

Week 2: Making Best Decisions in Settings with Low Uncertainty

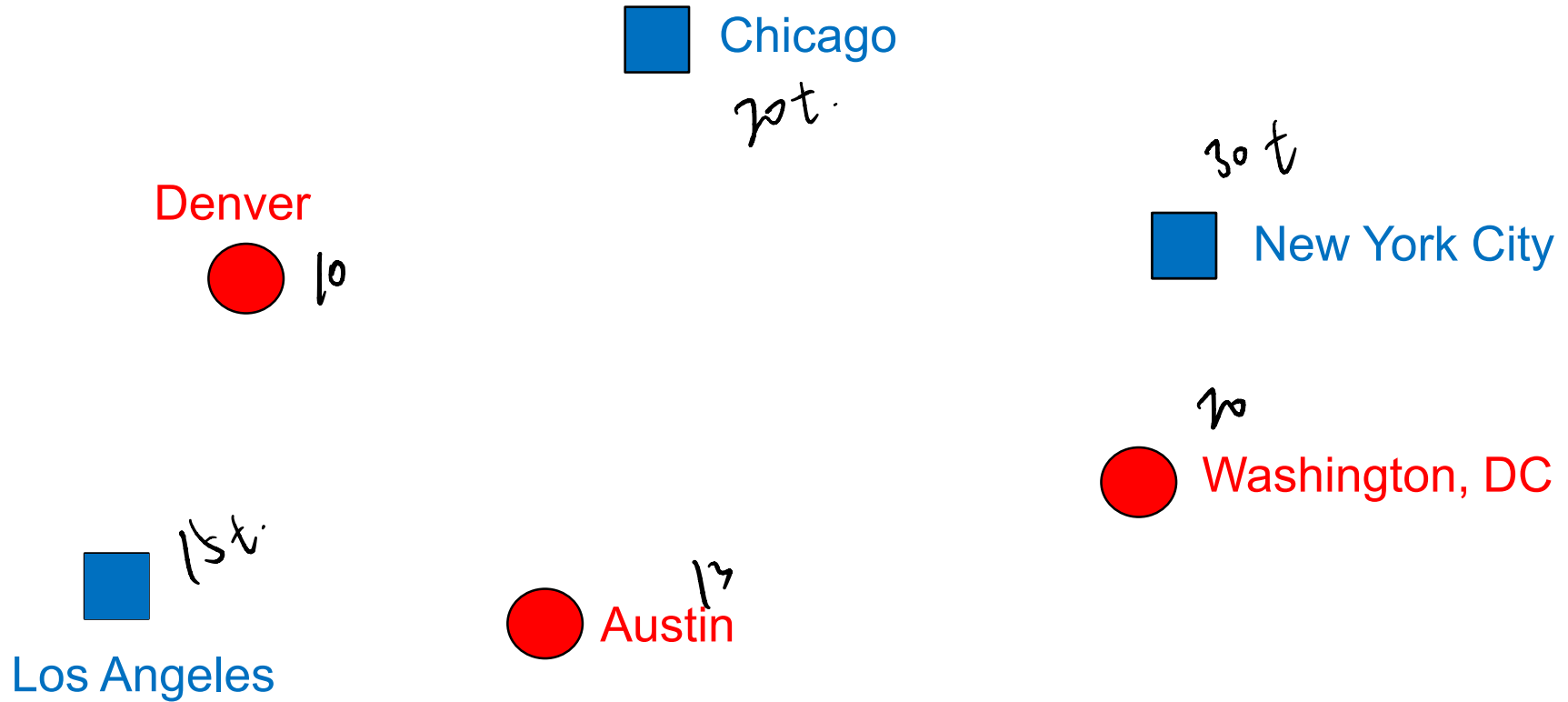
resource allocation
network optimization

- ◆ A resource allocation example: Zooter Industries
- ◆ Converting a verbal problem description into an algebraic model: decisions, objective, constraints
- ◆ From an algebraic model to a spreadsheet implementation: optimizing with Excel Solver
- ◆ Matching demand and supply across space: Keystone Dry Goods Logistics

Matching Demand and Supply Across a Network: Keystone Dry Goods Logistics

- ◆ The Keystone Dry Goods Logistics Company (KDGL) provides transportation and logistics services to a range of retail companies
- ◆ For the next month, KDGL needs to ensure on-time transportation of powdered drink shipments for one of its clients
- ◆ KDGL's client owns three warehouses, located in Los Angeles (L), Chicago (C), and New York City (N), and it uses three regional distribution centers located in Denver (D), Austin (A), and Washington, DC (W).

