MEAM 520
Lecture 6: DH Parameters

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Last Time

The **Denavit-Hartenberg transform** results from successive rotations and translations via the four DH parameters

a parameterization for homogeneous transformations

The transform from i to i-1 is

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Three DH parameters will be **constant** for each joint's transformation, and one will **vary**.

Plug DH parameters into the above formula to find each joint's transformation matrix.

The final transformation matrix from tip to base is

$$\mathbf{T}_n^0 = A_1(q_1) \cdots A_n(q_n)$$

There have been some questions about the lab report

This rubric was posted online with Lab 1

Completeness: Did the report address all assigned tasks?	5 pts
Method: Was the approach technically sound and reproducible?	5 pts
Evaluation: Were all relevant results reported? Are the test cases sufficient?	5 pts
Analysis: Was the analysis complete and free of error?	5 pts
Clarity: Was the report clear and organized?	5 pts

Methods:

5: Sound method that **identifies assumptions and limitations**; Methods described in sufficient detail to be **reproducible** by a classmate

Evaluation:

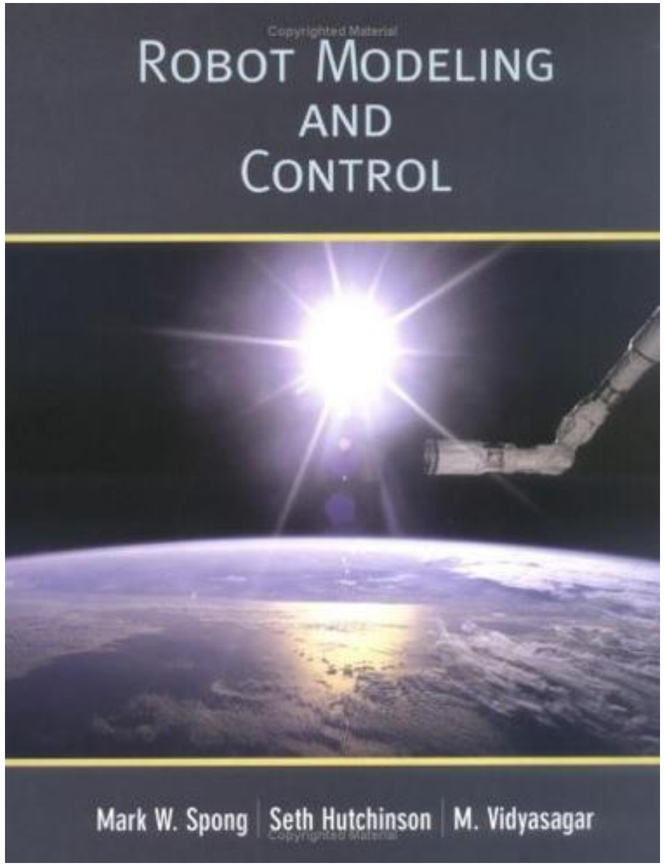
5: Test cases clearly designed to **demonstrate methods and limitations;** All relevant quantitative **data** and qualitative observations reported

Analysis:

5: Analysis is logical and **supported by data**; Relation between the reported results and the conclusions is clear

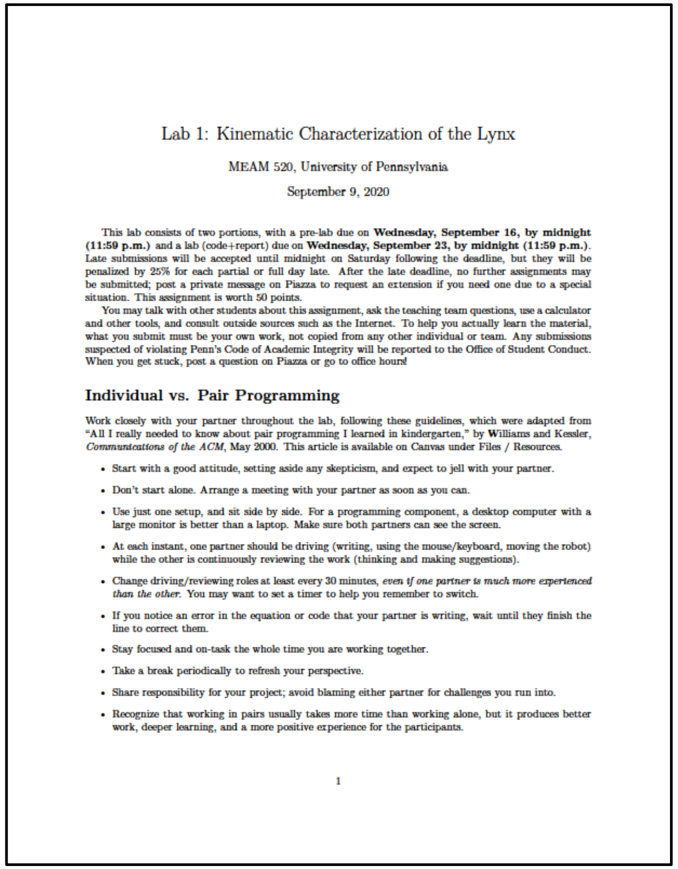
There is no requirement on how you accomplish this. Use tables, plots, screenshots of the simulation, textual description, depending on what works best for what you are trying to convey.

Today: DH parameter examples



Chapter 3: Forward and Inverse Kinematics

- Read Sec. 3.2



Lab 1: Kinematic Characterization of the Lynx

MEAM 520, University of Pennsylvania

September 9, 2020

This lab consists of two portions, with a pre-lab due on **Wednesday, September 16, by midnight (11:59 p.m.)** and a lab (code+report) due on **Wednesday, September 23, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation. This assignment is worth 50 points.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

Work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Williams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Lab 1 due 9/23, 11:59 p.m.

Today: DH parameter examples

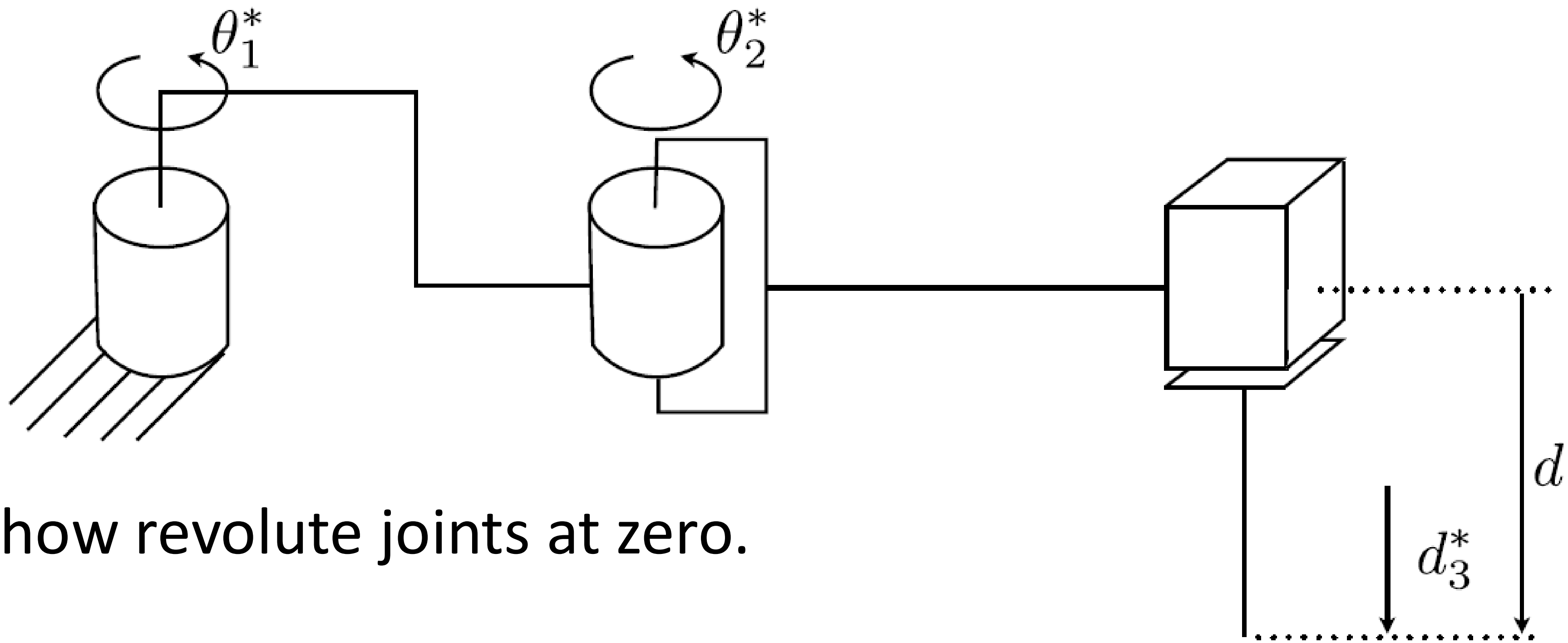
- When is $a_i \neq 0$?

- Help! My DH parameters don’t include all of my robot dimensions!

Example 1

SCARA

Shown at $\theta_1^* = 0, \theta_2^* = 0,$ and $d_3^* = d$



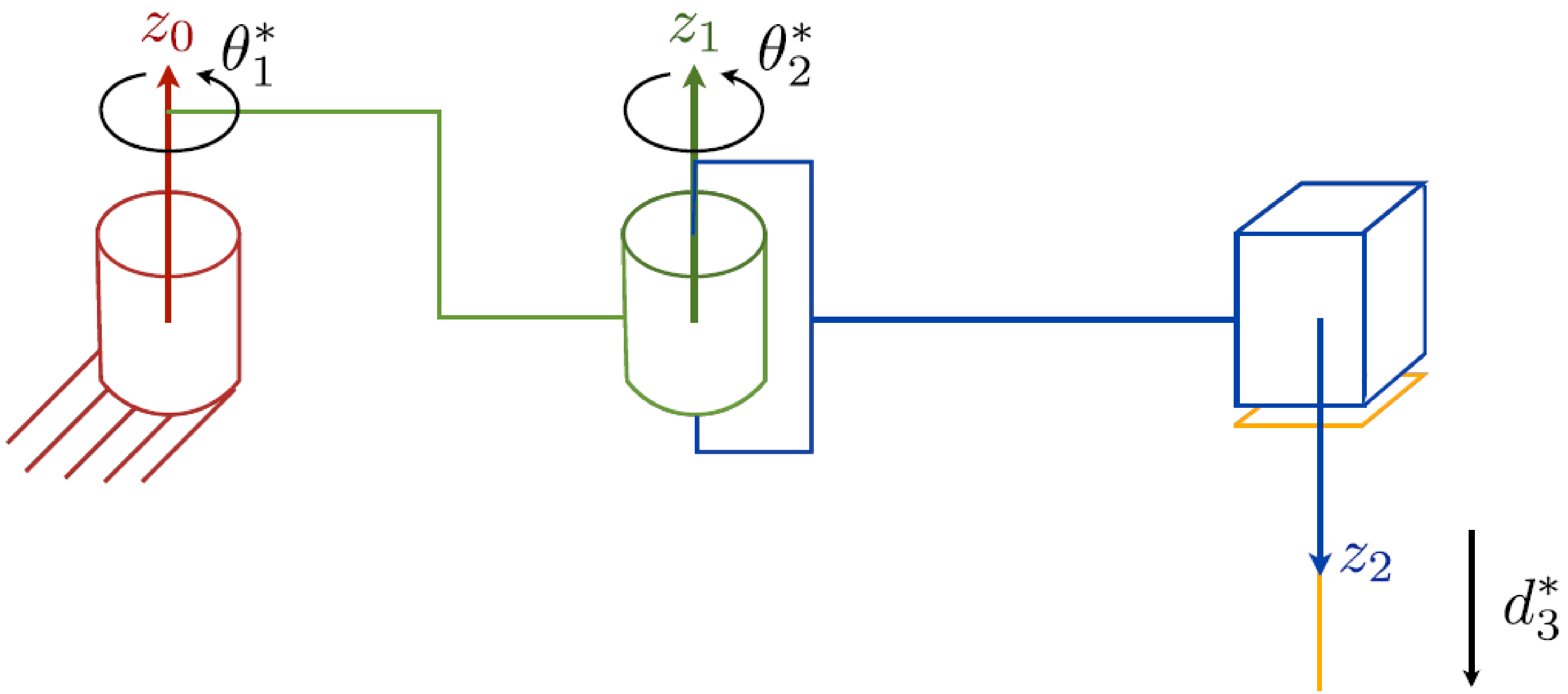
The diagram should show revolute joints at zero.

Prismatic joints are usually shown at a positive displacement instead of zero.

Example 1

Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

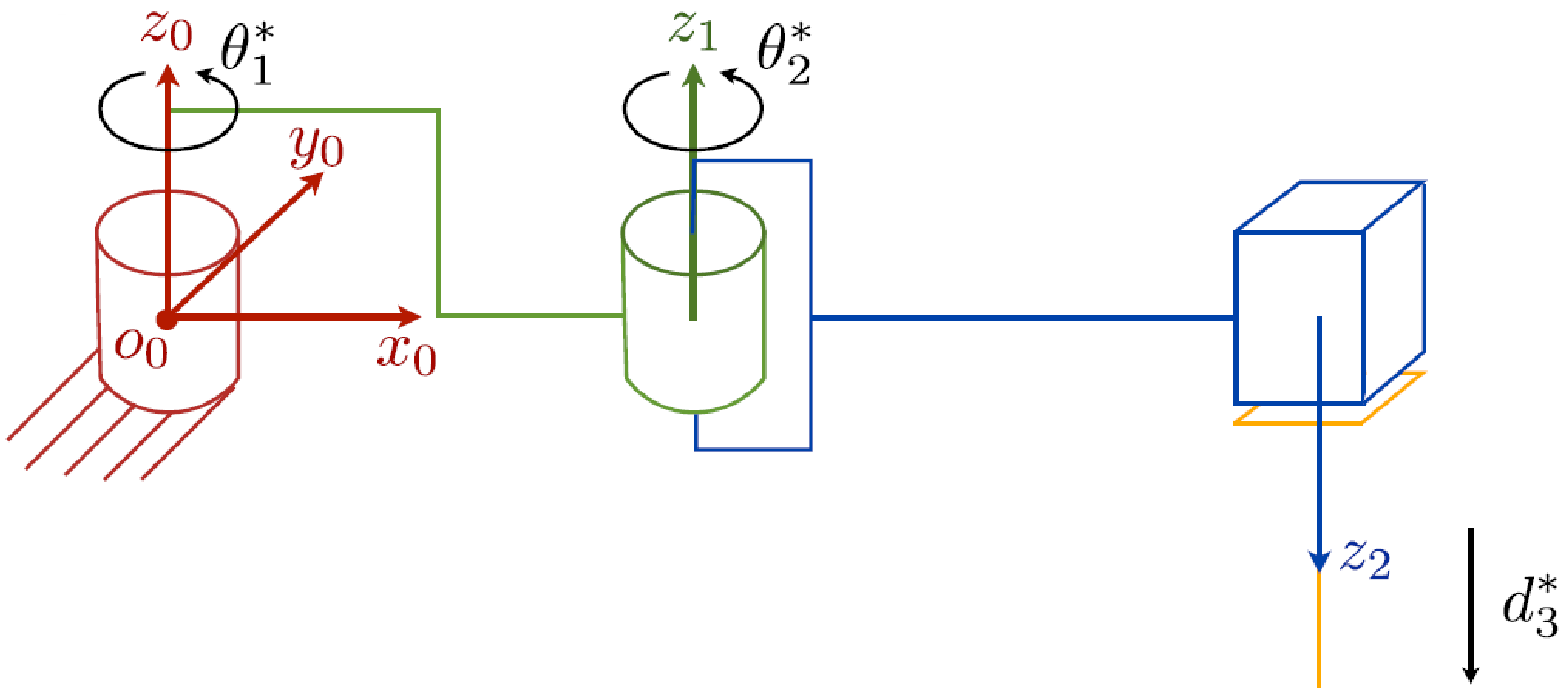
SCARA



SCARA

Example 1

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.



Example 1

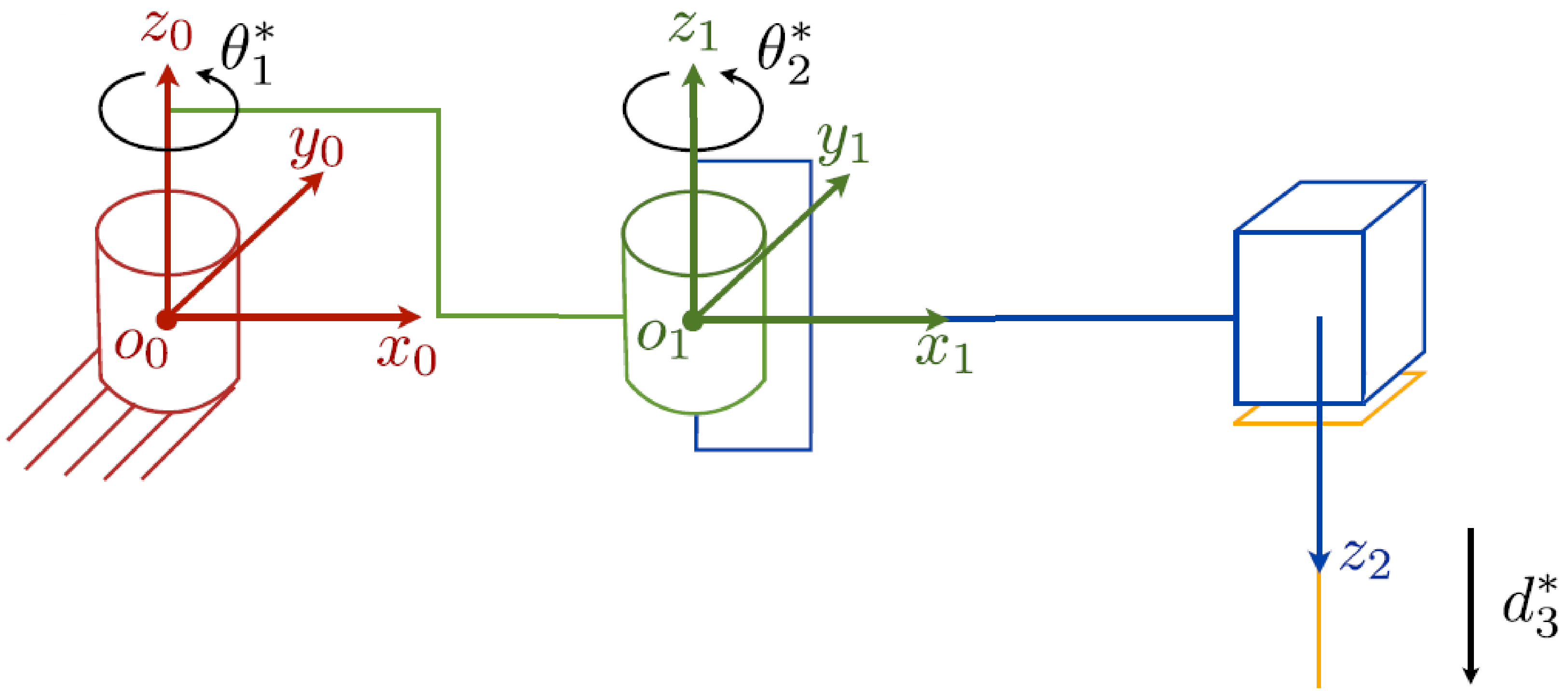
SCARA

For $i = 1, \ldots n - 1$,

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.



