

# Language Modeling: Attention

CSE 576: Topics in Natural Language Processing

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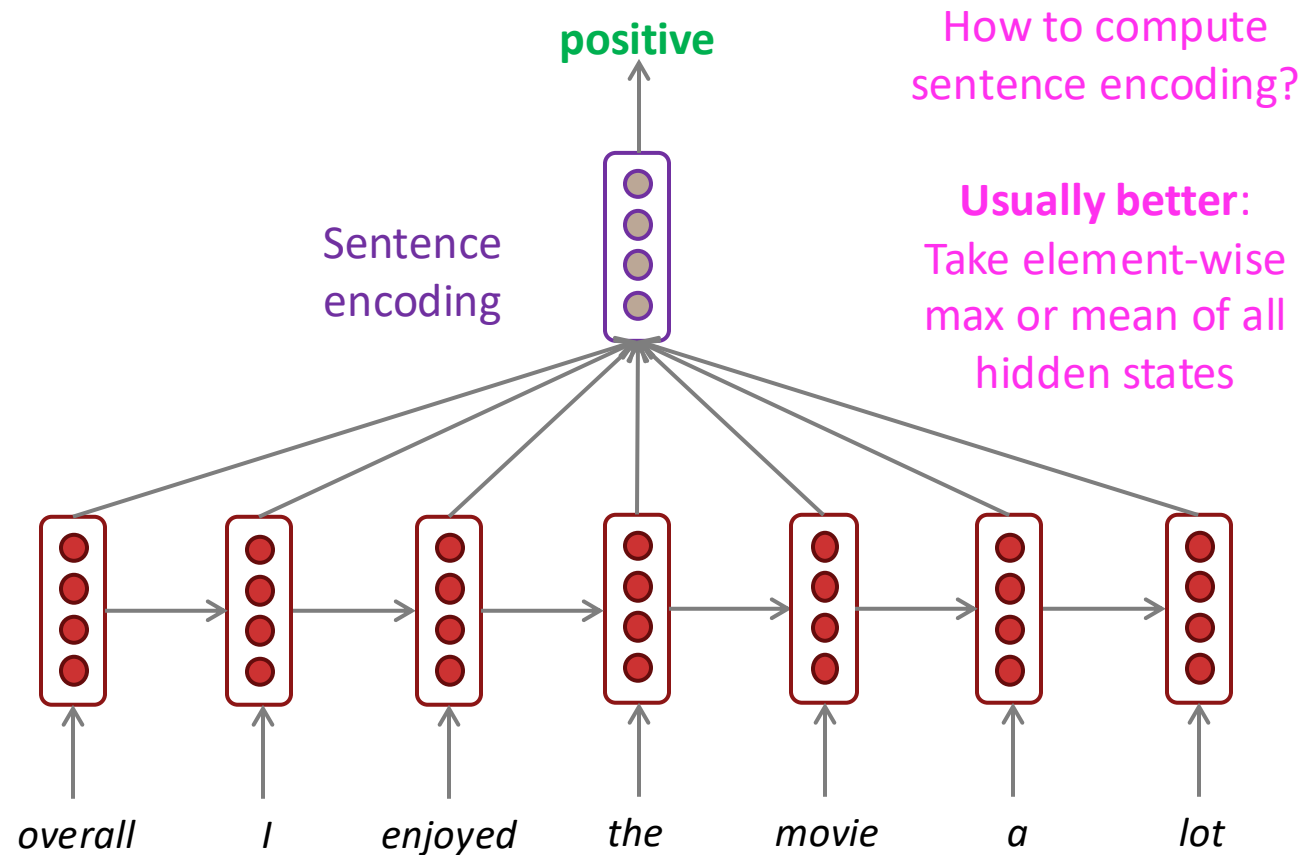
# Attention

- **Attention** provides a solution to the bottleneck problem.
- **Core idea:** on each step of the decoder, *use direct connection to the encoder to focus on a particular part* of the source sequence



- First, we will show via diagram (no equations), then we will show with equations

# The starting point: mean-pooling for RNNs

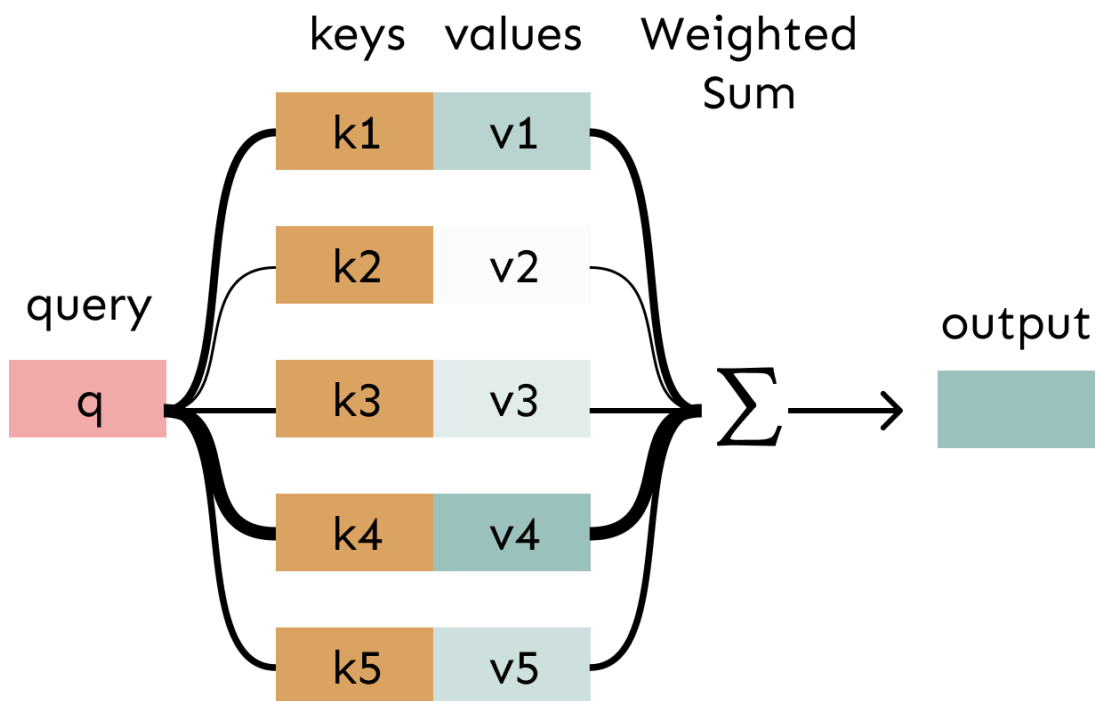


- Starting point: a *very* basic way of 'passing information from the encoder' is to *average*

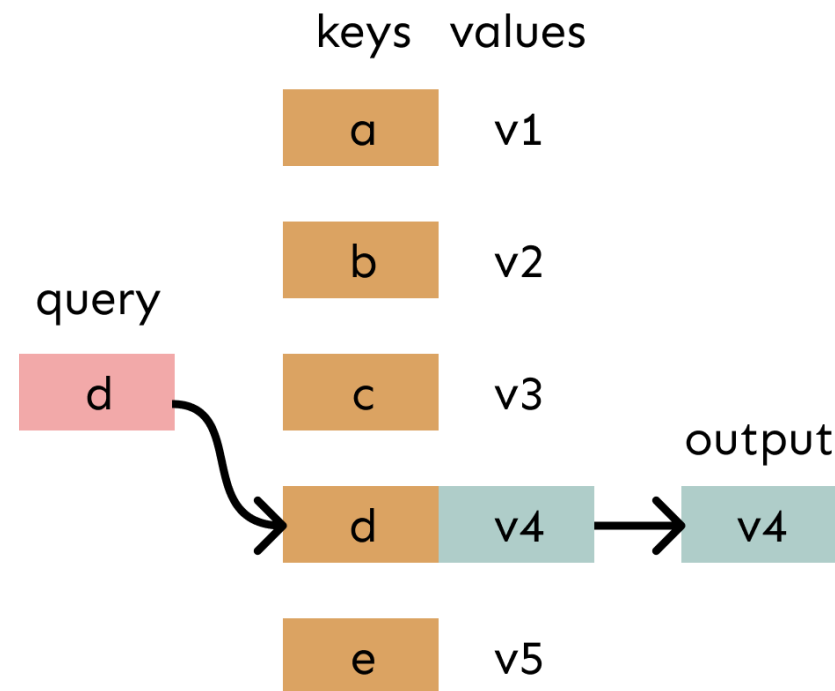
# Attention is *weighted* averaging, which lets you do lookups!

Attention is just a **weighted** average – this is very powerful if the weights are learned!

In **attention**, the **query** matches all **keys** *softly*, to a weight between 0 and 1. The keys' **values** are multiplied by the weights and summed.

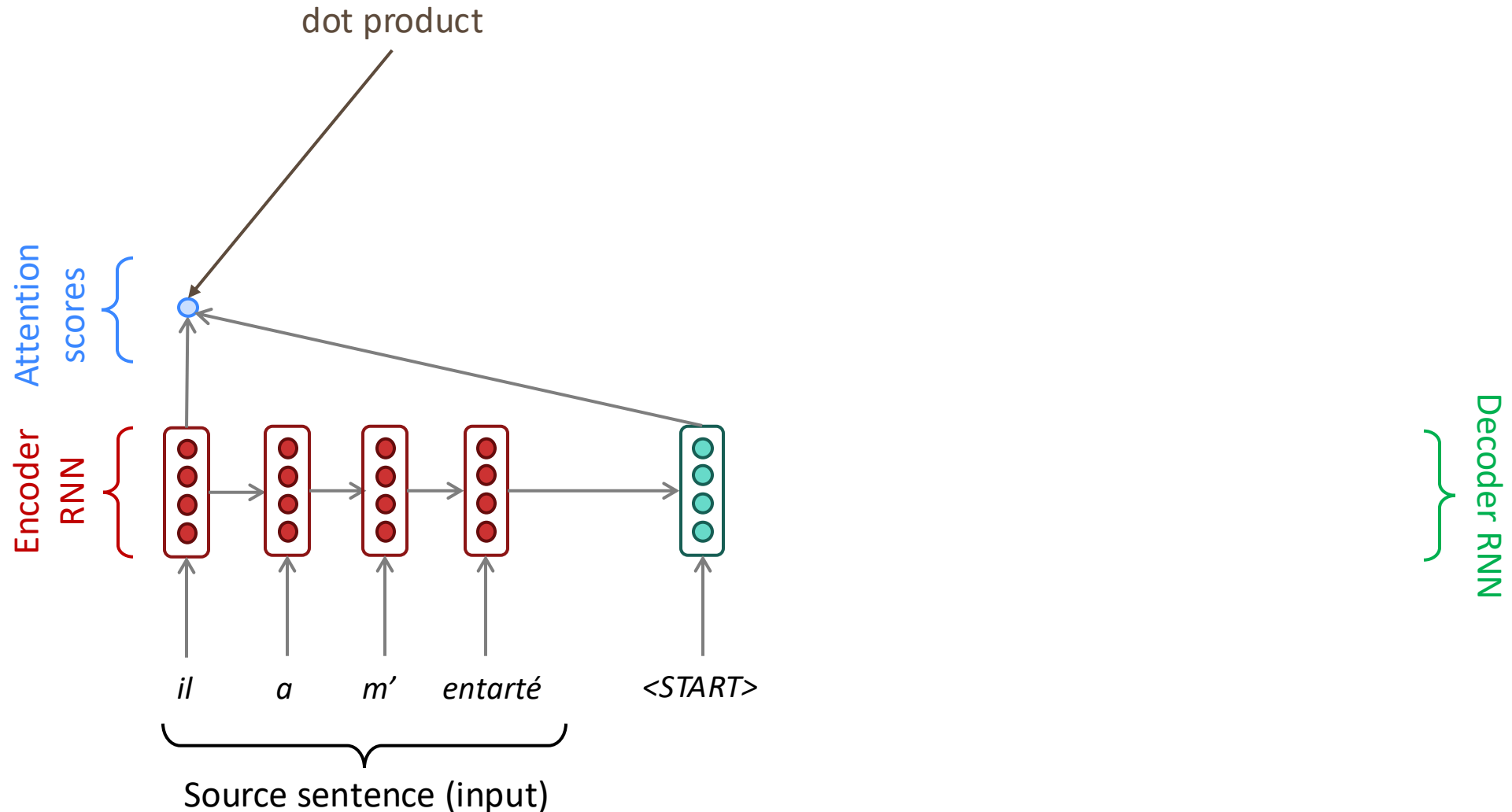


In a **lookup table**, we have a table of **keys** that map to **values**. The **query** matches one of the keys, returning its value.

