

MEAM 520

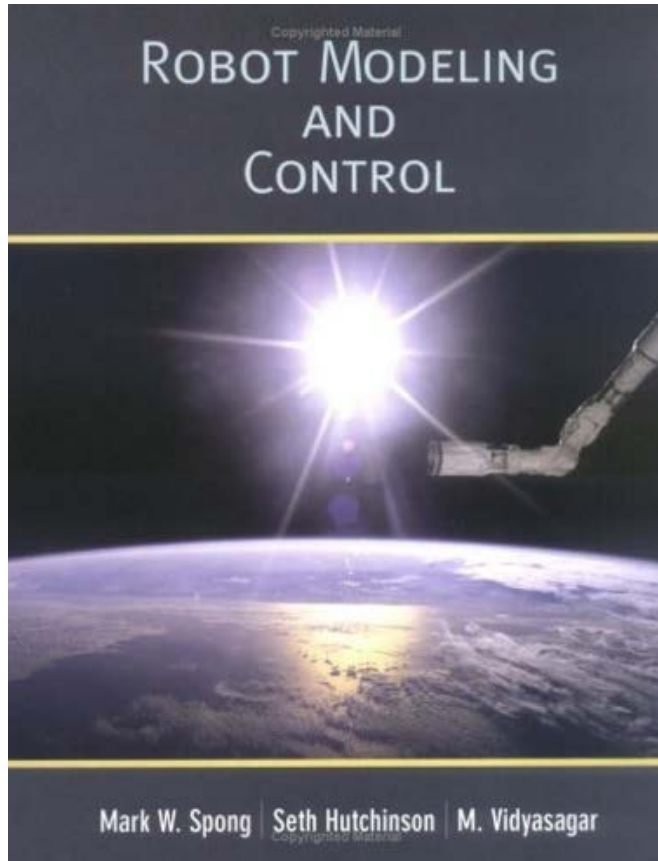
Lecture 8: Inverse Orientation Kinematics

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University of Pennsylvania

Today: Inverse Kinematics



Chapter 3: Forward and Inverse Kinematics

- Read Sec. 3.3 – 3.4

Lab 2: Inverse Kinematics for the Lynx

MEAM 520, University of Pennsylvania
September 23, 2020

This lab consists of two portions, with a pre-lab due on **Wednesday, September 30**, by midnight (11:59 p.m.) and a lab (code/report) due on **Wednesday, October 7**, by midnight (11:59 p.m.). Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation. This assignment is worth 50 points.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

Work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Williams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, even if one partner is much more experienced than the other. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

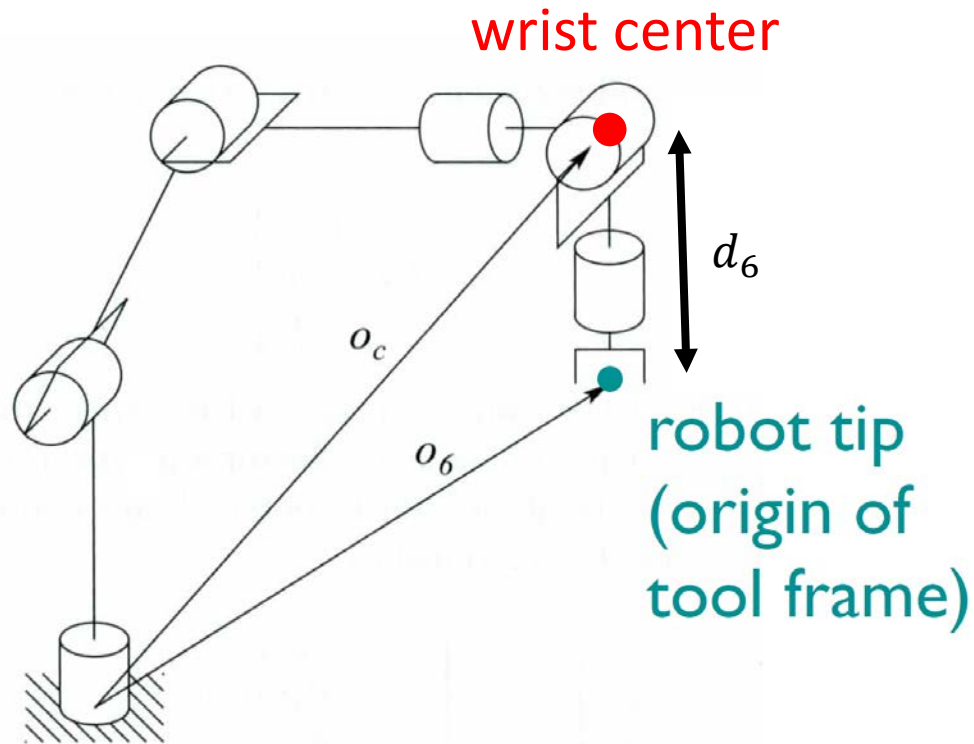
1

Lab 2 posted yesterday
Prelab due 9/30, 11:59 p.m.
Show your work!

Given $\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{o} \\ \mathbf{0} & 1 \end{bmatrix}$ and a certain manipulator with n joints,
find q_1, \dots, q_n such that $\mathbf{T}_n^0(q_1, \dots, q_n) = \mathbf{H}$

UGLY!

Last Time: Kinematic Decoupling



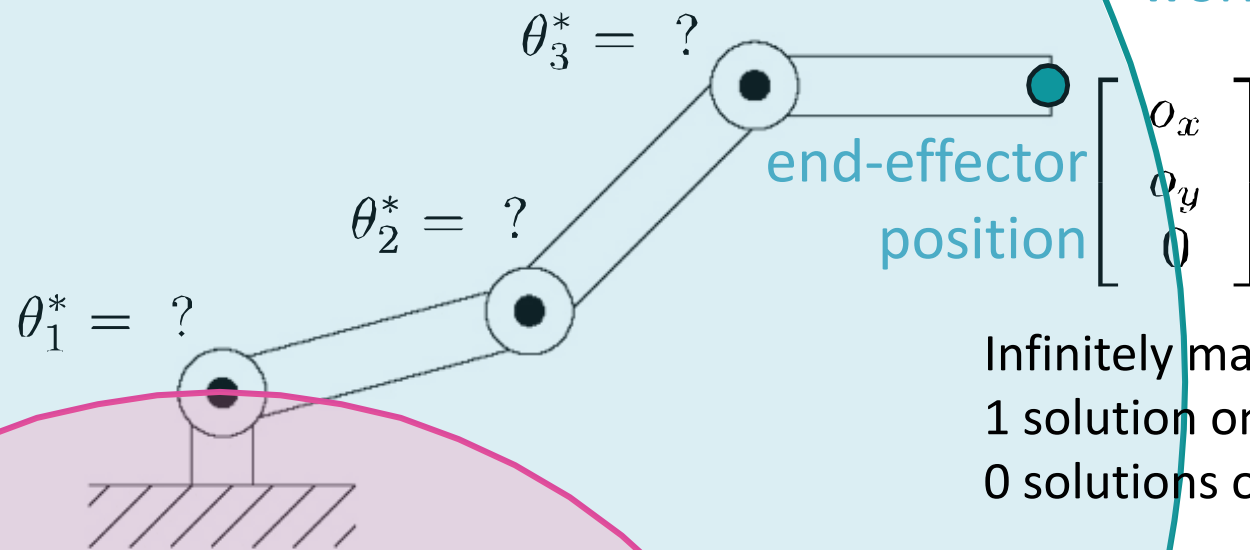
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1} \mathbf{R} = (\mathbf{R}_3^0)^T \mathbf{R}$$

orientation

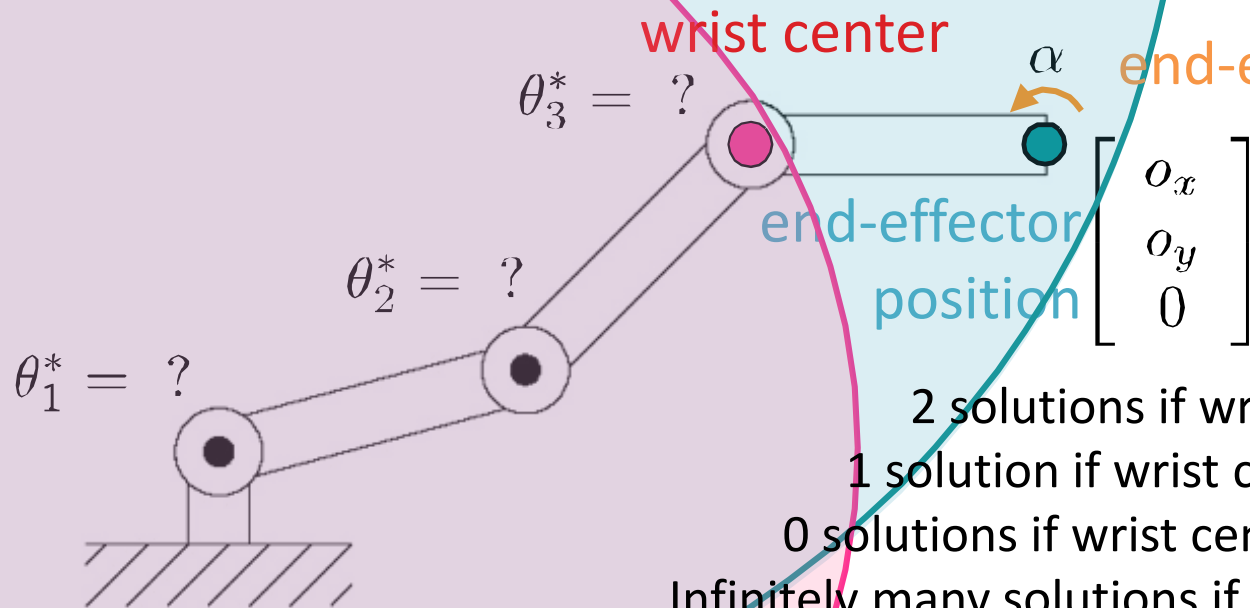
Multiple IK solutions



workspace

end-effector
position

Infinitely many solutions within 3-link workspace
1 solution on the 3-link workspace boundary
0 solutions outside the 3-link workspace