KDGL: Warehouses and Distribution Centers



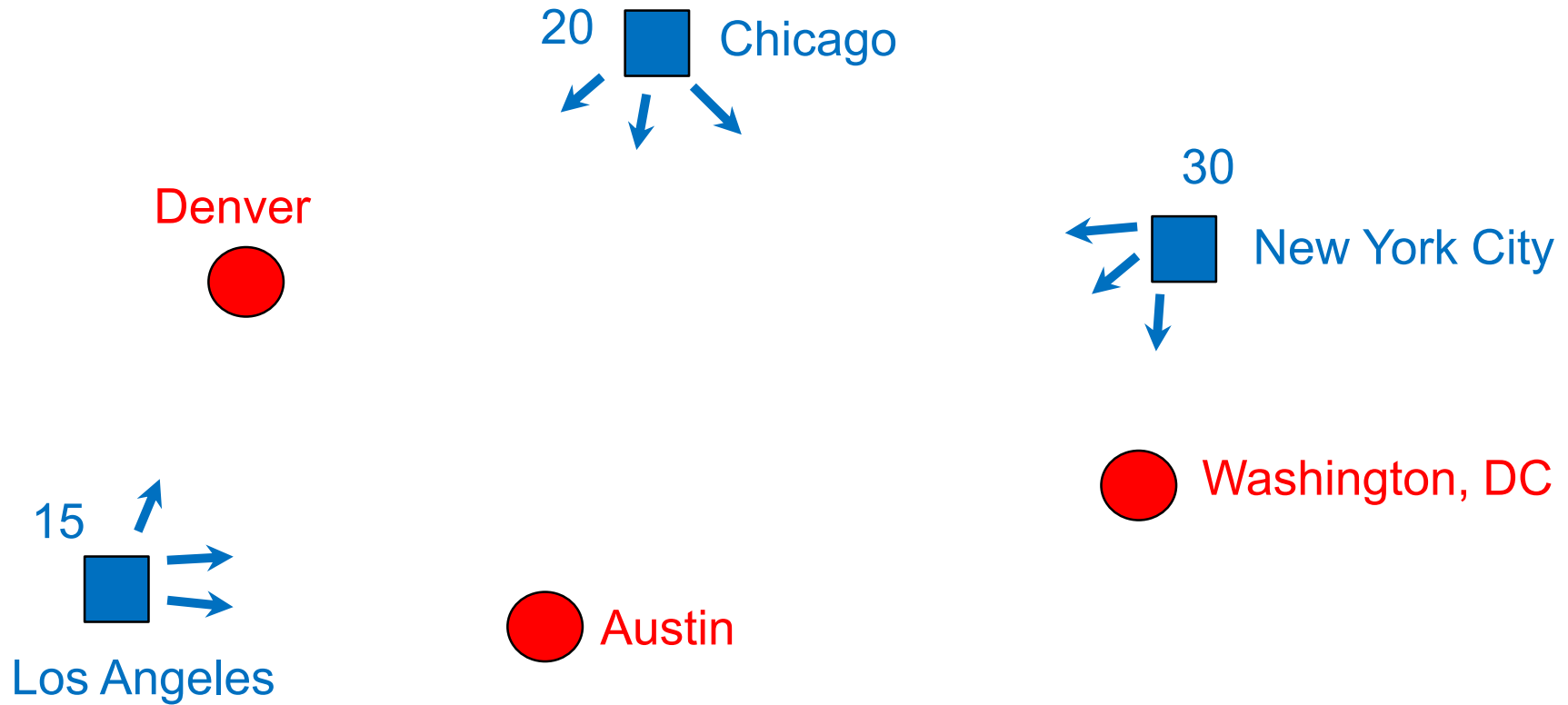
Amounts to Be Shipped Out of Warehouses

- ◆ The warehouses have a total supply of 65 tons of powdered drink that must be transported to the distribution centers next month to make room for new warehouse inventory.

Warehouse	To Be Shipped Out (tons)
Los Angeles (L)	15
Chicago (C)	20
New York City (N)	30

- ◆ For example, exactly 20 tons of powdered drink must be transported out of the Chicago warehouse

KDGL: Warehouses and Distribution Centers



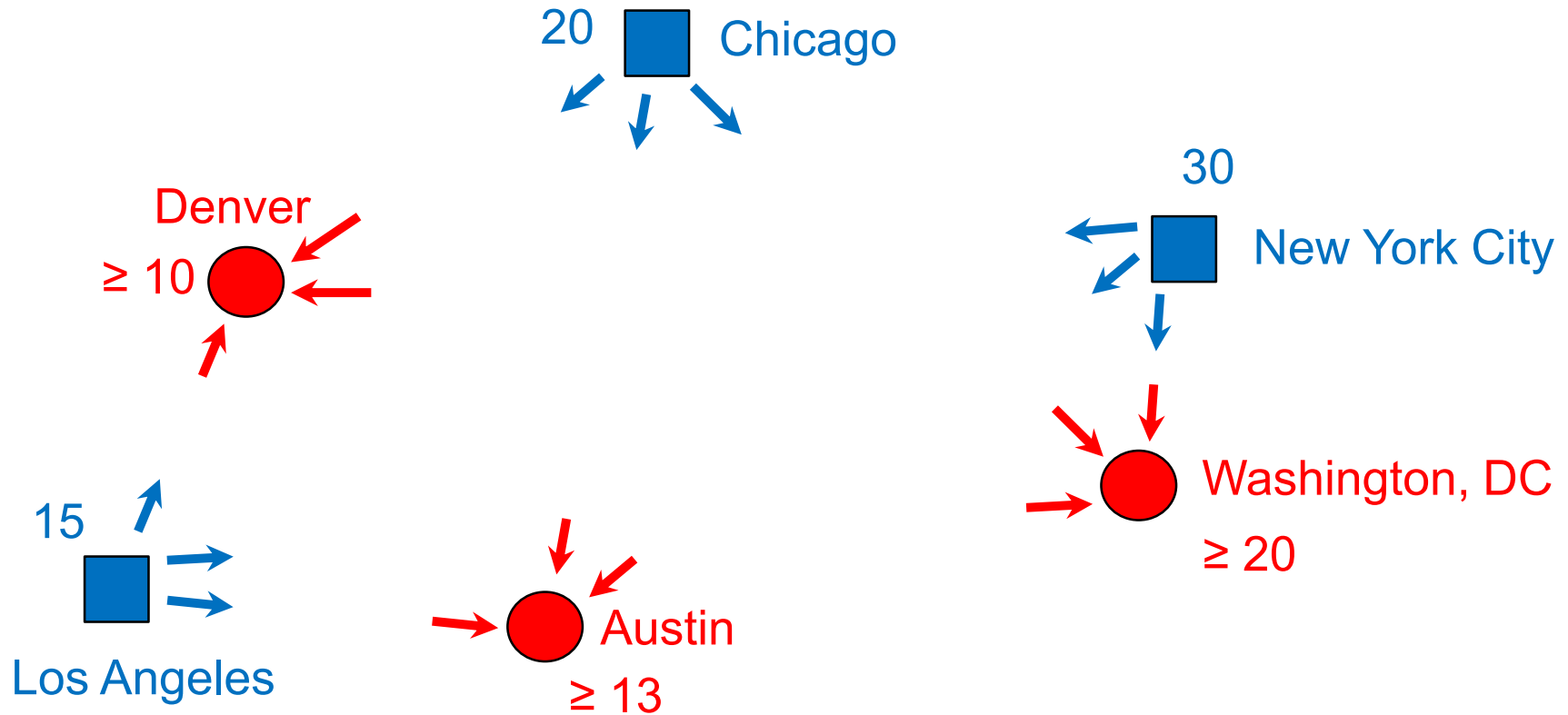
Minimum Amounts to Be Shipped to Distribution Centers

