Distances and Similarities

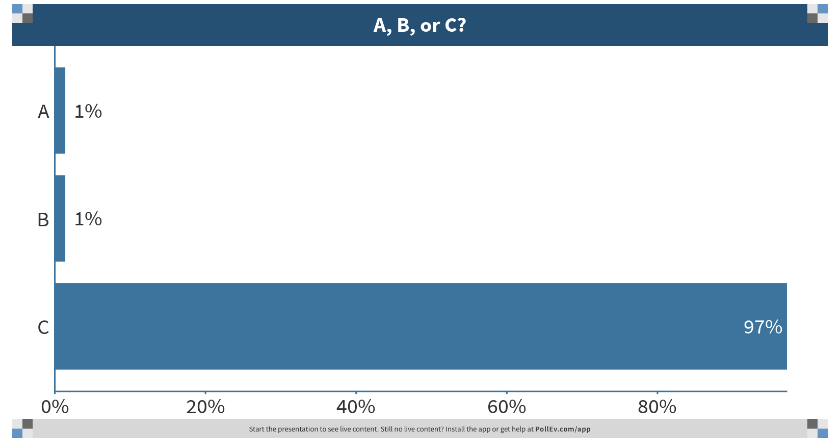
- **Distances**
  - What properties do they have?
- **Similarities**
  - How have we computed them?

KL-Divergence

A) **Distance**

B) **Similarity**

C) **Neither**



$$D_{\mathrm{KL}}(P\|Q) = -\sum_i P(i) \log \frac{Q(i)}{P(i)},$$

KL divergence properties

- **Non-negative*****: $D(P||Q) \geq 0$***
- **Divergence *0* if and only if *P* and *Q* are equal*:***
  - $D(P||Q) = 0$ *iff* $P = Q$
- **Non-symmetric*****: $D(P||Q) \neq D(Q||P)$***
- **Does not satisfy triangle inequality**
  - $D(P||Q) \leq D(P||R) + D(R||Q)$

Not a distance metric

Kullback Leibler divergence

- ***P*** = true distribution;
- ***Q*** = alternative distribution that is used to encode data
- **KL** divergence is the expected extra message length per datum that must be transmitted using ***Q***

$$D_{KL}(P \| Q) = \Sigma_i P(x_i) \log (P(x_i)/Q(x_i))$$

$$= - \Sigma_i P(x_i) \log Q(x_i) + \Sigma_i P(x_i) \log P(x_i)$$

$$= H(P,Q) \qquad - H(P)$$

= Cross-entropy - entropy

- **Measures how different the two distributions are**

KL divergence as info gain

- **The KL divergence of the posteriors measures the information gain expected from query (x')*:***

$$D(\, p(\theta \mid x, x') \,||\, p(\theta \mid x))$$

- **Goal: choose a query that *maximizes* the KL divergence between the updated posterior probability and the current posterior probability**
  - This represents the largest expected information gain