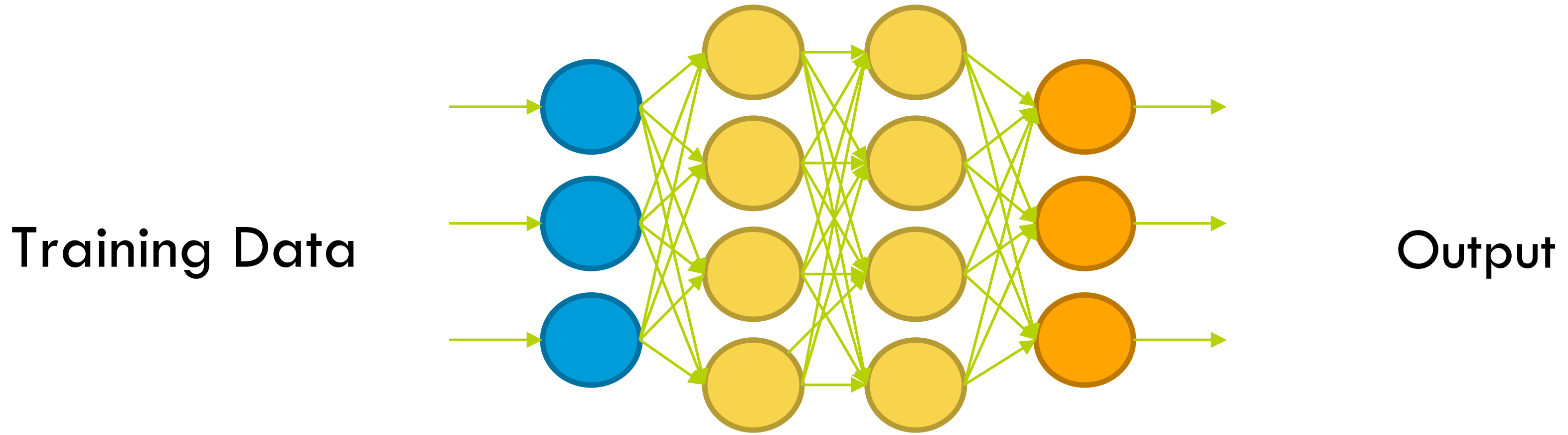
Backpropagation in Neural Nets

How to Train a Neural Net?



- Put in Training inputs, get the output
- Compare output to correct answers: Look at loss function J
- Adjust and repeat!
- Backpropagation tells us how to make a single adjustment using calculus.

How have we trained before?

- Gradient Descent!

1. Make prediction
2. Calculate Loss
3. Calculate gradient of the loss function w.r.t. parameters
4. Update parameters by taking a step in the opposite direction
5. Iterate

How have we trained before?

- Gradient Descent!

1. Make prediction
2. Calculate Loss

} 'The Forward Pass'

3. Calculate gradient of the loss function w.r.t. parameters
4. Update parameters by taking a step in the opposite direction
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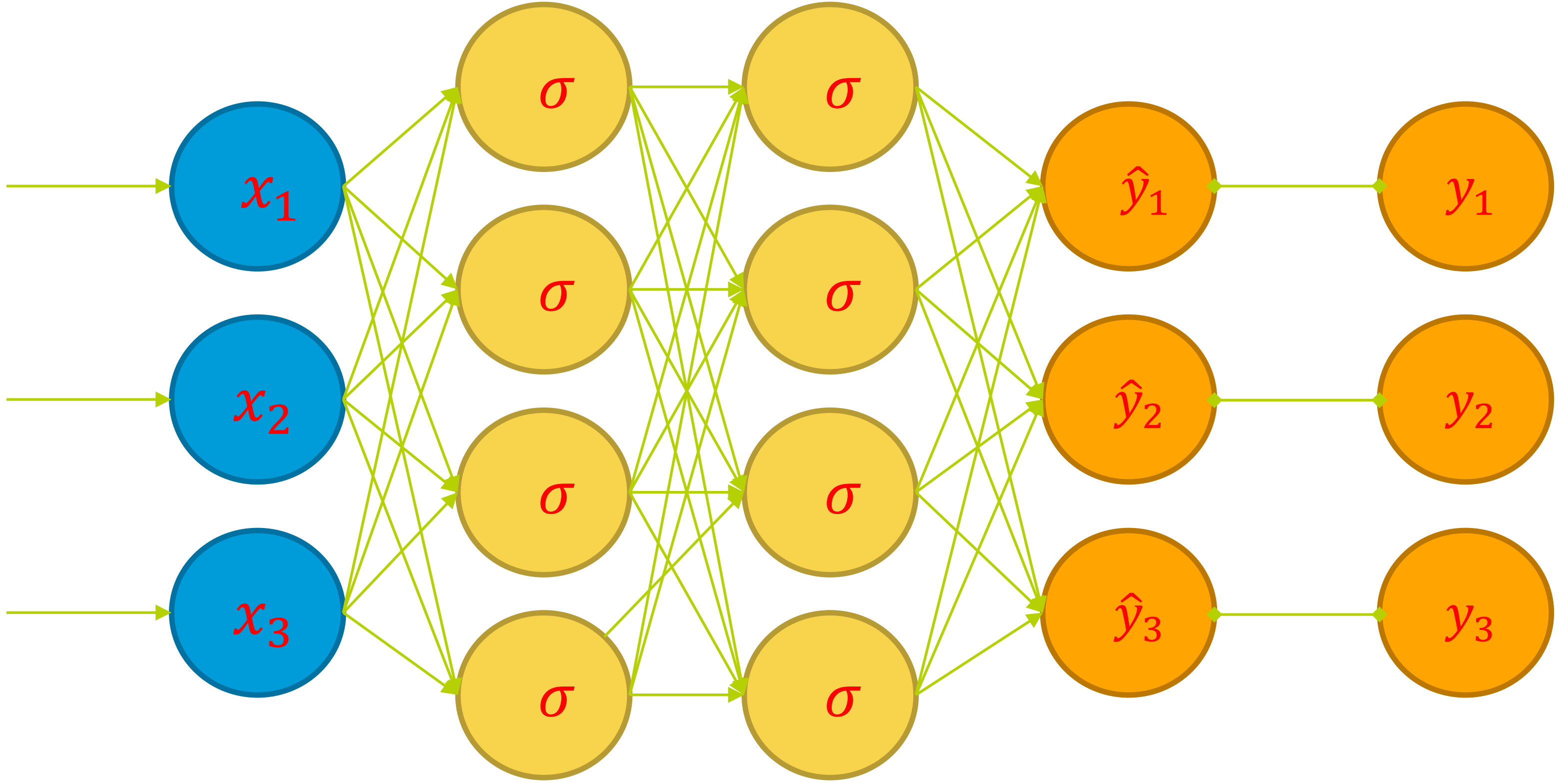
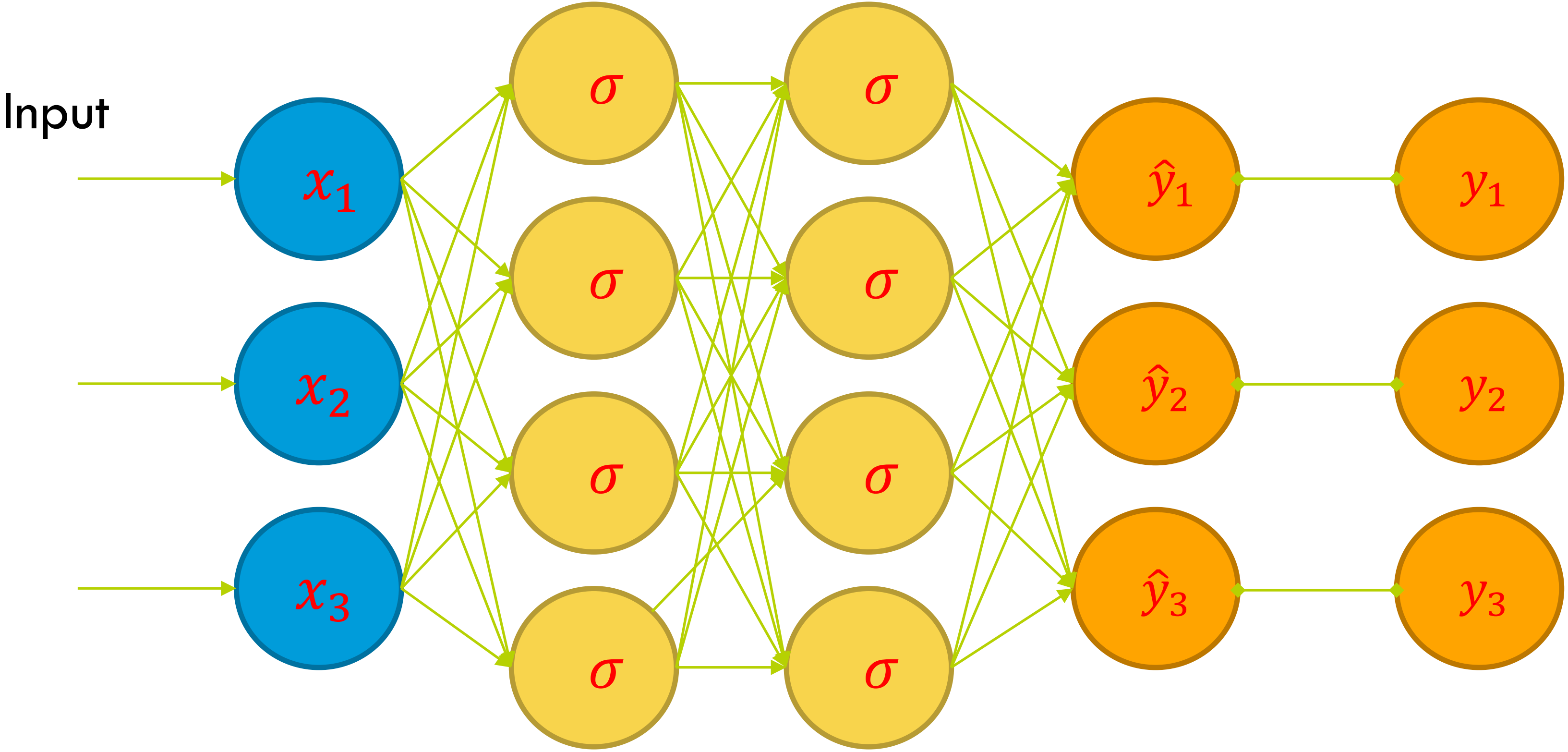
Feedforward Neural Network

