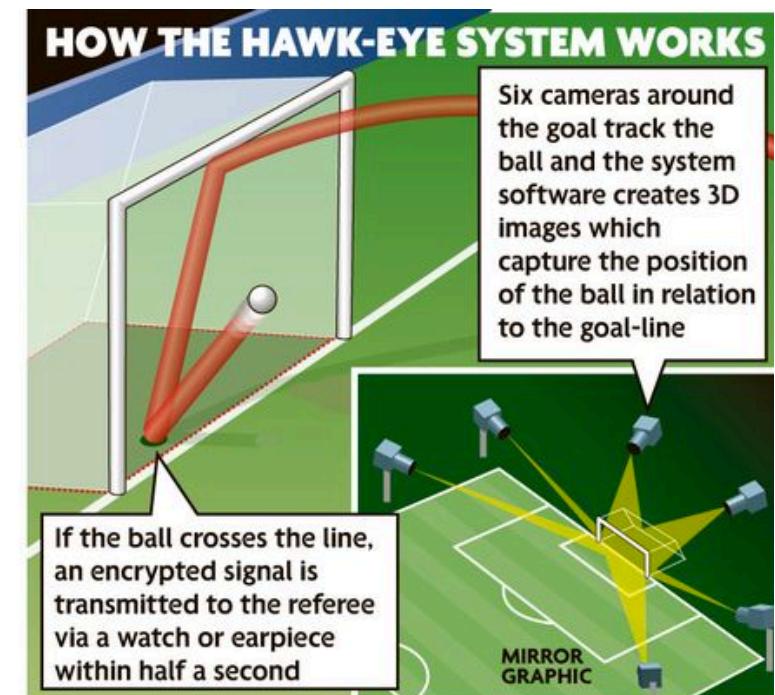
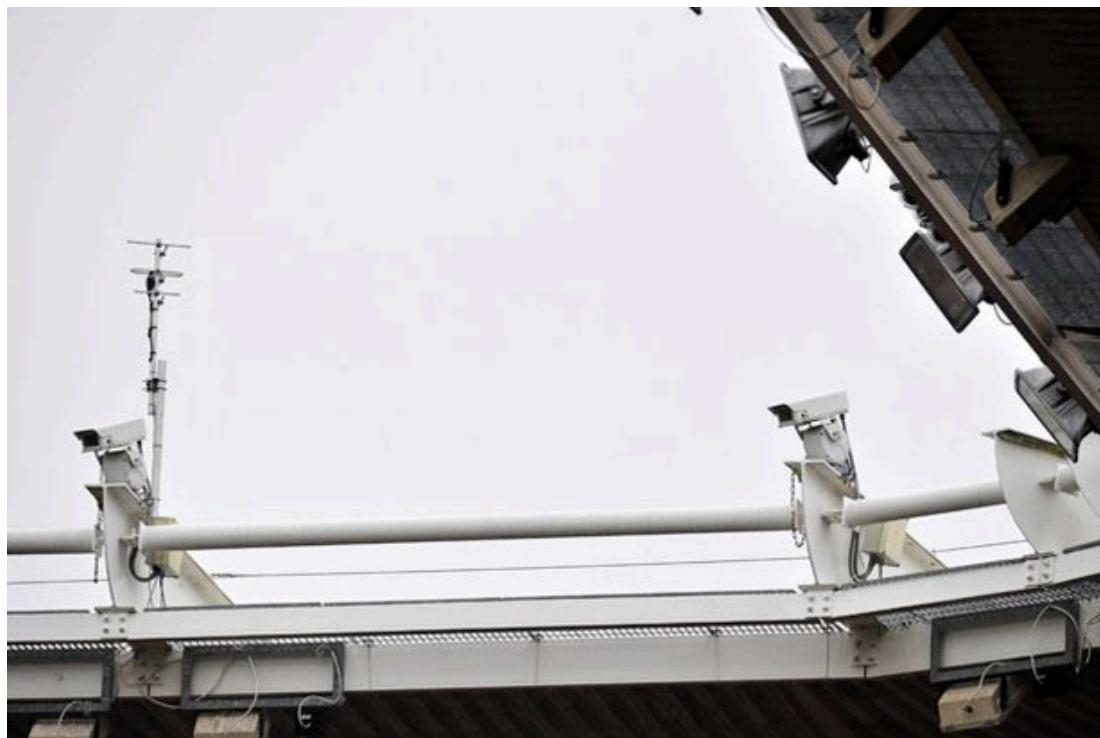
So, was it a goal ?

- No, the ball did not cross the line in full.
- A lot more engineering behind geometry:
  - Synchronizing frames cross cameras
  - Size of the ball
  - Motion blure

England – Germany 2010 !



Today, multiple cameras with very accurate relative pose tracked! The Hawk-Eye.



# Thanks to Reid and Zisserman!

<http://www.learnopencv.com/how-computer-vision-solved-the-greatest-soccer-mystery-of-all-times/>

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