<u>redo q. 2</u>

in reality it is a pure rotation $q \sim Rp$

we will show that if

$$a = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \quad \alpha, \beta, \gamma \text{ 3 free parameters}$$

then $E = \hat{a} R$ is always a solution, which means that we have a 3-parametric family of solution for E.

$$q^T \hat{a} \underbrace{Rp}_{q} = q^T \hat{a} q$$

$$= q^T (a \times q)$$

$$= 0 \quad \forall a, q$$

$\cos(\theta)$

8. $T = \begin{pmatrix} 0 \\ 0 \\ T_z \end{pmatrix}$ $\quad R = \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$

the only unknown is θ

$$E = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -s & -c & 0 \\ c & -s & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

condition on a general E to be of this form

① $E_{13} = E_{23} = E_{31} = E_{32} = E_{33} = 0$

$E_{11} = E_{22}$
$E_{21} = -E_{12}$

$\frac{3}{\sqrt{34}} \quad \frac{5}{\sqrt{34}}$

$\frac{5}{\sqrt{34}} \quad \frac{3}{\sqrt{34}}$

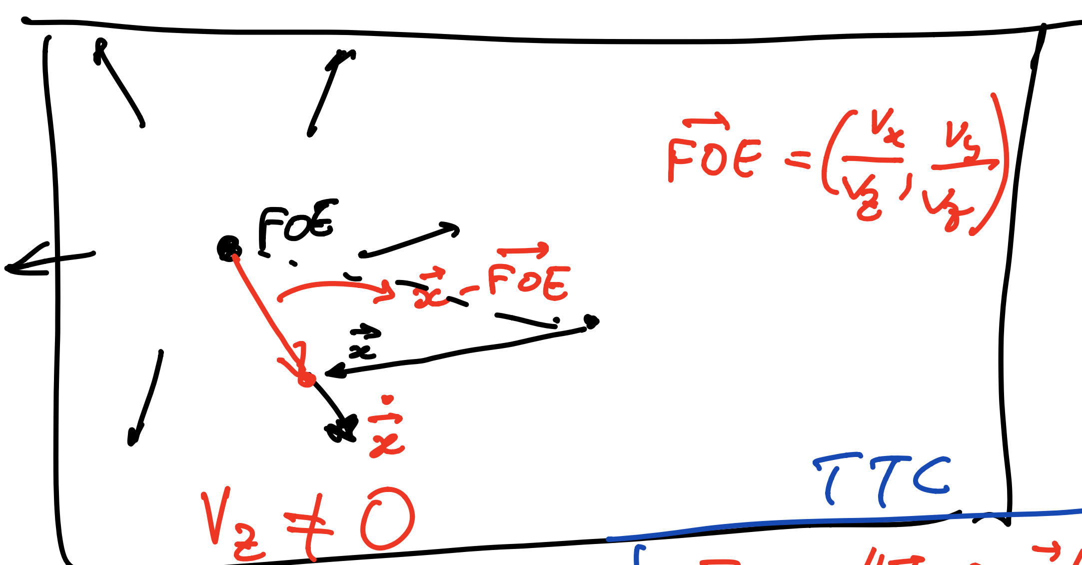
$$\begin{matrix} 3 & 5 & 0 \\ -5 & 3 & 0 \\ 0 & 0 & 0 \end{matrix}$$

② if E has this form

$$\tan\theta = \frac{E_{11}}{E_{12}}$$

3D velocities

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \underbrace{\frac{1}{Z}\underset{2\times 3}{A}\underset{3\times 1}{V}}_{\text{translational}} + \underbrace{\underset{2\times 3}{B}\underset{3\times 1}{\Omega}}_{\text{rotational}}$$