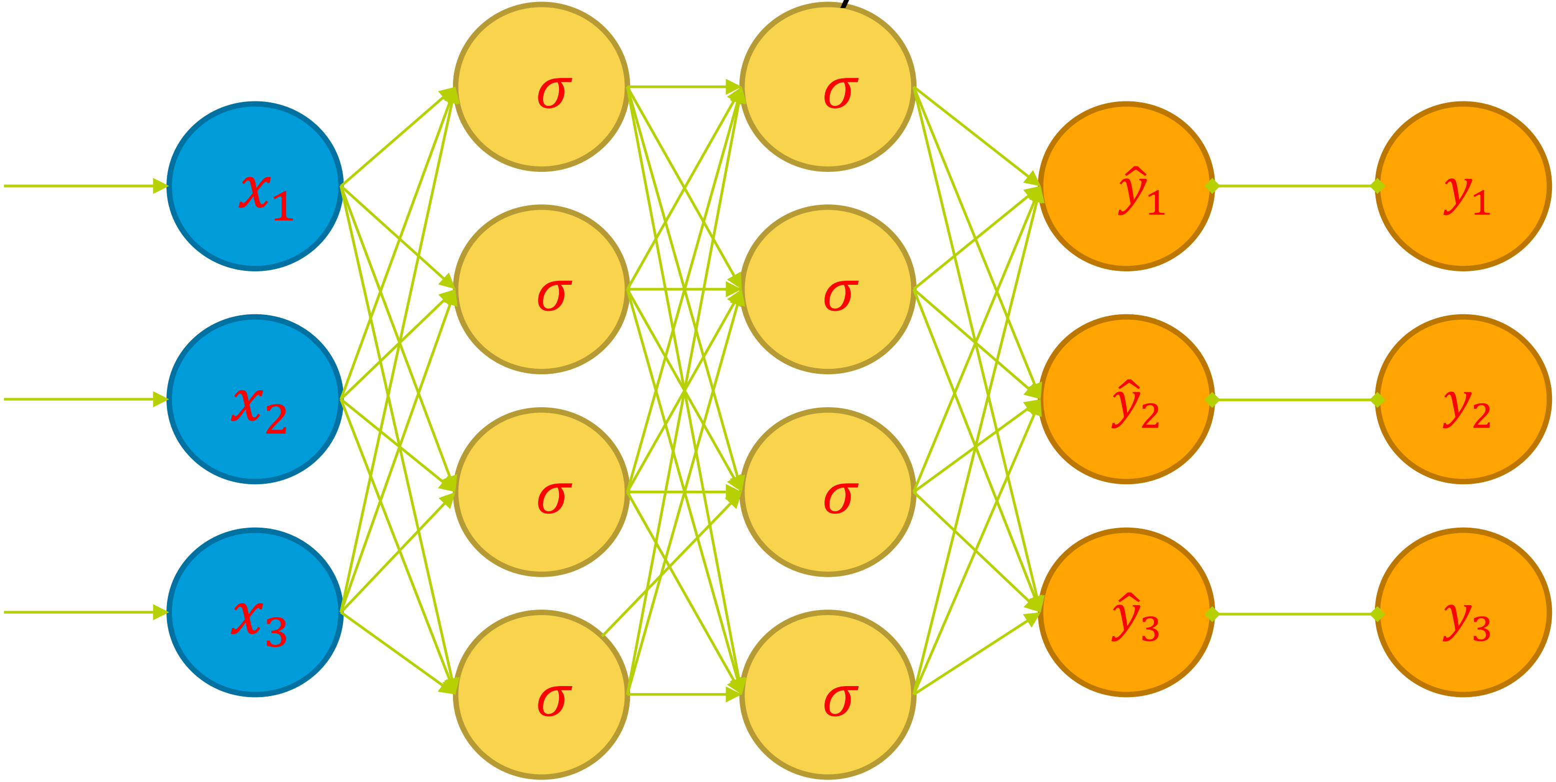


Forward Propagation

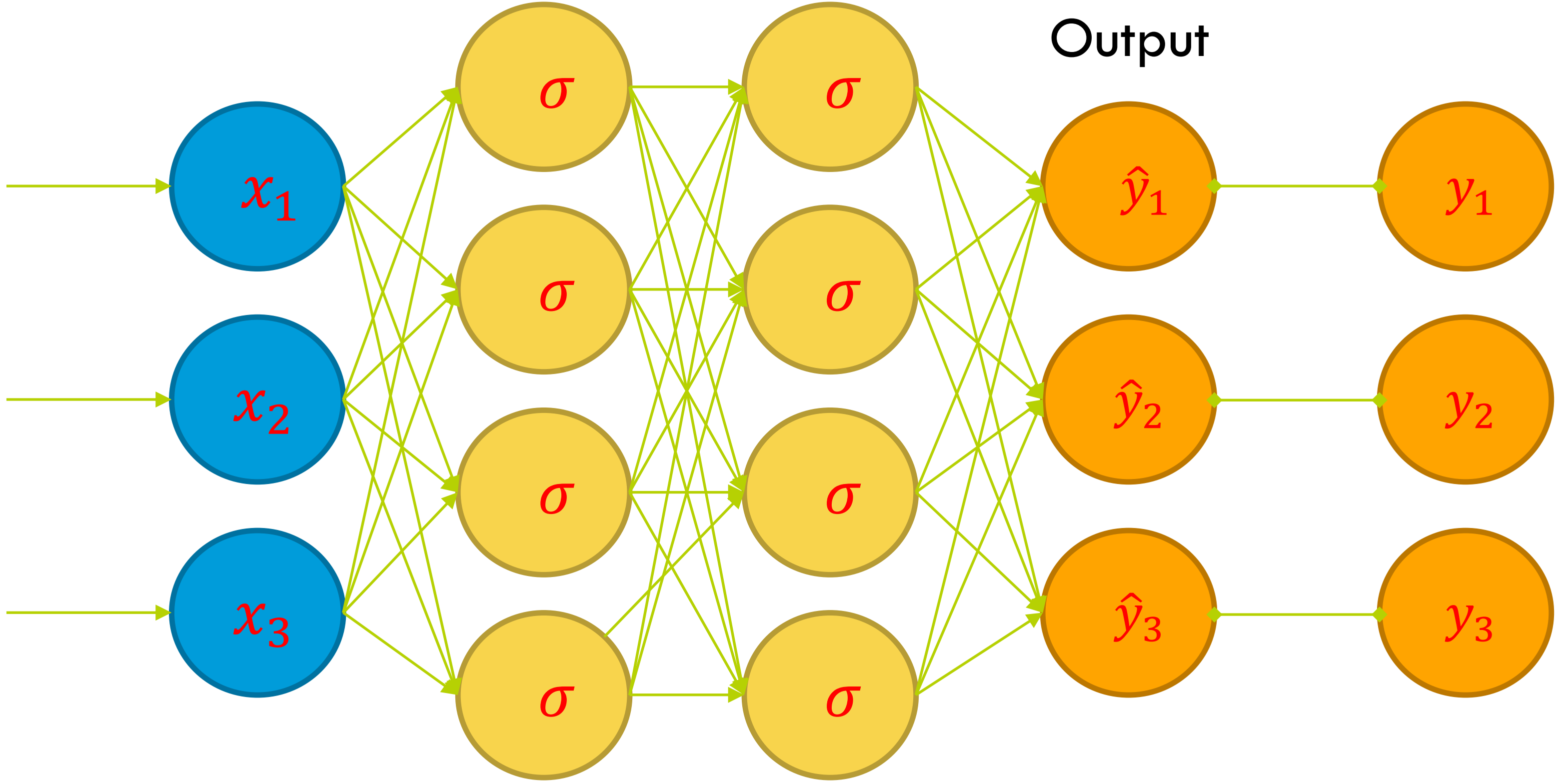


Forward Propagation

Calculate each Layer



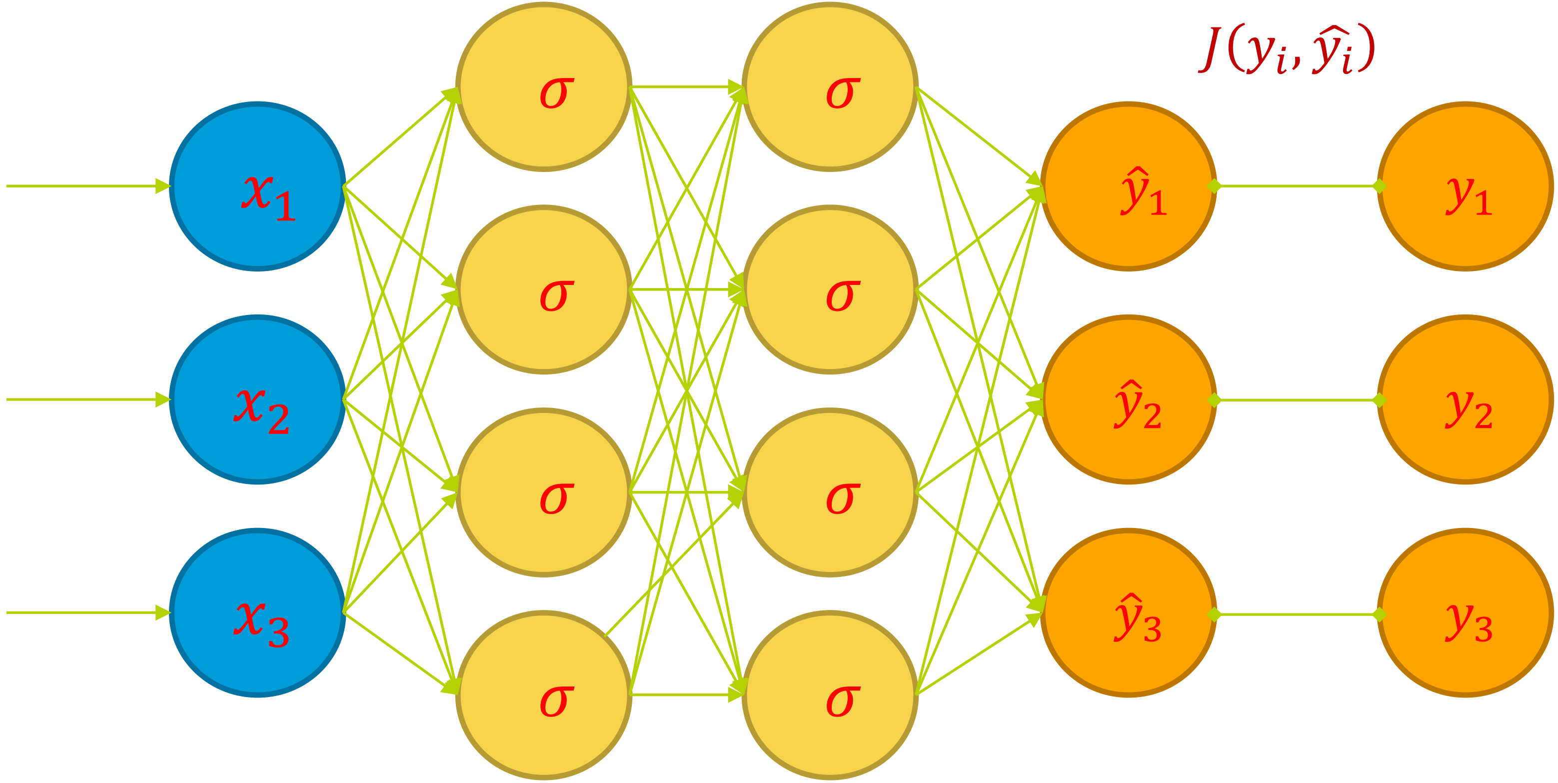
Forward Propagation



Forward Propagation

Evaluate:

$J(y_i, \hat{y_i})$



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How to calculate gradient?

