Language Modeling: Attention

CSE 576: Topics in Natural Language Processing

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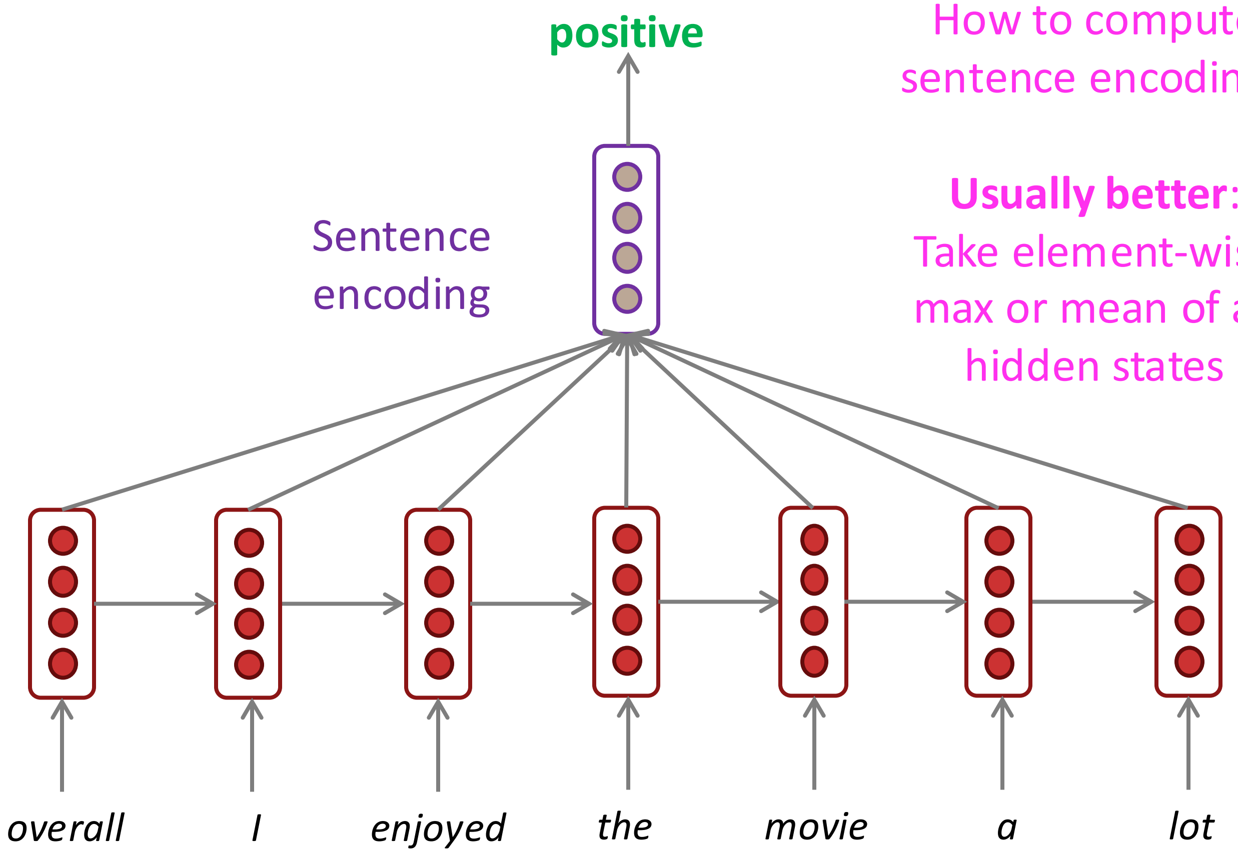
Attention

- **Attention** provides a solution to the bottleneck problem.

- **Core idea**: on each step of the decoder, use *direct connection to the encoder* to *focus on a particular part* of the source sequence

- First, we will show via diagram (no equations), then we will show with equations

The starting point: mean-pooling for RNNs



How to compute sentence encoding?

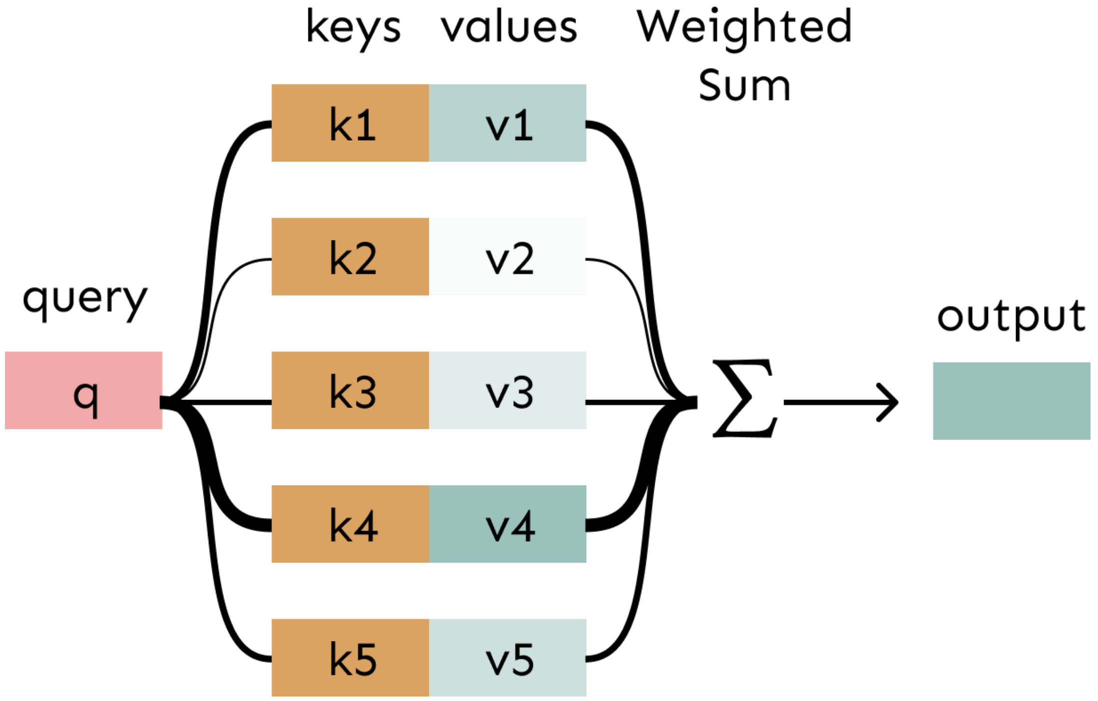
Usually better: Take element-wise max or mean of all hidden states

- Starting point: a *very* basic way of 'passing information from the encoder' is to *average*

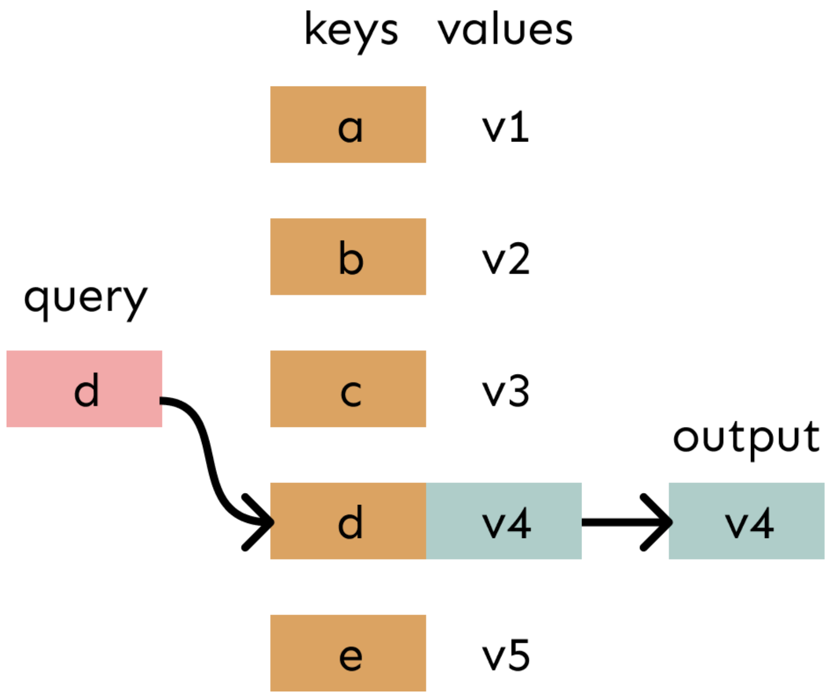
Attention is *weighted* averaging, which lets you do lookups!

Attention is just a **weighted** average – this is very powerful if the weights are learned!

In **attention**, the query matches all keys *softly*, to a weight between 0 and 1. The keys' values are multiplied by the weights and summed.

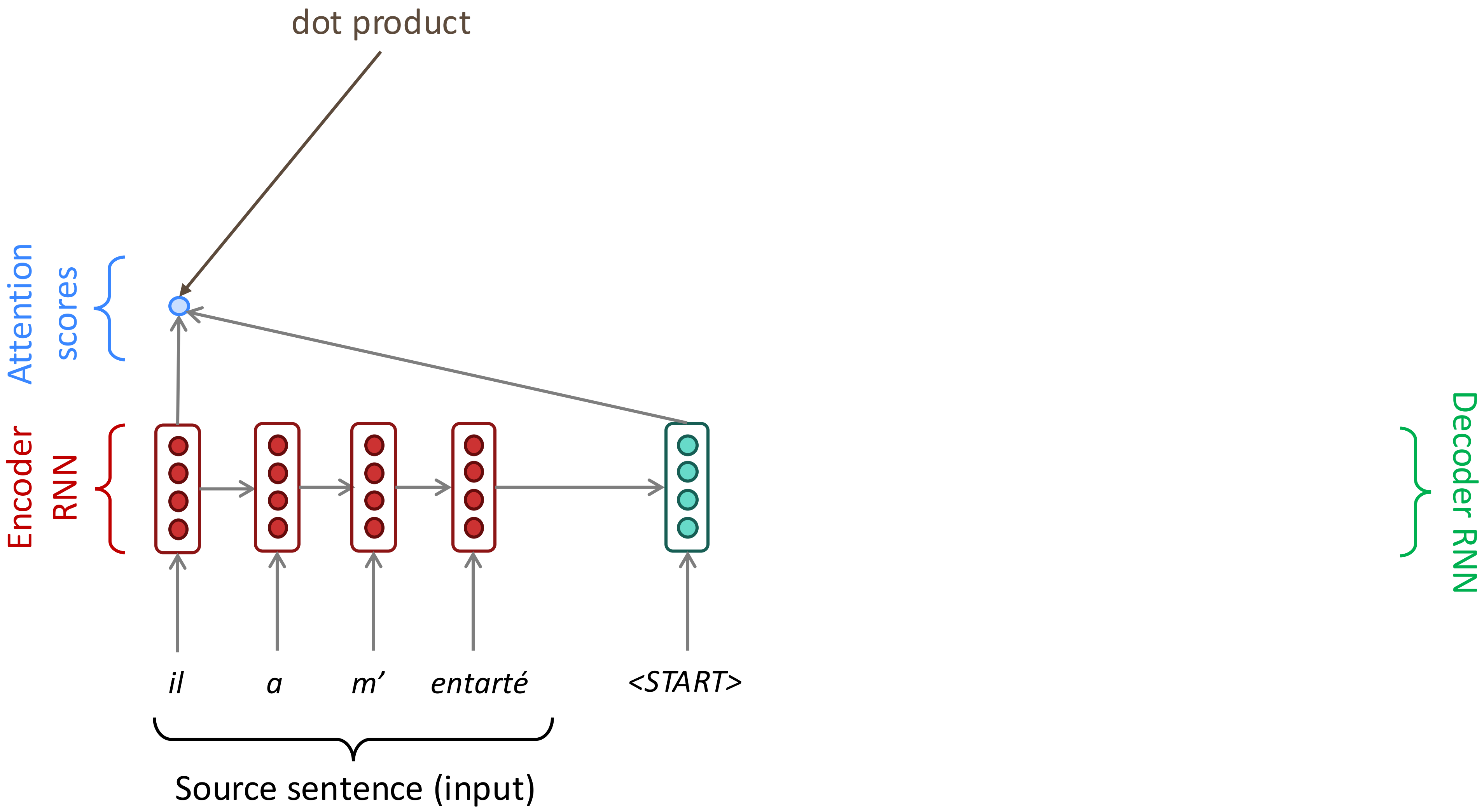


In a **lookup table**, we have a table of keys that map to values. The query matches one of the keys, returning its value.

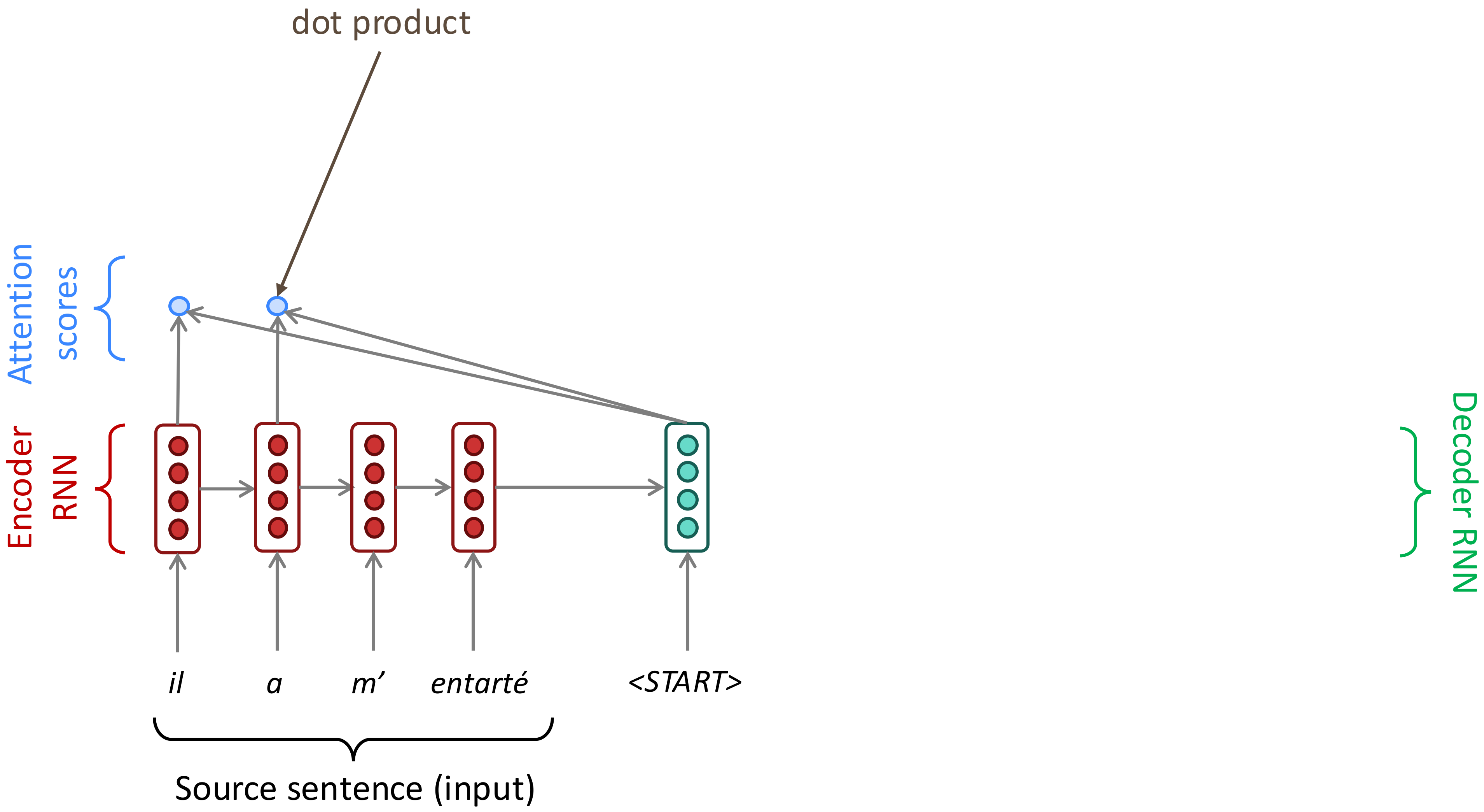


Sequence-to-sequence with attention

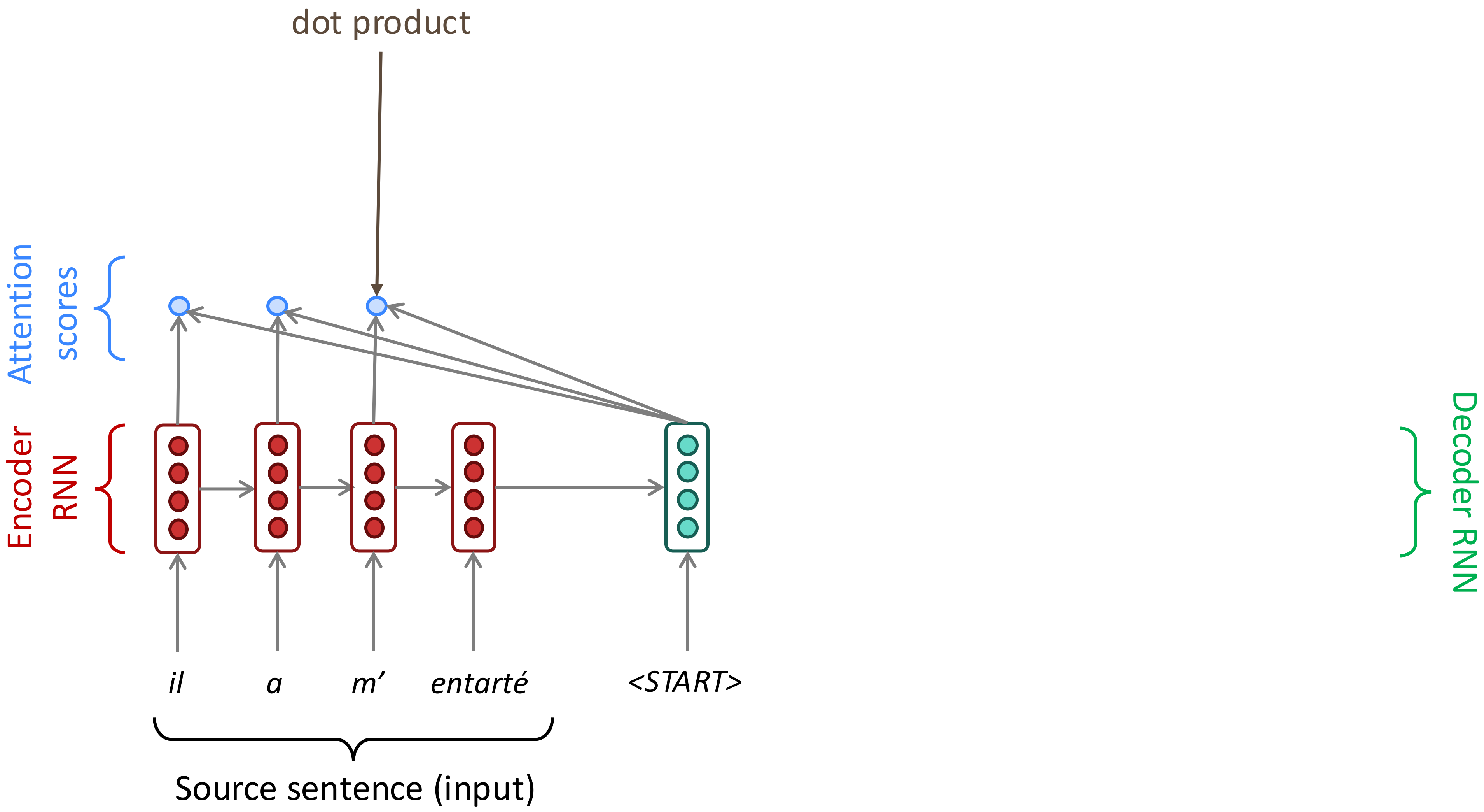
Core idea: on each step of the decoder, use *direct connection to the encoder* to *focus on a particular part* of the source sequence



