ECO316: Applied game theory

Lecture 1

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Game theory

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- Decision-makers can be individual humans, groups of humans, animals
- Used in economics, psychology, political science, sociology, computer science, biology
- Course covers basic theory, with emphasis on applications in economics

Applications: preview

- Competition between firms

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    - More firms $\Rightarrow$ better outcome for consumers?

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Voting in a jury (example of game-theoretic argument)

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- Assume that variations in interpretation are *random*
- All jurors share *same* goal: convict person if and only if probability they are guilty is high enough
- Jurors differ *only* in their (random) interpretation of evidence

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- And they should vote for conviction if and only if that probability is high enough

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- Suppose your interpretation is that defendant is innocent
- How should you vote?

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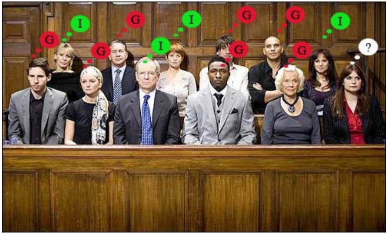

Voting in a jury (example of game-theoretic argument)

- When does your vote make a difference to the decision?
- Not if some other jurors' interpretations are that defendant is innocent, because they will vote to acquit
- Only if *all* other jurors' interpretations are that defendant is guilty, in which case they vote to convict

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- Thus if everyone else votes according to their interpretation of evidence, you should not do so

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- But if everyone else's interpretation of evidence is that defendant is guilty then it is highly likely she is guilty
- So you should vote for conviction unless your interpretation points *extremely strongly* to innocence!
- Thus if everyone else votes according to their interpretation of evidence, you should not do so
- Conclusion: outcome in which every juror votes according to her interpretation of evidence is *not* an "equilibrium"

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- It shows only that everyone's voting as if they alone were making the decision is not an equilibrium
- It does not tell us what *is* an equilibrium

Voting in a jury (example of game-theoretic argument)

- Note that argument does *not* show that a juror should always vote for conviction!
- It shows only that everyone’s voting as if they alone were making the decision is not an equilibrium
- It does not tell us what *is* an equilibrium
- Note also that argument ignores “deliberation” among jurors

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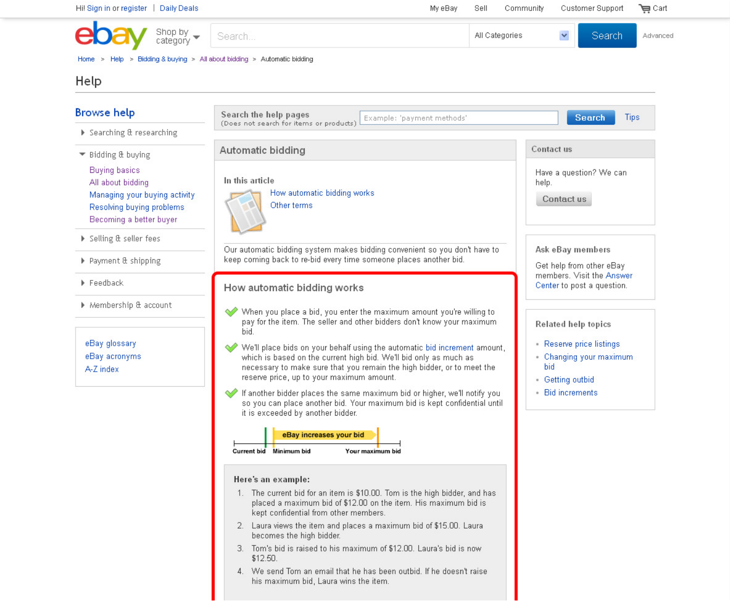
  - Which design produces an efficient outcome? High revenue for the auctioneer?

Auctions



Have a question? We can help.

Get help from other eBay members. Visit the Answer Center to post a question.

