Graphical Arithmetic for Learners with Dyscalculia

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ABSTRACT
We propose a model for arithmetic, based on graphical representations, to complement the symbolic language of mathematics. The focus is conceptual understanding of arithmetic. We argue that the graphical model supports understanding concepts known to be difficult for learners with dyscalculia, such as number-sense and decimal system. The proposed graphical representation share properties of the decimal system, but is closer to the semantic representation of numbers vital to the number-sense. The model is evaluated with school-children, but needs to be further tested by learners with dyscalculia.

Categories and Subject Descriptors
K.3.1[**Computers and Education**]: Computer Uses in Education.

General Terms
Human Factors, design.

Keywords
Dyscalculia, math disability, arithmetic, graphical model

1. INTRODUCTION
Numbers and numerical ideas are fundamental to everyday life. For learners with severe math disabilities, dyscalculia, the world of numbers may never be accessible. Of school-age children, 6-7% show persistent difficulties in learning arithmetic, which is not related to IQ, motivation or other learning factors [2]. For these learners, mathematical curriculum should focus on big ideas such as arithmetic. Skills required to master arithmetic include number-sense, counting, and the decimal system. The decimal system is essential but difficult for all learners [2]. The proposed model provides a graphical, analogical, constructive representation of the decimal-system and the arithmetic operations. It allows for exploration and self-regulation.

2. A GRAPHICAL MODEL
The idea is to represent arithmetic concepts by graphical components. Numbers are represented by graphical objects and operations by actions on these. The model is based on squares as building blocks. The model consists of colored squares and “square-boxes” that can be packed and unpacked, and a squared board where these are placed. The model provides representations for *digits*, *numbers*, *arithmetic operations*, and *computations*.

2.1 Representation of Numbers
A *digit* is represented by an adjacent group of squares. The number of squares denote the digit’s magnitude, and the color the unit it belongs to (hundreds, tens, ones). Beside the color there are additional marks on the squares that represent tens (one dot) and hundreds (two dots). Any formation of the squares is valid, as long as they form a group. The purpose is to focus on the magnitude, i.e., the number of squares, rather than the shape.
A *number* is represented by a sequence of digits. For instance, the graphical number in figure 1 denotes the number 446. The representation is based on a metaphor of square-boxes of different sizes. There are orange one-dot square-boxes which hold 10 red squares, yellow two-dot boxes holding 10 orange boxes, and so on. However, when seen from above, the height of the square-boxes do not show so they all look the same. There are two important properties of this representation: 1) *The significance of the position:* the 4's in 446 are represented by four squares but denote 400 and 40 respectively, as the symbolic representation. Often analogical represent-ations of numbers grow proportional to the magnitude. 2) *The quantitative understanding of the metaphor*: that notion of square-boxes is easy to picture and helps in understanding the magnitude of a number, as shown in figure 2.

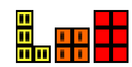

Figure 1. Graphical number 446

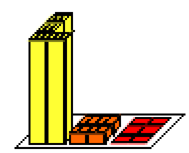

Figure 2. View from side showing the magnitude

2.2 Operations and Computations
The *operations of arithmetic* are represented by actions on the squares: Addition is represented by the act of adding squares to the board, subtraction by the act of removing squares from the board. Multiplication is represented by repeating addition, and division by repeating subtraction (only allowed when evens out).
A *computation* is a sequence of operations. It takes place on the board. The board consists of a number of compartments, corresponding to the different units (hundreds, tens, ones). The colored squares are associated with their corresponding compartment. In figure 3, the limit of the unit is explicit (the white boundary) to help the learner. When the unit is full, the next square will appear in the “packing-area” (the arrow) above. To proceed, the 10 squares must be packed into a square-box and placed in the compartment to the left (next higher unit). This procedure is animated so the packing become apparent to the learner. Squares/square-boxes can only be moved between compartments by packing or unpacking them.