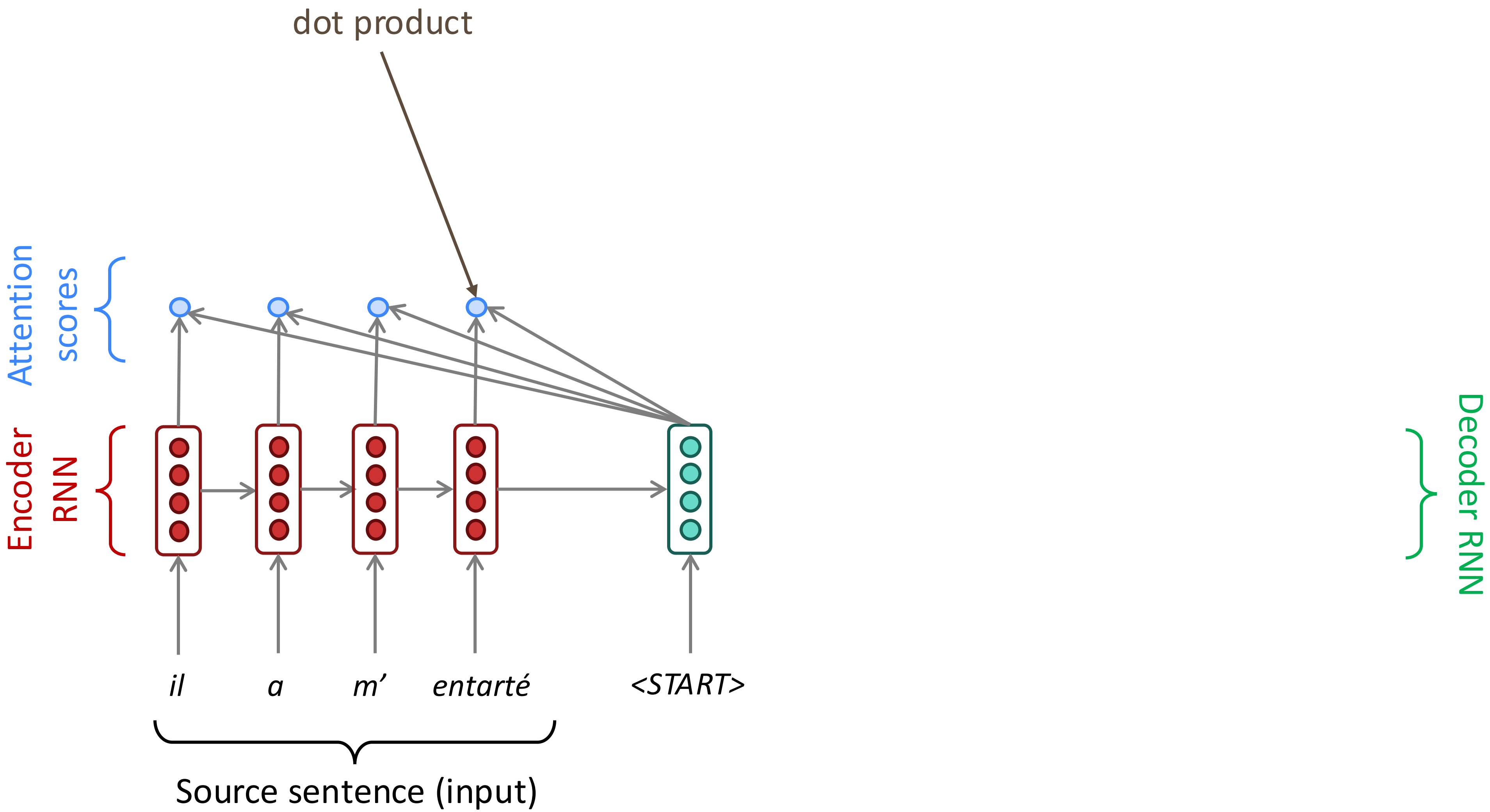
Sequence-to-sequence with attention



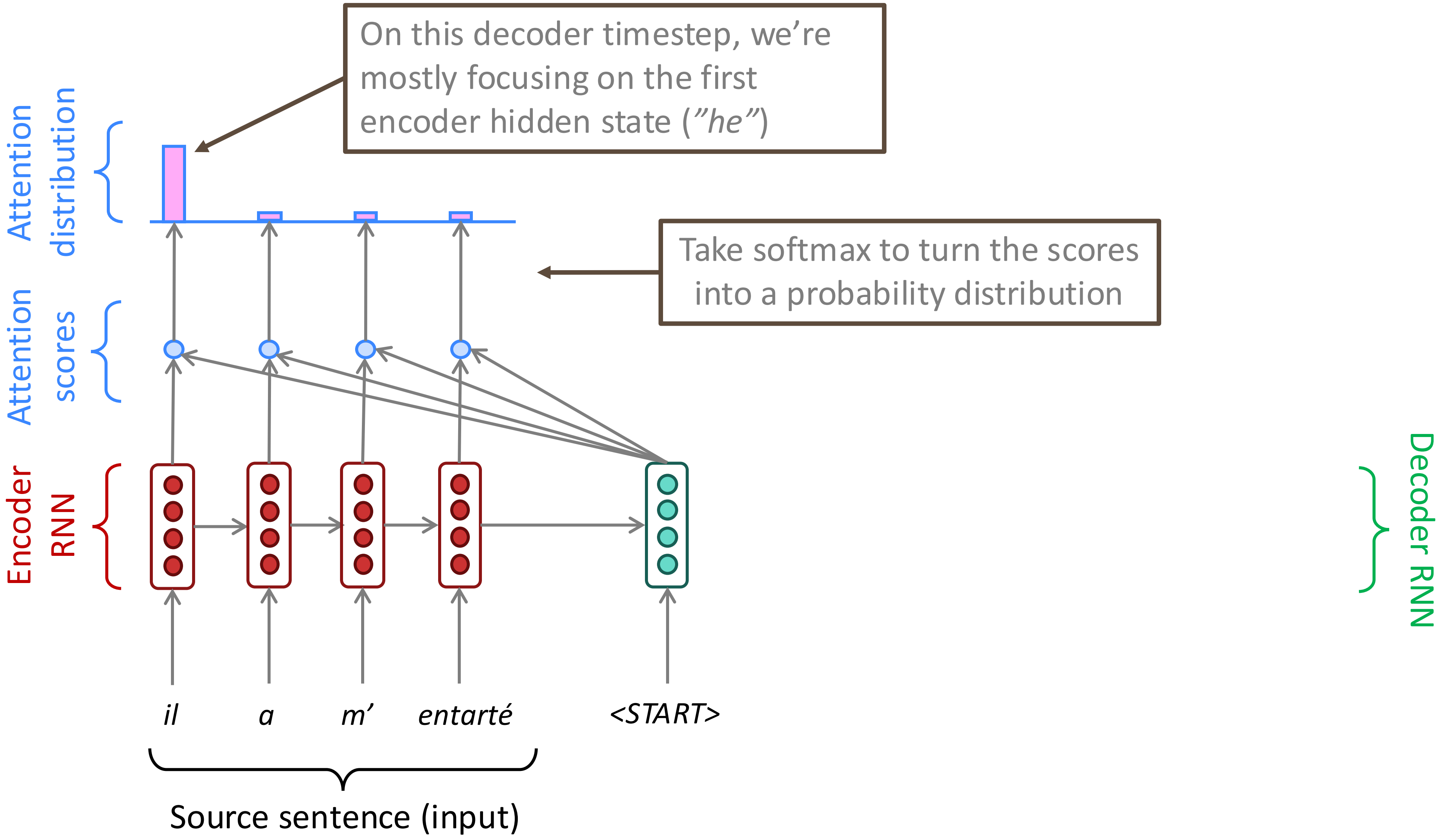
Sequence-to-sequence with attention



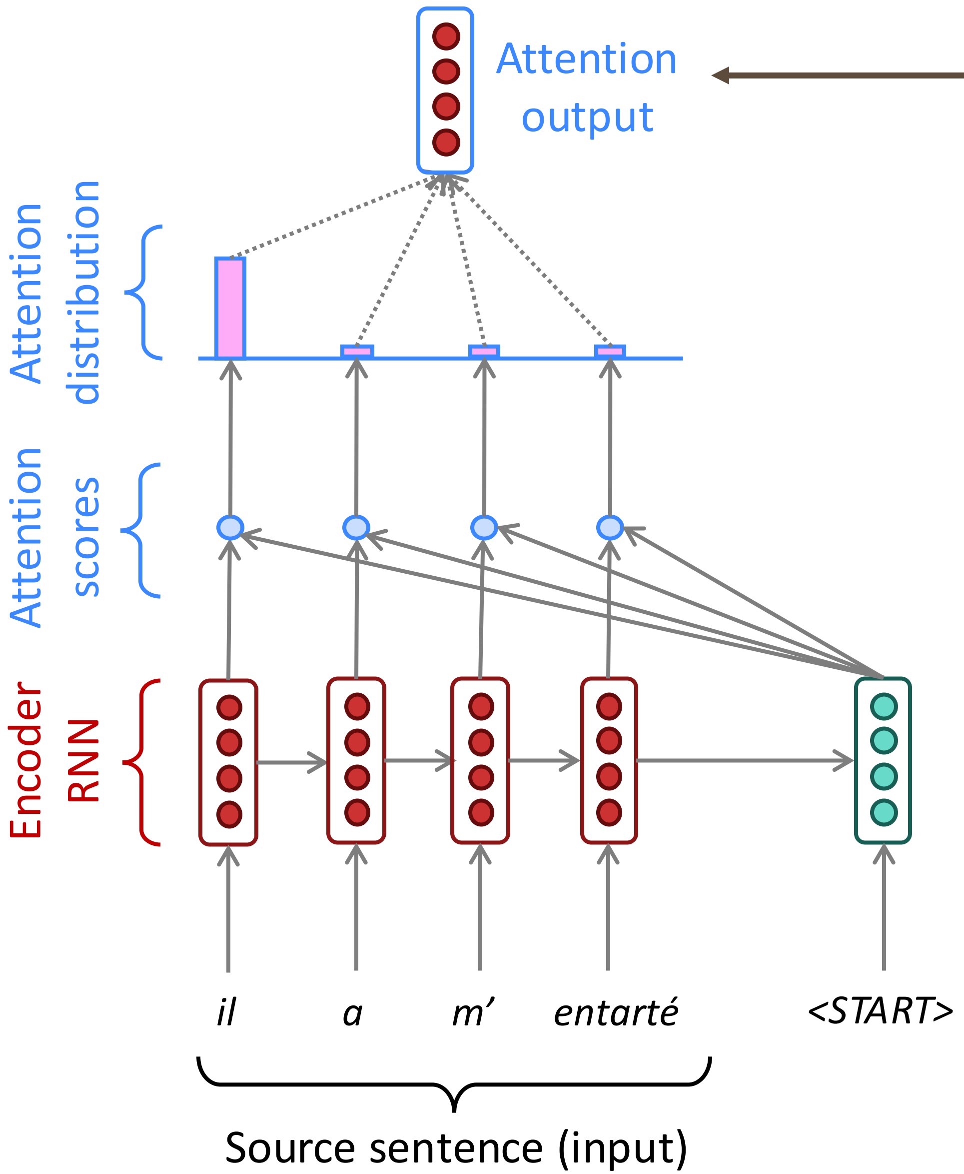
Sequence-to-sequence with attention



Sequence-to-sequence with attention



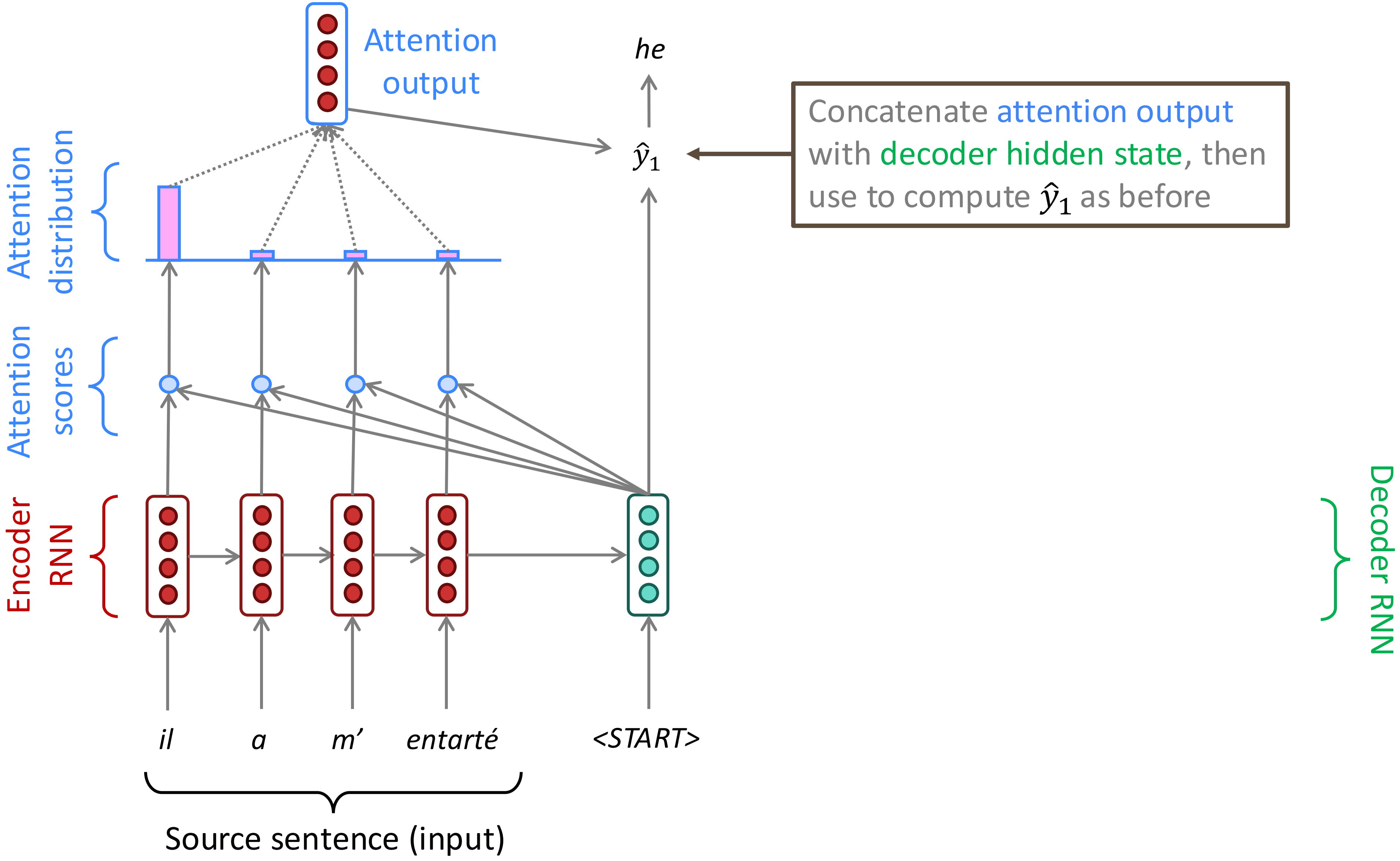
Sequence-to-sequence with attention



Use the attention distribution to take a **weighted sum** of the encoder hidden states.

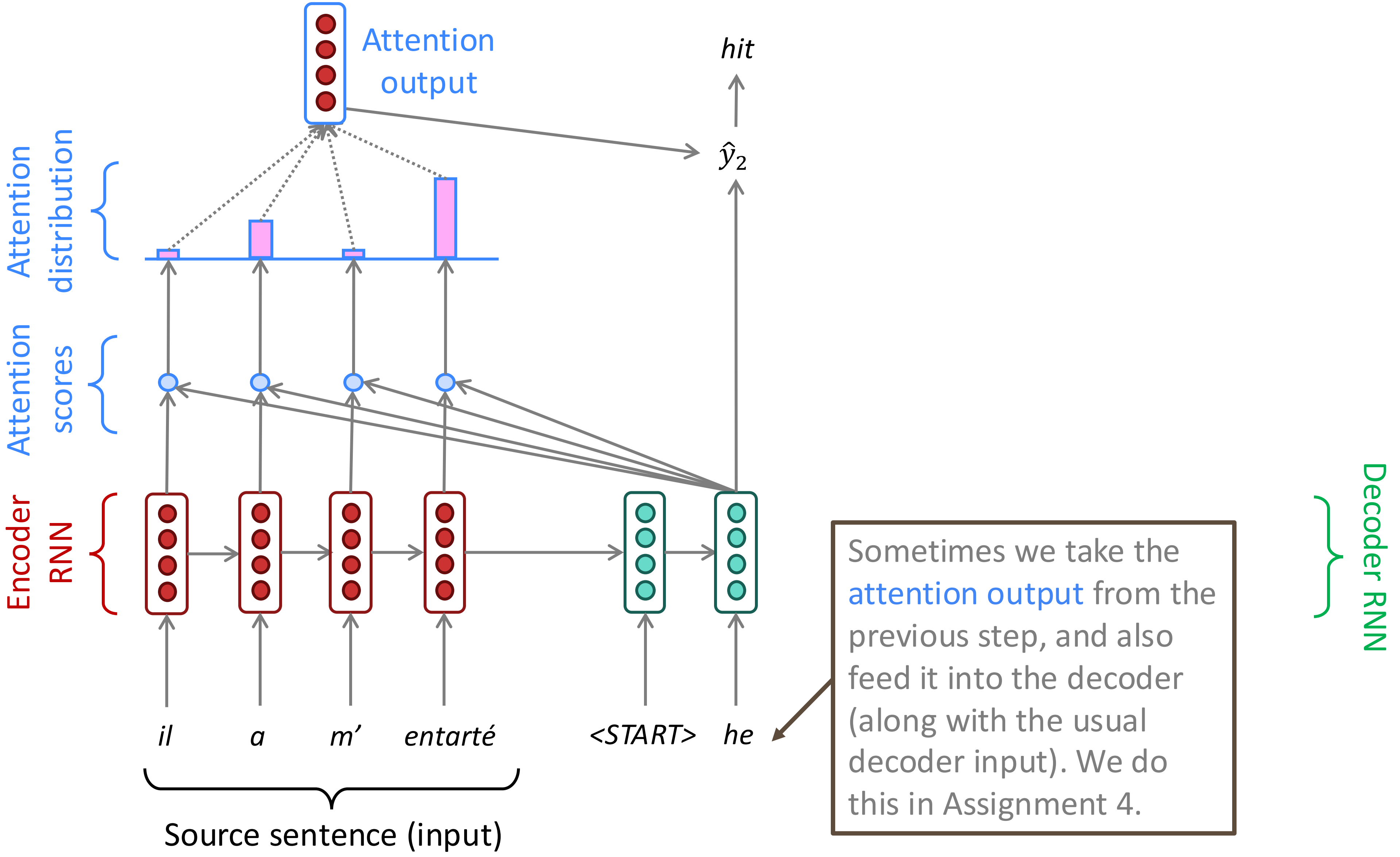
The attention output mostly contains information from the hidden states that received high attention.

