Hidden Markov Model (HMM)

Problem Statement

Consider a Hidden Markov Model (HMM) with the following parameters:

- Hidden States: S_1, S_2
- Observations: O, NFIDENTIAL
- Initial State Probabilities:

$$P(S_1) = 0.6, \quad P(S_2) = 0.4$$

• Transition Probabilities:

$$P(S_1 \to S_1) = 0.7$$
, $P(S_1 \to S_2) = 0.3$
 $P(S_2 \to S_1) = 0.4$, $P(S_2 \to S_2) = 0.6$

• Emission Probabilities:

$$P(O_1|S_1) = 0.5, \quad P(O_2|S_1) = 0.5 \ P(O_1|S_2) = 0.1, \quad P(O_2|S_2) = 0.9$$

Question 1: Forward Algorithm Calculation

Given the observation sequence $O = (O_1, O_2)$, compute the probability of observing this sequence using the **Forward Algorithm**. Specifically, compute:

$$P(O|\lambda) = \sum_{S_t} P(O, S_t|\lambda)$$

where λ represents the model parameters.

Question 2: Viterbi Algorithm Calculation

Given the same observation sequence $O = (O_1, O_2)$, determine the most probable hidden state sequence using the **Viterbi Algorithm**, i.e., find:

$$S^* = \arg\max_{S} P(S|O, \lambda)$$

where S^* is the most likely sequence of hidden states.