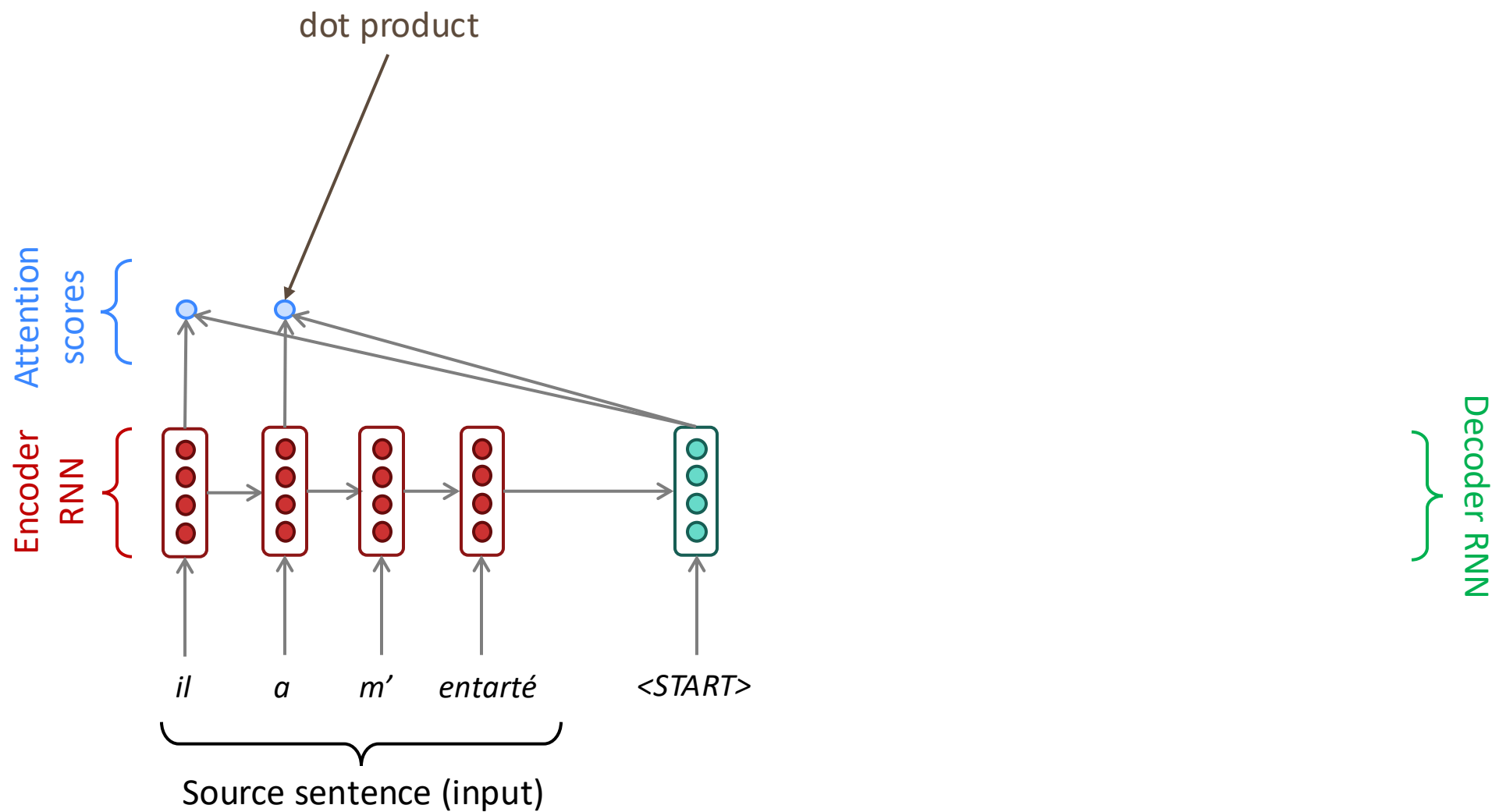


# Sequence-to-sequence with attention

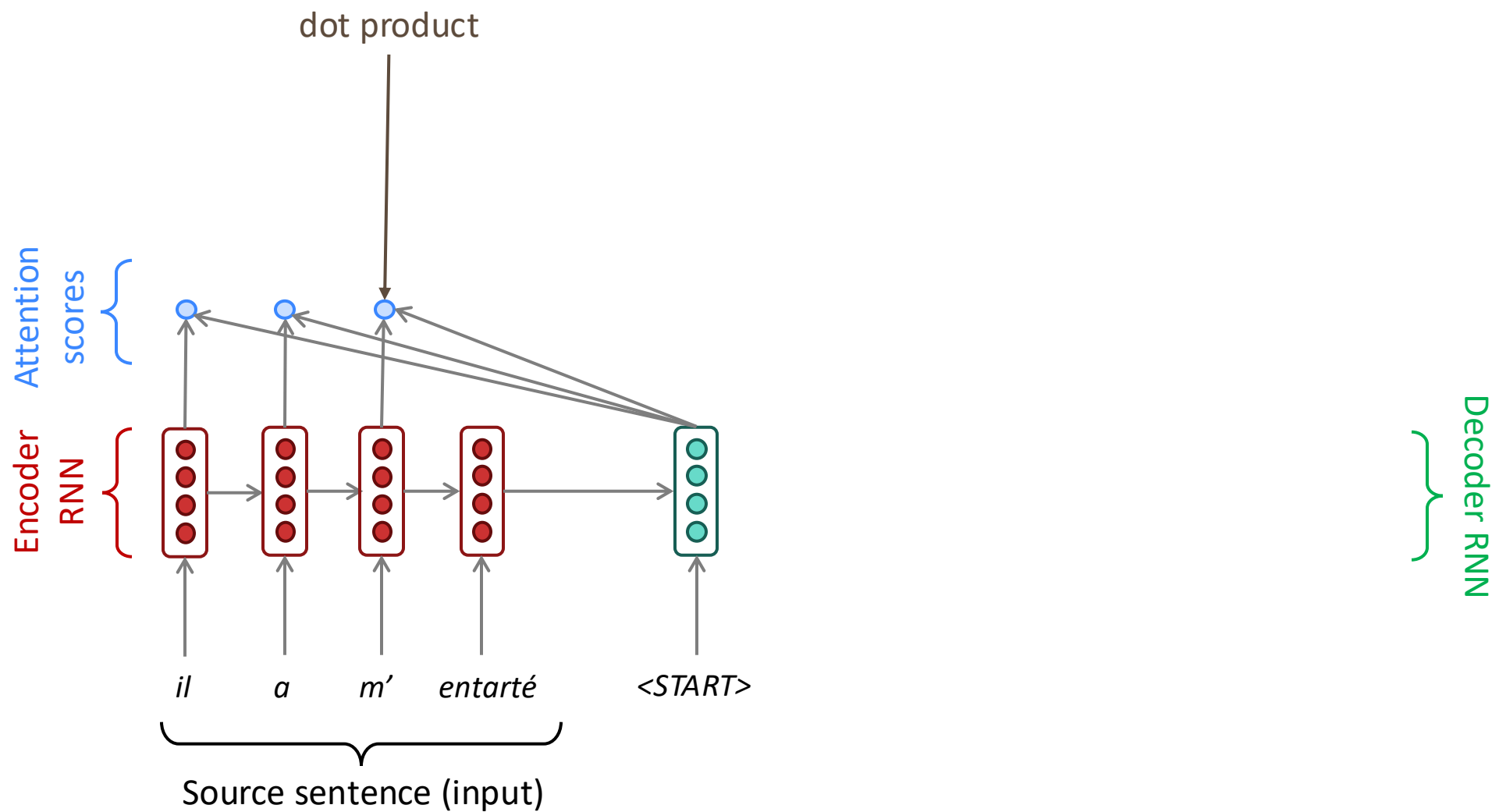
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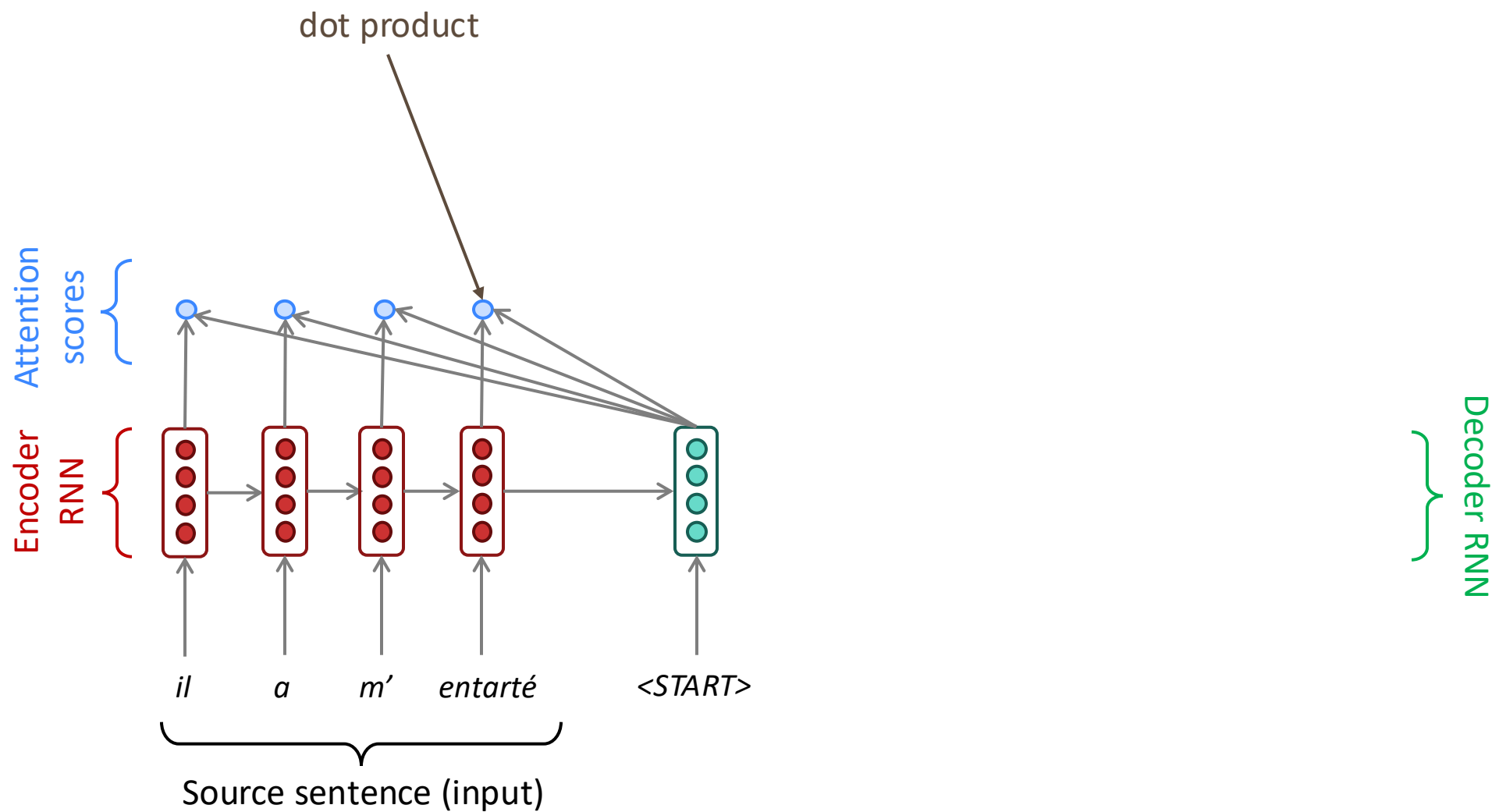
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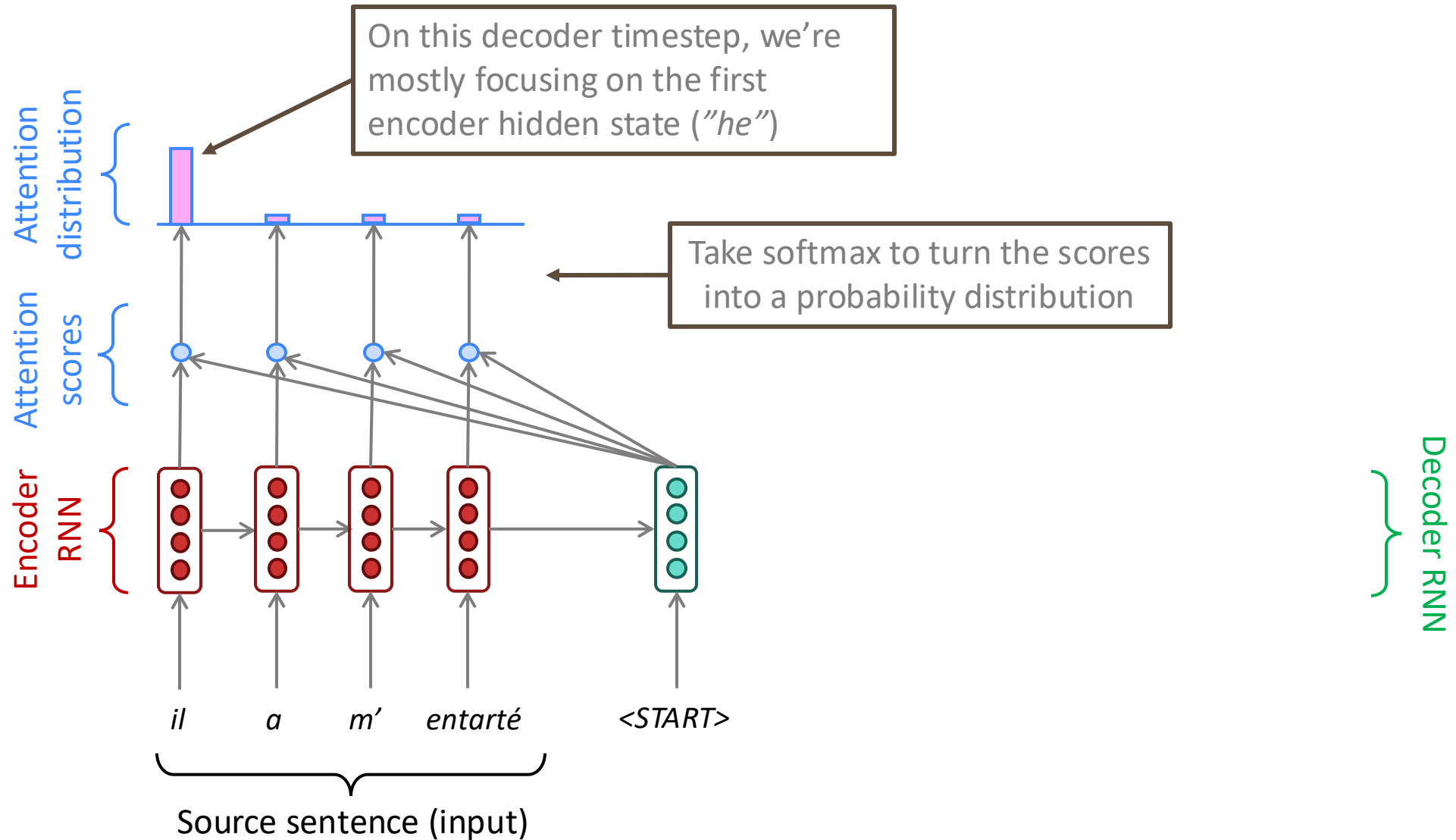


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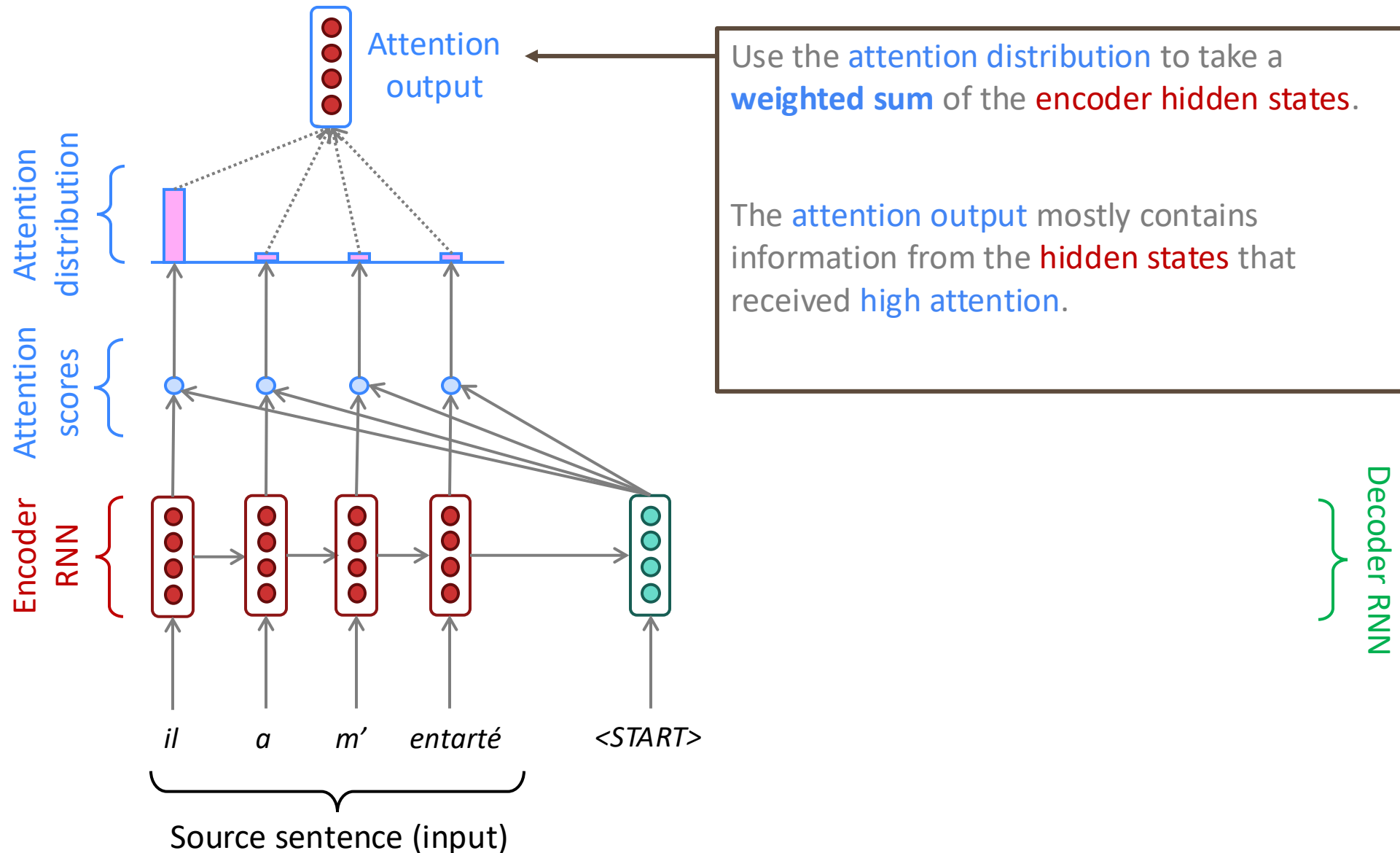