- Chain rule

How to Train a Neural Net?

- How could we change the weights to make our Loss Function lower?
- Think of neural net as a function F: X -> Y
- F is a complex computation involving many weights W_k
- Given the structure, the weights "define" the function F (and therefore define our model)
- Loss Function is J(y,F(x))

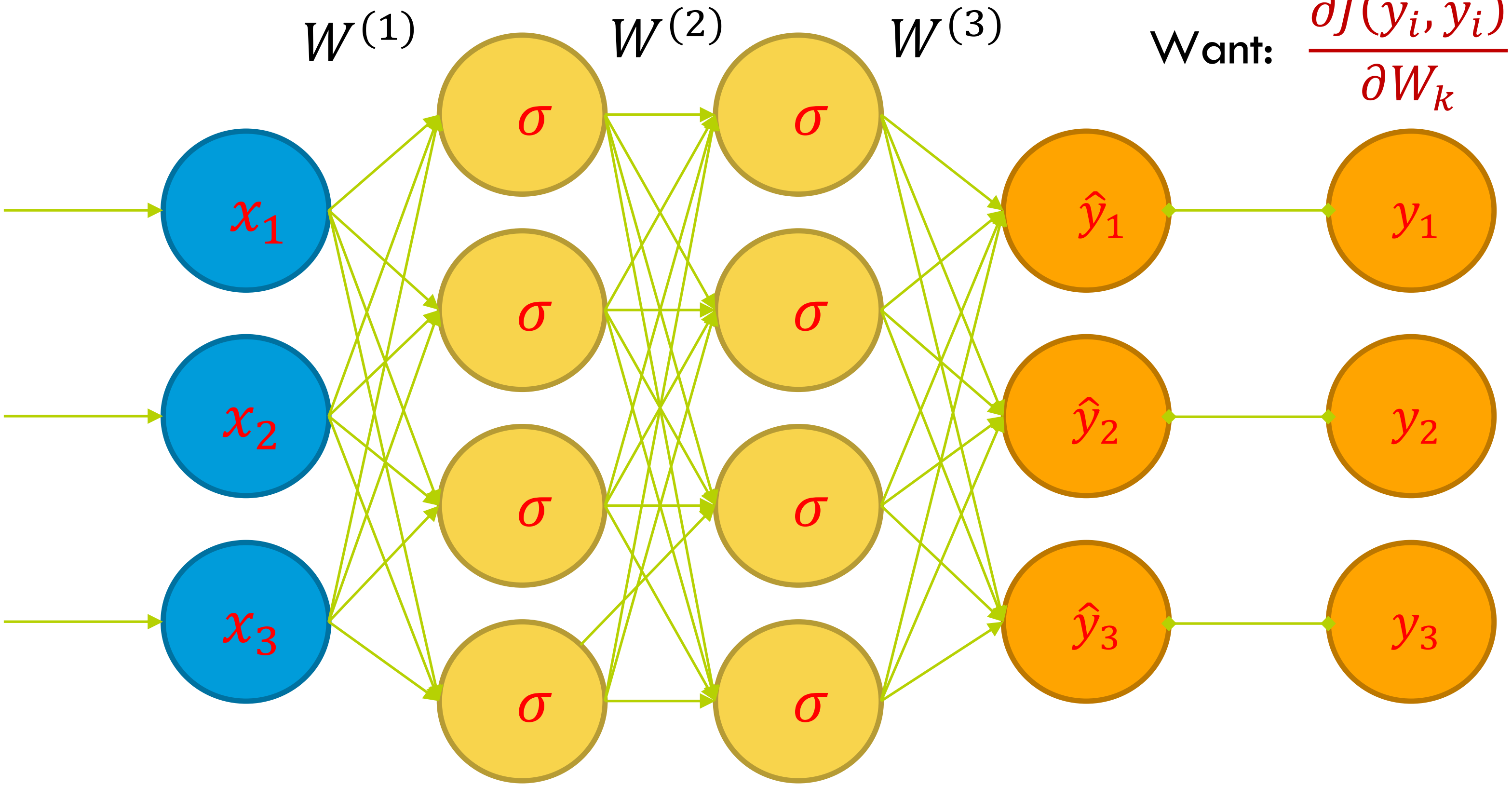
PS. Loss function may also look like L(y, F(x))
PS. F(x) is the current prediction or y^; y is the ground truth (label)

How to Train a Neural Net?

- Get $\frac{\partial J}{\partial W_k}$ for every weight in the network.
- This tells us what direction to adjust each W_k if we want to lower our loss function.
- Make an adjustment and repeat!

Feedforward Neural Network

Want: $\dfrac{\partial J(y_i, \hat{y_i})}{\partial W_k}$



Calculus to the Rescue

- Use calculus, chain rule, etc. etc.
- Functions are chosen to have "nice" derivatives
- Numerical issues to be considered

Punchline

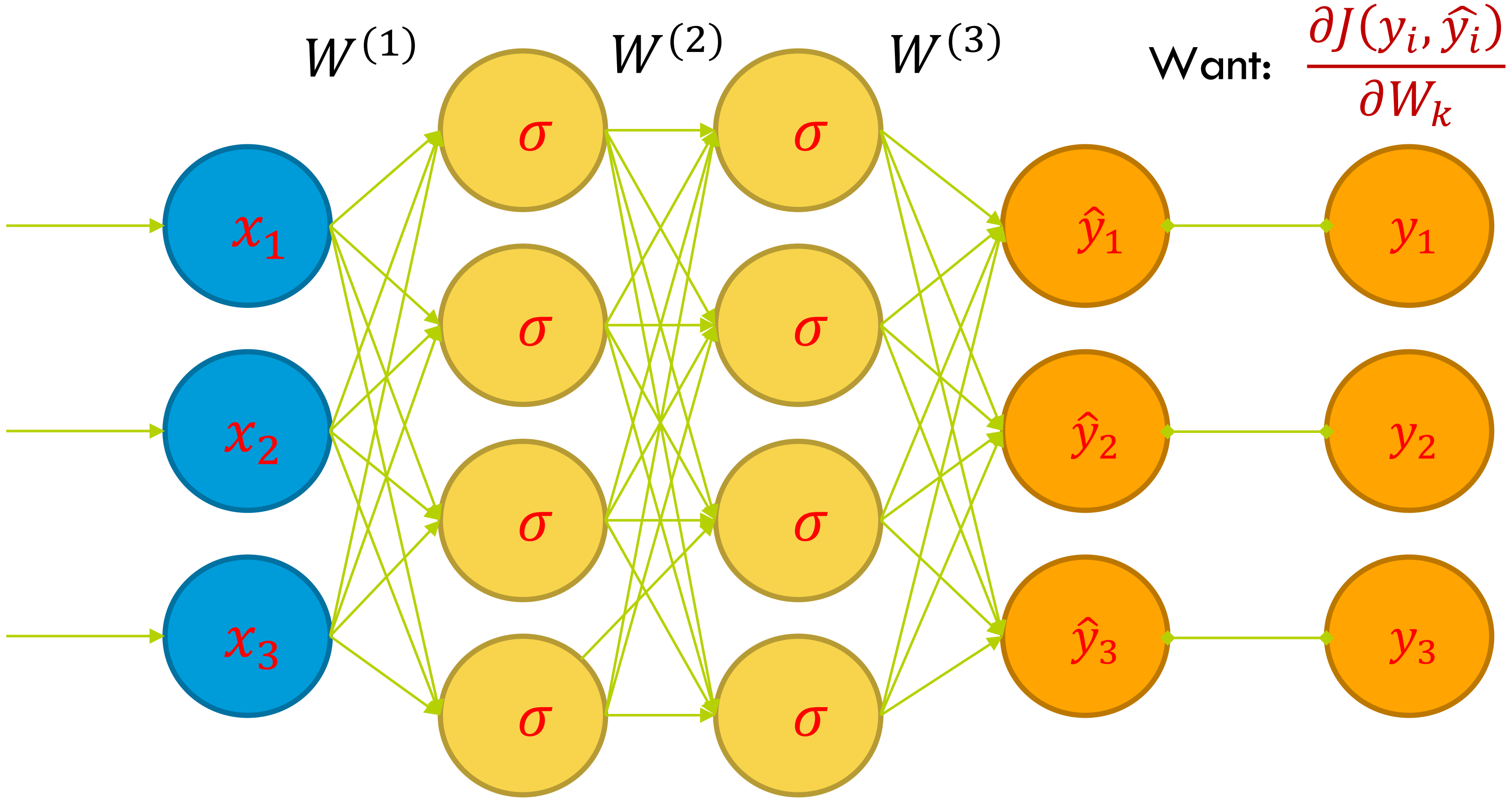
$$\frac{\partial J}{\partial W^{(3)}} = (\hat{y} - y) \cdot a^{(3)}$$