Automatic bidding

In this article

- How automatic bidding works
- Other terms

Our automatic bidding system makes bidding convenient so you don't have to keep coming back to re-bid every time someone places another bid.

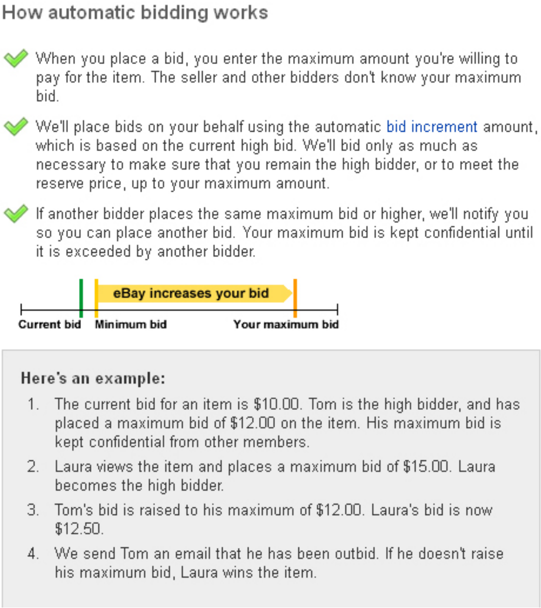
How automatic bidding works

- ✓ When you place a bid, you enter the maximum amount you're willing to pay for the item. The seller and other bidders don't know your maximum bid.
- ✓ We'll place bids on your behalf using the automatic bid increment amount, which is based on the current high bid. We'll bid only as much as necessary to make sure that you remain the high bidder, or to meet the reserve price, up to your maximum amount.
- ✓ If another bidder places the same maximum bid or higher, we'll notify you so you can place another bid. Your maximum bid is kept confidential until it is exceeded by another bidder.

Here's an example:

1. The current bid for an item is $10.00. Tom is the high bidder, and has placed a maximum bid of $12.00 on the item. His maximum bid is kept confidential from other members.
2. Laura views the item and places a maximum bid of $15.00. Laura becomes the high bidder.
3. Tom's bid is raised to his maximum of $12.00. Laura's bid is now $12.50.
4. We send Tom an email that he has been outbid. If he doesn't raise his maximum bid, Laura wins the item.

Auctions



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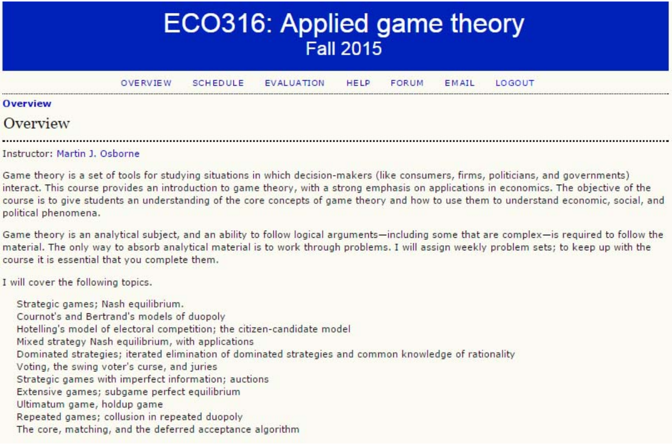
- Matching
  - Which algorithm produces a good outcome?

Course website

http://mjo.osborne.economics.utoronto.ca/index.php/course/index/5

Username: *your UTORid*
Password: *your U of T student number (without leading 0)*



ECO316: Applied game theory

Fall 2015

Overview

Instructor: Martin J. Osborne

Game theory is a set of tools for studying situations in which decision-makers (like consumers, firms, politicians, and governments) interact. This course provides an introduction to game theory, with a strong emphasis on applications in economics. The objective of the course is to give students an understanding of the core concepts of game theory and how to use them to understand economic, social, and political phenomena.

Game theory is an analytical subject, and an ability to follow logical arguments—including some that are complex—is required to follow the material. The only way to absorb analytical material is to work through problems. I will assign weekly problem sets; to keep up with the course it is essential that you complete them.

