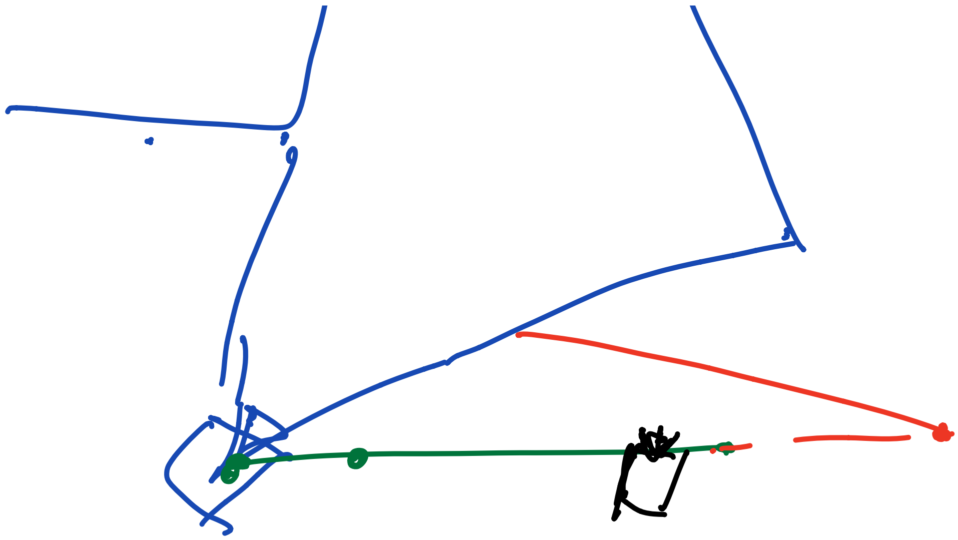
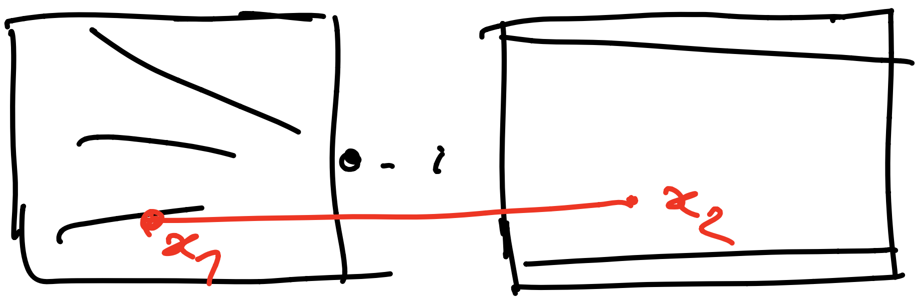


