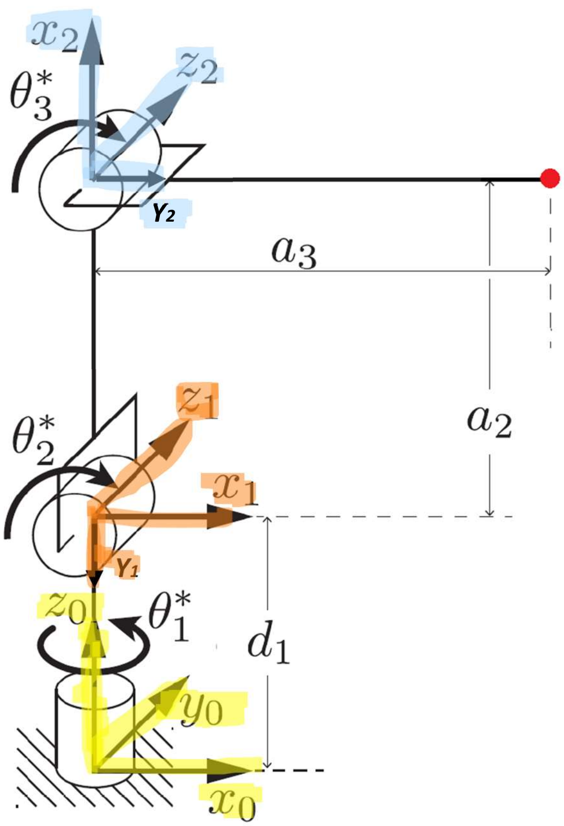
1. Write an equation for the position of the red dot at the end of the arm (i.e., the position of the center of the wrist) in frame 0 as a function of the joint variables.



Joint Variables:

- Theta1
- Theta2
- Theta3
- D_1
- A_2
- A_3

What does an equation for the position look like?

Forward Kinematics since we are determining the position of the end-effector as a function of the joint variables

1. Copy coordinate frame 3 from Lab 1 to Lab 2
2. Follow the DH convention to come up with the matrix of transformation

$$A_1^0 = \begin{bmatrix} \cos(\pi + \theta_1) & -\sin(\pi + \theta_1)\cos\left(-\frac{pi}{2}\right) & \sin(\pi + \theta_1)\sin\left(-\frac{\pi}{2}\right) & 0 \\ \sin(\pi + \theta_1) & \cos(\pi + \theta_1)\cos\left(-\frac{\pi}{2}\right) & -\cos(\pi + \theta_1)\sin\left(-\frac{\pi}{2}\right) & 0 \\ 0 & \sin\left(-\frac{\pi}{2}\right) & \cos\left(-\frac{\pi}{2}\right) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} -\cos\theta_1 & 0 & \sin\theta & 0 \\ -\sin\theta_1 & 0 & \cos\theta & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} \cos\left(\frac{\pi}{2} + \theta_2\right) & -\sin\left(\frac{\pi}{2} + \theta_2\right)\cos(0) & \sin\left(\frac{\pi}{2} + \theta_2\right)\sin(0) & -a_2 * \cos\left(\frac{\pi}{2} + \theta_2\right) \\ \sin\left(\frac{\pi}{2} + \theta_2\right) & \cos\left(\frac{\pi}{2} + \theta_2\right)\cos(0) & -\cos\left(\frac{\pi}{2} + \theta_2\right)\sin(0) & -a_2 * \sin\left(\frac{\pi}{2} + \theta_2\right) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} -\sin\theta_2 & -\cos\theta_2 & 0 & a_2\sin\theta_2 \\ \cos\theta_2 & -\sin\theta_2 & 0 & -a_2\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \cos(\theta_3 - \pi/2) & -\sin(\theta_3 - \pi/2)\cos(0) & \sin(\theta_3 - \pi/2)\sin(0) & -a_3 * \cos(\theta_3 - \pi/2) \\ \sin(\theta_3 - \pi/2) & \cos(\theta_3 - \pi/2)\cos(0) & -\cos(\theta_3 - \pi/2)\sin(0) & -a_3 * \sin(\theta_3 - \pi/2) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \sin\theta_3 & \cos\theta_3 & 0 & -a_3\sin\theta_3 \\ -\cos\theta_3 & \sin\theta_3 & 0 & a_3\cos\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{A_1^0 A_2^1}\, A_3^2 =$$