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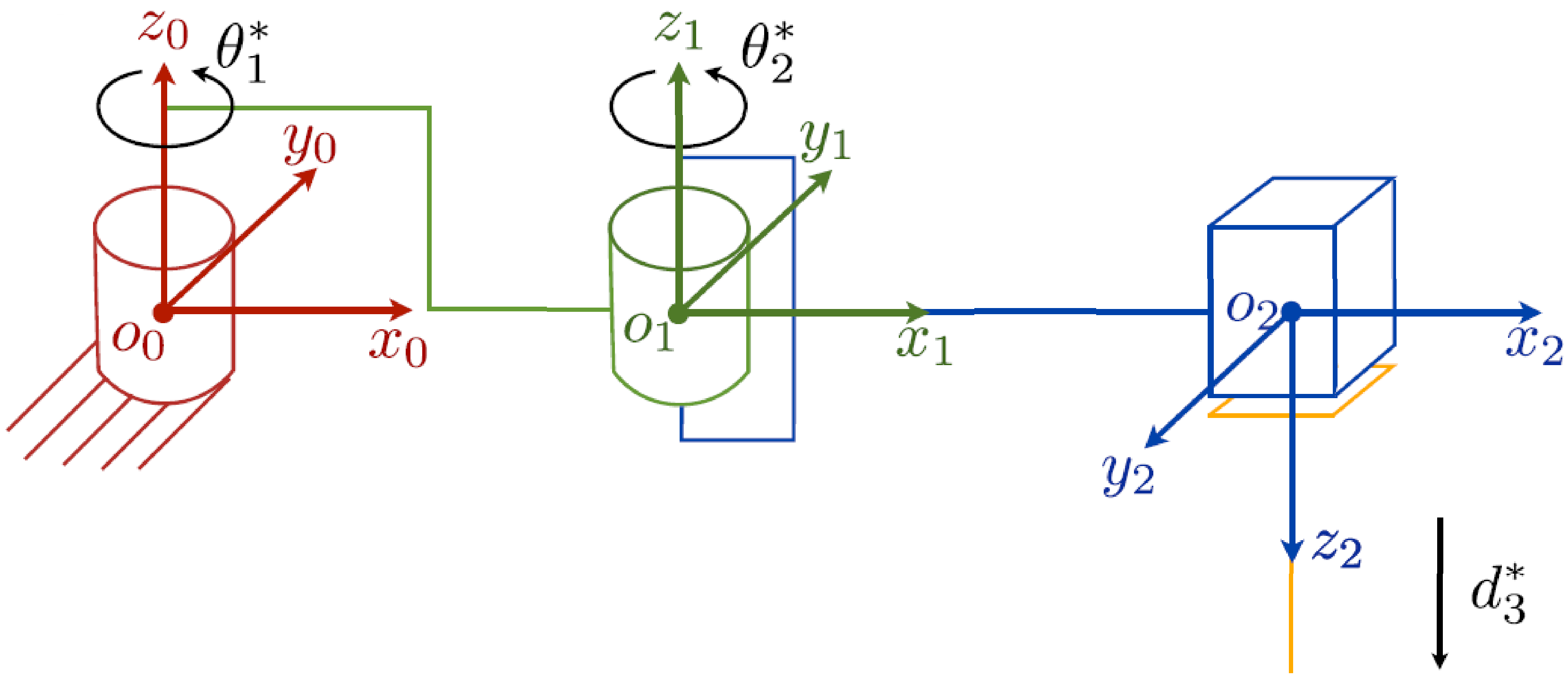
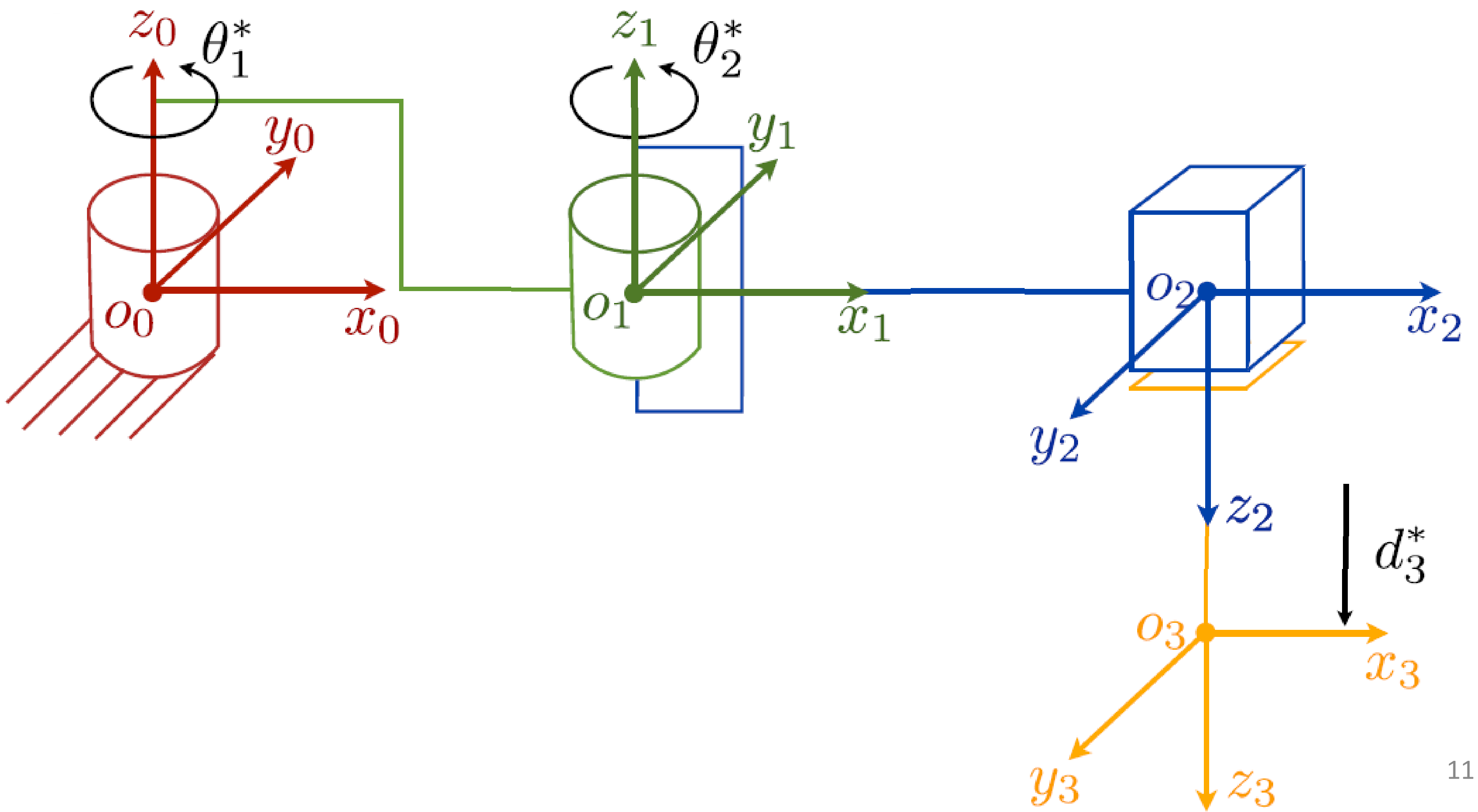


Example 1

SCARA

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n-th joint is revolute, set $z_n = a$ along the direction z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.



Example 1

SCARA

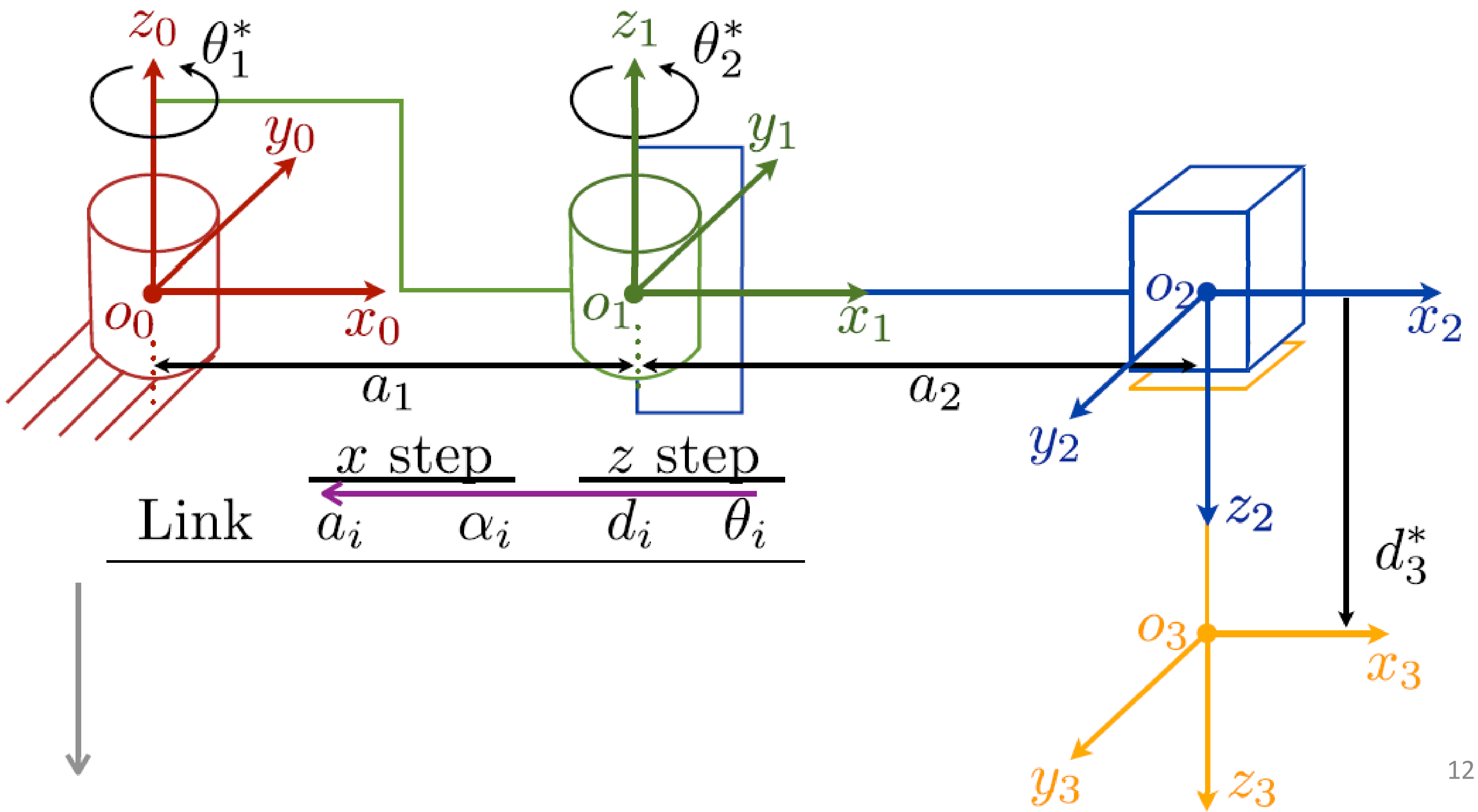
Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

$a_i =$ distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

$d_i =$ distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

$\alpha_i =$ the angle between z_{i-1} and z_i measured about x_i.

$\theta_i =$ the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.



Link	a_i	α_i	d_i	θ_i

Example 1

SCARA

Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

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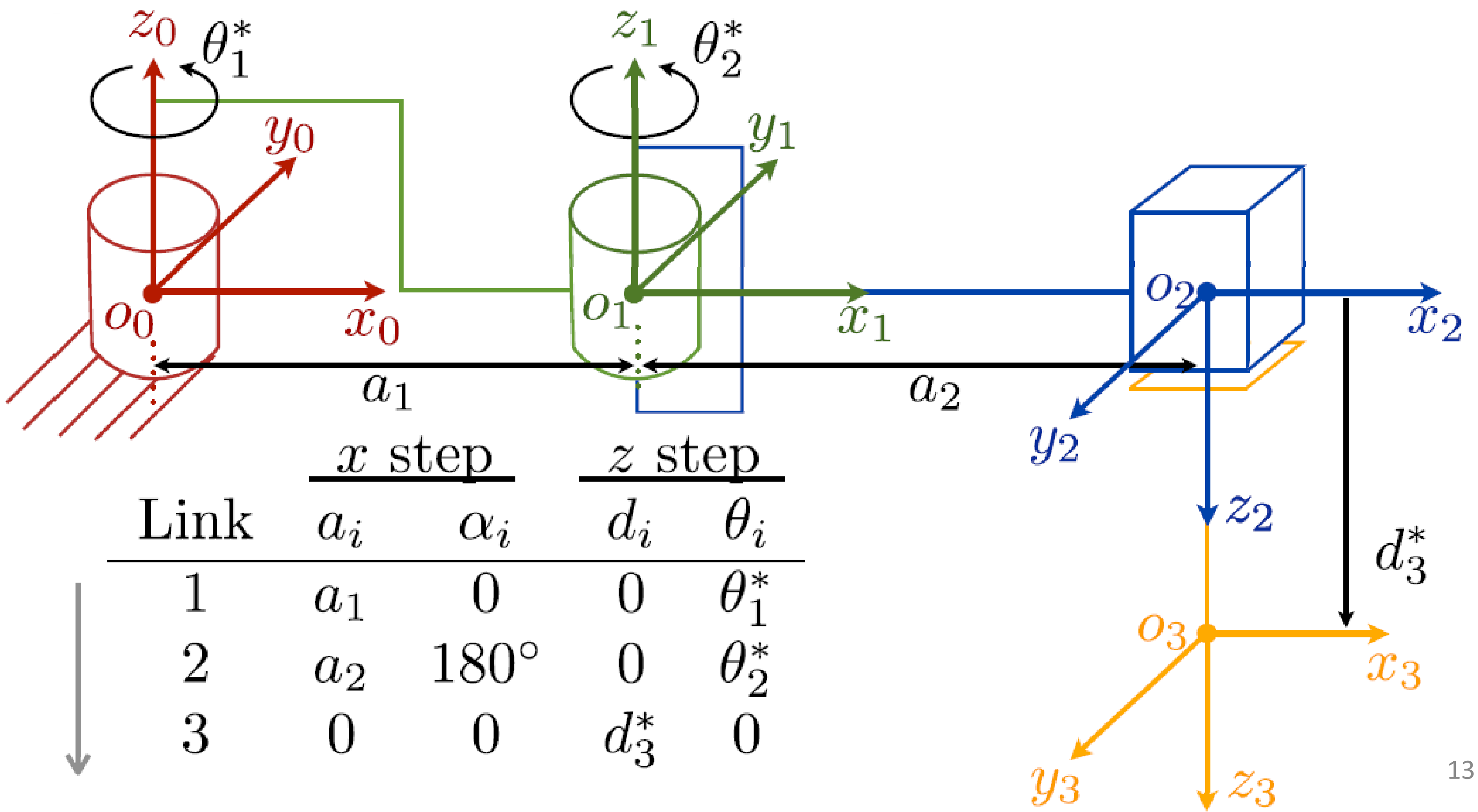


	x step		z step	
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	180°	0	θ_2^*
3	0	0	d_3^*	0

Example 1

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into (3.10).

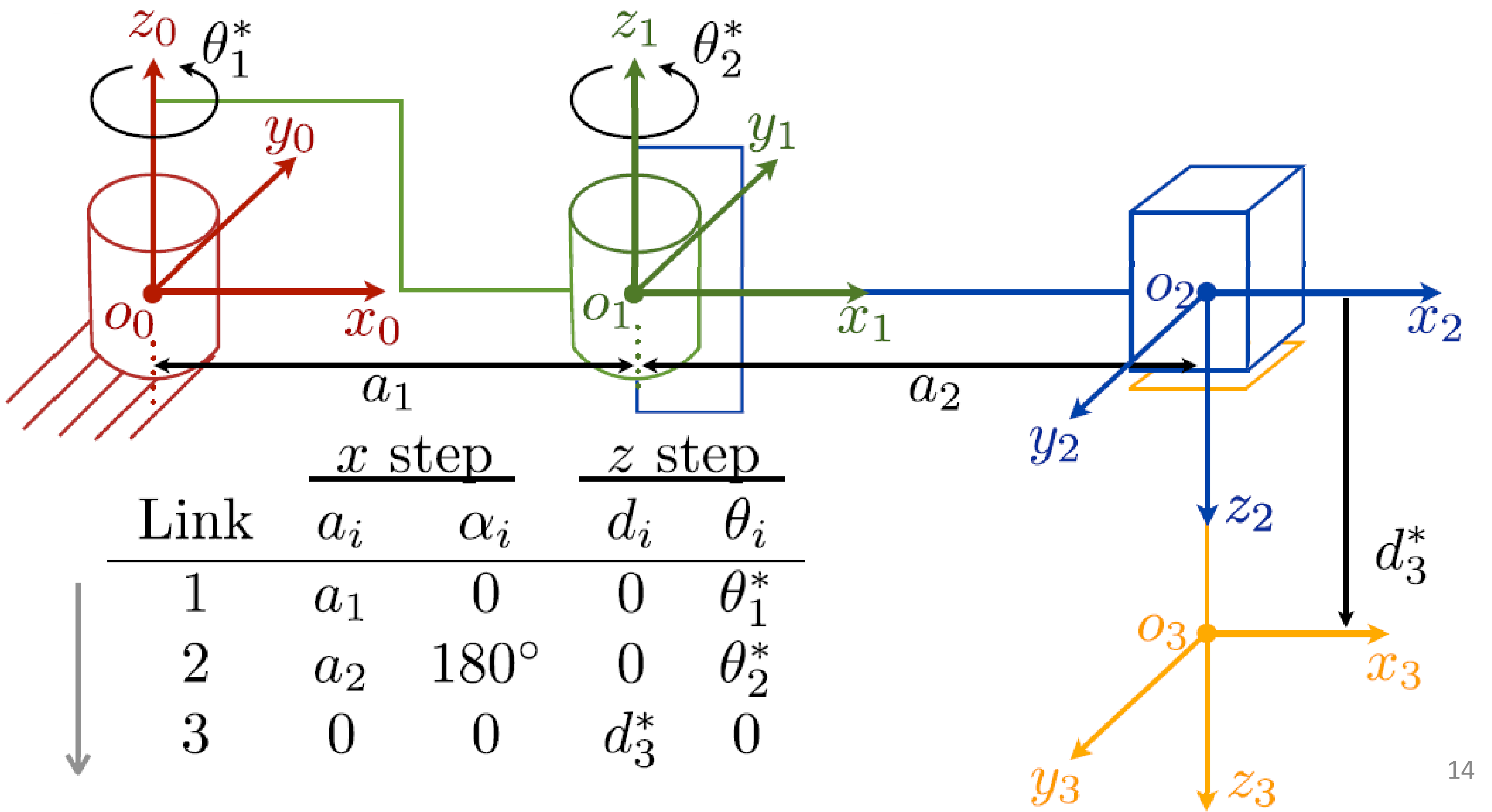
$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SCARA

