

Problems for Tutorial 1

1. Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive.
 - (a) Formulate this situation as a strategic game.
 - (b) If you call the aggressive action *Quiet* and the passive action *Fink*, is the resulting game the same as the *Prisoner's Dilemma*? (That is, are the players' preferences in the resulting game the same as their preferences in the *Prisoner's Dilemma*?) If not, how do they differ?
 - (c) Find the Nash equilibria (if any) of the game.
2. Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.
 - (a) Suppose that each person cares only about her own comfort. Model the situation as a strategic game. Is this game the same as the *Prisoner's Dilemma* (except for the names of the actions)? Find its Nash equilibrium (equilibria?).
 - (b) Suppose that each person is altruistic, ranking the outcomes according to the *other* person's comfort, but, out of politeness, prefers to stand than to sit if the other person stands. Model the situation as a strategic game. Is this game the *Prisoner's Dilemma* (except for the names of the actions)? Find its Nash equilibrium (equilibria?).
 - (c) Compare the people's comfort in the equilibria of the two games.
3. Two candidates, *A* and *B*, compete in an election. Of the n citizens, k support candidate *A* and $m (= n - k)$ support candidate *B*. Each citizen decides whether to vote, at a cost, for the candidate she supports,

or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs $2 - c$, $1 - c$, and $-c$ in these three cases, where $0 < c < 1$.

- (a) For $k = m = 1$, is the game the same (except for the names of the actions) as any we considered in class?
- (b) For $k = m$, find the set of Nash equilibria. (Is the action profile in which everyone votes a Nash equilibrium? Is there any Nash equilibrium in which the candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?)
- (c) What is the set of Nash equilibria for $k < m$?