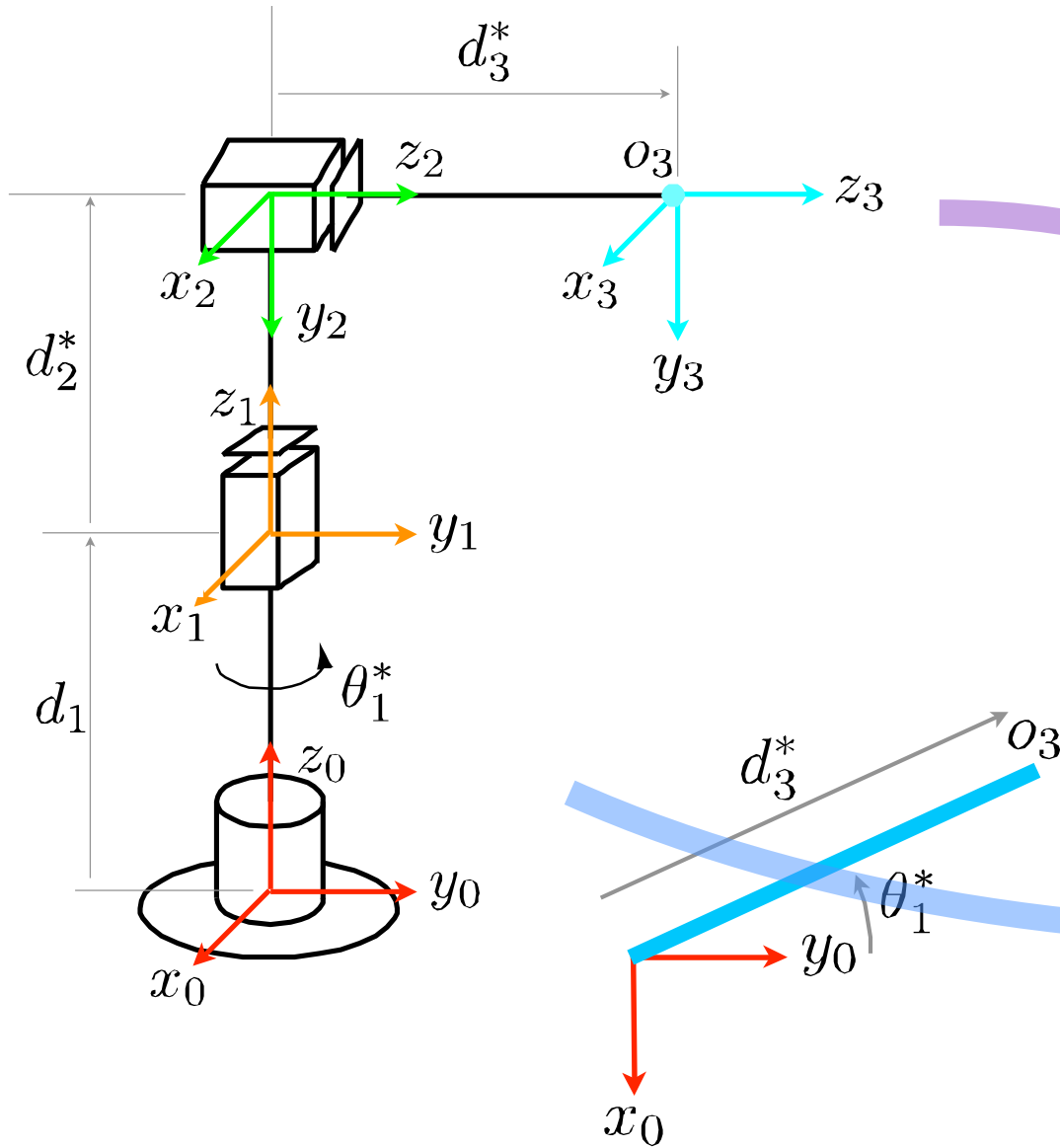
wrist center

end-effector orientation

*Kinematic
Decoupling*

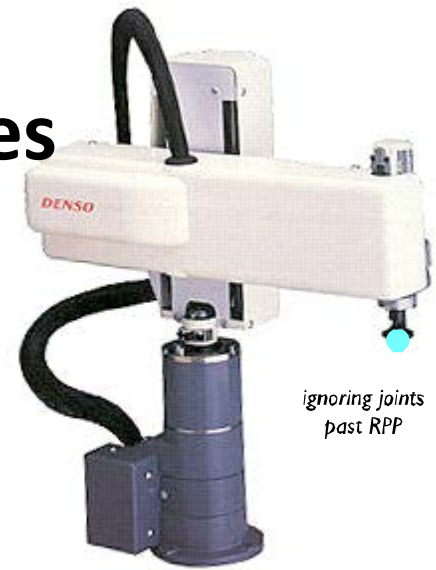
2 solutions if wrist center is inside 2-link workspace
1 solution if wrist center is on 2-link workspace boundary
0 solutions if wrist center needs to be outside 2-link workspace
Infinitely many solutions if wrist center is origin and links are equal length¹

Last Time: Two Inverse Position Kinematics Approaches



$$\begin{aligned}
 x &= -d_3^* \sin(\theta_1^*) \\
 y &= d_3^* \cos(\theta_1^*) \\
 z &= d_1 + d_2^*
 \end{aligned}$$

$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$$\begin{aligned}
 \theta_1^* &= ? \\
 d_2^* &= ? \\
 d_3^* &= ?
 \end{aligned}$$

Algebra

$$\theta_1^* = \text{atan2} \left(\frac{-x/d_3^*}{y/d_3^*} \right)$$

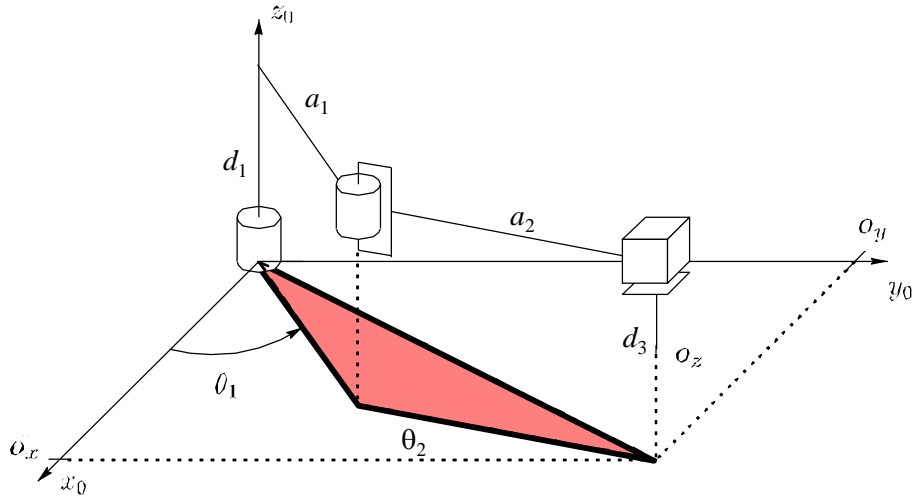
$$d_2^* = z - d_1$$

$$d_3^* = \pm \sqrt{x^2 + y^2}$$

Geometry

Complete SCARA IK Example

$${}^0_4O = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \Rightarrow \begin{matrix} \theta_1 = ? \\ \theta_2 = ? \\ d_3 = ? \end{matrix}$$



Start with Forward Position Kinematics

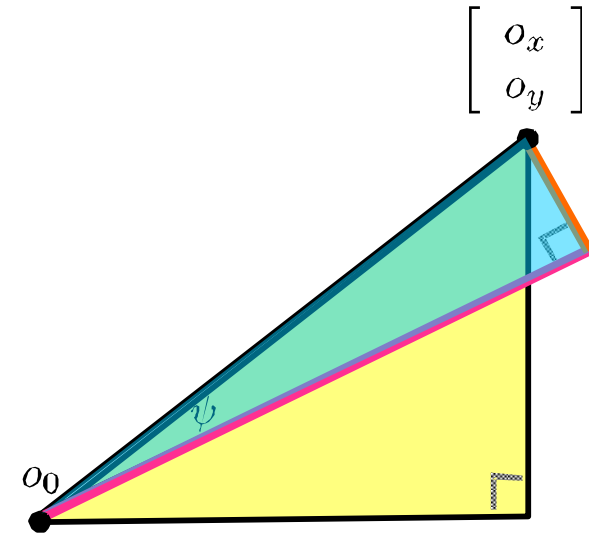
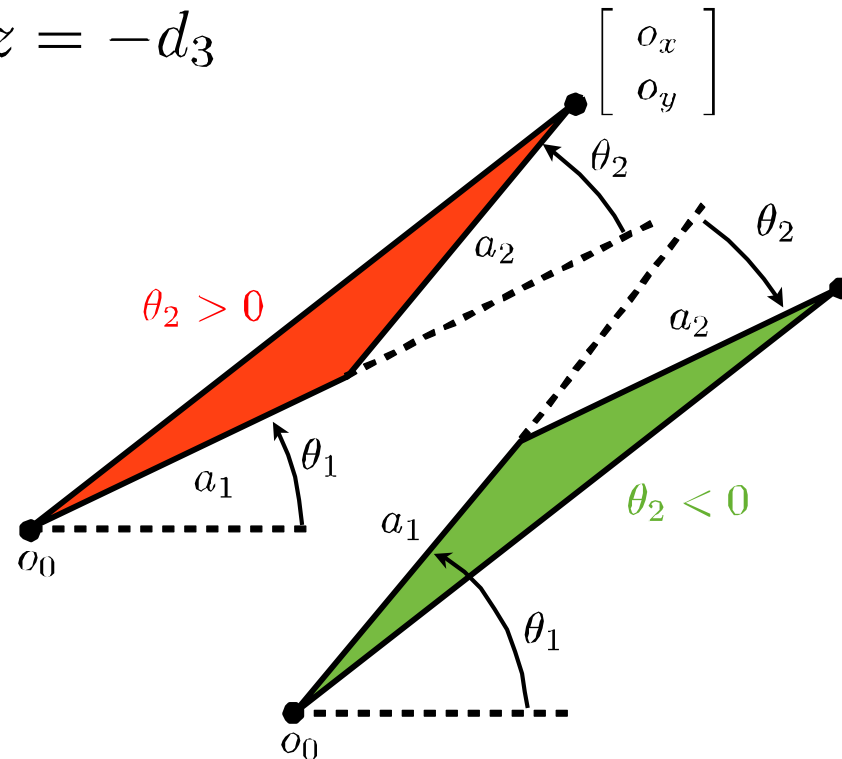
$$\begin{aligned} x &= a_1 c_1 + a_2 c_{12} \\ y &= a_1 s_1 + a_2 s_{12} \\ z &= -d_3 \end{aligned}$$

$$d_3 = -o_z$$

$$\cos \theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\theta_2 = \text{atan2} \left(\frac{\pm \sqrt{1 - \cos^2 \theta_2}}{\cos \theta_2} \right)$$