Sequence-to-sequence with attention



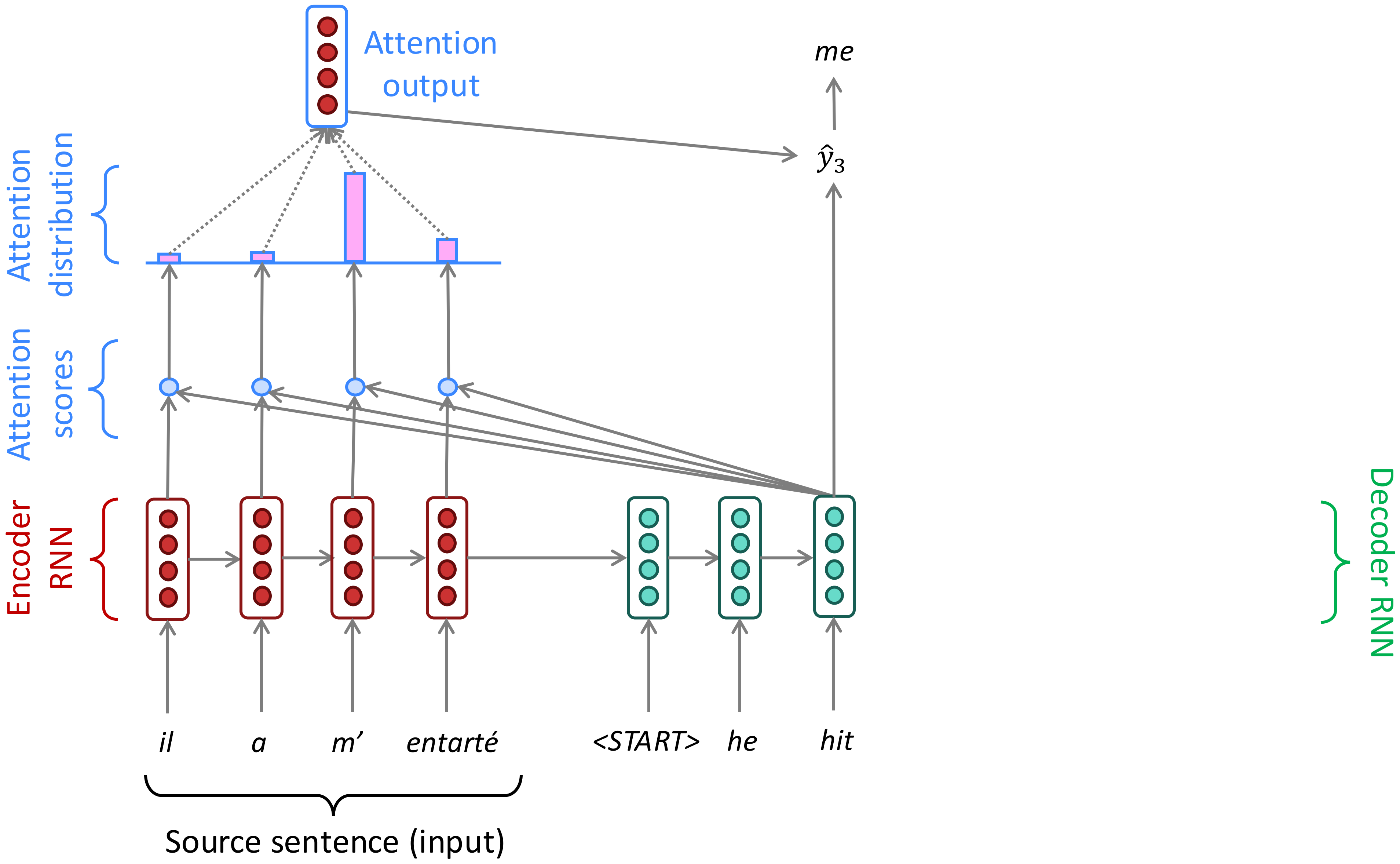
Concatenate attention output with decoder hidden state, then use to compute $\hat{y}_1$ as before

Sequence-to-sequence with attention

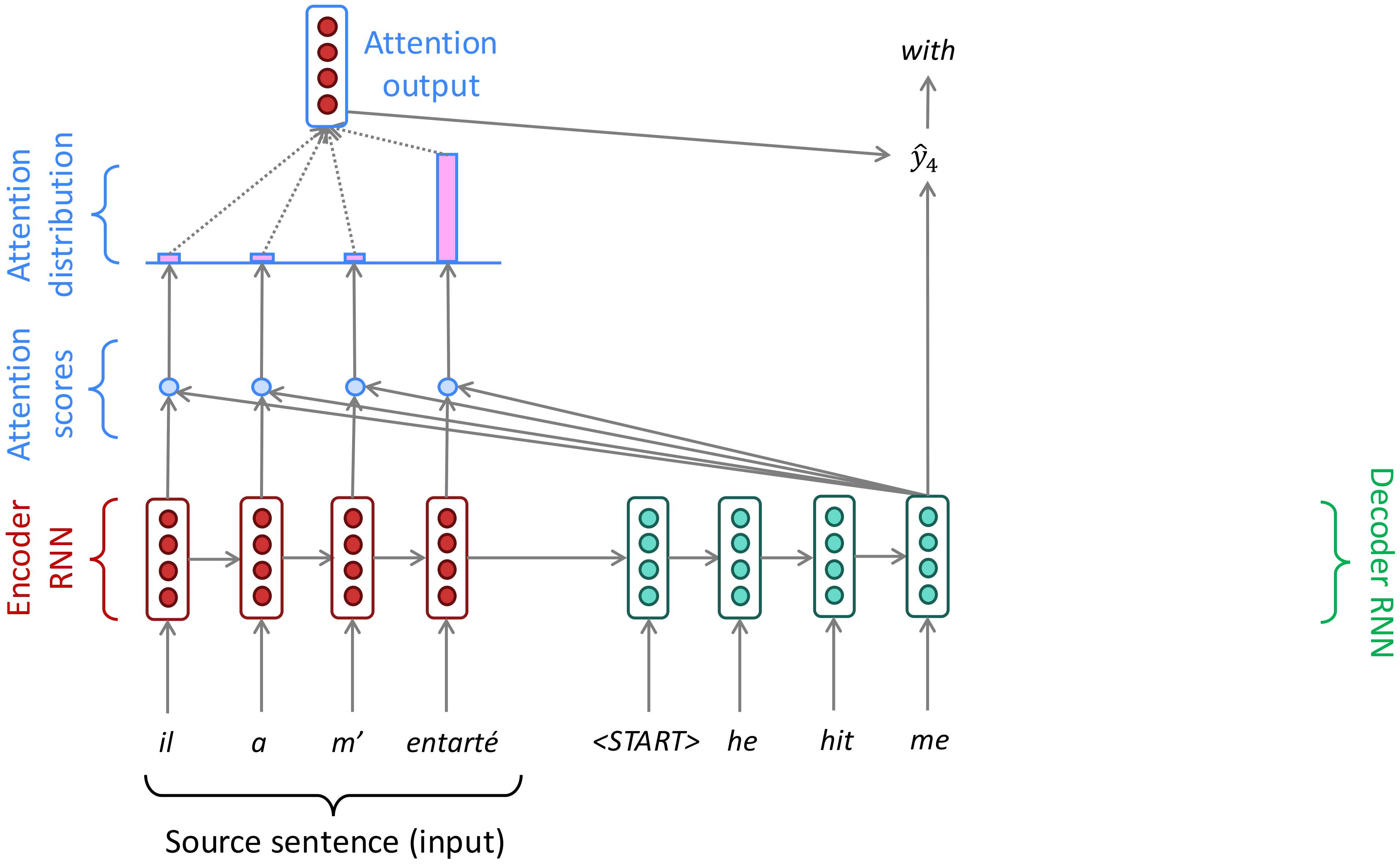


Sometimes we take the attention output from the previous step, and also feed it into the decoder (along with the usual decoder input). We do this in Assignment 4.

Sequence-to-sequence with attention



Sequence-to-sequence with attention



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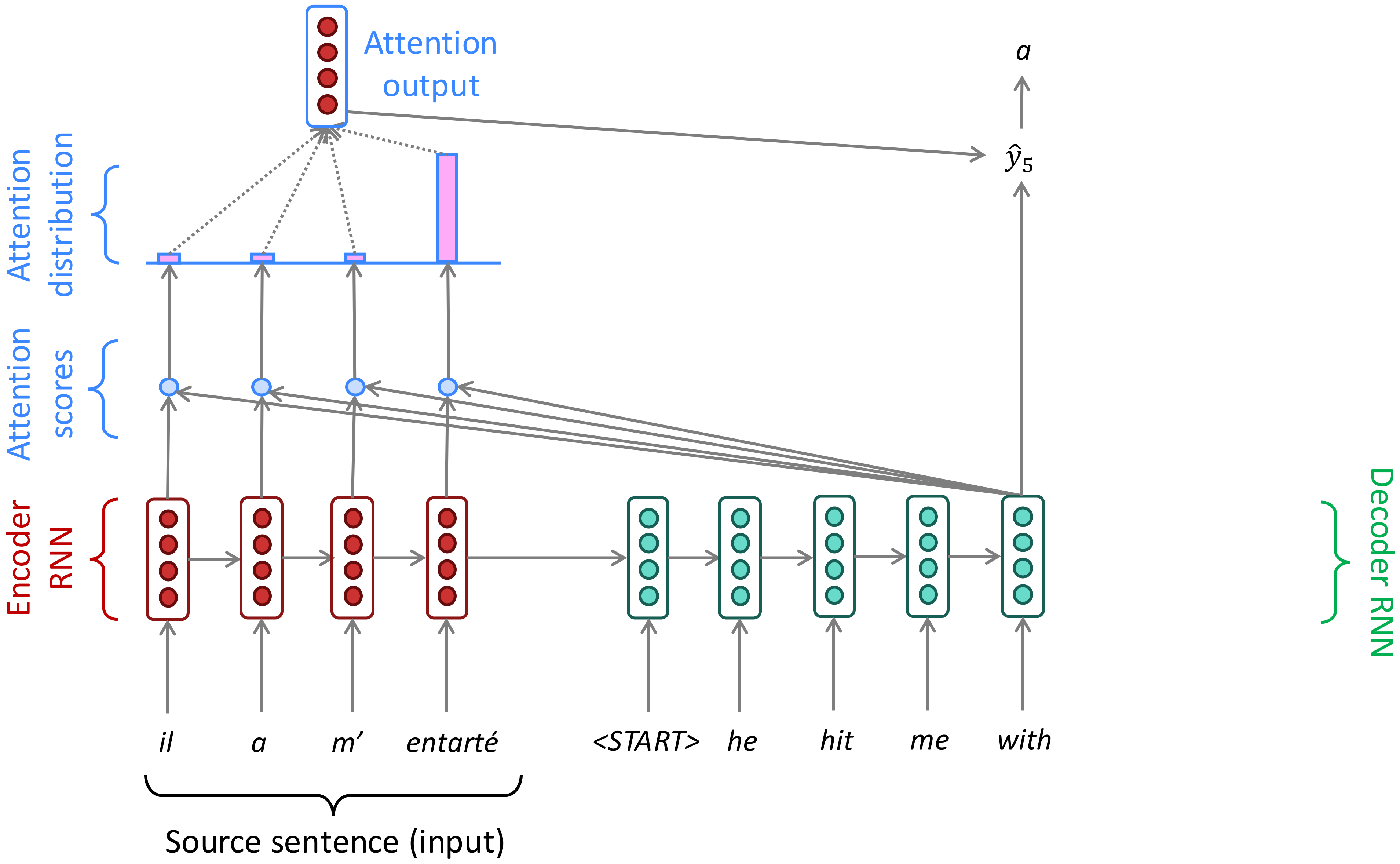
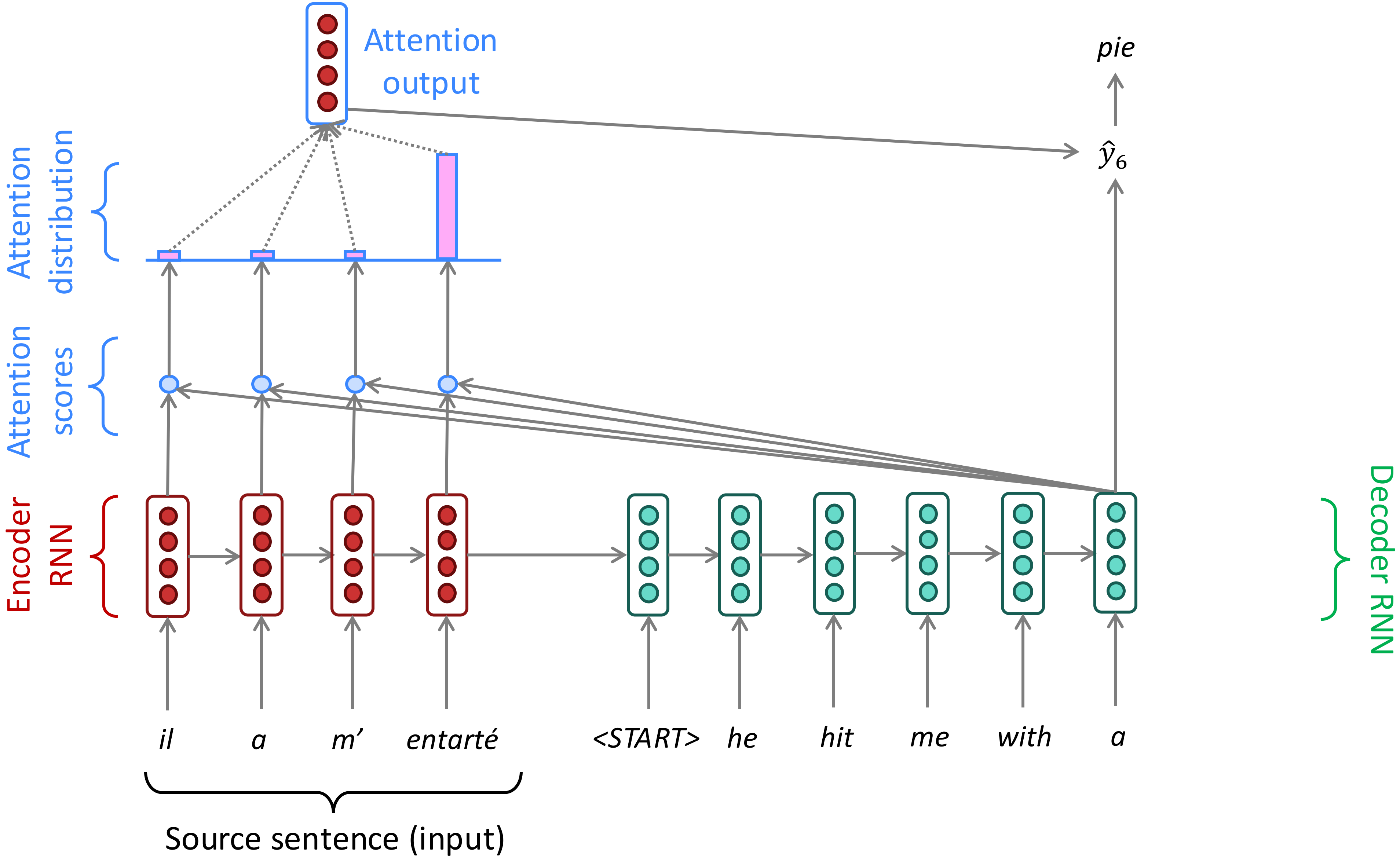


Sequence-to-sequence with attention



Attention: in equations

- We have encoder hidden states $h_1, \dots, h_N \in \mathbb{R}^h$
- On timestep *t*, we have decoder hidden state $s_t \in \mathbb{R}^h$
- We get the attention scores $\boldsymbol{e}^t$ for this step:

$$\boldsymbol{e}^t = [\boldsymbol{s}_t^T \boldsymbol{h}_1, \dots, \boldsymbol{s}_t^T \boldsymbol{h}_N] \in \mathbb{R}^N$$

- We take softmax to get the attention distribution α^t for this step (this is a probability distribution and sums to 1)

$$\alpha^t = \mathrm{softmax}(\boldsymbol{e}^t) \in \mathbb{R}^N$$

- We use α^t to take a weighted sum of the encoder hidden states to get the attention output $\boldsymbol{a}_t$

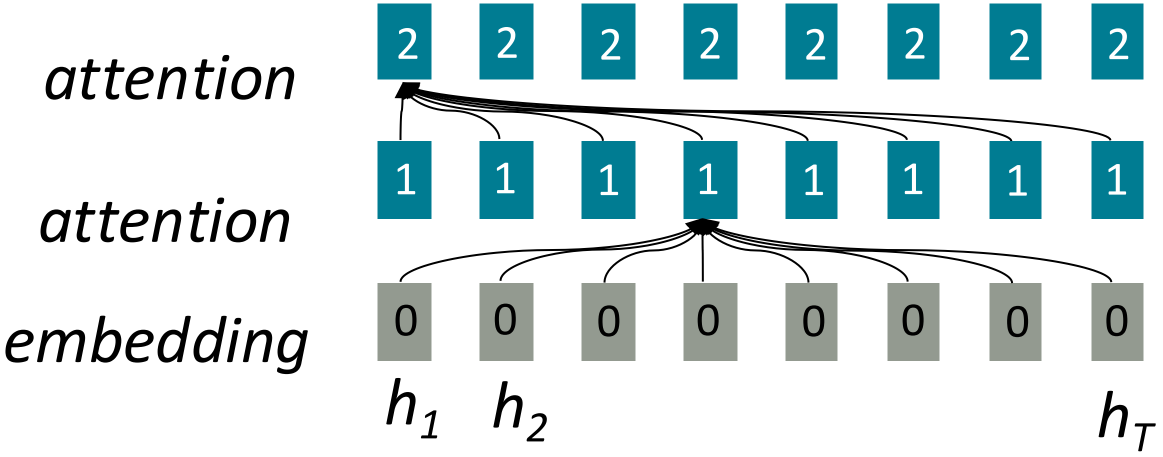
$$\boldsymbol{a}_t = \sum_{i=1}^{N} \alpha_i^t \boldsymbol{h}_i \in \mathbb{R}^h$$

- Finally we concatenate the attention output $\boldsymbol{a}_t$ with the decoder hidden state s_t and proceed as in the non-attention seq2seq model

$$[\boldsymbol{a}_t; \boldsymbol{s}_t] \in \mathbb{R}^{2h}$$

Attention is parallelizable, and solves bottleneck issues.

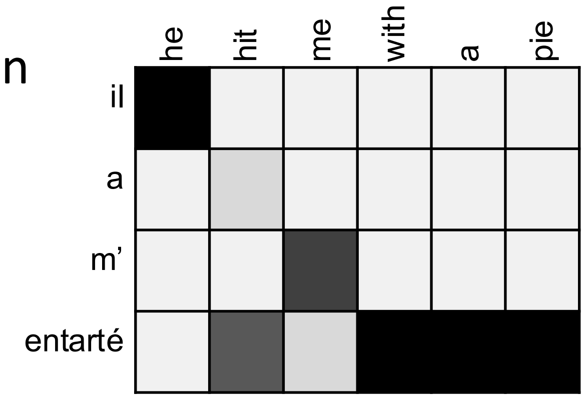
- **Attention** treats each word's representation as a **query** to access and incorporate information from **a set of values.**
  - We saw attention from the **decoder** to the **encoder**; today we'll think about attention **within a single sentence**.
- Number of unparallelizable operations does not increase with sequence length.
- Maximum interaction distance: O(1), since all words interact at every layer!