- ◆ There are minimum quantities that must be shipped to each of the distribution centers.
- ◆ KDGL may ship more than these quantities to ensure that all 65 tons of powdered drink are removed from the warehouses, but it cannot ship less

Distribution Center	Minimum To Be Shipped In (tons)
Denver	10
Austin	13
Washington, DC	20

- ◆ For example, at least 10 tons of powdered drink must be shipped to the Denver distribution center

KDGL: Warehouses and Distribution Centers



Shipping Costs

- ◆ The transportation costs (in \$ per ton) to ship powdered drink between each warehouse and each distribution center:

From / To	Denver (D)	Austin (A)	Washington, DC (W)
Los Angeles (L)	\$105.00	\$135.00	\$153.00
Chicago (C)	\$110.00	\$140.00	\$137.00
New York City (N)	\$130.00	\$132.00	\$115.00

- ◆ For example, if KDGL decides to ship 10 tons from the New York City warehouse to the Austin distribution center, it will incur a shipping cost of $\$132 \times 10 = \1320

Task Formulation

- ◆ KDGL's management needs to determine the amounts of cargo to transport from each warehouse to each distribution center to minimize the total shipping cost while satisfying the warehouse supply and distribution center demand constraints

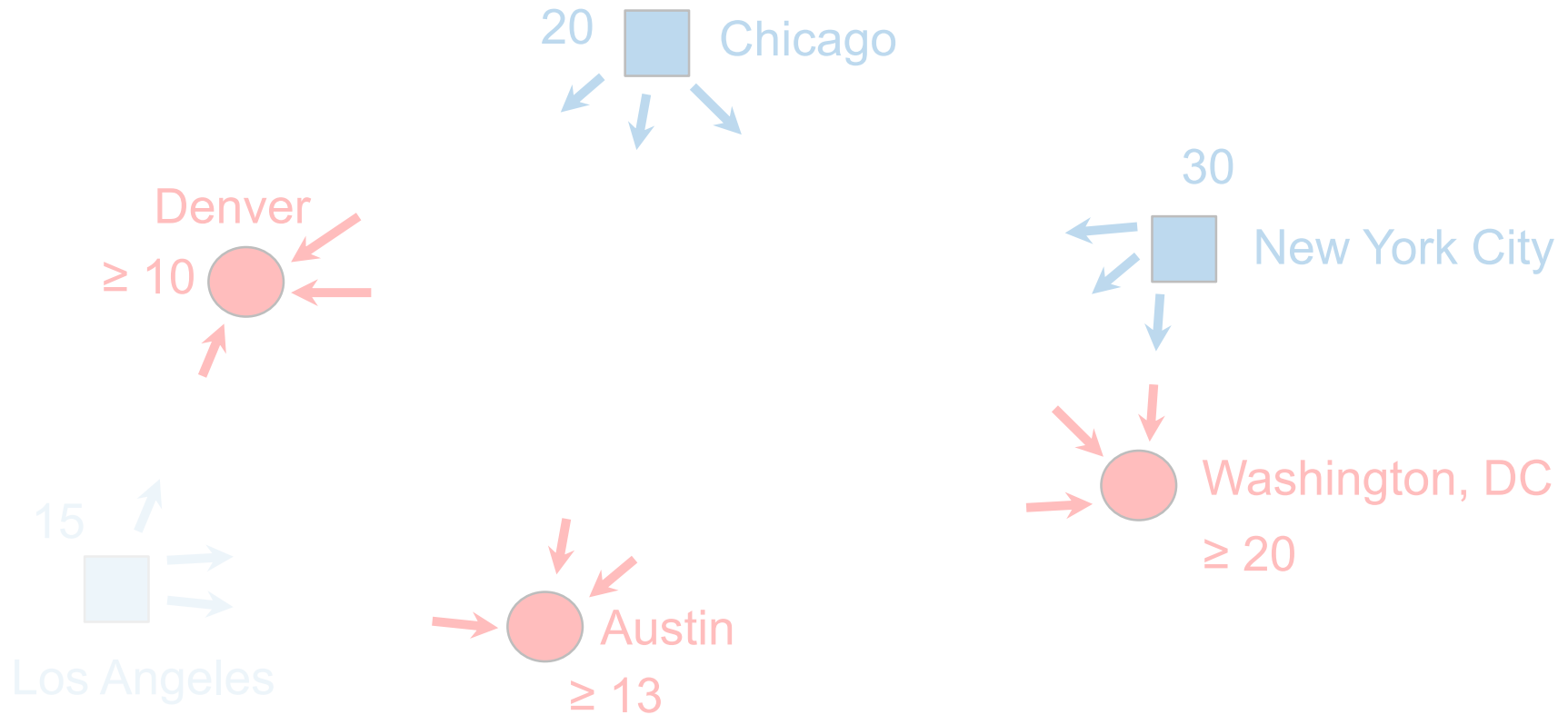
Verbal Formulation of KDGL Problem

- ◆ What does KDGL's management needs to determine?
 - Amounts of cargo to transport from each warehouse to each distribution center