I will cover the following topics.

Strategic games; Nash equilibrium.
Cournot's and Bertrand's models of duopoly
Hotelling's model of electoral competition; the citizen-candidate model
Mixed strategy Nash equilibrium, with applications
Dominated strategies; iterated elimination of dominated strategies and common knowledge of rationality
Voting, the swing voter's curse, and juries
Strategic games with imperfect information; auctions
Extensive games; subgame perfect equilibrium
Ultimatum game, holdup game
Repeated games; collusion in repeated duopoly
The core, matching, and the deferred acceptance algorithm

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  - fully informed
  - selfish
  - sensible in an objective sense

Rational decision-maker

Model

A decision problem consists of

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Theory

Decision-maker chooses the member of A that is best according to her preferences

Many decision-makers: Strategic games

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any decision-making entity: individual human being, group of individuals, animal, . . .

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any set (numbers, lists of numbers, functions, . . .)

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Action profile = list of actions, one for each player

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Preferences over action profiles $\Longrightarrow$ each player cares about actions taken by *other* players (as well as her own action)

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- Captures idea that undercutting is profitable

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Working with preferences

Use numbers to represent them:

Firm 2

		high	*low*
Firm 1	*high*	2	0
	low	3	1

Firm 1's payoffs

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Working with preferences

Combine tables:

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{Firm 2}} \\
 & & high & low \\ \cline{3-4}
\text{Firm 1} & high & 2,2 & 0,3 \\ \cline{3-4}
 & low & 3,0 & 1,1 \\ \cline{3-4}
\end{array}$$

Payoff of firm 1, payoff of firm 2

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Strategic game: Example

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Notes

- We could use other numbers to represent preferences (e.g. profits)

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- For current purposes, only *order* matters

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- What defines game are players' *preferences*, not specific payoff representations

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		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Same game:

		Firm 2	
		high	*low*
Firm 1	*high*	100, 100	−5, 101
	low	101, −5	0, 0

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Firm 1	*high*	0, 90	−100, 100
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Firm 1	*high*	0, 90	−100, 100
	low	100, −5	−1, 1

A game is determined by the players' *orderings* of the outcomes

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Strategic game: Example

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{Firm 2}} \\
 & & \textit{high} & \textit{low} \\ \cline{3-4}
\text{Firm 1} & \textit{high} & 2, 2 & 0, 3 \\ \cline{3-4}
 & \textit{low} & 3, 0 & 1, 1 \\ \cline{3-4}
\end{array}$$

- Game is called

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

- Game is called *Prisoner's Dilemma*

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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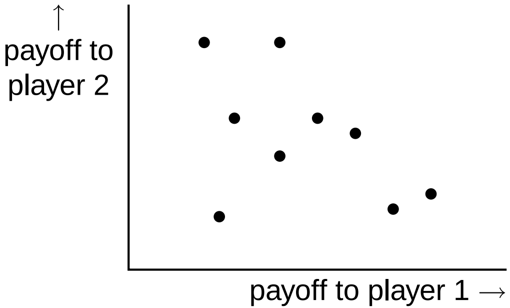
- Game is called *Prisoner's Dilemma*
- Structure of incentives in game is present in many situations

Strategic game: Example

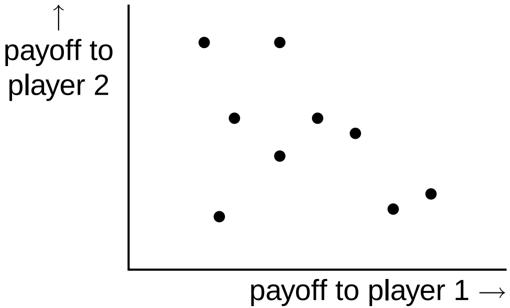
		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Game is called *Prisoner’s Dilemma*
- Structure of incentives in game is present in many situations
- Has been used to model a huge variety of situations in diverse fields