All words attend to all words in previous layer; most arrows here are omitted

Attention is **great!**

- Attention significantly improves NMT performance
  - It's very useful to allow decoder to focus on certain parts of the source
- Attention provides a more "human-like" model of the MT process
  - You can look back at the source sentence while translating, rather than needing to remember it all
- Attention solves the bottleneck problem
  - Attention allows decoder to look directly at source; bypass bottleneck
- Attention helps with the vanishing gradient problem
  - Provides shortcut to faraway states
- Attention provides some interpretability
  - By inspecting attention distribution, we see what the decoder was focusing on
  - We get (soft) alignment for free!
  - The network just learned alignment by itself
- (One issue – attention has *quadratic* cost with respect to sequence length)



Attention is a *general* Deep Learning technique

- We've seen that attention is a great way to improve the sequence-to-sequence model for Machine Translation.
- However: You can use attention in many architectures (not just seq2seq) and many tasks (not just MT)

- **More general definition of attention**:
  - Given a set of vector *values*, and a vector *query*, **attention** is a technique to compute a weighted sum of the values, dependent on the query.

- We sometimes say that the query *attends to* the values.
- For example, in the seq2seq + attention model, each decoder hidden state (query) *attends to* all the encoder hidden states (values).

Attention is a *general* Deep Learning technique

- **More general definition of attention**:
  - Given a set of vector *values*, and a vector *query*, **attention** is a technique to compute a weighted sum of the values, dependent on the query.

Intuition:

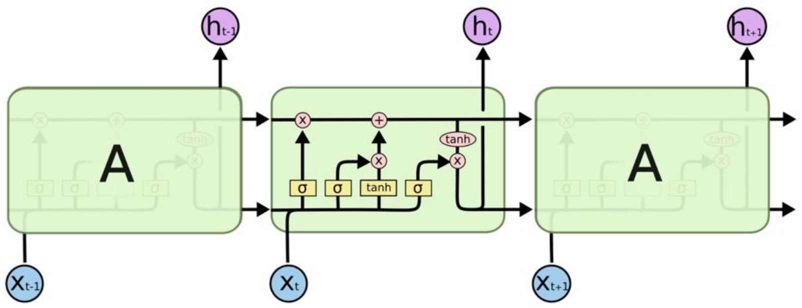
- The weighted sum is a *selective summary* of the information contained in the values, where the query determines which values to focus on.
- Attention is a way to obtain a *fixed-size representation of an arbitrary set of representations* (the values), dependent on some other representation (the query).

Upshot:

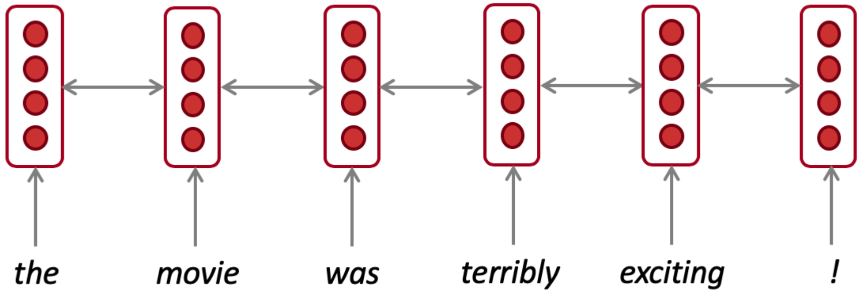
- Attention has become the powerful, flexible, general way pointer and memory manipulation in all deep learning models. A new idea from after 2010! From NMT!

In summary

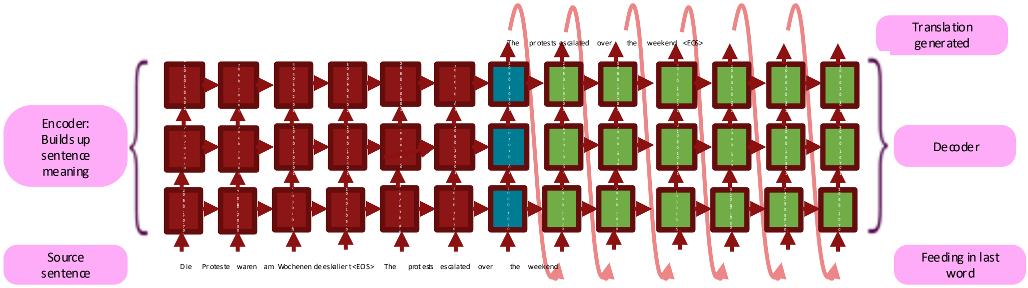
Lots of new information today! What are some of the practical takeaways?



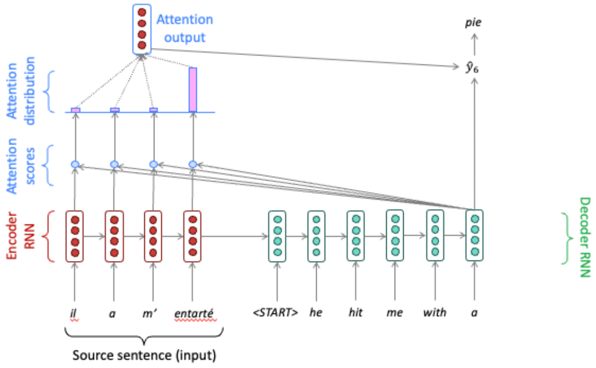
1. LSTMs are powerful



2. Use bidirectionality when possible



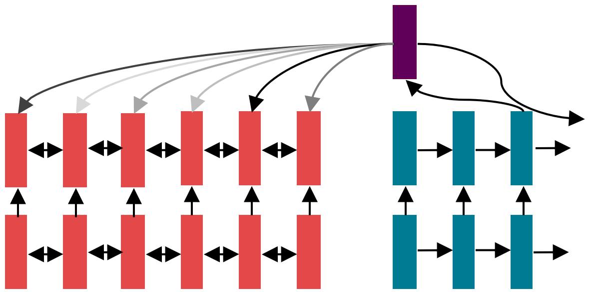
3. Encoder-Decoder Neural Machine Translation Systems work very well



4. Attention is a general, useful technique

Do we even need recurrence at all?

- Abstractly: Attention is a way to pass information from a sequence (x) to a neural network input. (h_t)
  - This is also *exactly* what RNNs are used for – to pass information!
  - **Can we just get rid of the RNN entirely?** Maybe attention is just a better way to pass information!



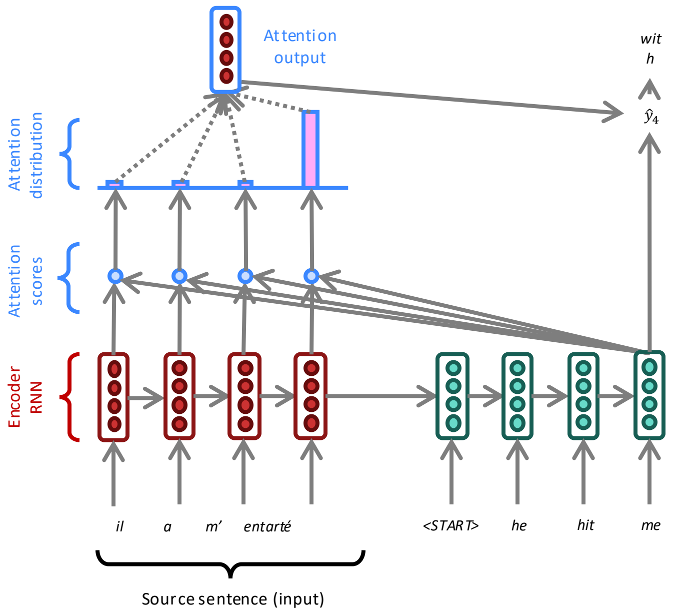
Lots of trial and error

2014-2017ish
Recurrence

2021
??????

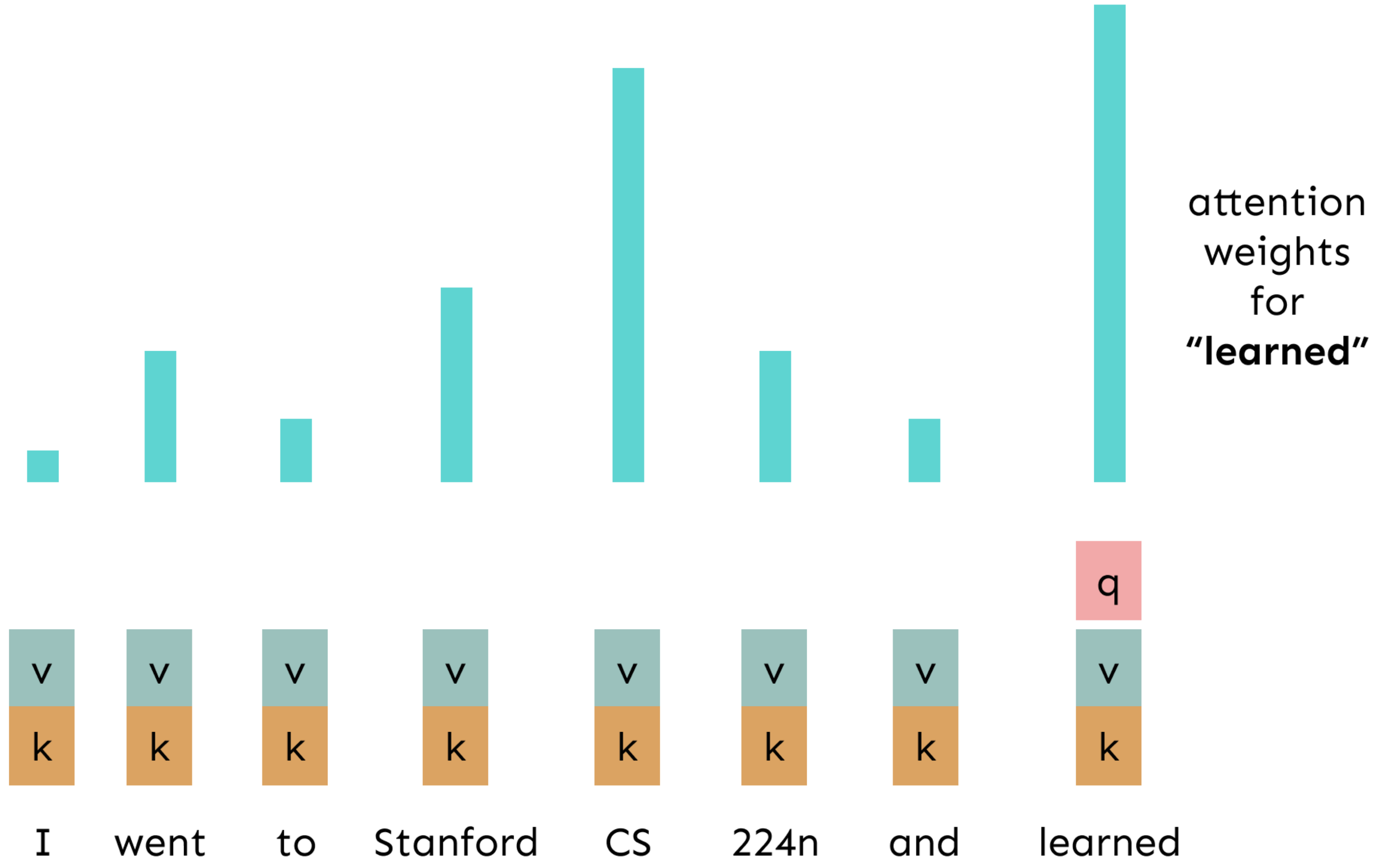
The building block we need: *self* attention

- What we talked about – **Cross** attention: paying attention to the input x to generate y_t



- What we need – **Self** attention: to generate y_t, we need to pay attention to $y_{<t}$

Self-Attention Hypothetical Example



Self-Attention: keys, queries, values from the same sequence

Let $\boldsymbol{w}_{1:n}$ be a sequence of words in vocabulary V, like *Zuko made his uncle tea*.

For each $\boldsymbol{w}_i$, let $\boldsymbol{x}_i = E\boldsymbol{w}_{\boldsymbol{i}}$, where $E \in \mathbb{R}^{d\times|V|}$ is an embedding matrix.

1. Transform each word embedding with weight matrices $\mathrm{Q, K, V}$, each in $\mathbb{R}^{d\times d}$

$$\boldsymbol{q}_i = Q\boldsymbol{x}_{\boldsymbol{i}} \quad \text{(queries)} \qquad \boldsymbol{k}_i = K\boldsymbol{x}_{\boldsymbol{i}} \quad \text{(keys)} \qquad \boldsymbol{v}_i = V\boldsymbol{x}_{\boldsymbol{i}} \quad \text{(values)}$$

2. Compute pairwise similarities between keys and queries; normalize with softmax

$$\boldsymbol{e}_{ij} = \boldsymbol{q}_{\boldsymbol{i}}^{\top}\boldsymbol{k}_{\boldsymbol{j}} \qquad \boldsymbol{\alpha}_{ij} = \frac{\exp(\boldsymbol{e}_{ij})}{\sum_{j'}\exp(\boldsymbol{e}_{ij'})}$$

3. Compute output for each word as weighted sum of values

$$\boldsymbol{o}_i = \sum_{\boldsymbol{j}} \boldsymbol{\alpha}_{ij}\,\boldsymbol{v}_i$$

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!

Solutions

Fixing the first self-attention problem: **sequence order**

- Since self-attention doesn't build in order information, we need to encode the order of the sentence in our keys, queries, and values.
- Consider representing each **sequence index** as a **vector**

$$\boldsymbol{p}_i \in \mathbb{R}^d, \text{ for } i \in \{1,2,\dots,n\} \text{ are position vectors}$$

- Don't worry about what the p_i are made of yet!
- Easy to incorporate this info into our self-attention block: just add the $\boldsymbol{p}_i$ to our inputs!
- Recall that $\boldsymbol{x}_i$ is the embedding of the word at index i. The positioned embedding is:

$$\widetilde{\boldsymbol{x}}_i = \boldsymbol{x}_i + \boldsymbol{p}_i$$

In deep self-attention networks, we do this at the first layer! You could concatenate them as well, but people mostly just add…

Position representation vectors through sinusoids

- **Sinusoidal position representations:** concatenate sinusoidal functions of varying periods:

$$\boldsymbol{p}_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$

- Pros:
  - Periodicity indicates that maybe "absolute position" isn't as important
  - Maybe can extrapolate to longer sequences as periods restart!
- Cons:
  - Not learnable; also the extrapolation doesn't really work!

Image: https://timodenk.com/blog/linear-relationships-in-the-transformers-positional-encoding/

Position representation vectors learned from scratch

- **Learned absolute position representations:** Let all p_i be learnable parameters!

  Learn a matrix $\boldsymbol{p} \in \mathbb{R}^{d\times n}$*, and let each* $\boldsymbol{p}_i$ *be a column of that matrix!*

- Pros:
  - Flexibility: each position gets to be learned to fit the data
- Cons:
  - Definitely can't extrapolate to indices outside $1, \dots, n$.
- Most systems use this!

- Sometimes people try more flexible representations of position:
  - Relative linear position attention [Shaw et al., 2018]
  - Dependency syntax-based position [Wang et al., 2019]

Common, modern position embeddings - RoPE

High level thought process: a *relative* position embedding should be some $f(x, i)$ s.t.

$$\langle f(x, i), f(y, j)\rangle = g(x, y, i - j)$$

That is, the attention function *only* gets to depend on the relative position (i-j). How do existing embeddings not fulfill this goal?

•**Sine:** Has various cross-terms that are not relative

- **Absolute:**

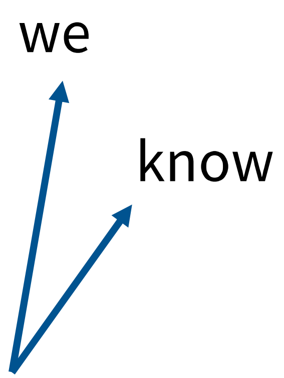
$$e_{ij} = \frac{x_i W^Q (x_j W^K + a_{ij}^K)^T}{\sqrt{d_z}}$$

is not an inner product

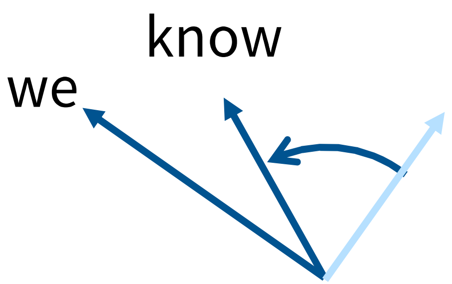
RoPE – Embedding via rotation

How can we solve this problem?

- We want our embeddings to be invariant to absolute position
- We know that inner products are invariant to arbitrary rotation.

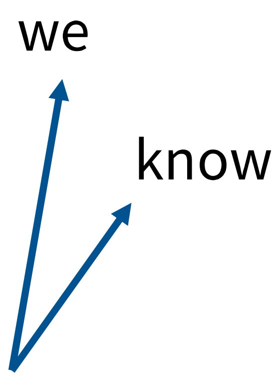


Position independent embedding



Embedding
“of course we know”

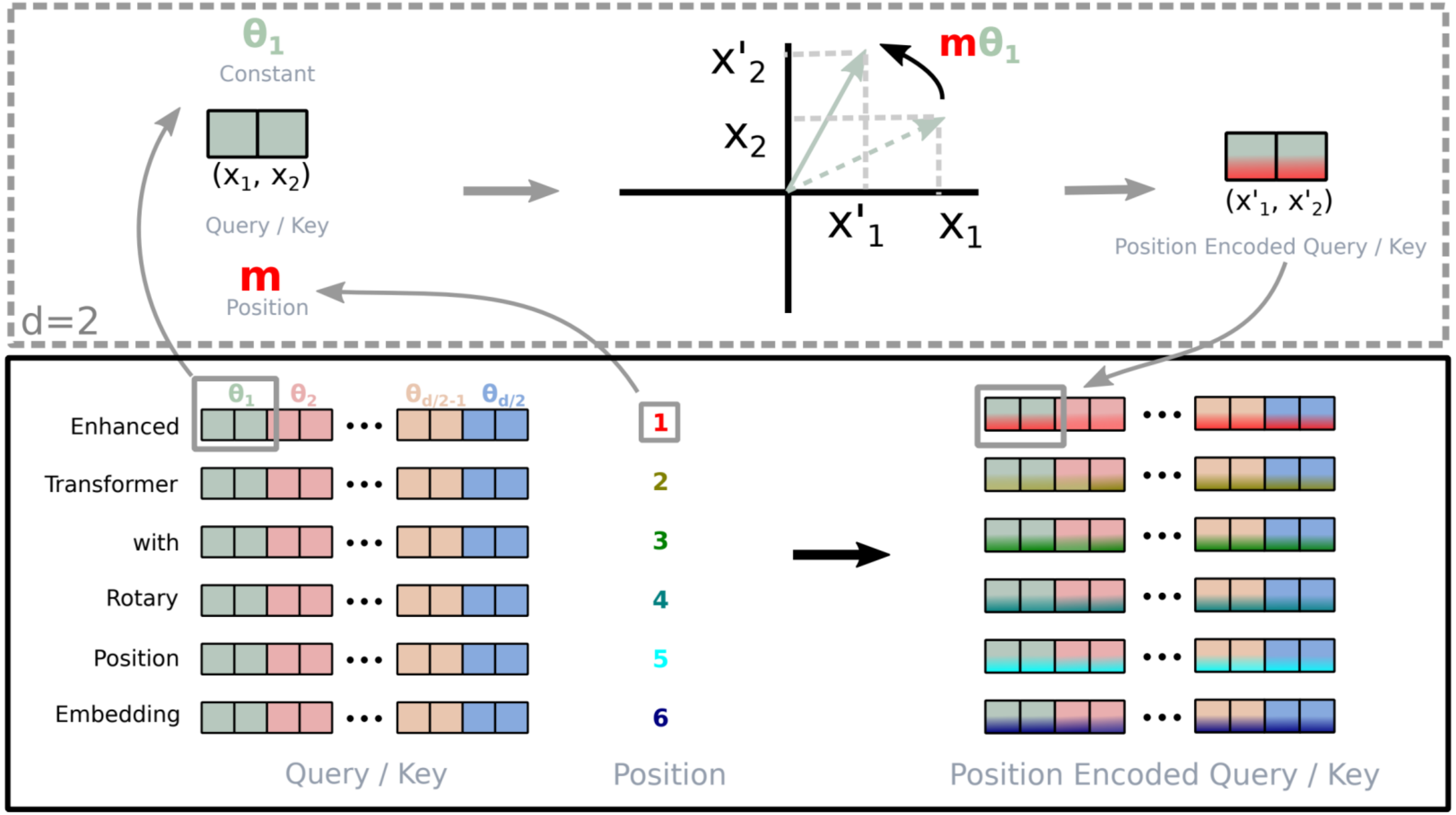
Rotate by ‘2 positions’



