

Week 3: Risk and Evaluation of Alternatives

- ◆ Making Decisions in Low-Uncertainty vs. High-Uncertainty Settings
- ◆ Example: Evaluating a Wireless Data Plan
- ◆ Reward and Risk
- ◆ Connecting Random Inputs and Random Outputs
- ◆ Simulating Uncertain Outcomes in Excel
- ◆ Interpreting Simulation Results: “Short” vs. “Long” Simulations
- ◆ Using Histograms to Visualize Simulation Results

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ See DataPlan10.xlsx

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ We are interested in analyzing the distribution of the **monthly payment**

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ But first, let us look at the simulated values of **monthly data usage**

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ Why? Because we know its true probability distribution...

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ ... So we can compare the sample mean and standard deviation with the true values

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)
1	11.9319952
2	24.0282690
3	25.6828047
4	21.7321587
5	34.2335329
6	16.5820597
7	30.7079676
8	36.9010808
9	20.3471859
10	28.3229996
Sample Mean	25.0470054
Sample St. Dev.	7.787935101

- ◆ ... So we can compare the sample mean and standard deviation with the true values

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)
1	11.9319952
2	24.0282690
3	25.6828047
4	21.7321587
5	34.2335329
6	16.5820597
7	30.7079676
8	36.9010808
9	20.3471859
10	28.3229996
Sample Mean	25.0470054
Sample St. Dev.	7.787935101

In this simulation, **10** values for monthly data usage “drawn” from the normal distribution with mean of 23 and standard deviation of 5 averaged to about **25.047**.

- ◆ Sample mean is an approximation to the true value of the expected data usage

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)
1	11.9319952
2	24.0282690
3	25.6828047
4	21.7321587
5	34.2335329
6	16.5820597
7	30.7079676
8	36.9010808
9	20.3471859
10	28.3229996
Sample Mean	25.0470054
Sample St. Dev.	7.787935101

In this simulation, **10** values for monthly data usage “drawn” from the normal distribution with mean of 23 and standard deviation of 5 produced a sample standard deviation of about **7.788**.

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- The sample mean of the simulated values for monthly payment is about \$253, and the sample standard deviation is about \$92

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ The more simulation runs we conduct...

Simulated Data Usage Values and Corresponding Monthly Payment Values ($n=10$ simulation runs, seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

- ◆ ... the closer the sample mean and standard deviation will be to the true values

Simulated Data Usage Values and Corresponding Monthly Payment Values (**n=1000** simulation runs, seed = 123)

	A	B	C	D	E	F
1	DataPlan_0.xlsx	Wireless Data Plan	Simulation Run	Data Usage, U (GB)	Payment, P (\$)	
2	Operations Analytics MOOC		1	11.93199518	160	
3			2	24.02826903	220.4240354	
4	Data Allowance (GB)	20	3	25.68280473	245.242071	
5	Fixed Payment (\$)	160	4	21.7321587	185.9823805	
6	Rate Above Allowance (\$/GB)	15	5	34.23353286	373.5029929	
7			6	16.58205969	160	
8	Expected Data Usage (GB)	23	7	30.7079676	320.619514	
9	St. Dev. of Data Usage (GB)	5	8	36.90108082	413.5162123	
10			9	20.34718585	165.2077878	
1001			1000	23.1895728	207.843592	
1002						
1003			Sample Mean	23.28418394	220.1594691	
1004			Sample St. Dev.	4.877547328	58.23620041	

◆ DataPlan1000.xlsx

Comparing Results for $n=10$ and $n=1000$ simulation runs (seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.93199518	160
2	24.02826903	220.4240354
3	25.68280473	245.242071
4	21.7321587	185.9823805
5	34.23353286	373.5029929
6	16.58205969	160
7	30.7079676	320.619514
8	36.90108082	413.5162123
9	20.34718585	165.2077878
1000	23.1895728	207.843592
Sample Mean	23.28418394	220.1594691
Sample St. Dev.	4.877547328	58.23620041