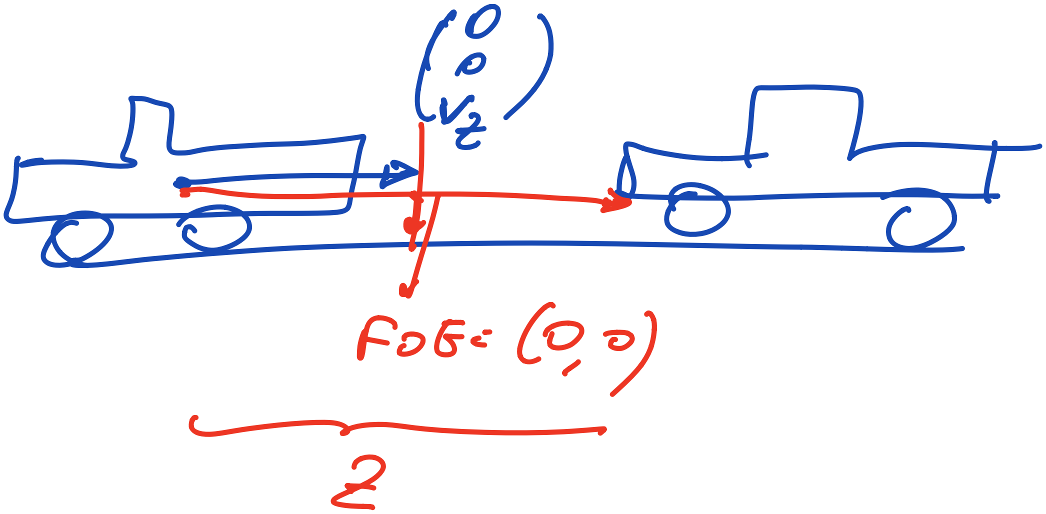
$$\vec{FOE} = \left(\frac{V_x}{V_z}, \frac{V_y}{V_z}\right)$$

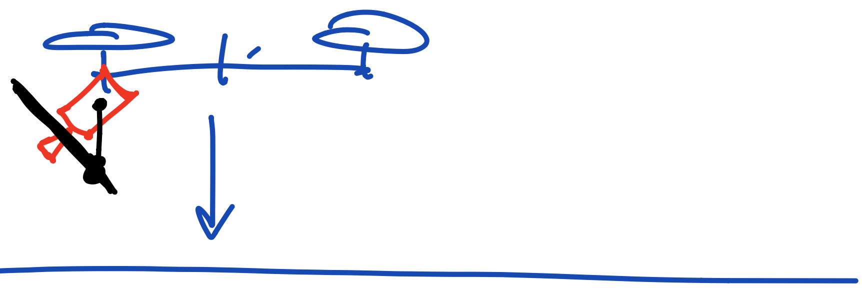
$$V_z \neq 0$$

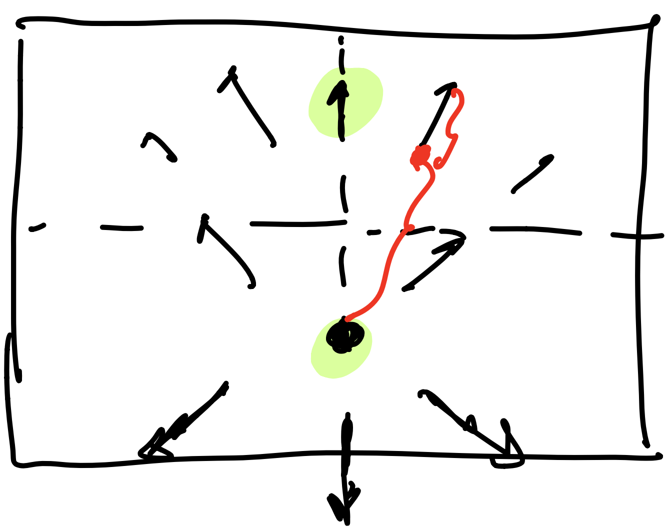
$$\left(\vec{x} - \vec{FOE}\right)\frac{V_z}{Z} = \dot{\vec{x}}$$