- ◆ What does it want to achieve?
 - Lowest possible total shipping cost

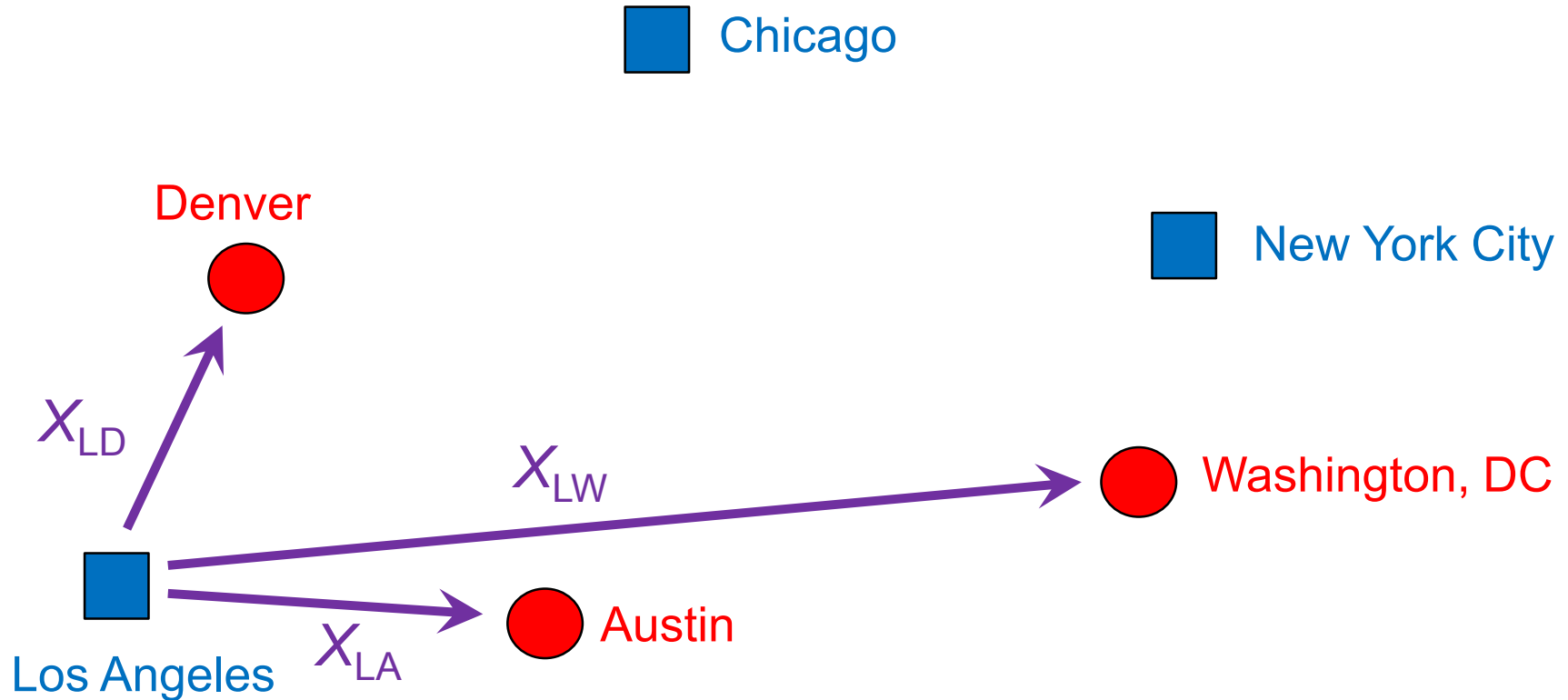
- ◆ What business requirements must it satisfy?
 - It has to satisfy warehouse supply constraints and distribution center minimum demand constraints