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



	x step		z step	
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	180°	0	θ_2^*
3	0	0	d_3^*	0

Example 1

Step 9: Form $T_n^0 = A_1 \cdots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

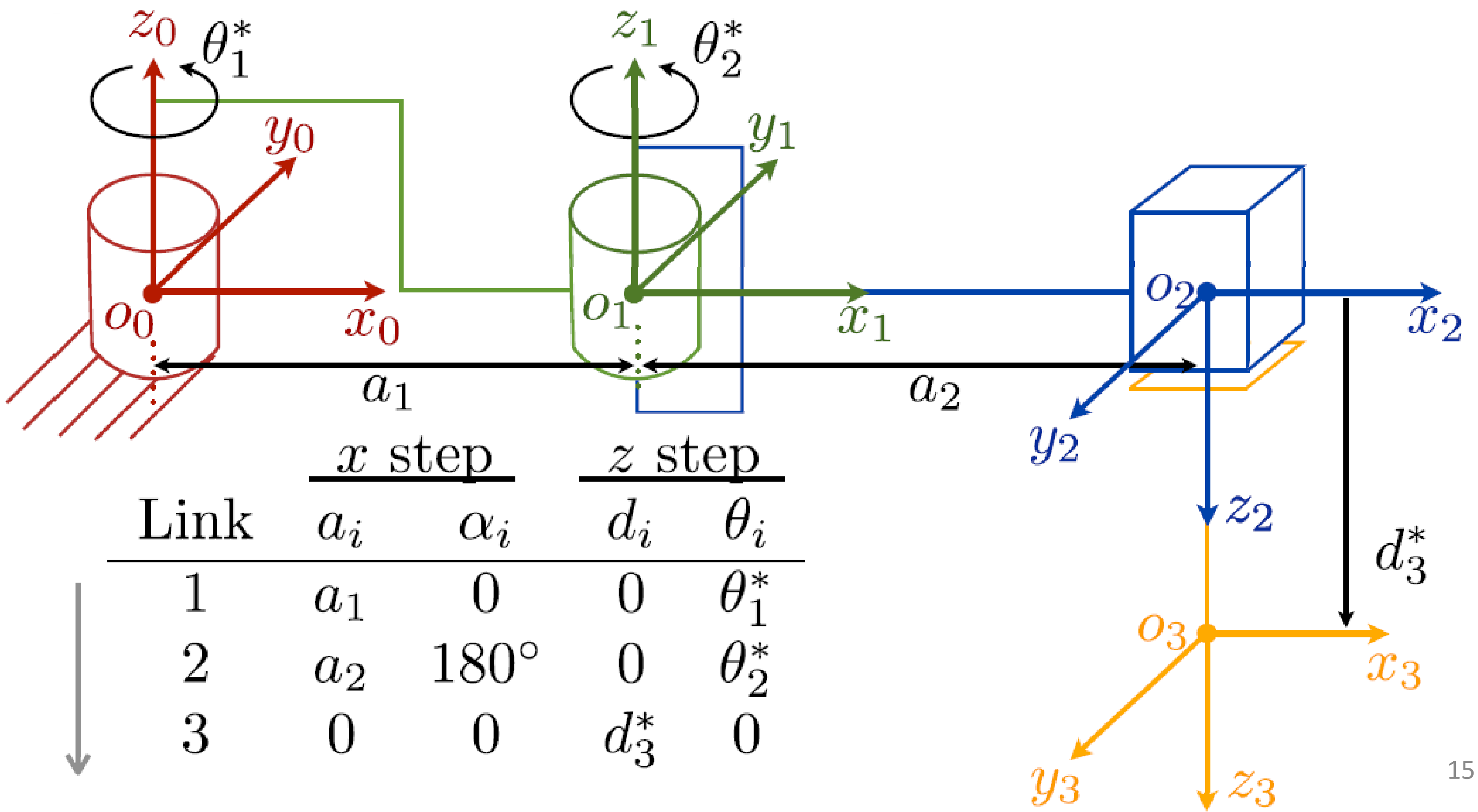
$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_{12}^* & s_{12}^* & 0 & a_1 c_1^* + a_2 c_{12}^* \\ s_{12}^* & -c_{12}^* & 0 & a_1 s_1^* + a_2 s_{12}^* \\ 0 & 0 & -1 & -d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SCARA

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

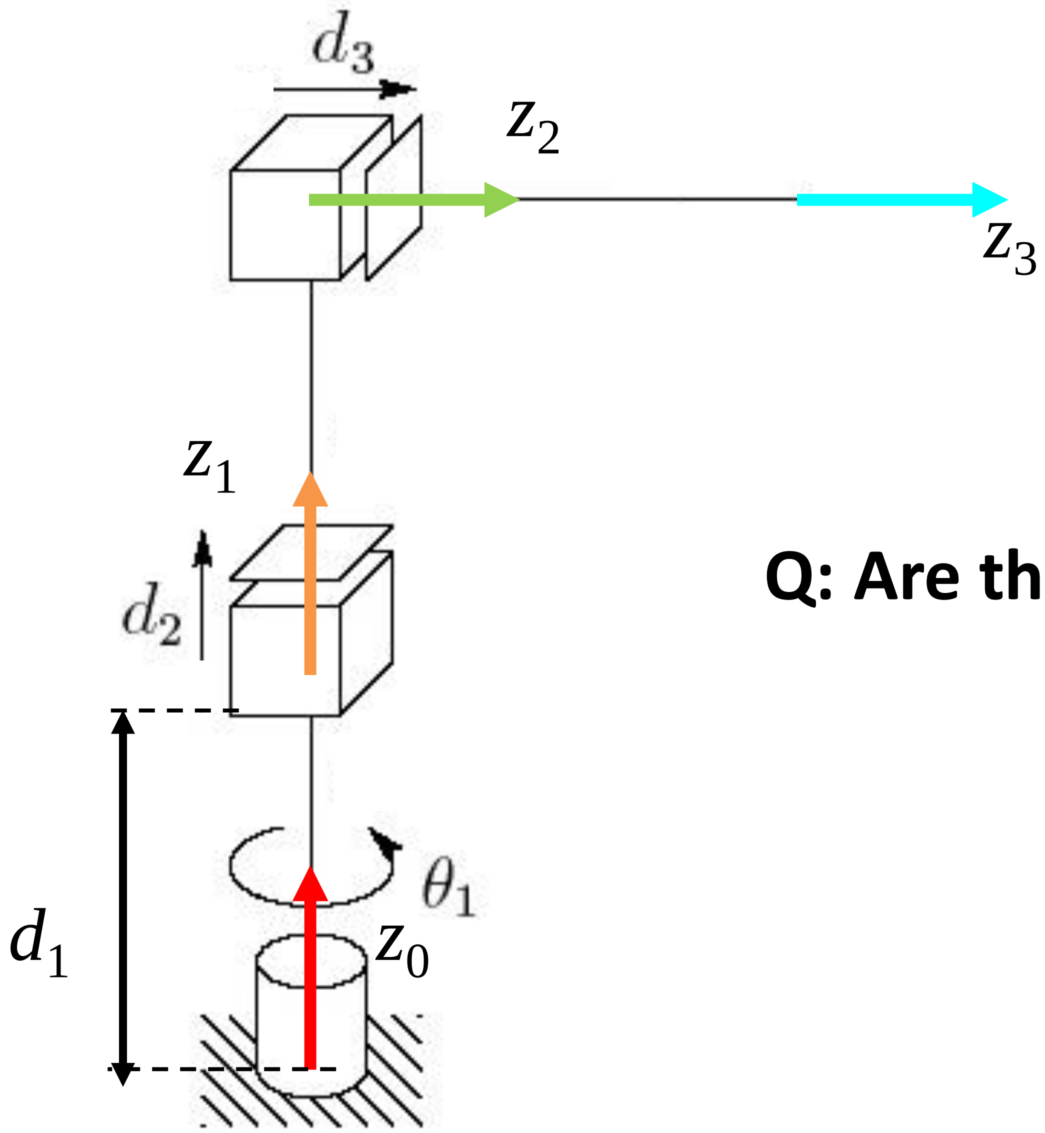
$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



	x step		z step	
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	180°	0	θ_2^*
3	0	0	d_3^*	0

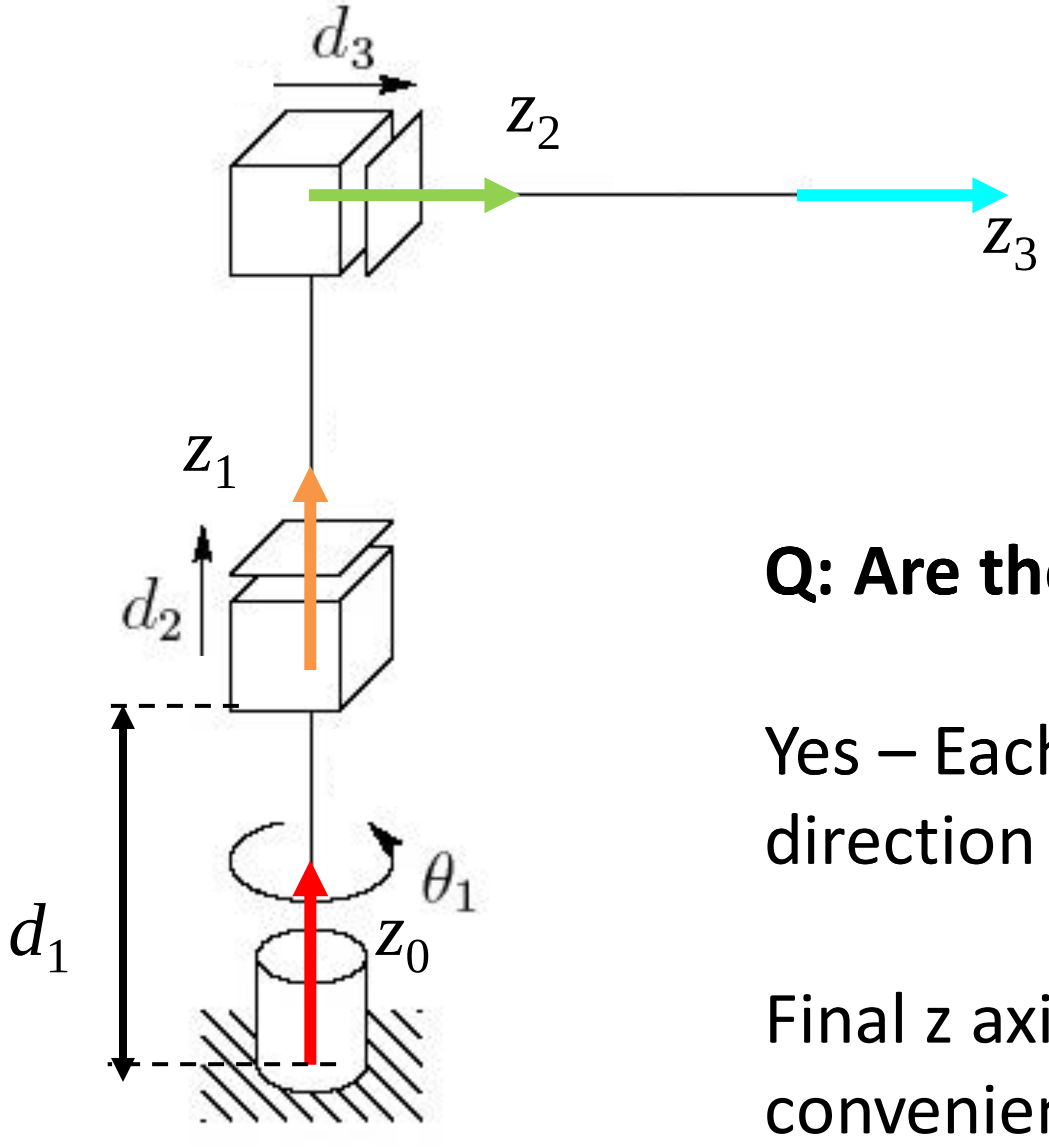
Example 2



cylindrical

Q: Are these z-axis locations correct?

Example 2



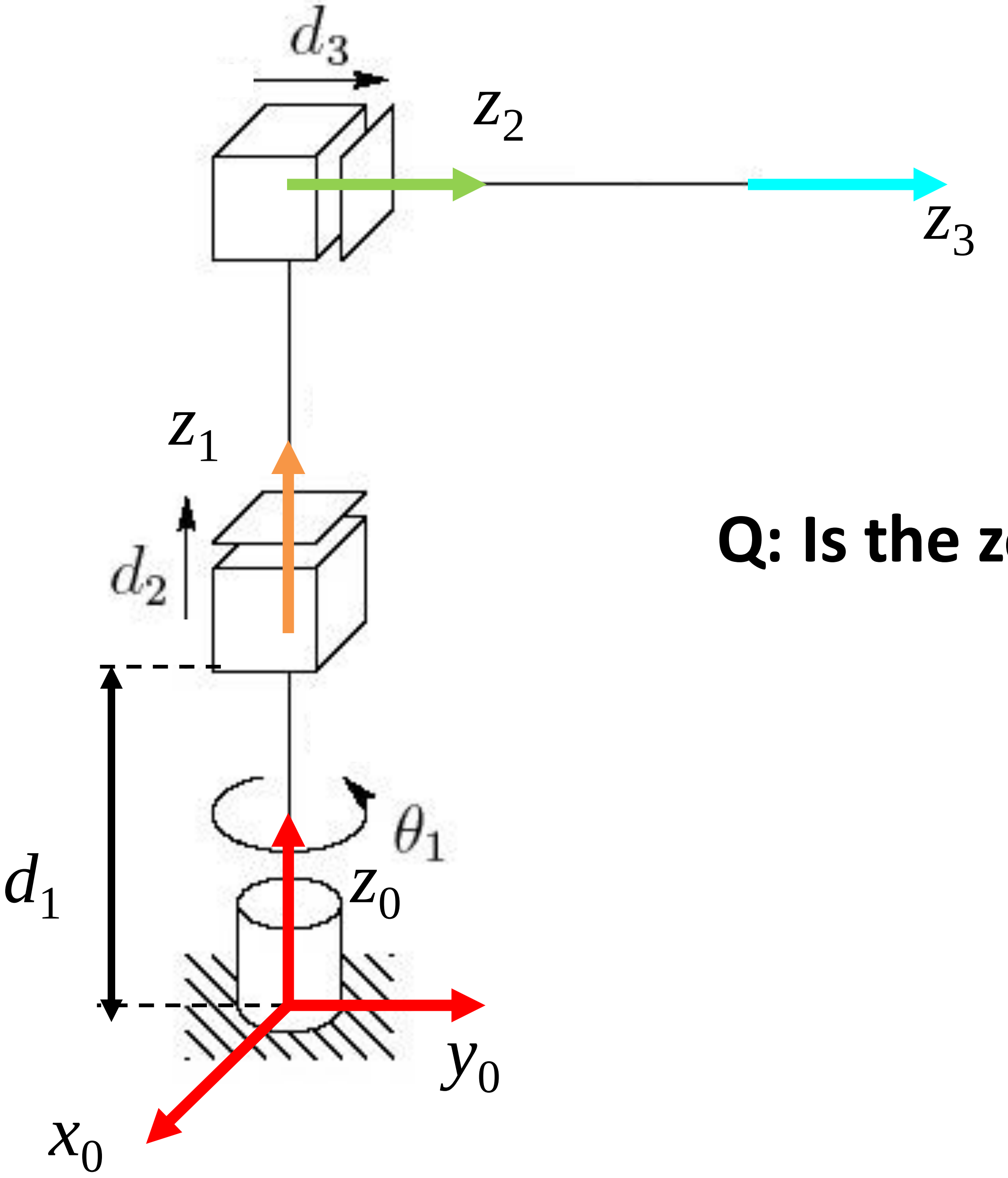
cylindrical

Q: Are these z-axis locations correct?

Yes – Each z axis defines the positive axial direction for the following joint variable.

Final z axis is parallel to the previous one for convenience. Could have chosen other.

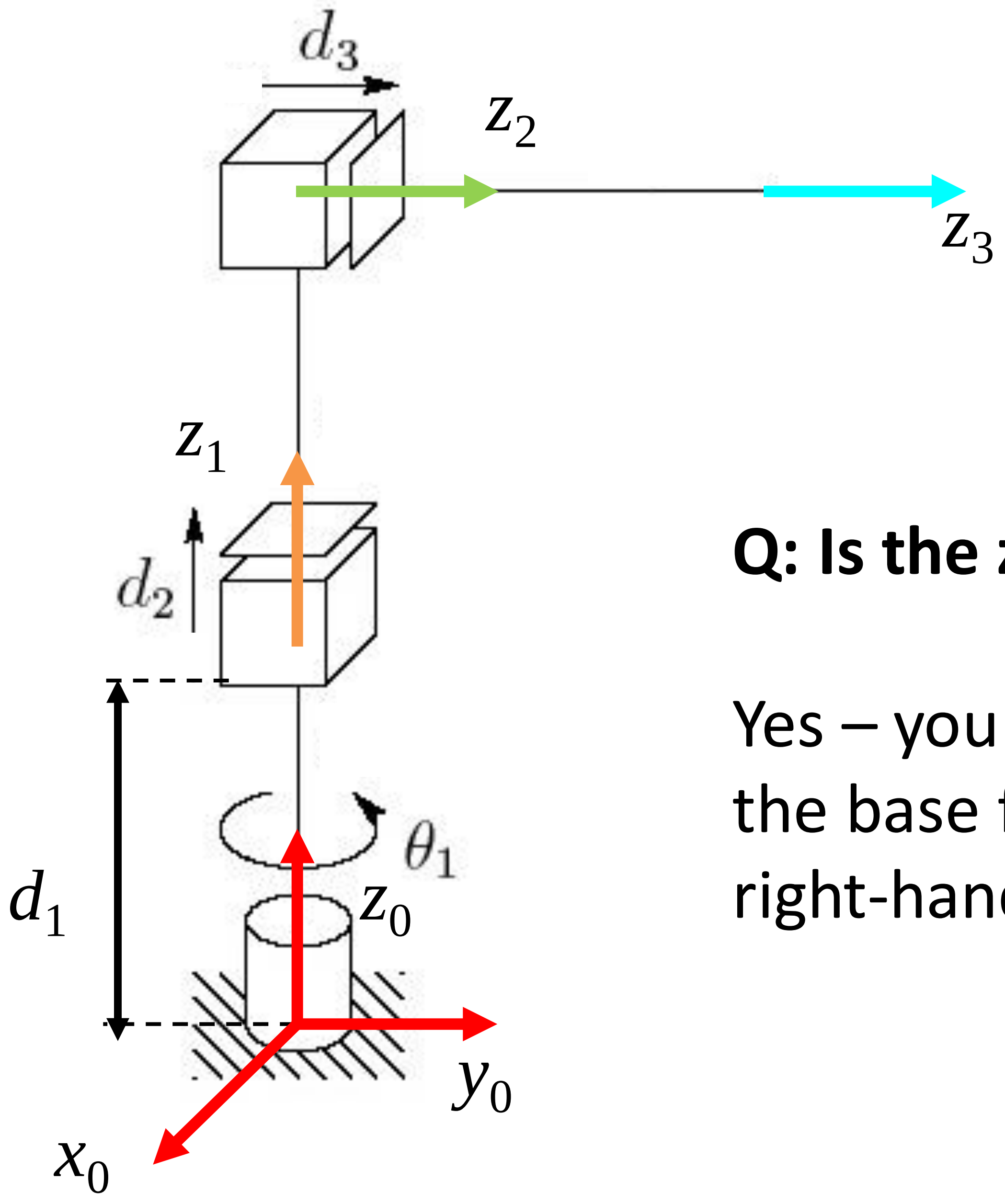
Example 2



Q: Is the zero frame correct?

