Week 2 Review: Making Best Decisions in Settings with Low Uncertainty

- Resource Allocation Example: Natural Disaster Housing Relief

- Network Optimization Example: Colombi Parquet

Natural Disaster Housing Relief: Context

- A UN agency that manages relief efforts is organizing medium-term housing for survivors in a country that recently suffered from a natural disaster. The agency is considering building four types of housing units: A, B, C, and D

- Each of four types of housing uses the same basic set of building materials (bricks, corrugated iron sheets, and wooden poles), as well as labor, in the following amounts:

Unit Type	Bricks	Iron Sheets	Wooden Poles	Labor Hours
A	400	20	35	300
B	600	10	32	200
C	400	25	26	200
D	300	18	30	400

- Once built, housing units will provide shelter for 7, 6, 5 and 7 people for each unit of A, B, C, and D, respectively

Natural Disaster Housing Relief: Context

- The availability of building materials and labor is limited by the following amounts:

Resource	Available
Bricks	1500000
Iron Sheets	100000
Wooden Poles	125000
Labor Hours	1500000

- The agency would like to decide **how many units of each type of housing to build** to **maximize the number of people receiving shelter** without exceeding the **resource availabilities**

Natural Disaster Housing Relief: Decision Variables

- How many units of each type to build?

N_A = number of type A units to build,
N_B = number of type B units to build,
N_C = number of type C units to build,
N_D = number of type D units to build.

Natural Disaster Housing Relief: Objective Function

- Maximize the number of people receiving shelter:

  - Each type A unit provides shelter for 7 people. N_A type A units provide shelter for $7*N_A$ people

  - Each type B unit provides shelter for 6 people. N_B type B units provide shelter for $6*N_B$ people

  - Each type C unit provides shelter for 5 people. N_C type C units provide shelter for $5*N_C$ people

  - Each type D unit provides shelter for 7 people. N_D type D units provide shelter for $7*N_D$ people

- The total number of people receiving shelter: $7*N_A + 6*N_B + 5*N_C + 7*N_D$

Natural Disaster Housing Relief: Constraints

- Resource Availabilities
  - Number of required bricks cannot exceed brick availability

N_A type A units require $400*N_A$ bricks

N_B type B units require $600*N_B$ bricks

N_C type C units require $400*N_C$ bricks

N_D type D units require $300*N_D$ bricks

$$400*N_A + 600*N_B + 400*N_C + 300*N_D \leq 1500000$$

  - Number of required iron sheets cannot exceed iron sheet availability

N_A type A units require $20*N_A$ iron sheets

N_B type B units require $10*N_B$ iron sheets

N_C type C units require $25*N_C$ iron sheets

N_D type D units require $18*N_D$ iron sheets

$$20*N_A + 10*N_B + 25*N_C + 18*N_D \leq 100000$$

Natural Disaster Housing Relief: Constraints

- Resource Availabilities
  - Number of required wooden poles cannot exceed wooden pole availability.

N_A type A units require $35*N_A$ wooden poles.

N_B type B units require $32*N_B$ wooden poles.

N_C type C units require $26*N_C$ wooden poles.

N_D type D units require $30*N_D$ wooden poles.

$$35*N_A + 32*N_B + 26*N_C + 30*N_D \leq 125000$$

  - Number of required labor hours cannot exceed labor hours availability

N_A type A units require $300*N_A$ labor hours

N_B type B units require $200*N_B$ labor hours

N_C type C units require $200*N_C$ labor hours

N_D type D units require $400*N_D$ labor hours

$$300*N_A +200*N_B + 200*N_C + 400*N_D \leq 1500000$$

Natural Disaster Housing Relief: Constraints

- Resource Availabilities

$400*N_A + 600*N_B + 400*N_C + 300*N_D \leq 1500000$ (bricks)

$20*N_A + 10*N_B + 25*N_C + 18*N_D \leq 100000$ (iron sheets)

$35*N_A + 32*N_B + 26*N_C + 30*N_D \leq 125000$ (wooden poles)

$300*N_A + 200*N_B + 200*N_C + 400*N_D \leq 1500000$ (labor hours)

Natural Disaster Housing Relief: Constraints

- All decision variables must be integer

N_A, N_B, N_C, N_D = integer

- All decision variables must be non-negative

$N_A, N_B, N_C, N_D \geq 0$

Natural Disaster Housing Relief: Complete Model

Maximize $7*N_A + 6*N_B + 5*N_C + 7*N_D$

Subject to

$$400*N_A + 600*N_B + 400*N_C + 300*N_D \leq 1500000$$

$$20*N_A + 10*N_B + 25*N_C + 18*N_D \leq 100000$$

$$35*N_A + 32*N_B + 26*N_C + 30*N_D \leq 125000$$

$$300*N_A + 200*N_B + 200*N_C + 400*N_D \leq 1500000$$

$$N_A, N_B, N_C, N_D = \text{integer}$$

$$N_A, N_B, N_C, N_D \geq 0$$

Natural Disaster Housing Relief: Excel Implementation

	A	B	C	D	E	F	G	H	I
1	Housing Relief.xlsx								
2	Operations Analytics MOOC								
3									
4			Type A	Type B	Type C	Type D		Total People Sheltered	
5	Number To Build		0	734	2	3382		28088	
6	Number of People Sheltered		7	6	5	7			
7									
8			Resource Requirements						
9			A	B	C	D	Total Used		Total Available
10	Bricks		400	600	400	300	1455800	<=	1500000
11	Iron Sheets		20	10	25	18	68266	<=	100000
12	Wooden Poles		35	32	26	30	125000	<=	125000
13	Labor Hours		300	200	200	400	1500000	<=	1500000

