ESE 546, FALL 2020
PROBLEM SET 1
INSTRUCTOR SOLUTIONS

Solution 1. See Jupyter notebook.

Solution 2. **Proof**. To Prove :

$$\mathop{\mathbb{E}}_{X}[\varphi(X)] \geq \varphi(\mu)$$

We know that the curve of a convex function always lies above its tangent at that point. Let the tangent at a point x be given by $L(X)$, then we have

$$\begin{aligned} \varphi(x) &\geq L(x) \\ \mathop{\mathbb{E}}_{X}[\varphi(X)] &\geq \mathop{\mathbb{E}}_{X}[L(x)] \\ \mathop{\mathbb{E}}_{X}[\varphi(X)] &\geq \mathop{\mathbb{E}}_{X}[mx + c] \\ \mathop{\mathbb{E}}_{X}[\varphi(X)] &\geq \mathop{\mathbb{E}}_{X}[mx] + c \\ \mathop{\mathbb{E}}_{X}[\varphi(X)] &\geq m \mathop{\mathbb{E}}_{X}[x] + c \\ \mathop{\mathbb{E}}_{X}[\varphi(X)] &\geq L(\mathop{\mathbb{E}}_{X}[x]) \end{aligned}$$

Since the value of tangent at a given point on the curve is the same as the value of the curve itself, we have :

$$\mathop{\mathbb{E}}_{X}[\varphi(X)] \geq \varphi(\mathop{\mathbb{E}}_{X}[x])$$

□

Solution 3. See Jupyter notebook.