KDGL: Decision Variables



- ◆ KDGL's management needs to determine the amounts of cargo to transport

KDGL: Decision Variables

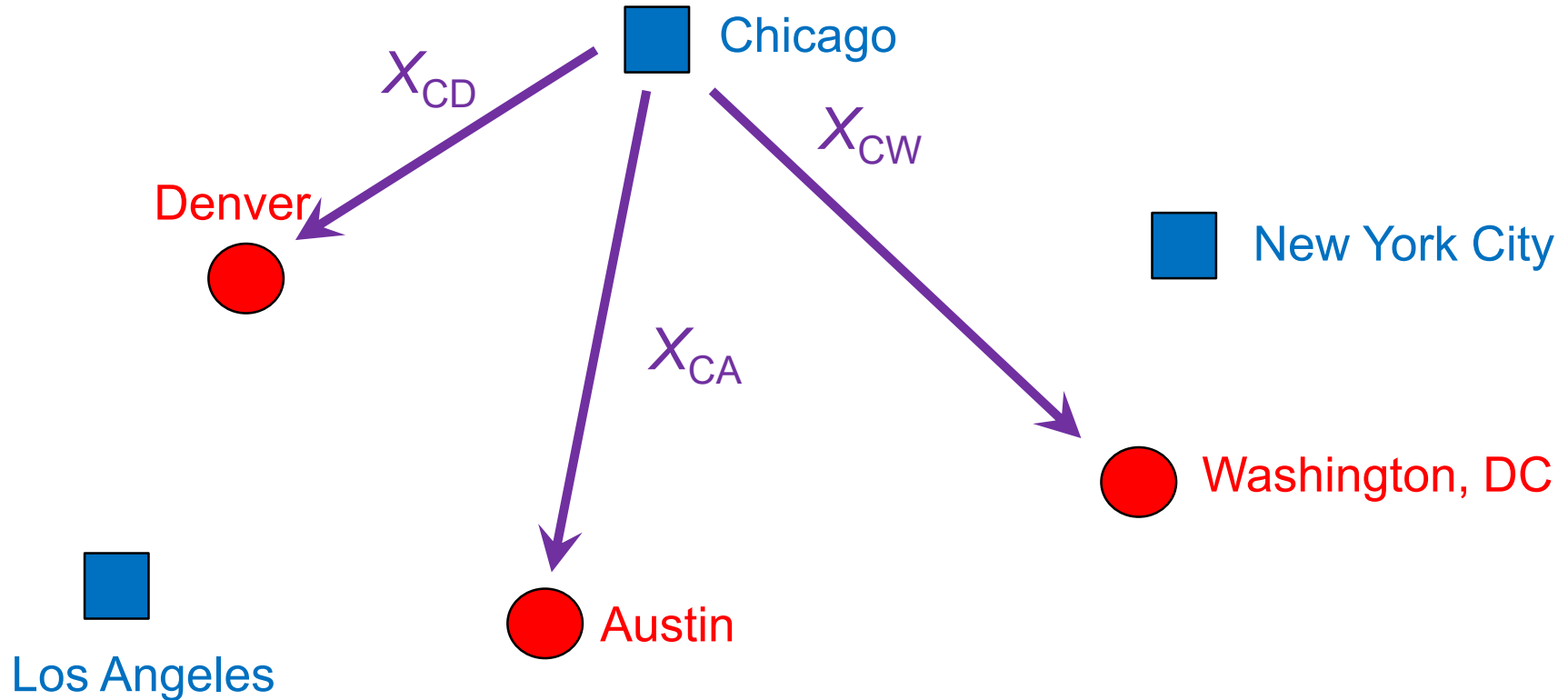


X_{LD} = number of tons shipped from Los Angeles to Denver

X_{LA} = number of tons shipped from Los Angeles to Austin

X_{LW} = number of tons shipped from Los Angeles to Washington, DC

KDGL: Decision Variables

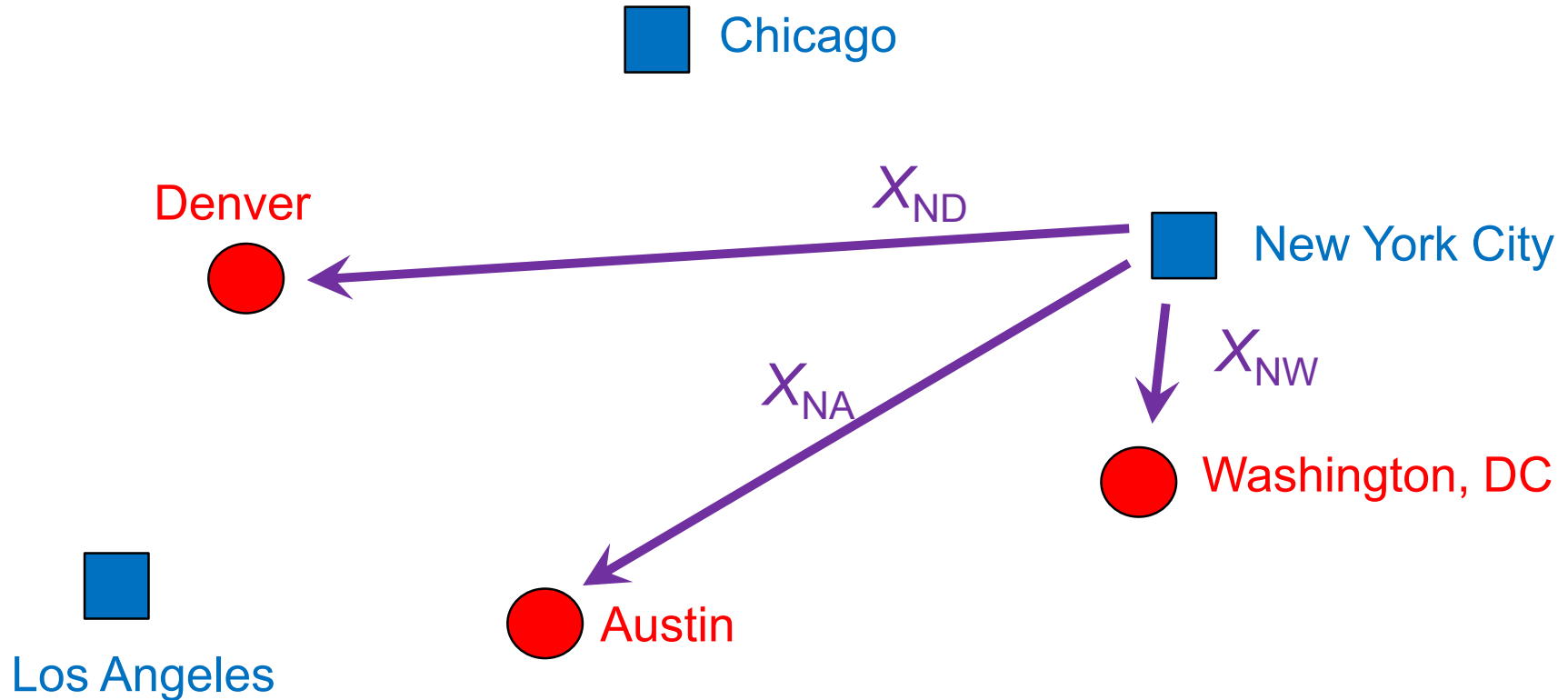


X_{CD} = number of tons shipped from Chicago to Denver

X_{CA} = number of tons shipped from Chicago to Austin

X_{CW} = number of tons shipped from Chicago to Washington, DC

KDGL: Decision Variables



X_{ND} = number of tons shipped from New York City to Denver

X_{NA} = number of tons shipped from New York City to Austin

X_{NW} = number of tons shipped from New York City to Washington, DC

KDGL: 9 Decision Variables

number of tons.