=SUMPRODUCT(C5:F5,C6:F6)

=SUMPRODUCT($C$5:$F$5,C10:F10)

- According to the Solver, the optimal solution is to build 734 units of type B, 2 units of type C, and 3382 units of type D
- The total number of sheltered people corresponding to this decision is 28088

Colombi Parquet: Context

- Colombi Parquet (CP) is a German company that specializes in manufacturing high-quality hardwood floors
- CP operates two plants: one located in Saxony-Anhalt (plant A) and another one located in Baden-Württemberg (plant B)
- The capacities (in thousands of square feet) of these two plants for the next quarter are shown below

Plant	Capacity
A	2500
B	3000

Colombi Parquet: Context

- CP’s hardwood floors are distributed within three marketing regions in Northern and Central Europe

- The minimum demand requirements (in thousands of square feet) for each marketing region for the next quarter are shown below:

Region 1	Region 2	Region 3
2000	930	2200

Colombi Parquet: Context

- The shipping costs (in € per thousand square feet) from the plants to the marketing regions are shown below:

From	to Region 1	to Region 2	to Region 3
Plant A	15	21	17
Plant B	23.5	25.5	22

- The company wants to determine the **quantities of hardwood floors it should ship from each plant to each marketing region** in order to **minimize the total transportation cost** for the next quarter while satisfying **the demand requirements at each marketing region** and **the capacity limitations at each plant**

Colombi Parquet: Decision Variables

- How many square feet of hardwood floors should it ship from each plant to each marketing region?

X_{A1} = 1000’s of square feet of hardwood floors to ship from plant A to region 1,
X_{B1} = 1000’s of square feet of hardwood floors to ship from plant B to region 1,
X_{A2} = 1000’s of square feet of hardwood floors to ship from plant A to region 2,
X_{B2} = 1000’s of square feet of hardwood floors to ship from plant B to region 2,
X_{A3} = 1000’s of square feet of hardwood floors to ship from plant A to region 3,
X_{B3} = 1000’s of square feet of hardwood floors to ship from plant B to region 3

Colombi Parquet: Objective Function

- Minimize the total transportation cost
  - Total Transportation Cost:

$$15*X_{A1} + 23.5*X_{B1} + 21*X_{A2} + 25.5*X_{B2} + 17*X_{A3} + 22*X_{B3}$$

Colombi Parquet: Constraints

- Supply Constraints:

  – The amount shipped from plant A cannot exceed plant A's capacity

    $X_{A1} + X_{A2} + X_{A3} \leq 2500$

  – The amount shipped from plant B cannot exceed plant B's capacity

    $X_{B1} + X_{B2} + X_{B3} \leq 3000$

Colombi Parquet: Constraints

- Demand Constraints:
  - The amount shipped to region 1 cannot be less than 2000
    $X_{A1} + X_{B1} \geq 2000$
  - The amount shipped to region 2 cannot be less than 930
    $X_{A2} + X_{B2} \geq 930$
  - The amount shipped to region 3 cannot be less than 2200
    $X_{A3} + X_{B3} \geq 2200$

Colombi Parquet: Constraints

- Shipping amounts cannot be negative:

$$X_{A1}, X_{B1}, X_{A2}, X_{B2}, X_{A3}, X_{B3} \geq 0$$

Colombi Parquet: Complete Model

Minimize $15*X_{A1} + 23.5*X_{B1} + 21*X_{A2} + 25.5*X_{B2} + 17*X_{A3} + 22*X_{B3}$

Subject to:

$X_{A1} + X_{A2} + X_{A3} \leq 2500$

$X_{B1} + X_{B2} + X_{B3} \leq 3000$

$X_{A1} + X_{B1} \geq 2000$

$X_{A2} + X_{B2} \geq 930$

$X_{A3} + X_{B3} \geq 2200$

$X_{A1}, X_{B1}, X_{A2}, X_{B2}, X_{A3}, X_{B3} \geq 0$

Colombi Parquet: Excel Implementation

	A	B	C	D	E	F	G
1	Colombi.xlsx		Colombi Parquet Company				
2	Operations Analytics MOOC					Shipping Cost (in €)	
3						99615.00	
4	Transportation Cost						
5	(€/1000 square feet)	to Region 1	Region 2	Region 3	=SUMPRODUCT(B11:D12,B6:D7)		
6	from Plant A	15.00	21.00	17.00			
7	Plant B	23.50	25.50	22.00			
8					=SUM(B11:D11)		
9	Shipping Quantity	to	to	to			
10	(in 1000 square feet)	Region 1	Region 2	Region 3	Shipped		Capacity
11	from Plant A	2,000.00	0.00	500.00	2,500	<=	2,500
12	Plant B	0.00	930.00	1,700.00	2,630	<=	3,000
13	Total shipment	2,000	930	2,200			
14	=SUM(B11:B12)	>=	>=	>=			
15	Demand	2,000	930	2,200			

- According to the Solver, the optimal solution is to
  - produce 2500 thousand square feet in Plant A and split this production between regions 1 and 3
  - produce 2630 thousand square feet in Plant B and split this production between regions 2 and 3
- The optimal cost is 99615€