cylindrical

Example 2

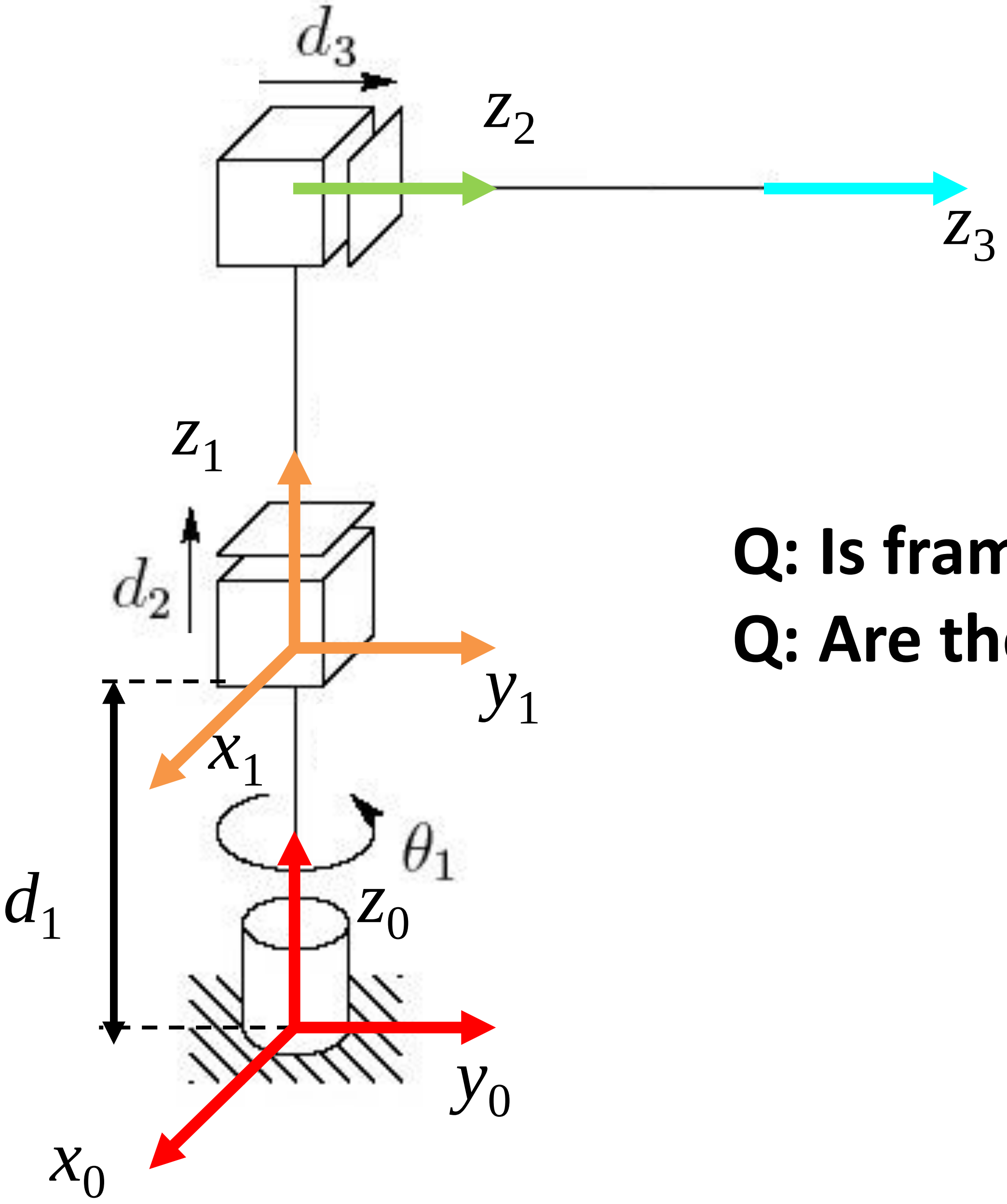


Q: Is the zero frame correct?

Yes – you can pick any x and y axes for the base frame as long as the frame is right-handed.

cylindrical

Example 2

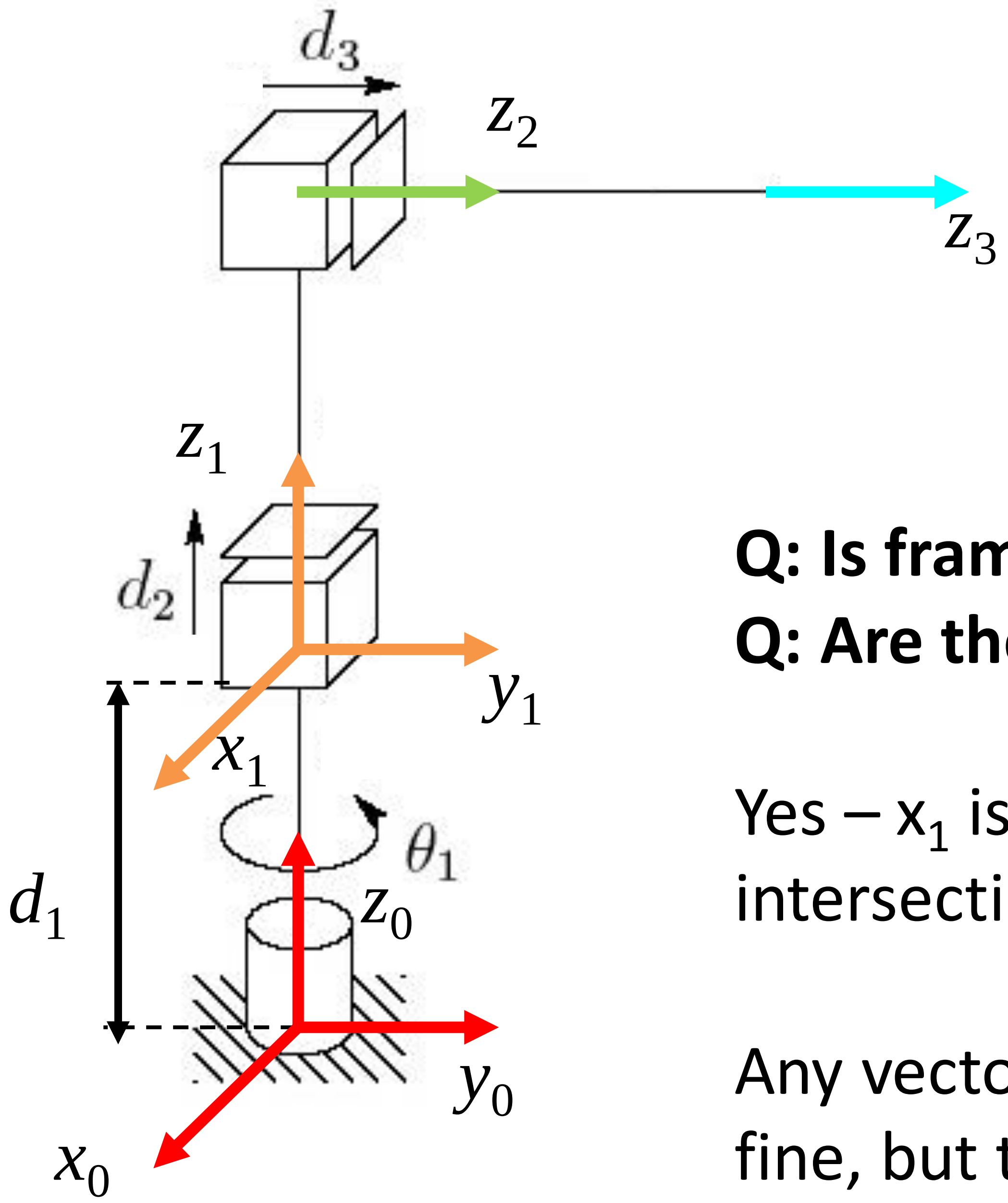


Q: Is frame 1 correct?

Q: Are there other options?

cylindrical

Example 2



cylindrical

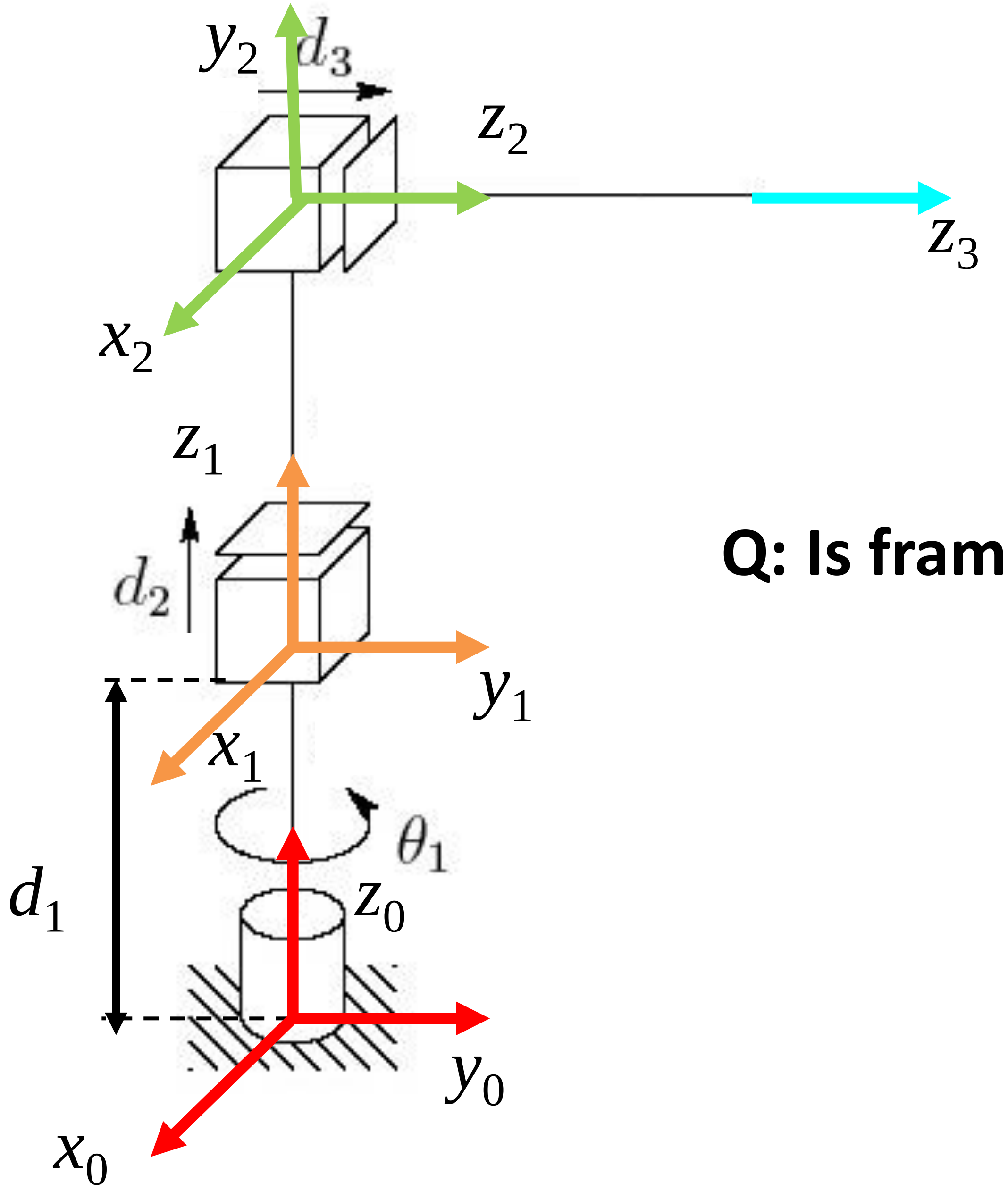
Q: Is frame 1 correct?

Q: Are there other options?

Yes – x_1 is perpendicular to and intersecting both z_0 and z_1.

Any vector perpendicular to z_1 would work fine, but the parameters would differ.

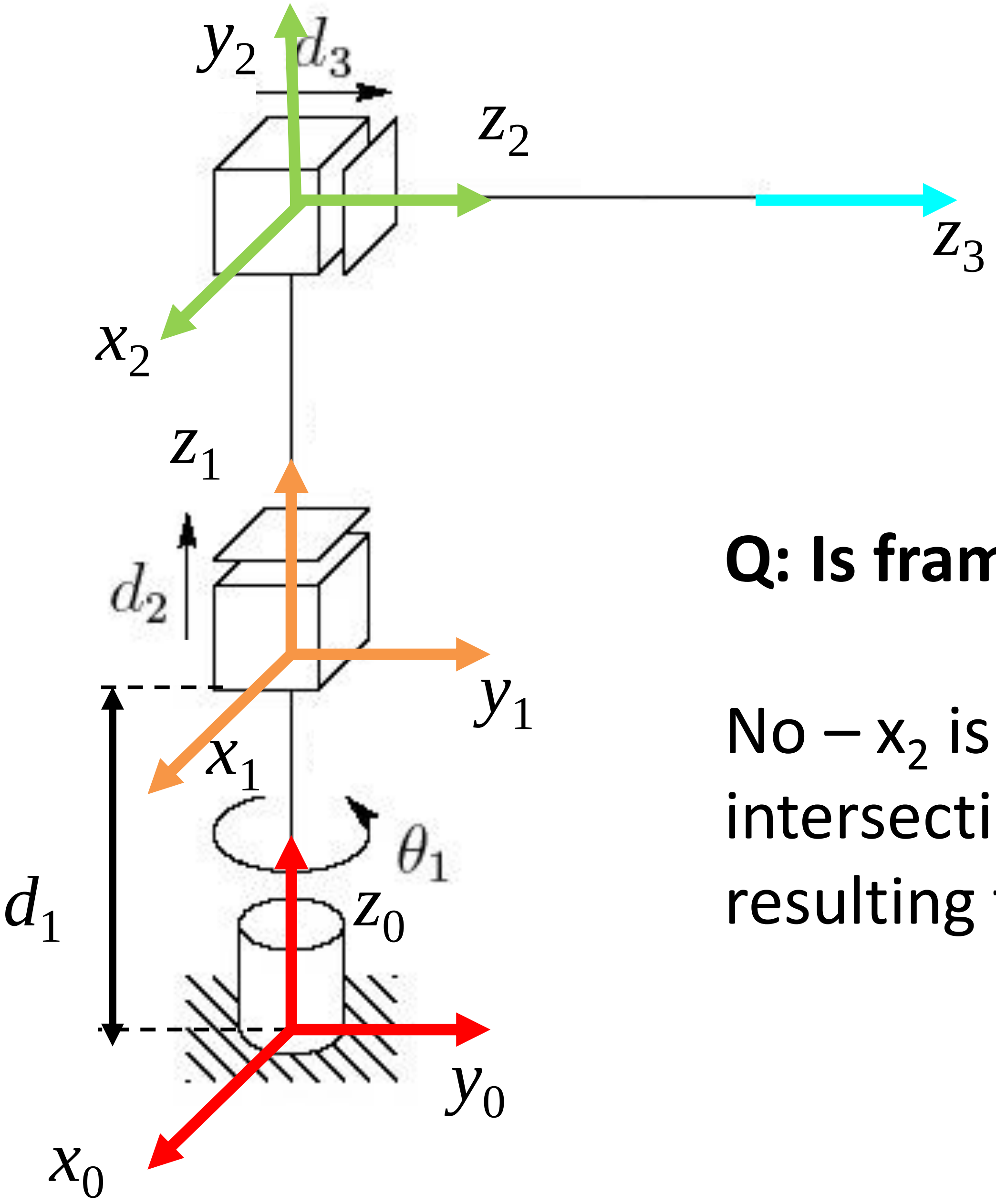
Example 2



Q: Is frame 2 correct?

cylindrical

Example 2

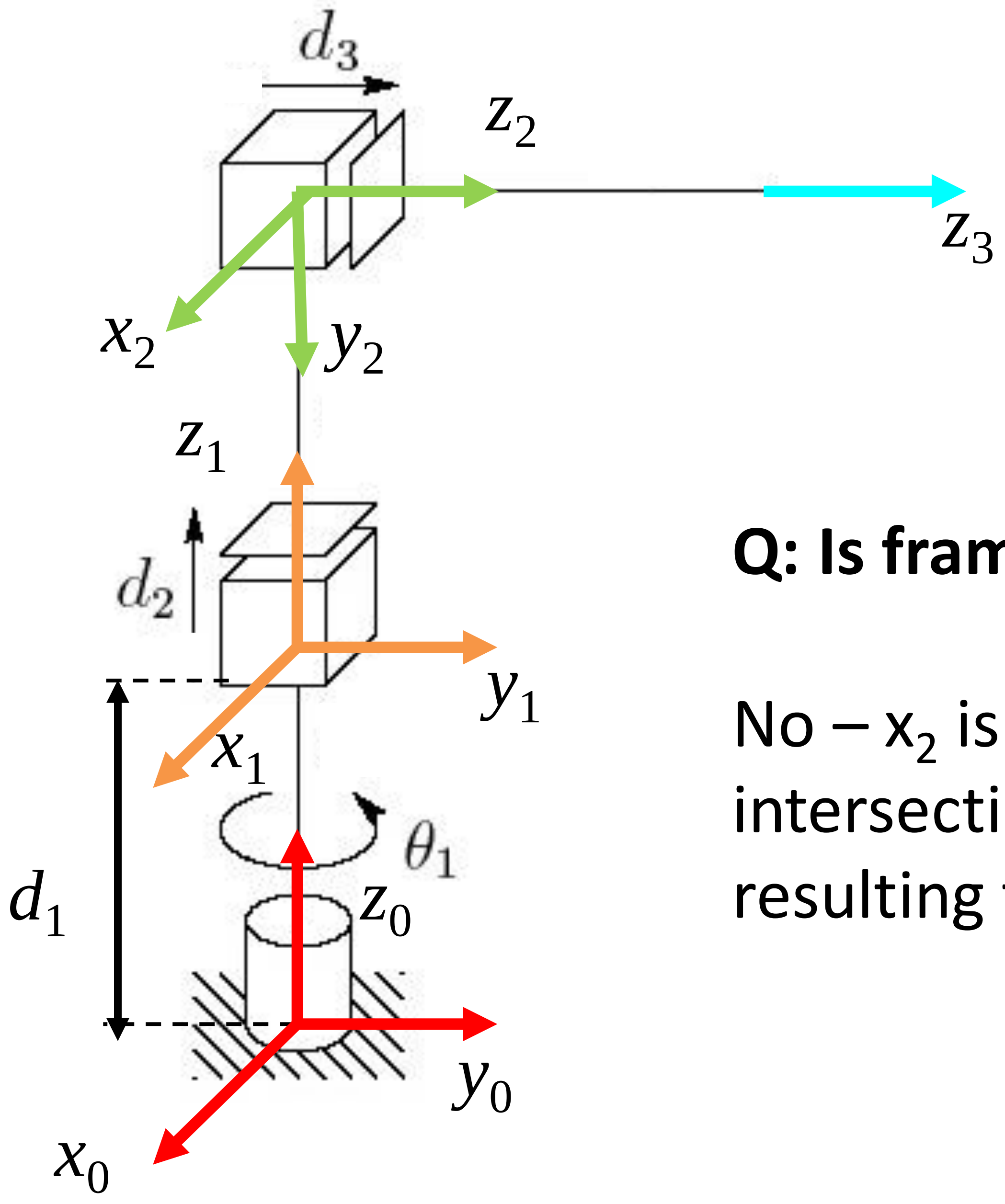


cylindrical

Q: Is frame 2 correct?