Digression: Pareto efficiency

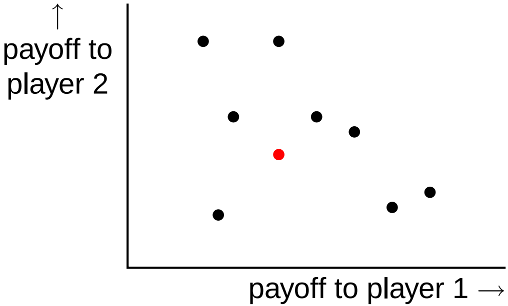


Digression: Pareto efficiency



- Which outcomes are Pareto efficient?

Digression: Pareto efficiency



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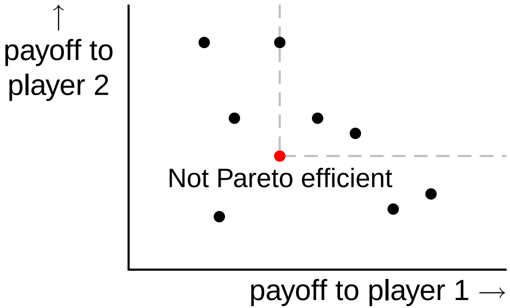

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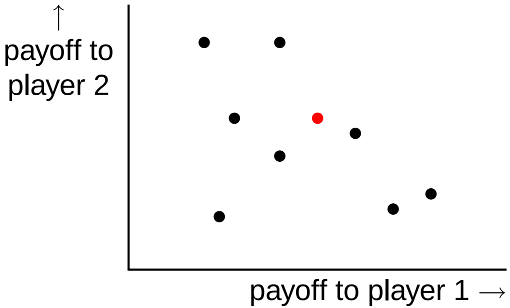

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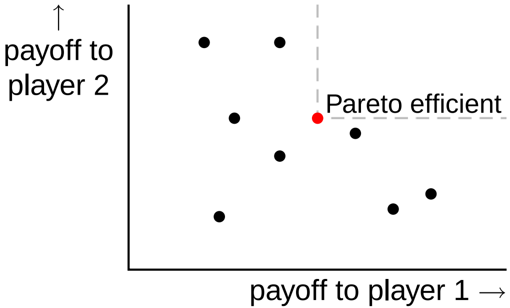

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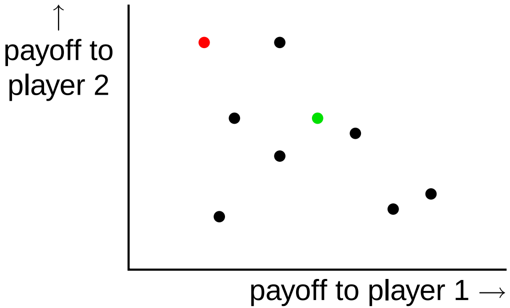
Digression: Pareto efficiency



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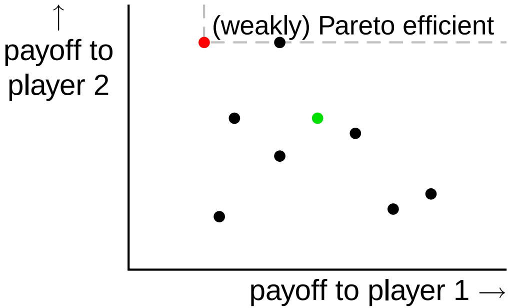

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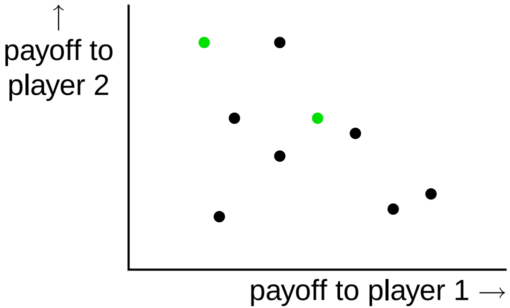
Digression: Pareto efficiency