$$\frac{\partial J}{\partial W^{(2)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot a^{(2)}$$

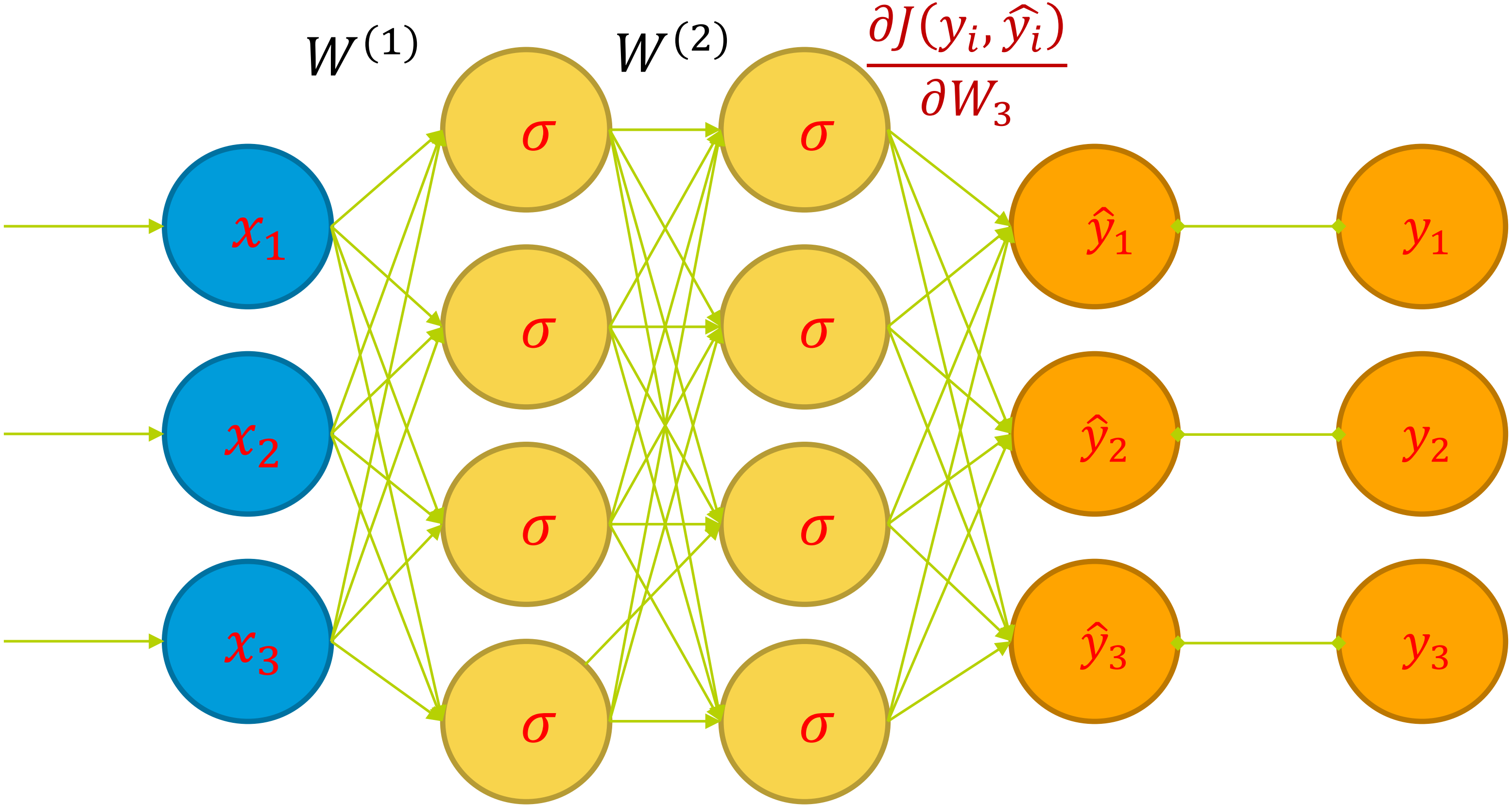
$$\frac{\partial J}{\partial W^{(1)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot W^{(2)} \cdot \sigma'\left(z^{(2)}\right) \cdot X$$

- Recall that: $\sigma'(z) = \sigma(z)(1 - \sigma(z))$
- Though they appear complex, above are easy to compute!

