Lab 1: Kinematic Characterization of the Lynx

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Introduction

The objective of this lab was to create a kinematic representation of the robot from lab 0 while understanding the theoretical reachable workspace of its end effector. The robot's symbolic representation is shown in figure 1 in the zero configuration as well as the frame of references at each joint. These frames would be used to determine kinematic relations to determine the positions of each joint as well as the transformation matrix. The transformation matrix was manually determined from the prelab before it was compared with the method from lab 1. The last objective of the lab was to find the reachable workspace of the robot using a simulation and to compare it to our expectations of the reachable workspace from lab 0.

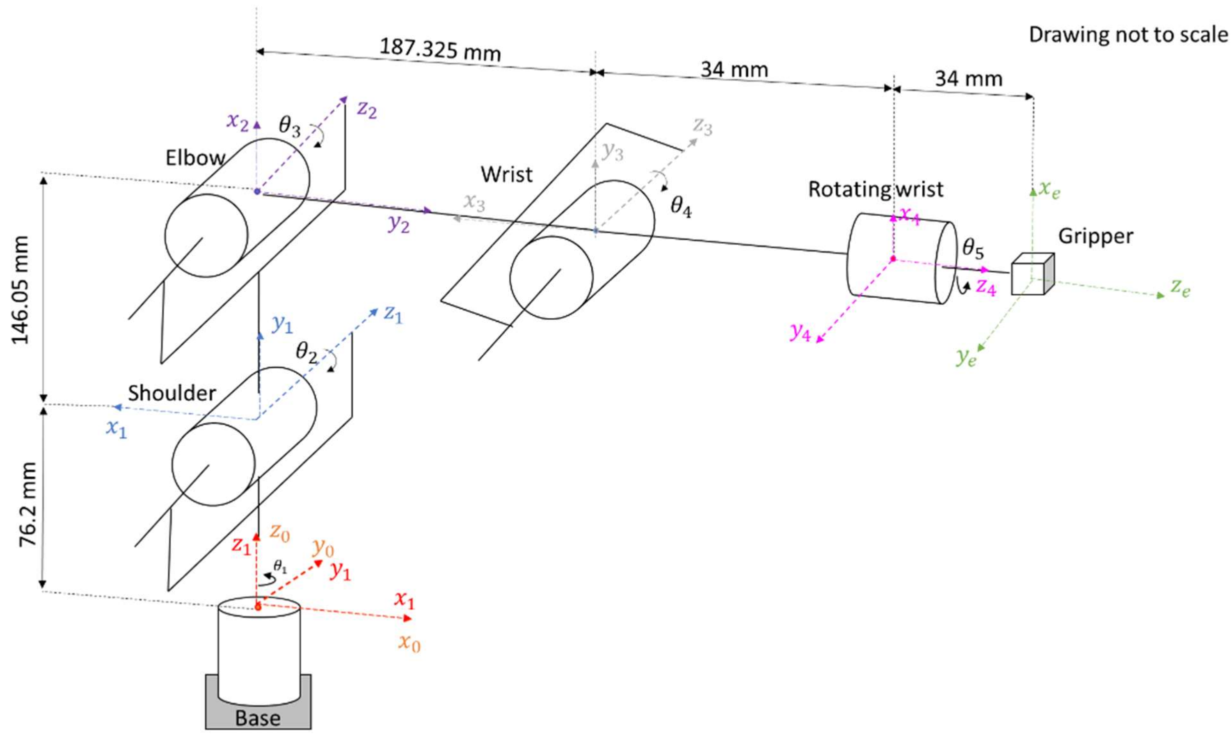

Figure 1. Robot with joint reference frames in the zero configuration

Method

Three different approaches were taken to determine the relation of the gripper's position to the universal frame 0. The first method was to determine the position geometrically. The second method involved finding the homogenous transformation matrix by using frames as shown in figure 3. The last method involved using the DH convention to establish new frames of references.