- ◆ See DataPlan10.xlsx and DataPlan1000.xlsx

Comparing Results for $n=10$ and $n=1000$ simulation runs (seed = 123)

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.93199518	160
2	24.02826903	220.4240354
3	25.68280473	245.242071
4	21.7321587	185.9823805
5	34.23353286	373.5029929
6	16.58205969	160
7	30.7079676	320.619514
8	36.90108082	413.5162123
9	20.34718585	165.2077878
1000	23.1895728	207.843592
Sample Mean	23.28418394	220.1594691
Sample St. Dev.	4.877547328	58.23620041

- ◆ The sample mean and sample standard deviation for **monthly data usage** simulated for $n=1000$ runs (approximately, 23.2842 and 4.8775) are much closer to the true values of 23 and 5 than the corresponding sample mean and standard deviation for $n=10$ simulation runs

Comparing Results for $n=10$ and $n=1000$ simulation runs (seed = 123)

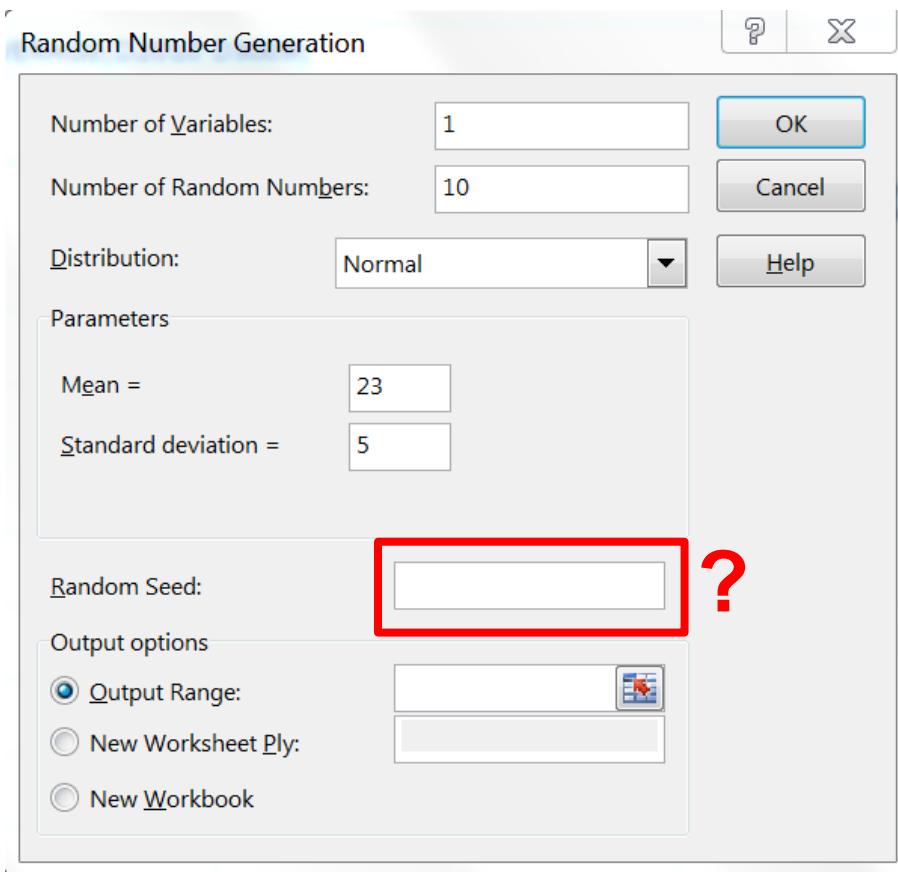
Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.9319952	160
2	24.0282690	220.4240354
3	25.6828047	245.242071
4	21.7321587	185.9823805
5	34.2335329	373.5029929
6	16.5820597	160
7	30.7079676	320.619514
8	36.9010808	413.5162123
9	20.3471859	165.2077878
10	28.3229996	284.8449946
Sample Mean	25.0470054	252.9339988
Sample St. Dev.	7.787935101	92.19007977