X_{LD} = number of tons shipped from Los Angeles to Denver

X_{LA} = number of tons shipped from Los Angeles to Austin

X_{LW} = number of tons shipped from Los Angeles to Washington, DC

X_{CD} = number of tons shipped from Chicago to Denver

X_{CA} = number of tons shipped from Chicago to Austin

X_{CW} = number of tons shipped from Chicago to Washington, DC

X_{ND} = number of tons shipped from New York City to Denver

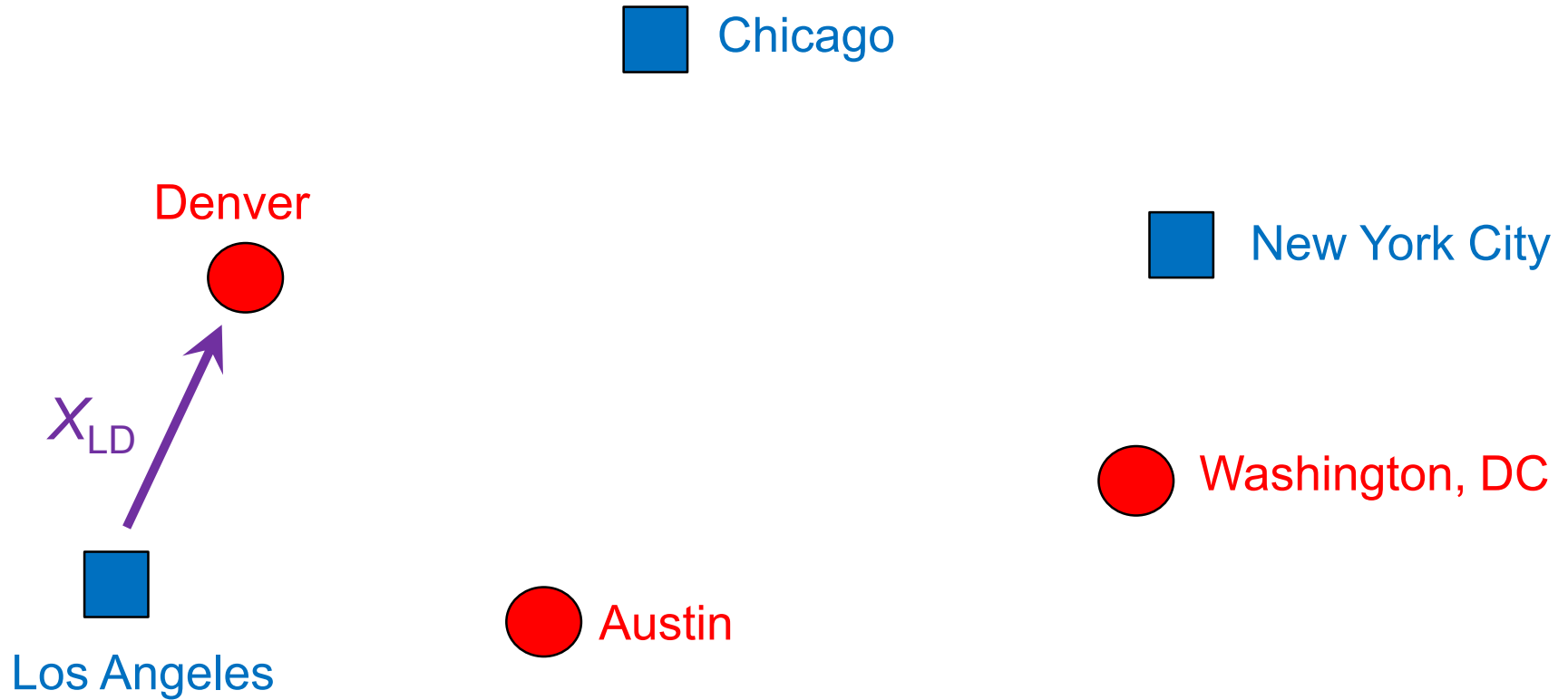
X_{NA} = number of tons shipped from New York City to Austin