Determining Joint positions geometrically

To find the position of each joint with respect to the 0th frame, a 2D representation of the robot was developed with angles $\theta_2 - \theta_5$ having non-zero angles as shown in figure 2. From this arbitrary configuration (figure 2), expressions were developed to estimate the position at each joint of the robot. The position of each joint was found by finding its relative position relation with the previous joint first before combining the joints before it. For example, the position of joint 3 with respect to joint 2 is found to be 146sin(t2) in the horizontal direction and 146cos(t2) in the vertical direction. After this was determined, joint 1's angle (t1) was incorporated to find joint 2's location in the 0th frame. This was found to be $146.05\sin(\theta_2) * \cos(\theta_1)$ in the x_0 direction and$146.05\sin(\theta_2) * \sin(\theta_1)$ in the y_0 direction. The z direction was found to be $146.05\cos(\theta_2) + 76.2$. This was similarly done with the remaining joints and was combined to form table 1 which shows the kinematic relations that were derived. The code used for the kinematic calculation is shown in lines 46-60.

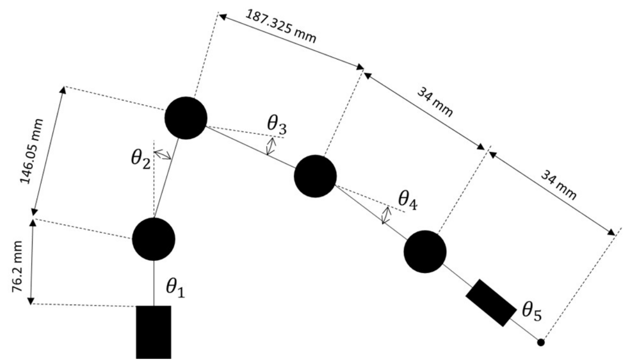

Figure 2. X-Z view of robot with non-zero theta values

Joint	x_0	y_0	z_0
1	0	0	0
2	0	0	76.2
3	$146.05\sin(\theta_2)\cos(\theta_1)$	$146.05\sin(\theta_2)\cos(\theta_1)$	$146.05\sin(\theta_2) + 76.2$
4	$\mathrm{Cos}(\theta_1)(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2))$	$\sin(\mathrm{t1})(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2))$	$146.05\cos(\theta_2) + 76.2 - 187\sin(\theta_3+\theta_2)$
5	$\mathrm{Cos}(\theta_1)(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2) + 34\cos(\theta_4 + \theta_3+\theta_2))$	$\mathrm{Sin}(\theta_1)(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2) + 34\cos(\theta_4 + \theta_3+\theta_2))$	$146.05\cos(\theta_2) + 76.2 - 187\sin(\theta_3+\theta_2) + 34\sin(\theta_4+\theta_3+\theta_2)$
6/e	$\mathrm{Cos}(\theta_1)(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2) + 68\cos(\theta_4 + \theta_3+\theta_2))$	$\sin(\theta_1)(187.325\cos(\theta_3+\theta_2) + 146.05\sin(\theta_2) + 68\cos(\theta_4 + \theta_3+\theta_2))$	$146.05\cos(\theta_2) + 76.2 - 187\sin(\theta_3+\theta_2) + 68\sin(\mathrm{t4}+\theta_3+\theta_2)$

Table 1. Joint position kinematic relations with reference to the 0th frame

Determining Joint positions with a transformation matrix

Transformation matrices were also developed to describe the representation of the joint's positions in space. By using the same figure 3 and its frame orientations, homogeneous transformation matrices were developed for each pair of joints before post-multiplying to find the final joint matrix.