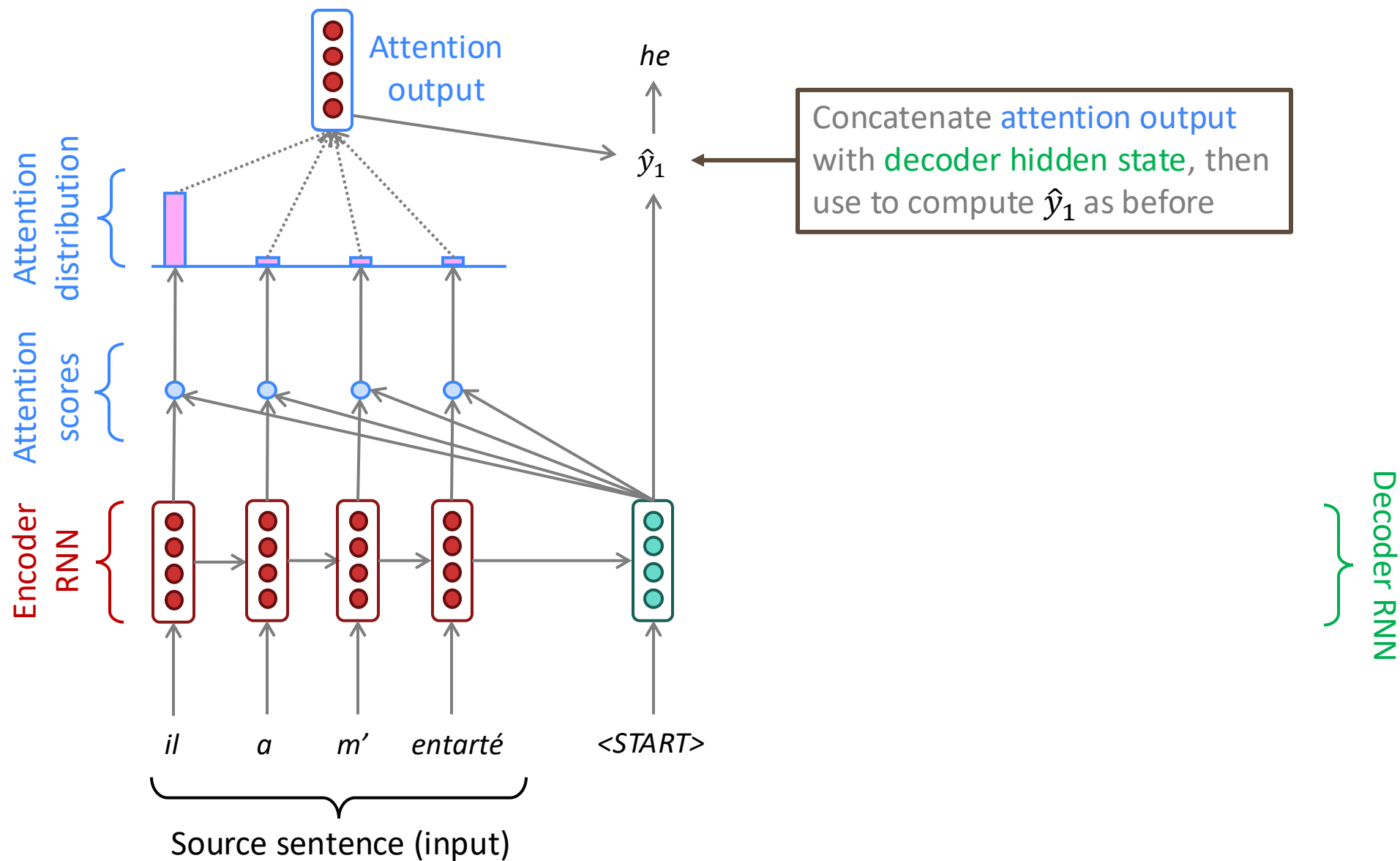
# Sequence-to-sequence with attention



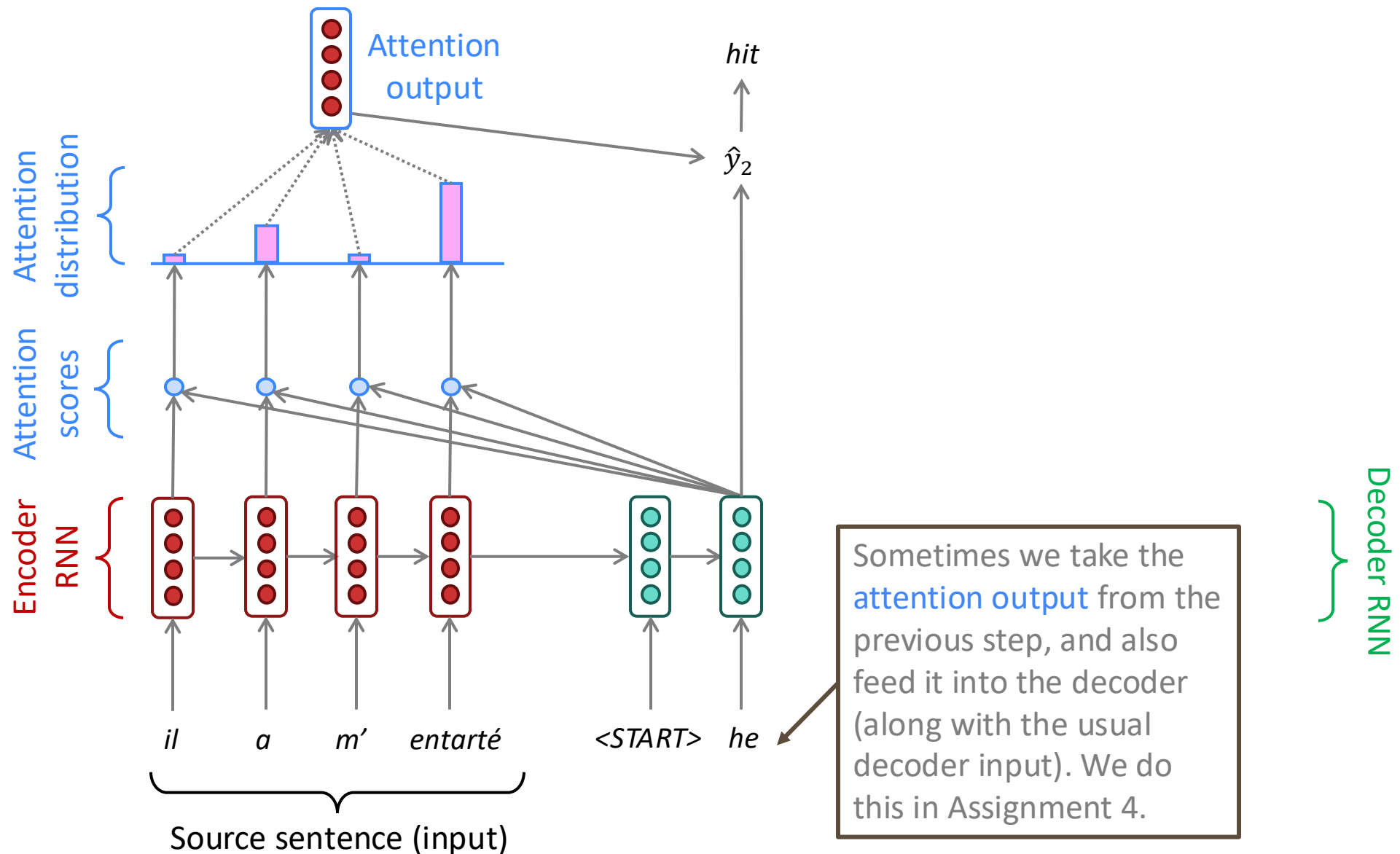
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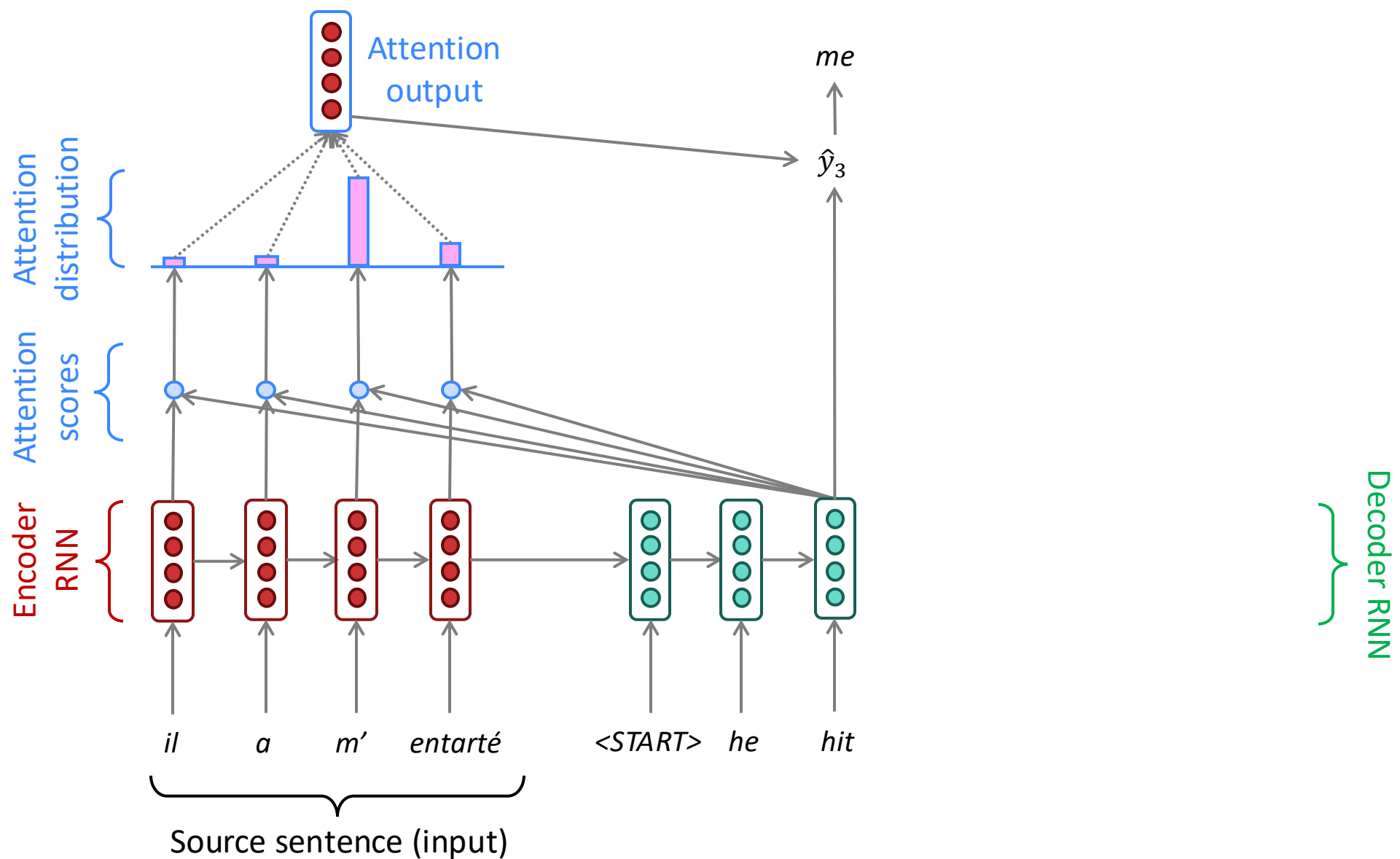
# Sequence-to-sequence with attention



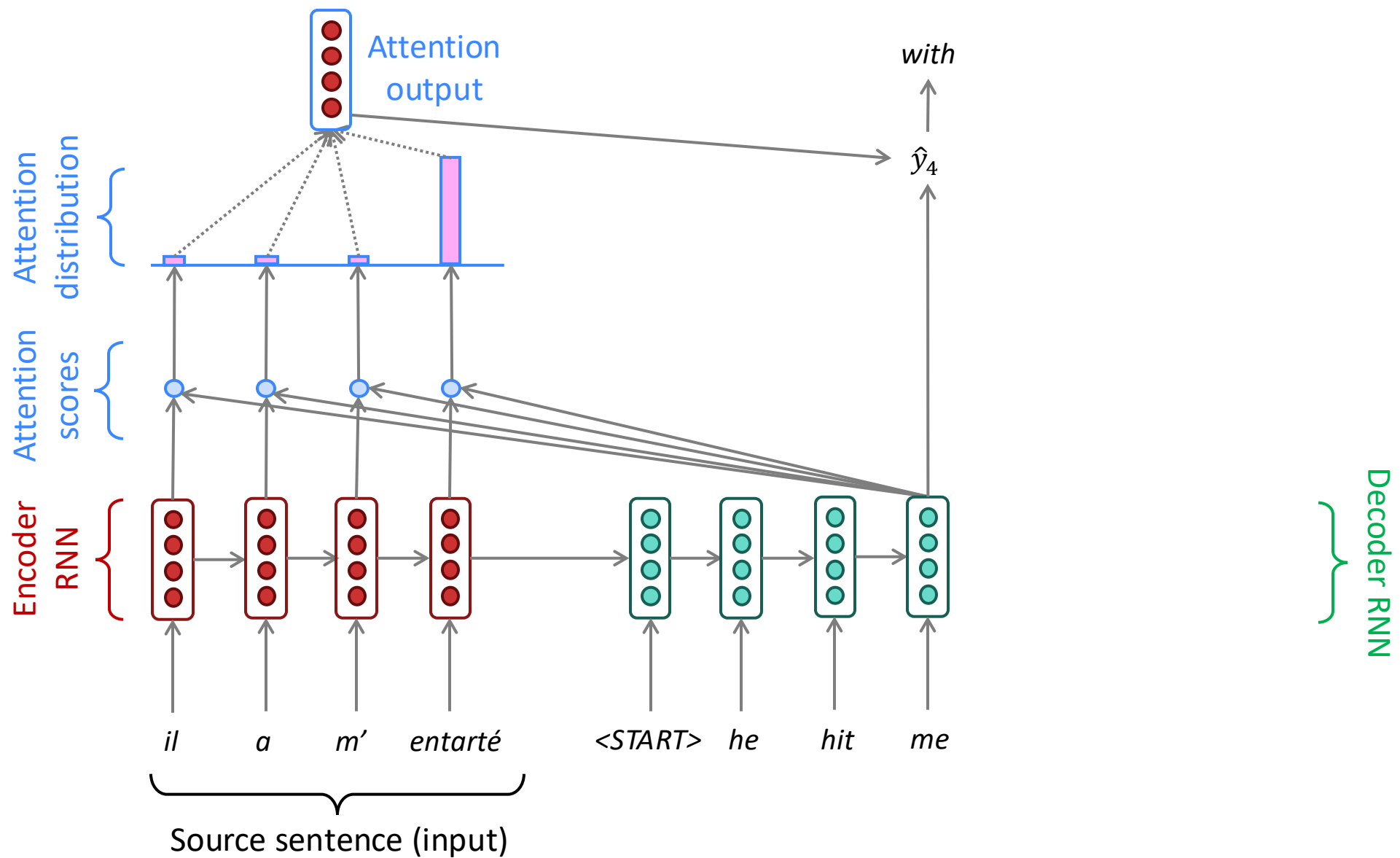
# Sequence-to-sequence with attention



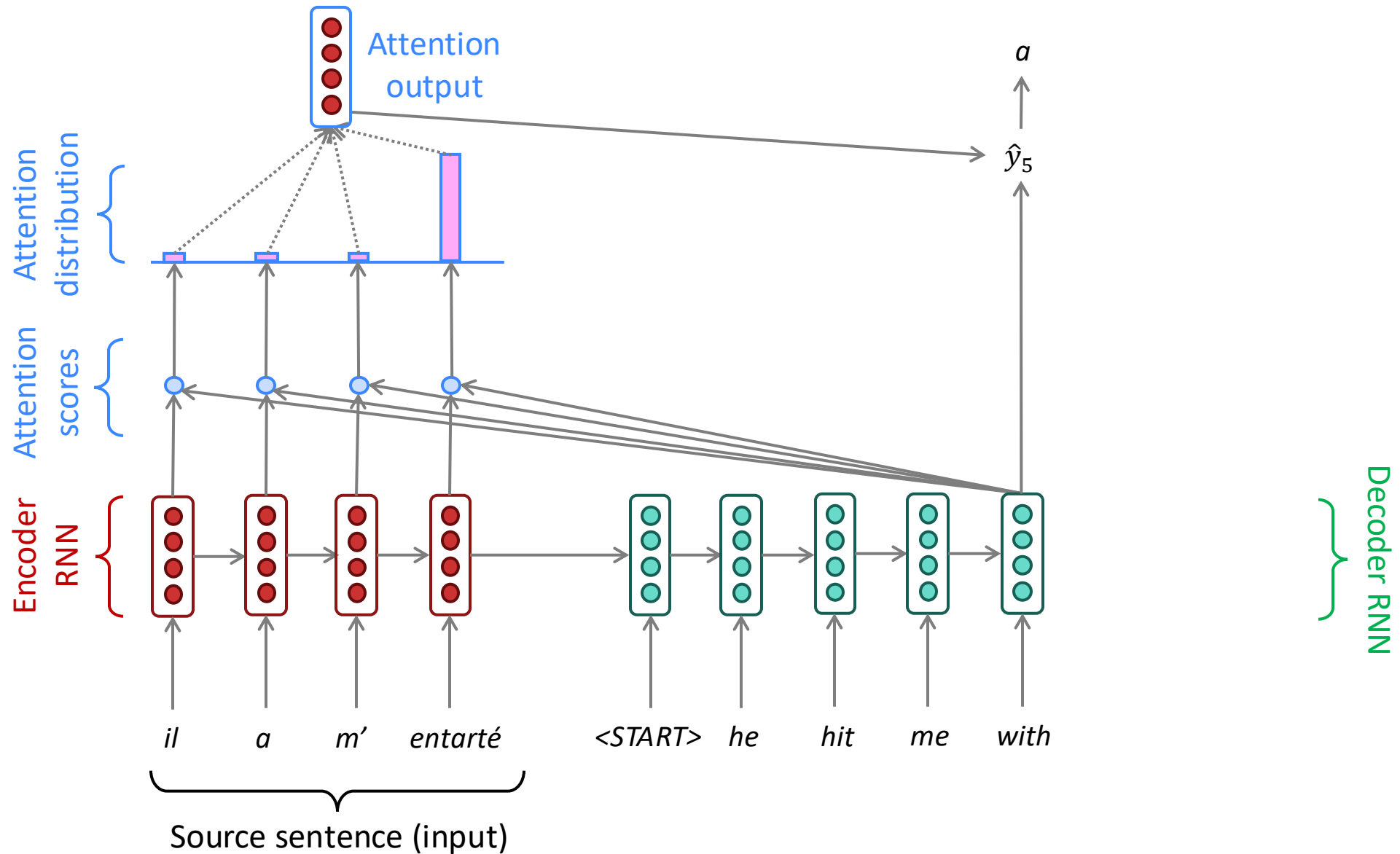
# Sequence-to-sequence with attention



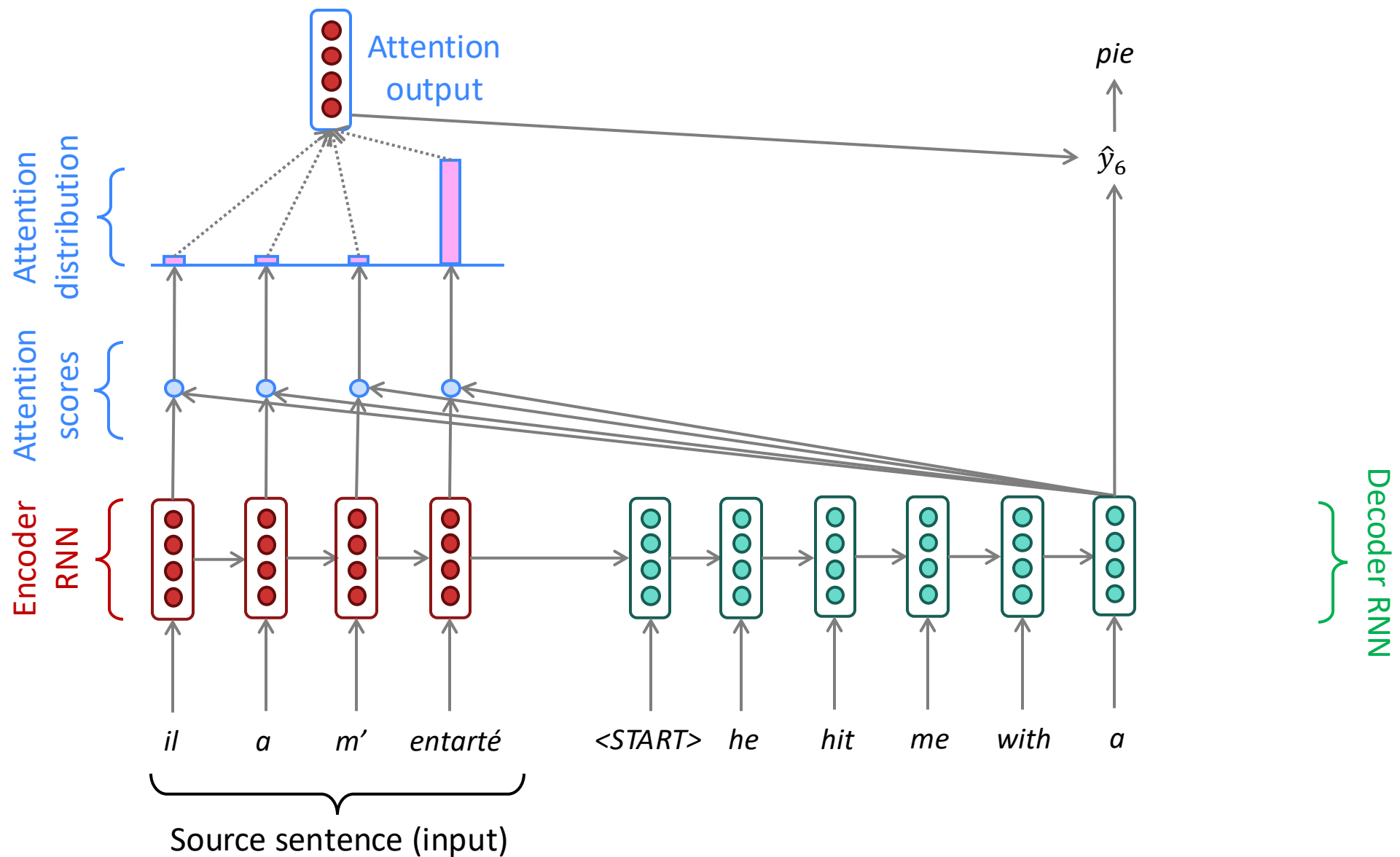
# Sequence-to-sequence with attention



# Sequence-to-sequence with attention



# Sequence-to-sequence with attention





# Attention: in equations

- We have encoder hidden states  $h_1, \dots, h_N \in \mathbb{R}^h$
- On timestep  $t$ , we have decoder hidden state  $s_t \in \mathbb{R}^h$
- We get the attention scores  $e^t$  for this step:

$$e^t = [s_t^T h_1, \dots, s_t^T h_N] \in \mathbb{R}^N$$

- We take softmax to get the attention distribution  $\alpha^t$  for this step (this is a probability distribution and sums to 1)

$$\alpha^t = \text{softmax}(e^t) \in \mathbb{R}^N$$

- We use  $\alpha^t$  to take a weighted sum of the encoder hidden states to get the attention output  $a_t$

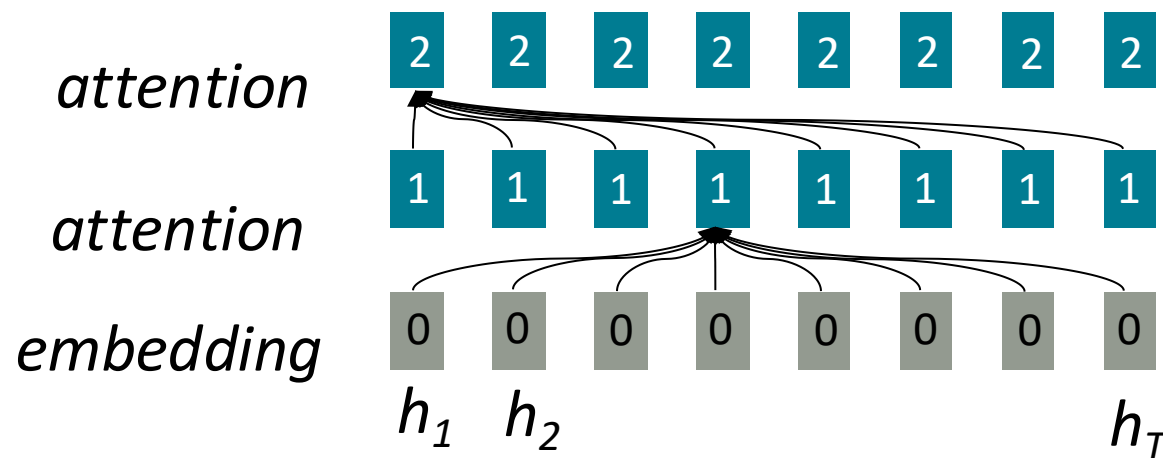
$$a_t = \sum_{i=1}^N \alpha_i^t h_i \in \mathbb{R}^h$$

- Finally we concatenate the attention output  $a_t$  with the decoder hidden state  $s_t$  and proceed as in the non-attention seq2seq model

$$[a_t; s_t] \in \mathbb{R}^{2h}$$

# Attention is parallelizable, and solves bottleneck issues.

- **Attention** treats each word's representation as a **query** to access and incorporate information from **a set of values**.
  - We saw attention from the **decoder** to the **encoder**; today we'll think about attention **within a single sentence**.
- Number of unparallelizable operations does not increase with sequence length.
- Maximum interaction distance:  $O(1)$ , since all words interact at every layer!



