

3C

$$h^*(x) = \arg \min_{y \in \{-1, 1\}} P(Y=y | X=x)$$

$$\Rightarrow \quad \frac{P(X=x | Y=y) P(Y=y)}{P(X=x)}$$

$$\Rightarrow \quad P(X=x | Y=y) P(Y=y)$$

$$\Rightarrow \quad \frac{P(X=x | Y=y)}{2}$$

$$\Rightarrow \arg \min_y \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-s_y)^2}{2}}$$

( $\because$  denominator does not impact outcome)

(We can take away the  $\frac{1}{2} = P(Y=y)$  as well since it's a constant)

This results in the classifier

$$h^*(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

$$\because P(X=x | Y=-1) >$$

$$P(X=x | Y=1) \text{ when}$$

$$x < 0 \text{ \&}$$

vice versa