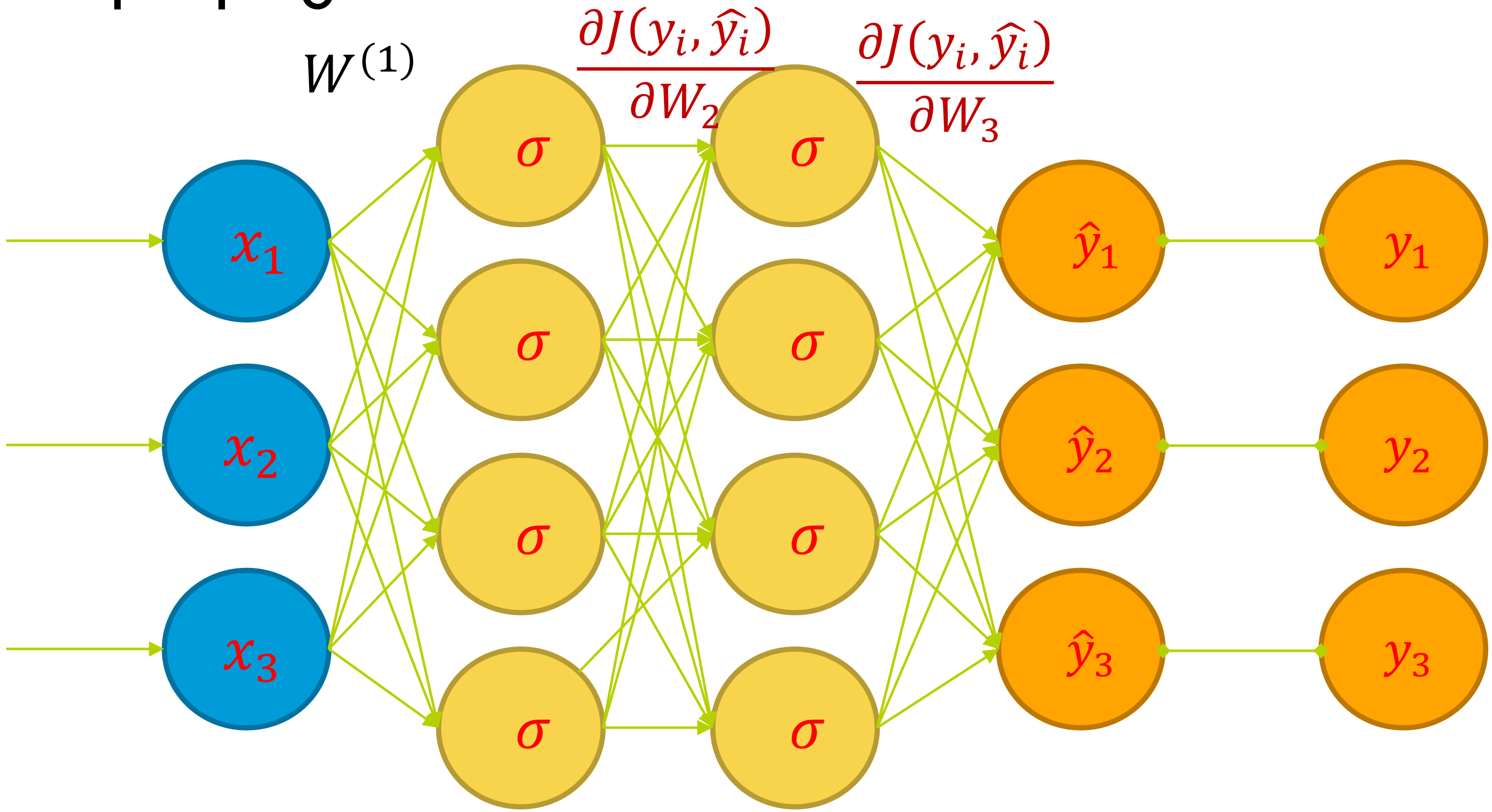
Backpropagation



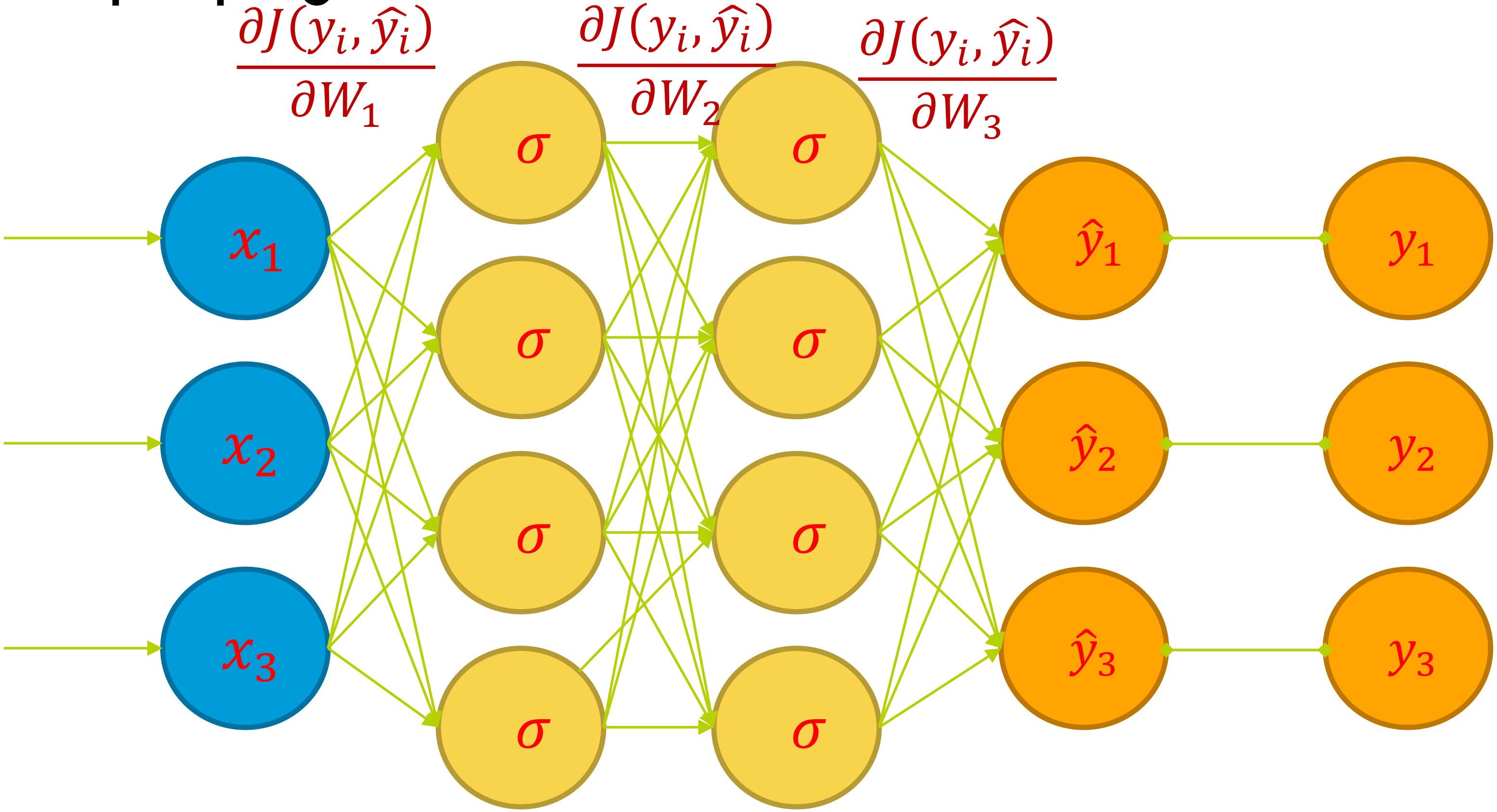
Backpropagation



Backpropagation



Backpropagation



How have we trained before?

- Gradient Descent!