$$\begin{bmatrix} \cos\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_2 & \sin\theta_1 & -a_2\cos\theta_1\sin\theta_2 \\ \sin\theta_1\sin\theta_2 & \sin\theta_1\cos\theta_2 & \cos\theta_1 & -a_2\sin\theta_1\sin\theta_2 \\ -\cos\theta_2 & \sin\theta_2 & 0 & d_1 + a_2\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Product:

$$\begin{bmatrix} C_1S_2S_3 - C_1C_2C_3 & C_1S_2C_3 + C_1C_2S_3 & S_1 \; \cancel{\sin\theta} & -a_3C_1S_2S_3 + a_3C_1C_2C_3 - a_2C_1S_2 \\ S_1S_2S_3 - S_1C_2C_3 & S_1S_2C_2 + S_1C_2S_3 & C_1 & -a_3S_1S_2S_3 + a_3S_1C_2C_3 + a_2S_1S_2 \\ -C_2S_3 - S_2C_3 & -C_2C_3 + S_2S_3 & 0 & +a_3C_2S_3 + a_3S_2C_3 + d_1 + a_2C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Assume this robot has no joint limits and will not collide with itself in any configuration. As a step toward solving this robot's IK, draw a **diagram that shows how many inverse position kinematics solutions exist** in different regions around this robot; the number of solutions might be 0, 1, 2, 3, 4, . . . , ∞. Count the number of unique physical configurations; adding 2π to a joint angle is a trivial modification of a solution, so it does not increase the number of solutions.

Since the wrist center is in the reachable workspace of the 2-link system (L_1, L_2) so there are 2 solutions

visualized

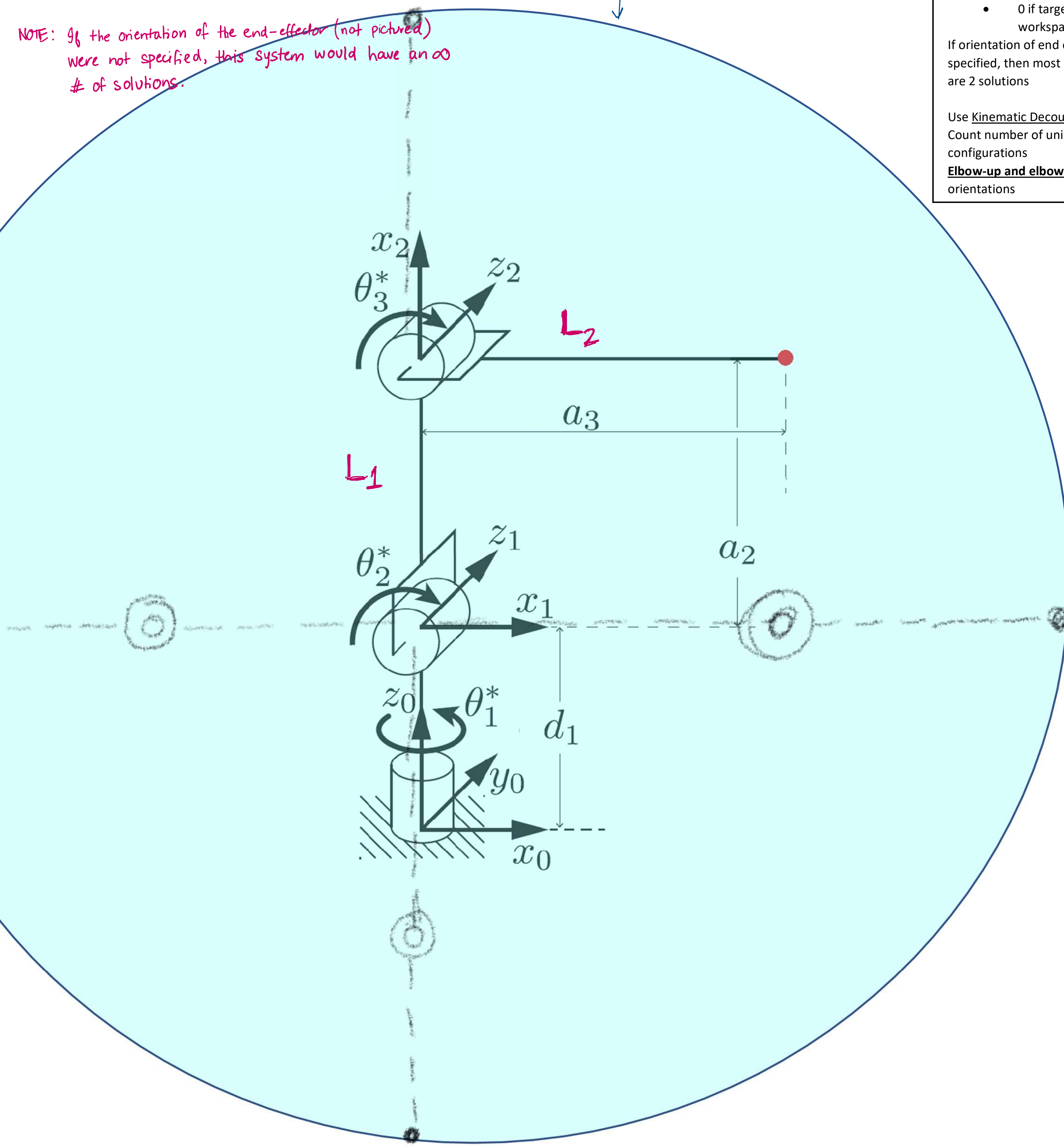
<u>How many IK solutions exist in different regions around the robot?</u>