No – x_2 is perpendicular to and intersecting both z_1 and z_2, but the resulting frame is left-handed.

Example 2



cylindrical

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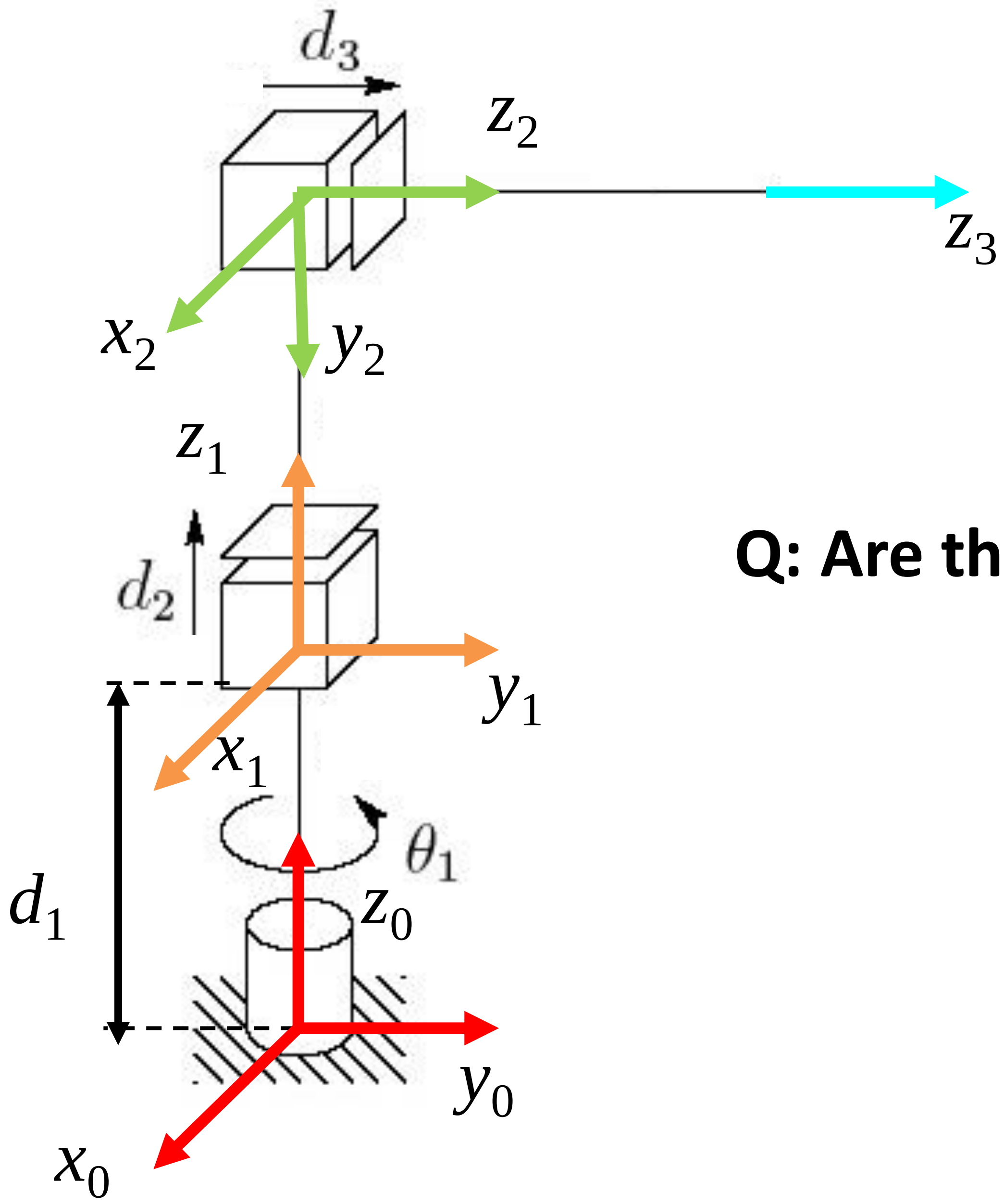
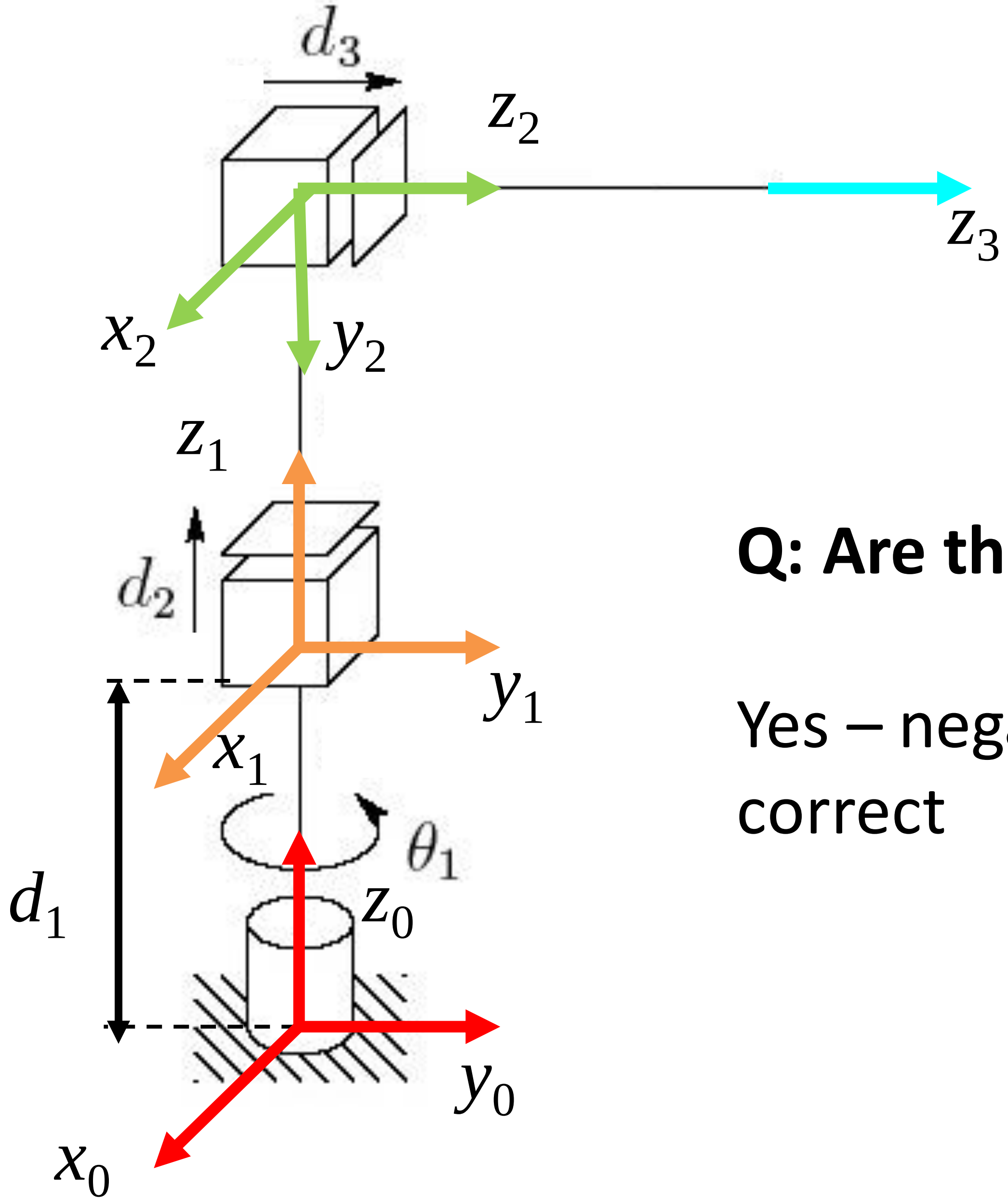
No – x_2 is perpendicular to and intersecting both z_1 and z_2, but the resulting frame is left-handed.

Example 2



Q: Are there other options?

cylindrical



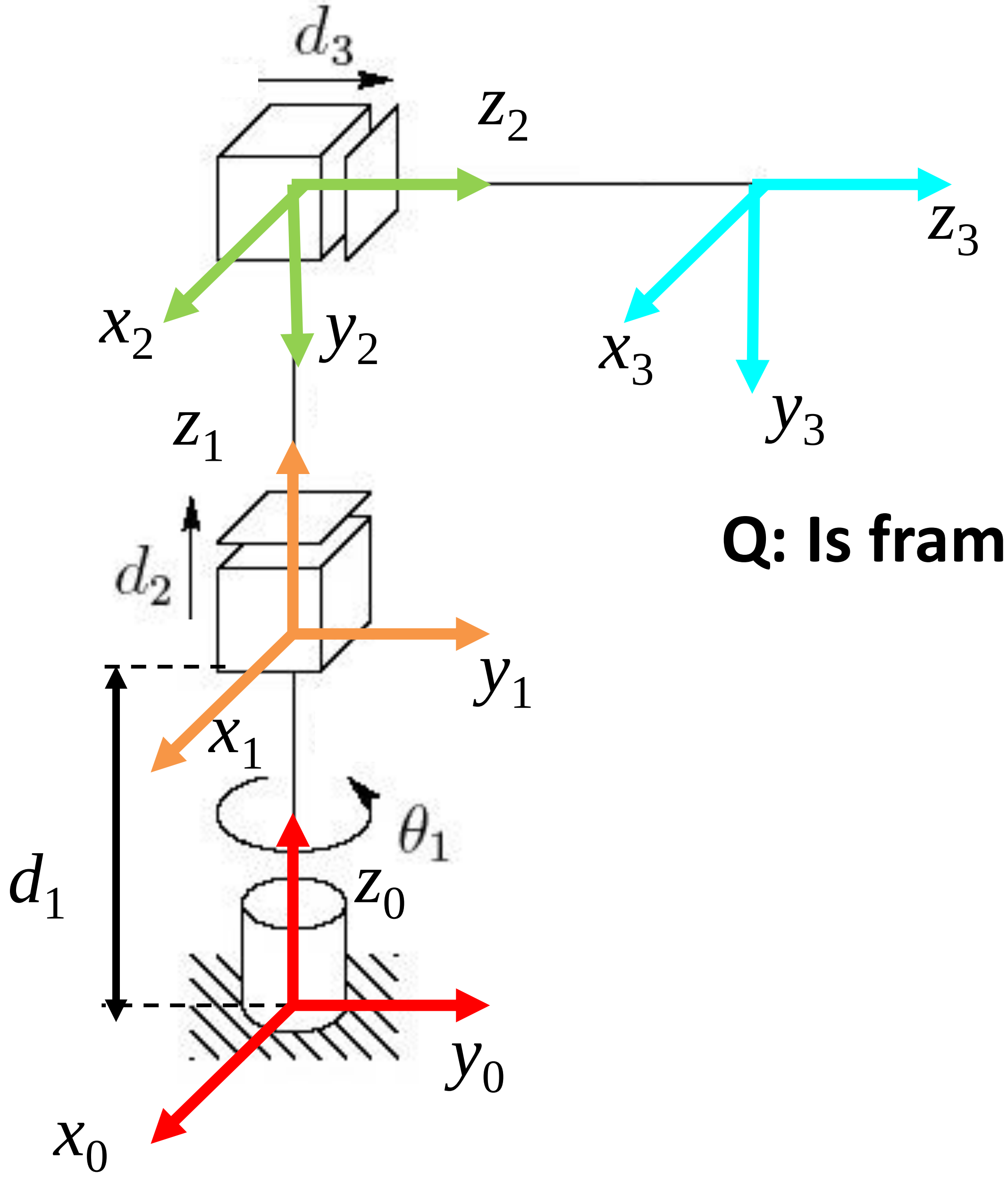
Example 2

cylindrical

Q: Are there other options?

Yes – negative of what is shown is also correct

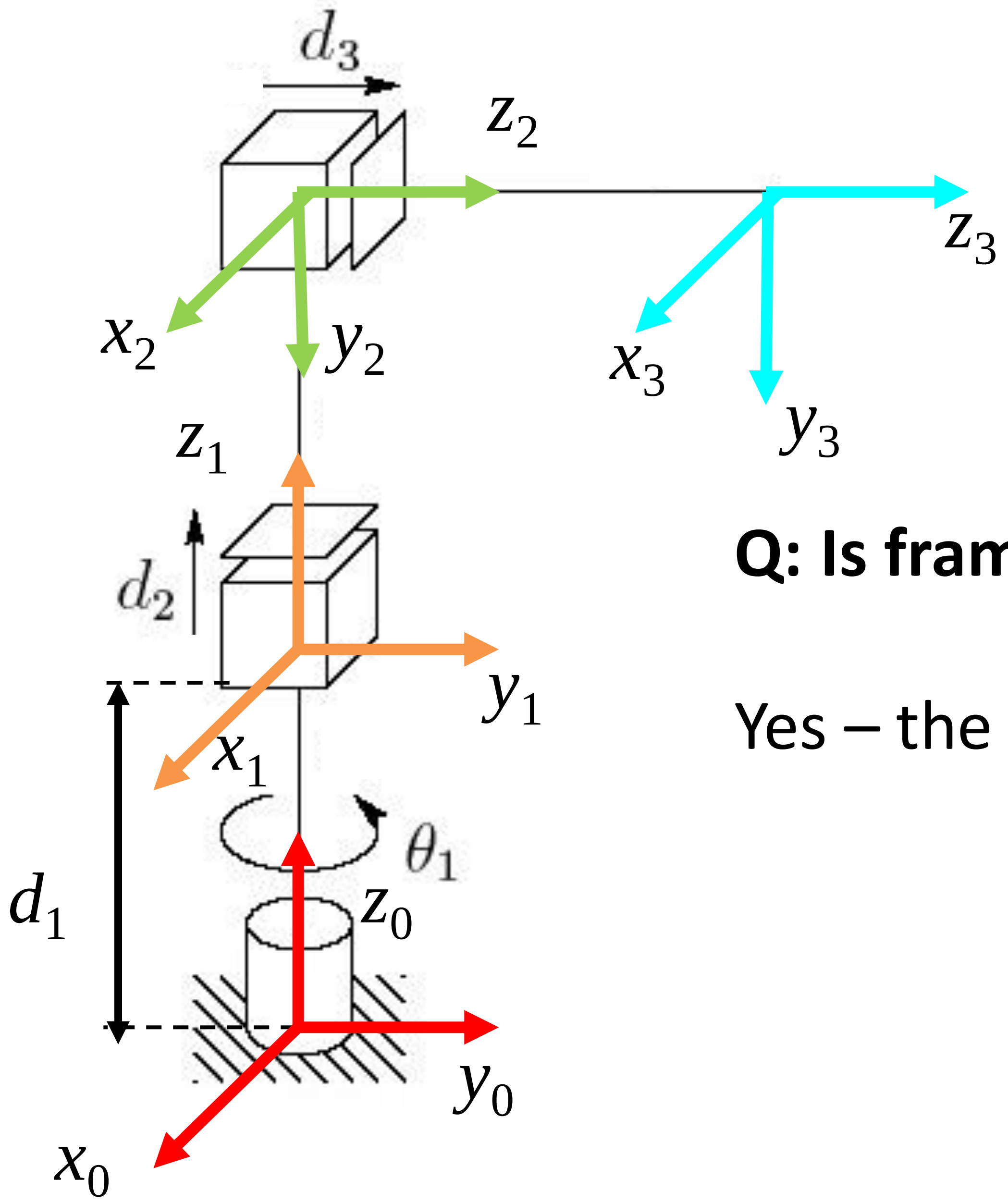
Example 2



Q: Is frame 3 correct?

cylindrical

Example 2

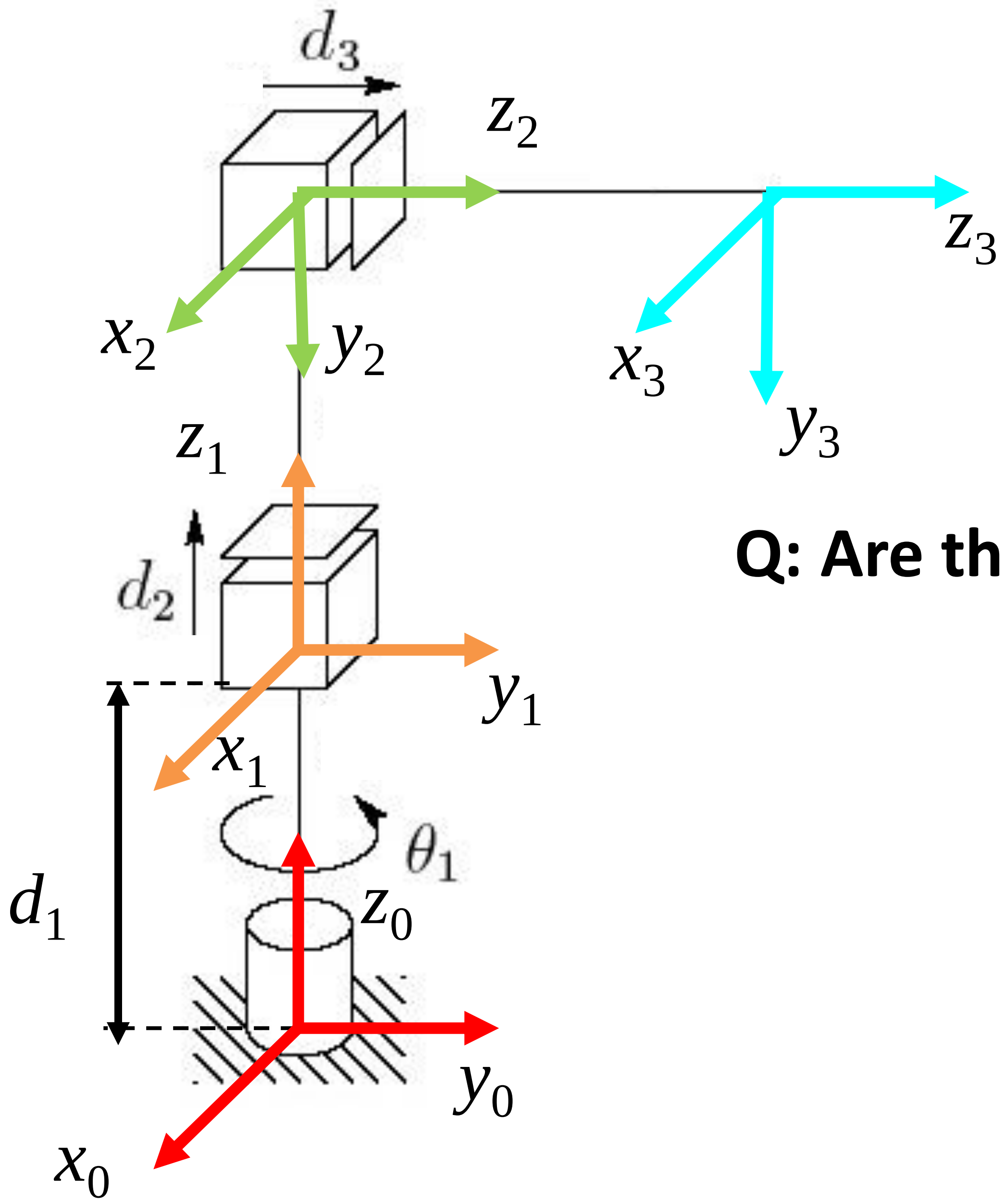


cylindrical

Q: Is frame 3 correct?