All words attend to all words in previous layer; most arrows here are omitted

# Attention is great!



- Attention significantly **improves NMT performance**
  - It's very useful to allow decoder to focus on certain parts of the source
- Attention provides a **more “human-like” model** of the MT process
  - You can look back at the source sentence while translating, rather than needing to remember it all
- Attention **solves the bottleneck problem**
  - Attention allows decoder to look directly at source; bypass bottleneck
- Attention **helps with the vanishing gradient problem**
  - Provides shortcut to faraway states
- Attention provides **some interpretability**
  - By inspecting attention distribution, we see what the decoder was focusing on
  - We get (soft) **alignment for free!**
  - The network just learned alignment by itself
- (**One issue** – attention has *quadratic* cost with respect to sequence length)

	he	hit	me	with	a	pie
il						
a						
m'						
entarté						

# Attention is a *general* Deep Learning technique

- We've seen that attention is a great way to improve the sequence-to-sequence model for Machine Translation.
- However: You can use attention in **many architectures** (not just seq2seq) and **many tasks** (not just MT)
- More general definition of attention:
  - Given a set of vector **values**, and a vector **query**, attention is a technique to compute a weighted sum of the values, dependent on the query.
- We sometimes say that the **query attends to the values**.
- For example, in the seq2seq + attention model, each decoder hidden state (query) *attends to* all the encoder hidden states (values).

# Attention is a *general* Deep Learning technique

- More general definition of attention:
  - Given a set of vector *values*, and a vector *query*, attention is a technique to compute a weighted sum of the values, dependent on the query.

## Intuition:

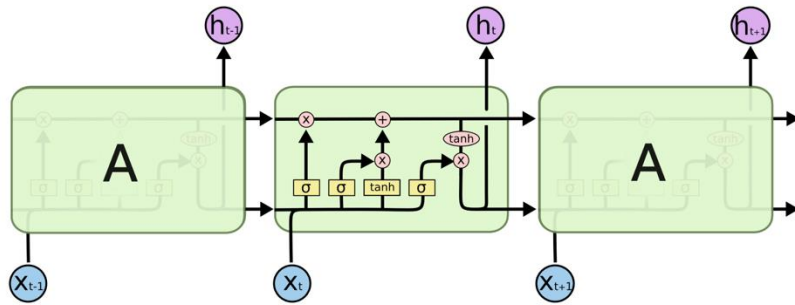
- The weighted sum is a *selective summary* of the information contained in the values, where the query determines which values to focus on.
- Attention is a way to obtain a *fixed-size representation of an arbitrary set of representations* (the values), dependent on some other representation (the query).

## Upshot:

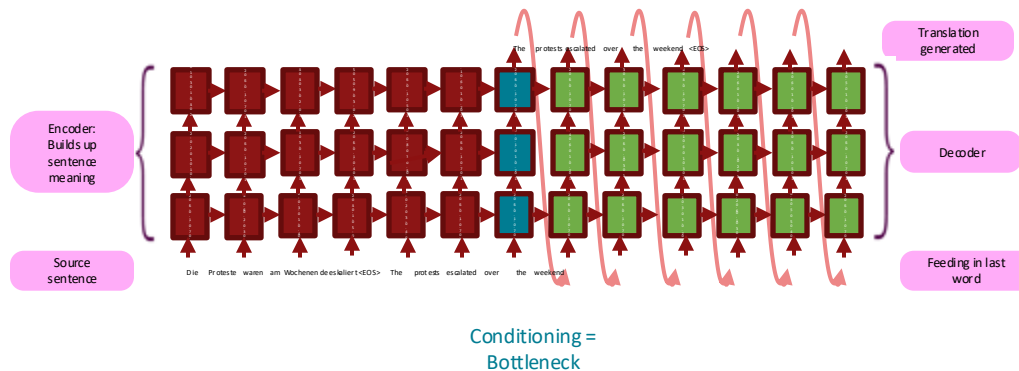
- Attention has become the powerful, flexible, general way pointer and memory manipulation in all deep learning models. A new idea from after 2010! From NMT!

# In summary

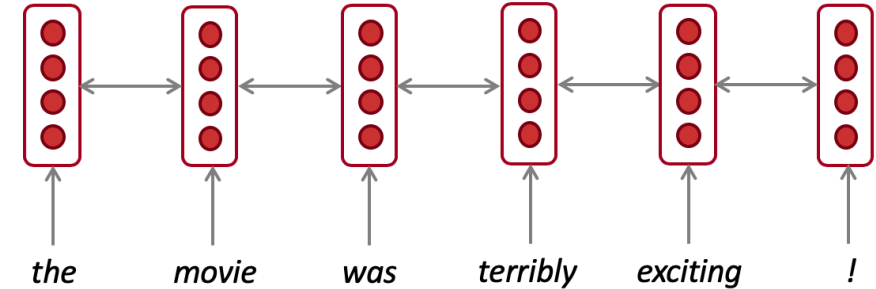
Lots of new information today! What are some of the **practical takeaways?**



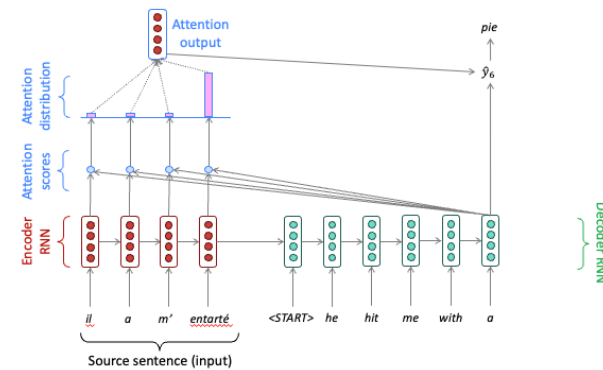
## 1. LSTMs are powerful



## 3. Encoder-Decoder Neural Machine Translation Systems work very well



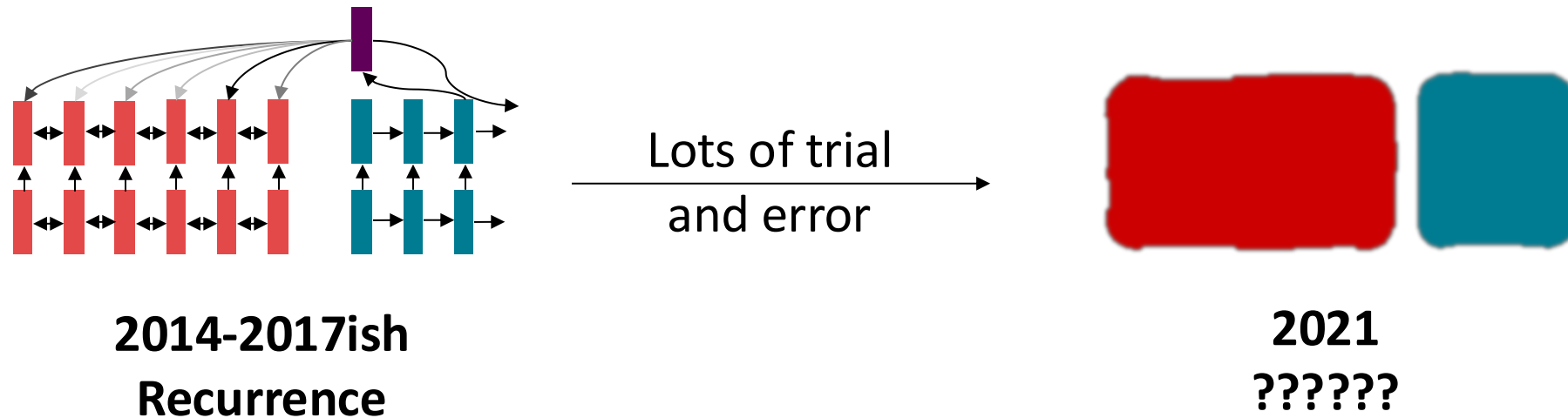
## 2. Use bidirectionality when possible



## 4. Attention is a general, useful technique

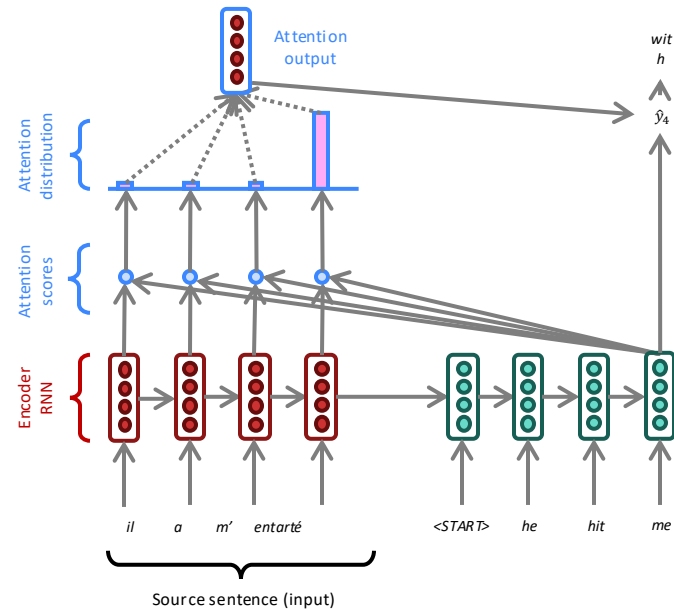
# Do we even need recurrence at all?

- Abstractly: Attention is a way to pass information from a sequence ( $x$ ) to a neural network input. ( $h_t$ )
  - This is also *exactly* what RNNs are used for – to pass information!
  - **Can we just get rid of the RNN entirely?** Maybe attention is just a better way to pass information!



# The building block we need: *self* attention

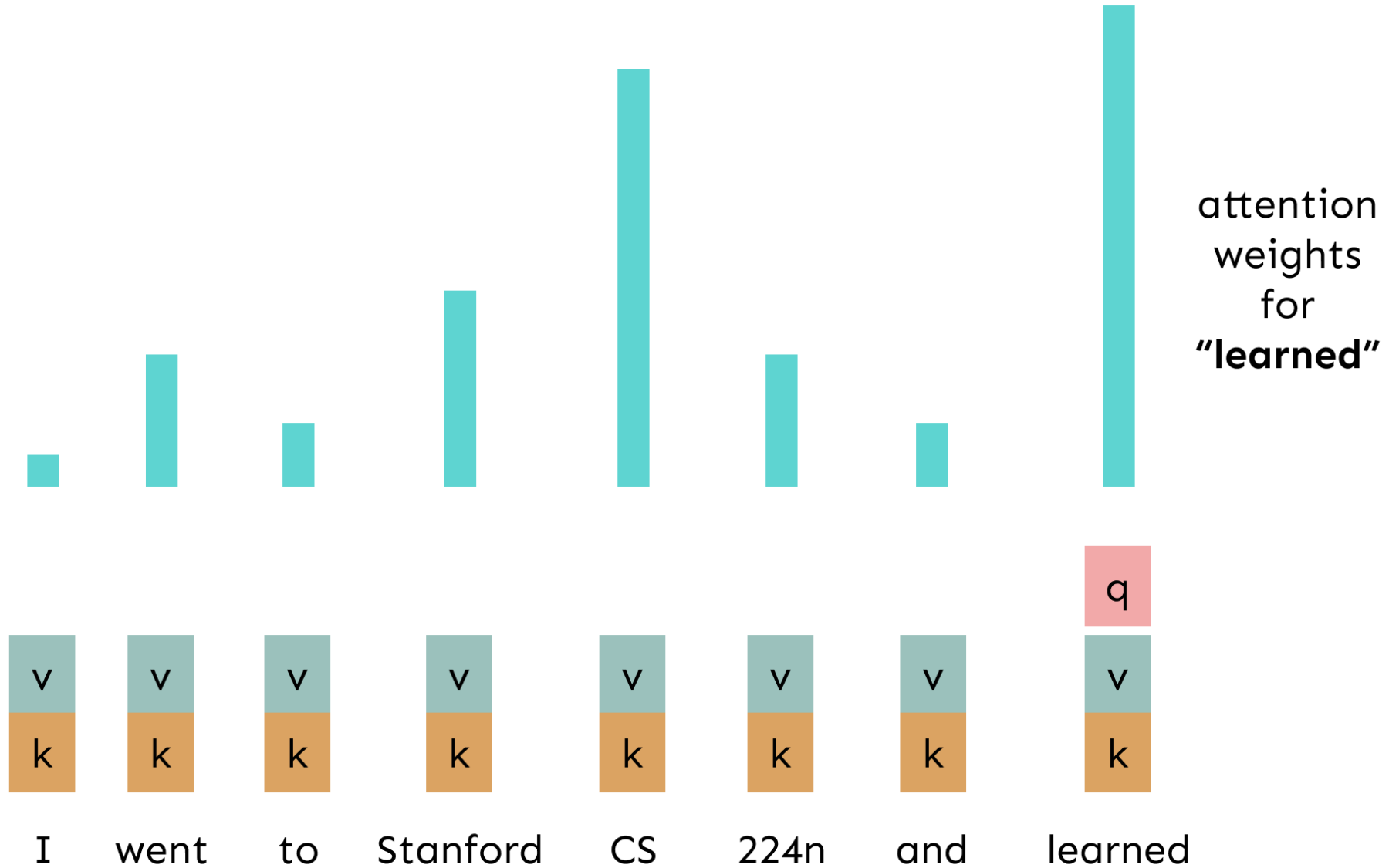
- What we talked about – **Cross** attention: paying attention to the input  $x$  to generate  $y_t$



- What we need – **Self** attention: to generate  $y_t$ , we need to pay attention to  $y_{<t}$



# Self-Attention Hypothetical Example



# Self-Attention: keys, queries, values from the same sequence

Let  $\mathbf{w}_{1:n}$  be a sequence of words in vocabulary  $V$ , like *Zuko made his uncle tea*.

For each  $\mathbf{w}_i$ , let  $\mathbf{x}_i = E\mathbf{w}_i$ , where  $E \in \mathbb{R}^{d \times |V|}$  is an embedding matrix.

1. Transform each word embedding with weight matrices  $Q, K, V$ , each in  $\mathbb{R}^{d \times d}$

$$\mathbf{q}_i = Q\mathbf{x}_i \text{ (queries)} \quad \mathbf{k}_i = K\mathbf{x}_i \text{ (keys)} \quad \mathbf{v}_i = V\mathbf{x}_i \text{ (values)}$$

2. Compute pairwise similarities between keys and queries; normalize with softmax

$$\mathbf{e}_{ij} = \mathbf{q}_i^\top \mathbf{k}_j \quad \alpha_{ij} = \frac{\exp(\mathbf{e}_{ij})}{\sum_{j'} \exp(\mathbf{e}_{ij'})}$$

3. Compute output for each word as weighted sum of values

$$\mathbf{o}_i = \sum_j \alpha_{ij} \mathbf{v}_j$$

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!



## Solutions

# Fixing the first self-attention problem: **sequence order**

- Since self-attention doesn't build in order information, we need to encode the order of the sentence in our keys, queries, and values.
- Consider representing each **sequence index** as a **vector**

$\mathbf{p}_i \in \mathbb{R}^d$ , for  $i \in \{1, 2, \dots, n\}$  are position vectors

- Don't worry about what the  $\mathbf{p}_i$  are made of yet!
- Easy to incorporate this info into our self-attention block: just add the  $\mathbf{p}_i$  to our inputs!
- Recall that  $\mathbf{x}_i$  is the embedding of the word at index  $i$ . The positioned embedding is:

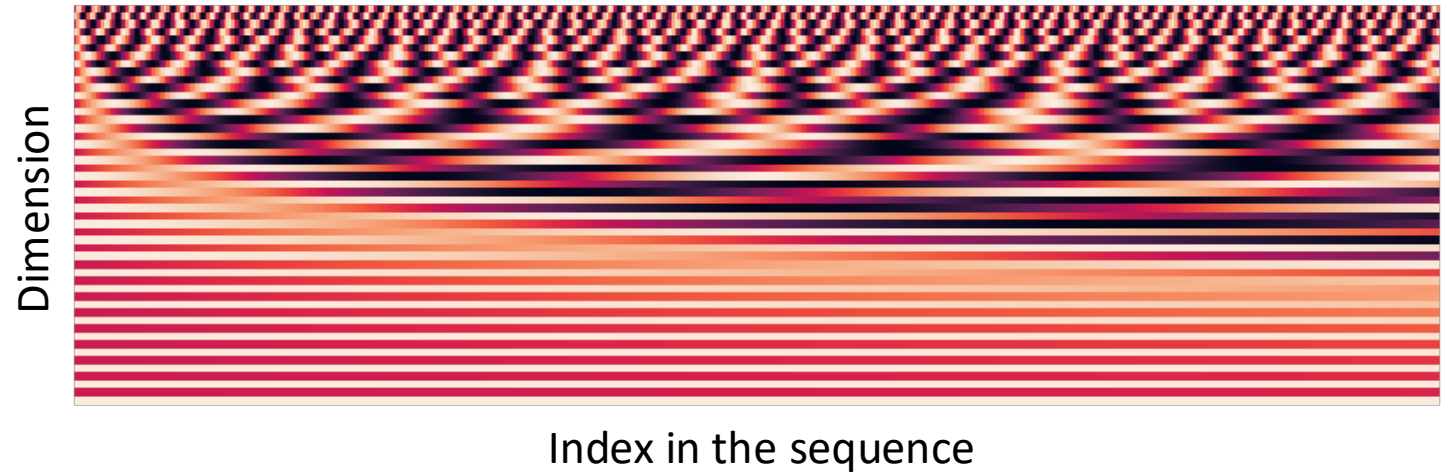
$$\tilde{\mathbf{x}}_i = \mathbf{x}_i + \mathbf{p}_i$$

In deep self-attention networks, we do this at the first layer! You could concatenate them as well, but people mostly just add...

