

CIS 580, Machine Perception, Spring 2021  
Homework 6: Image Processing Basics,  
version of Saturday 10<sup>th</sup> April, 2021 21:14GMT  
Due: Tuesday April 13 2021, 11:59pm

Deliverable is a pdf report including your calculations, plots, and explanations. No code submission but you can include code in your pdf if it helps in the explanation. You are allowed to use the Fourier Table (link in the homework website).

### 1 Convolution of image with a Gaussian

Consider the **the grey-value version of the** statue.jpg. Using the equation

$$g(x, y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (1)$$

with  $\sigma = 1$  construct a 5x5 Gaussian filter and convolve it with your image. What happens to the image? Do this again with  $\sigma = 0.1$ . **Plot the images and** explain the difference.

### 2 Convolution of Gaussians

Assume the 1D Gaussian:

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$

Prove that the convolution of two Gaussians with  $\sigma_1, \sigma_2$  is a Gaussian. Find its  $\sigma$  in terms of  $\sigma_1, \sigma_2$ . You can use all Fourier pairs in the Fourier table without proving them.

### 3 Convolution of Step Edge with Gaussian derivative

Compute (**“by hand”**) the convolution of a step edge

$$h(t) = \begin{cases} H/2 & \text{if } t \geq 0, \\ -H/2 & \text{if } t < 0 \end{cases}.$$

with the first derivative of a Gaussian with standard deviation  $\sigma$ . Write a program that performs this convolution. Use  $H$  and  $\sigma$  as arguments. Keep  $H$  fixed equal to 1 and vary  $\sigma$  from 0.5 to 5 with step 0.5. Plot the 10 curves arising from convolving the step edge with these 10 Gaussian derivatives. **What do you observe?**