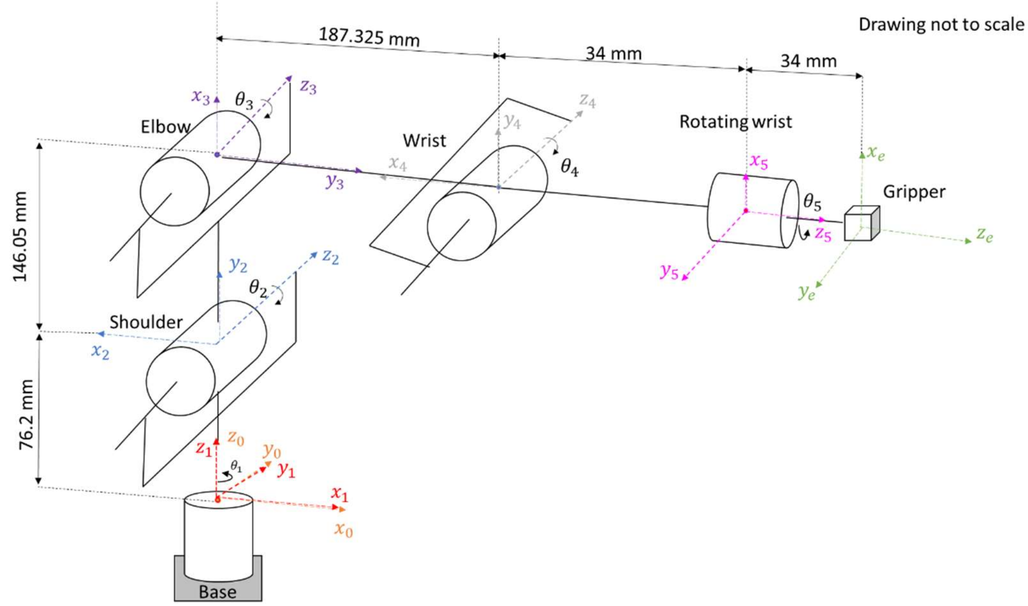

Figure 3. Figure with frames used to find the homogeneous transformation matrix

The transformation matrices for each pair of joints is shown below:

$$H_1^0 = \begin{bmatrix} \cos(\theta_1) & \cos(\theta_1 + \pi/2) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} -\cos(\theta_2) & \cos(\theta_2 + \pi/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 76.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} -\sin(\theta_3) & -\cos(\theta_3) & 0 & 0 \\ \cos(\theta_3) & -\sin(\theta_3) & 0 & 146.05 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^3 = \begin{bmatrix} \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ -\cos(\theta_4) & \sin(\theta_4) & 0 & 187.325 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5^4 = \begin{bmatrix} 0 & 0 & -1 & -34 \\ \cos(\theta_5) & 0 & 0 & 0 \\ -\sin(\theta_5) & -\cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_e^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 34 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determining a transformation matrix using DH convention

We chose to use DH convention to assign the coordinate frames to each joint because it:

- Simplifies the kinematic analysis by representing each homogenous transformation as a product of four basic transformations
- Provides a universal language for engineers to communicate, using 4 basic parameters: link length, link twist, link offset and joint angle

We followed these steps to locate the coordinate frames (from SHV p110-111):

Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.

For $i = 1, \ldots, n-1$, perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n-th joint is revolute, set $z_n = a$ parallel to z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.

Once we created and verified the coordinate frames for each joint, we followed the below procedure to identify the four basic parameters (link length, link twist, link offset and joint angle) for each joint:

Step 7: Create a table of DH parameters a_i, d_i, α_i, θ_i.

$a_i =$ distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

$d_i =$ distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

$\alpha_i =$ the angle from z_{i-1} to z_i measured about x_i.

$\theta_i =$ the angle from x_{i-1} to x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

The representation of the robot with the new reference frames is shown below:

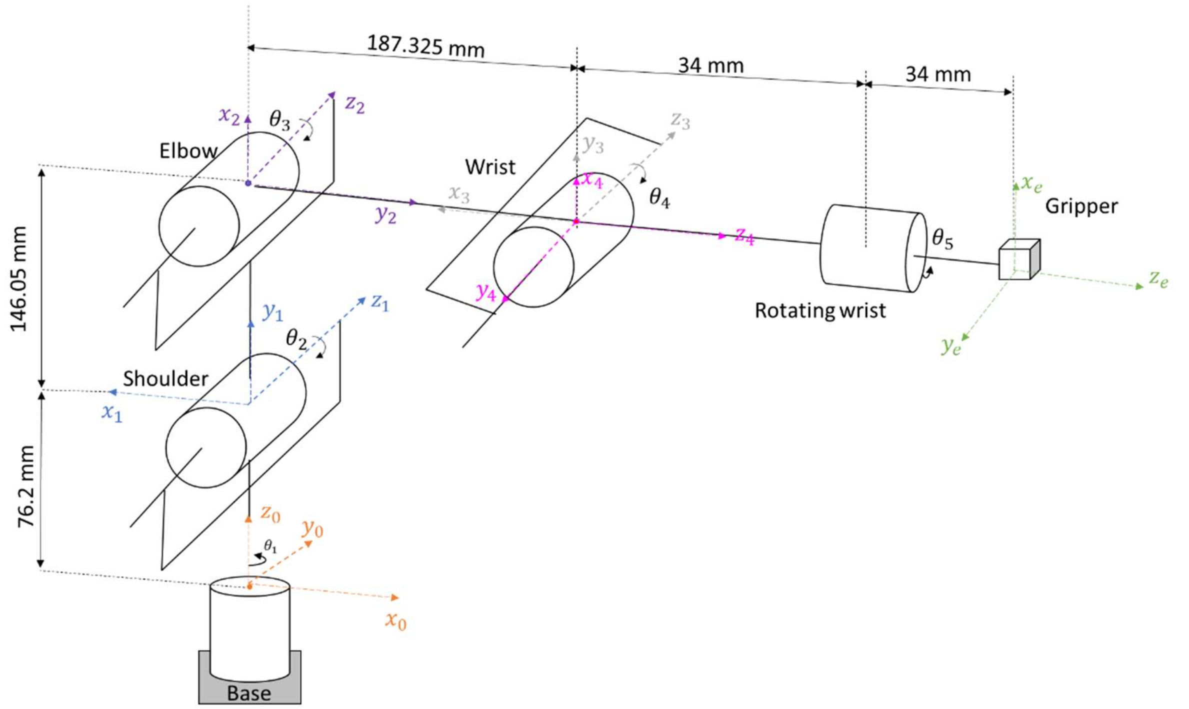

Figure 4. Reference frames used for DH convention

Link number	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76.2	$\pi + \theta_1{}^*$
2	146.05	0	0	$\pi/2 + \theta_2{}^*$
3	-187.325	0	0	$\theta_3 - \pi/2^*$
4	0	$-\pi/2$	0	$\theta_4 + \pi/2^*$
e	0	0	68	θ_5

Table 2. DH table

We then used the generalization of the post-multiplication rule to create our transformation matrices:

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} \cos(\pi + \theta_1) & -\sin(\pi + \theta_1)\, cos(-pi/2) & \sin(\pi + \theta_1)\, sin(-\pi/2) & 0 \\ \sin(\pi + \theta_1) & \cos(\pi + \theta_1)\, cos(-\pi/2) & -\cos(\pi + \theta_1)\, sin(-\pi/2) & 0 \\ 0 & sin(-\pi/2) & cos(-\pi/2) & 76.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} \cos(\pi/2 + \theta_2) & -\sin(\pi/2 + \theta_2)\cos(0) & \sin(\pi/2 + \theta_2)\sin(0) & -146.05 * \cos(\pi/2 + \theta_2) \\ \sin(\pi/2 + \theta_2) & \cos(\pi/2 + \theta_2)\cos(0) & -\cos(\pi/2 + \theta_2)\sin(0) & -146.05 * \sin(\pi/2 + \theta_2) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \cos(\theta_3 - \pi/2) & -\sin(\theta_3 - \pi/2)\cos(0) & \sin(\theta_3 - \pi/2)\sin(0) & -187.325 * \cos(\theta_3 - \pi/2) \\ \sin(\theta_3 - \pi/2) & \cos(\theta_3 - \pi/2)\cos(0) & -\cos(\theta_3 - \pi/2)\sin(0) & -187.325 * \sin(\theta_3 - \pi/2) \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^4 = \begin{bmatrix} \cos(\theta_4 + \pi/2) & -\sin(\theta_4 + \pi/2)\cos(-\pi/2) & \sin(\theta_4 + \pi/2)\sin(-\pi/2) & 0 \\ \sin(\theta_4 + \pi/2) & \cos(\theta_4 + \pi/2)\cos(-\pi/2) & -\cos(\theta_4 + \pi/2)\sin(-\pi/2) & 0 \\ 0 & \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_e^4 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5)\cos(0) & \sin(\theta_5)\sin(0) & 0 \\ \sin(\theta_5) & \cos(\theta_5)\cos(0) & -\cos(\theta_5)\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 68 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To program the transformation matrices in Python, we utilized the sin, cosine, zeroes function from the `numpy` library. We accessed the theta values for each joint by indexing into the q array and used the constants `self.L1 -L5` for link length and link offset.