TTC

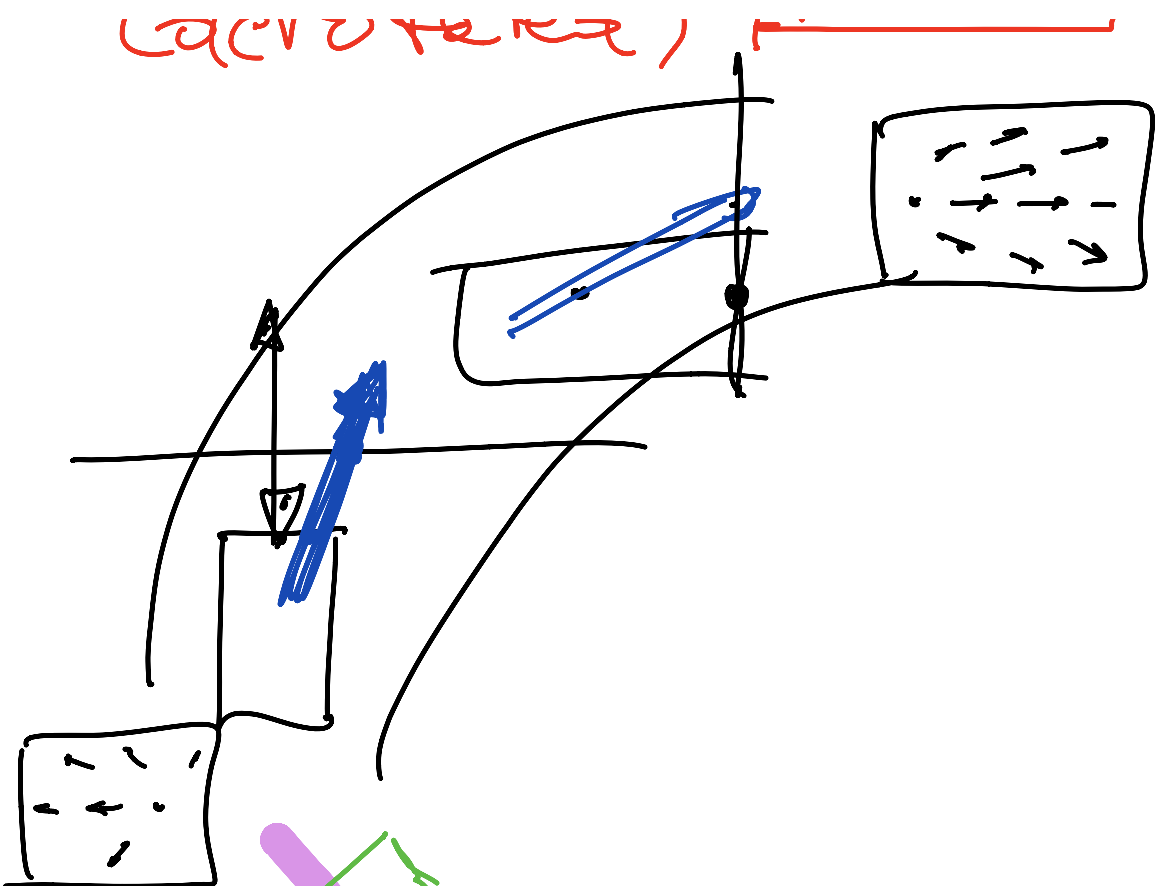
$$\boxed{\frac{Z\ [m]}{V_z\ [m/sec]} = \frac{\|\vec{x} - \vec{FOE}\|}{\|\dot{\vec{x}}\|}}$$

