# JULIA SUNLET COORDINATES KEY

SHELBY COX

This is a reference for the output of Sampling.ipynb, which computes the chambers of the 3-sunlet arrangement and their ranks. As of 12/13/2022, I have only implemented the code for $G = \mathbb{Z}/3\mathbb{Z}$ and $G = \mathbb{Z}/4\mathbb{Z}$. Note that the coordinate choices are special for cyclic $G$. The coordinates are ordered as in (1).

## 1. Hyperplanes in Coordinates

$$\mu_0\ \mu_1\ \cdots\ \mu_{n-1}\ \eta_{10}\ \eta_{21}\ \cdots\ \eta_{n(n-1)}\ \eta_{0n} \tag{1}$$

The hyperplanes for $G = \mathbb{Z}/3\mathbb{Z}$ and $\mathbb{Z}/4\mathbb{Z}$ in the order they appear in the code are given in 1 and 2, respectively.

| | $\mu_0$ | $\mu_1$ | $\mu_2$ | $\eta_{10}$ | $\eta_{21}$ | $\eta_{02}$ | equation |
|---|---|---|---|---|---|---|---|
| $H_1$ | 1 | 0 | 0 | 0 | 0 | 0 | $\mu_0 = 0$ |
| $H_2$ | 0 | 1 | 0 | -1 | 0 | 0 | $\mu_1 - \eta_{10} = 0$ |
| $H_3$ | 0 | 1 | 0 | 0 | -1 | 0 | $\mu_1 - \eta_{21} = 0$ |
| $H_4$ | 0 | 1 | 0 | 0 | 0 | -1 | $\mu_1 - \eta_{02} = 0$ |
| $H_5$ | 0 | 0 | 1 | 1 | 0 | 0 | $\mu_2 + \eta_{10} = 0$ |
| $H_6$ | 0 | 0 | 1 | 0 | 1 | 0 | $\mu_2 + \eta_{21} = 0$ |
| $H_7$ | 0 | 0 | 1 | 0 | 0 | 1 | $\mu_2 + \eta_{02} = 0$ |

Table 1. Hyperplanes defining the 3-Sunlet arrangement for $G = \mathbb{Z}/3\mathbb{Z}$.

| | $\mu_0$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\eta_{10}$ | $\eta_{21}$ | $\eta_{32}$ | $\eta_{03}$ | equation |
|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\mu_0 = 0$ |
| $H_2$ | 0 | 0 | 1 | 0 | 0 | -1 | -1 | 0 | $\mu_2 - \eta_{21} - \eta_{32} = 0$ |
| $H_3$ | 0 | 0 | 1 | 0 | -1 | -1 | 0 | 0 | $\mu_2 - \eta_{10} - \eta_{21} = 0$ |
| $H_4$ | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | $\mu_2 + \eta_{21} + \eta_{32} = 0$ |
| $H_5$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | $\mu_2 + \eta_{10} + \eta_{21} = 0$ |
| $H_6$ | 0 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | $\mu_1 - \eta_{32} = 0$ |
| $H_7$ | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | $\mu_1 - \eta_{21} = 0$ |
| $H_8$ | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | $\mu_1 - \eta_{10} = 0$ |
| $H_9$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | $\mu_1 - \eta_{03} = 0$ |
| $H_{10}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mu_3 + \eta_{03} = 0$ |
| $H_{11}$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mu_3 + \eta_{32} = 0$ |
| $H_{12}$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mu_3 + \eta_{21} = 0$ |
| $H_{13}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | $\mu_3 + \eta_{10} = 0$ |

Table 2. Hyperplanes defining the 3-Sunlet arrangement for $G = \mathbb{Z}/4\mathbb{Z}$.