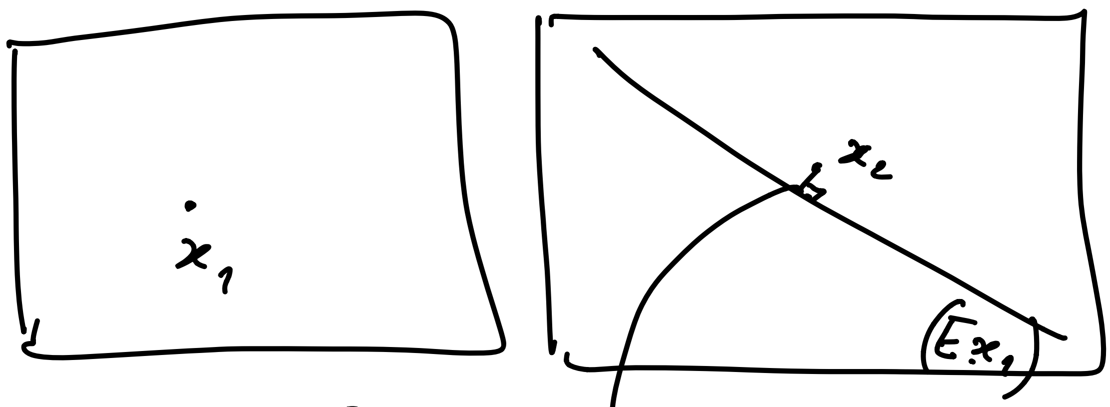


1.1. Given correspondences (x_1, x_2)

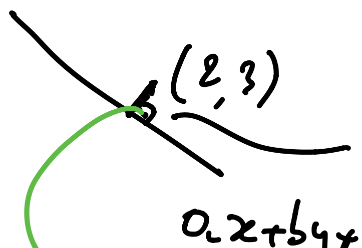
① 8-point $\Rightarrow E$

$$\Rightarrow U\begin{pmatrix}1 & & \\ & 1 & \\ & & 0\end{pmatrix}V^T$$

② 8-point $\boxed{\text{RANSAC}}$



$e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\quad$ dist $\neq (x_2^T E x_1)$



$\neq a2+b3+c$

$ax+by+c=0$

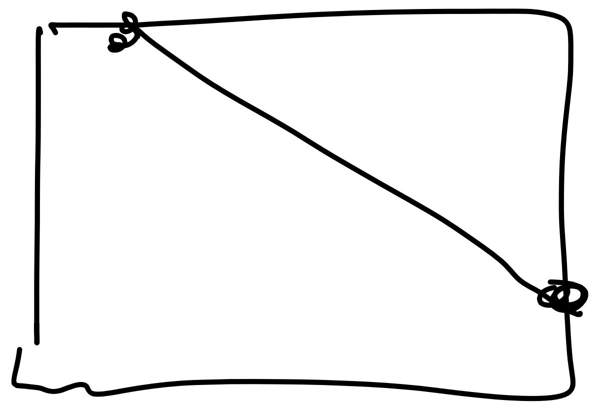
$$\frac{a2+b3+c}{\sqrt{a^2+b^2}} = 2\cos\theta + 3\sin\theta + \rho$$

$$\left\| \begin{pmatrix}0\\0\\1\end{pmatrix} E x_1 \right\|^2$$

"gut feeling" of calibrated coordinates: what is a distance of 10^{-2} in calibrated coordinates