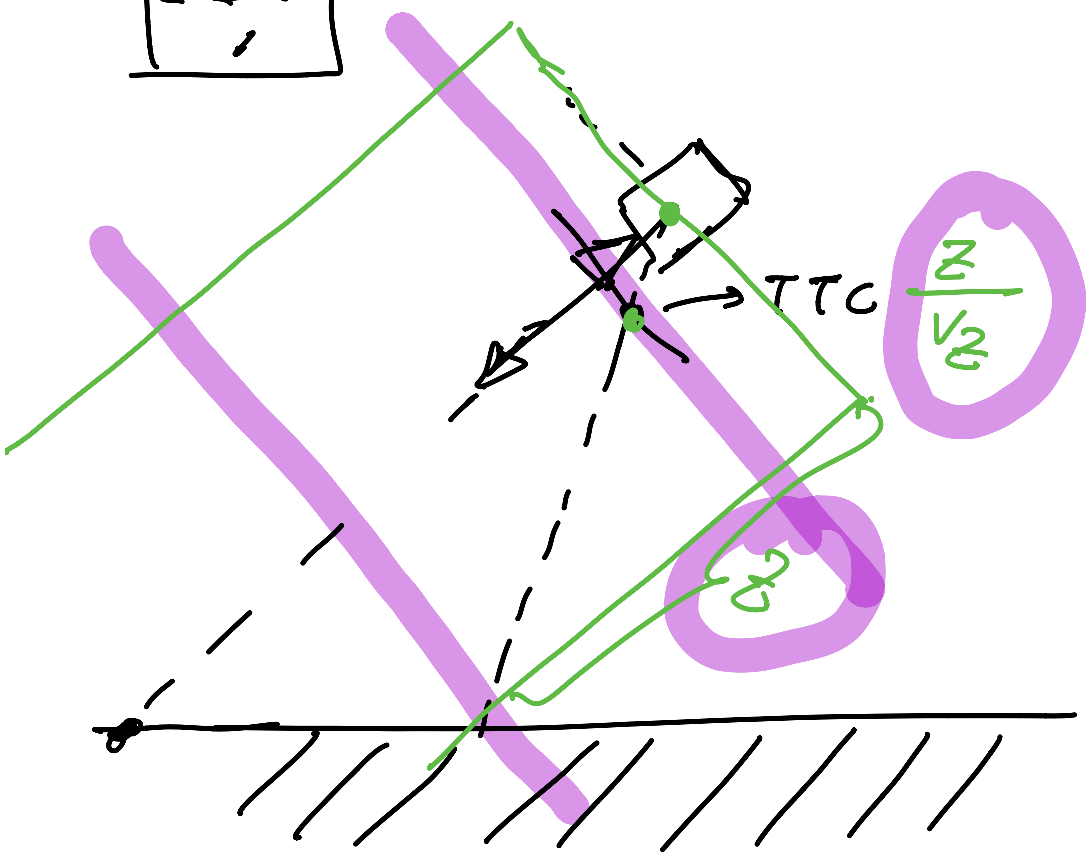






pure translational flow fields




TTC
Z/V_Z
Z

Convolution of Gaussians

$$g_{\sigma_1} * g_{\sigma_2} = g_{\sqrt{\sigma_1^2+\sigma_2^2}}$$

$$\frac{\partial g}{\partial x} = \frac{1}{\sqrt{2\pi}\,\sigma}\left(-\frac{x}{\sigma^2}\right)e^{-\frac{x^2}{2\sigma^2}}$$

$$g(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{x^2}{2\sigma^2}} \quad \text{1D}$$

$$g(x,y) = \frac{1}{2\pi\sigma^2}\, e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \text{2D}$$

extremely crucial for

DoG = LoG

DoG

$$\frac{\partial g(x,y)}{\partial \sigma} = \frac{1}{2\pi}\left(-\frac{2}{\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{1}{\sigma^2}\left(\frac{x^2+y^2}{\sigma^3}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}\right)$$

$$= \frac{1}{2\pi\sigma^3}\left(-\cancel{1}\,2 + \frac{x^2+y^2}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\underbrace{\qquad}_{x^2+y^2 < \cancel{1}\,2}$$

$\ominus$

DoG

$$\frac{\partial g(x,y)}{\partial \sigma} \approx \frac{g(\sigma+\Delta\sigma) - g(\sigma)}{\Delta\sigma}$$

does not have to be tiny e.g. $\Delta\sigma = 0.7$

2D
$$\frac{\partial g(x,y)}{\partial x} = \frac{1}{2\pi\sigma^2}\left(-\frac{x}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 g(x,y)}{\partial x^2} = \frac{-1}{2\pi\sigma^4}\left(e^{-\frac{x^2+y^2}{2\sigma^2}} + x\left(-\frac{x}{\sigma^2}\right)e^{-\quad}\right)$$

$$= \frac{1}{2\pi\sigma^4}\left(-1+\frac{x^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 g}{\partial y^2} = \frac{1}{2\pi\sigma^4}\left(-1+\frac{y^2}{\sigma^2}\right)e^{-\quad}$$

$$\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2} = \frac{1}{2\pi\sigma^4}\left(-2+\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$DOG = \sigma LOG$$

$$LOG = \frac{DOG}{\sigma} = \frac{g(\sigma+\Delta\sigma)-g(\sigma)}{\sigma\Delta\sigma}$$

heat equation $\qquad \dfrac{\partial^2 f}{\partial x^2} = \dfrac{1}{t}\dfrac{\partial f}{\partial t}$

