Homework 1

APM 523

Arizona State University

Problem 1

Implement the Golden Section Search algorithm for finding a minimum of of

$$f(x) = 10 + x^2 - 10\cos(2\pi x), \quad x \in [a, b],$$

where $a = 1.6$ and $b = 2.4$. You can use a tolerance $\epsilon = 10^{-4}$ and maximum iterations 100. What is $x_{\min}$ and what is $f(x_{\min})$?

Problem 2

Implement the Nelder-Mead simplex algorithm and test it on the even Rosenbrock function

$$f(x) = \sum_{i=1}^{n/2} 100(x_{2i-1}^2 - x_{2i})^2 + (1 - x_{2i-1})^2$$

You can use the following parameters

$$X_0 = \begin{bmatrix} 0 & I_n \end{bmatrix} \text{ (initial verticies)}, \quad \epsilon = 10^{-4} \text{ (stopping)}, \quad M = 1000 \text{ (max iter)}$$

$$\alpha = 1, \quad \beta = \frac{1}{2}, \quad \gamma = 1$$

1. Test the method when $n = 2$
2. Also, test the method when $n = 8$

Problem 3

The contraction of the simplex in the Nelder-Mead algorithm happens when no point improves on $x^{(n)}$. This part of the algorithm overwrites the previous vertices by

$$x^{(i)} \leftarrow \frac{1}{2}(x^{(i)} + x^{(n)}), \qquad 0 \le i \le n$$

Show that if f is a convex function, the contraction step will not increase the average value of the function over the simplex vertices defined by $\frac{1}{n+1}\sum_{i=0}^{n} f(x^{(i)})$. Show that unless $f(x^{(0)}) = f(x^{(1)}) = \cdots = f(x^{(n)})$, the average value will in fact decrease. Hint: You may use the definition of convexity: $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha)f(y)$ for all $\alpha \in [0,1]$ and here $x, y \in \mathbb{R}^n$.

Problem 4

Show that the steepest descent method with an "exact" line-search converges in one iteration to x^* when $f(x)$ is a convex quadratic function, and $x_0 - x^*$ is parallel to an eigenvector of $A \in \mathbb{R}^{n\times n}$

$$f(x) = \frac{1}{2}x^T Ax - b^T x$$

Hint: Assume that $x_0 = x^* + q$ where q is some eigenvector of A with eigenvalue λ. Moreover, the value for $\alpha_k = \min_{\alpha>0} f(x_k - \alpha\nabla f(x_k))$ in the line-search can be analytically computed.

Problem 5

Consider a logistic regression problem with two classes. Given a training set P consisting of datapoint and label pairs (z, y) where $z \in \mathbb{R}^n$ and $y \in \{-1, +1\}$, we define the objective function f (or loss) for variables $x \in \mathbb{R}^n$ (or weight vector) to be

$$f(x) = \sum_{(z,y)\in P} -\ln\left(\sigma(yx^T z)\right),$$

where $\sigma(s) = 1/(1 + exp(-s))$. One says that the weight vector x is a separator for P if for all $(z, y) \in P$, $y(x^T z) \geq 0$. A separator is said to be trivial if for all $(x, y) \in P$, $y(x^T z) = 0$. For example $x = 0$ is a trivial separator. Depending on the data P, there may be other trivial separators. Prove whether $f(x)$ is convex.