Simulation Run	Data Usage, U (GB)	Payment, P (\$)
1	11.93199518	160
2	24.02826903	220.4240354
3	25.68280473	245.242071
4	21.7321587	185.9823805
5	34.23353286	373.5029929
6	16.58205969	160
7	30.7079676	320.619514
8	36.90108082	413.5162123
9	20.34718585	165.2077878
1000	23.1895728	207.843592
Sample Mean	23.28418394	220.1594691
Sample St. Dev.	4.877547328	58.23620041

- ◆ In a similar way, the sample mean and sample standard deviation for **monthly payment** simulated for $n=1000$ runs (approximately, \$220.1995 and \$58.2362) are much closer to the true (unknown to us) values than the corresponding sample mean and standard deviation for $n=10$ simulation runs

- ◆ Longer simulations produce more precise estimates for the reward and risk measures

Random Seed Value?



- ◆ So, what **random seed value** should one use when running a simulation?

Comparing Results for $n=10$ and $n=1000$ simulation runs for different seed values

- ◆ Simulated data usage values

$n=10$	seed = 123	seed = 1826	seed = 19104
Sample Mean, GB	25.05	19.48	24.72
Sample St. Dev., GB	7.79	5.21	3.20

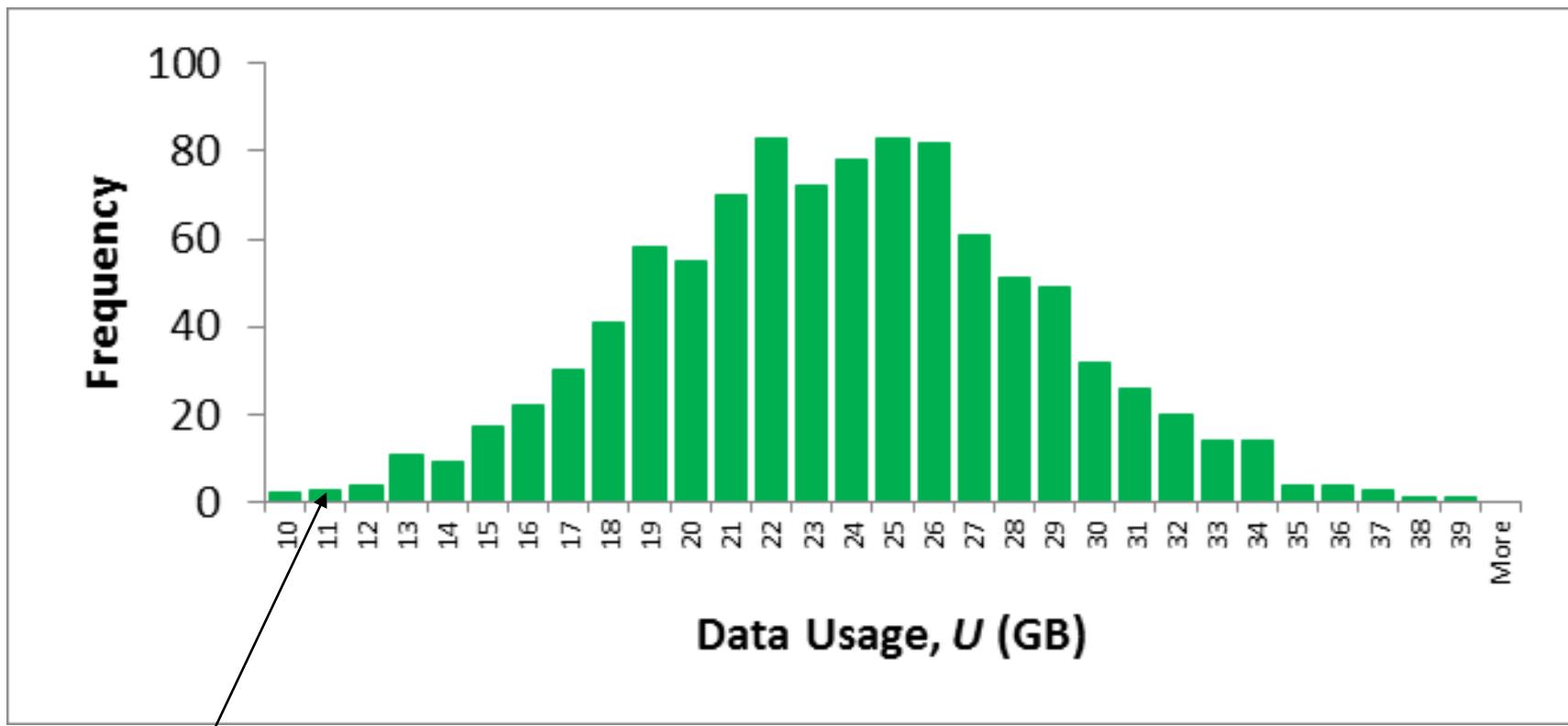
$n=1000$	seed = 123	seed = 1826	seed = 19104
Sample Mean, GB	23.28	23.08	23.04
Sample St. Dev., GB	4.88	4.90	4.96