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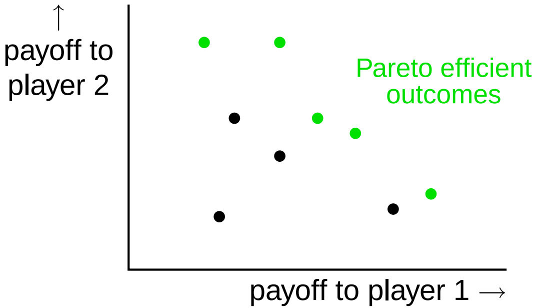
Digression: Pareto efficiency



- Which outcomes are Pareto efficient?
- An outcome is *Pareto efficient* if there is no outcome in which *every* player is better off

Strategic game: Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?

Strategic game: Prisoner’s Dilemma

		Firm 2	
		high	*low*
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		Firm 2	
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	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(\mathit{high}, \mathit{high})$

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Strategic game: Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$, $(high, low)$

Strategic game: Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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- Which outcomes (action pairs) are Pareto efficient?
- (*high*, *high*), (*high*, *low*)

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	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(\mathit{high}, \mathit{high})$, $(\mathit{high}, \mathit{low})$, $(\mathit{low}, \mathit{high})$

Strategic game: Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$, $(high, low)$, $(low, high)$

in these situations, deviations can only make one player better off, not both

Strategic game: More examples

In other examples, the pattern of incentives is different

	X	*Y*
X	2, 2	1, 1
Y	1, 1	0, 0

Strategic game: More examples

In other examples, the pattern of incentives is different

	X	*Y*
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Y	1, 1	0, 0

	X	*Y*
X	2, 2	0, 0
Y	0, 0	1, 1

Coordination game

Strategic game: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
Y	$1, 1$	$0, 0$

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Coordination game

	X	Y
X	$2, 1$	$0, 0$
Y	$0, 0$	$1, 2$

Bach or Stravinsky?

Strategic game: More examples

In other examples, the pattern of incentives is different

	X	*Y*
X	2, 2	1, 1
Y	1, 1	0, 0

	X	*Y*
X	2, 2	0, 0
Y	0, 0	1, 1

Coordination game

	X	*Y*
X	2, 1	0, 0
Y	0, 0	1, 2

Bach or Stravinsky?

	X	*Y*
X	1, −1	−1, 1
Y	−1, 1	1, −1

Matching pennies

A richer example

In examples so far, only two players, each with only two actions

A richer example

In examples so far, only two players, each with only two actions

Cournot’s oligopoly game

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: *n* firms

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: *n* firms
- For each firm
    - possible actions:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: *n* firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

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In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

Notes

- Many players, each with continuum of actions

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

Notes

- Many players, each with continuum of actions
- *Cannot* represent game in a table

Equilibrium outcomes

- We want to assume each player is rational

Equilibrium outcomes

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- But each player doesn’t know what others will do—so how to choose action?

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Equilibrium outcomes

- We want to assume each player is rational
- But each player doesn’t know what others will do—so how to choose action?
- Form *beliefs* about others’ actions
- Where do beliefs come from?
- Assume players have experience playing the game, or similar games—in fact, assume that their beliefs are *correct*

Equilibrium outcomes

- Each player’s action is optimal given her beliefs

Equilibrium outcomes

- Each player’s action is optimal given her beliefs
- Each player’s belief is correct

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Definition

A Nash equilibrium of a strategic game is an
action profile

Equilibrium outcomes

- Each player’s action is optimal given her beliefs
- Each player’s belief is correct

$\Rightarrow$ each player’s action is optimal given other players’ actions

Definition

A Nash equilibrium of a strategic game is an action profile

list of actions, one for each player

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal

Equilibrium outcomes

- Each player’s action is optimal given her beliefs
- Each player’s belief is correct

$\Rightarrow$ each player’s action is optimal given other players’ actions

Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player’s action is optimal, given the other players’ actions.