# Position representation vectors through sinusoids

- **Sinusoidal position representations:** concatenate sinusoidal functions of varying periods:

$$p_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$



- Pros:
  - Periodicity indicates that maybe “absolute position” isn’t as important
  - Maybe can extrapolate to longer sequences as periods restart!
- Cons:
  - Not learnable; also the extrapolation doesn’t really work!

# Position representation vectors learned from scratch

- **Learned absolute position representations:** Let all  $p_i$  be learnable parameters!  
Learn a matrix  $\mathbf{p} \in \mathbb{R}^{d \times n}$ , and let each  $\mathbf{p}_i$  be a column of that matrix!
- Pros:
  - Flexibility: each position gets to be learned to fit the data
- Cons:
  - Definitely can't extrapolate to indices outside  $1, \dots, n$ .
- Most systems use this!
- Sometimes people try more flexible representations of position:
  - Relative linear position attention [\[Shaw et al., 2018\]](#)
  - Dependency syntax-based position [\[Wang et al., 2019\]](#)

# Common, modern position embeddings - RoPE

**High level thought process:** a *relative* position embedding should be some  $f(x, i)$  s.t.

$$\langle f(x, i), f(y, j) \rangle = g(x, y, i - j)$$

That is, the attention function *only* gets to depend on the relative position (i-j). How do existing embeddings not fulfill this goal?

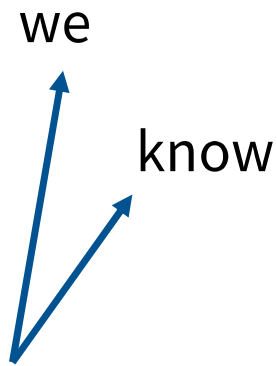
- **Sine:** Has various cross-terms that are not relative
- **Absolute:**

$$e_{ij} = \frac{x_i W^Q (x_j W^K + a_{ij}^K)^T}{\sqrt{d_z}} \quad \text{is not an inner product}$$

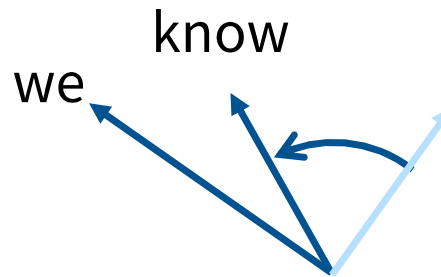
# RoPE – Embedding via rotation

## How can we solve this problem?

- We want our embeddings to be invariant to absolute position
- We know that inner products are invariant to arbitrary rotation.

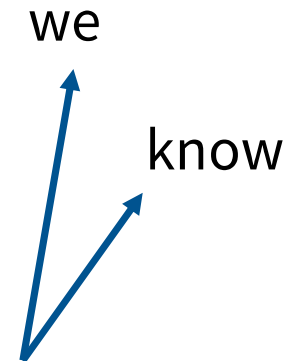


Position independent  
embedding



Embedding  
“of course we know”

Rotate by ‘2 positions’

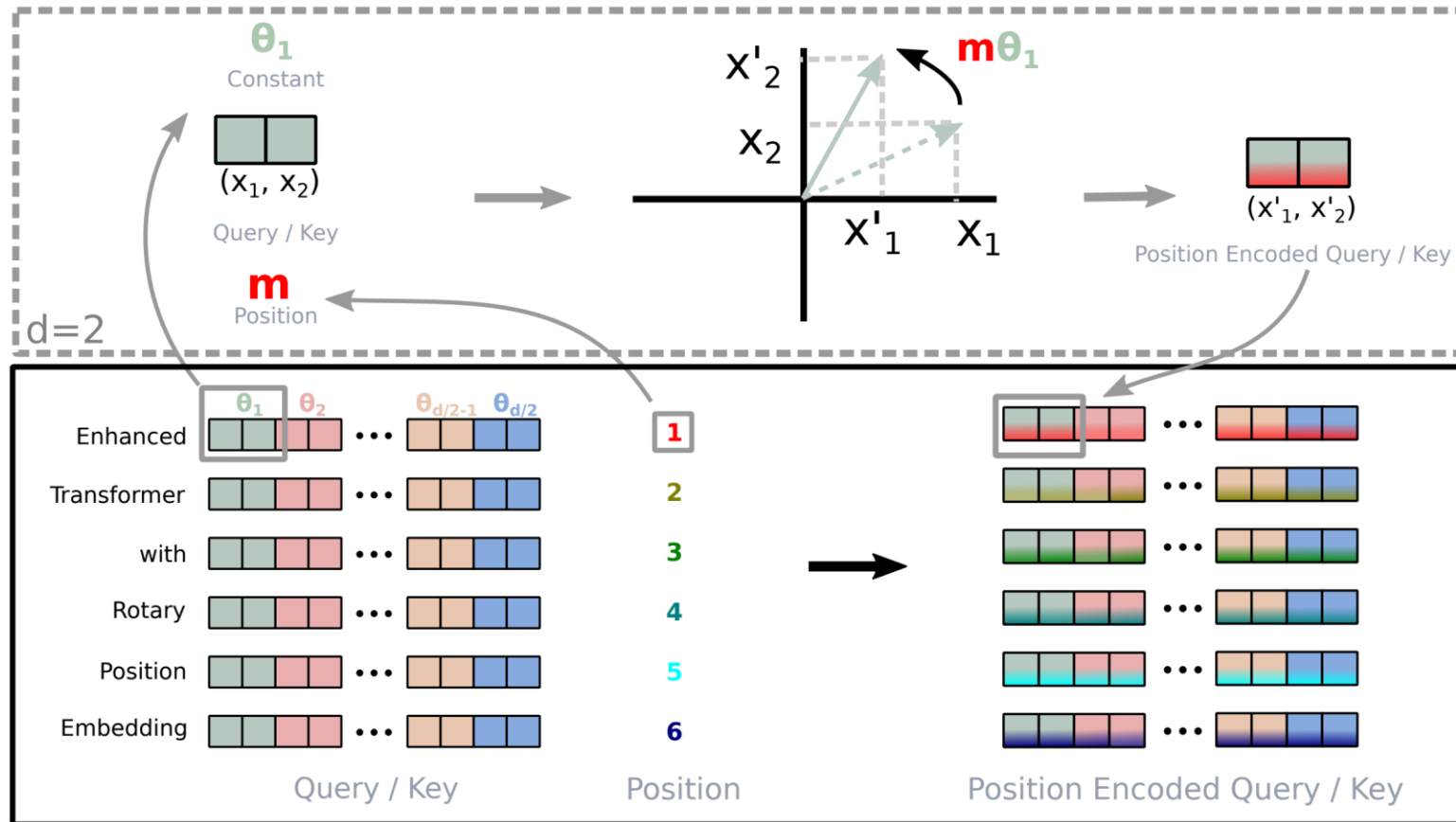


Embedding  
“we know that”

Rotate by ‘0 positions’



# RoPE – From 2 to many dimensions



[Su et al 2021]

Just pair up the coordinates and rotate them in 2d (motivation: complex numbers)

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning! It's all just weighted averages



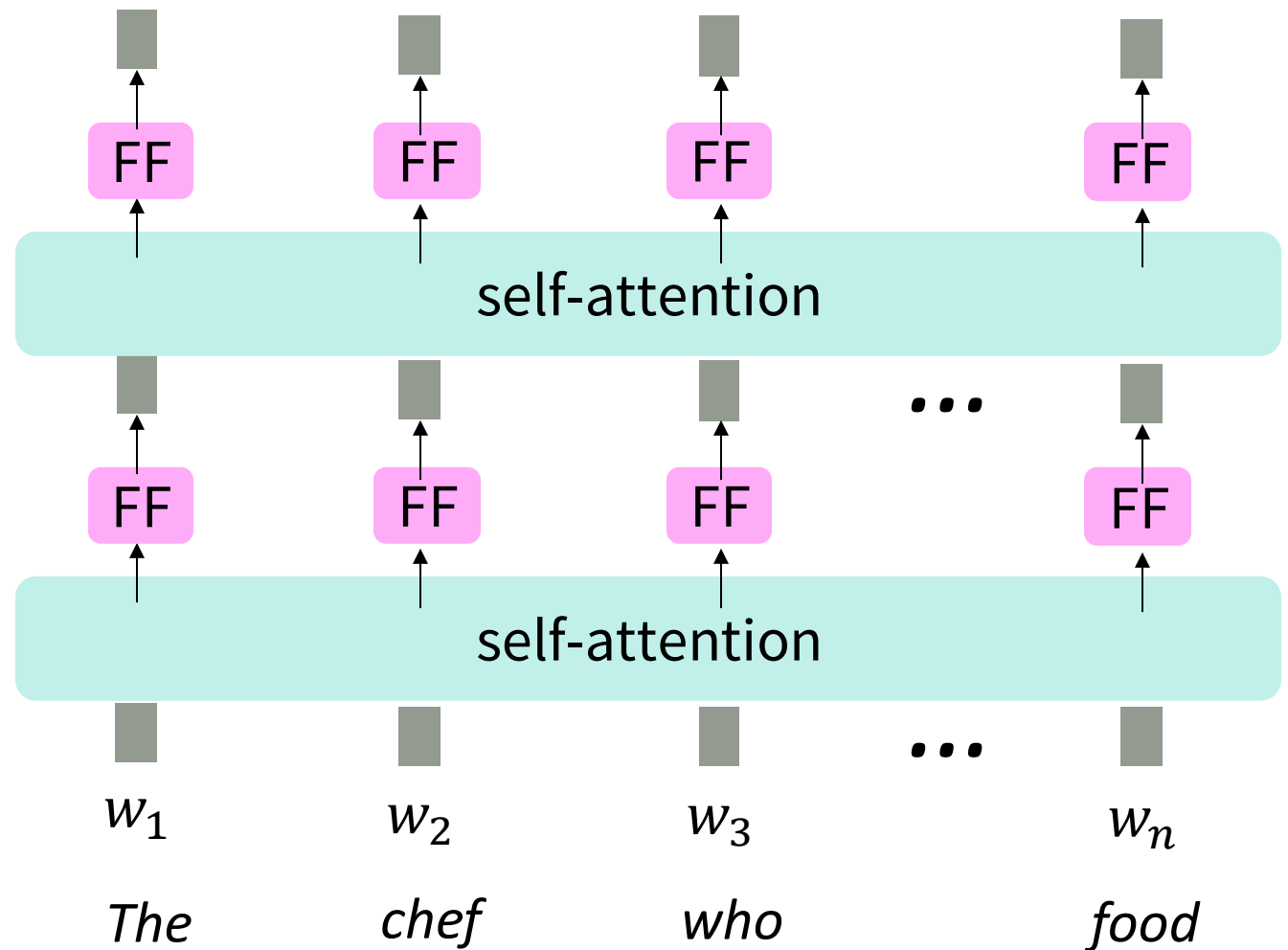
## Solutions

- Add position representations to the inputs

# Adding nonlinearities in self-attention

- Note that there are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages **value** vectors (Why? Look at the notes!)
- Easy fix: add a **feed-forward network** to post-process each output vector.

$$\begin{aligned} m_i &= MLP(\text{output}_i) \\ &= W_2 * \text{ReLU}(W_1 \text{output}_i + b_1) + b_2 \end{aligned}$$



Intuition: the FF network processes the result of attention

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
  - Like in machine translation
  - Or language modeling



## Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.



# Masking the future in self-attention

- To use self-attention in **decoders**, we need to ensure we can't peek at the future.
- At every timestep, we could change the set of **keys and queries** to include only past words. (Inefficient!)
- To enable parallelization, we **mask out attention** to future words by setting attention scores to  $-\infty$ .

$$e_{ij} = \begin{cases} q_i^\top k_j, j \leq i \\ -\infty, j > i \end{cases}$$

For encoding these words

We can look at these (not greyed out) words

	[START]	The	chef	who
[START]		$-\infty$	$-\infty$	$-\infty$
The			$-\infty$	$-\infty$
chef				$-\infty$
who				

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
  - Like in machine translation
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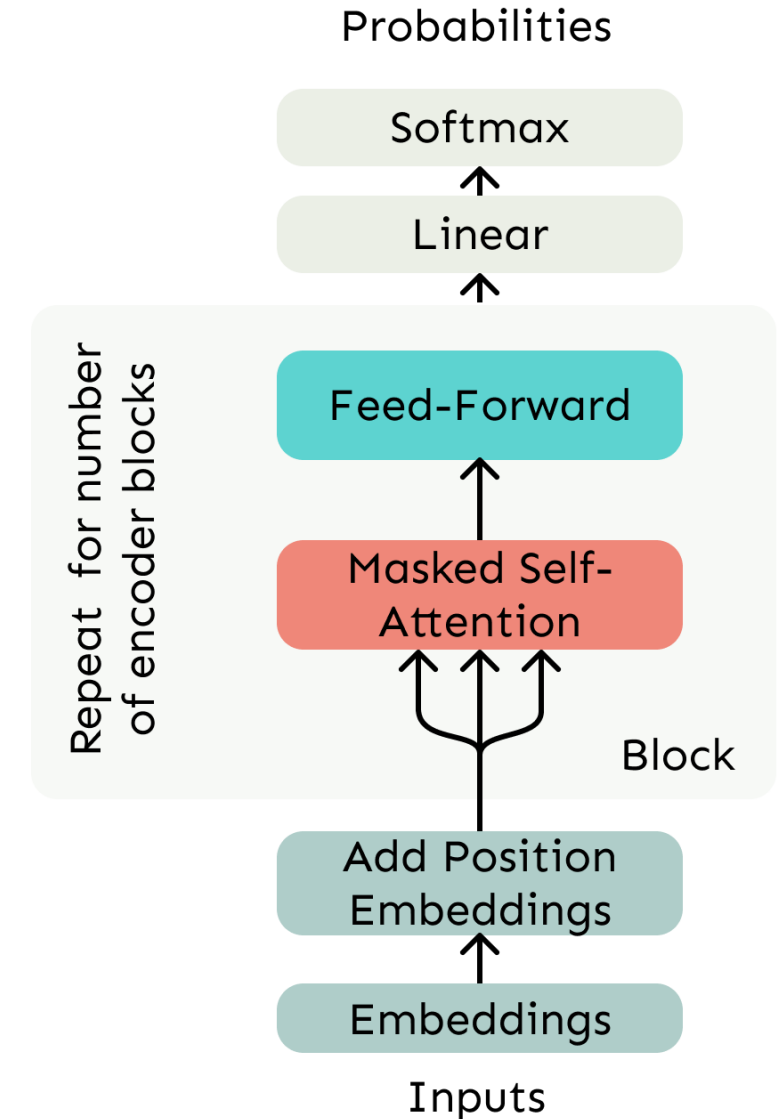


## Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.
- Mask out the future by artificially setting attention weights to 0!

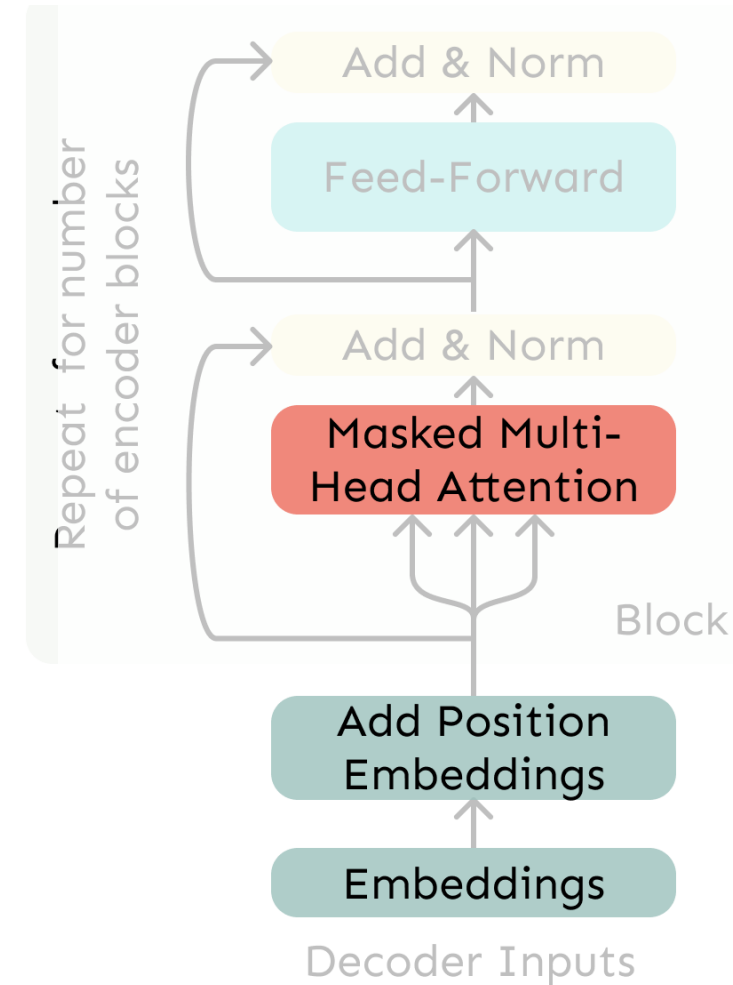
# Necessities for a self-attention building block:

- **Self-attention:**
  - the basis of the method.
- **Position representations:**
  - Specify the sequence order, since self-attention is an unordered function of its inputs.
- **Nonlinearities:**
  - At the output of the self-attention block
  - Frequently implemented as a simple feed-forward network.
- **Masking:**
  - In order to parallelize operations while not looking at the future.
  - Keeps information about the future from “leaking” to the past.