Embedding
“we know that”

Rotate by ‘0 positions’

RoPE – From 2 to many dimensions



[Su et al 2021]

Just pair up the coordinates and rotate them in 2d (motivation: complex numbers)

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning! It's all just weighted averages

Solutions

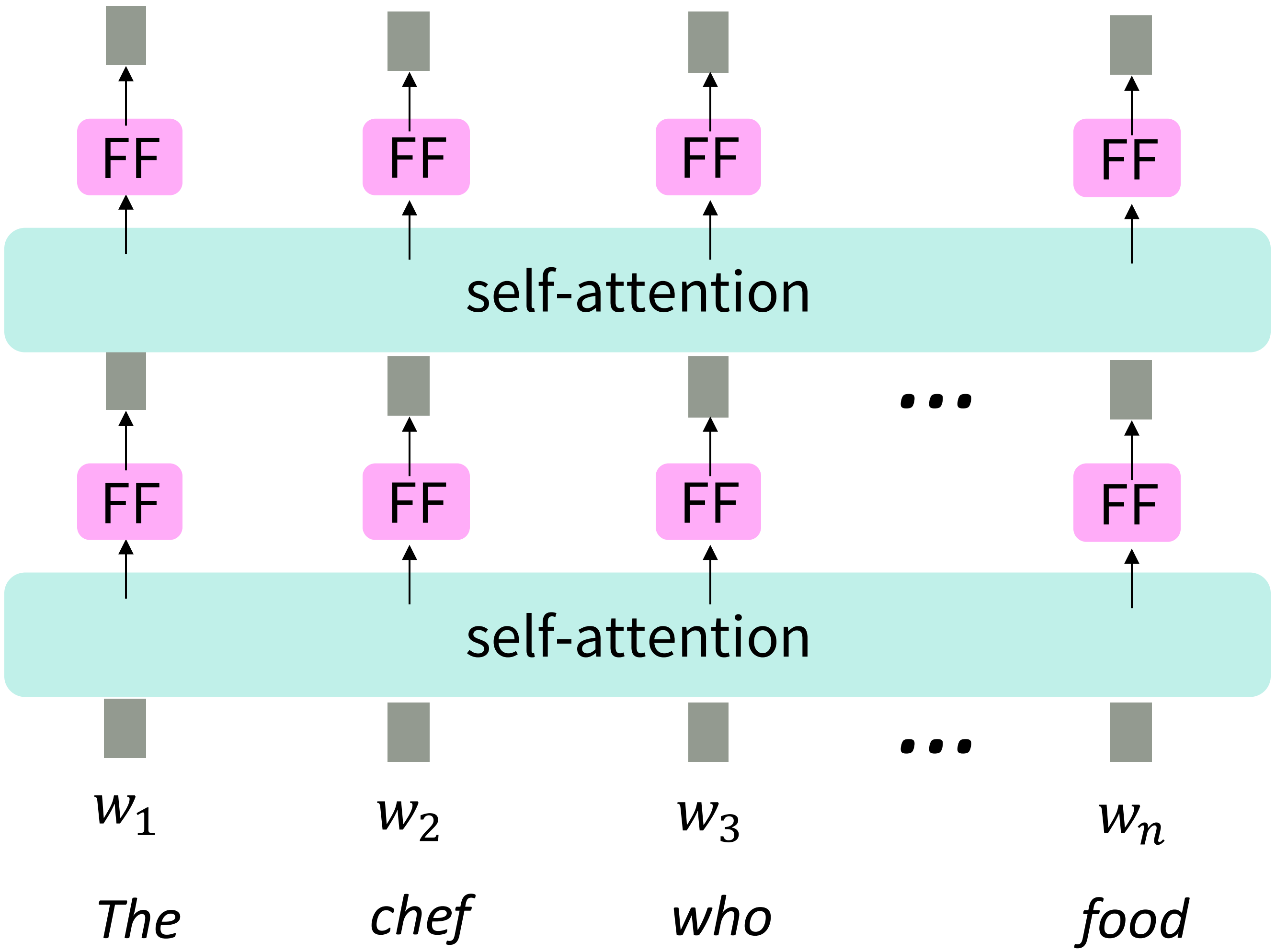
- Add position representations to the inputs

Adding nonlinearities in self-attention

- Note that there are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages **value** vectors (Why? Look at the notes!)

- Easy fix: add a **feed-forward network** to post-process each output vector.

$$m_i = MLP(\text{output}_i)$$
$$= W_2 * \text{ReLU}(W_1 \text{ output}_i + b_1) + b_2$$



Intuition: the FF network processes the result of attention

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn’t have an inherent notion of order!
- No nonlinearities for deep learning magic! It’s all just weighted averages
- Need to ensure we don’t “look at the future” when predicting a sequence
  - Like in machine translation
  - Or language modeling

Solutions

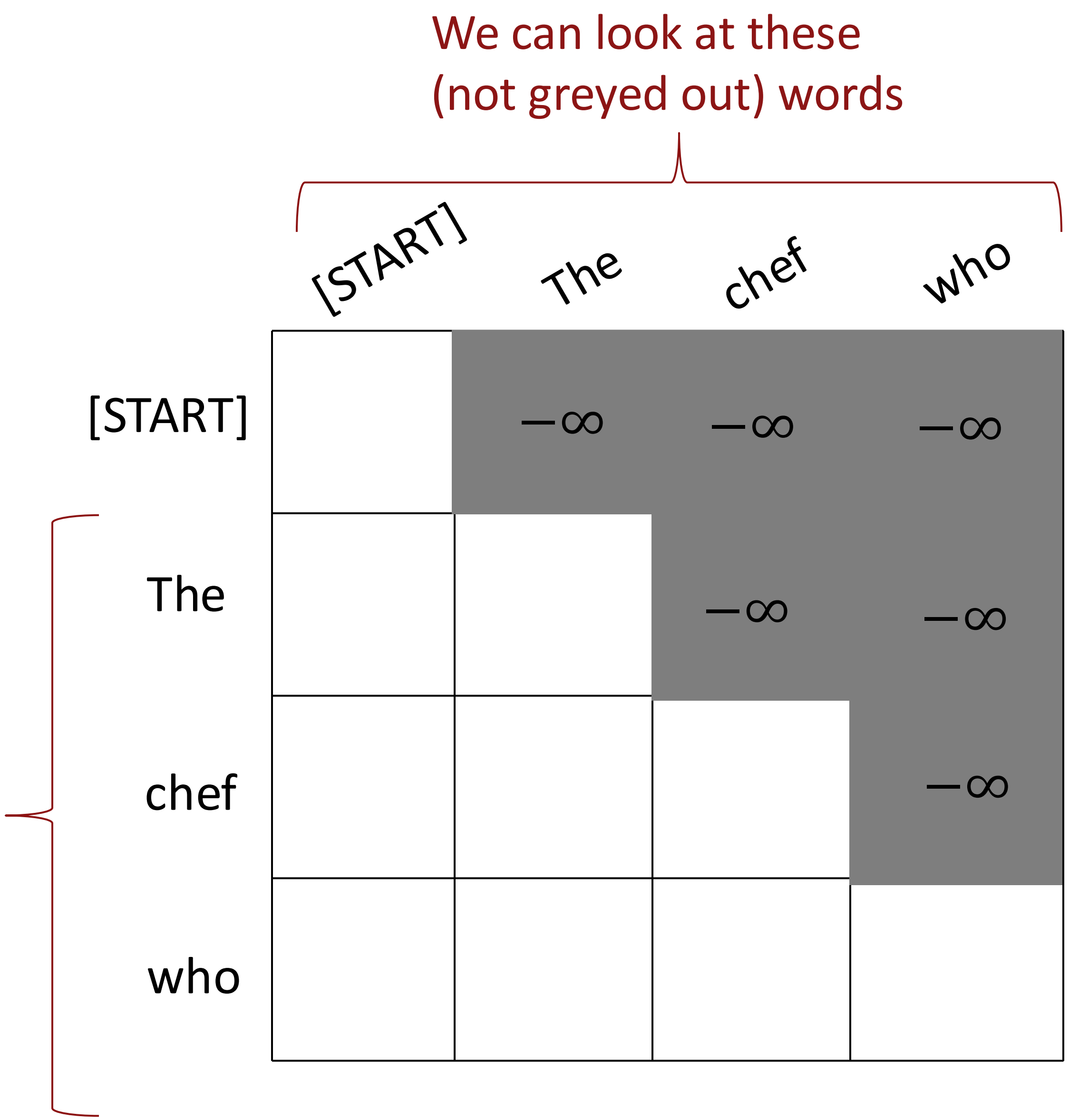
- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.

Masking the future in self-attention

- To use self-attention in **decoders**, we need to ensure we can't peek at the future.
- At every timestep, we could change the set of **keys and queries** to include only past words. (Inefficient!)
- To enable parallelization, we **mask out attention** to future words by setting attention scores to $-\infty$.

$$e_{ij} = \begin{cases} q_i^\top k_j, j \leq i \\ -\infty, j > i \end{cases}$$

For encoding these words



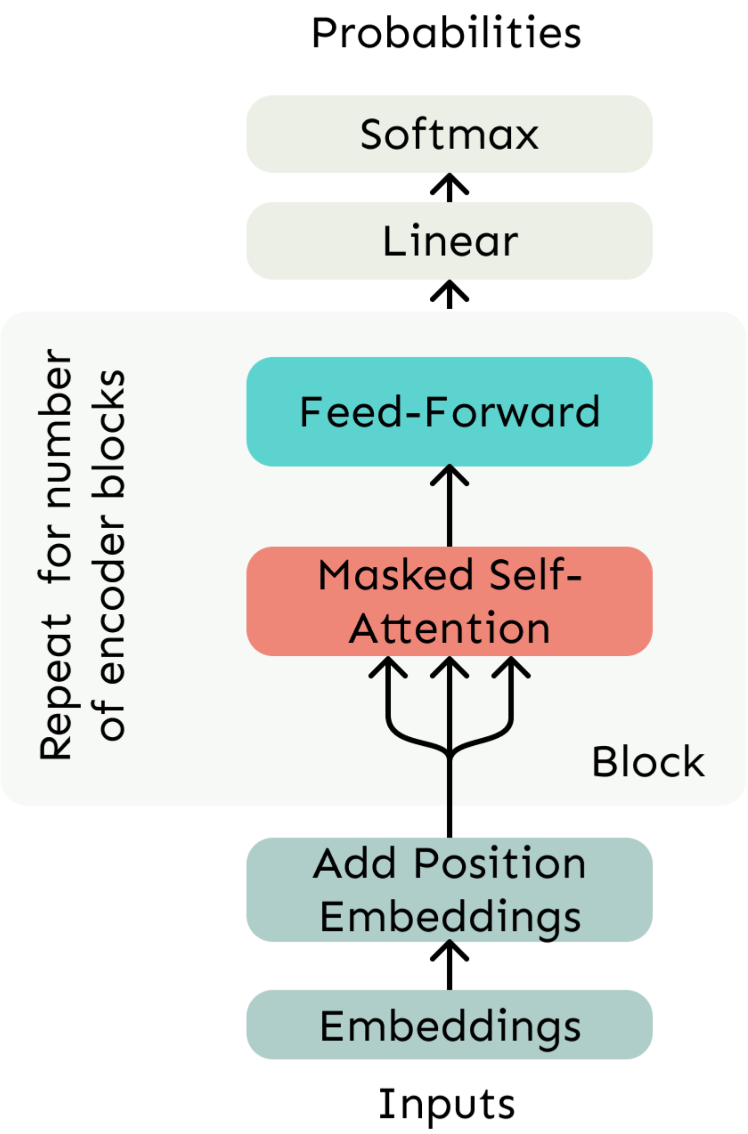
Barriers and solutions for Self-Attention as a building block

Barriers		Solutions
• Doesn't have an inherent notion of order!	→	• Add position representations to the inputs
• No nonlinearities for deep learning magic! It's all just weighted averages	→	• Easy fix: apply the same feedforward network to each self-attention output.

• Need to ensure we don't "look at the future" when predicting a sequence
  • Like in machine translation
  • Or language modeling
	→	• Mask out the future by artificially setting attention weights to 0!

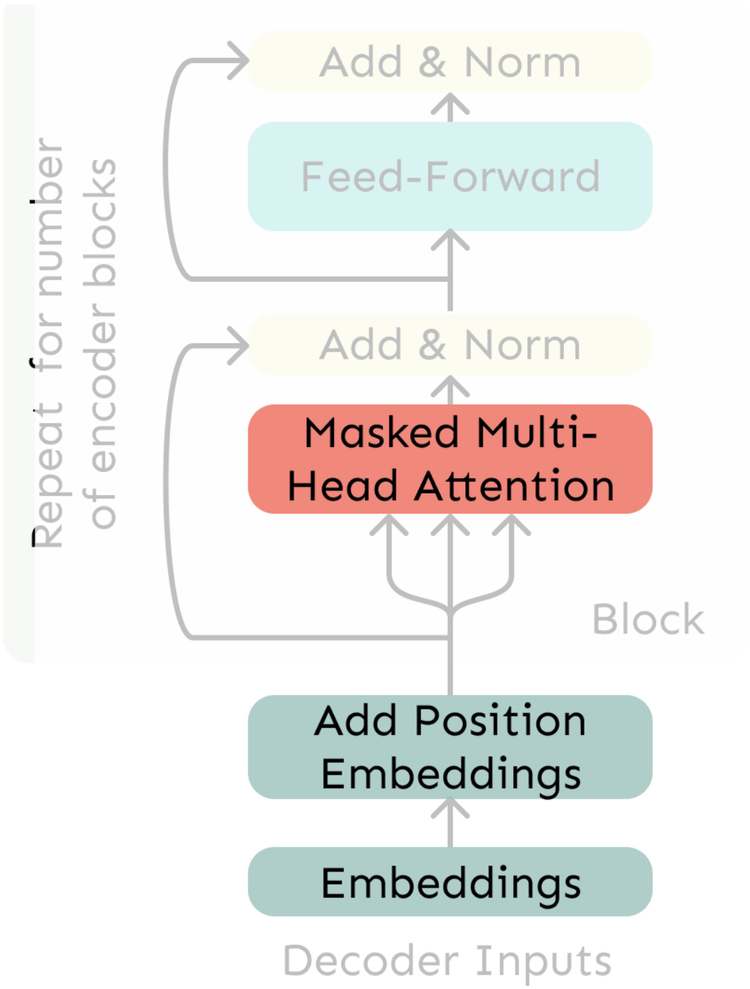
Necessities for a self-attention building block:

- **Self-attention**:
  - the basis of the method.
- **Position representations**:
  - Specify the sequence order, since self-attention is an unordered function of its inputs.
- **Nonlinearities**:
  - At the output of the self-attention block
  - Frequently implemented as a simple feed-forward network.
- **Masking**:
  - In order to parallelize operations while not looking at the future.
  - Keeps information about the future from "leaking" to the past.



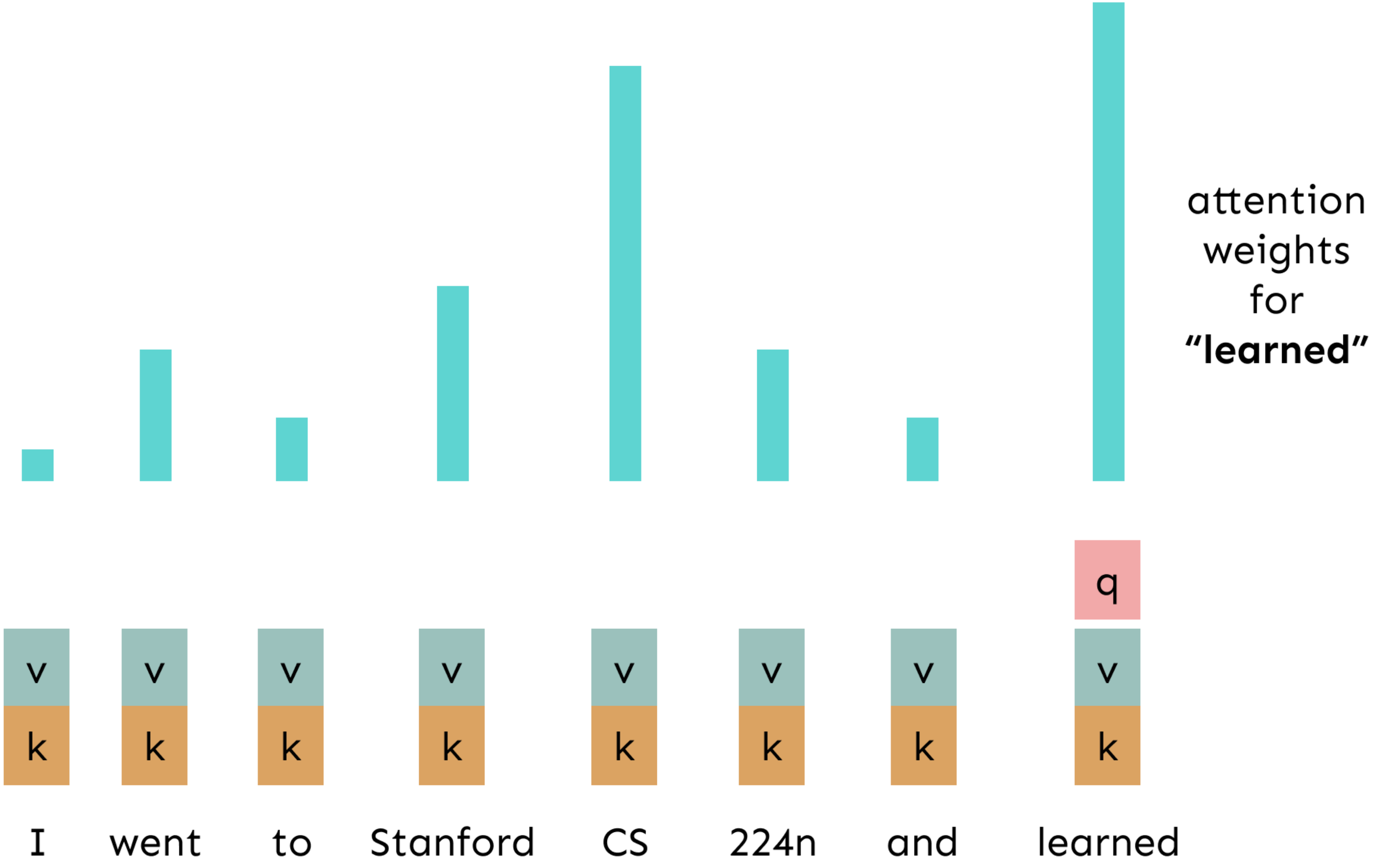
The Transformer Decoder

- A Transformer decoder is how we'll build systems like **language models**.
- It's a lot like our minimal self-attention architecture, but with a few more components.
- The embeddings and position embeddings are identical.
- We'll next replace our self-attention with **multi-head self-attention.**