### 4 Box Function

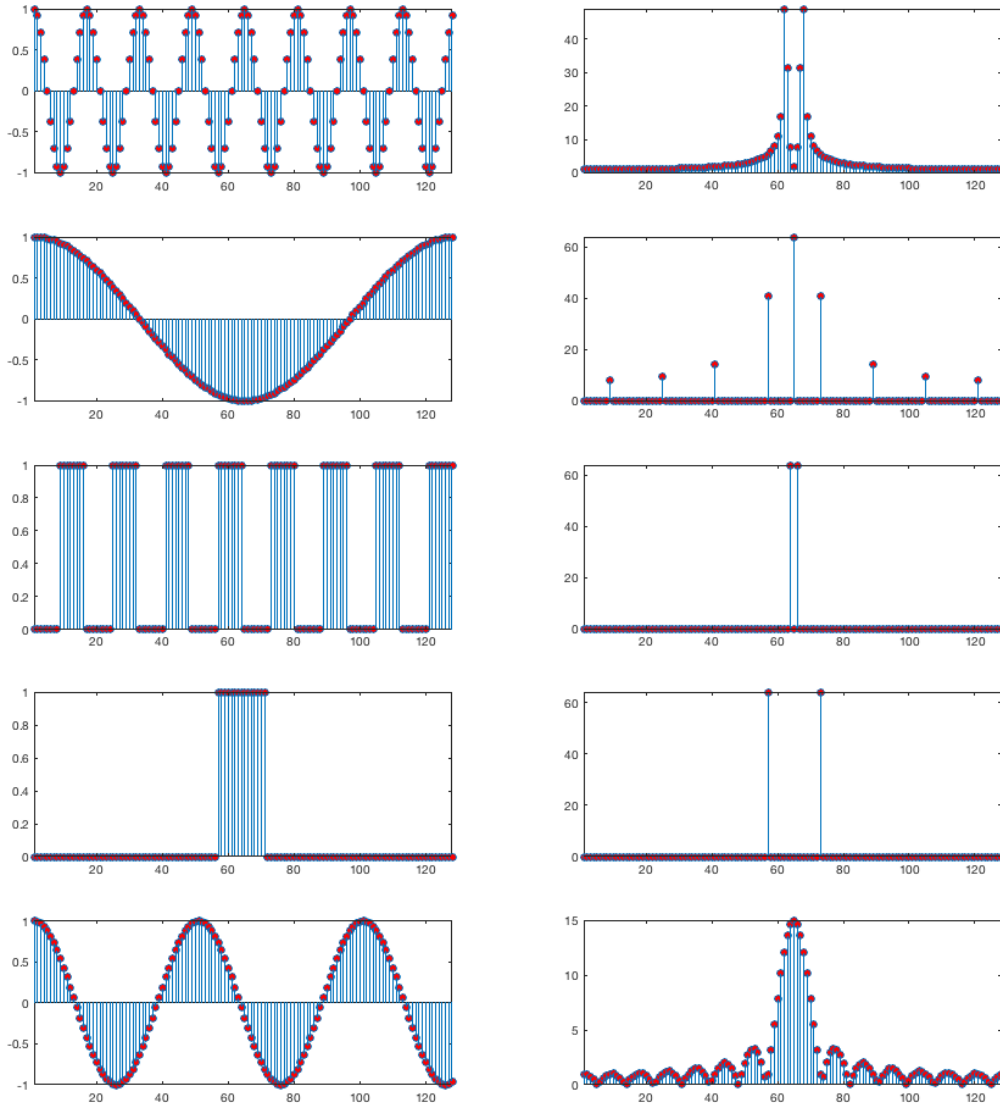
The 1D box function

$$h(t) = \begin{cases} 1/a, & \text{if } |t| \leq a/2 \\ 0, & \text{otherwise.} \end{cases}.$$

is the section of the image of a white line on black background. Compute (“by hand”) its convolution with the 1st derivative of the Gaussian. Write a program that performs this convolution. Fix  $a = 1$  and vary  $\sigma$  from 0.1 to 3.0 with step  $\sigma$  and plot all curves. What do you observe?

## 5 1D FFT Quiz

Match the original signals with the corresponding Fourier transform (DFT). On the right we show the magnitude of the `fftshift` of the transform. **Explain your choices.**



## 6 2D Fourier Transform [10 pts]

Match the following pictures and 2D Fourier transform shown in figure 1. Explain your choices.

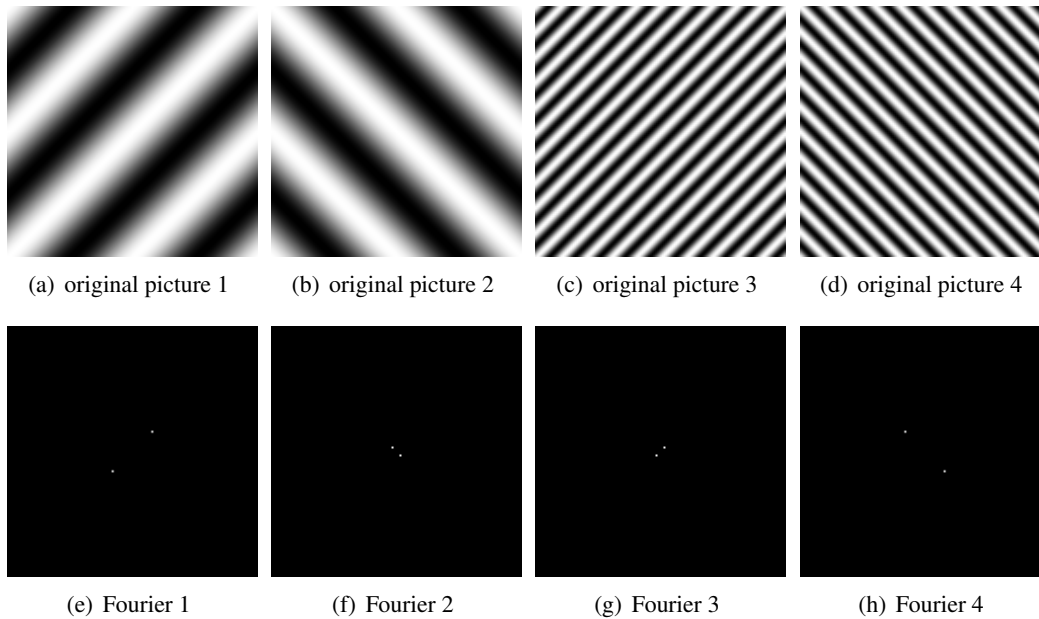


Figure 1: images and corresponding Fourier transform

## 7 Filter Design

Suppose you are given signal  $y(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$  with  $\omega_1 = 2\pi/4$ ,  $\omega_2 = 2\pi/8$ . Define a Gaussian filter  $g(t) = 2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}}$ .

1. Compute “by hand” the output signal  $s(t) = (y * g)(t)$ , after we convolve  $y$  with  $g$ . You can use the Fourier tables.
2. Compute  $\sigma$  such that the output of the convolution is

$$s(t) = \frac{1}{2} \cos(\omega_1 t) + a * \cos(\omega_2 t)$$

for some  $a \geq 0$ . In your solution what is the value of  $a$  in terms of  $\omega_1$  and  $\omega_2$ ?