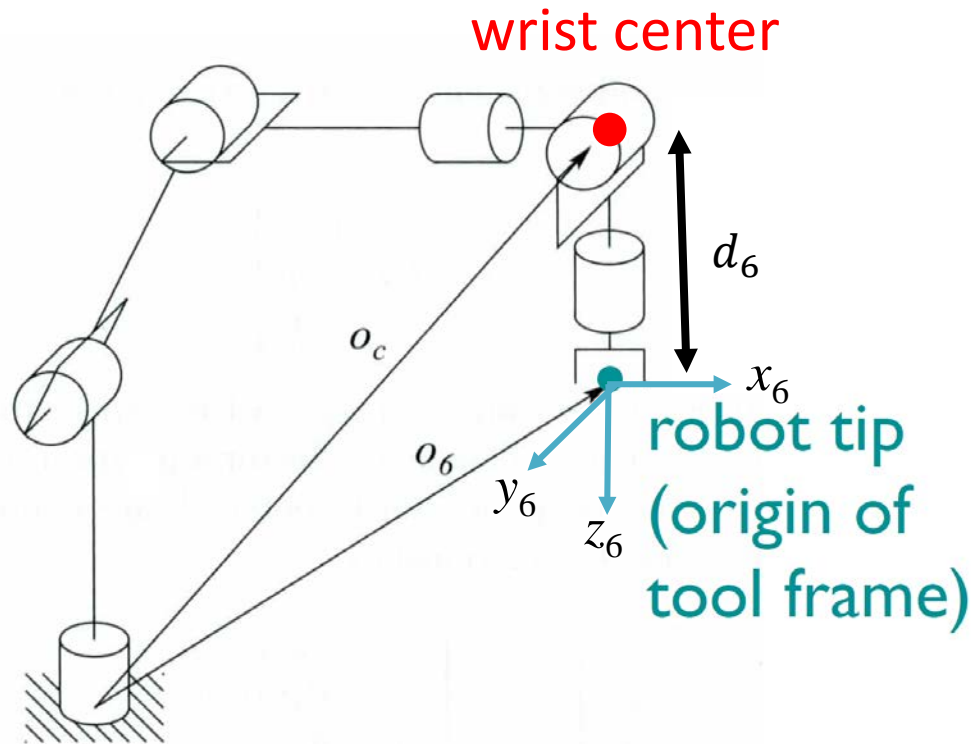
$$\theta_1 = \text{atan2} \left(\frac{o_y}{o_x} \right) - \text{atan2} \left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2} \right)$$



Given $\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{o} \\ \mathbf{0} & 1 \end{bmatrix}$ and a certain manipulator with n joints,
find q_1, \dots, q_n such that $\mathbf{T}_n^0(q_1, \dots, q_n) = \mathbf{H}$

UGLY!

Last Time: Kinematic Decoupling



Last Time

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

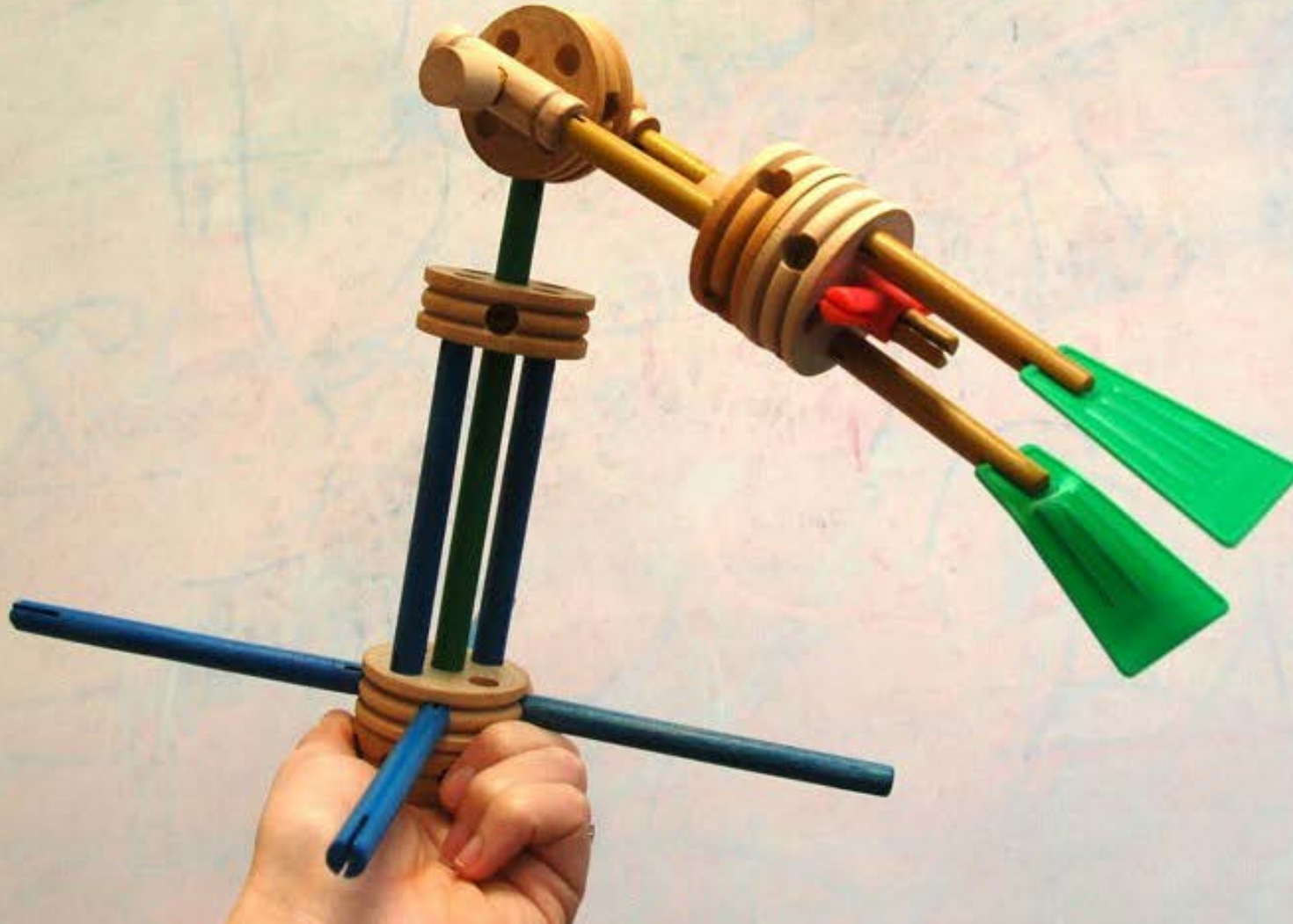
Today

$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1} \mathbf{R} = (\mathbf{R}_3^0)^T \mathbf{R}$$

orientation

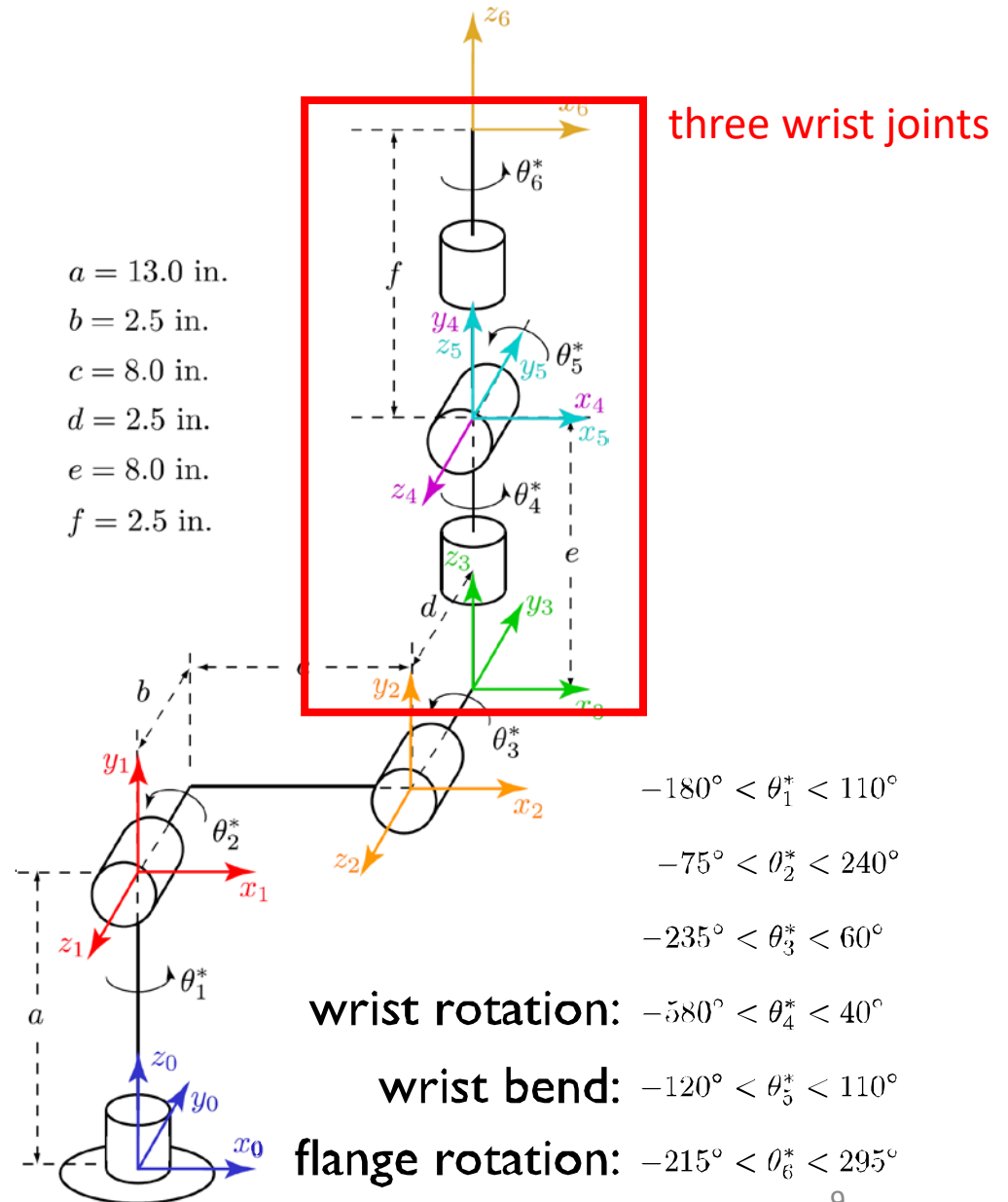
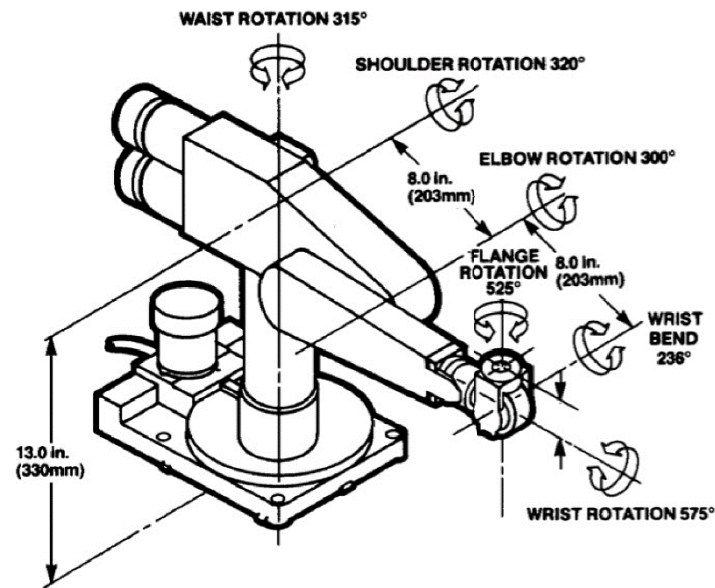
Spherical Wrist