Overview of `calculateFK.py`

- Lines 38-39: initialize empty matrices for `jointPositions` and `T0e`
- Lines 43-60: populate `jointPositions` matrix with the analytically computed result
- Line 65-128: transformation matrices between adjacent frames
- Line 130: post-order matrix multiplication of transformation matrices to arrive at `T0e`

How the reachable workspace was simulated

The workspace was simulated by using python. The Matplotlib library was used to plot the x-y, x-y and y-z planes of the reachable workspace. It was done by creating nested for loops which ran between joint limits. The iterative process ran through the loops with the angle increment of 0.05 radians and collected the x, y and z values using the kinematic equations from the table above for joint 6. After collecting all the x, y and z data points, scatter plots were created to represent the workspace.

Results

Zero joint angles

We obtain the following homogenous transformation and joint positions for the zero pose. Note that the simulation **T0e** matches the predicted and our transformation matrix from the pre-lab.

```
Simulation T0e =
[[   0.      -0.       1.     255.325]
 [   0.      -1.      -0.      -0.   ]
 [   1.       0.      -0.     222.25 ]
 [   0.       0.       0.       1.   ]]
Predicted =
[[  -0.      -0.       1.     255.325]
 [  -0.      -1.      -0.      -0.   ]
 [   1.      -0.       0.     222.25 ]
 [   0.       0.       0.       1.   ]]
```

```
Simulation Joint Positions =
[[   0.       0.       0.   ]
 [   0.       0.      76.2  ]
 [   0.       0.     222.25 ]
 [ 187.325   -0.     222.25 ]
 [ 221.325   -0.     222.25 ]
 [ 255.325   -0.     222.25 ]]
Predicted Joint Positions =
[[   0.       0.       0.   ]
 [   0.       0.      76.2  ]
 [   0.       0.     222.25 ]
 [ 187.325    0.     222.25 ]
 [ 221.325    0.     222.25 ]
 [ 255.325    0.     222.25 ]]
```

Evaluate specific joint angles

Input 1:

$$q = [\pi/4, 0, 0, 0, 0, 0]$$

The results match our expected transformation matrices from the pre-lab.

Pre-Lab T0e

$$\begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 180.542 \\ 0 & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 180.542 \\ 1 & 0 & 0 & 222.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
Simulation T0e =
[[  -0.       0.707    0.707  180.542]
 [   0.      -0.707    0.707  180.542]
 [   1.       0.      -0.     222.25 ]
 [   0.       0.       0.       1.   ]]
```

Input 2:

$$q = [-\pi/2, 0, \pi/4, 0, \pi/2, 0]$$

The predicted results match our expected transformation matrices from the pre-lab.

Pre-Lab T0e

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & -180.542 \\ 0 & \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & 41.71 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
Simulation T0e =
[[  -1.       0.       0.      -0.   ]
 [  -0.       0.707   -0.707 -180.542]
 [  -0.      -0.707   -0.707   41.708]
 [   0.       0.       0.       1.   ]]
```

Input 3:

$$q = [0, 0, 0, 1.7, 0, 0]$$

The simulation results match our predicted transformation matrices.

```
Simulation T0e =
[[   0.992   -0.       -0.129  178.564]
 [   0.      -1.        0.      -0.   ]
 [  -0.129   -0.       -0.992  154.817]
 [   0.       0.        0.       1.   ]]
Predicted =
[[   0.992   -0.       -0.129  178.564]
 [  -0.      -1.        0.      -0.   ]
 [  -0.129   -0.       -0.992  154.817]
 [   0.       0.        0.       1.   ]]
```

```
Simulation Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [   0.       -0.      222.25 ]
 [ 187.325   -0.      222.25 ]
 [ 182.945   -0.      188.534]
 [ 178.564   -0.      154.817]]
Predicted Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [   0.        0.      222.25 ]
 [ 187.325    0.      222.25 ]
 [ 182.944    0.      188.533]
 [ 178.564    0.      154.817]]
```