X_{NW} = number of tons shipped from New York City to Washington, DC

KDGL: Objective Function

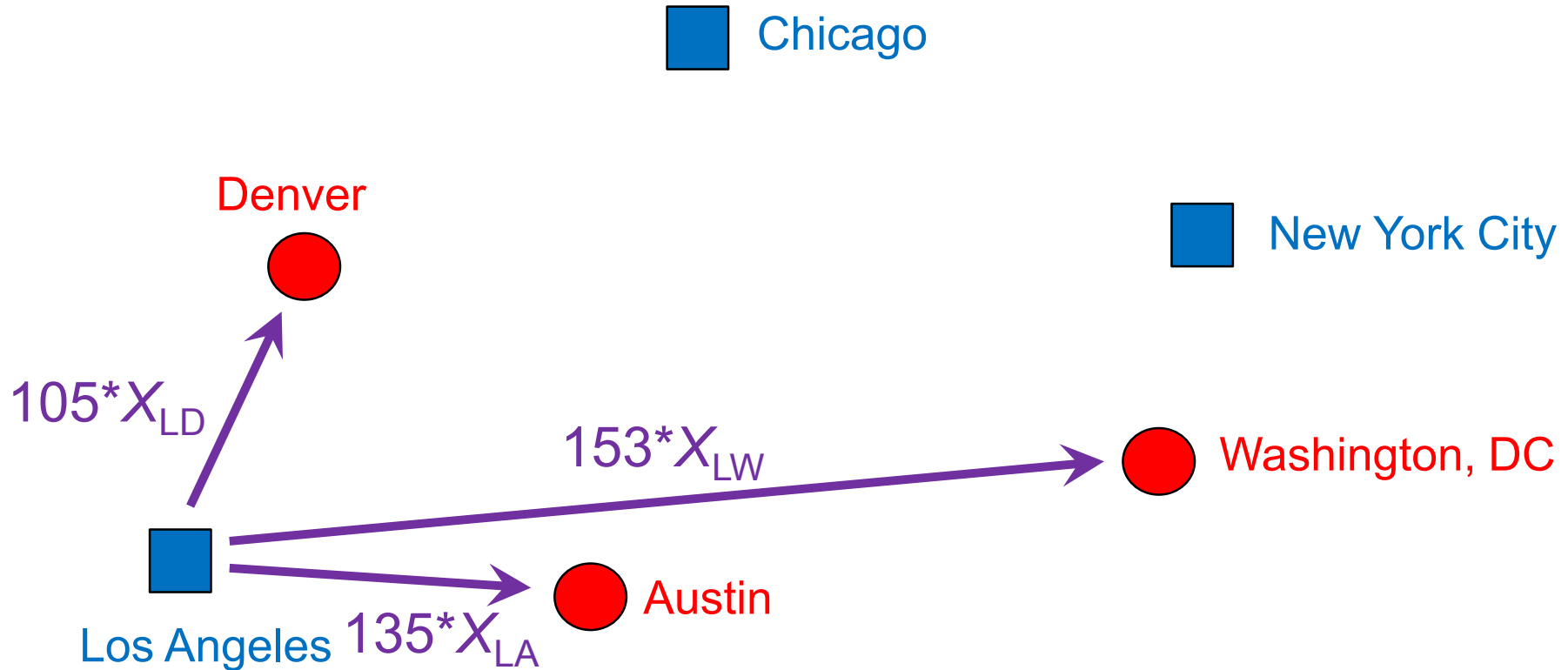
- ◆ KDGL's management needs to determine the **amounts of cargo to transport from each warehouse to each distribution center** to

KDGL: Objective Function



From / To	Denver (D)	Austin (A)	Washington, DC (W)
Los Angeles (L)	\$105.00	\$135.00	\$153.00
Chicago (C)	\$110.00	\$140.00	\$137.00
New York City (N)	\$130.00	\$132.00	\$115.00

KDGL: Objective Function



$105 * X_{LD}$ = the cost to ship X_{LD} tons from Los Angeles to Denver

$135 * X_{LA}$ = the cost to ship X_{LA} tons from Los Angeles to Austin

$153 * X_{LW}$ = the cost to ship X_{LW} tons from Los Angeles to Washington, DC

KDGL: Objective Function

Total Shipping Cost to Be Minimized:

$$\begin{aligned} &105 * X_{LD} + 135 * X_{LA} + 153 * X_{LW} \quad (\text{Cost for shipping from Los Angeles}) \\ &+ 110 * X_{CD} + 140 * X_{CA} + 137 * X_{CW} \quad (\text{Cost for shipping from Chicago}) \\ &+ 130 * X_{ND} + 132 * X_{NA} + 115 * X_{NW} \quad (\text{Cost for shipping from New York City}) \end{aligned}$$

KDGL: Constraints

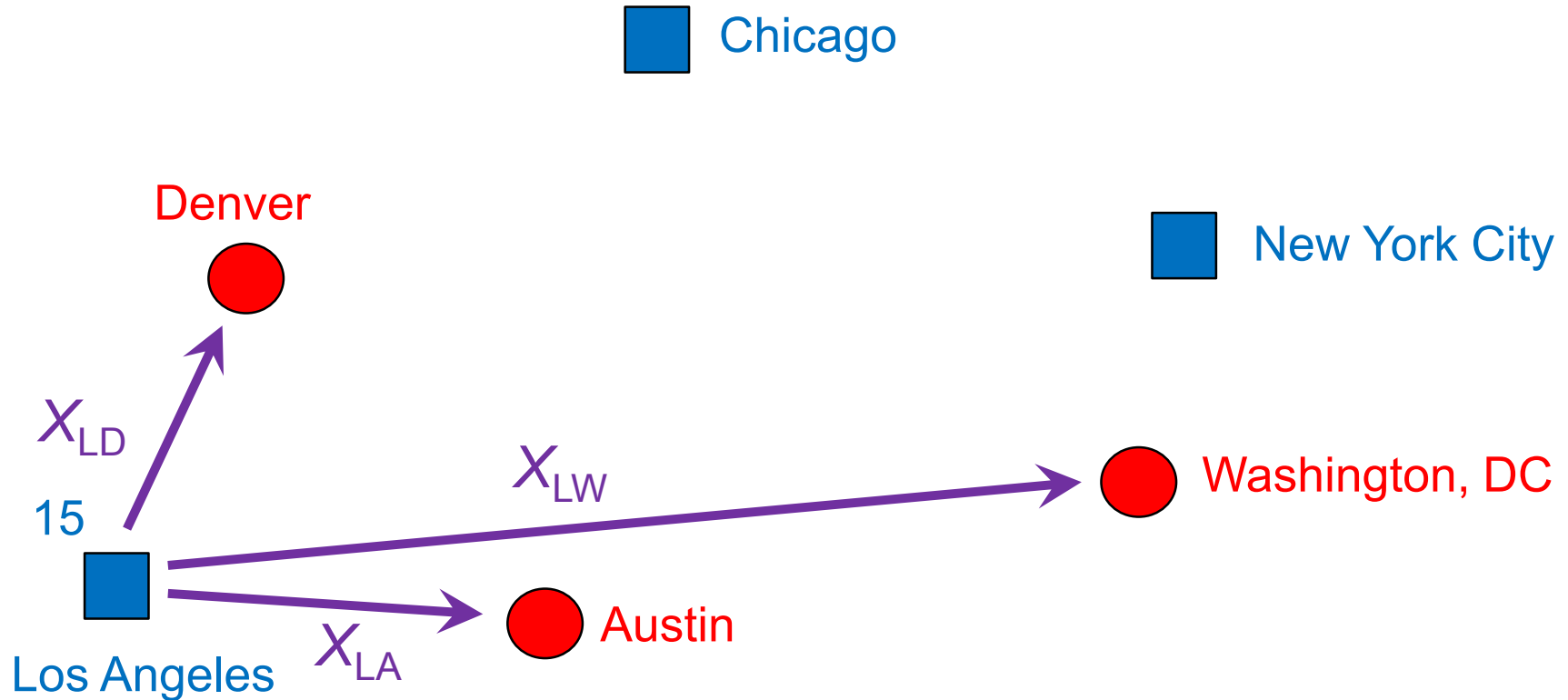
- ◆ KDGL's management needs to determine the **amounts of cargo to transport from each warehouse to each distribution center to minimize the total shipping cost**

