Pre lab 2

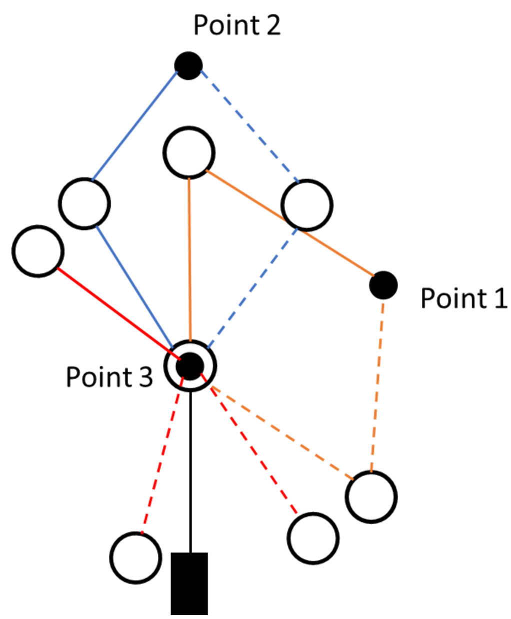
1.

$$z = d_1 + d_2\cos(\theta_2) - a_3\cos(\theta_3 + \theta_2)$$

$$x = \cos(\theta_1)\,[a_2\sin(\theta_2) + a_3\cos(\theta_3 + \theta_2)]$$

$$y = \sin(\theta_1)\,[a_2\sin(\theta_2) + a_3\cos(\theta_3 + \theta_2)]$$

2. Drawing shown:



Given a certain angle θ_1, points 1 and 2 shown in the diagram have two configurations for a point in space. However, there are also certain points in space that have infinite θ_2 and θ_3 angles. Point 3 represents this by its end effector being behind joint 2. The dashed red lines indicate some of the infinite positions that joint 2 and 3 can reach to place the end effector at the area. If θ_1 can move, points 1 and 2 also have unlimited configurations possible.

3.

$$\theta_1 = Atan2^{-1}(\frac{y}{x})$$

$$\theta_2 = \cos^{-1}(\frac{x^2 + z^2 - a_1^2 - a_2^2}{2a_1a_2})$$

$$\theta_3 = sin^{-1}\left(\frac{x^2 + (z - a1)^2 - a_2^2 - a_3^2}{-2a_2a_3}\right)$$

I took a geometric approach to solve for the angles. θ_1 was derived with the intuition that the end effector and the axis of θ_1 are always on the same plane so the tangent inverse of (y/x) would give us the solution of θ_1. θ_2 was derived from the law of cosines as it was done in class with the SCARA robot. $l^2 = x^2 + z^2$ and the law of cosines is: $l^2 = {a_1}^2 + {a_2}^2 - 2a_1a_2\cos(\gamma)$ where a1 was the link length from frame 0-1, a2 was the link length from frame 1-2 and a3 is the link length from frame 2-3. θ_3 was similar to $\theta_2's$ derivation but was modified to use coordinates from the 0th frame instead of frame 1. Figures 2 and 3 show the triangle that was used to help find θ_2 and θ_3.

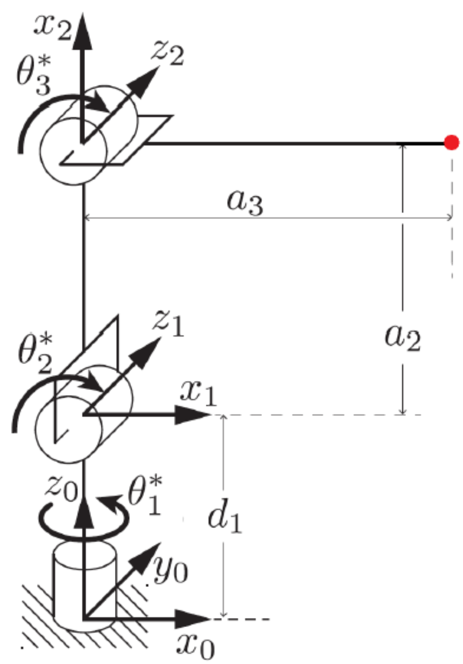


Figure 1. 3D representation of robot

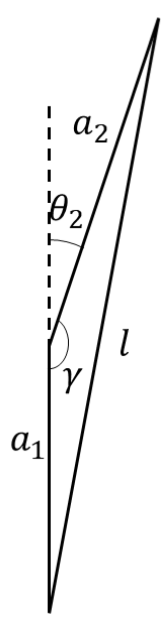


Figure 2. Using the cosine law to determine equation of theta 2

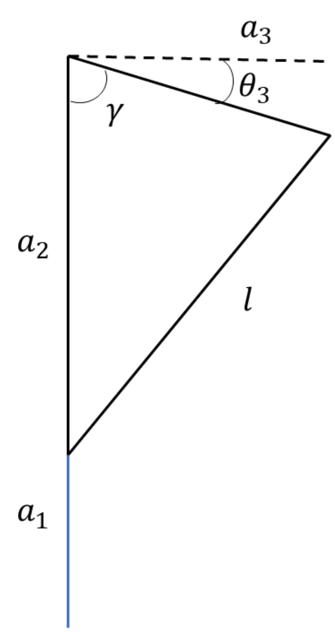


Figure 3. Using cosine law to find theta 3 equation