Input 4:

$$q = [0, 1.4, 0, 0, 0, 0]$$

The simulation results match our predicted transformation matrices.

```
Simulation T0e =
[[   0.985    0.        0.17   187.32 ]
 [   0.      -1.        0.       0.001]
 [   0.17     0.       -0.985 -150.59 ]
 [   0.       0.        0.       1.   ]]
Predicted =
[[   0.985   -0.        0.17   187.322]
 [  -0.      -1.        0.      -0.   ]
 [   0.17    -0.       -0.985 -150.586]
 [   0.       0.        0.       1.   ]]
```

```
Simulation Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [ 143.926    0.      101.02 ]
 [ 175.762    0.001   -83.58 ]
 [ 181.541    0.001 -117.085]
 [ 187.32     0.001 -150.59 ]]
Predicted Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [ 143.925    0.      101.024]
 [ 175.764    0.      -83.576]
 [ 181.543    0.     -117.081]
 [ 187.322    0.     -150.586]]
```

Input 5:

$$q = [0, 1.4, -1.8, 0, 0, 0]$$

The simulation results match our predicted transformation matrices.

```
Simulation T0e =
[[  -0.389   -0.        0.921  379.095]
 [  -0.      -1.       -0.      -0.   ]
 [   0.921   -0.        0.389  200.452]
 [   0.       0.        0.       1.   ]]
Predicted =
[[  -0.389   -0.        0.921  379.095]
 [  -0.      -1.       -0.      -0.   ]
 [   0.921   -0.        0.389  200.452]
 [   0.       0.        0.       1.   ]]
```

```
Simulation Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [ 143.925   -0.      101.022]
 [ 316.464   -0.      173.967]
 [ 347.78    -0.      187.208]
 [ 379.096   -0.      200.449]]
Predicted Joint Positions =
[[   0.        0.        0.   ]
 [   0.        0.       76.2  ]
 [ 143.925    0.      101.024]
 [ 316.463    0.      173.971]
 [ 347.779    0.      187.212]
 [ 379.095    0.      200.452]]
```

Workspace plots

X-Y plot:

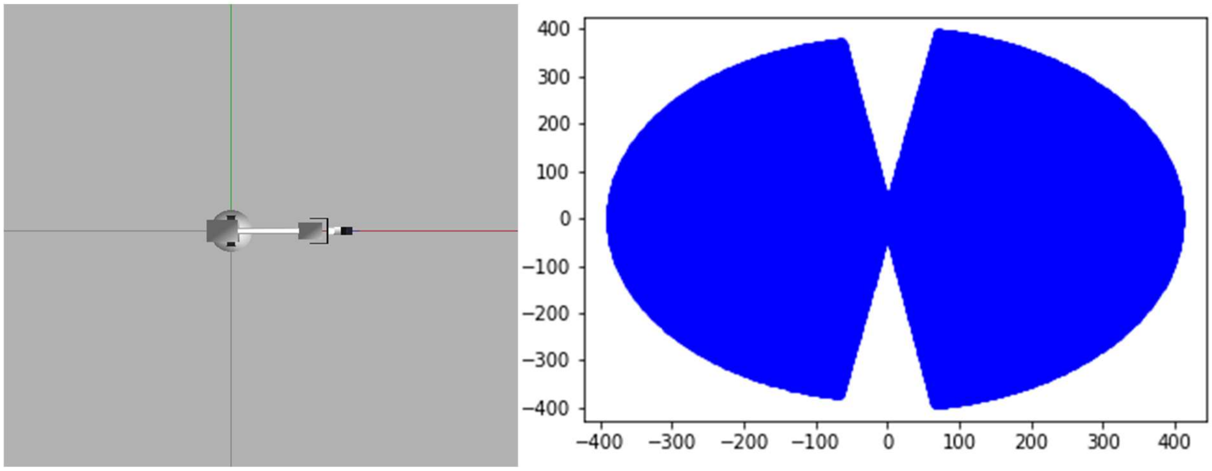

Figure 5. X-Y plot of reachable workspace

X-Z plot:

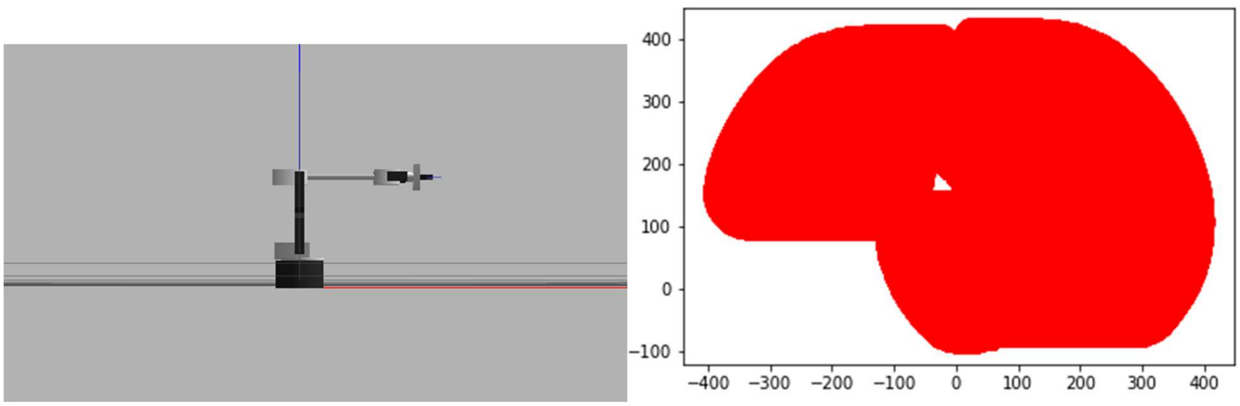

Figure 6. X-Z plot of reachable workspace

Y-Z plot:

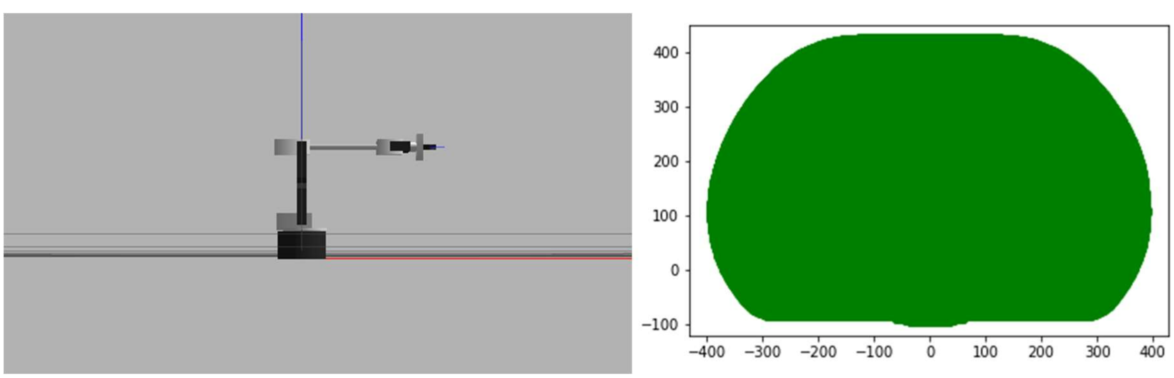

Figure 7. Y-Z plot of reachable workspace

Discussion

The robotic model was characterized geometrically and through the usage of transformation matrices. The transformation matrices were found yielded the same joint positions when comparing to each other and the kinematic relation. In each of the inputs specified, the joint positions were identical with +-0.005 accuracy. The transformation matrix T_0^e was also identical with each input given, which proves the validity of the transformation matrices and kinematic relations that were determined analytically.

When plotting the workspace, it could be seen in figure 5 that there was a blind spot where the gripper could not reach in white. When completing lab 0, this was not expected. Only the simulation was able to show this representation. It would have been geometrically difficult to determine this without using a simulation tool. It was noticed that the step size that was used to plot figures 4-6 was important to understand the reachability. The resolution of the charts was dependent on having a small angle step size for each iteration. However, if the step size was made too small, the computation time became too large.

We chose our angles in the evaluation section to be at the boundary of the reachable workspace for the robot. We did so to better understand where self-collisions occur and help us visualize the orientation of the robot at the workspace boundary at locations such as the region near the robot base described above.

Self-collisions are hard to detect from the joint position calculation since it does not tell us when links collide. Hence, we tried to push the robot to near its joint limits to visually find any potential self-collisions in the reachable workspace.