ESE 504-542 : Statistics for Data Science
Instructor: Hamed Hassani, Shirin Saeedi
Spring 2019

Midterm Examination

NAME	

One two-sided note-sheet allowed.

	Grade (y/n)	Score	Max. Score
Problem 1			50
Problem 2			50
TOTAL			100

Problem 1 (50 points) [**Estimating Parameters of Distribution**]
Let $X_1, X_2, \ldots, X_n$ be an independent and identically distributed (i.i.d.) sample from a distribution with density function

$$f(x|\theta) = (\theta + 1)x^{\theta}, \quad 0 \leq x \leq 1.$$

1. Find the maximum likelihood estimate (MLE) of θ.

2. Find the asymptotic variance of the MLE.

3. Find a sufficient statistic for θ.

Problem 2 (50 points) [**Hypothesis testing for uniform samples**]
Let $X_1, X_2, \cdots, X_n$ be generated i.i.d. from the distribution Uniform$(0, \theta)$. We know that θ can possibly take two values θ_0 and θ_1 where $\theta_1 < \theta_0$. Our goal is to find out from the data sample which one is the true value of θ.

1. Formulate this task as a hypothesis testing problem in which the null hypothesis corresponds to the value θ_0.

2. Find the most powerful test of H_0 against the alternative hypothesis H_1 at significance level $\alpha = (\frac{\theta_1}{\theta_0})^n$.

3. What is the power of this test as a function of θ_0, θ_1, and n?

4. Consider a (not necessarily optimal) test at significance level $\alpha < (\frac{\theta_1}{\theta_0})^n$ that rejects H_0 when $X_{(n)} < k$. Find the appropriate value of k.