diffusion

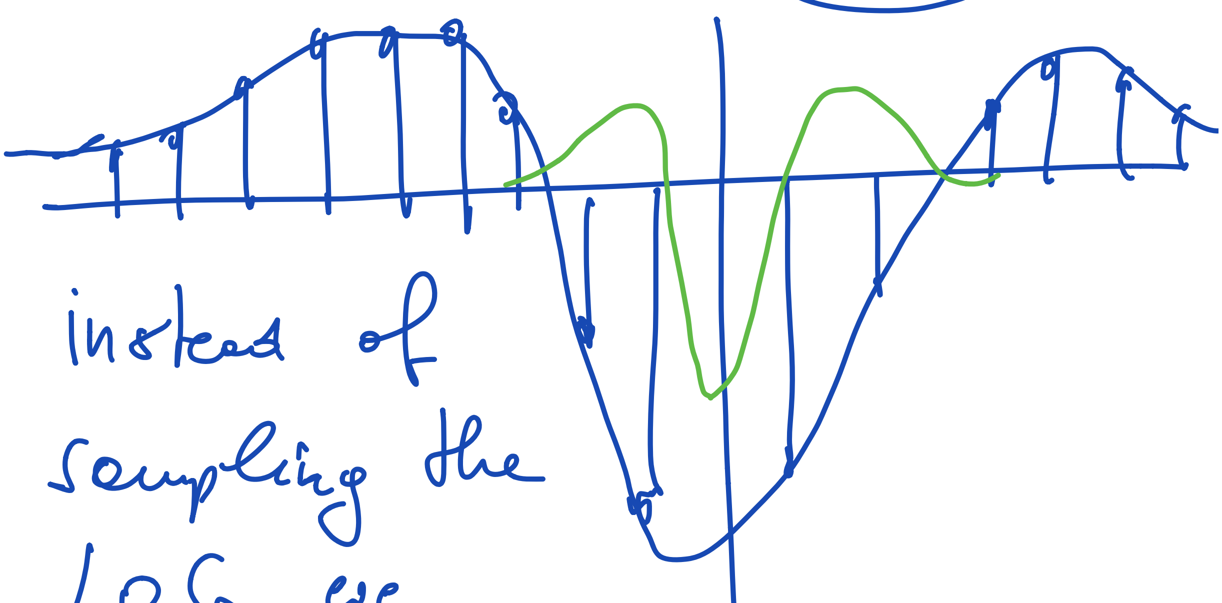
$$f(x, t=0) = f_0(x)$$

solution is $\quad f(x,t) = \text{gauss} * f_0(x)$

practically :

compute LoG for big σ !

$\sigma = 8$



instead of sampling the LoG we approximate the DOG.

$$g_{\sigma=\frac{1}{\sqrt{2}}} = \frac{1}{16}\begin{pmatrix} 1 & 4 & 6 & 4 & 1 \end{pmatrix}$$