# The Transformer Decoder

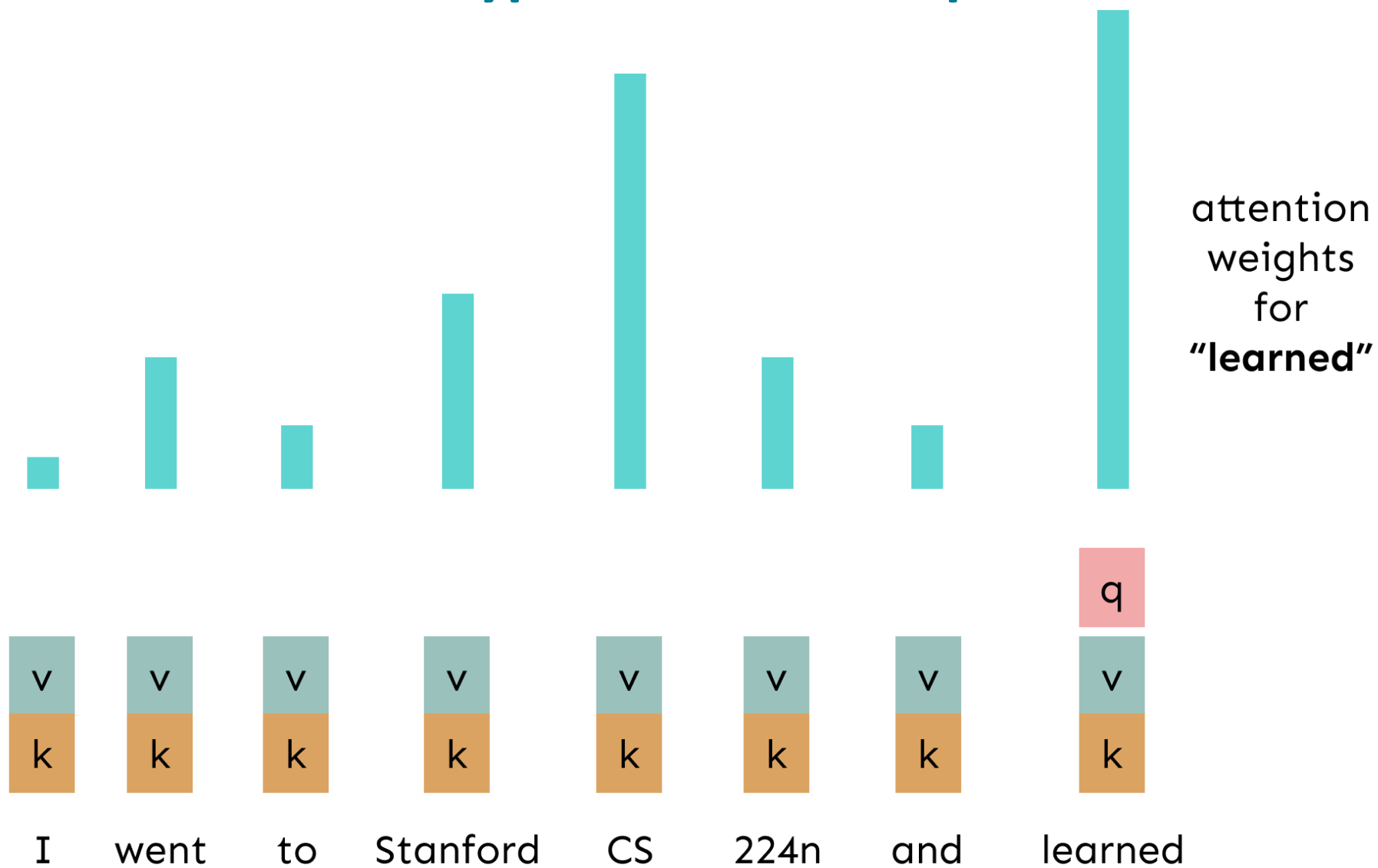
- A Transformer decoder is how we'll build systems like **language models**.
- It's a lot like our minimal self-attention architecture, but with a few more components.
- The embeddings and position embeddings are identical.
- We'll next replace our self-attention with **multi-head self-attention**.



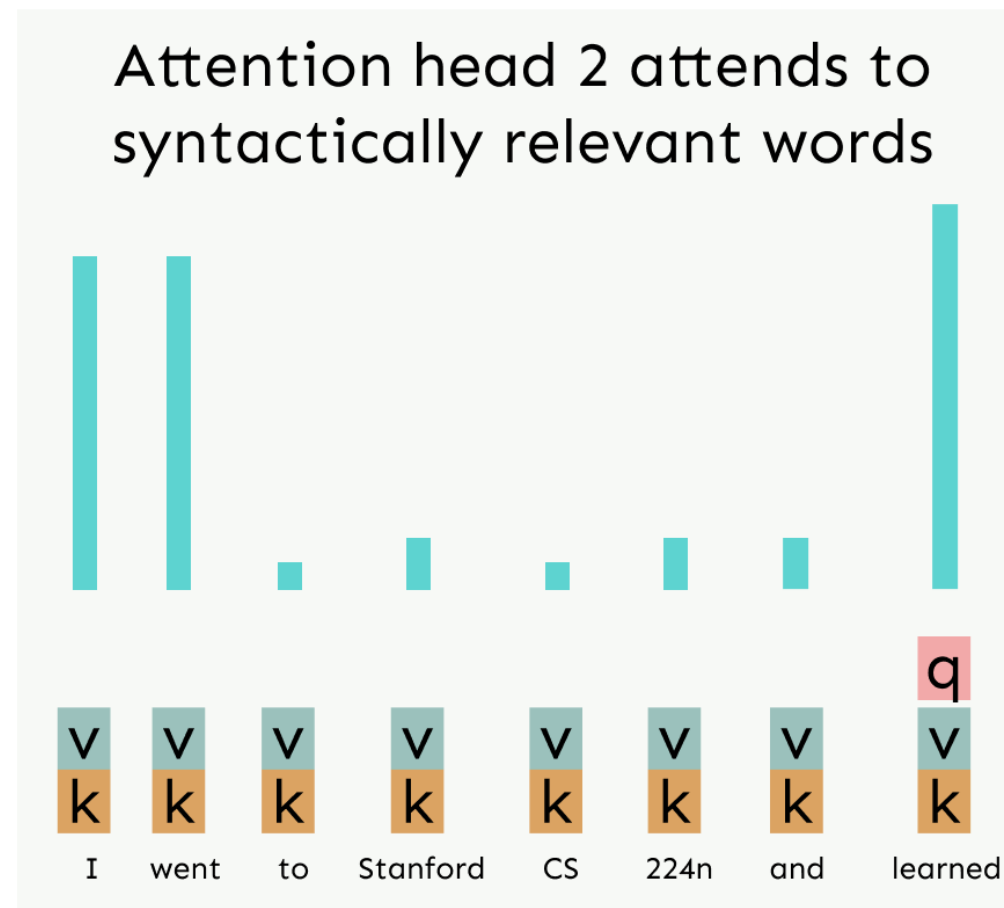
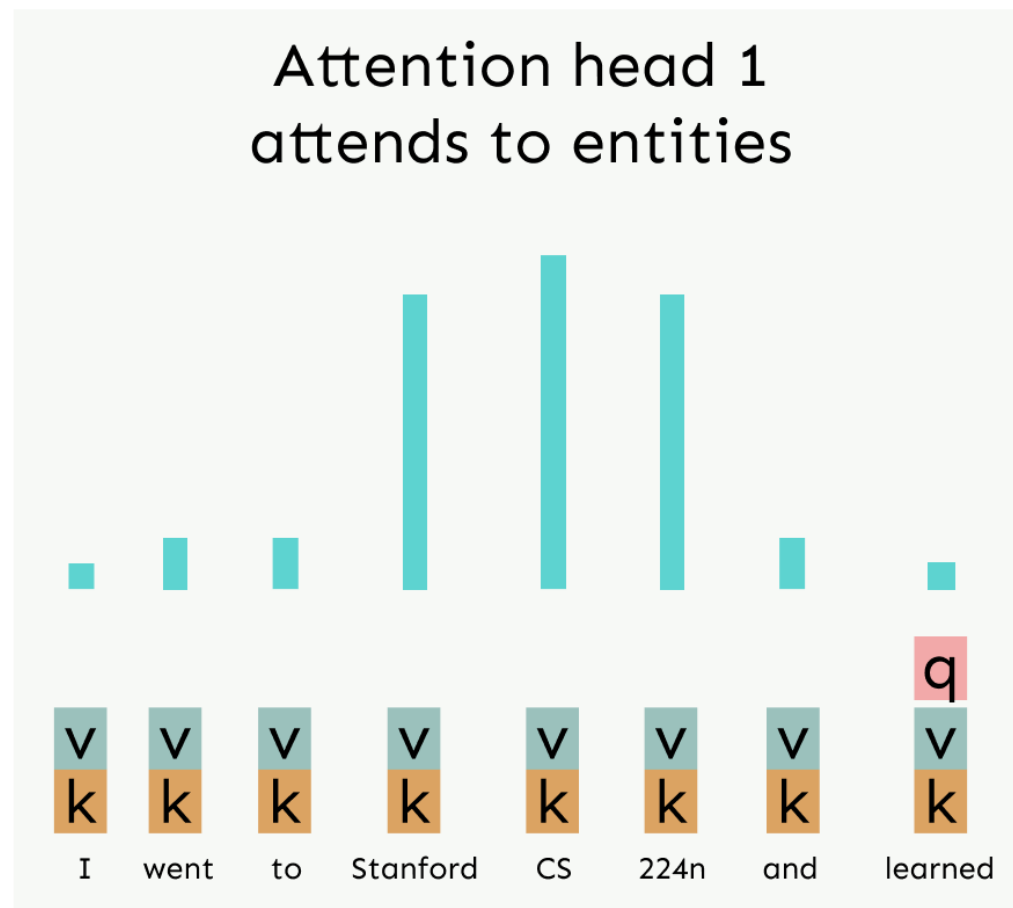
Transformer Decoder



# Recall the Self-Attention Hypothetical Example



# Hypothetical Example of Multi-Head Attention



I went to Stanford

CS 224n and learned

# Sequence-Stacked form of Attention

- Let's look at how key-query-value attention is computed, in matrices.
  - Let  $X = [x_1; \dots; x_n] \in \mathbb{R}^{n \times d}$  be the concatenation of input vectors.
  - First, note that  $XK \in \mathbb{R}^{n \times d}$ ,  $XQ \in \mathbb{R}^{n \times d}$ ,  $XV \in \mathbb{R}^{n \times d}$ .
  - The output is defined as  $\text{output} = \text{softmax}(XQ(XK)^T)XV \in \mathbb{R}^{n \times d}$ .

First, take the query-key dot products in one matrix multiplication:  $XQ(XK)^T$

$$XQ \quad K^T X^T = XQK^T X^T \in \mathbb{R}^{n \times n}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$$\text{softmax} \left( XQK^T X^T \right) XV = \text{output} \in \mathbb{R}^{n \times d}$$

# Multi-headed attention

- What if we want to look in multiple places in the sentence at once?
  - For word  $i$ , self-attention “looks” where  $x_i^\top Q^\top K x_j$  is high, but maybe we want to focus on different  $j$  for different reasons?
- We’ll define **multiple attention “heads”** through multiple Q,K,V matrices
- Let,  $Q_\ell, K_\ell, V_\ell \in \mathbb{R}^{d \times \frac{d}{h}}$ , where  $h$  is the number of attention heads, and  $\ell$  ranges from 1 to  $h$ .
- Each attention head performs attention independently:
  - $\text{output}_\ell = \text{softmax}(X Q_\ell K_\ell^\top X^\top) * X V_\ell$ , where  $\text{output}_\ell \in \mathbb{R}^{d/h}$
- Then the outputs of all the heads are combined!
  - $\text{output} = [\text{output}_1; \dots; \text{output}_h] Y$ , where  $Y \in \mathbb{R}^{d \times d}$
- Each head gets to “look” at different things, and construct value vectors differently.

# Multi-head self-attention is computationally efficient

- Even though we compute  $h$  many attention heads, it's not really more costly.
  - We compute  $XQ \in \mathbb{R}^{n \times d}$ , and then reshape to  $\mathbb{R}^{n \times h \times d/h}$ . (Likewise for  $XK, XV$ .)
  - Then we transpose to  $\mathbb{R}^{h \times n \times d/h}$ ; now the head axis is like a batch axis.
  - Almost everything else is identical, and the **matrices are the same sizes**.

First, take the query-key dot products in one matrix multiplication:  $XQ(XK)^\top$

$$XQ \quad K^\top X^\top = XQK^\top X^\top \in \mathbb{R}^{3 \times n \times n}$$

3 sets of all pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$$\text{softmax} \left( XQK^\top X^\top \right) XV = \text{mix} = \text{output} \in \mathbb{R}^{n \times d}$$

# Scaled Dot Product [Vaswani et al., 2017]

- “**Scaled Dot Product**” attention aids in training.
- When dimensionality  $d$  becomes large, dot products between vectors tend to become large.
  - Because of this, inputs to the softmax function can be large, making the gradients small.

- Instead of the self-attention function we’ve seen:

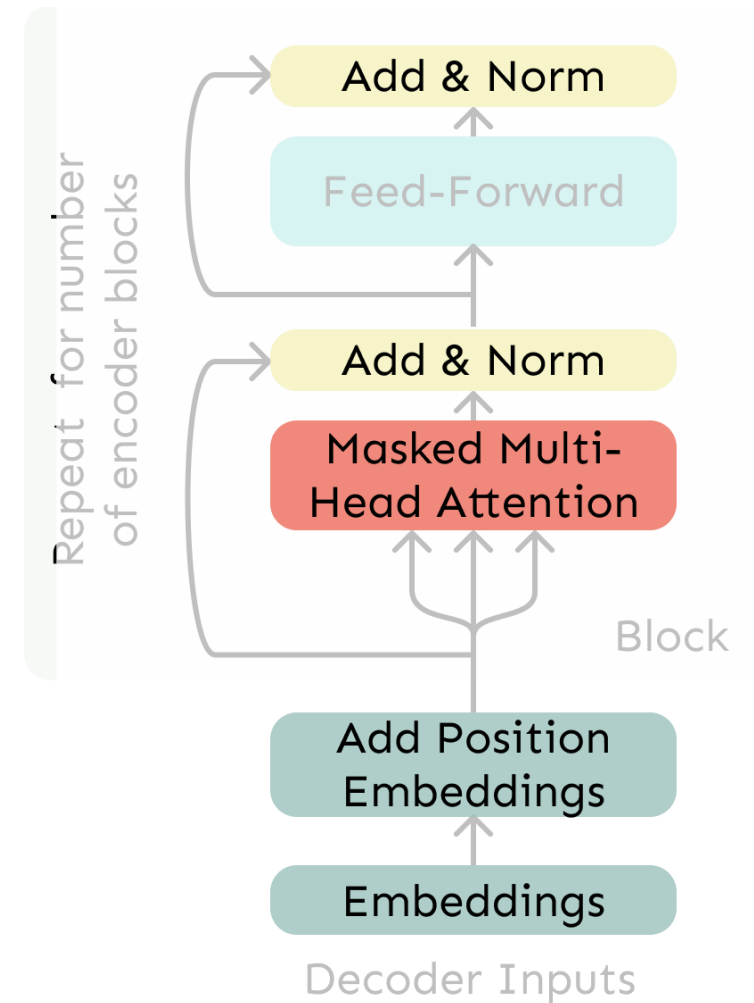
$$\text{output}_\ell = \text{softmax}(XQ_\ell K_\ell^\top X^\top) * XV_\ell$$

- We divide the attention scores by  $\sqrt{d/h}$ , to stop the scores from becoming large just as a function of  $d/h$  (The dimensionality divided by the number of heads.)

$$\text{output}_\ell = \text{softmax}\left(\frac{XQ_\ell K_\ell^\top X^\top}{\sqrt{d/h}}\right) * XV_\ell$$

# The Transformer Decoder

- Now that we've replaced self-attention with multi-head self-attention, we'll go through two **optimization tricks** that end up being :
  - **Residual Connections**
  - **Layer Normalization**
- In most Transformer diagrams, these are often written together as "Add & Norm"



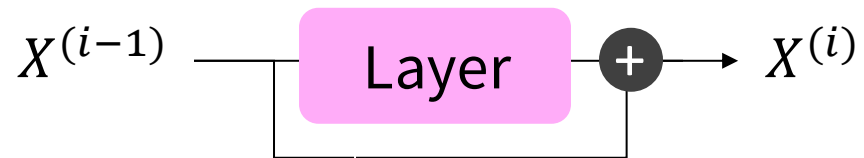
Transformer Decoder

# The Transformer Encoder: **Residual connections** [[He et al., 2016](#)]

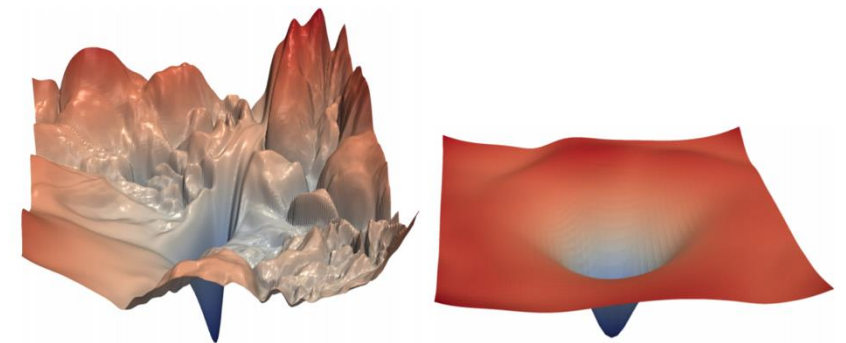
- **Residual connections** are a trick to help models train better.
  - Instead of  $X^{(i)} = \text{Layer}(X^{(i-1)})$  (where  $i$  represents the layer)



- We let  $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$  (so we only have to learn “the residual” from the previous layer)



- Gradient is **great** through the residual connection; it's 1!
- Bias towards the identity function!