Intersecting RRR with middle joint perpendicular to the other two.



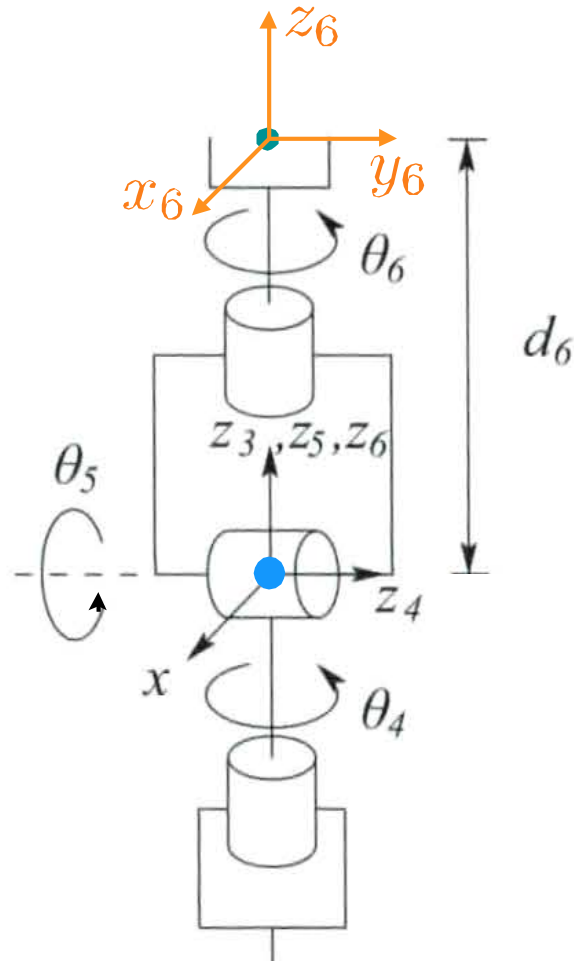
(let's redefine the zero config so the wrist lines up)

PUMA example



Given a rotation matrix \mathbf{R} , find a set of joint angles that puts the end-effector in the desired orientation.

Spherical Wrist



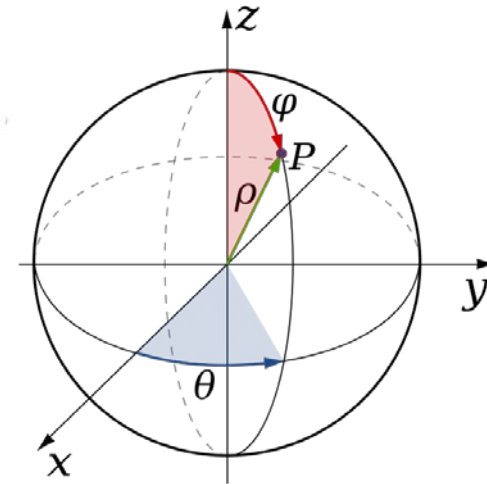
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

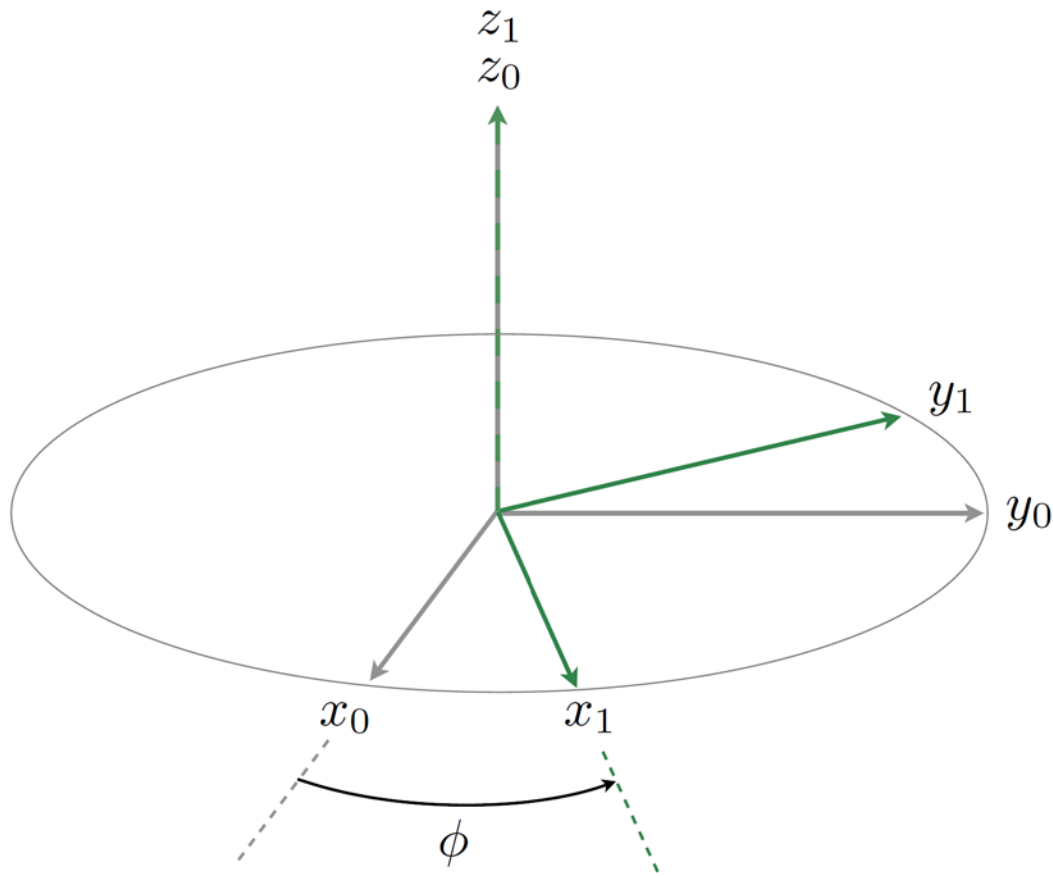
$$T_6^3 = A_4 A_5 A_6 = \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \text{ Our Focus Today}$$

$$= \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ , θ , ψ to go from one frame to another by rotating around the axes of the **current frame**.

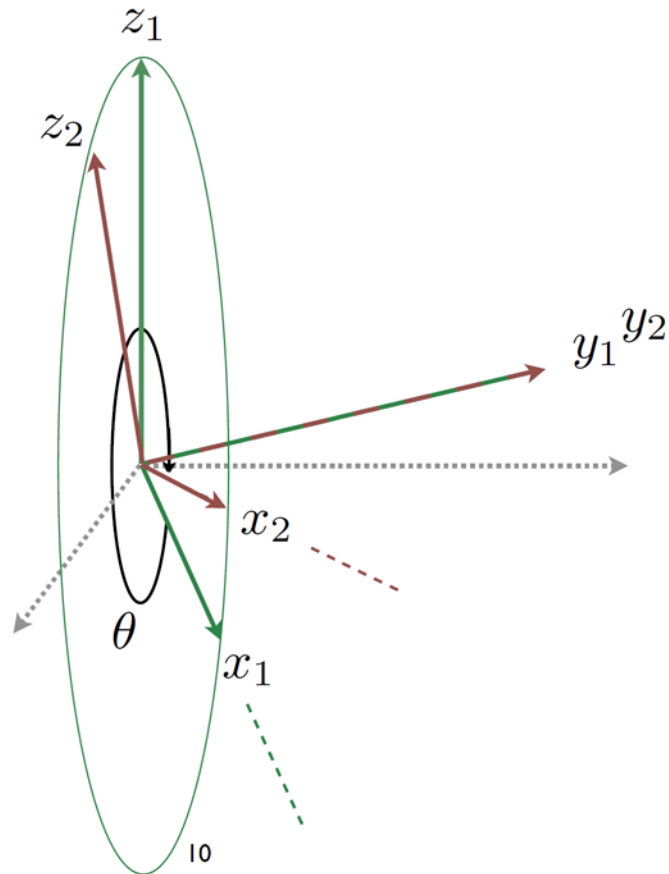


Using Z-Y-Z convention:

1. Rotate by ϕ about z_0

Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ , θ , ψ to go from $0 \rightarrow 3$ by rotating around the axes of the **current frame**.

