Hidden Markov Model (HMM)

Problem Statement

Consider a Hidden Markov Model (HMM) with the following parameters:

- **Hidden States:** S_1, S_2
- **Observations:** O_1, O_2
- **Initial State Probabilities:**

$$P(S_1) = 0.6, \quad P(S_2) = 0.4$$

- **Transition Probabilities:**

$$\begin{aligned} P(S_1 \to S_1) = 0.7, &\quad P(S_1 \to S_2) = 0.3 \\ P(S_2 \to S_1) = 0.4, &\quad P(S_2 \to S_2) = 0.6 \end{aligned}$$

- **Emission Probabilities:**

$$\begin{aligned} P(O_1|S_1) = 0.5, &\quad P(O_2|S_1) = 0.5 \\ P(O_1|S_2) = 0.1, &\quad P(O_2|S_2) = 0.9 \end{aligned}$$

Question 1: Forward Algorithm Calculation

Given the observation sequence $O = (O_1, O_2)$, compute the probability of observing this sequence using the **Forward Algorithm**. Specifically, compute:

$$P(O|\lambda) = \sum_{S_t} P(O, S_t|\lambda)$$

where λ represents the model parameters.

Question 2: Viterbi Algorithm Calculation

Given the same observation sequence $O = (O_1, O_2)$, determine the most probable hidden state sequence using the **Viterbi Algorithm**, i.e., find:

$$S^* = \arg\max_S P(S|O, \lambda)$$

where S^* is the most likely sequence of hidden states.