[no residuals]

[residuals]

[Loss landscape visualization,  
[Li et al., 2018](#), on a ResNet]



# The Transformer Encoder: **Layer normalization** [[Ba et al., 2016](#)]

- **Layer normalization** is a trick to help models train faster.
- Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean and standard deviation **within each layer**.
  - LayerNorm's success may be due to its normalizing gradients [[Xu et al., 2019](#)]
- Let  $x \in \mathbb{R}^d$  be an individual (word) vector in the model.
- Let  $\mu = \sum_{j=1}^d x_j$ ; this is the mean;  $\mu \in \mathbb{R}$ .
- Let  $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$ ; this is the standard deviation;  $\sigma \in \mathbb{R}$ .
- Let  $\gamma \in \mathbb{R}^d$  and  $\beta \in \mathbb{R}^d$  be learned “gain” and “bias” parameters. (Can omit!)
- Then layer normalization computes:

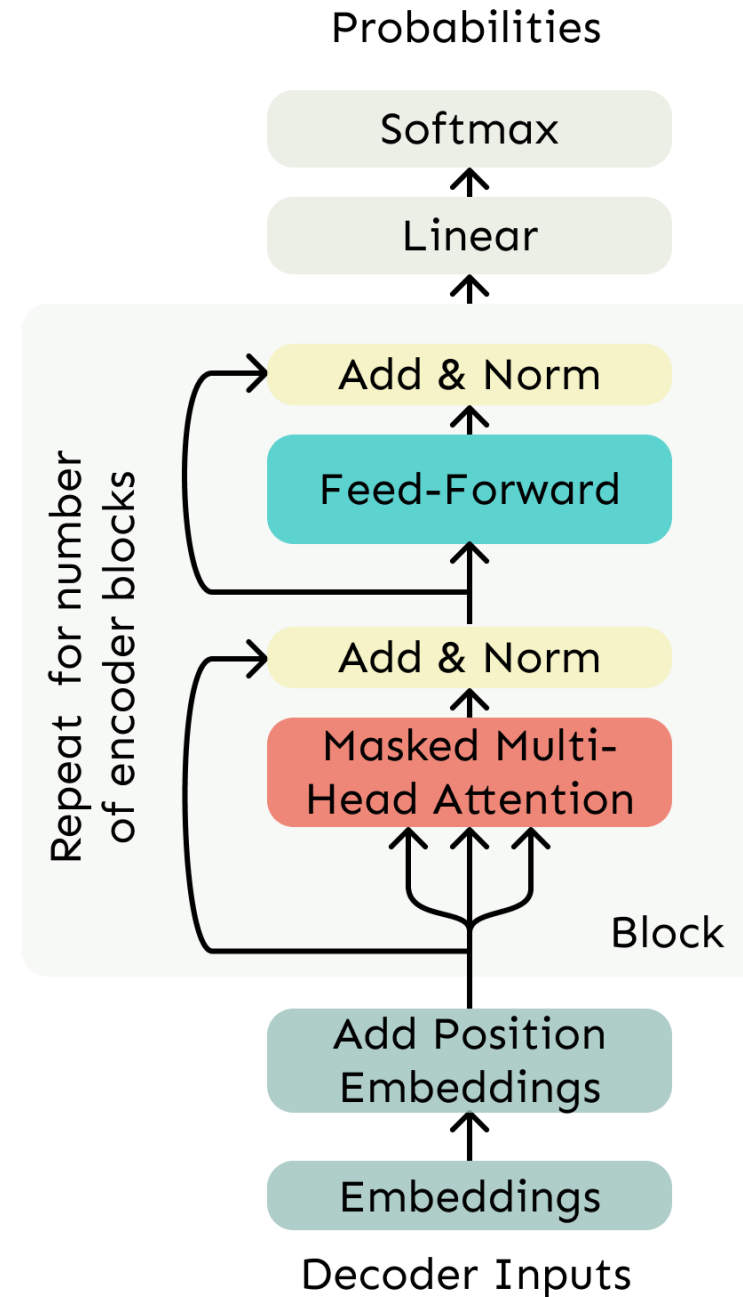
$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

Modulate by learned elementwise gain and bias

# The Transformer Decoder

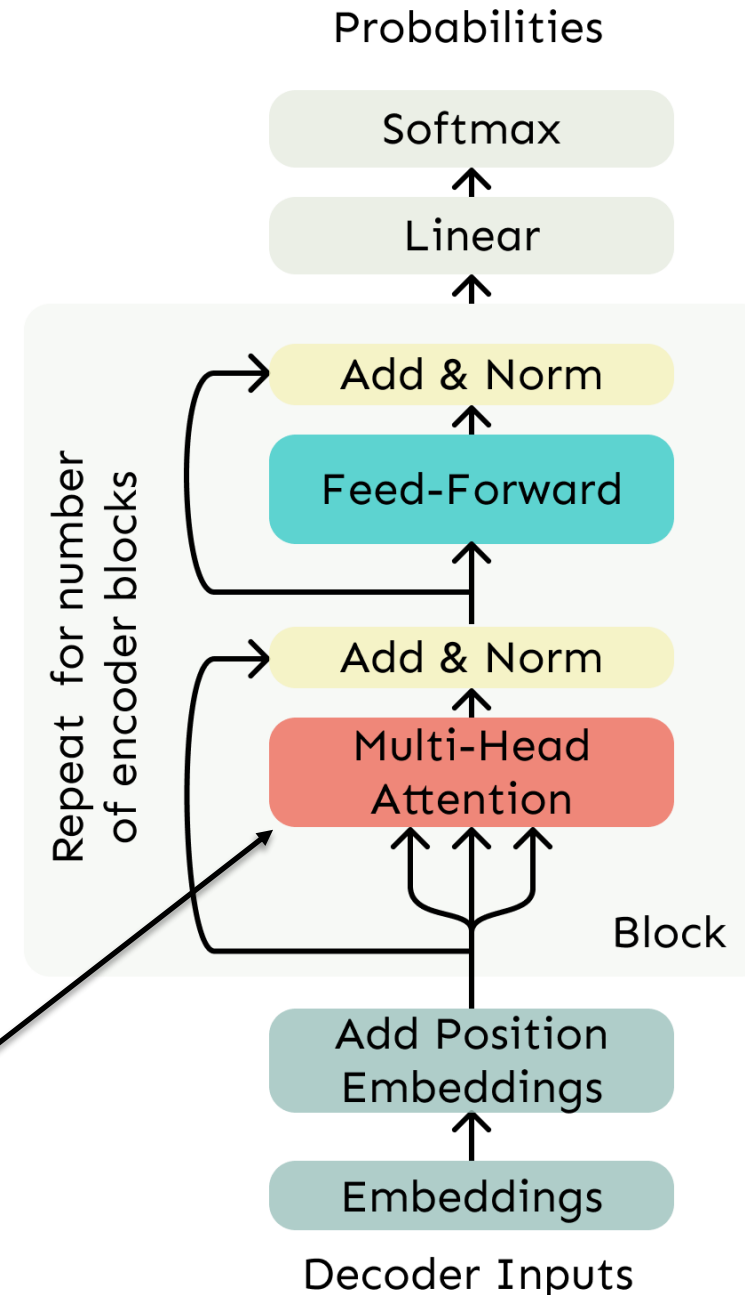
- The Transformer Decoder is a stack of Transformer Decoder **Blocks**.
- Each Block consists of:
  - Self-attention
  - Add & Norm
  - Feed-Forward
  - Add & Norm
- That's it! We've gone through the Transformer Decoder.



# The Transformer Encoder

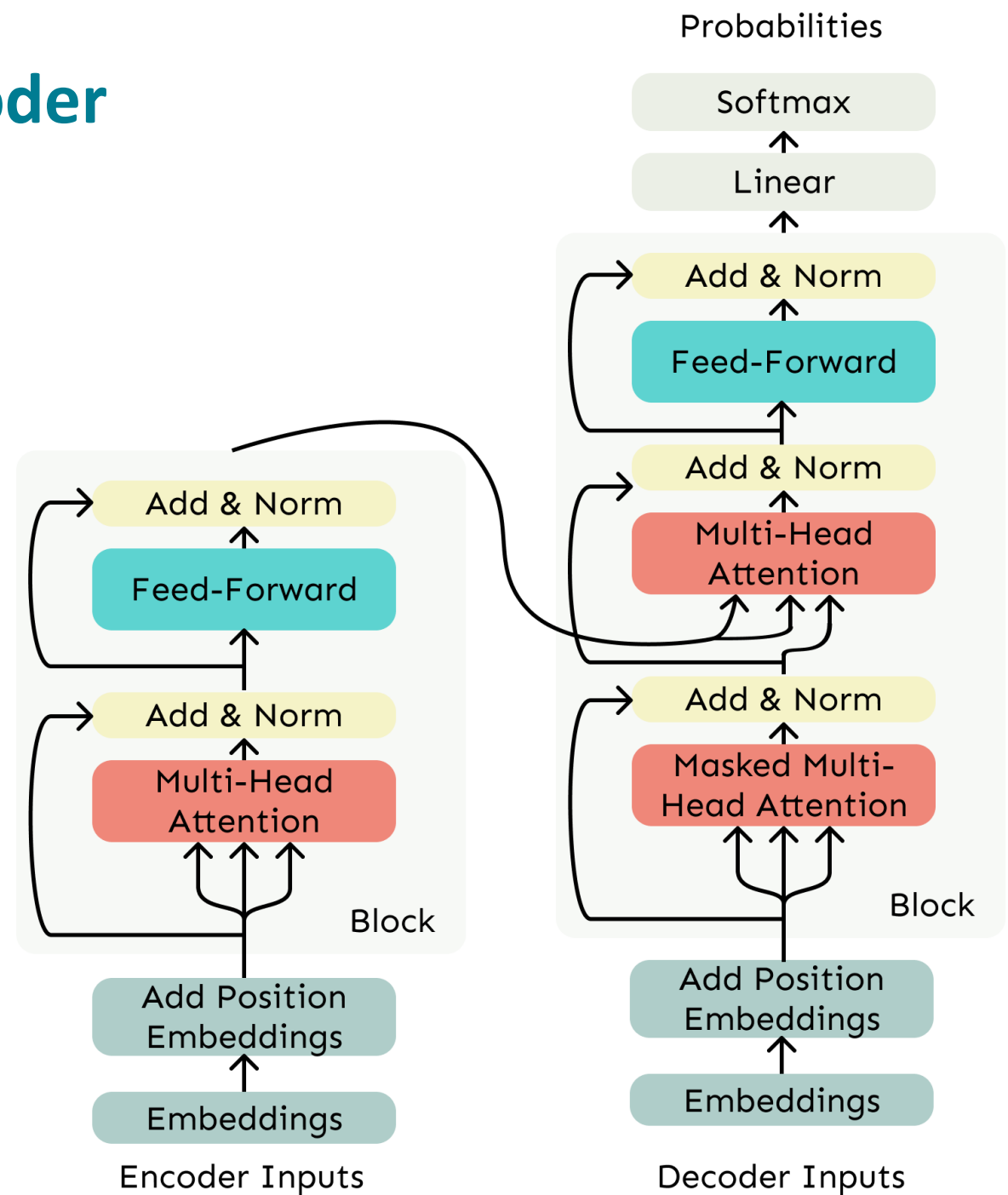
- The Transformer Decoder constrains to **unidirectional context**, as for **language models**.
- What if we want **bidirectional context**, like in a bidirectional RNN?
- This is the Transformer Encoder. The only difference is that we **remove the masking** in the self-attention.

**No Masking!**



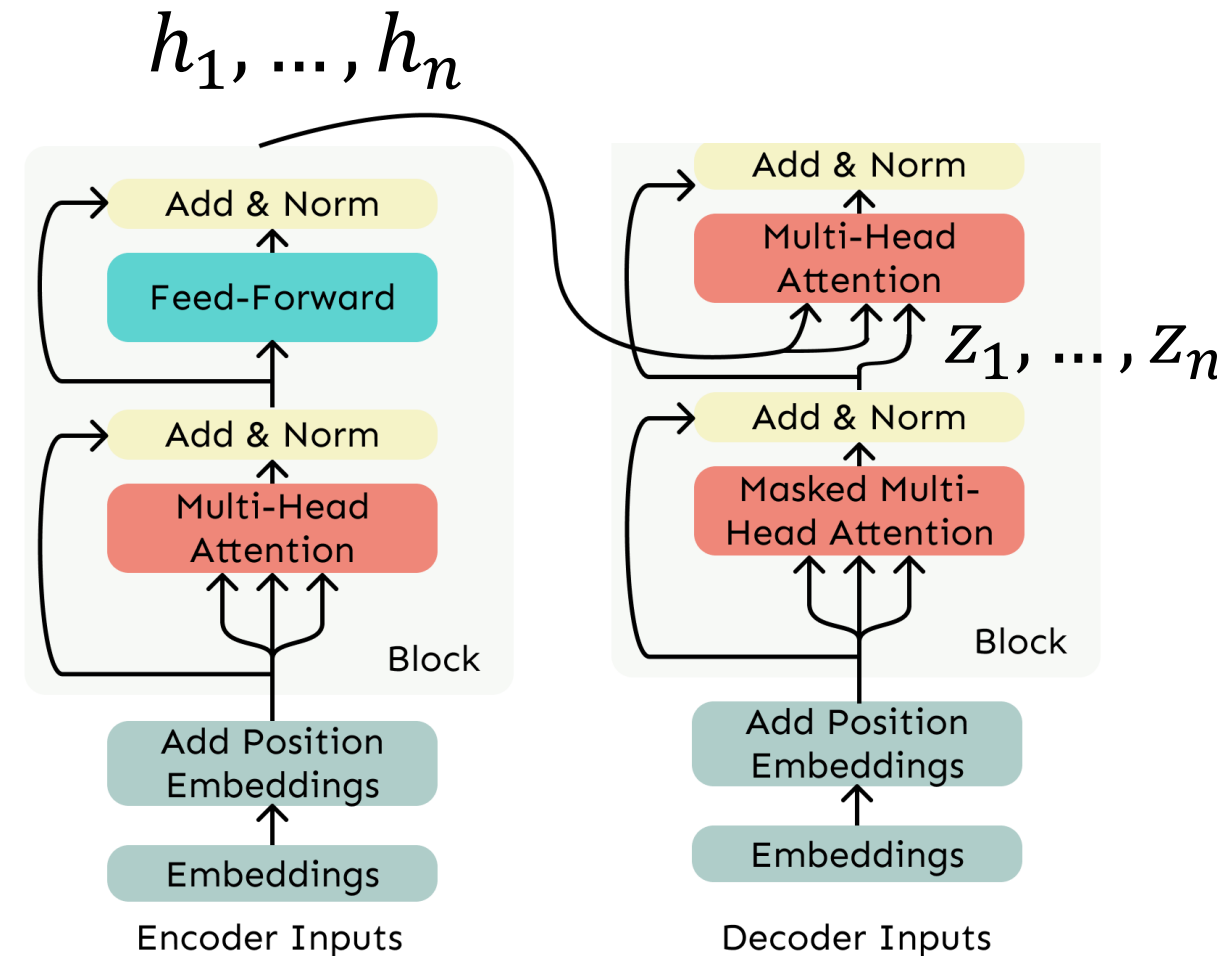
# The Transformer Encoder-Decoder

- Recall that in machine translation, we processed the source sentence with a **bidirectional** model and generated the target with a **unidirectional model**.
- For this kind of seq2seq format, we often use a Transformer Encoder-Decoder.
- We use a normal Transformer Encoder.
- Our Transformer Decoder is modified to perform **cross-attention** to the output of the Encoder.



# Cross-attention (details)

- We saw that self-attention is when keys, queries, and values come from the same source.
- In the decoder, we have attention that looks more like what we saw last week.
- Let  $h_1, \dots, h_n$  be **output** vectors **from** the Transformer **encoder**;  $x_i \in \mathbb{R}^d$
- Let  $z_1, \dots, z_n$  be input vectors from the Transformer **decoder**,  $z_i \in \mathbb{R}^d$
- Then keys and values are drawn from the **encoder** (like a memory):
  - $k_i = Kh_i, v_i = Vh_i$ .
- And the queries are drawn from the **decoder**,  $q_i = Qz_i$ .



# Cross-attention (details)

- Let's look at how cross-attention is computed, in matrices.
  - Let  $H = [h_1; \dots; h_T] \in \mathbb{R}^{T \times d}$  be the concatenation of encoder vectors.
  - Let  $Z = [z_1; \dots; z_T] \in \mathbb{R}^{T \times d}$  be the concatenation of decoder vectors.
  - The output is defined as  $\text{output} = \text{softmax}(ZQ(HK)^\top) \times HV$ .

First, take the query-key dot products in one matrix multiplication:  $ZQ(HK)^\top$

A diagram illustrating the first step of cross-attention. It shows a teal box labeled  $ZQ$  multiplied by an orange box labeled  $K^\top H^\top$ , resulting in a grey box labeled  $ZQK^\top H^\top$ . To the right of the result box is the text  $\in \mathbb{R}^{T \times T}$ . A blue annotation "All pairs of attention scores!" points to the result box.

$$ZQ \cdot K^\top H^\top = ZQK^\top H^\top \in \mathbb{R}^{T \times T}$$

Next, softmax, and compute the weighted average with another matrix multiplication.

A diagram illustrating the second step of cross-attention. It shows a grey box labeled  $ZQK^\top H^\top$  inside a large left-facing square bracket, with the word "softmax" to its left. This is followed by a red box labeled  $HV$ , an equals sign, and a final grey box representing the output. To the right of the output box is the text  $\text{output} \in \mathbb{R}^{T \times d}$ . An arrow points from the  $ZQK^\top H^\top$  box in the previous diagram to this one.

$$\text{softmax} \left( ZQK^\top H^\top \right) \cdot HV = \text{output} \in \mathbb{R}^{T \times d}$$

# Great Results with Transformers

First, Machine Translation from the original Transformers paper!

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	<b>41.29</b>	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$

# Great Results with Transformers

Next, document generation!

Model	Test perplexity	ROUGE-L
<i>seq2seq-attention, <math>L = 500</math></i>	5.04952	12.7
<i>Transformer-ED, <math>L = 500</math></i>	2.46645	34.2
<i>Transformer-D, <math>L = 4000</math></i>	2.22216	33.6
<i>Transformer-DMCA, no MoE-layer, <math>L = 11000</math></i>	2.05159	36.2
<i>Transformer-DMCA, MoE-128, <math>L = 11000</math></i>	1.92871	37.9
<i>Transformer-DMCA, MoE-256, <math>L = 7500</math></i>	1.90325	38.8

The old standard



Transformers all the way down.





# Great Results with Transformers

Before too long, most Transformers results also included **pretraining**, a method we'll go over next.

Transformers' parallelizability allows for efficient pretraining, and have made them the de-facto standard.

On this popular aggregate benchmark, for example:



**All** top models are Transformer (and pretraining)-based.

Rank Name		Model	URL	Score
1	DeBERTa Team - Microsoft	DeBERTa / TuringNLRv4	<a href="#">↗</a>	90.8
2	HFL iFLYTEK	MacALBERT + DKM		90.7
<b>+</b> 3	Alibaba DAMO NLP	StructBERT + TAPT	<a href="#">↗</a>	90.6
<b>+</b> 4	PING-AN Omni-Sinitic	ALBERT + DAAF + NAS		90.6
5	ERNIE Team - Baidu	ERNIE	<a href="#">↗</a>	90.4
6	T5 Team - Google	T5	<a href="#">↗</a>	90.3

# What would we like to fix about the Transformer?

- **Training instabilities (Pre vs Post norm)**
- **Quadratic compute in self-attention :**
  - Computing all pairs of interactions means our computation grows **quadratically** with the sequence length!
  - For recurrent models, it only grew linearly!