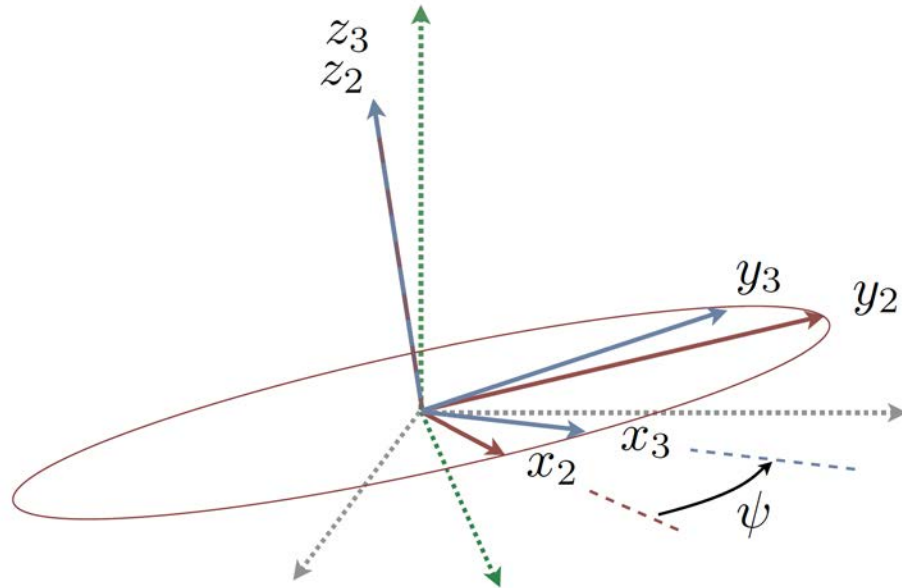


Using Z-Y-Z convention:

1. Rotate by ϕ about z_0
2. Rotate by θ about y_1

Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ , θ , ψ to go from $0 \rightarrow 3$ by rotating around the axes of the **current frame**.



Using Z-Y-Z convention:

1. Rotate by ϕ about z_0
2. Rotate by θ about y_1
3. Rotate by ψ about z_2

Euler Angles (from Lecture 3)

Post-multiply using the **basic rotation matrices**

$$\mathbf{R} = \mathbf{R}_{z,\phi} \mathbf{R}_{y,\theta} \mathbf{R}_{z,\psi}$$

$$s_\theta = \sin \theta, c_\theta = \cos \theta$$

$$\mathbf{R} = \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Euler Angles vs Spherical Wrists

$$\begin{aligned}
 T_6^3 &= A_4 A_5 A_6 \\
 &= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$\theta_4 = \phi \quad \theta_5 = \theta \quad \theta_6 = \psi$$

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

How to calculate the three angles given \mathbf{R} ?
See Lecture 3 and SHV pages 55-56.

Rotation Matrices to Euler Angles

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Plug in to solve for ψ

Solve for θ

Plug in to solve for ϕ

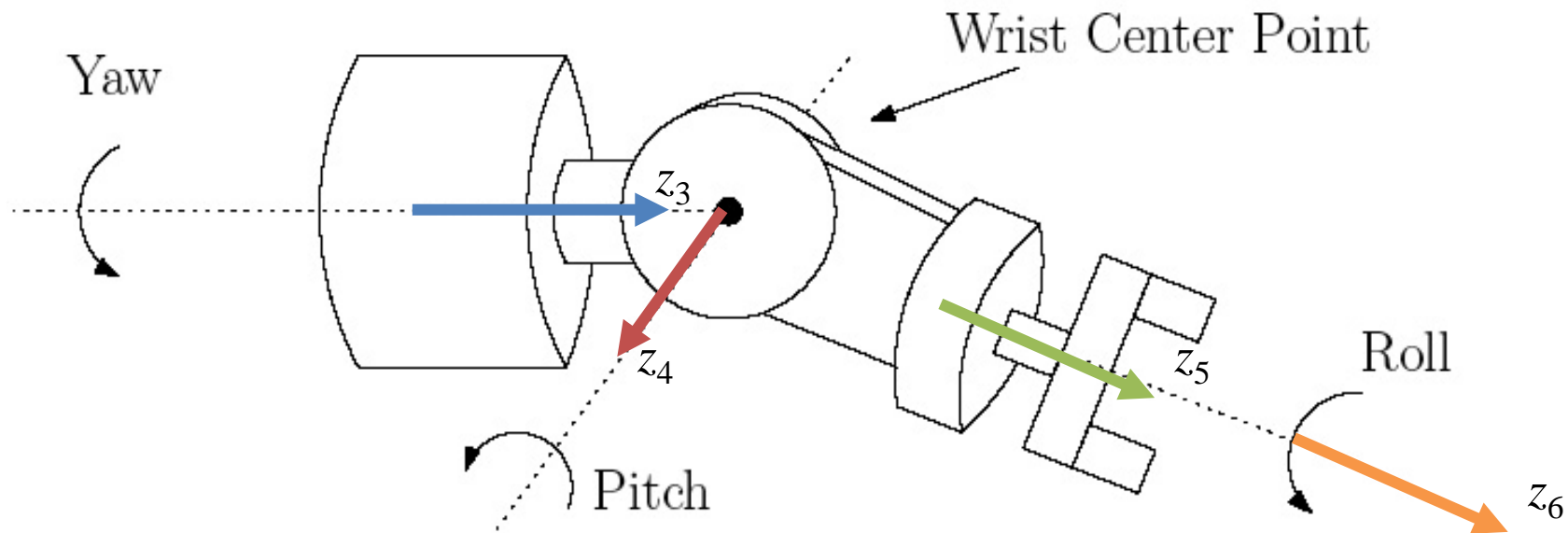
Two solutions for θ because sign of s_θ is not known.

Geometric Interpretation of Solution Method

$$\mathbf{R}_6^3 = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

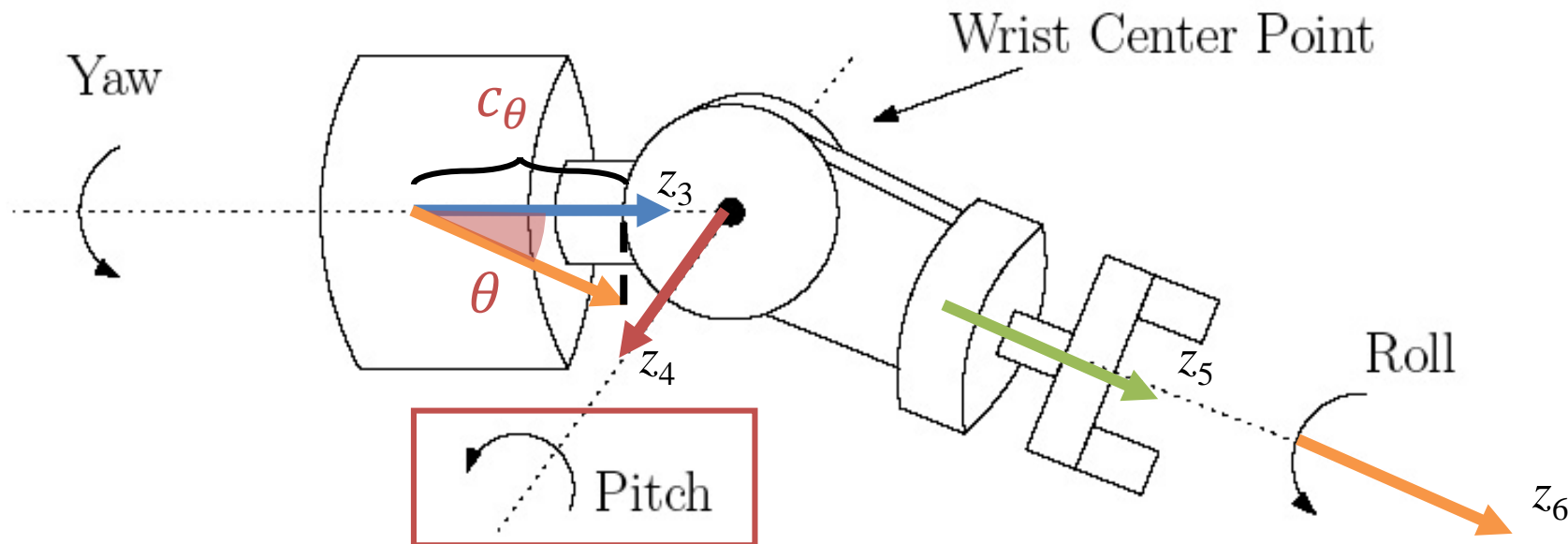
\hat{x}_6^3 \hat{y}_6^3 \hat{z}_6^3

Plug in to solve for ψ Plug in to solve for ϕ Solve for θ



Geometric Interpretation of Solution Method

$$\begin{array}{c} \hat{x}_6^3 \qquad \hat{y}_6^3 \qquad \hat{z}_6^3 \\ \mathbf{R}_6^3 = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ \text{Plug in to solve for } \psi & s_\theta s_\psi & \text{Solve for } \theta \end{bmatrix} \begin{array}{l} \text{Plug in to} \\ \text{solve for } \phi \end{array} \end{array}$$

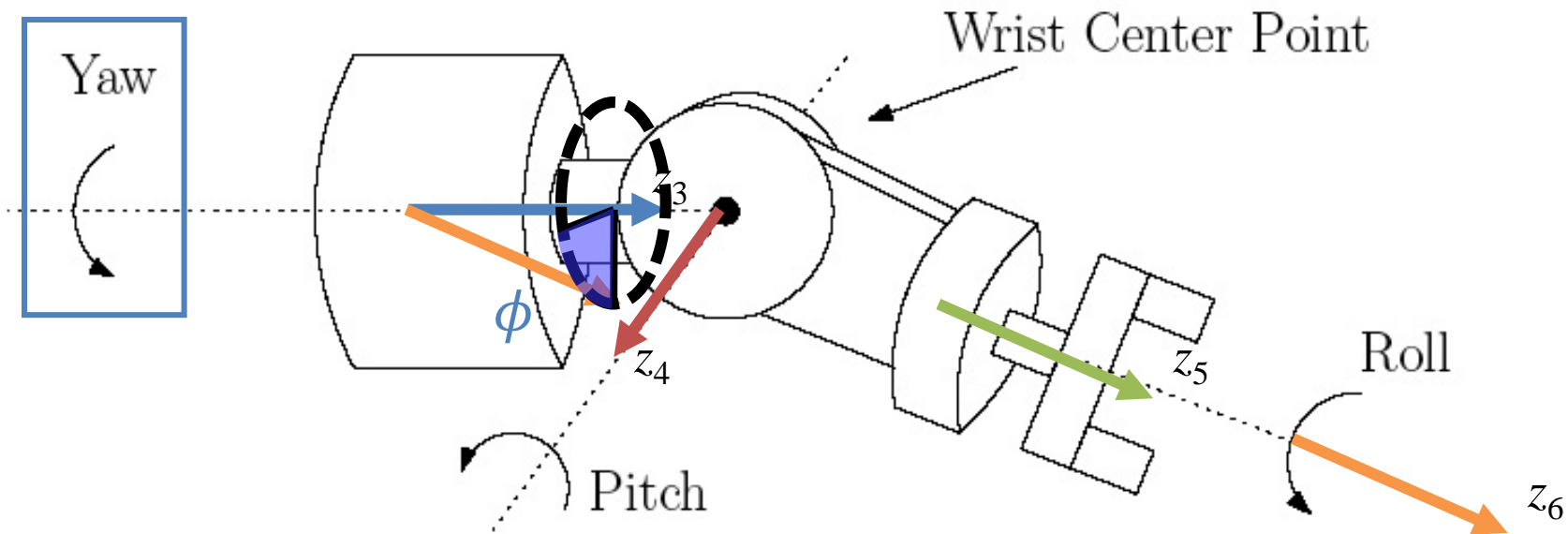


Geometric Interpretation of Solution Method

$$\begin{array}{c} \hat{x}_6^3 \qquad \hat{y}_6^3 \qquad \hat{z}_6^3 \end{array}$$