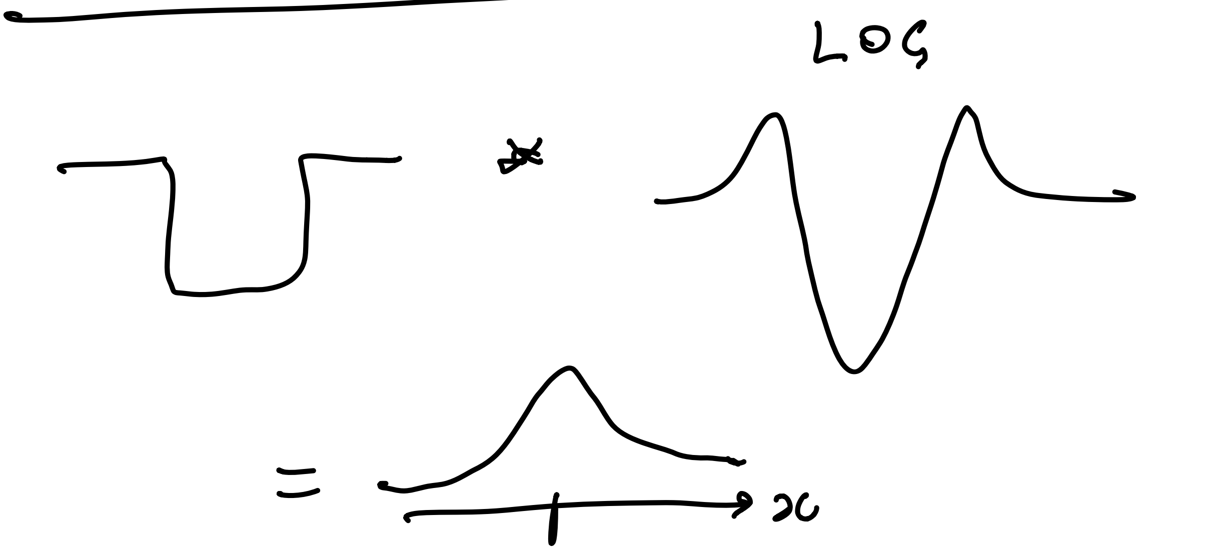
with itself

$\vdots$

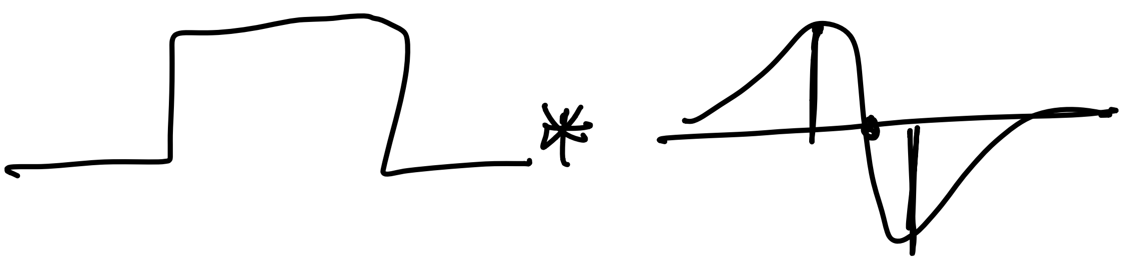
$$g_{\sigma=8}$$

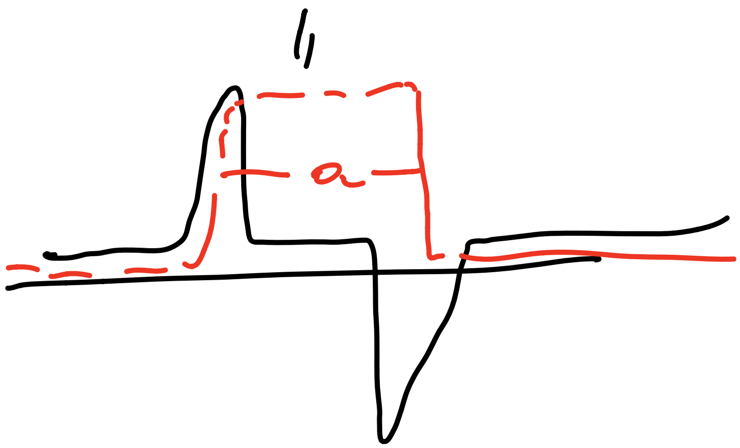
$$g_{\sigma=\sqrt{8^2+\left(\frac{1}{\sqrt{2}}\right)^2}}$$

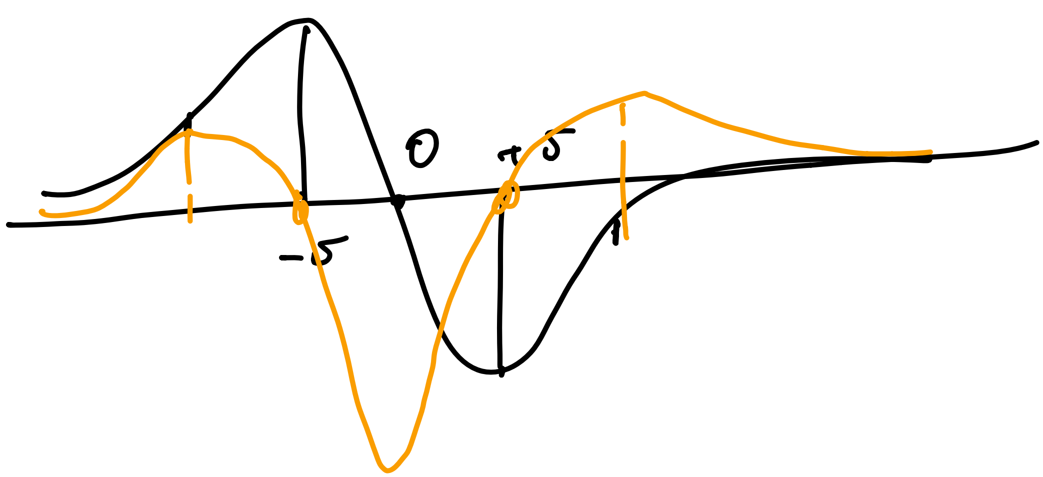
$$\frac{g_{\sigma=\sqrt{8^2+\left(\frac{1}{\sqrt{2}}\right)^2}} - g_{\sigma=8}}{\sigma\ (\doteq \sigma)}$$



max over σ makes only sense if LOG is normalized with σ^2 ($= \sigma^2 LOG$)







$$\arg\max_x x e^{-\frac{x^2}{2\sigma^2}} \qquad \frac{\partial}{\partial x} = e^{-\frac{x^2}{2\sigma^2}} + x\left(-\frac{x}{\sigma^2}\right) e^{-\frac{x^2}{2\sigma^2}} = 0$$

$$\left(1 - \frac{x^2}{2\sigma^2}\right) e^{-\frac{x^2}{2\sigma^2}}$$