- ◆ Random seed value does not matter much when you run a simulation with large number of simulation runs

Visualizing Simulation Results Using Histograms

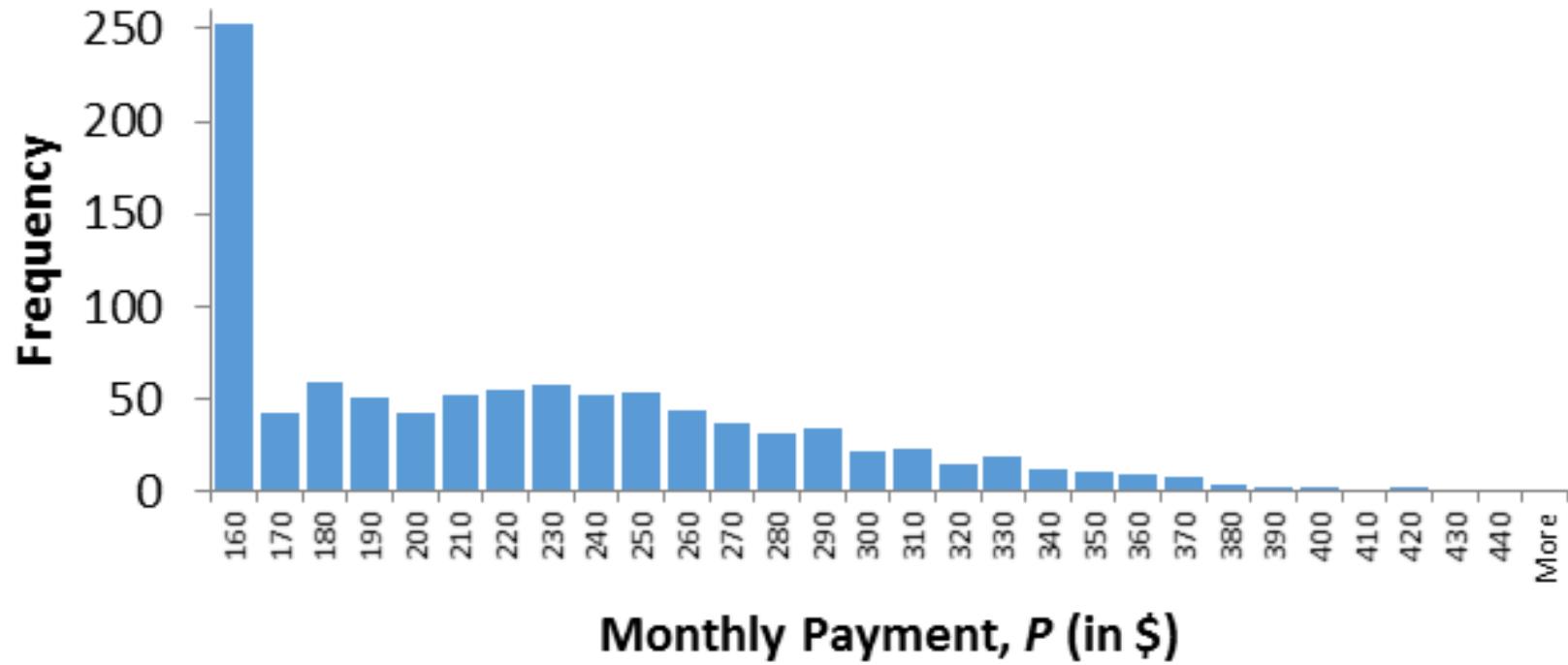
- ◆ Histograms are often useful for gaining intuition about the random inputs and the random outputs involved in a simulation
- ◆ In the data plan example, the random input is the data usage U , and the random output is the monthly payment P

