Boosting

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Learning objectives
Review stagewise regression
Know adaboost and
gradient boosting algorithms

Stagewise Regression

- **Sequentially learn the weights α_t**
  - Never readjust previously learned weights
  - $h(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})$

h(**x**) = *0* **or** *average*(**y**)

For *t* =*1:T*

$\mathbf{r}_t = \mathbf{y} - h(\mathbf{x})$

regress $\mathbf{r}_t = \alpha_t h(\mathbf{x})$ **to find** α_t

$h(\mathbf{x}) = h(\mathbf{x}) + \alpha_t h_t(\mathbf{x})$

Boosting

- **Ensemble method**
  - Weighted combination of weak learners $h_t(\boldsymbol{x})$

$$h(\mathbf{x}) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right)$$

- **Learned stagewise**
  - At each stage, boosting gives more weight to what it got wrong before

Adaboost

Given: n examples $(\mathbf{x}_i, y_i)$, where $\mathbf{x} \in \mathcal{X}, y \in \pm 1$.

Initialize: $D_1(i) = \frac{1}{n}$

For $t = 1 \ldots T$

- Train weak classifier on distribution $D(i)$, $h_t(\mathbf{x}) : \mathcal{X} \mapsto \pm 1$
- Choose weight α_t (see how below)
- Update: $D_{t+1}(i) = \frac{D_t(i) \exp\{-\alpha_t y_i h_t(\mathbf{x}_i)\}}{Z_t}$, for all i, where $Z_t = \sum_i D_t(i) \exp\{-\alpha_t y_i h_t(\mathbf{x}_i)\}$

Output classifier: $h(\mathbf{x}) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right)$

Where α_t is the log-odds of the weighted probability of the prediction being wrong

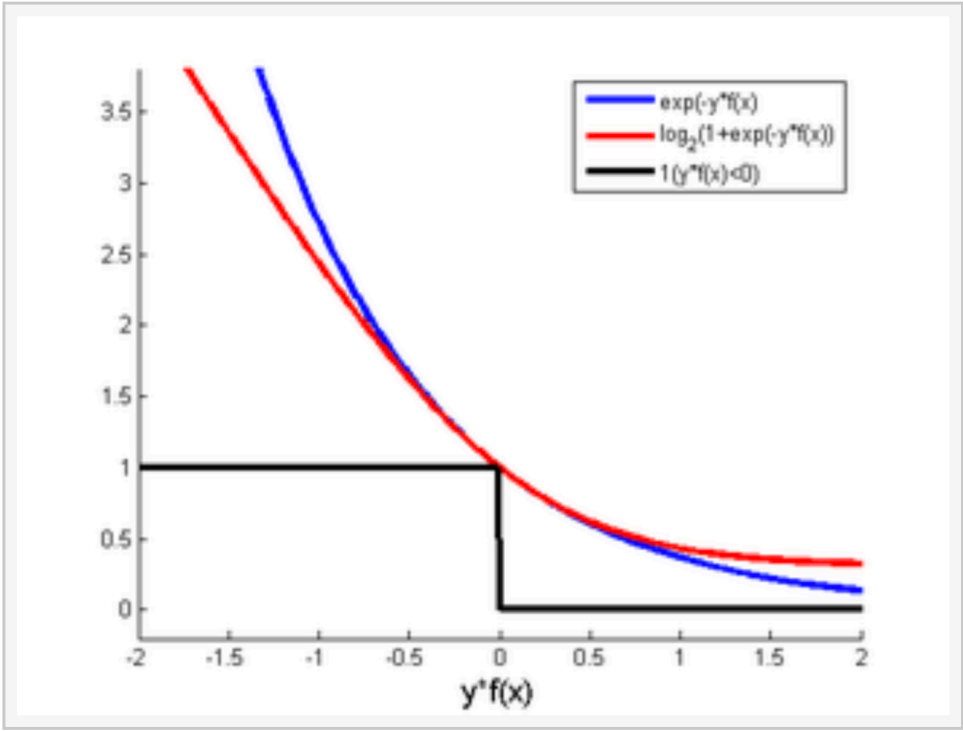
$$\alpha_t = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t} \qquad \epsilon_t = \sum_i D_t(i) \mathbf{1}(y_i \neq h_t(\mathbf{x_i}))$$

Adaboost example

- https://alliance.seas.upenn.edu/~cis520/dynamic/2019/wiki/index.php?n=Lectures.Boosting

Adaboost minimizes exponential loss

Boosting : $\exp(-y_i f_\alpha(\mathbf{x}_i))$ **Logistic** : $\log(1+\exp(-y_i f_\mathbf{w}(\mathbf{x}_i)))$



And it learns it exponentially fast

$$\frac{1}{n}\sum_i \mathbf{1}(y_i \neq h(\mathbf{x}_i)) \leq \prod_{t=1}^{T} Z_t \leq \exp\{\sum_t -2(0.5-\epsilon_t)^2\} \leq \exp\{-2T\gamma^2\}$$

Average Error

where $\gamma = \min_t(0.5 - \epsilon_t)$.