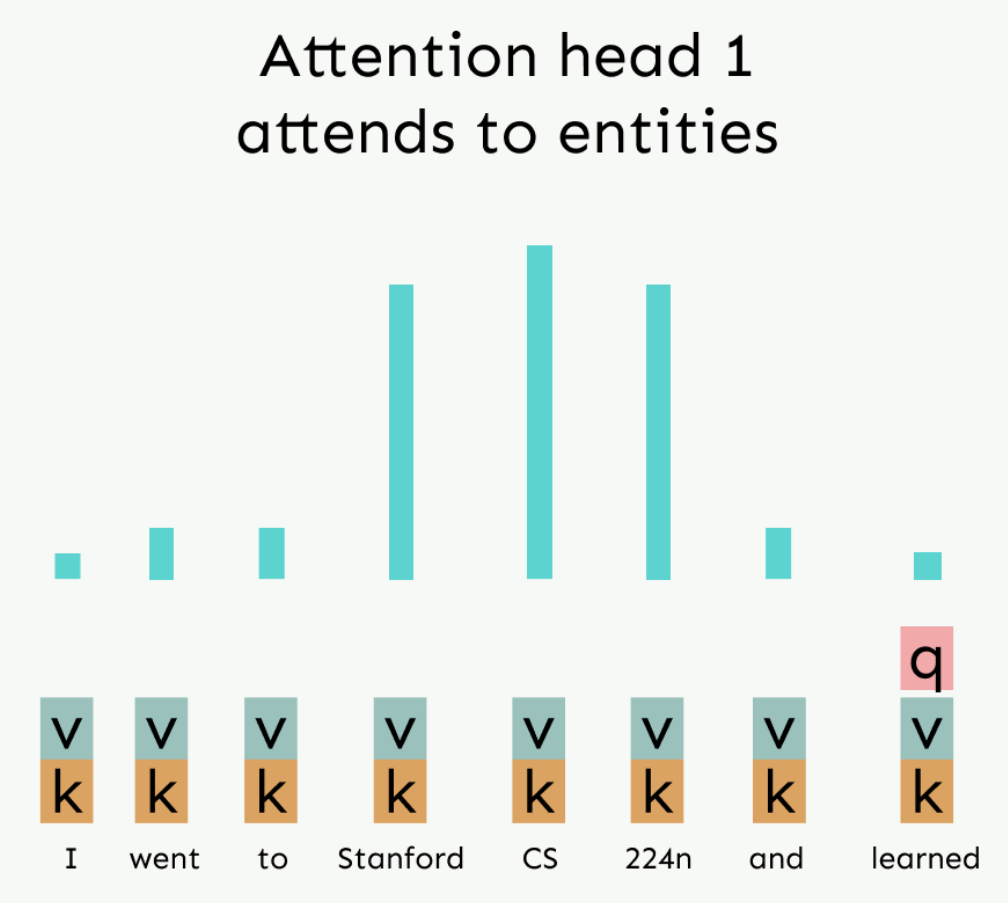


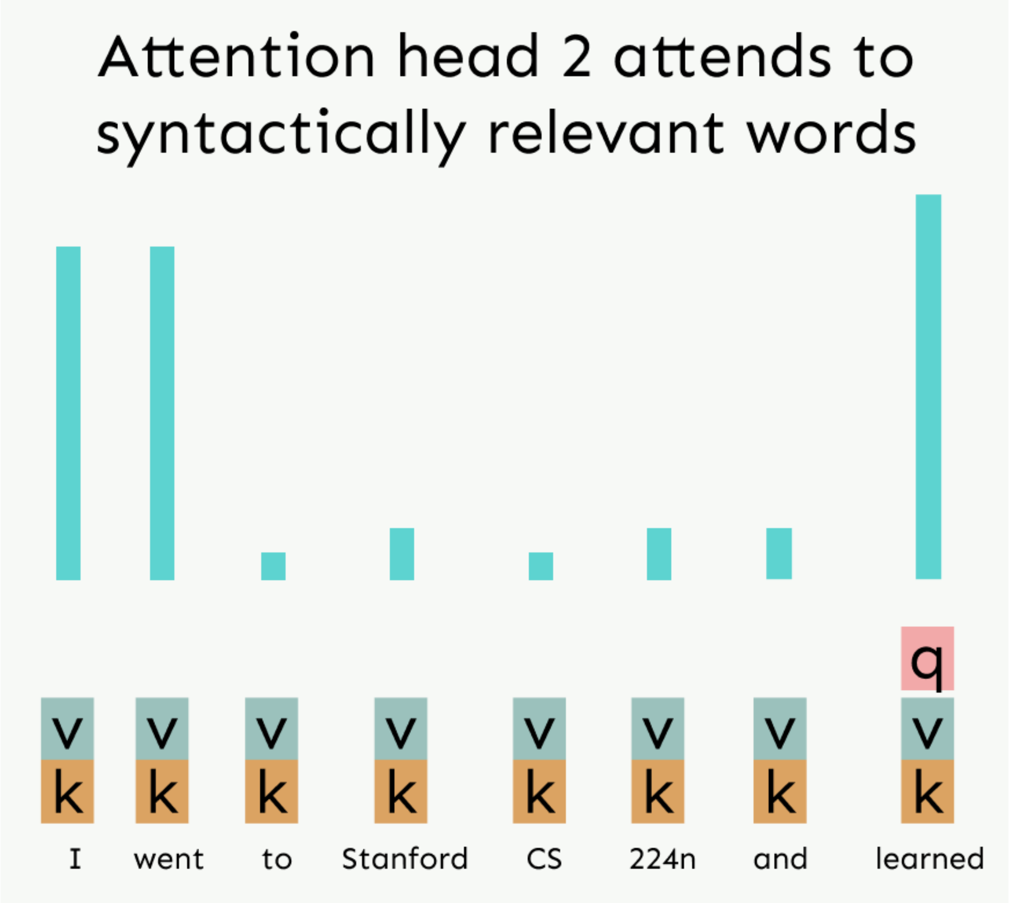
Transformer Decoder

Recall the Self-Attention Hypothetical Example



Hypothetical Example of Multi-Head Attention





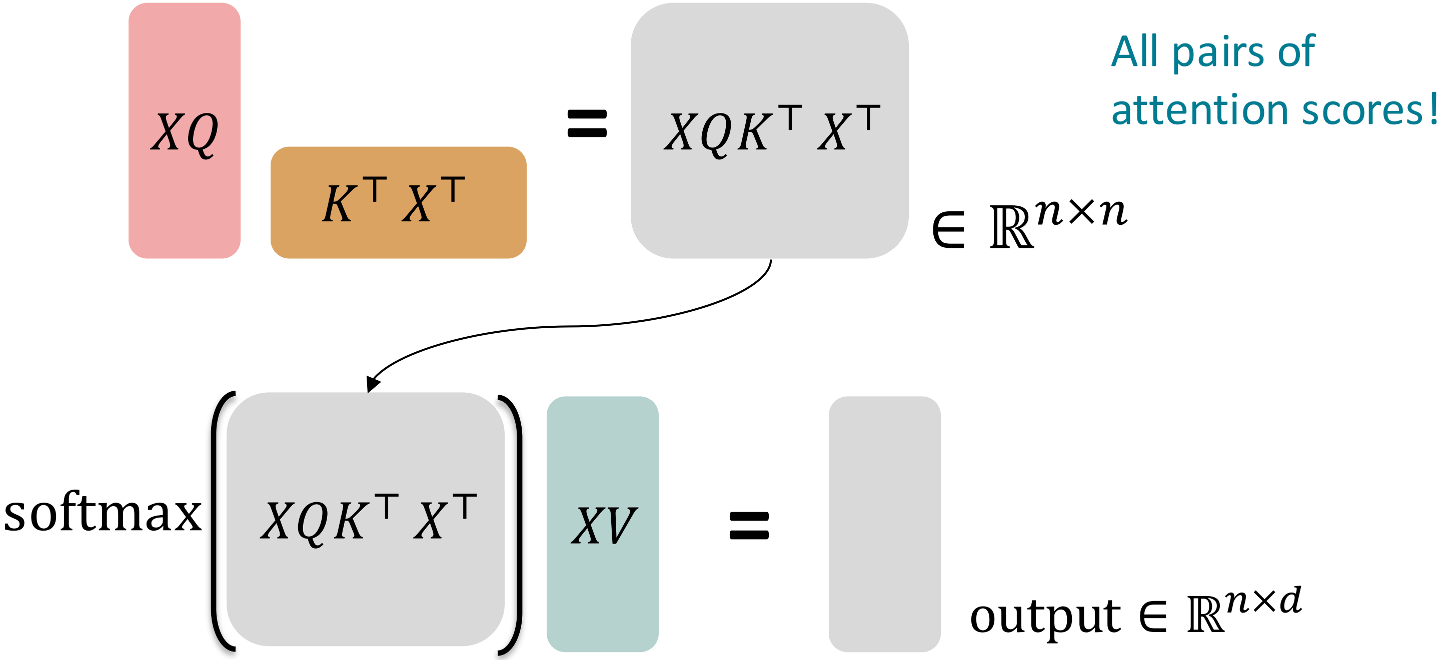
I went to Stanford CS 224n and learned

Sequence-Stacked form of Attention

- Let's look at how key-query-value attention is computed, in matrices.
  - Let $X = [x_1; \ldots; x_n] \in \mathbb{R}^{n\times d}$ be the concatenation of input vectors.
  - First, note that $XK \in \mathbb{R}^{n\times d}$, $XQ \in \mathbb{R}^{n\times d}$, $XV \in \mathbb{R}^{n\times d}$.
  - The output is defined as output $=$ $\mathrm{softmax}(XQ(XK)^\top)XV \in\in \mathbb{R}^{n\times d}$.

First, take the query-key dot products in one matrix multiplication: $XQ(XK)^\top$

Next, softmax, and compute the weighted average with another matrix multiplication.



Multi-headed attention

- What if we want to look in multiple places in the sentence at once?
  - For word i, self-attention "looks" where $x_i^\top Q^\top K x_j$ is high, but maybe we want to focus on different j for different reasons?
- We'll define **multiple attention "heads"** through multiple Q,K,V matrices
- Let, $Q_\ell, K_\ell, V_\ell \in \mathbb{R}^{d\times\frac{d}{h}}$, where h is the number of attention heads, and ℓ ranges from 1 to h.
- Each attention head performs attention independently:
  - $\text{output}_\ell = \text{softmax}\left(XQ_\ell K_\ell^\top X^\top\right) * XV_\ell$, where $\text{output}_\ell \in \mathbb{R}^{d/h}$
- Then the outputs of all the heads are combined!
  - $\text{output} = [\text{output}_1; \ldots; \text{output}_h]Y$, where $Y \in \mathbb{R}^{d\times d}$

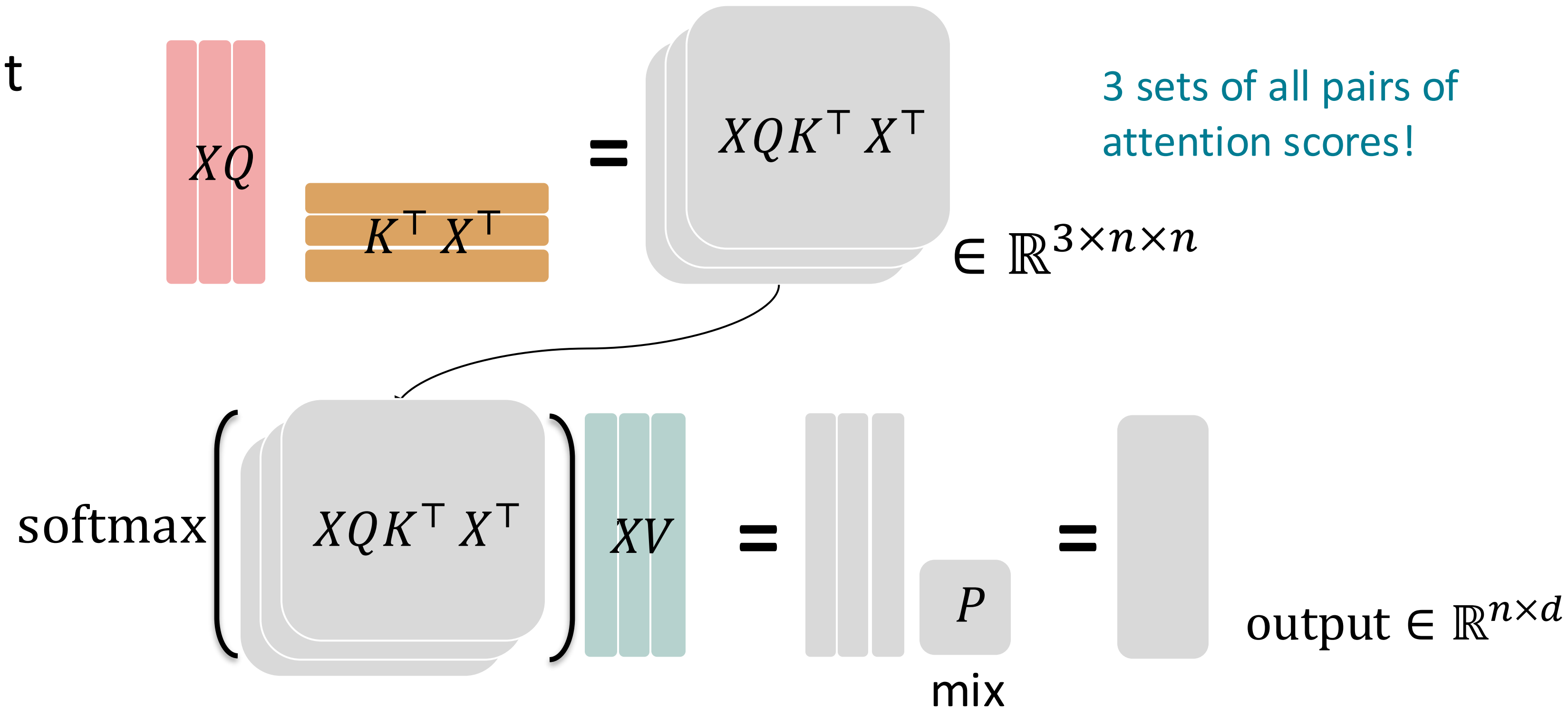
- Each head gets to "look" at different things, and construct value vectors differently.

Multi-head self-attention is computationally efficient

- Even though we compute h many attention heads, it's not really more costly.
  - We compute $XQ \in \mathbb{R}^{n \times d}$, and then reshape to $\mathbb{R}^{n \times h \times d/h}$. (Likewise for XK, XV.)
  - Then we transpose to $\mathbb{R}^{h \times n \times d/h}$; now the head axis is like a batch axis.
  - Almost everything else is identical, and the **matrices are the same sizes.**

First, take the query-key dot products in one matrix multiplication: $XQ(XK)^\top$

Next, softmax, and compute the weighted average with another matrix multiplication.



Scaled Dot Product [Vaswani et al., 2017]

- **"Scaled Dot Product"** attention aids in training.
- When dimensionality d becomes large, dot products between vectors tend to become large.
  - Because of this, inputs to the softmax function can be large, making the gradients small.
- Instead of the self-attention function we've seen:

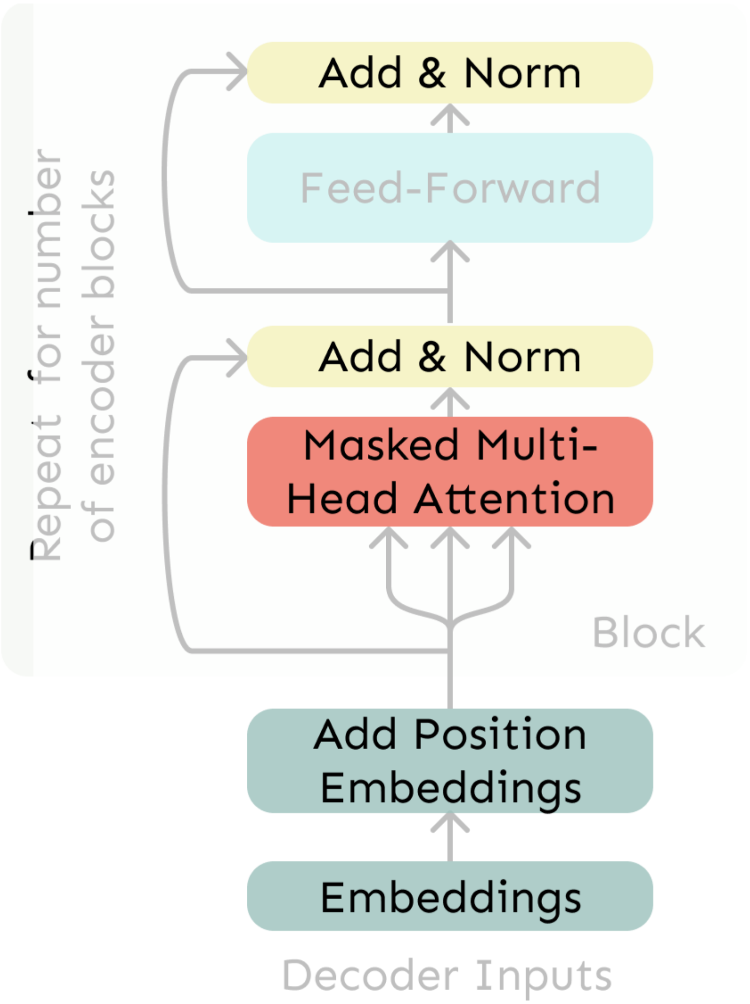
$$\mathrm{output}_\ell = \mathrm{softmax}\left(XQ_\ell K_\ell^\top X^\top\right) * XV_\ell$$

- We divide the attention scores by $\sqrt{d/h}$, to stop the scores from becoming large just as a function of d/h (The dimensionality divided by the number of heads.)

$$\mathrm{output}_\ell = \mathrm{softmax}\left(\frac{XQ_\ell K_\ell^\top X^\top}{\sqrt{d/h}}\right) * XV_\ell$$

The Transformer Decoder

- Now that we've replaced self-attention with multi-head self-attention, we'll go through two **optimization tricks** that end up being :
  - **Residual Connections**
  - **Layer Normalization**
- In most Transformer diagrams, these are often written together as "Add & Norm"

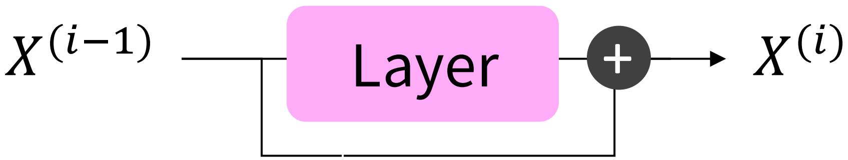


Transformer Decoder

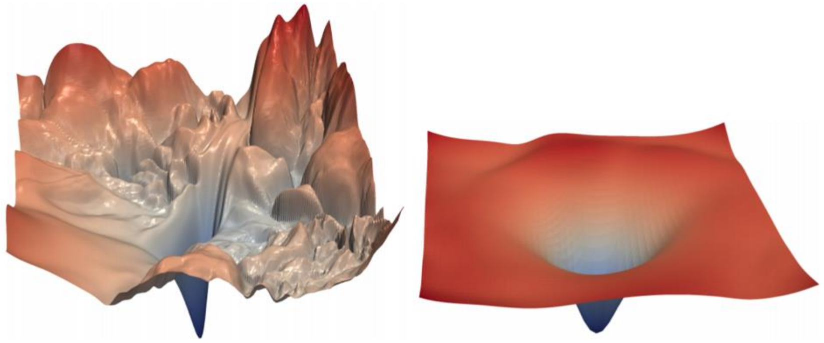
The Transformer Encoder: **Residual connections** [He et al., 2016]

- **Residual connections** are a trick to help models train better.
  - Instead of $X^{(i)} = \text{Layer}(X^{(i-1)})$ (where i represents the layer)

  - We let $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$ (so we only have to learn "the residual" from the previous layer)



  - Gradient is **great** through the residual connection; it's 1!
  - Bias towards the identity function!