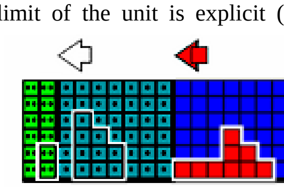

Figure 3. Computation board

Negative numbers are included in the model as an option. They are represented as "the opposites" of positive numbers: they have opposite colors and they occur at the opposite side of the zero line (see figure 4), to capture the symmetry of arithmetic. The unit compartments have opposite colors to their corresponding digit, to mimic arithmetic symmetry in the model: In arithmetic, subtracting –2 and adding 2, yields the same result. Normally, this is taught as a rule $a-(-b) = a+b$. In our graphical model, this symmetry is built in: removing 4 "negative" squares, *has the same effect* as adding 4 positive. This important rule can thus be discovered using the model, instead of memorized as a rule.

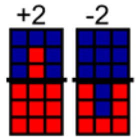

Figure 4.

2.3 Application of the model

The graphical model can be used to understand arithmetic concepts, without symbolic representations. The incremental shift to symbolic representation is done by decorating graphical components. The model is sound; numbers, operations and computations can be translated between the two representations. A game and story based educational environment is developed using the model [4]. The games practice arithmetic skills and understanding, and the story provides intuition for the model. A prototype implementation of the environment has been tested by school-children through out the development, and the results show that the model is directly adopted by children and that the graphical language is suitable for solving problems and "talking mathematics" with. The tests indicate that performance levels in the graphical representation differ from performance with symbolic representations. Some weak math students performed very well using the model. Our hypothesis is that the model is suitable for many learners with math learning disabilities.

3. LEARNERS WITH DYSCALCULIA

Dyscalculia is a learning disability concerning mathematics. The following dyscalculia related problems are discussed in the literature [1,2,3]: *Misconception of counting principles*, *weak number-sense*, and *difficulty abstracting principles*.

3.1 Misconception of Counting Principles

Counting is the most basic skill in arithmetic, since it relates quantities to numbers. Learners with dyscalculia have difficulties with the order-irrelevance principle and often believe that objects must be counted from left to right: they do not understand counting [2]. The graphical representation of squares encourages counting and different counting strategies, since the squares come in different formations. The order-irrelevance principle is explicitly practiced in the model.

3.2 Weak or Lacking Number-Sense.

The sense of numbers is fundamental to mathematics. Learners with dyscalculia have difficulties seeing the "threeness" of three objects (referred to as *disability to subitize*), and must always rely on counting. Moreover, the number word their counting results in has no meaning to them. Dyscalculia is closely related to the disability to subitize. The graphical representation provides the opportunity to practice subitization: every digit is represented by a group of squares, which area is proportional to the magnitude. Most learners can distinguish a larger area from a smaller.

Children must learn to associate the three representations of a number: the symbol "2", the word "two" and the quantity (magnitude) "2". It is recognized that many children with dyscalculia have brain deficits in their quantity system, and that these representational deficits are likely to persist. Learners with dyscalculia often acquire the magnitude representation easier than the symbolic; the two being functionally independent aspects of number knowledge. Our graphical representation is somewhere between these representations: sharing properties of both. It shares the following with the symbolic representation: the structure of the decimal system; the significance of position; and the visual similarity of digits in different positions. It also shares the following properties with the quantitative aspect of numbers: the appearance of a digit is proportional to its magnitude; the learner can rely on counting the squares, and the magnitude is implicit in the model, by the square-box metaphor. The graphical and the symbolic representations can be present simultaneously, making the association explicit.

The relationships between units are explicit: the different compartments, the packing and unpacking of square-boxes. The model provides low-stress algorithms [3], where every step of a computation is explicit. The distinction between numbers and operation is clear: numbers are graphical objects and operations are animated actions. Operations are consequent: addition is to put squares; subtraction to remove squares; multiplication is repeated addition; and subtraction repeated division.

3.3 Great Difficulty Abstracting Principles

Students with learning disabilities have great difficulties abstracting principles from experiences [2]. In the model rules of arithmetic are built in: learners can experience rather than learn the symmetry of numbers and operations, the sign rules, and the "crossing zero line". These aspects are known to be difficult for all students. Learners may discover these principles themselves.

4. CONCLUSION

The model has potential to help learners with dyscalculia. The following practices are recommended for these learners: encourage and practice to "talk math" [1], using motivational practices such as games [1], and using board-games and instructional software [3]. All these exist in the prototype microworld based on the model [4]. However, the model needs to be validated by user tests with dyscalculia learners.

5. REFERENCES

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