Exponential in stages T and the accuracy of the weak learner γ

Gradient Tree Boosting

- **Current state-of-the-art for moderate-sized data sets**
  - i.e. on average very slightly better than random forests when you don't have enough data to do deep learning

Gradient Boosting

- **Model:** $F(\mathbf{x}) = \Sigma_i \gamma_i h_i(\mathbf{x}) + const$
- **Loss function:** $L(\mathbf{y}, F(\mathbf{x}))$
  - L_2 or logistic or …
- **Base learner:** $h_i(\mathbf{x})$
  - Decision tree of specified depth
- **Optionally subsample features**
  - "stochastic gradient boosting"
- **Do stagewise estimation of F(x)**
  - Estimate $h_i(x)$ *and* γ_i at each iteration i

1. Initialize model with a constant value:

$$F_0(x) = \arg\min_{\gamma} \sum_{i=1}^{n} L(y_i, \gamma).$$

2. For m = 1 to M:

   1. Compute so-called *pseudo-residuals*:

$$r_{im} = -\left[\frac{\partial L(y_i, F(x_i))}{\partial F(x_i)}\right]_{F(x)=F_{m-1}(x)} \quad \text{for } i = 1, \ldots, n.$$

   For squared error, this is just the standard residual

   2. Fit a base learner (e.g. tree) $h_m(x)$ to pseudo-residuals, i.e. train it using the training set $\{(x_i, r_{im})\}_{i=1}^{n}$.
   3. Compute multiplier γ_m by solving the following one-dimensional optimization problem:

$$\gamma_m = \arg\min_{\gamma} \sum_{i=1}^{n} L\left(y_i, F_{m-1}(x_i) + \gamma h_m(x_i)\right).$$

   4. Update the model:

$$F_m(x) = F_{m-1}(x) + \gamma_m h_m(x).$$

3. Output $F_M(x)$.

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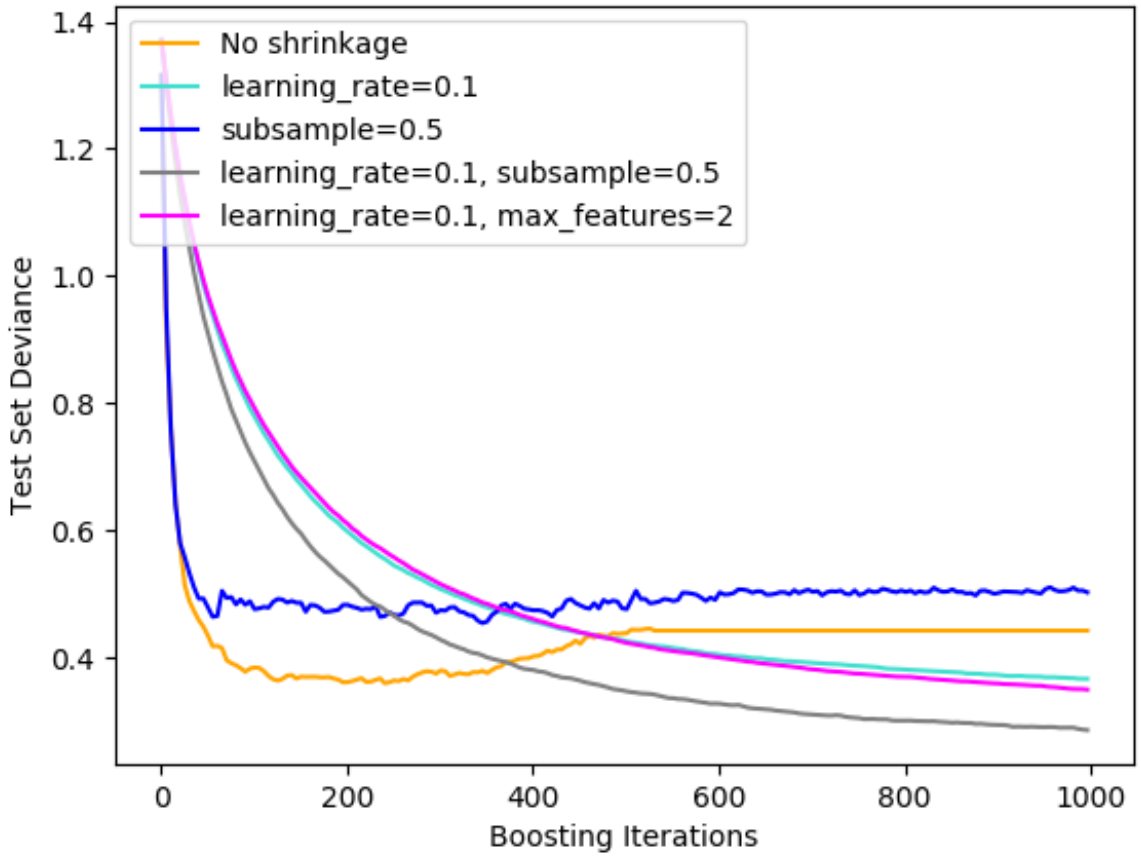
https://en.wikipedia.org/wiki/Gradient_boosting

Gradient Tree Boosting for Regression

- **Loss function: L_2**
- **Base learners $h_i(\mathbf{x})$**
  - Fixed depth regression tree fit on pseudo-residual
  - Gives a constant prediction for each leaf of the tree
- **Stagewise: find weights on each $h_i(\mathbf{x})$**
  - Fancy version: fit different weights for each leaf of tree

Regularization helps



Subsample = stochastic gradient boosting

Learning rate = shrinkage on γ

http://scikit-learn.org/stable/auto_examples/ensemble/plot_gradient_boosting_regularization.html

What you should know

- **Boosting**
  - Stagewise regression upweighting previous errors
  - Gives highly accurate ensemble models
  - Relatively fast
  - Tends not to overfit (but still: use early stopping!)
- **Gradient Tree Boosting**
  - Uses pseudo-residuals
  - Very accurate!!!