2. $f(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-1}{2\sigma_i^2}\varepsilon_i^2}$

$$= \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-1}{2\sigma_i^2}(Y_i - X_i\omega)^2}$$

Likelihood

$$\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-1}{2\sigma_i^2}(Y_i - X_i\omega_i)^2}$$

Log-likelihood

$$\ln\left[\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma_i}}\right] - \sum_{i=1}^{m} \frac{1}{2\sigma_i^2}(Y_i - X_i\omega_i)^2$$

Derive WRT ω_i

$$0 = \sum_{i=1}^{m} \frac{X_i}{\sigma_i^2}(Y_i - X_i\omega_i)$$

Matrix Notation: let $\beta = \begin{pmatrix} 1/\sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_m^2 \end{pmatrix}$

$$0 = X^T\beta(Y - X\omega), \quad X \in \mathbb{R}^{m\times n}, \quad \beta \in \mathbb{R}^{m\times m}$$

$$0 = X^T\beta Y - X^T\beta X\omega$$

$$\hat{\omega} = (X^T\beta X)^{-1} X^T\beta Y$$