Yes – the frame is right-handed

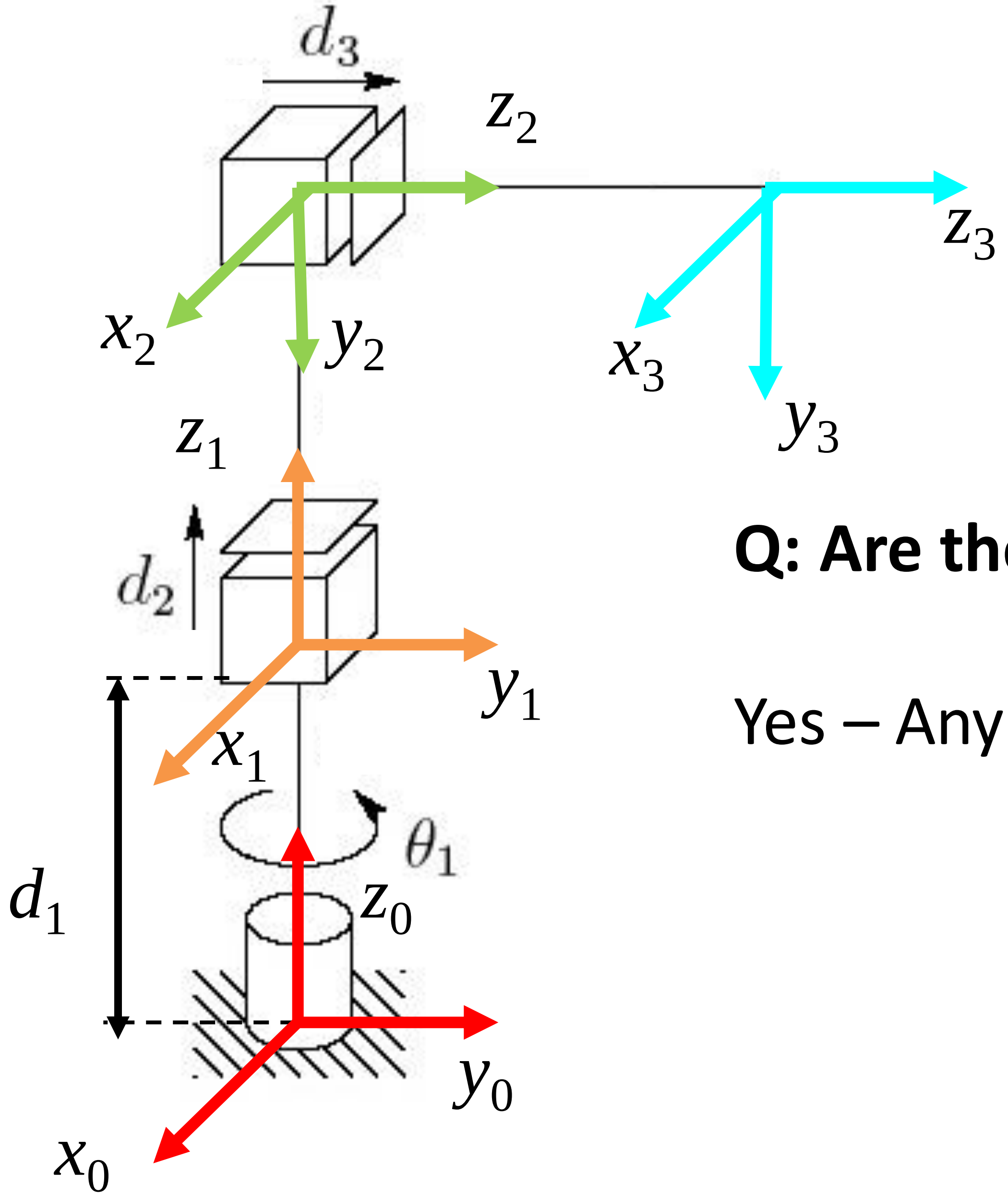
Example 2



Q: Are there other options?

cylindrical

Example 2



cylindrical

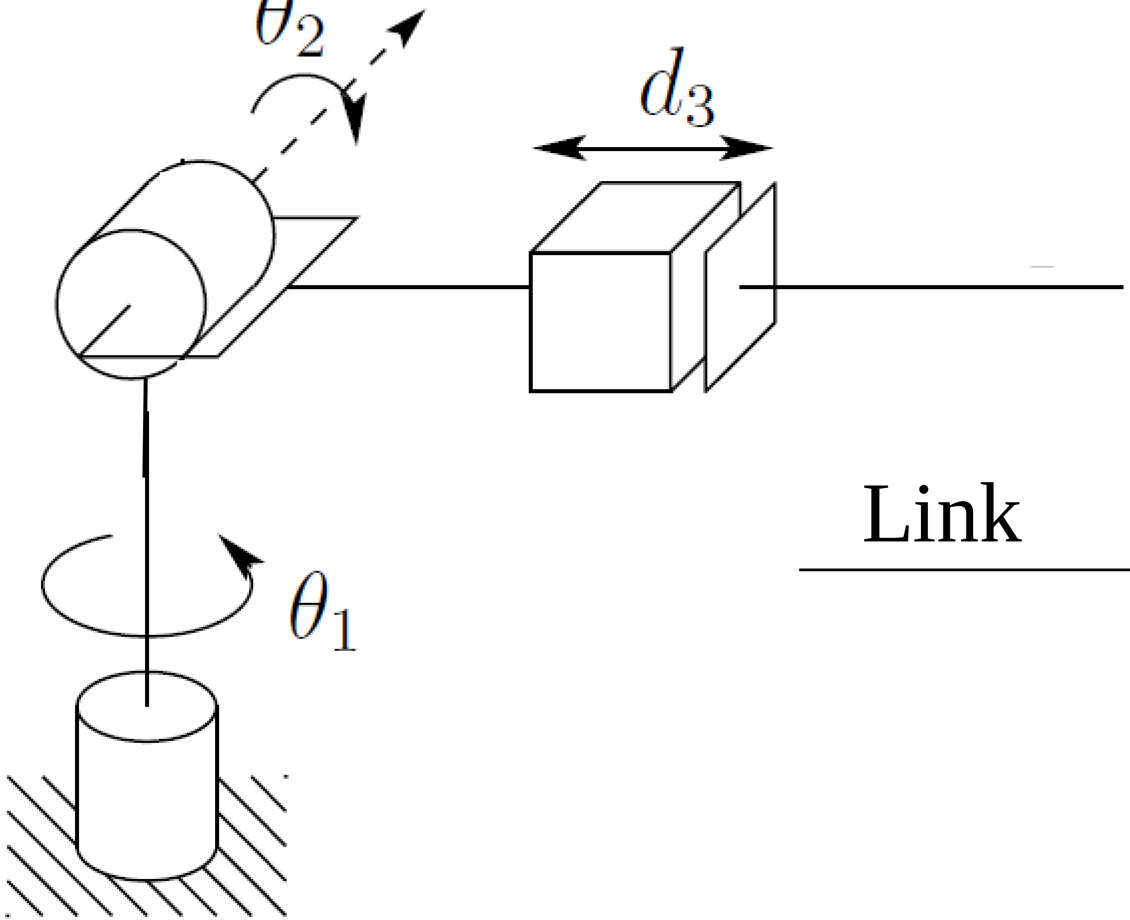
Q: Are there other options?

Yes – Any other right-handed option is fine.

Example 3

Your coordinate frame may not lie on the joint?!?!

spherical



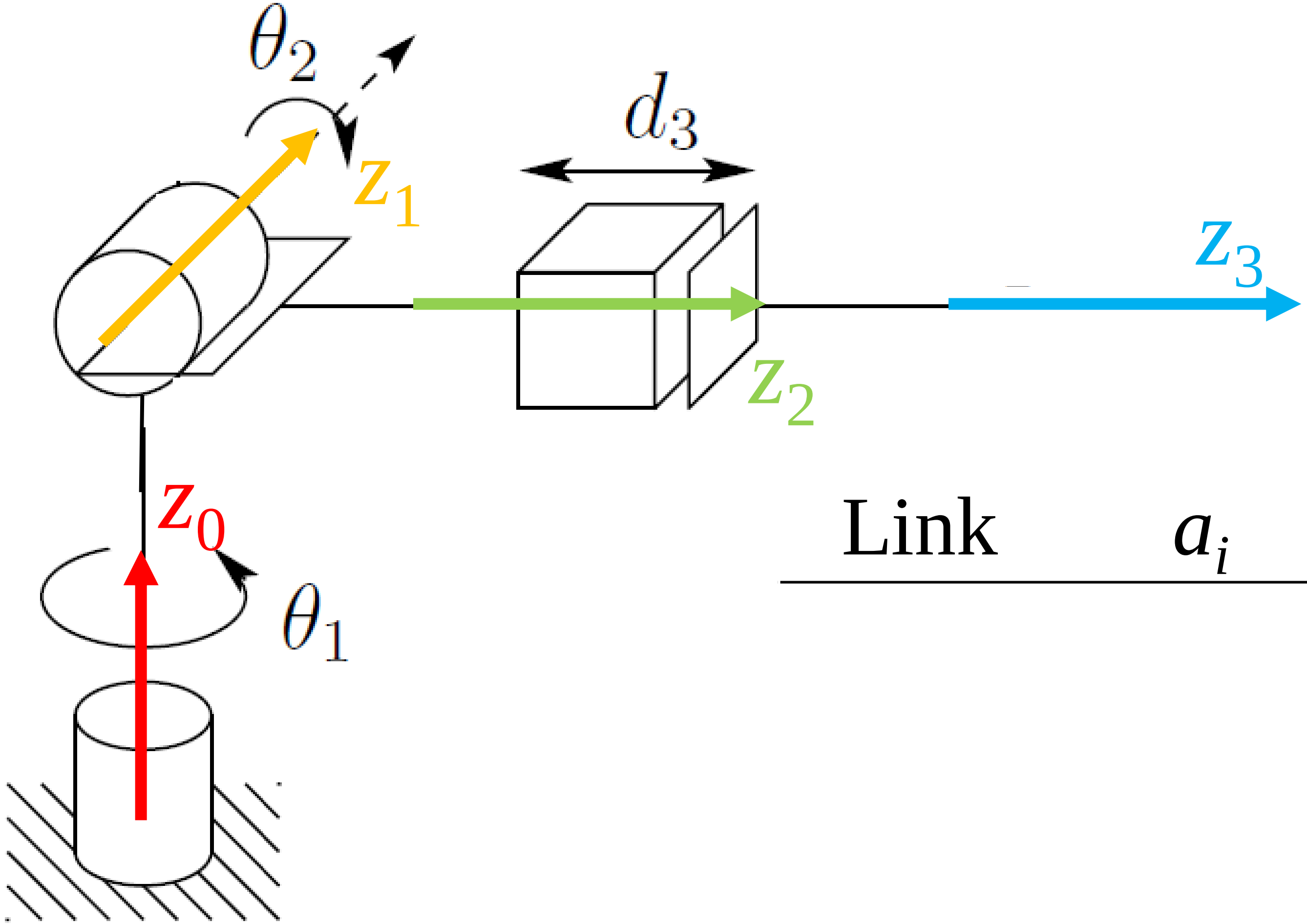
Link	a_i	α_i	d_i	θ_i

Example 3

Your coordinate frame may not lie on the joint?!?!

spherical

Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.



Link	a_i	α_i	d_i	θ_i

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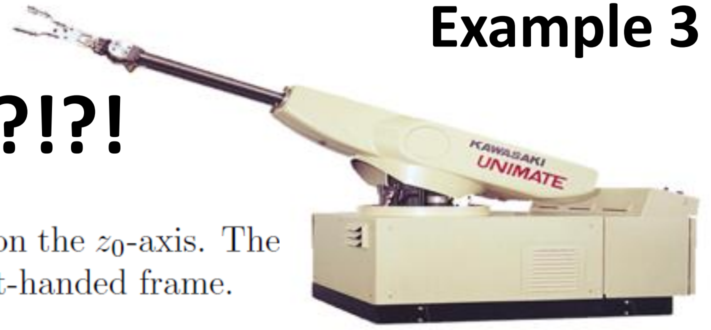
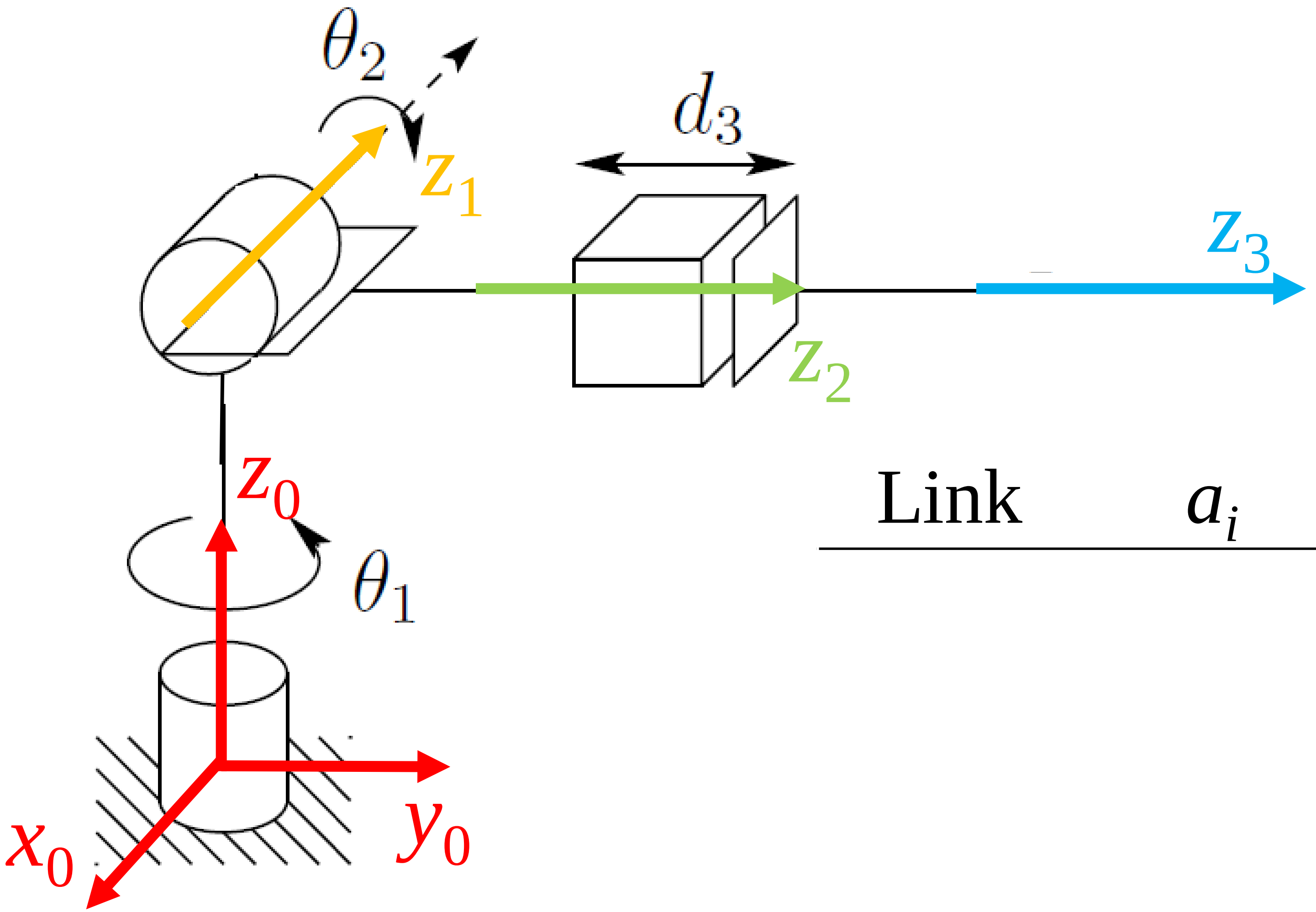


spherical

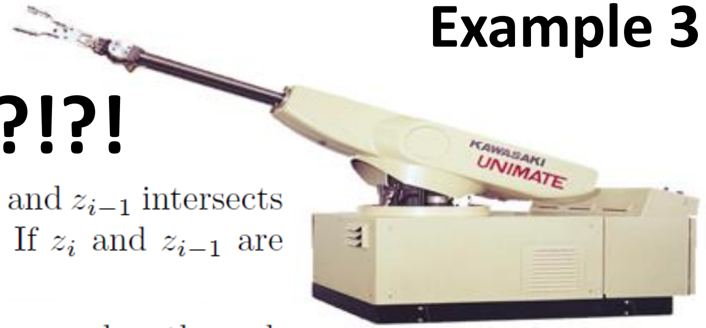
Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.