$$\mathbf{R}_6^3 = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

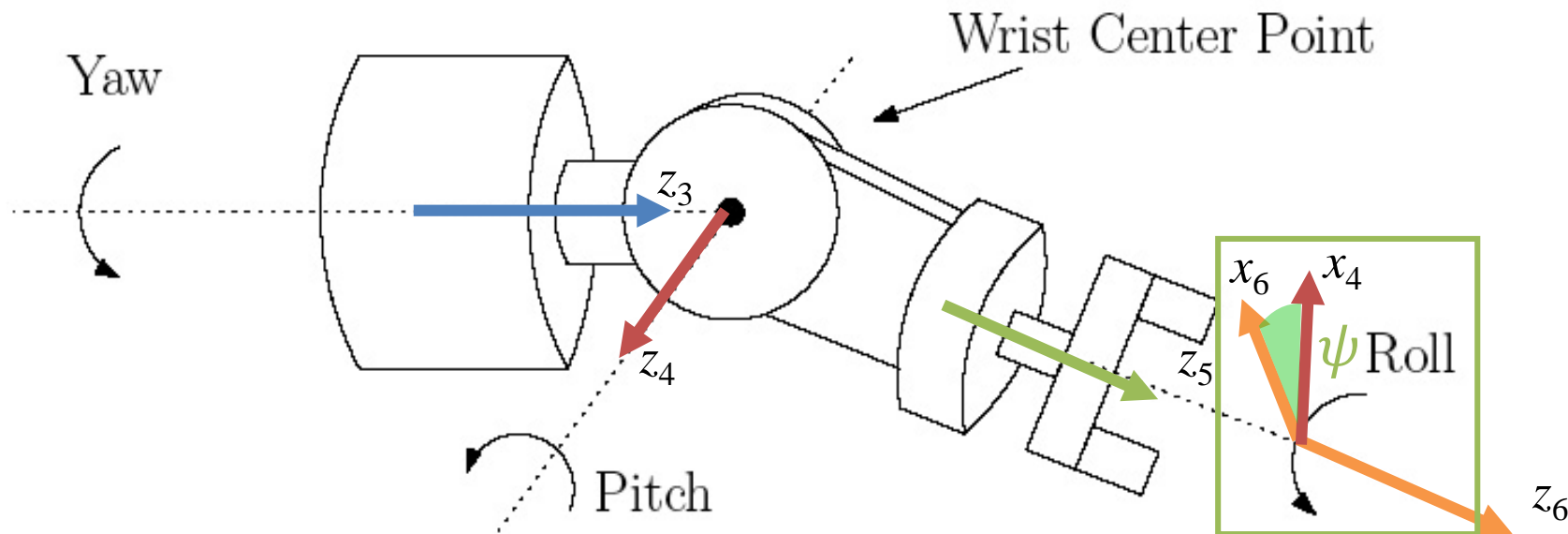
Plug in to solve for ψ (green oval around the third row)
 Plug in to solve for ϕ (blue oval around the first two columns)
 Solve for θ (red oval around c_θ)



Geometric Interpretation of Solution Method

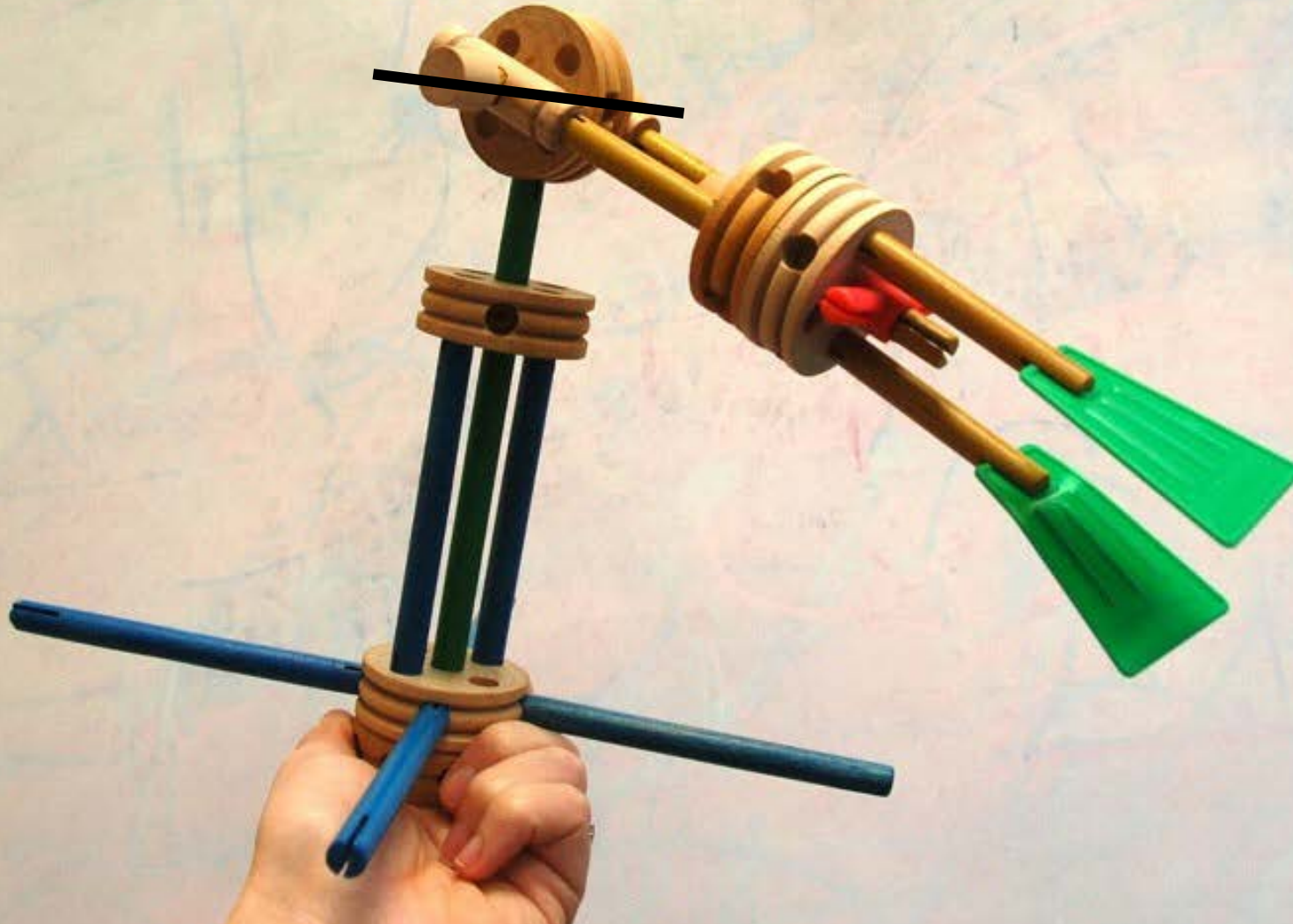
$$\begin{array}{c} \hat{x}_6^3 \qquad \qquad \hat{y}_6^3 \qquad \qquad \hat{z}_6^3 \\ \mathbf{R}_6^3 = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ \underbrace{-s_\theta c_\psi}_{\text{Plug in to solve for } \psi} & \underbrace{s_\theta s_\psi}_{\text{Plug in to solve for } \psi} & \underbrace{\begin{bmatrix} c_\phi s_\theta \\ s_\phi s_\theta \\ c_\theta \end{bmatrix}}_{\text{Solve for } \theta} \end{bmatrix} \end{array}$$

Plug in to solve for ϕ



What do the two solution sets mean geometrically?

Talk to a partner



Special case for $\cos \theta = \pm 1$

2.5. PARAMETERIZATIONS OF ROTATIONS

55

To find a solution for this problem we break it down into two cases. First, suppose that not both of r_{13} , r_{23} are zero. Then from Equation (2.26) we deduce that $s_\theta \neq 0$, and hence that not both of r_{31} , r_{32} are zero. If not both r_{13} and r_{23} are zero, then $r_{33} \neq \pm 1$, and we have $c_\theta = r_{33}$, $s_\theta = \pm\sqrt{1 - r_{33}^2}$ so

$$\theta = \text{Atan2}\left(r_{33}, \sqrt{1 - r_{33}^2}\right) \quad (2.28)$$

or

$$\theta = \text{Atan2}\left(r_{33}, -\sqrt{1 - r_{33}^2}\right) \quad (2.29)$$

where the function Atan2 is the **two-argument arctangent function** defined in Appendix A.

If we choose the value for θ given by Equation (2.28), then $s_\theta > 0$, and

$$\phi = \text{Atan2}(r_{13}, r_{23}) \quad (2.30)$$

$$\psi = \text{Atan2}(-r_{31}, r_{32}) \quad (2.31)$$

If we choose the value for θ given by Equation (2.29), then $s_\theta < 0$, and

$$\phi = \text{Atan2}(-r_{13}, -r_{23}) \quad (2.32)$$

$$\psi = \text{Atan2}(r_{31}, -r_{32}) \quad (2.33)$$

Thus, there are two solutions depending on the sign chosen for θ .