1. Make prediction
2. Calculate Loss
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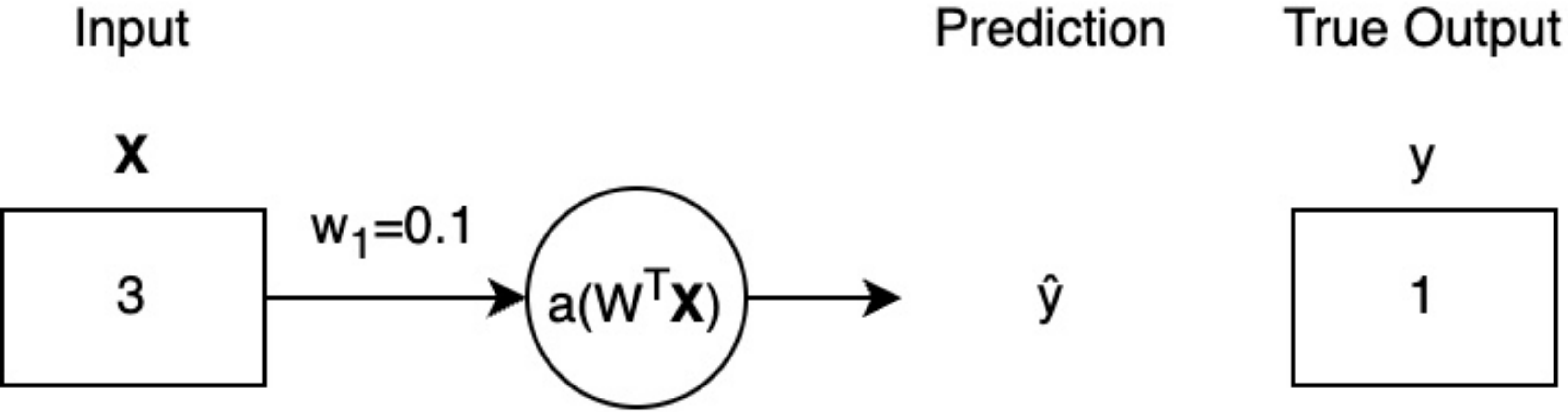
In short, Backpropagation ~= passing back the loss



Activation Function: $a(z) = z$

Mean Squared Error Function: $L(\hat{y}, y) = (\hat{y} - y)^2$

Learning Rate: $\eta = 1$

Let's use the above example, 1D input, 1 scalar weight w1, and identity activation function.

First, it's Forward-passing. We have the MSE loss regarding our input x as

$$L = (y - \hat{y})^2 = \left(y - W^T X\right)^2 = (1 - 0.3)^2 = 0.49$$

Next, backpropagation, we will solve the partial gradient of L regarding w1 using chain rule.

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_1} = 2\,(\hat{y} - y)\,x = 2\,(-0.7)\,3 = -4.2$$

Last, we will update w1 with the opposite of the gradient * learning rate. The reason we go the opposite of the gradient is because the gradient points to the direction of the largest increase in L.

$$w_1^{new} = w_1 - \eta \frac{\partial L}{\partial w_1} = 0.1 + 1 \times 4.2 = 4.3$$

Vanishing Gradients

Recall that:

$$\frac{\partial J}{\partial W^{(1)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot W^{(2)} \cdot \sigma'\left(z^{(2)}\right) \cdot X$$

- Remember: $\sigma'(z) = \sigma(z)\left(1 - \sigma(z)\right) \leq .25$
- As we have more layers, the gradient gets very small at the early layers.
- This is known as the "vanishing gradient" problem.
- For this reason, other activations (such as ReLU) have become more common.

Other Activation Functions

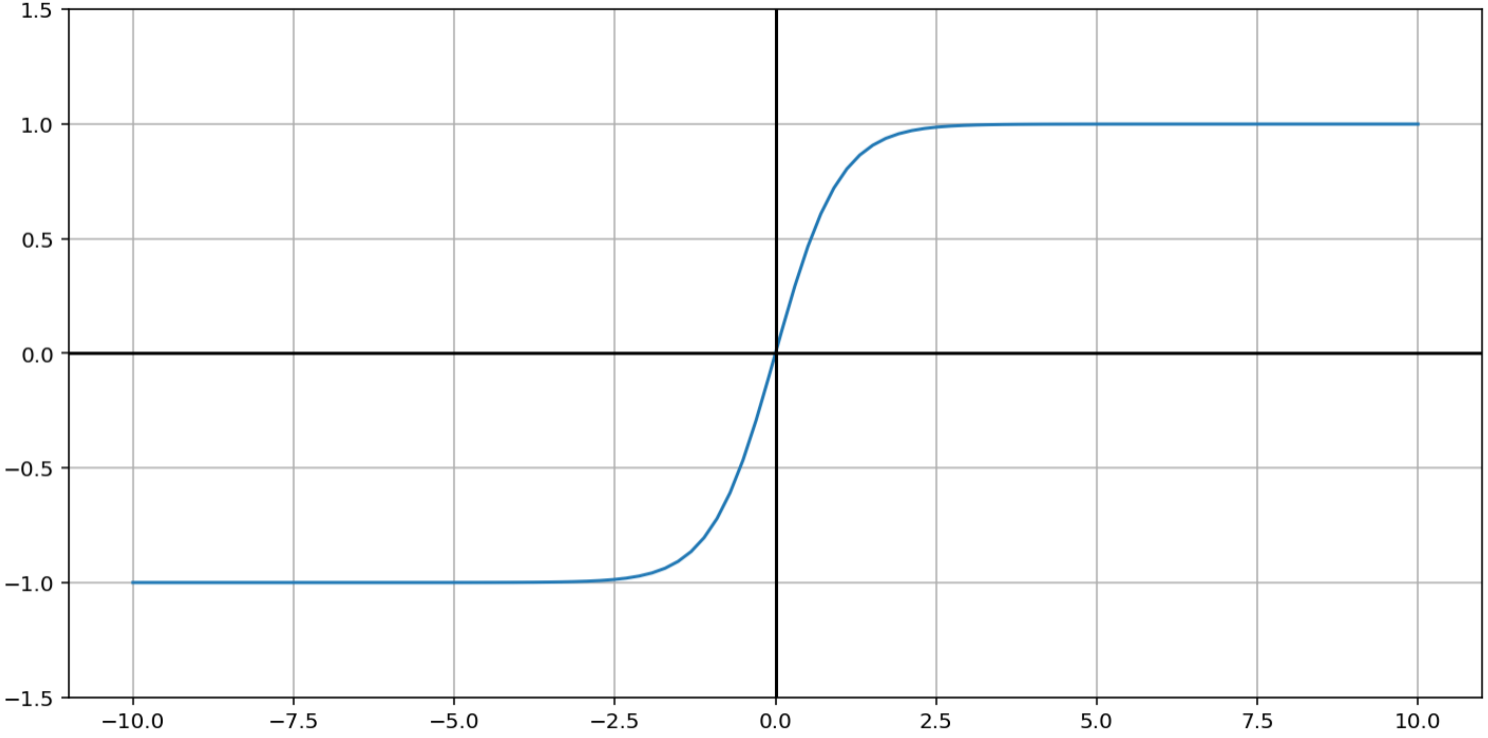
Hyperbolic Tangent Function

- Hyperbolic tangent function
- Pronounced "tanch"

$$tanh(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$tanh(0) = 0$$
$$tanh(\infty) = 1$$
$$tanh(-\infty) = -1$$