Link	a_i	α_i	d_i	θ_i

Example 3

Your coordinate frame may not lie on the joint?!?!



spherical

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

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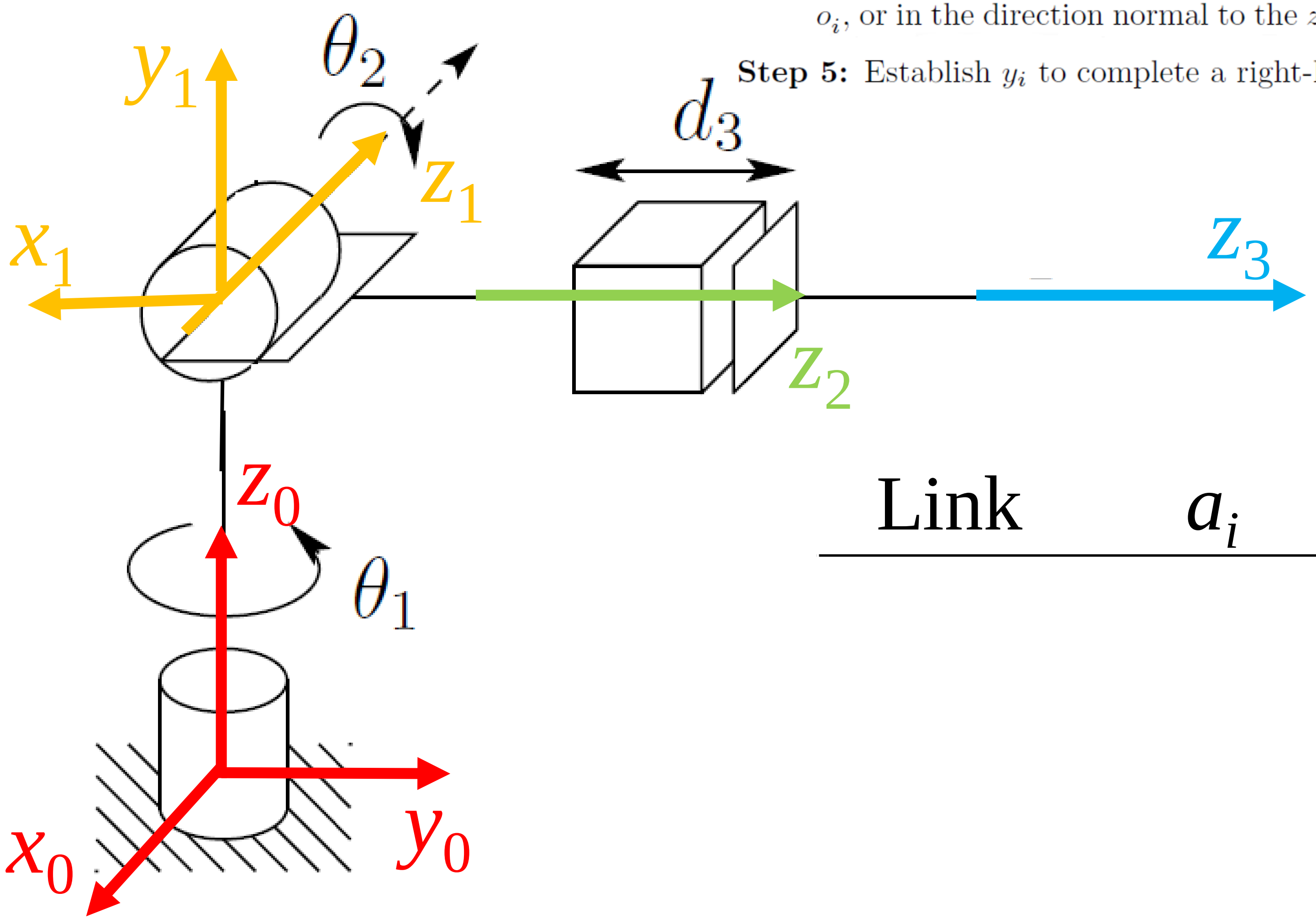
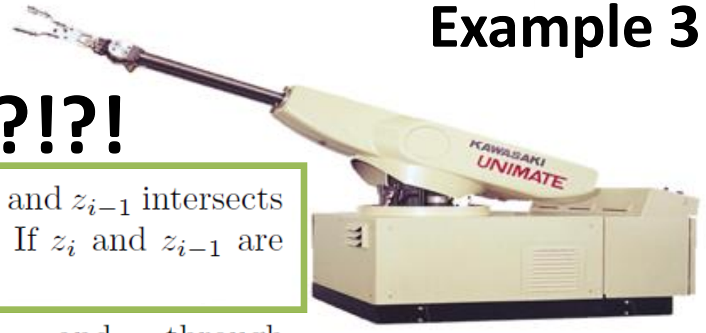
Step 5: Establish y_i to complete a right-handed frame.



Link	a_i	α_i	d_i	θ_i

Example 3



spherical

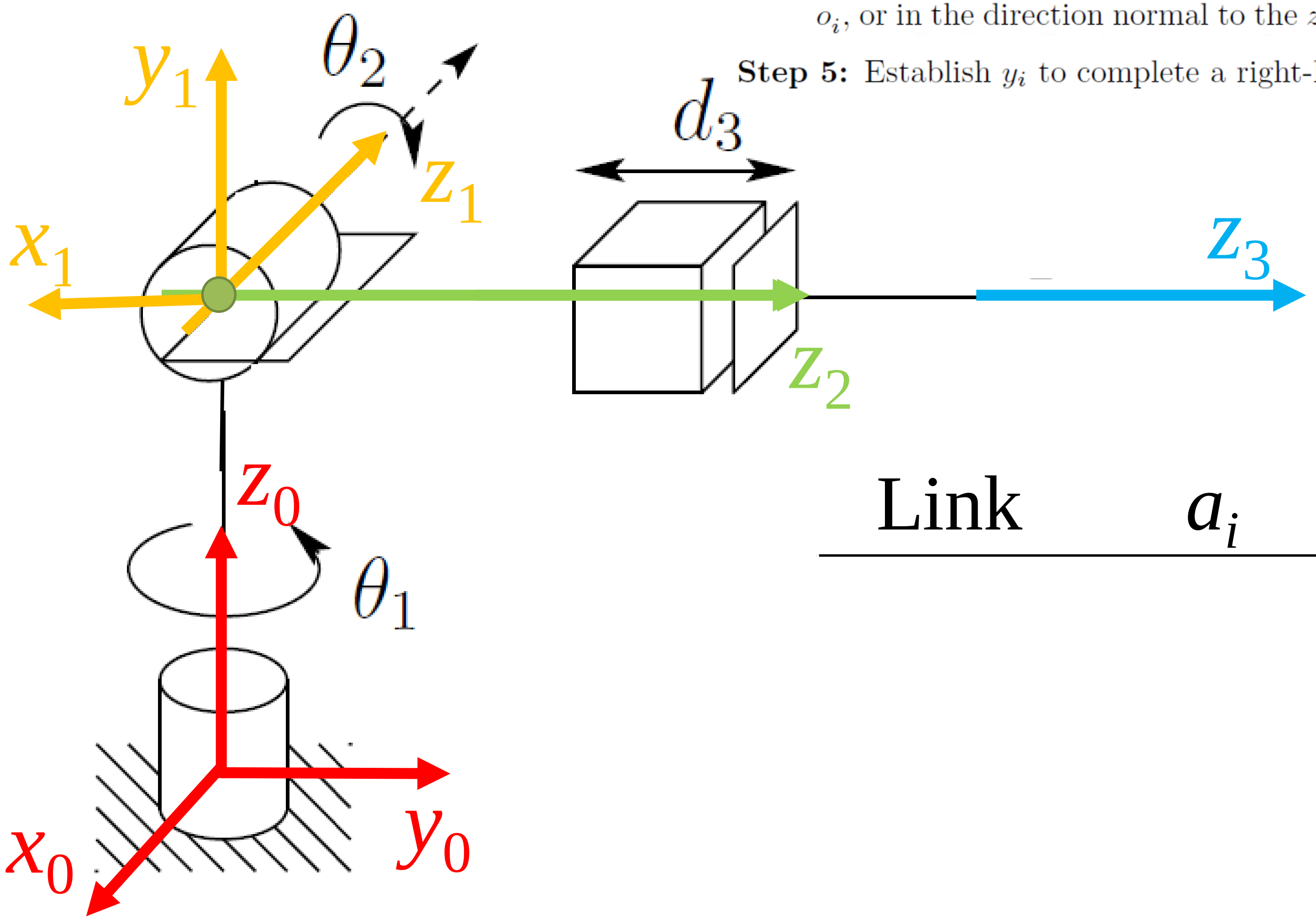
Your coordinate frame may not lie on the joint?!?!

o_2 is coincident with o_1!

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.



Link	a_i	α_i	d_i	θ_i

Example 3

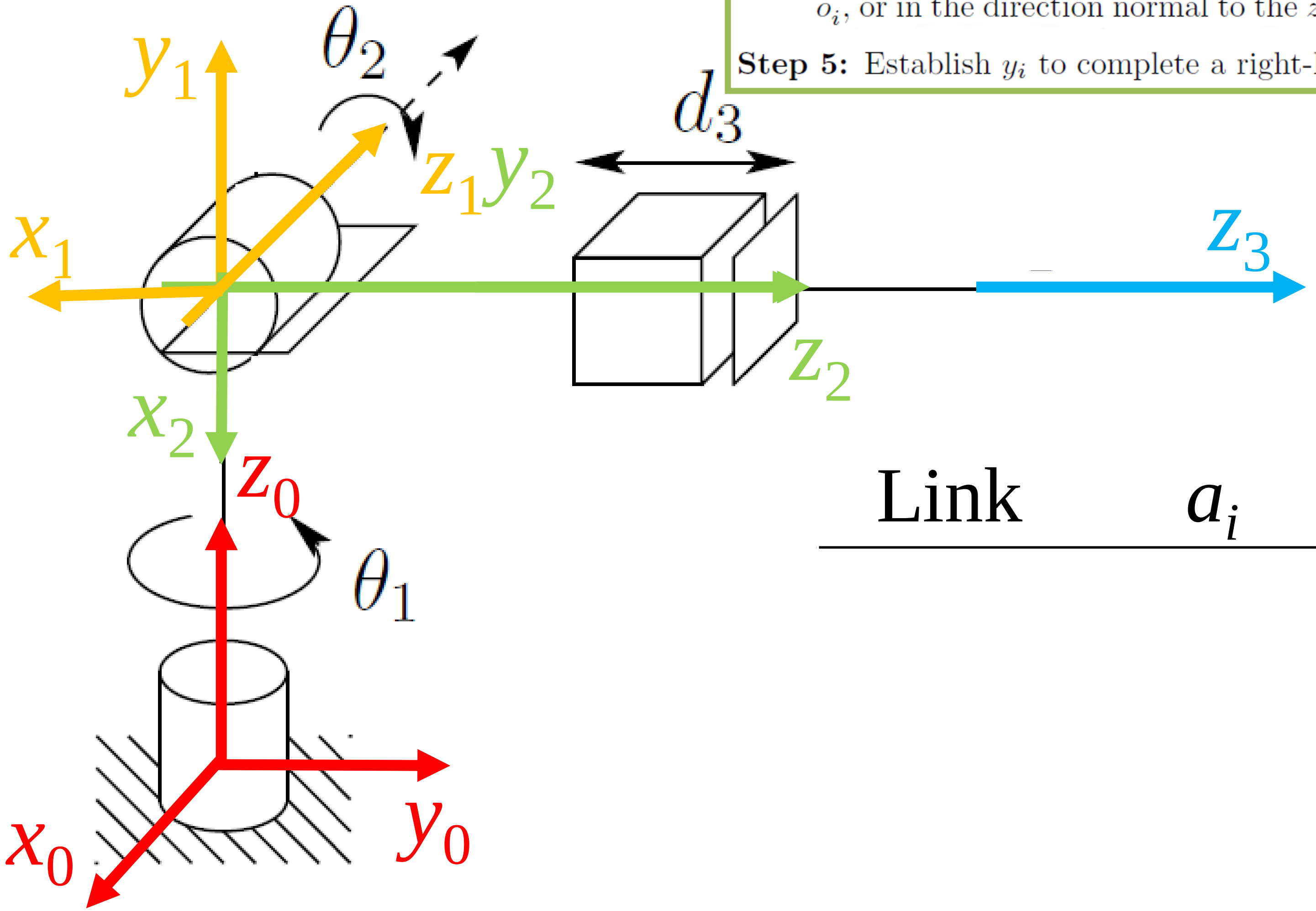
Your coordinate frame may not lie on the joint?!?!

spherical

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.



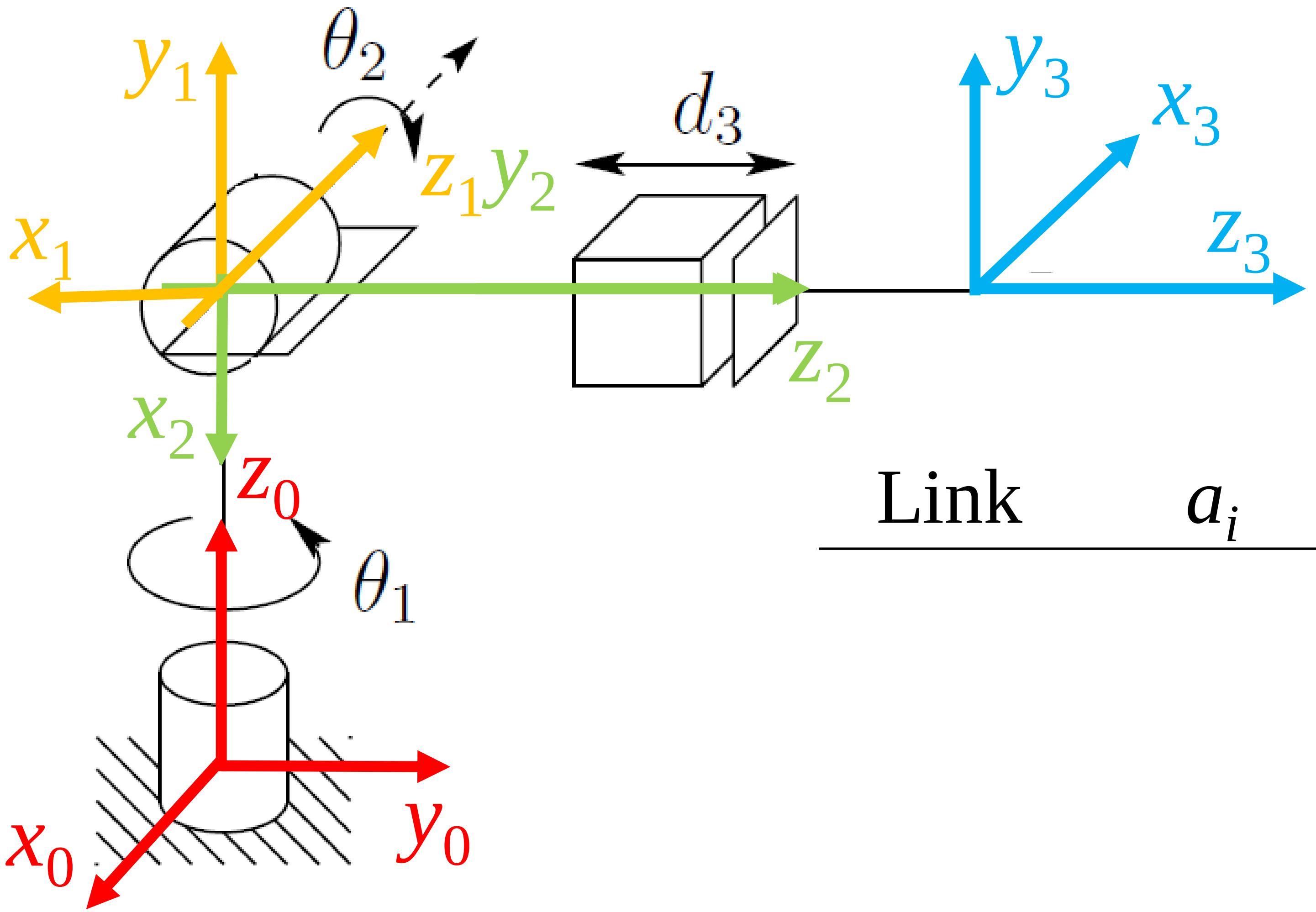
Link	a_i	α_i	d_i	θ_i

Example 3

Your coordinate frame may not lie on the joint?!?!

spherical

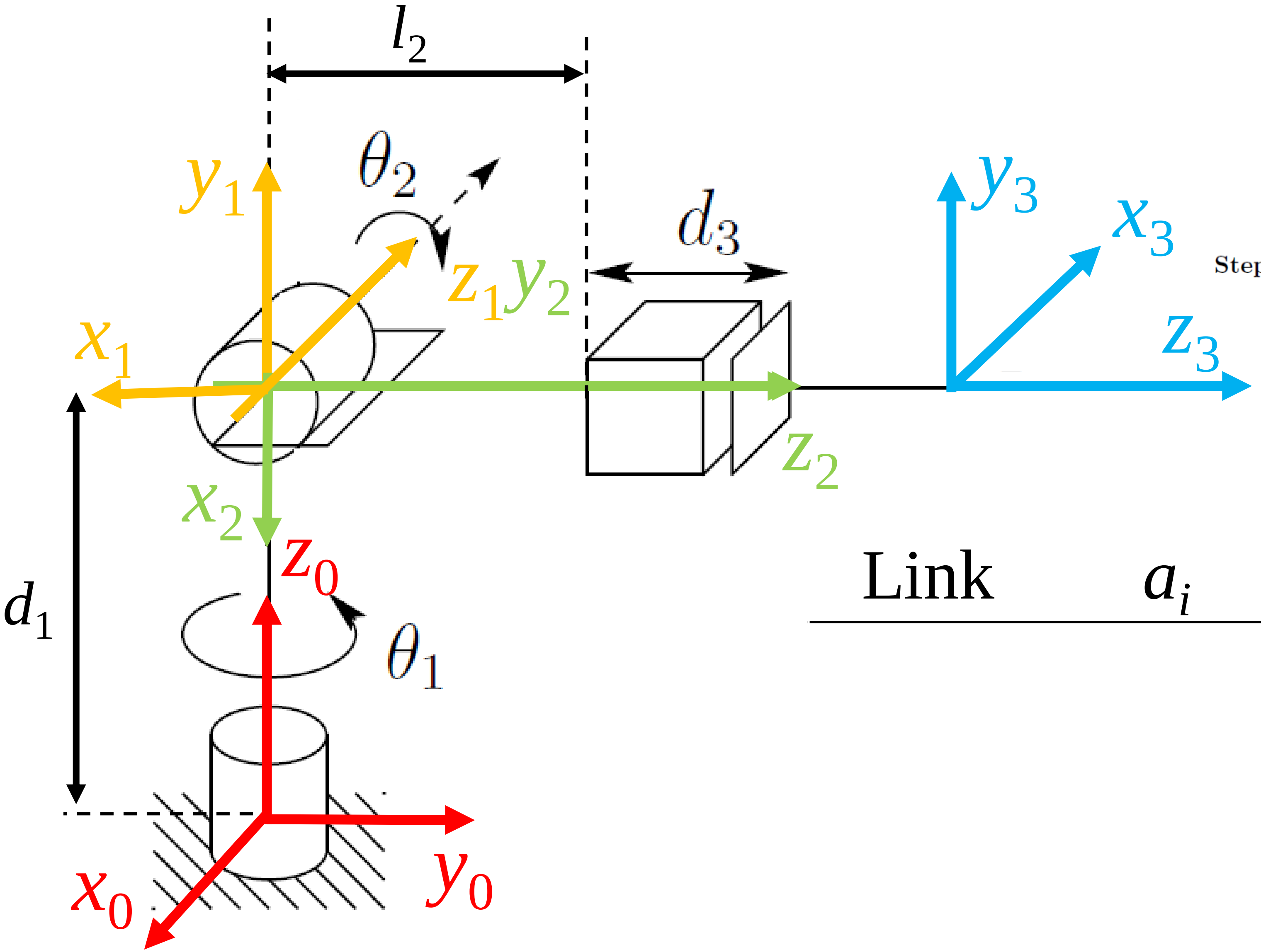
Step 6: Establish the end-effector frame $o_n x_n y_n z_n$.