# Pre vs Post norm

The one thing everyone agrees on (in 2024)

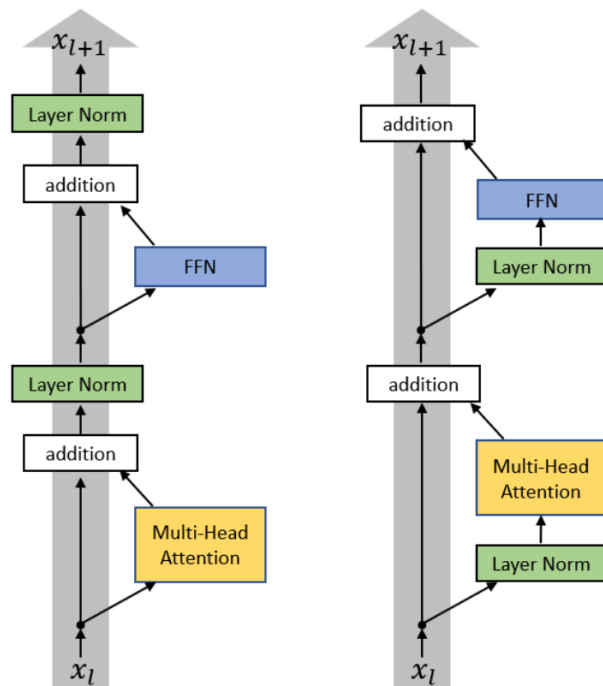


Figure from Xiong 2020

## Post-LN Transformer

$$\begin{aligned}x_{l,i}^{post,1} &= \text{MultiHeadAtt}(x_{l,i}^{post}, [x_{l,1}^{post}, \dots, x_{l,n}^{post}]) \\x_{l,i}^{post,2} &= x_{l,i}^{post} + x_{l,i}^{post,1} \\x_{l,i}^{post,3} &= \text{LayerNorm}(x_{l,i}^{post,2}) \\x_{l,i}^{post,4} &= \text{ReLU}(x_{l,i}^{post,3} W^{1,l} + b^{1,l}) W^{2,l} + b^{2,l} \\x_{l,i}^{post,5} &= x_{l,i}^{post,3} + x_{l,i}^{post,4} \\x_{l+1,i}^{post} &= \text{LayerNorm}(x_{l,i}^{post,5})\end{aligned}$$

## Pre-LN Transformer

$$\begin{aligned}x_{l,i}^{pre,1} &= \text{LayerNorm}(x_{l,i}^{pre}) \\x_{l,i}^{pre,2} &= \text{MultiHeadAtt}(x_{l,i}^{pre,1}, [x_{l,1}^{pre,1}, \dots, x_{l,n}^{pre,1}]) \\x_{l,i}^{pre,3} &= x_{l,i}^{pre} + x_{l,i}^{pre,2} \\x_{l,i}^{pre,4} &= \text{LayerNorm}(x_{l,i}^{pre,3}) \\x_{l,i}^{pre,5} &= \text{ReLU}(x_{l,i}^{pre,4} W^{1,l} + b^{1,l}) W^{2,l} + b^{2,l} \\x_{l+1,i}^{pre} &= x_{l,i}^{pre,5} + x_{l,i}^{pre,3}\end{aligned}$$

$$\text{Final LayerNorm: } x_{Final,i}^{pre} \leftarrow \text{LayerNorm}(x_{L+1,i}^{pre})$$

Set up LayerNorm so that it doesn't affect the main residual signal path (on the left)

**Almost all modern LMs use pre-norm (but BERT was post-norm)**

(One somewhat funny exception – OPT350M. I don't know why this is post-norm)

# Quadratic computation as a function of sequence length

- One of the benefits of self-attention over recurrence was that it's highly parallelizable.
- However, its total number of operations grows as  $O(n^2d)$ , where  $n$  is the sequence length, and  $d$  is the dimensionality.

$$\begin{matrix} \boxed{XQ} & \boxed{K^T X^T} & = & \boxed{XQK^T X^T} & \in \mathbb{R}^{n \times n} \end{matrix}$$

Need to compute all pairs of interactions!  
 $O(n^2d)$

- Think of  $d$  as around **1,000** (though for large language models it's much larger!).
  - So, for a single (shortish) sentence,  $n \leq 30$ ;  $n^2 \leq \mathbf{900}$ .
  - In practice, we set a bound like  $n = 512$ .
  - **But what if we'd like  $n \geq 50,000$ ?** For example, to work on long documents?

# Back to the future – RNNs are back!

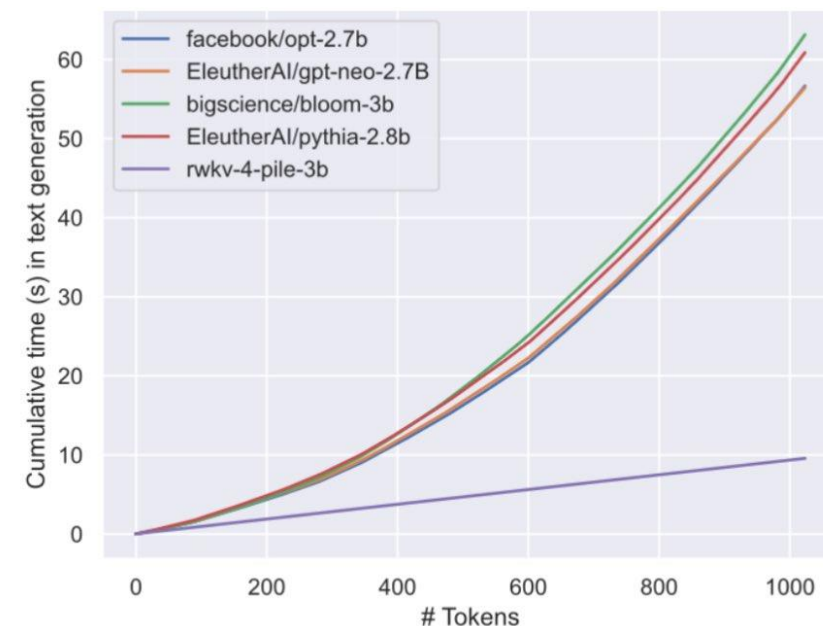
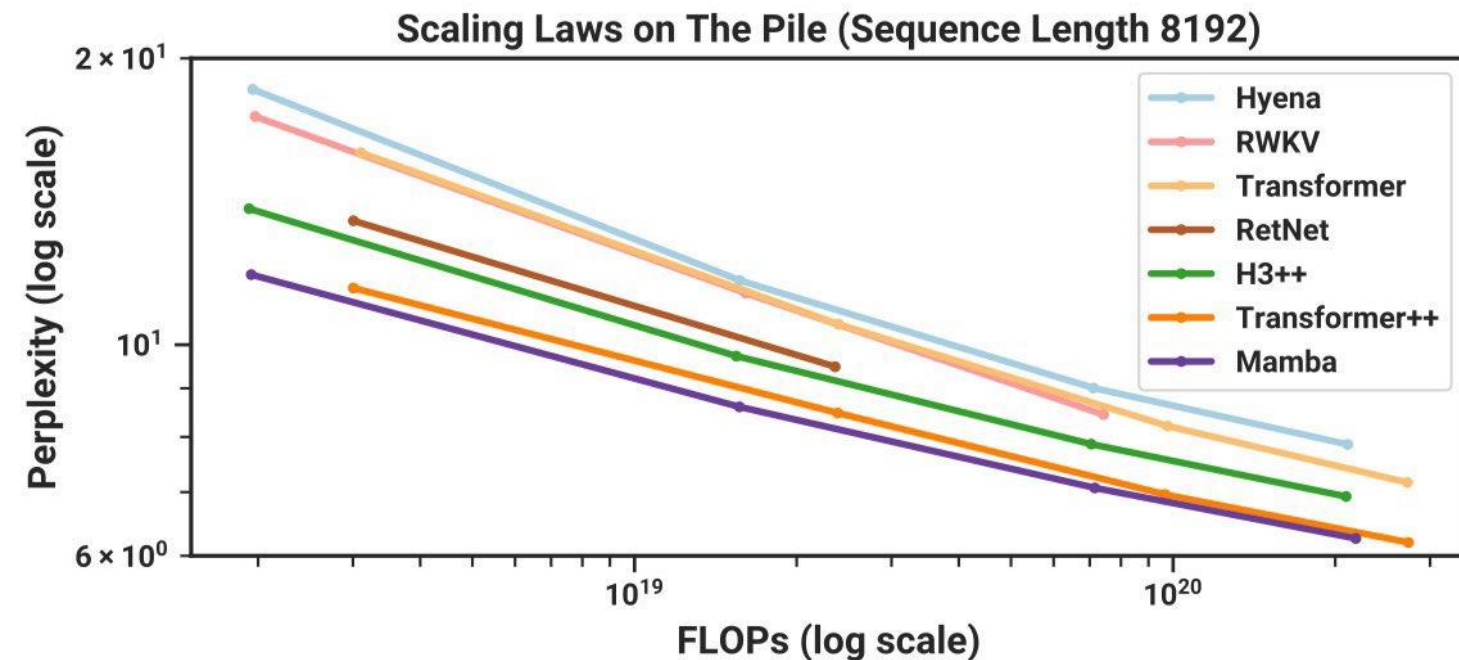


Figure 7: Cumulative time on text generation for LLM  
Unlike transformers, RWKV exhibits linear scaling.

If you want *really* long context, RNNs provide this (linear complexity).  
Modern RNNs (RWKV, Mamba, etc) are getting better!

# Do we even need to remove the quadratic cost of attention?

- As Transformers grow larger, a larger and larger percent of compute is **outside** the self-attention portion, despite the quadratic cost.
- In practice, **production Transformer language models use quadratic cost attention**
  - The cheaper methods tend not to work as well at scale.
  - Systems optimizations work well (Flash attention – Jun 2022)

Foundation Model Context Length



# Do Transformer Modifications Transfer?

- "Surprisingly, we find that most modifications do not meaningfully improve performance."

Model	Params	Ops	Step/s	Early loss	Final loss	SGLUE	XSum	WebQ	WMT EnDe
Vanilla Transformer	223M	11.1T	3.50	2.182 ± 0.005	1.838	71.66	17.78	23.02	26.62
GeLU	223M	11.1T	3.58	2.179 ± 0.003	1.838	<b>75.79</b>	<b>17.86</b>	<b>25.13</b>	26.47
Swish	223M	11.1T	3.62	2.186 ± 0.003	1.847	<b>73.77</b>	17.74	<b>24.34</b>	<b>26.75</b>
ELU	223M	11.1T	3.56	2.270 ± 0.007	1.932	67.83	16.73	23.02	26.08
GLU	223M	11.1T	3.59	2.174 ± 0.003	<b>1.814</b>	<b>74.20</b>	<b>17.42</b>	24.34	<b>27.12</b>
GoGLU	223M	11.1T	3.55	2.130 ± 0.006	<b>1.792</b>	<b>75.96</b>	<b>18.27</b>	<b>24.87</b>	<b>26.87</b>
ReGLU	223M	11.1T	3.57	2.145 ± 0.004	<b>1.803</b>	<b>76.17</b>	<b>18.36</b>	<b>24.87</b>	<b>27.02</b>
SeLU	223M	11.1T	3.55	2.315 ± 0.004	1.948	68.76	16.76	22.75	25.99
SwiGLU	223M	11.1T	3.53	2.127 ± 0.003	<b>1.789</b>	<b>76.00</b>	<b>18.20</b>	<b>24.34</b>	<b>27.02</b>
LaGLU	223M	11.1T	3.59	2.149 ± 0.005	<b>1.798</b>	<b>75.34</b>	<b>17.97</b>	<b>24.34</b>	26.53
Sigmoid	223M	11.1T	3.63	2.291 ± 0.019	1.867	<b>74.31</b>	17.51	23.02	26.30
Softplus	223M	11.1T	3.47	2.207 ± 0.011	1.850	<b>72.45</b>	17.65	<b>24.34</b>	<b>26.89</b>
RMS Norm	223M	11.1T	3.68	2.167 ± 0.008	<b>1.821</b>	<b>75.45</b>	<b>17.94</b>	<b>24.07</b>	<b>27.14</b>
Resero	223M	11.1T	3.51	2.262 ± 0.003	1.939	61.69	15.64	20.90	26.37
Resero + LayerNorm	223M	11.1T	3.26	2.223 ± 0.006	1.858	70.42	17.58	23.02	26.29
Resero + RMS Norm	223M	11.1T	3.34	2.221 ± 0.009	1.875	70.33	17.32	23.02	26.19
Fixup	223M	11.1T	2.95	2.382 ± 0.012	2.067	58.56	14.42	23.02	26.31
24 layers, $d_k = 1536, H = 6$	224M	11.1T	3.33	2.200 ± 0.007	1.843	<b>74.89</b>	17.75	<b>25.13</b>	<b>26.89</b>
18 layers, $d_k = 2048, H = 8$	223M	11.1T	3.38	2.185 ± 0.005	<b>1.831</b>	<b>76.45</b>	16.83	<b>24.34</b>	<b>27.10</b>
8 layers, $d_k = 4096, H = 18$	223M	11.1T	3.69	2.190 ± 0.005	1.847	<b>74.58</b>	17.69	<b>23.28</b>	<b>26.85</b>
6 layers, $d_k = 6144, H = 24$	223M	11.1T	3.70	2.201 ± 0.010	1.857	<b>73.55</b>	17.59	<b>24.60</b>	<b>26.66</b>
Block sharing	65M	11.1T	3.91	2.407 ± 0.037	2.164	64.50	14.53	21.96	25.48
+ Factorized embeddings	45M	9.4T	4.21	2.631 ± 0.305	2.183	60.84	14.00	19.84	25.27
+ Factorized & shared embeddings	20M	9.1T	4.37	2.907 ± 0.313	2.385	53.95	11.37	19.84	25.19
Encoder only block sharing	170M	11.1T	3.68	2.298 ± 0.023	1.929	69.60	16.23	23.02	26.23
Decoder only block sharing	144M	11.1T	3.70	2.352 ± 0.029	2.082	67.93	16.13	<b>23.81</b>	26.08
Factorized Embedding	227M	9.4T	3.80	2.208 ± 0.006	1.855	70.41	15.92	22.75	26.50
Factorized & shared embeddings	202M	9.1T	3.92	2.320 ± 0.010	1.952	68.69	16.33	22.22	26.44
Tied encoder/decoder input embeddings	248M	11.1T	3.55	2.192 ± 0.002	1.840	<b>71.70</b>	17.72	<b>24.34</b>	26.49
Tied decoder input and output embeddings	248M	11.1T	3.57	2.187 ± 0.007	<b>1.827</b>	<b>74.86</b>	17.74	<b>24.87</b>	<b>26.67</b>
Unified embeddings	273M	11.1T	3.53	2.195 ± 0.005	<b>1.834</b>	<b>72.99</b>	17.58	<b>23.28</b>	26.48
Adaptive input embeddings	204M	9.2T	3.55	2.250 ± 0.002	1.899	66.57	16.21	<b>24.07</b>	<b>26.66</b>
Adaptive softmax	204M	9.2T	3.60	2.364 ± 0.005	1.982	<b>72.91</b>	16.67	21.16	25.56
Adaptive softmax without projection	223M	10.8T	3.43	2.229 ± 0.009	1.914	<b>71.82</b>	17.10	23.02	25.72
Mixture of softmaxes	232M	16.3T	2.24	2.227 ± 0.017	<b>1.821</b>	<b>76.77</b>	17.62	22.75	<b>26.82</b>
Transparent attention	223M	11.1T	3.33	2.181 ± 0.014	1.874	54.31	10.40	21.16	<b>26.80</b>
Lightweight convolution	257M	11.8T	2.65	2.403 ± 0.009	2.047	58.30	12.67	21.16	17.03
Envelop Transformer	224M	10.4T	4.07	2.370 ± 0.010	1.989	63.07	14.86	23.02	24.73
Synthesizer (dense)	217M	9.9T	3.69	2.220 ± 0.003	1.863	<b>73.47</b>	10.76	<b>24.07</b>	26.58
Synthesizer (dense plus)	224M	11.4T	3.47	2.334 ± 0.021	1.962	61.03	14.27	16.14	<b>26.63</b>
Synthesizer (dense plus alpha)	243M	12.6T	3.22	2.191 ± 0.010	1.840	<b>73.98</b>	16.96	<b>23.81</b>	<b>26.71</b>
Synthesizer (dense plus alpha)	243M	12.6T	3.01	2.180 ± 0.007	<b>1.828</b>	<b>74.25</b>	17.02	<b>23.28</b>	26.61
Synthesizer (factorized)	207M	10.1T	3.94	2.341 ± 0.017	1.968	62.78	15.39	<b>23.55</b>	26.42
Synthesizer (random)	254M	10.1T	4.08	2.326 ± 0.012	2.009	54.27	10.35	19.56	26.44
Synthesizer (random plus)	292M	12.0T	3.63	2.189 ± 0.004	1.842	<b>73.32</b>	17.04	<b>24.87</b>	26.43
Synthesizer (random plus alpha)	292M	12.0T	3.42	2.186 ± 0.007	<b>1.828</b>	<b>75.24</b>	17.08	<b>24.08</b>	26.39
Universal Transformer	84M	40.0T	0.88	2.406 ± 0.036	2.053	70.13	14.09	19.05	23.91
Mixture of experts	648M	11.7T	3.20	2.146 ± 0.006	1.785	<b>74.55</b>	<b>18.13</b>	<b>24.08</b>	<b>26.84</b>
Switch Transformer	1100M	11.7T	3.18	2.135 ± 0.007	<b>1.758</b>	<b>75.38</b>	<b>18.02</b>	<b>26.19</b>	<b>26.81</b>
Funnel Transformer	223M	1.9T	4.30	2.288 ± 0.008	1.918	67.34	16.26	22.75	23.20
Weighted Transformer	280M	71.0T	0.59	2.378 ± 0.021	1.989	69.04	16.98	23.02	26.30
Product key memory	421M	386.6T	0.25	2.155 ± 0.003	<b>1.798</b>	<b>75.16</b>	17.04	<b>23.55</b>	<b>26.73</b>

## Do Transformer Modifications Transfer Across Implementations and Applications?

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