If $r_{13} = r_{23} = 0$, then the fact that R is orthogonal implies that $r_{33} = \pm 1$, and that $r_{31} = r_{32} = 0$. Thus, R has the form

$$R = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix} \quad (2.34)$$

If $r_{33} = 1$, then $c_\theta = 1$ and $s_\theta = 0$, so that $\theta = 0$. In this case, Equation (2.26) becomes

$$\begin{bmatrix} c_\phi c_\psi - s_\phi s_\psi & -c_\phi s_\psi - s_\phi c_\psi & 0 \\ s_\phi c_\psi + c_\phi s_\psi & -s_\phi s_\psi + c_\phi c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\phi+\psi} & -s_{\phi+\psi} & 0 \\ s_{\phi+\psi} & c_{\phi+\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, the sum $\phi + \psi$ can be determined as

$$\phi + \psi = \text{Atan2}(r_{11}, r_{21}) = \text{Atan2}(r_{11}, -r_{12}) \quad (2.35)$$

Since only the sum $\phi + \psi$ can be determined in this case, there are infinitely many solutions. In this case, we may take $\phi = 0$ by convention. If $r_{33} = -1$, then $c_\theta = -1$ and $s_\theta = 0$, so that $\theta = \pi$. In this case Equation (2.26) becomes

$$\begin{bmatrix} -c_{\phi-\psi} & -s_{\phi-\psi} & 0 \\ s_{\phi-\psi} & c_{\phi-\psi} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (2.36)$$

The solution is thus

$$\phi - \psi = \text{Atan2}(-r_{11}, -r_{12}) \quad (2.37)$$

As before there are infinitely many solutions.

What does it mean
when $\cos \theta = +1$?

What does it mean
when $\cos \theta = -1$?

Special case for $\cos \theta = \pm 1$

2.5. PARAMETERIZATIONS OF ROTATIONS

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To find a solution for this problem we break it down into two cases. First, suppose that not both of r_{13} , r_{23} are zero. Then from Equation (2.26) we deduce that $s_\theta \neq 0$, and hence that not both of r_{31} , r_{32} are zero. If not both r_{13} and r_{23} are zero, then $r_{33} \neq \pm 1$, and we have $c_\theta = r_{33}$, $s_\theta = \pm\sqrt{1 - r_{33}^2}$ so

$$\theta = \text{Atan2}\left(r_{33}, \sqrt{1 - r_{33}^2}\right) \quad (2.28)$$

or

$$\theta = \text{Atan2}\left(r_{33}, -\sqrt{1 - r_{33}^2}\right) \quad (2.29)$$

where the function Atan2 is the **two-argument arctangent function** defined in Appendix A.

If we choose the value for θ given by Equation (2.28), then $s_\theta > 0$, and

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$$\psi = \text{Atan2}(-r_{31}, r_{32}) \quad (2.31)$$

If we choose the value for θ given by Equation (2.29), then $s_\theta < 0$, and

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Thus, there are two solutions depending on the sign chosen for θ .

If $r_{13} = r_{23} = 0$, then the fact that R is orthogonal implies that $r_{33} = \pm 1$, and that $r_{31} = r_{32} = 0$. Thus, R has the form

$$R = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix} \quad (2.34)$$

If $r_{33} = 1$, then $c_\theta = 1$ and $s_\theta = 0$, so that $\theta = 0$. In this case, Equation (2.26) becomes

$$\begin{bmatrix} c_\phi c_\psi - s_\phi s_\psi & -c_\phi s_\psi - s_\phi c_\psi & 0 \\ s_\phi c_\psi + c_\phi s_\psi & -s_\phi s_\psi + c_\phi c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\phi+\psi} & -s_{\phi+\psi} & 0 \\ s_{\phi+\psi} & c_{\phi+\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, the sum $\phi + \psi$ can be determined as

$$\phi + \psi = \text{Atan2}(r_{11}, r_{21}) = \text{Atan2}(r_{11}, -r_{12}) \quad (2.35)$$

Since only the sum $\phi + \psi$ can be determined in this case, there are infinitely many solutions. In this case, we may take $\phi = 0$ by convention. If $r_{33} = -1$, then $c_\theta = -1$ and $s_\theta = 0$, so that $\theta = \pi$. In this case Equation (2.26) becomes

$$\begin{bmatrix} -c_{\phi-\psi} & -s_{\phi-\psi} & 0 \\ s_{\phi-\psi} & c_{\phi-\psi} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (2.36)$$

The solution is thus

$$\phi - \psi = \text{Atan2}(-r_{11}, -r_{12}) \quad (2.37)$$

As before there are infinitely many solutions.

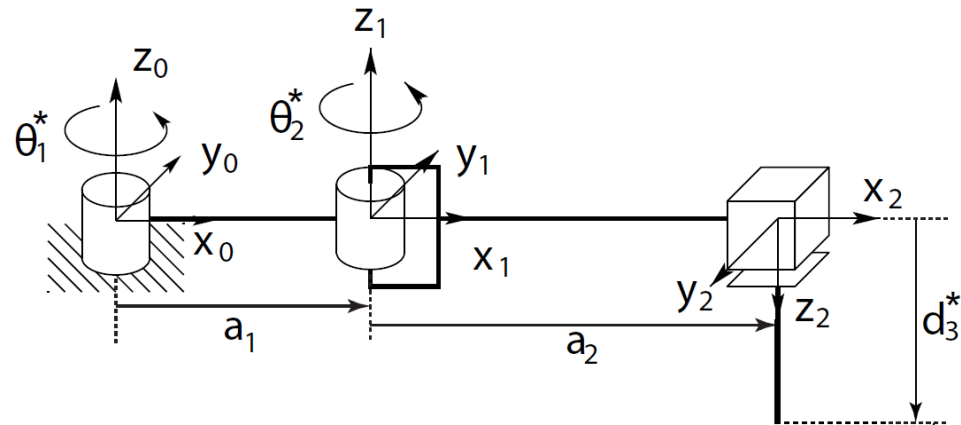
What does it mean
when $\cos \theta = +1$?

Wrist is straight

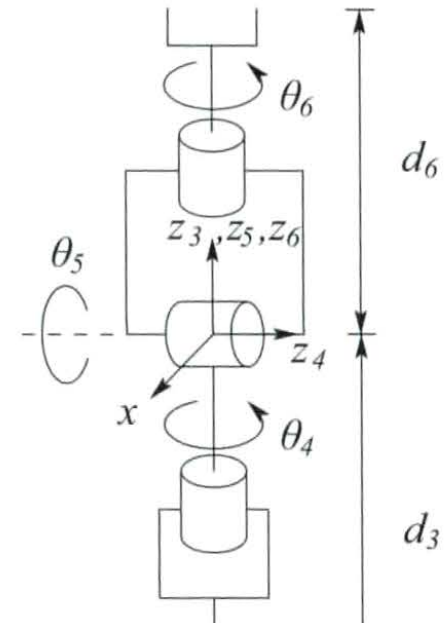
What does it mean
when $\cos \theta = -1$?

Wrist is bent all the
way back.

SCARA + spherical wrist tracing out a circle



+



SCARA + spherical wrist tracing out a circle

1	<code>%% scara_circle_starter.m</code>	
2	<code>%</code>	
3	<code>% This Matlab script provides solves the SCARA robot's inverse kinematics.</code>	
4	<code>% It was modified from the original starter code provided by</code>	
5	<code>% Professor Katherine J. Kuchenbecker (kuchenbe@seas.upenn.edu)</code>	
6	<code>% for Homework 6 in MEAM 520.</code>	
7	<code>%</code>	
8		
9		
10	<code>%% SETUP</code>	
11		
12	<code>% Define our time vector.</code>	
13	<code>tStart = 0; % The time at which the simulation starts, in seconds.</code>	
14	<code>tStep = 0.01; % The simulation's time step, in seconds.</code>	
15	<code>tEnd = 2*pi; % The time at which the simulation ends, in seconds.</code>	
16	<code>t = (tStart:tStep:tEnd)'; % The time vector (a column vector).</code>	
17		
18	<code>% Set whether to animate the robot's movement and how much to slow it down.</code>	
19	<code>pause on; % Set this to off if you don't want to watch the animation.</code>	
20	<code>GraphingTimeDelay = 0.002; % How long Matlab should pause between positions</code>	when graphing, if
21		
22		
23	<code>%% ROBOT PARAMETERS</code>	
24		
25	<code>% This problem is about the first three joints (RRP) of a SCARA</code>	
26	<code>% manipulator. This robot's forward kinematics are worked out on</code>	
27	<code>% pages 91 to 93 of the SHV textbook, although we are replacing the</code>	
28	<code>% fourth joint with a full spherical wrist.</code>	
29		
30	<code>% Define robot link lengths.</code>	
31	<code>a1 = 1.0; % Distance between joints 1 and 2, in meters.</code>	
32	<code>a2 = 0.7 * a1; % Distance between joints 2 and 3, in meters.</code>	
33	<code>d6 = 0.25 * a1; % Offset from wrist center to end-effector, in meters.</code>	
34		

SCARA + spherical wrist tracing out a circle

```
36 %% DEFINE CIRCULAR MOTION
37
38 % We want the SCARA to draw a vertical circle parallel to the x-z
39 % plane.
40
41 % Define the radius of the circle.
42 radius = .5; % meters
43
44 % Define the y-value for the plane that contains the circle.
45 y_offset = -1; % meters
46
47 % Define the x and z coordinates for the center of the circle.
48 x_center = -1; % meters
49 z_center = -1; % meters
50
51 % Set the desired x, y, and z positions over time given the circle
52 % parameters.
53 ox_history = x_center + radius * sin(t);
54 oy_history = y_offset * ones(size(t));
55 oz_history = z_center + radius * cos(t);
56
```