[no residuals] [residuals]

[Loss landscape visualization, Li et al., 2018, on a ResNet]

The Transformer Encoder: **Layer normalization** [Ba et al., 2016]

- **Layer normalization** is a trick to help models train faster.
- Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean and standard deviation **within each layer**.
  - LayerNorm's success may be due to its normalizing gradients [Xu et al., 2019]
- Let $x \in \mathbb{R}^d$ be an individual (word) vector in the model.
- Let $\mu = \sum_{j=1}^{d} x_j$; this is the mean; $\mu \in \mathbb{R}$.
- Let $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^{d} \left(x_j - \mu\right)^2}$; this is the standard deviation; $\sigma \in \mathbb{R}$.
- Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned "gain" and "bias" parameters. (Can omit!)
- Then layer normalization computes:

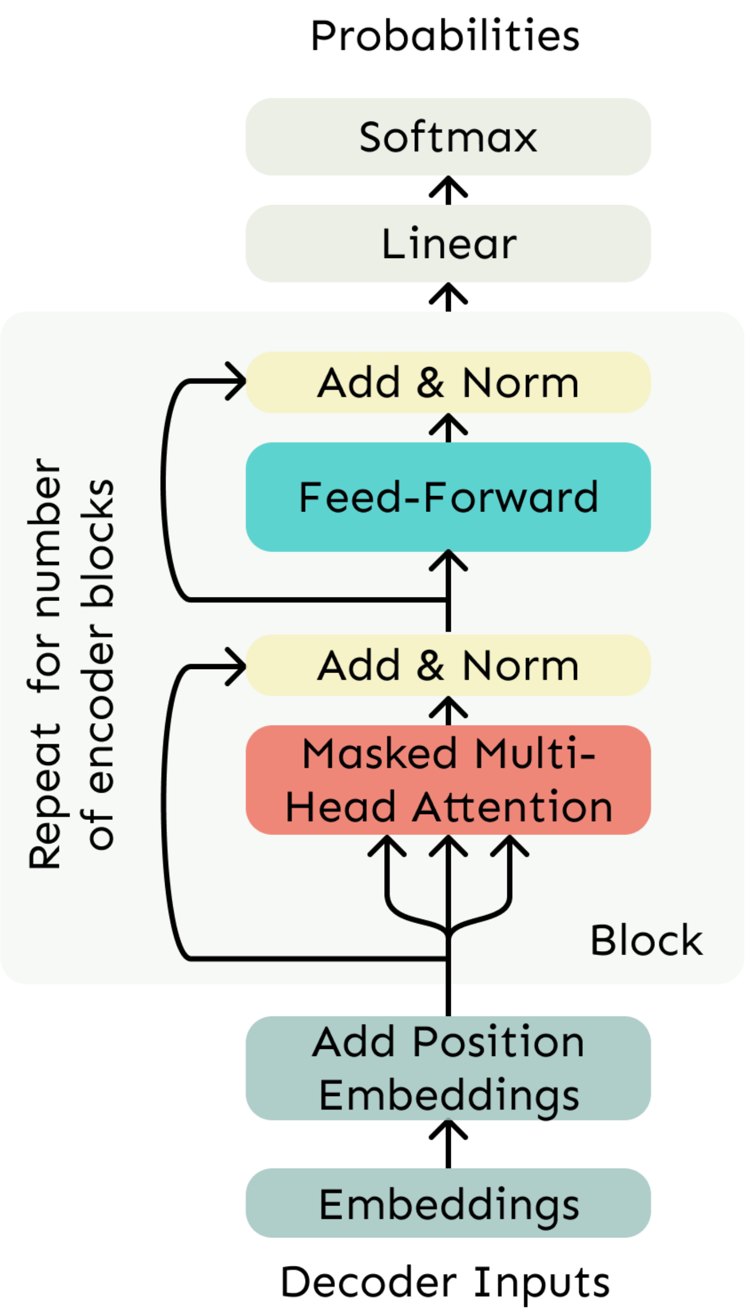
$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

Modulate by learned elementwise gain and bias

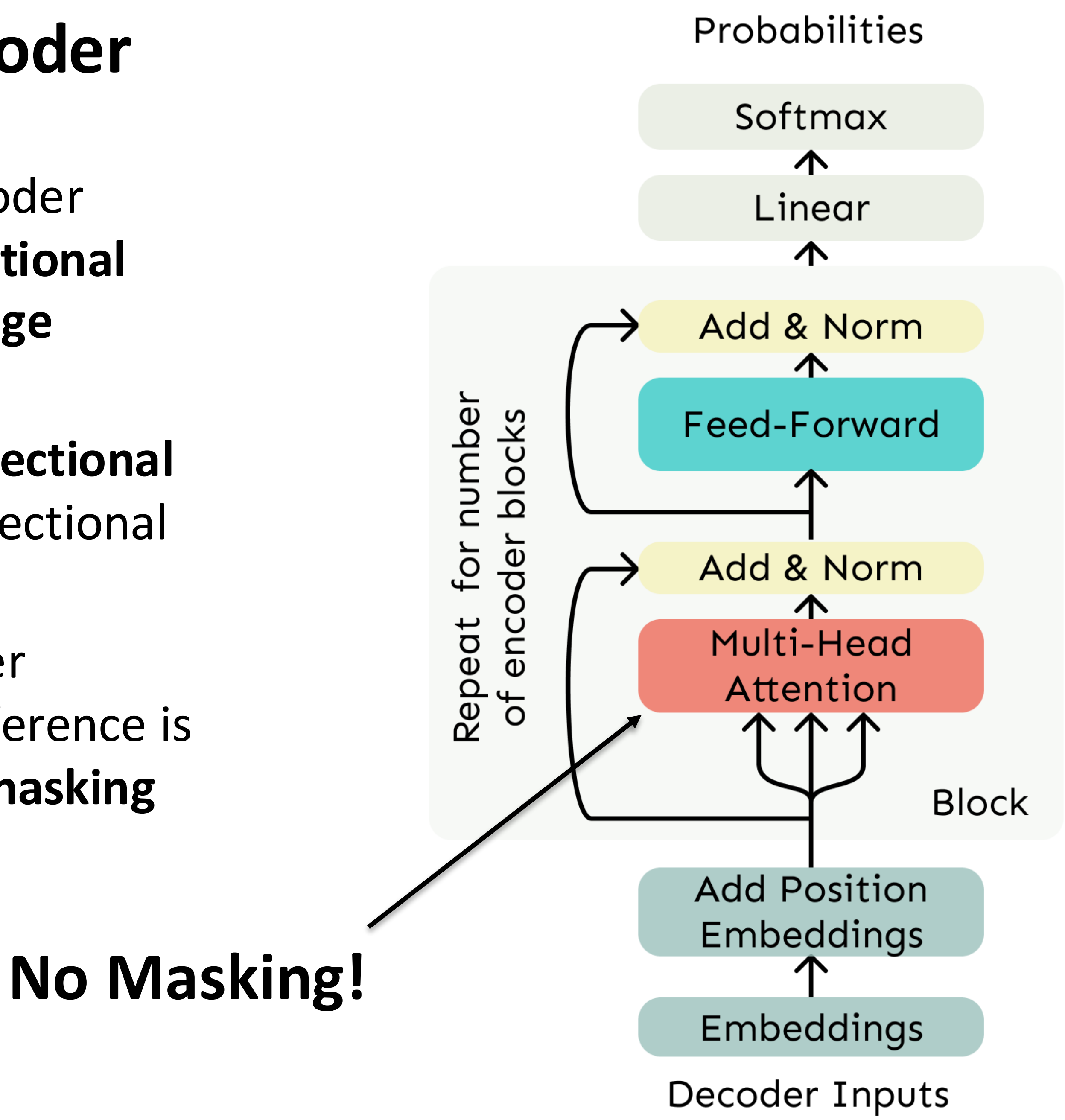
The Transformer Decoder

- The Transformer Decoder is a stack of Transformer Decoder **Blocks**.
- Each Block consists of:
  - Self-attention
  - Add & Norm
  - Feed-Forward
  - Add & Norm
- That’s it! We’ve gone through the Transformer Decoder.



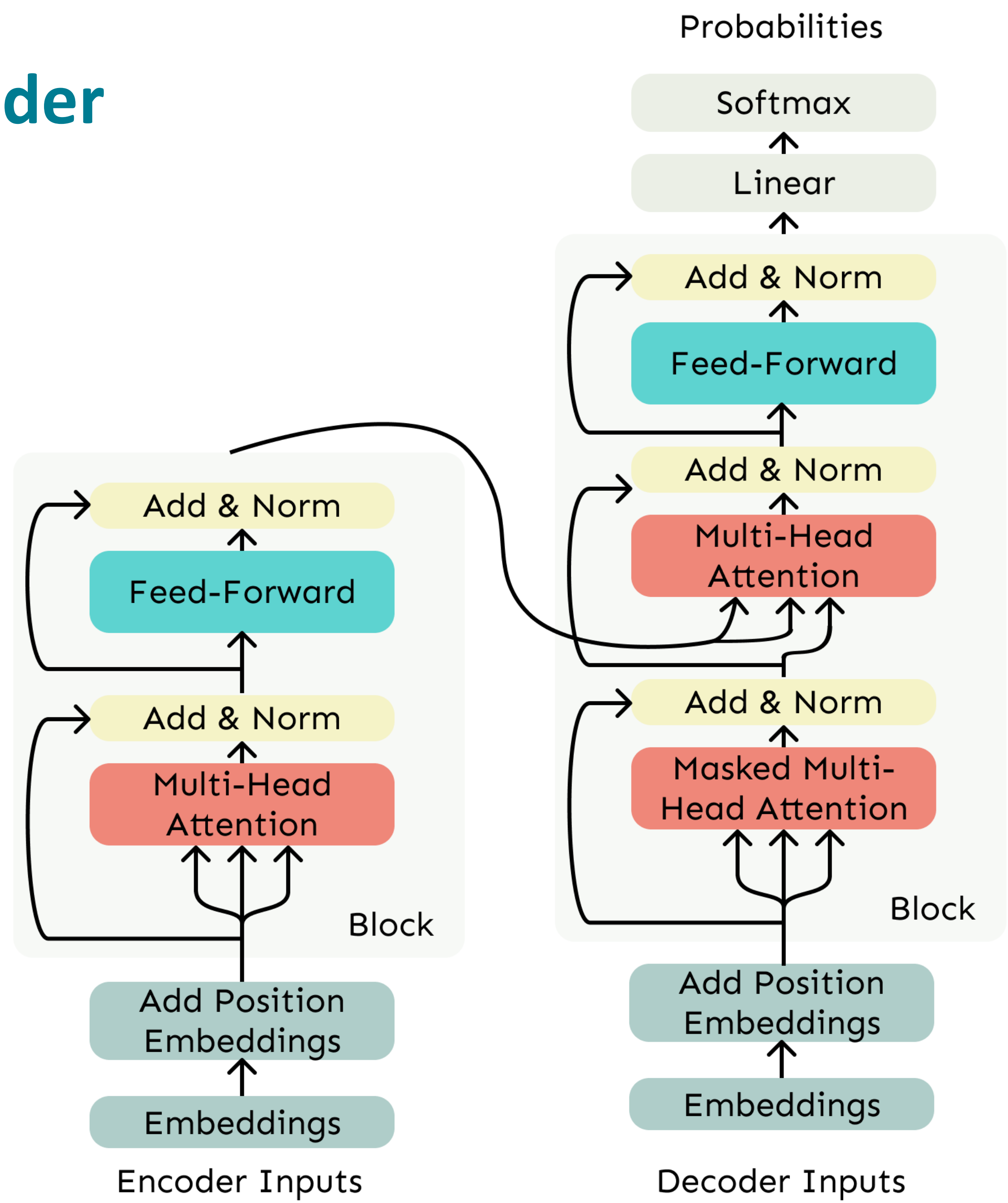
The Transformer Encoder

- The Transformer Decoder constrains to **unidirectional context**, as for **language models.**
- What if we want **bidirectional context**, like in a bidirectional RNN?
- This is the Transformer Encoder. The only difference is that we **remove the masking** in the self-attention.



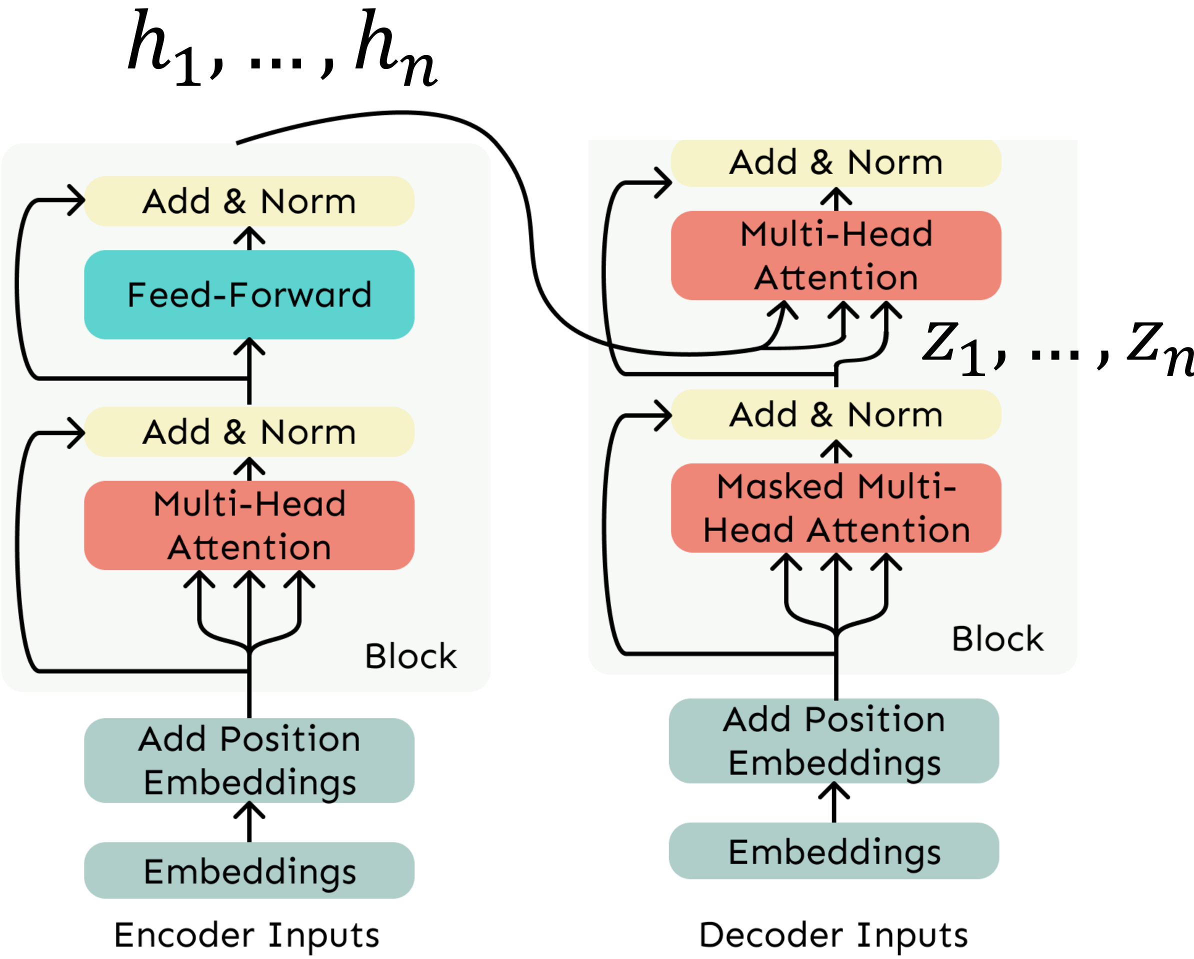
The Transformer Encoder-Decoder

- Recall that in machine translation, we processed the source sentence with a **bidirectional** model and generated the target with a **unidirectional model**.
- For this kind of seq2seq format, we often use a Transformer Encoder-Decoder.
- We use a normal Transformer Encoder.
- Our Transformer Decoder is modified to perform **cross-attention** to the output of the Encoder.



Cross-attention (details)

- We saw that self-attention is when keys, queries, and values come from the same source.
- In the decoder, we have attention that looks more like what we saw last week.
- Let $h_1, \dots, h_n$ be **output** vectors **from** the Transformer **encoder**; $x_i \in \mathbb{R}^d$
- Let $z_1, \dots, z_n$ be input vectors from the Transformer **decoder**, $z_i \in \mathbb{R}^d$
- Then keys and values are drawn from the **encoder** (like a memory):
  - $k_i = Kh_i, v_i = Vh_i$.
- And the queries are drawn from the **decoder**, $q_i = Qz_i$.



Cross-attention (details)

- Let's look at how cross-attention is computed, in matrices.
  - Let $\mathrm{H} = [h_1; \dots; h_T] \in \mathbb{R}^{T\times d}$ be the concatenation of encoder vectors.
  - Let $\mathrm{Z} = [z_1; \dots; z_T] \in \mathbb{R}^{T\times d}$ be the concatenation of decoder vectors.
  - The output is defined as output $= \mathrm{softmax}(ZQ(HK)^\top) \times HV$.

First, take the query-key dot products in one matrix multiplication: $ZQ(HK)^\top$

$$ZQ \quad K^\top H^\top = ZQK^\top H^\top \in \mathbb{R}^{T\times T}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$$\mathrm{softmax}\left(ZQK^\top H^\top\right) HV = \quad \text{output} \in \mathbb{R}^{T\times d}$$

Great Results with Transformers

First, Machine Translation from the original Transformers paper!

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	**41.29**	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$

[Test sets: WMT 2014 English-German and English-French]

[Vaswani et al., 2017]

Great Results with Transformers

Next, document generation!

Model	Test perplexity	ROUGE-L
seq2seq-attention, $L = 500$	5.04952	12.7
Transformer-ED, $L = 500$	2.46645	34.2
Transformer-D, $L = 4000$	2.22216	33.6
Transformer-DMCA, no MoE-layer, $L = 11000$	2.05159	36.2
Transformer-DMCA, MoE-128, $L = 11000$	1.92871	37.9
Transformer-DMCA, MoE-256, $L = 7500$	1.90325	38.8

The old standard

Transformers all the way down.

[Liu et al., 2018]; WikiSum dataset

Great Results with Transformers

Before too long, most Transformers results also included **pretraining**, a method we'll go over next.

Transformers' parallelizability allows for efficient pretraining, and have made them the de-facto standard.

On this popular aggregate benchmark, for example:

GLUE

All top models are Transformer (and pretraining)-based.

Rank	Name	Model	URL	Score
1	DeBERTa Team - Microsoft	DeBERTa / TuringNLRv4		90.8
2	HFL iFLYTEK	MacALBERT + DKM		90.7
3	Alibaba DAMO NLP	StructBERT + TAPT		90.6
4	PING-AN Omni-Sinitic	ALBERT + DAAF + NAS		90.6
5	ERNIE Team - Baidu	ERNIE		90.4
6	T5 Team - Google	T5		90.3

[Liu et al., 2018]

What would we like to fix about the Transformer?

- **Training instabilities (Pre vs Post norm)**
- **Quadratic compute in self-attention** :
  - Computing all pairs of interactions means our computation grows **quadratically** with the sequence length!
  - For recurrent models, it only grew linearly!