## 2. Sample Points in Coordinates

The $i$th entry of the inequality vector for the sample points indicates the sign of the non-zero side of the $i$th hyperplane equation. A 1 means the non-zero side is positive, and a 0 means the non-zero side is negative. For example, $s = [0\ 1\ 1\ 1\ 1\ 1\ 1]$ is the inequality vector for a point in the complement of the 3-Sunlet arrangement when $G = \mathbb{Z}/3\mathbb{Z}$ that satisfies the inequalities in (2).

$$\mu_0 < 0,\ \mu_1 > \eta_{10},\ \mu_1 > \eta_{21},\ \mu_1 > \eta_{02},\ \mu_2 > -\eta_{10},\ \mu_2 > -\eta_{21},\ \mu_2 > -\eta_{02} \tag{2}$$

2.1. $G = \mathbb{Z}/3\mathbb{Z}$**.** Table 3 records the inequality vectors of sample points and the rank of the corresponding $A_\lambda$ matrix. Note that there are three possible matrix ranks: 7, 8, and 9. There are four chambers that achieve rank 7 (the lowest possible). Two of the lowest rank chambers are parochial (inequality vectors are all 0's or all 1's), and the other two lowest rank chambers neighbor a parochial chamber along the hyperplane $\mu_0 = 0$.

| sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | rank | # / rank |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 4 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | |
| 5 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 8 | 24 |
| 6 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 8 | |
| 7 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 8 | |
| 8 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 8 | |
| 9 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 8 | |
| 10 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 8 | |
| 11 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 8 | |
| 12 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 8 | |
| 13 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 8 | |
| 14 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 8 | |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 8 | |
| 16 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 8 | |
| 17 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 8 | |
| 18 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 8 | |
| 19 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 8 | |
| 20 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 8 | |
| 21 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 8 | |
| 22 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 8 | |
| 23 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 8 | |
| 24 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 8 | |
| 25 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 8 | |
| 26 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 8 | |
| 27 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 8 | |
| 28 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 8 | |
| 29 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 9 | 64 |
| 30 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 9 | |
| 31 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 9 | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 32 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 9 | |
| 33 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 9 | |
| 34 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 9 | |
| 35 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 9 | |
| 36 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 9 | |
| 37 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 9 | |
| 38 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 9 | |
| 39 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 9 | |
| 40 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 9 | |
| 41 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 9 | |
| 42 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 9 | |
| 43 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 9 | |
| 44 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 9 | |
| 45 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 9 | |
| 46 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 9 | |
| 47 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 9 | |
| 48 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 9 | |
| 49 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 9 | |
| 50 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 9 | |
| 51 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 9 | |
| 52 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 9 | |
| 53 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 9 | |
| 54 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 9 | |
| 55 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 9 | |
| 56 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 9 | |
| 57 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 9 | |
| 58 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 9 | |
| 59 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 9 | |
| 60 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 9 | |
| 61 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 9 | |
| 62 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 9 | |
| 63 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 9 | |
| 64 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 9 | |
| 65 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 9 | |
| 66 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 9 | |
| 67 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 9 | |
| 68 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 9 | |
| 69 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 9 | |
| 70 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 9 | |
| 71 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 9 | |
| 72 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 9 | |
| 73 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 9 | |
| 74 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 9 | |
| 75 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 9 | |
| 76 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 9 | |
| 77 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 9 | |
| 78 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 9 | |
| 79 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 9 | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 9 | |
| 81 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 9 | |
| 82 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 9 | |
| 83 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 9 | |
| 84 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 9 | |
| 85 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 9 | |
| 86 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 9 | |
| 87 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 9 | |
| 88 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 9 | |
| 89 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 9 | |
| 90 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 9 | |
| 91 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 9 | |
| 92 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 9 | |

Table 3: Inequality vectors of chambers in the 3-Sunlet arrangement with $G = \mathbb{Z}/3\mathbb{Z}$, ordered by rank. The last column records the number of chambers of each rank.

2.2. $G = \mathbb{Z}/4\mathbb{Z}$. There are too many chambers (¿ 2k) to record the inequality vectors of sample points and the rank of the corresponding $A_\lambda$ matrix in a LaTeX document. See the file ranks_ineq_4.txt for the complete list of inequality vectors sorted by rank. Note that there are three possible matrix ranks: 10, 11, 12, 13, 14, 15, and 16. There are four chambers that achieve rank 10 (the lowest possible). Two of the lowest rank chambers are parochial (inequality vectors are all 0's or all 1's), and the other two lowest rank chambers neighbor a parochial chamber along the hyperplane $\mu_0 = 0$.

## 3. Translating to Lambda Coordinates

Under Construction.