Histogram of Simulated Values of Data Usage U ($n=1000$, seed = 123)



- ◆ See DataPlan1000_Histogram.xlsx
- ◆ This “bar” indicates the frequency (number of occurrences) for the values of $10 < U \leq 11$

Histogram of Simulated Values of Monthly Payment P ($n=1000$, seed = 123)



- ◆ See DataPlan1000_Histogram.xlsx

Histogram of Simulated Values of Monthly Payment P ($n=1000$, seed = 123)



- ◆ The input (values of U) was drawn from a normal distribution – but the output looks nothing like a normal distribution

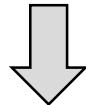
Histogram of Simulated Values of Monthly Payment P ($n=1000$, seed = 123)



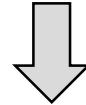
- ◆ In general, one must use simulation to understand the shape of the distribution and its parameters for an output random variable

Making Best Decisions in High-Uncertainty Settings: A Roadmap

Decide upon **reward** and **risk** measures



For each competing decision, use **simulation** to estimate reward and risk measures



Use **reward** as an **objective** and **risk measures** as **constraints** to find the best decision

Data Plan Example:

- Reward measure = expected monthly payment
- Risk measure = standard deviation of monthly payment

Data Plan Example:

- Estimate of expected monthly payment = \$220.1995
- Estimate of standard deviation of monthly payment = \$58.2362

Simulation in Practice: Commercial Simulation Packages

- ◆ If you are interested in going beyond Excel's Analysis ToolPak add-in, there exists a number of commercial simulation packages
- ◆ A recent comparison of simulation software packages by the *OR/MS Today* can be accessed here: <http://www.orms-today.org/surveys/Simulation/Simulation.html>
- ◆ Some commercial packages, in addition to simulating uncertain outcomes, provide capabilities to fit data to a wide range of probability distributions

Simulation in Practice

- ◆ Wireless Data Plan example uses one random input (monthly data usage), and analyzes one random output (monthly payment)
- ◆ In practice, simulation can be used in the models with many random inputs and many random outputs
- ◆ Examples of the use of simulation: articles published in *Interfaces*



[Optimizing Capital Investment Decisions at Intel Corporation](#)

Karl G. Kempf, Feryal Erhun, Erik F. Hertzler, Timothy R. Rosenberg,
and Chen Peng
Interfaces 2013, 43:1, 62-78.



[Kroger Uses Simulation-Optimization to Improve Pharmacy Inventory Management](#)

Xinhui Zhang, Doug Meiser, Yan Liu, Brett Bonner, Lebin Lin
Interfaces 2014, 44:1, 70-84.

- ◆ As is the case with optimization, simulation in practice is often used hand-in-hand with other analytics techniques