Pre vs Post norm

The one thing *everyone* agrees on (in 2024)

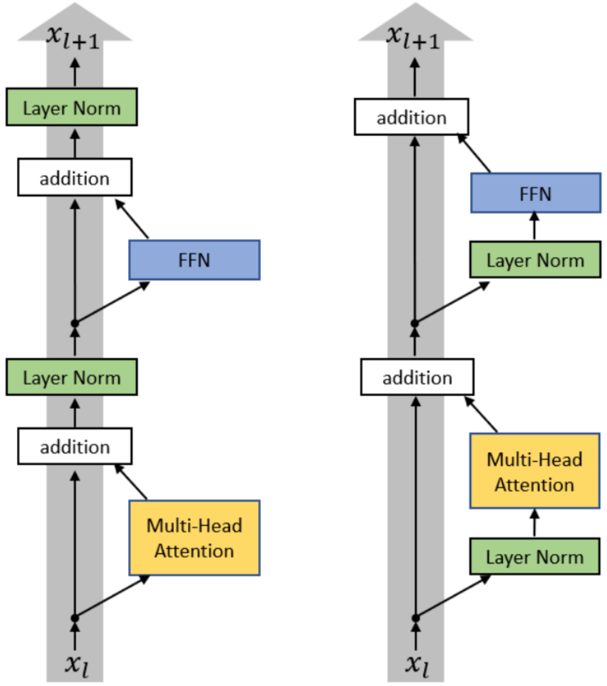


Figure from Xiong 2020

Post-LN Transformer	Pre-LN Transformer
$x^{post,1}_{l,i} = \text{MultiHeadAtt}(x^{post}_{l,i}, [x^{post}_{l,1}, \cdots, x^{post}_{l,n}])$	$x^{pre,1}_{l,i} = \text{LayerNorm}(x^{pre}_{l,i})$
$x^{post,2}_{l,i} = x^{post}_{l,i} + x^{post,1}_{l,i}$	$x^{pre,2}_{l,i} = \text{MultiHeadAtt}(x^{pre,1}_{l,i}, [x^{pre,1}_{l,1}, \cdots, x^{pre,1}_{l,n}])$
$x^{post,3}_{l,i} = \text{LayerNorm}(x^{post,2}_{l,i})$	$x^{pre,3}_{l,i} = x^{pre}_{l,i} + x^{pre,2}_{l,i}$
$x^{post,4}_{l,i} = \text{ReLU}(x^{post,3}_{l,i} W^{1,l} + b^{1,l})W^{2,l} + b^{2,l}$	$x^{pre,4}_{l,i} = \text{LayerNorm}(x^{pre,3}_{l,i})$
$x^{post,5}_{l,i} = x^{post,3}_{l,i} + x^{post,4}_{l,i}$	$x^{pre,5}_{l,i} = \text{ReLU}(x^{pre,4}_{l,i} W^{1,l} + b^{1,l})W^{2,l} + b^{2,l}$
$x^{post}_{l+1,i} = \text{LayerNorm}(x^{post,5}_{l,i})$	$x^{pre}_{l+1,i} = x^{pre,5}_{l,i} + x^{pre,3}_{l,i}$
	Final LayerNorm: $x^{pre}_{Final,i} \leftarrow \text{LayerNorm}(x^{pre}_{L+1,i})$

Set up LayerNorm so that it doesn't affect the main residual signal path (on the left)

Almost all modern LMs use pre-norm (but BERT was post-norm)

(One somewhat funny exception – OPT350M. I don't know why this is post-norm)

Quadratic computation as a function of sequence length

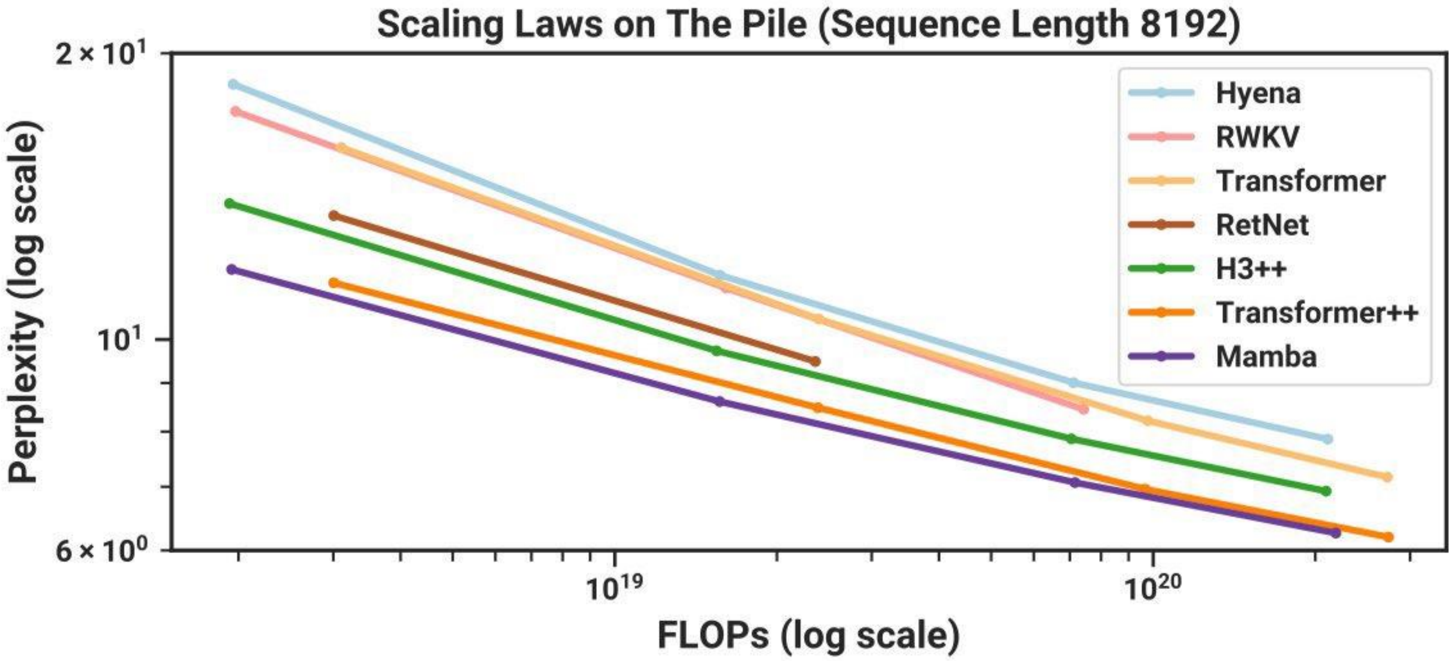
- One of the benefits of self-attention over recurrence was that it's highly parallelizable.
- However, its total number of operations grows as $O(n^2 d)$, where n is the sequence length, and d is the dimensionality.

$$XQ \; K^\top X^\top = XQK^\top X^\top \in \mathbb{R}^{n \times n}$$

Need to compute all pairs of interactions! $O(n^2 d)$

- Think of d as around $\mathbf{1,000}$ (though for large language models it's much larger!).
  - So, for a single (shortish) sentence, $n \leq 30; n^2 \leq \mathbf{900}$**.**
  - In practice, we set a bound like $n = 512$.
  - **But what if we'd like $\boldsymbol{n} \geq \mathbf{50,000}$?** For example, to work on long documents?

Back to the future – RNNs are back!



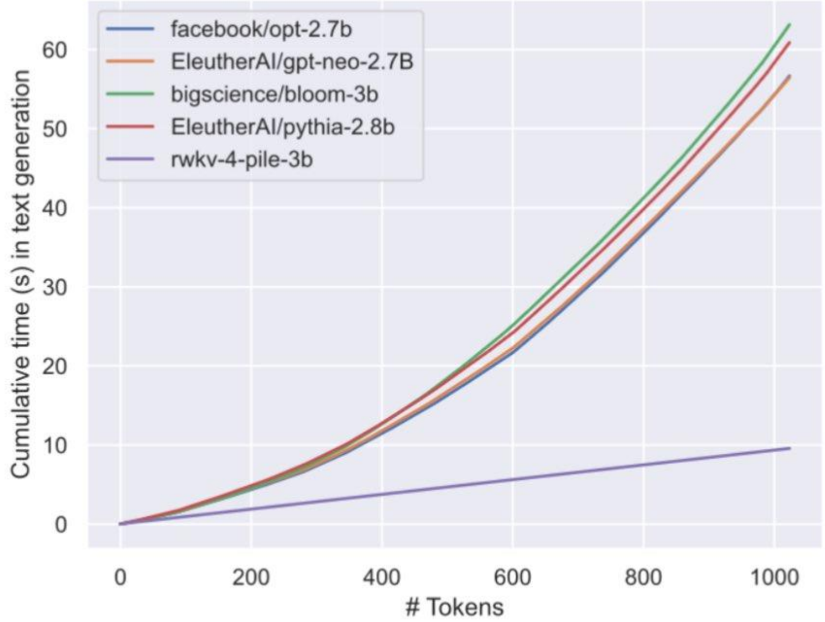
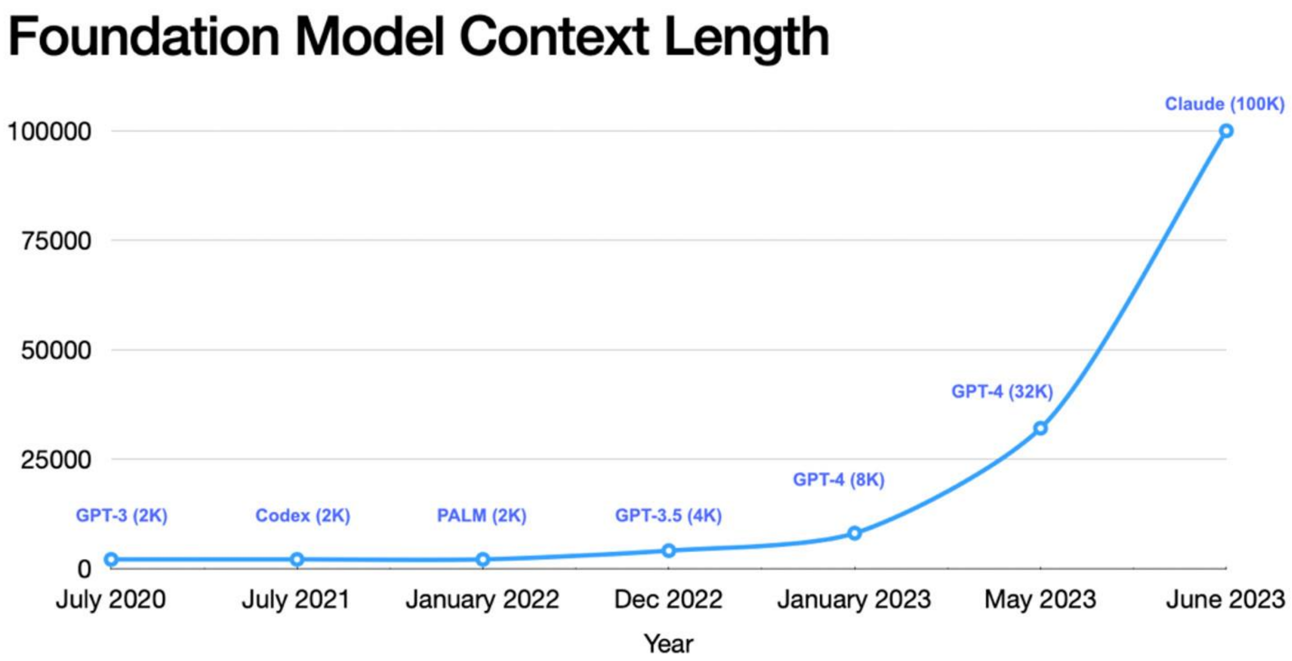


Figure 7: Cumulative time on text generation for LLM
Unlike transformers, RWKV exhibits linear scaling.

If you want *really* long context, RNNs provide this (linear complexity).
Modern RNNs (RWKV, Mamba, etc) are getting better!

Do we even need to remove the quadratic cost of attention?

- As Transformers grow larger, a larger and larger percent of compute is **outside** the self-attention portion, despite the quadratic cost.
- In practice, **production Transformer language models use quadratic cost attention**
  - The cheaper methods tend not to work as well at scale.
  - Systems optimizations work well (Flash attention – Jun 2022)



Do Transformer Modifications Transfer?

- "Surprisingly, we find that most modifications do not meaningfully improve performance."

Model	Params	Ops	Step/s	Early loss	Final loss	SGLUE	XSum	WebQ	WMT EnDe
Vanilla Transformer	223M	11.1T	3.50	2.182 ± 0.005	1.838	71.66	17.78	23.02	26.62
GeLU	223M	11.1T	3.58	2.179 ± 0.003	1.838	**75.79**	**17.86**	**25.13**	26.47
Swish	223M	11.1T	3.62	2.186 ± 0.003	1.847	**73.77**	17.74	**24.34**	**26.75**
ELU	223M	11.1T	3.56	2.270 ± 0.007	1.932	67.83	16.73	23.02	26.08
GLU	223M	11.1T	3.59	2.174 ± 0.003	**1.814**	**74.20**	**17.42**	24.34	**27.12**
GeGLU	223M	11.1T	3.55	2.130 ± 0.006	**1.792**	**75.96**	**18.27**	**24.87**	**26.87**
ReGLU	223M	11.1T	3.57	2.145 ± 0.004	**1.803**	**76.17**	**18.36**	**24.87**	**27.02**
SeLU	223M	11.1T	3.55	2.315 ± 0.004	1.948	68.76	16.76	22.75	25.99
SwiGLU	223M	11.1T	3.53	2.127 ± 0.003	**1.789**	**76.00**	**18.20**	**24.34**	**27.02**
LiGLU	223M	11.1T	3.59	2.149 ± 0.005	**1.798**	**75.34**	**17.97**	**24.34**	26.53
Sigmoid	223M	11.1T	3.63	2.291 ± 0.019	1.867	**74.31**	17.51	23.02	26.30
Softplus	223M	11.1T	3.47	2.207 ± 0.011	1.850	**72.45**	17.65	**24.34**	**26.89**
RMS Norm	223M	11.1T	3.68	2.167 ± 0.008	**1.821**	**75.45**	**17.94**	**24.07**	**27.14**
Rezero	223M	11.1T	3.51	2.262 ± 0.003	1.939	61.69	15.64	20.90	26.37
Rezero + LayerNorm	223M	11.1T	3.26	2.223 ± 0.006	1.858	70.42	17.58	23.02	26.29
Rezero + RMS Norm	223M	11.1T	3.34	2.221 ± 0.009	1.875	70.33	17.32	23.02	26.19
Fixup	223M	11.1T	2.95	2.382 ± 0.012	2.067	58.56	14.42	23.02	26.31
24 layers, $d_{ff} = 1536, H = 6$	224M	11.1T	3.33	2.200 ± 0.007	1.843	**74.89**	17.75	**25.13**	**26.89**
18 layers, $d_{ff} = 2048, H = 8$	223M	11.1T	3.38	2.185 ± 0.005	**1.831**	**76.45**	16.83	**24.34**	**27.10**
8 layers, $d_{ff} = 4608, H = 18$	223M	11.1T	3.69	2.190 ± 0.005	1.847	**74.58**	17.69	**23.28**	**26.85**
6 layers, $d_{ff} = 6144, H = 24$	223M	11.1T	3.70	2.201 ± 0.010	1.857	**73.55**	17.59	**24.60**	**26.66**
Block sharing	65M	11.1T	3.91	2.497 ± 0.037	2.164	64.50	14.53	21.96	25.48
+ Factorized embeddings	45M	9.4T	4.21	2.631 ± 0.305	2.183	60.84	14.00	19.84	25.27
+ Factorized & shared embeddings	20M	9.1T	4.37	2.907 ± 0.313	2.385	53.95	11.37	19.84	25.19
Encoder only block sharing	170M	11.1T	3.68	2.298 ± 0.023	1.929	69.60	16.23	23.02	26.23
Decoder only block sharing	144M	11.1T	3.70	2.352 ± 0.029	2.082	67.93	16.13	**23.81**	26.08
Factorized Embedding	227M	9.4T	3.80	2.208 ± 0.006	1.855	70.41	15.92	22.75	26.50
Factorized & shared embeddings	202M	9.1T	3.92	2.320 ± 0.010	1.952	68.69	16.33	22.22	26.44
Tied encoder/decoder input embeddings	248M	11.1T	3.55	2.192 ± 0.002	1.840	**71.70**	17.72	**24.34**	26.49
Tied decoder input and output embeddings	248M	11.1T	3.57	2.187 ± 0.007	**1.827**	**74.86**	17.74	**24.87**	**26.67**
Untied embeddings	273M	11.1T	3.53	2.195 ± 0.005	**1.834**	**72.99**	17.58	**23.28**	26.48
Adaptive input embeddings	204M	9.2T	3.55	2.250 ± 0.002	1.899	66.57	16.21	**24.07**	**26.66**
Adaptive softmax	204M	9.2T	3.60	2.364 ± 0.005	1.982	**72.91**	16.67	21.16	25.56
Adaptive softmax without projection	223M	10.8T	3.43	2.229 ± 0.009	1.914	**71.82**	17.10	23.02	25.72
Mixture of softmaxes	232M	16.3T	2.24	2.227 ± 0.017	**1.821**	**76.77**	17.62	22.75	**26.82**
Transparent attention	223M	11.1T	3.33	2.181 ± 0.014	1.874	54.31	10.40	21.16	**26.80**
Dynamic convolution	257M	11.8T	2.65	2.403 ± 0.009	2.047	58.30	12.67	21.16	17.03
Lightweight convolution	224M	10.4T	4.07	2.370 ± 0.010	1.989	63.07	14.86	23.02	24.73
Evolved Transformer	217M	9.9T	3.09	2.220 ± 0.003	1.863	**73.67**	10.76	**24.07**	26.58
Synthesizer (dense)	224M	11.4T	3.47	2.334 ± 0.021	1.962	61.03	14.27	16.14	**26.63**
Synthesizer (dense plus)	243M	12.6T	3.22	2.191 ± 0.010	1.840	**73.98**	16.96	**23.81**	**26.71**
Synthesizer (dense plus alpha)	243M	12.6T	3.01	2.180 ± 0.007	**1.828**	**74.25**	17.02	**23.28**	26.61
Synthesizer (factorized)	207M	10.1T	3.94	2.341 ± 0.017	1.968	62.78	15.39	**23.55**	26.42
Synthesizer (random)	254M	10.1T	4.08	2.326 ± 0.012	2.009	54.27	10.35	19.56	26.44
Synthesizer (random plus)	292M	12.0T	3.63	2.189 ± 0.004	1.842	**73.32**	17.04	**24.87**	26.43
Synthesizer (random plus alpha)	292M	12.0T	3.42	2.186 ± 0.007	**1.828**	**75.24**	17.08	**24.08**	26.39
Universal Transformer	84M	40.0T	0.88	2.406 ± 0.036	2.053	70.13	14.09	19.05	23.91
Mixture of experts	648M	11.7T	3.20	2.148 ± 0.006	**1.785**	**74.55**	**18.13**	**24.08**	**26.94**
Switch Transformer	1100M	11.7T	3.18	2.135 ± 0.007	**1.758**	**75.38**	**18.02**	**26.19**	**26.81**
Funnel Transformer	223M	1.9T	4.30	2.288 ± 0.008	1.918	67.34	16.26	22.75	23.20
Weighted Transformer	280M	71.0T	0.59	2.378 ± 0.021	1.989	69.04	16.98	23.02	26.30
Product key memory	421M	386.6T	0.25	2.155 ± 0.003	**1.798**	**75.16**	17.04	**23.55**	**26.73**

Do Transformer Modifications Transfer Across Implementations and Applications?

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