Link	a_i	α_i	d_i	θ_i

Your coordinate frame may not lie on the joint?!?!

spherical



Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

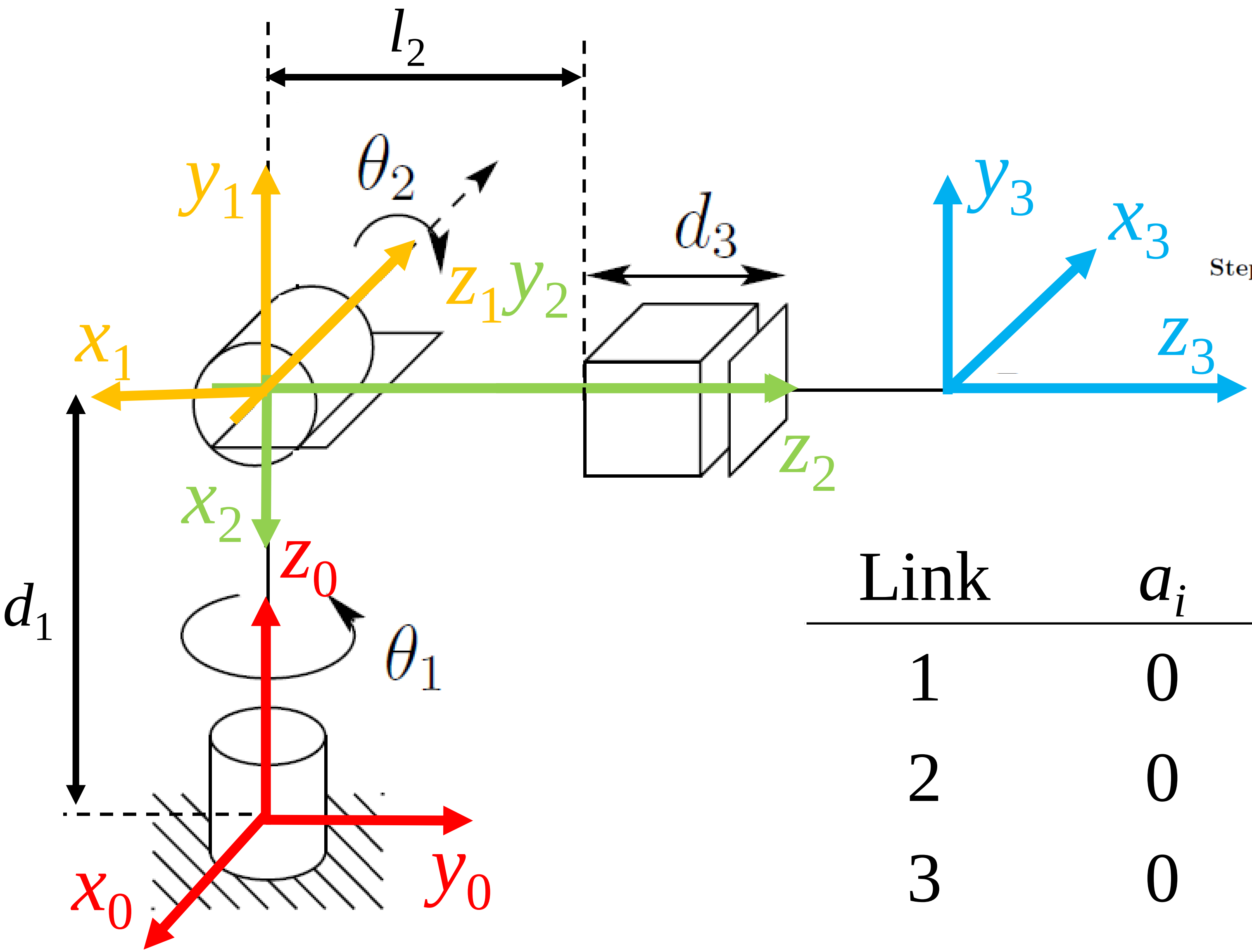
α_i = the angle between z_{i-1} and z_i measured about x_i.

θ_i = the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

Link	a_i	α_i	d_i	θ_i

Example 3

Your coordinate frame may not lie on the joint?!?!



spherical

Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

α_i = the angle between z_{i-1} and z_i measured about x_i.

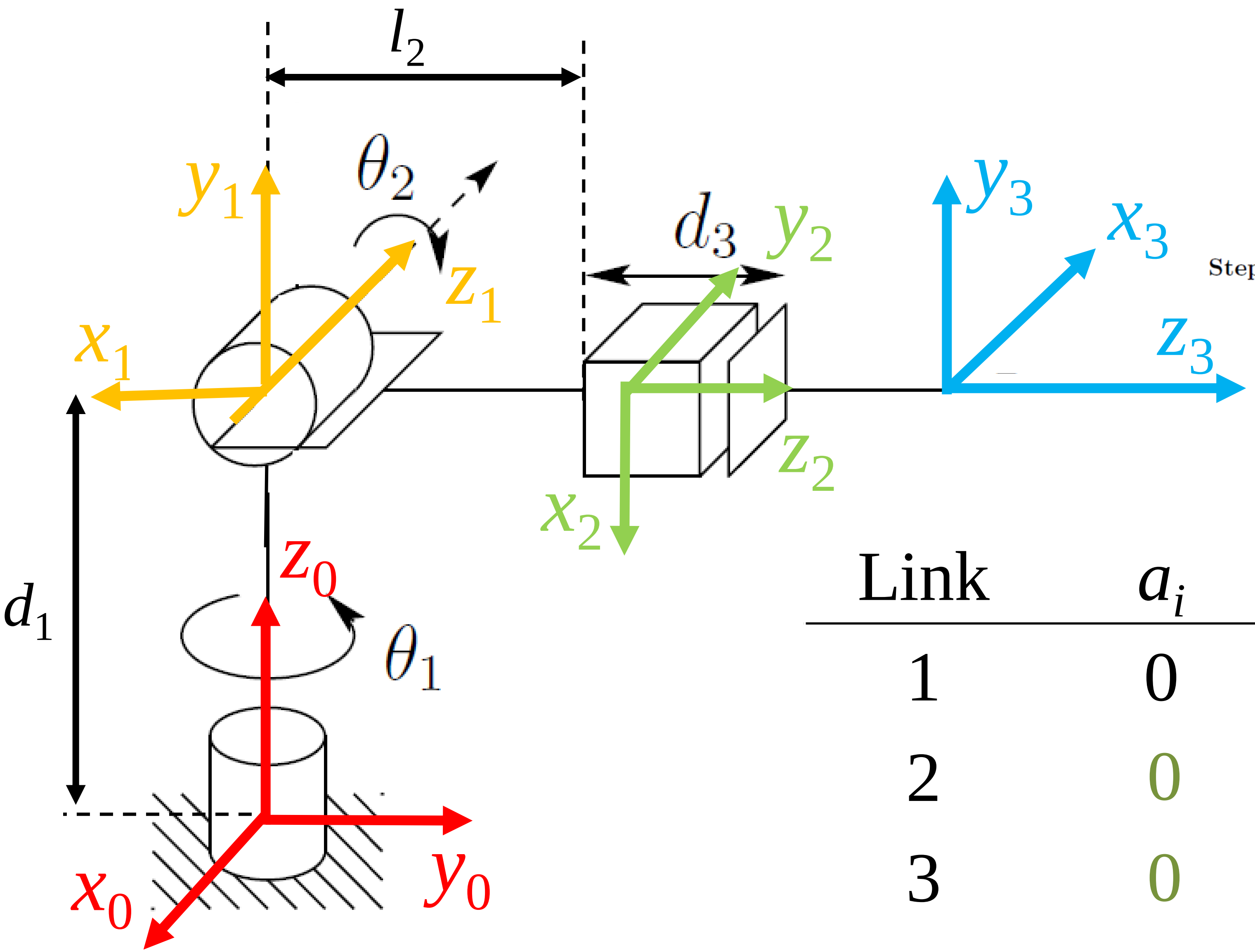
θ_i = the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	$\theta_1^*-\pi/2$
2	0	$\pi/2$	0	$\theta_2^*-\pi/2$
3	0	0	$\boxed{d_3^*+l_2}$	$\pi/2$

l_2 shows up in link 3, not link 2!!

Example 3

What goes wrong if we force frames onto joints?



Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

$a_i =$ distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

$d_i =$ distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

$\alpha_i =$ the angle between z_{i-1} and z_i measured about x_i.

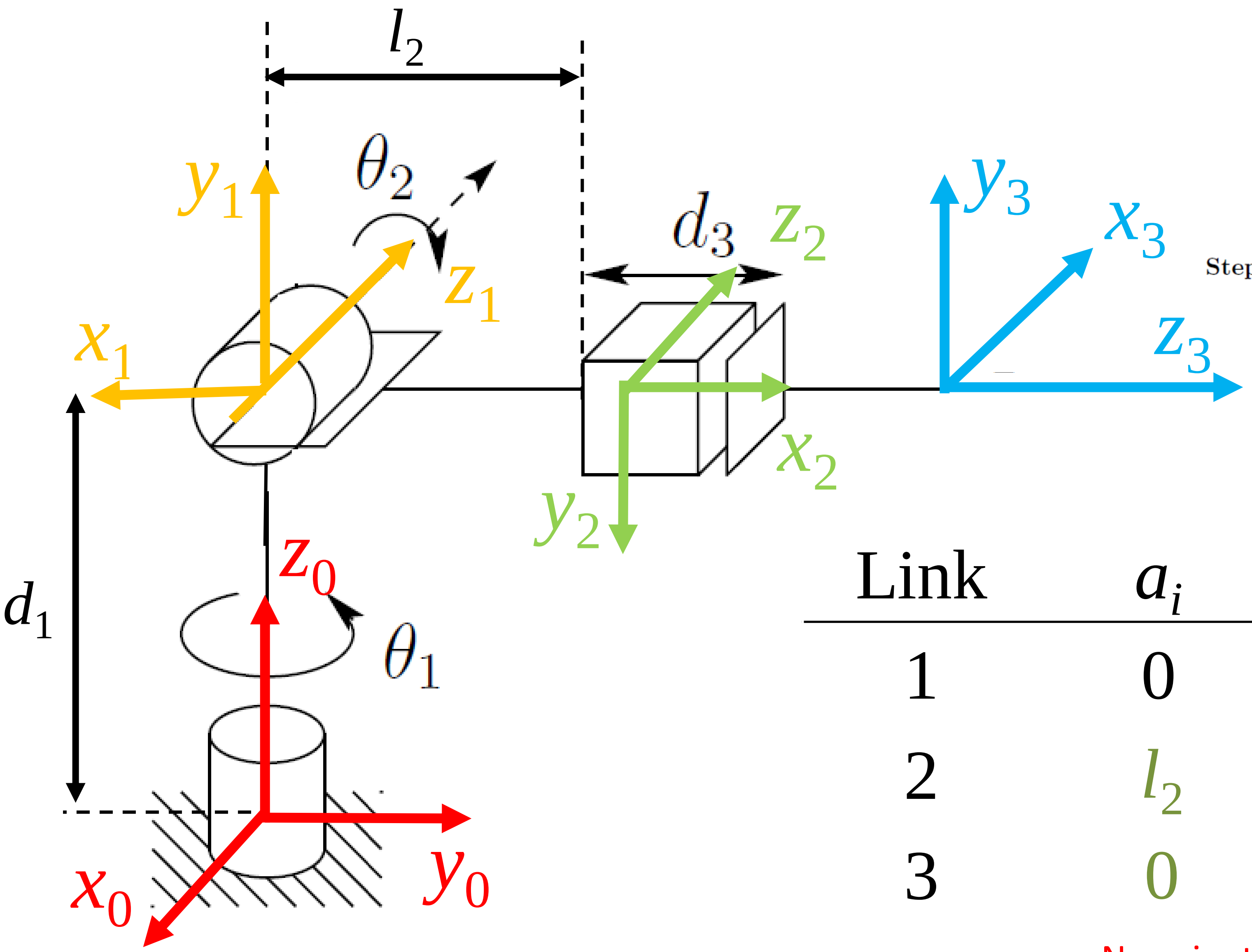
$\theta_i =$ the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	$\theta_1^* - \pi/2$
2	0	$\pi/2$	0	$\theta_2^* - \pi/2$
3	0	0	d_3^*	$\pi/2$

We lost l_2!!!

Example 3

What goes wrong if we change orientations?



Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

α_i = the angle between z_{i-1} and z_i measured about x_i.

θ_i = the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	$\theta_1^* - \pi/2$
2	l_2	0	0	$\theta_2^* + \pi$
3	0	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$

No orientation change! We lost d_3^*!!!

Devanit-Hartenberg convention allows you to describe transformations using **only four parameters**.

General transformations represent **6 degrees of freedom**.

In order to **ignore the extra two parameters** in a general transformation, you must follow the rules for frame assignment:

1. $z_{i\text{-}1}$ is the direction of motion for joint i
2. x_i is both perpendicular to and intersects $z_{i\text{-}1}$

Practice for you

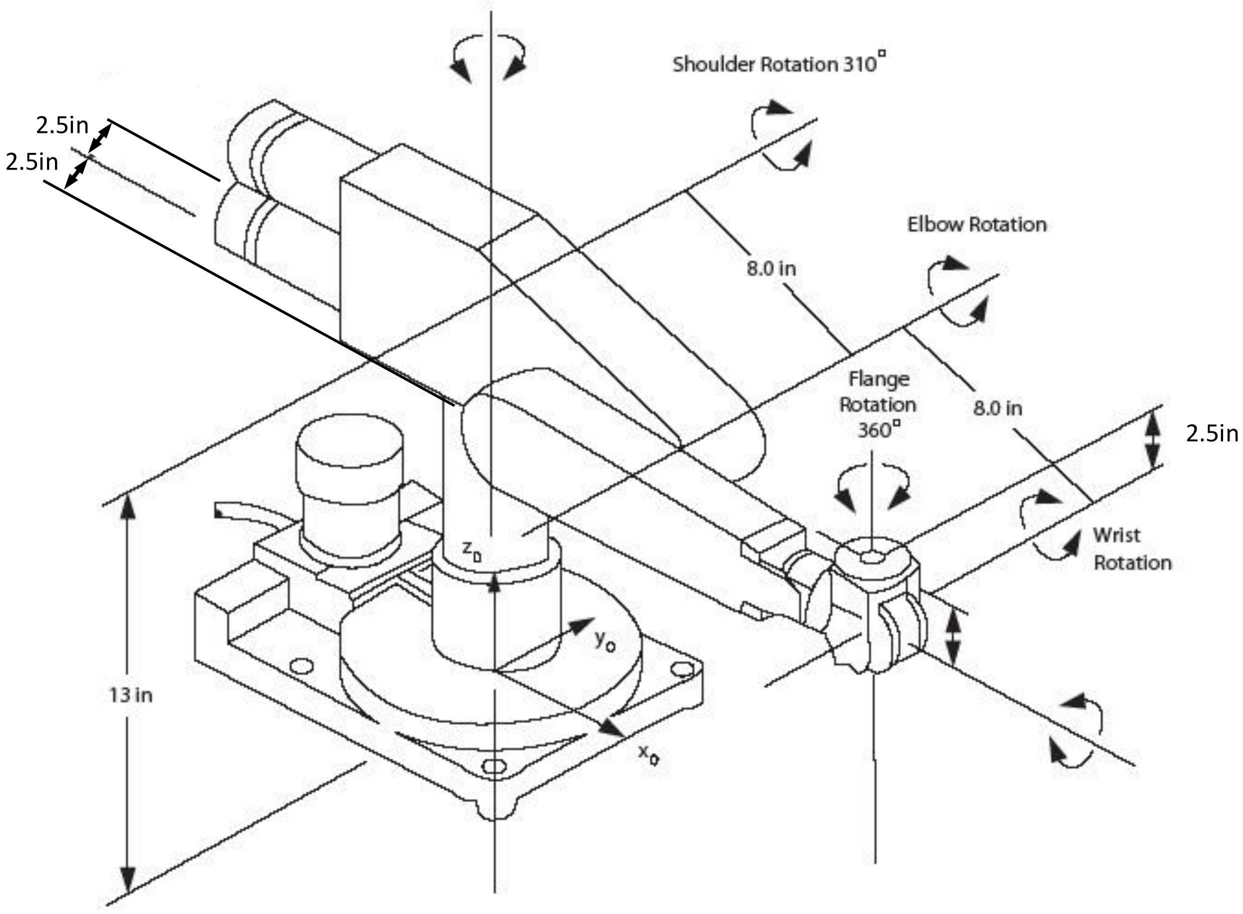
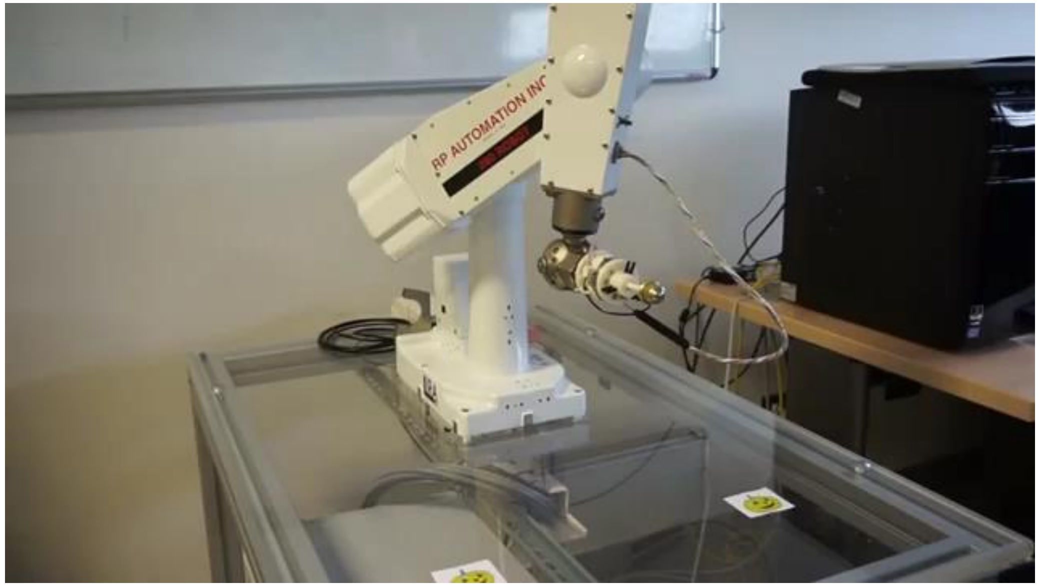


Figure 3.31: PUMA 260 manipulator.



https://www.youtube.com/watch?v=3zhTopWv2oI

Next time: Inverse Position Kinematics

Chapter 3: Forward and Inverse Kinematics

- Read 3.3 – 3.4