1

(a)

$$\sigma(-x) = \frac{1}{1+e^{x}}$$

$$\begin{aligned} 1-\sigma(x) &= \frac{e^{-x}}{1+e^{-x}} \\ &= \frac{e^{-x}e^{x}}{e^{x}(1+e^{-x})} \\ &= \frac{1}{1+e^{x}} \end{aligned}$$

(b)

$$\begin{aligned} \frac{d}{dx}\sigma(x) &= -\frac{(1+e^{-x})^{'}}{(1+e^{-x})^{2}} \\ &= \frac{e^{-x}}{1+2e^{-x}+e^{-2x}} \\ &= \frac{1}{e^{x}+2+e^{-x}} \\ &= \frac{1}{(1+e^{-x})(1+e^{x})} \\ &= \sigma(x)(1-\sigma(x)) \end{aligned}$$

2

$$f^{'}(c) = \frac{2}{n}\sum_{i=1}^{n}(c-x_i)$$

Let $f^{'}(c) = 0$, we can have

$$c = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$f^{''}(c) = 2 > 0$$

So it must be the minimum.

3

I choose 2, there is no information about religious leaders.

4

Let A represents the event of breast cancer. And let B represents the event of the positive, and we can get the following formulas.

$$\begin{aligned} P(A) &= 0.01 \\ P(B|A) &= 0.8 \\ P(B|\bar{A}) &= 0.096 \end{aligned}$$

$$\begin{aligned} P(A|B) &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} \\ &= \frac{0.8 * 0.01}{0.8 * 0.01 + 0.096 * 0.99} \\ &= 0.078 \end{aligned}$$

5

The distribution is Normal distribution.

$$\begin{aligned} \frac{1}{\sqrt{2\pi}\sigma} &= 0.063 \\ \sigma^2 &= 6.38 \end{aligned}$$

So choose B.

6

(a)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R(A) = 2$$

(b)

$$0\vec{v_1} = \vec{v_2}$$

(c)

$$10\vec{v_1} + 2\vec{v_2} = \vec{3}$$

(d)

$$\vec{v_1} - \vec{v_2} = \vec{v_3}$$