Hyperbolic Tangent Function



Rectified Linear Unit (ReLU)

$$ReLU(z) = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases}$$

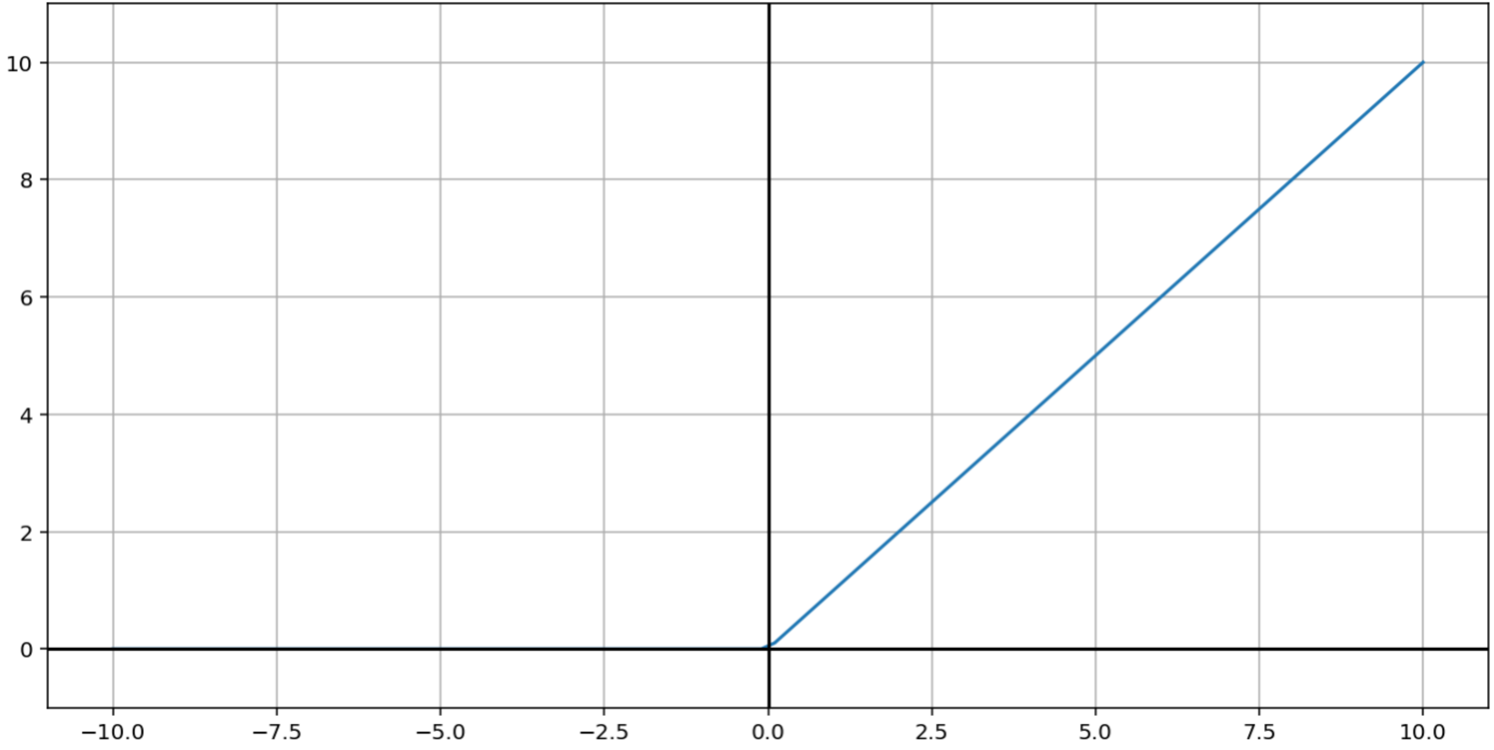
$$= \max(0, z)$$

$$ReLU(0) = 0$$

$$ReLU(z) = z \qquad \text{for } (z \gg 0)$$

$$ReLU(-z) = 0$$

Rectified Linear Unit (ReLU)



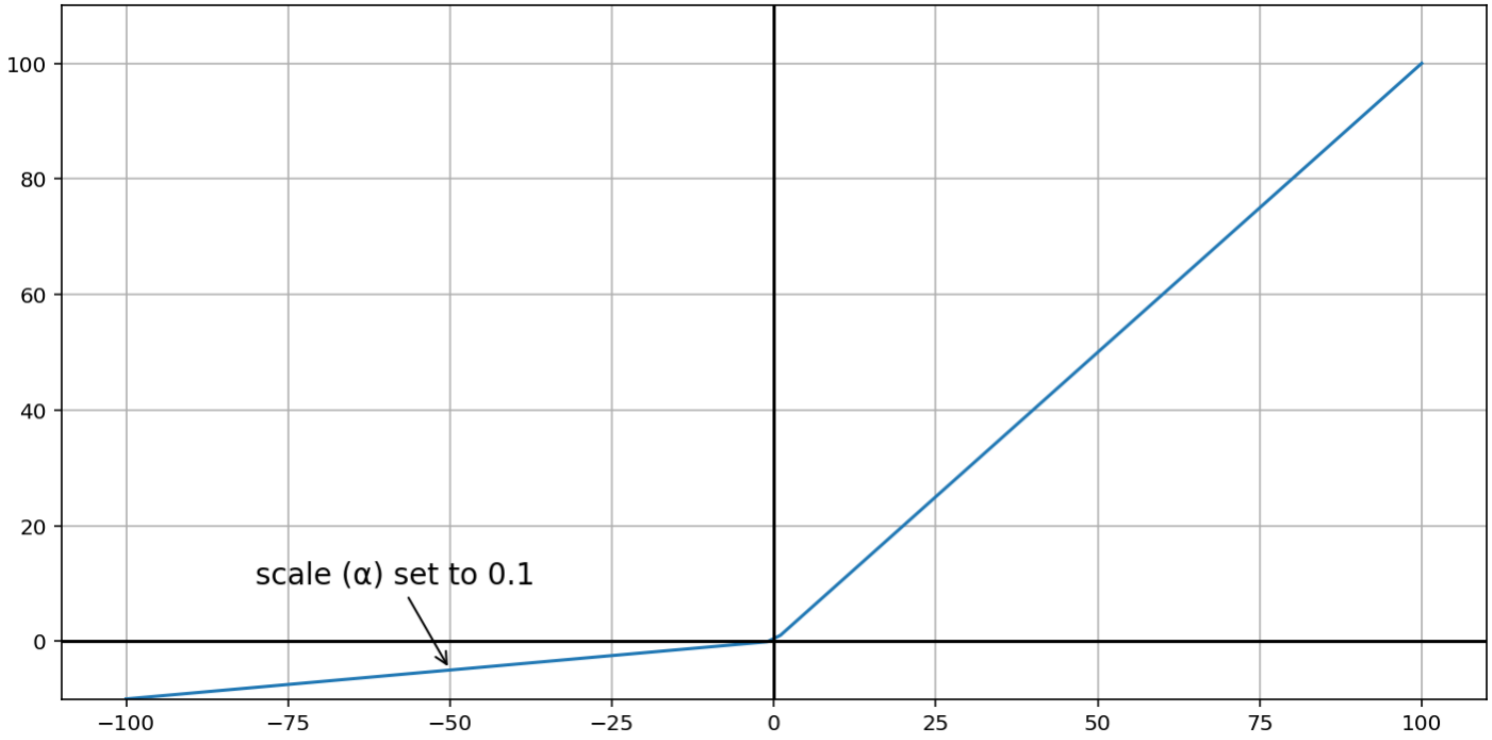
"Leaky" Rectified Linear Unit (ReLU)

$$LReLU(z) = \begin{cases} \alpha z, & z < 0 \\ z, & z \geq 0 \end{cases}$$

$$= \max(\alpha z, z) \qquad \text{for } (\alpha < 1)$$

$$LReLU(0) = 0$$
$$LReLU(z) = z \qquad \text{for } (z \gg 0)$$
$$LReLU(-z) = -\alpha z$$

“Leaky” Rectified Linear Unit (ReLU)



What next?

- We now know how to make a single update to a model given some data.
- But how do we do the full training?