SCARA + spherical wrist tracing out a circle

```
91 % Step through the time vector to animate the robot.
92 for i = 1:length(t)
93
94     % Pull the current values of ox, oy, and oz from their histories.
95     ox = ox_history(i);
96     oy = oy_history(i);
97     oz = oz_history(i);
98
99     % Calculate where the wrist center position needs to be.
100    oc = [ox oy oz]' - d6*R(:,3);
101
102    % Pull out the components of wrist center and store in ox, oy, oz.
103    ox = oc(1);
104    oy = oc(2);
105    oz = oc(3);
106
107    % Calculate theta1, theta2, and d3 given the robot's parameters
108    % (a1 and a2) and the current desired position for its tip ([ox oy
109    % oz]').
110
111    % Calculate the cosine of theta2 using law of cosines.
112    c2 = (ox.^2 + oy.^2 - a1^2 - a2^2)/(2*a1*a2);
113
114    % Calculate the positive and negative solutions for theta2.
115    theta2_pos = atan2(sqrt(1-c2^2),c2); % Corrected solution from our derivation.
116    theta2_neg = atan2(-sqrt(1-c2^2),c2); % Corrected solution from our derivation.
117
118    % Choose the positive solution.
119    theta2 = theta2_pos;
120
```

$$d_3 = -o_z$$

$$\cos \theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$\theta_2 = \text{atan2}\left(\frac{\pm\sqrt{1 - \cos^2 \theta_2}}{\cos \theta_2}\right)$$

$$\theta_1 = \text{atan2}\left(\frac{o_y}{o_x}\right) - \text{atan2}\left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2}\right)$$

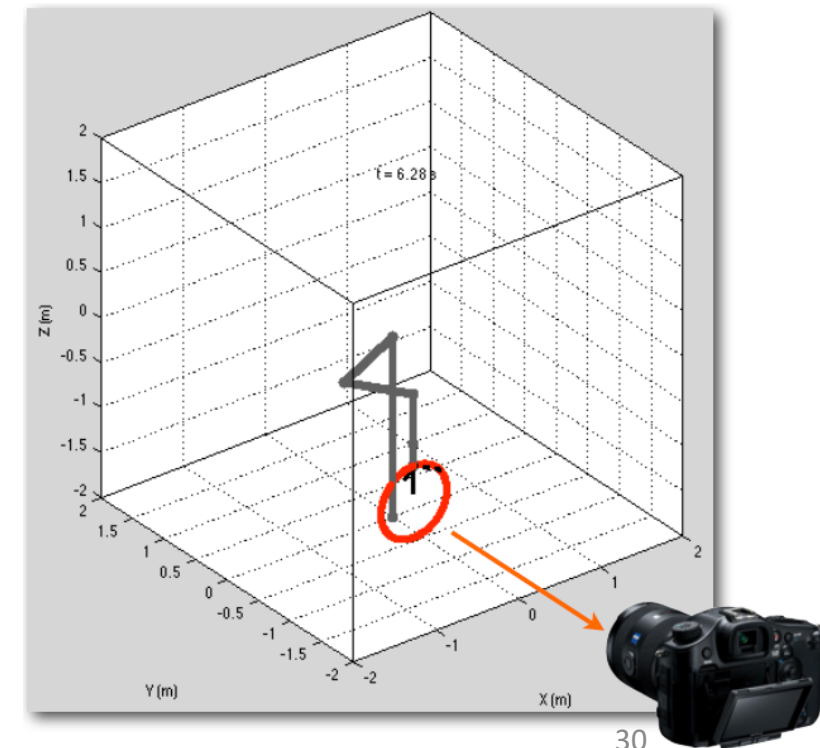
SCARA + spherical wrist tracing out a circle

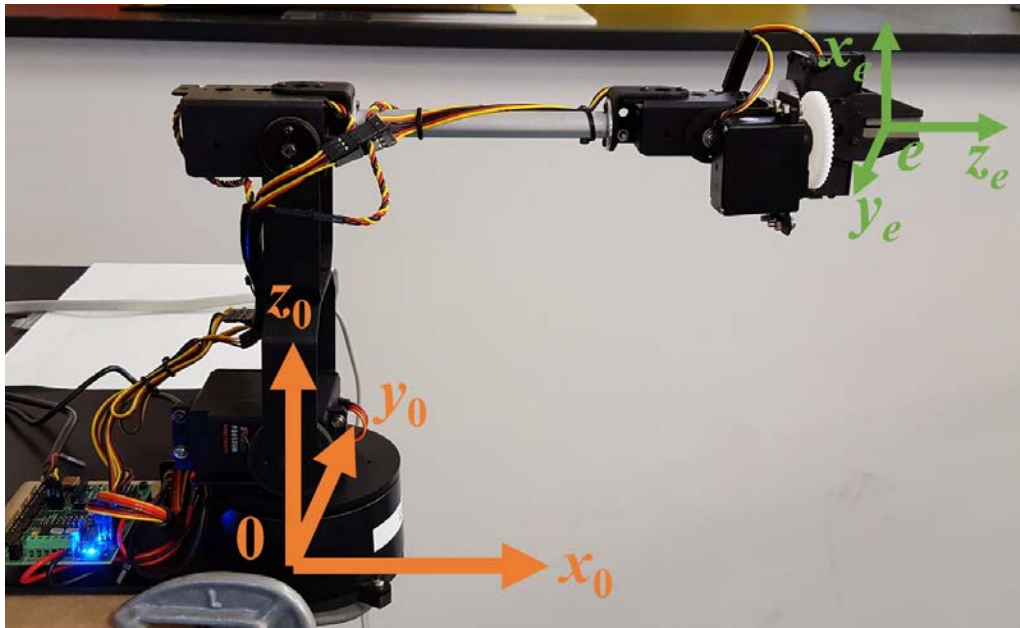
```
128 % Calculate theta1 using a pair of inverse tangents. Note that
129 % MATLAB atan2 takes the numerator and then the denominator.
130 - theta1 = atan2(oy,ox) - atan2(a2*sin(theta2),a1+a2*cos(theta2));
131
132 % Calculate d3.
133 - d3 = -oz; % Corrected solution from our derivation.
134
135 % *****
136 % Solve for the values of theta4, theta5, and theta6 that will put
137 % the end-effector frame at the desired orientation R relative to
138 % the base frame, given the present values of theta1, theta2, and
139 % d3.
140
141 - [theta4, theta5, theta6] = scara_ik_orientation(R, theta1, theta2, a1, a2, d3, d6);
142
143 % *****
```

solve for thetas using math from today

Questions for you

- The SCARA has been programmed to place its wrist center at the location needed to draw a flat circle. When instead it keeps all three of its wrist joints at zero, how does the orientation of the end-effector frame change over time? Sketch / explain.
- What shape will the tip of the SCARA draw if we set the 5th joint at 90 deg. And keep joints 4 and 6 at 0 deg.?
- We want the end-effector's z axis to point out toward the camera with its y-axis straight up (vertical). What rotation matrix \mathbf{R} should we give to the inverse orientation kinematics?



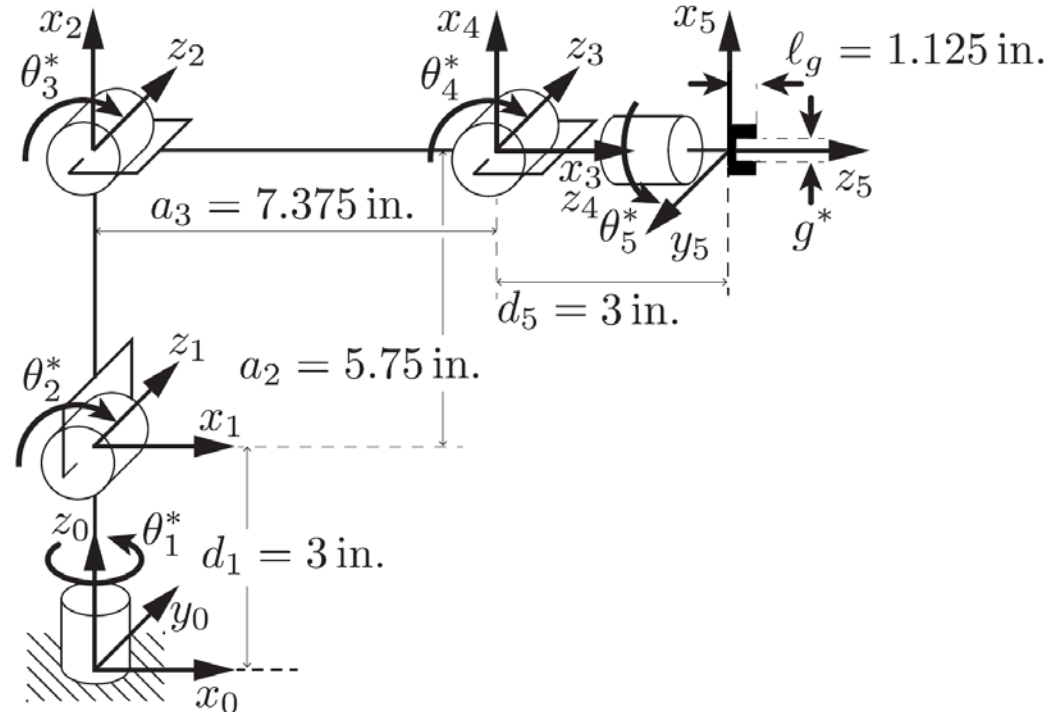
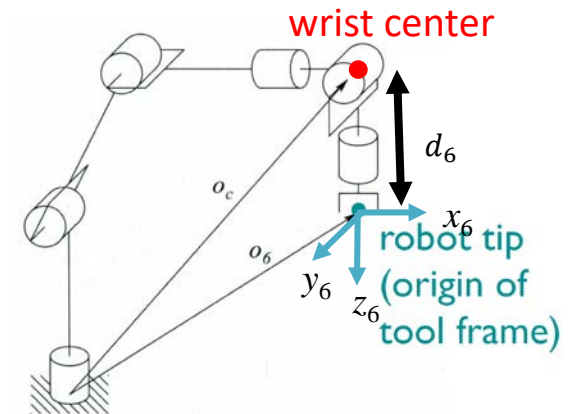


What if the wrist is not spherical?

Can we use Kinematic Decoupling?

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position



$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1} \mathbf{R} = (\mathbf{R}_3^0)^T \mathbf{R}$$

orientation

Next time: Quaternions!



“A comparison of transforms and quaternions in robotics” (ICRA 1998)

Lab 2: Inverse Kinematics for the Lynx

MEAM 520, University of Pennsylvania

September 23, 2020

This lab consists of two portions, with a pre-lab due on **Wednesday, September 30, by midnight (11:59 p.m.)** and a lab (code + report) due on **Wednesday, October 7, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation. This assignment is worth 50 points.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

Work closely with your partner throughout the lab, following these guidelines, which were adapted from “All I really needed to know about pair programming I learned in kindergarten,” by Williams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Lab 2: Inverse Kinematics due 10/7

- You can now do all the tasks