$\longrightarrow f \cdot 10^{-2} = 500 \cdot 10^{-2} = 5$ pix

Draw $ax+by+c=0$



Multiple views

BUNDLER

CAPTURE REALITY

ORB-SLAM

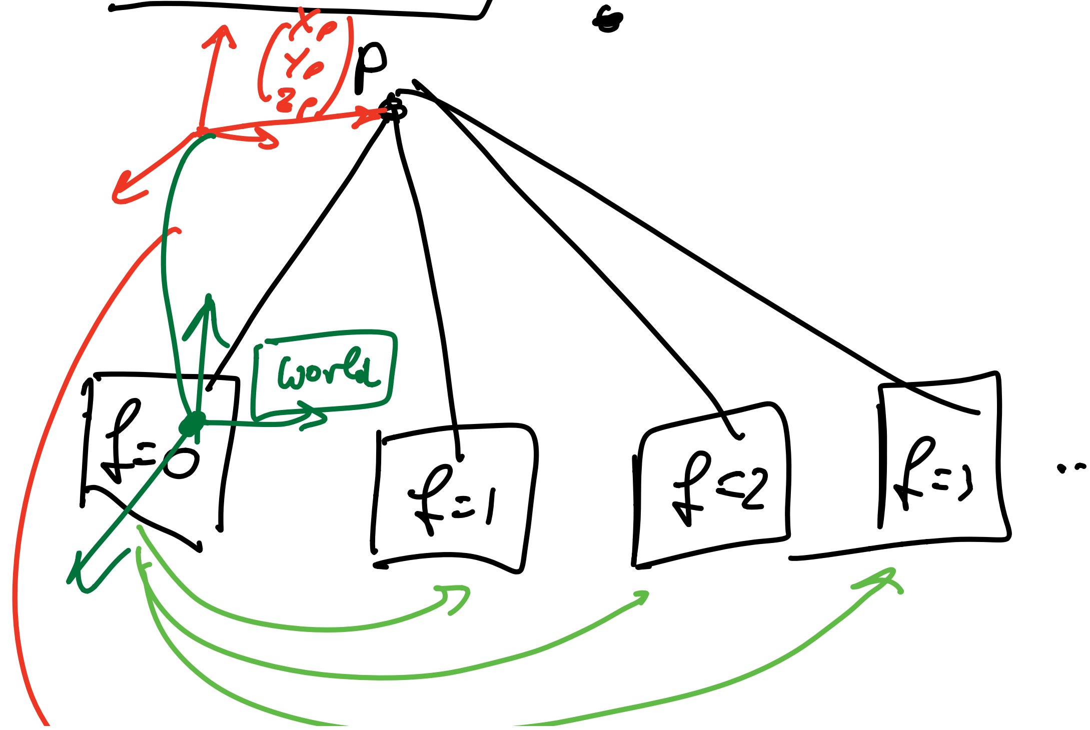
SLAM

unorganized views

Bundle Adjustment

3D-Modeling

2-views



BUT in SfM we set frame 0 to be the world coordinate system

unknowns for motion transformation in F frames : $6(F-1)$

6 for $0 \to 1$

6 for $0 \to 2$

$\vdots$

3 rotation

3 translation

unknowns for N points : $3N$

1 scale which is non-recoverable

fix the depth of one point in 0^{th} frame.

unknowns: $6(F-1)$ motion

$+ 3N-1$ point

scale amb

measurement equation

p-th point
f-th frame

$$\lambda_p^f \begin{pmatrix} x_p^f \\ y_p^f \\ 1 \end{pmatrix} = R^f \begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} + T^f$$

$x_p^f =$ ______

$y_p^f =$ ______

2NF equations

$$\boxed{2NF \geq 6(F-1) + 3N - 1}$$

$F=2$ what we learnt: 2-view SfM

$$4N \geq 6\cdot(2-1)+3N-1$$

$$\boxed{N \geq 5}$$ 5-points ✓

$F=3$ $$6N \geq 6(3-1)+3N-1$$

$$3N \geq 11$$

$$\boxed{N \geq 4}$$

$$\boxed{N \geq \frac{6F-7}{2F-3}}$$

$$= \frac{6F-9+2}{2F-3} = 3 + \frac{2}{2F-3}$$ >0

$$\boxed{N \geq 4}$$

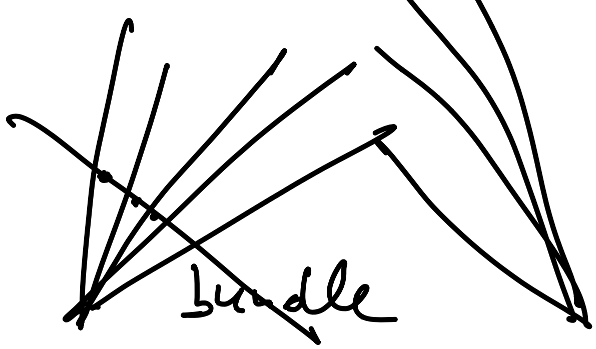
Least squares minimization

$$\varphi(u) = \sum_{f=0}^{F}\sum_{p=1}^{N}\left(\varepsilon_p^f\right)^2$$

unknowns

$$\left(x_p^f - \_\_\_\_\right)^2 + \left(y_p^f - \_\_\right)^2$$

minimization of reprojection error



iteration $\boxed{Hess(\varphi)\,\Delta u = -\nabla\varphi}$

Jacobian $\dfrac{\partial \mathcal{E}}{\partial u}$ $\mathcal{E} \rightarrow NF$ reprojection error

$u \rightarrow 6(F-1)+3N-1$

$NF \times 6(F-1)+3N-1$

iteration $\boxed{\underbrace{(J^T J + \lambda I)}_{(6(F-1)+3N-1) \times (6(F-1)+3N-1)} \Delta u = -J^T \mathcal{E}}$ $NF \times 1$

Whole challeng is to invert efficiently $J^T J$!

$F = 101$

$N = 100$

(899×899) inverted at each iteration

$(600 \times 600)\, \Delta\text{motion} = \cdot$

$(3 \times 3)\, \Delta\text{point} = \cdot$