- Infinitely many if target is in the workspace
- 1 if target is on workspace boundary
- 0 if target is outside workspace

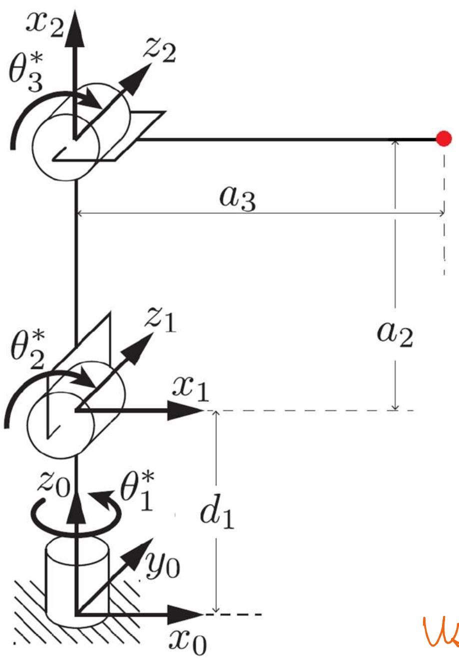
If orientation of end effector also specified, then most of the time there are 2 solutions

Use <u>Kinematic Decoupling</u> to solve
Count number of unique physical configurations
<u>Elbow-up and elbow-down</u> orientations

NOTE: If the orientation of the end-effector (not pictured) were not specified, this system would have an ∞ # of solutions.



3. Given a desired position of the red dot $[x \;\; y \;\; z]^\intercal$ for which at least one solution exists, find **all possible solutions** to this arm's inverse position kinematics. Derive closed-form equations for the joint variables in terms of x, y, and z and any needed robot parameters. Explain your steps - did you take a geometric or algebraic approach?



Using Algebraic Decomposition to solve for joint angles, given the forward transformation matrix developed in Q1.

$$O_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

	x-step		z-step	
i	a	α	d	θ
1	0	-90°	d_1	θ_1^*
2	a_2	0°	0	θ_2^*
3	a_3	0°	0	θ_3^*

$$x = -a_3 C_1 S_2 S_3 + a_3 C_1 C_2 C_3 - a_2 C_1 S_2$$

$$y = -a_3 S_1 S_2 S_3 + a_3 S_1 C_2 C_3 - a_2 S_1 S_2$$

$$z = a_3 C_2 S_3 + a_3 S_2 C_3 + d_1 + a_2 C_2$$

Using the Algebraic approach, if we find θ_1^*, θ_2^* & θ_3^* which satisfy the above system of equations, we will get all possible configurations of the robot where the end effector matches the specified pos^n & orientation.

$$\mathbf{q} = \begin{bmatrix} q_1(d_x, d_y, d_z) \\ q_2(d_x, d_y, d_z) \\ \vdots \\ q_n(d_x, d_y, d_z) \end{bmatrix}$$

Give any sets of joint angles that will produce the prescribed position.

Do not have to give angles that are identical up to the point of adding / subtracting integer multiples of 2 pi.