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

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Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal, given the other players' actions.

Equilibrium outcomes

- Each player’s action is optimal given her beliefs
- Each player’s belief is correct

$\Rightarrow$ each player’s action is optimal given other players’ actions

Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player’s action is optimal, given the other players’ actions.

Equivalently: an action profile is a Nash equilibrium if no player can increase her payoff by changing her action, given the other players’ actions other's action is fixed

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Check each action pair in turn

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*):

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*):

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		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .

Prisoner's Dilemma

		Firm 2	
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
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		Firm 2	
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*):

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn assume other player remain the same choice

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*): Nash equilibrium because each player is worse off switching to *high* if other player’s action is *low*.

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*): Nash equilibrium because each player is worse off switching to *high* if other player's action is *low*.

So: unique Nash equilibrium, (*low*, *low*).

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
  - (*low*, *high*), (*high*, *high*), (*high*, *low*)

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
  - (*low*, *high*), (*high*, *high*), (*high*, *low*)
- Note that the unique Nash equilibrium, (*low*, *low*), is *not* Pareto efficient

Prisoner’s Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

- Which outcomes (action pairs) are Pareto efficient?
  - $(low, high)$, $(high, high)$, $(high, low)$
- Note that the unique Nash equilibrium, (low, low), is *not* Pareto efficient

http://www.youtube.com/watch?v=p3Uos2fzIJ0

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria?

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(\mathit{Split}, \mathit{Steal})$, $(\mathit{Steal}, \mathit{Split})$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(\mathit{Split}, \mathit{Steal})$, $(\mathit{Steal}, \mathit{Split})$, and $(\mathit{Steal}, \mathit{Steal})$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
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Nash equilibria? $(\textit{Split}, \textit{Steal})$, $(\textit{Steal}, \textit{Split})$, and $(\textit{Steal}, \textit{Steal})$

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? (*Split*, *Steal*), (*Steal*, *Split*), and (*Steal*, *Steal*)

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Compare with *Prisoner's Dilemma*:

	X	Y
X	$2, 2$	$0, 3$
Y	$3, 0$	$1, 1$

Only difference between games:

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? (*Split*, *Steal*), (*Steal*, *Split*), and (*Steal*, *Steal*)

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
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Compare with *Prisoner's Dilemma*:

	X	Y
X	$2, 2$	$0, 3$
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Only difference between games: indicated preferences

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? (*Split*, *Steal*), (*Steal*, *Split*), and (*Steal*, *Steal*)

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Compare with *Prisoner's Dilemma*:

	X	*Y*
X	2, 2	0, 3
Y	3, 0	1, 1

Only difference between games: indicated preferences

Example of Nash equilibrium: Coordination game

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Example of Nash equilibrium: Coordination game

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Two Nash equilibria, (X, X) and (Y, Y)

Example of Nash equilibrium: *Bach or Stravinsky?*

	Bach	*Stravinsky*
Bach	$2, 1$	$0, 0$
Stravinsky	$0, 0$	$1, 2$

Example of Nash equilibrium: *Bach or Stravinsky?*

compare only the same column for player 1

compare only the same row for player 2

	Bach	*Stravinsky*
Bach	$2, 1$	$0, 0$
Stravinsky	$0, 0$	$1, 2$

Two Nash equilibria, (B, B) and (S, S)

Example of Nash equilibrium: *Matching Pennies*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Example of Nash equilibrium: *Matching Pennies*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

No Nash equilibrium!

Example of Nash equilibrium

$$\begin{array}{c|c|c|} & L & R \\ \hline T & 1, 1 & 2, 1 \\ \hline B & 0, 0 & 2, 4 \\ \hline \end{array}$$

Example of Nash equilibrium

	L	R
T	$1, 1$	$2, 1$
B	$0, 0$	$2, 4$

Nash equilibria: (T, L), (T, R), and (B, R)