KDGL: Supply Constraints

- ◆ The warehouses have a total supply of 65 tons of powdered drink that must be transported to the distribution centers next month to make room for new warehouse inventory.

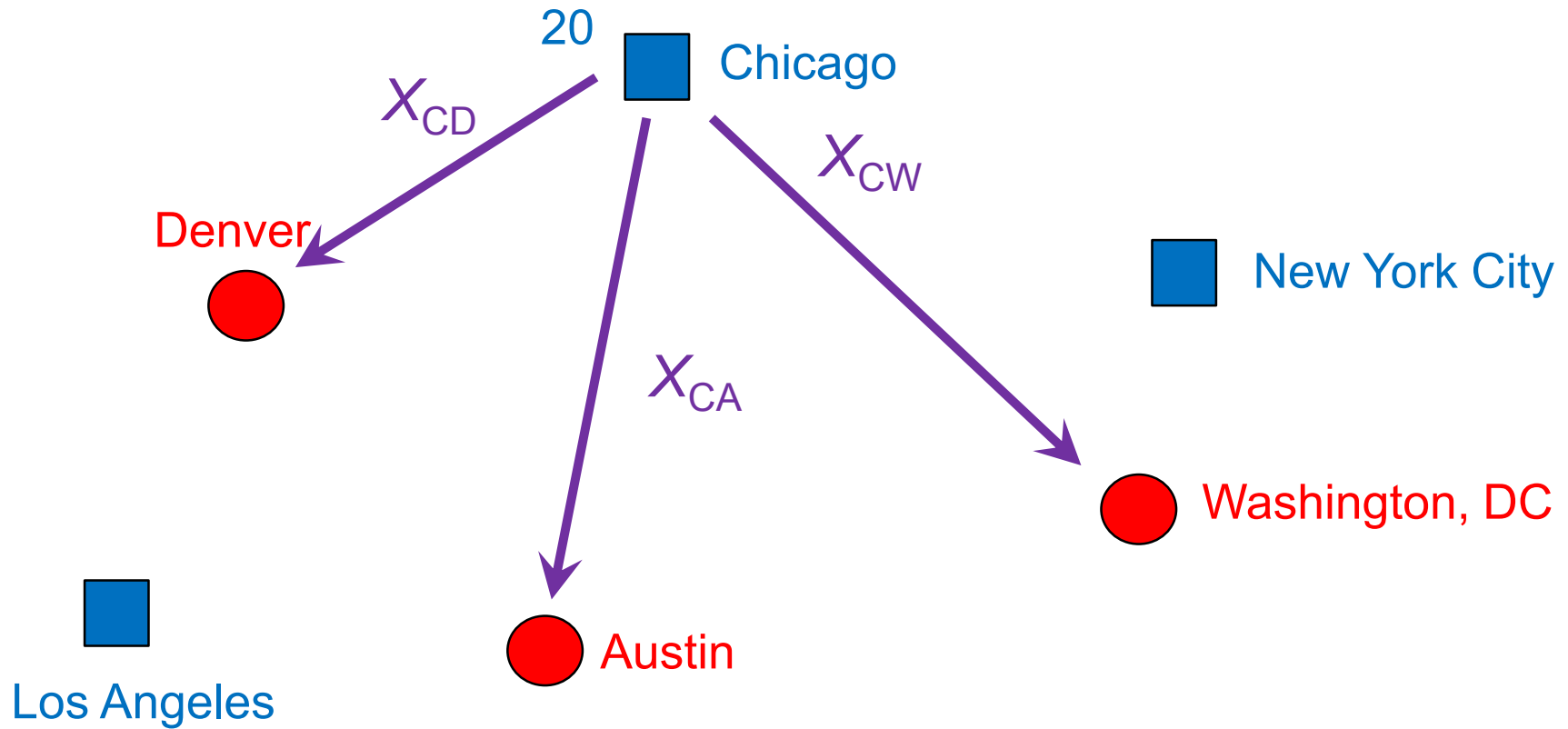
Warehouse	To Be Shipped Out (tons)
Los Angeles (L)	15
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KDGL: Supply Constraints



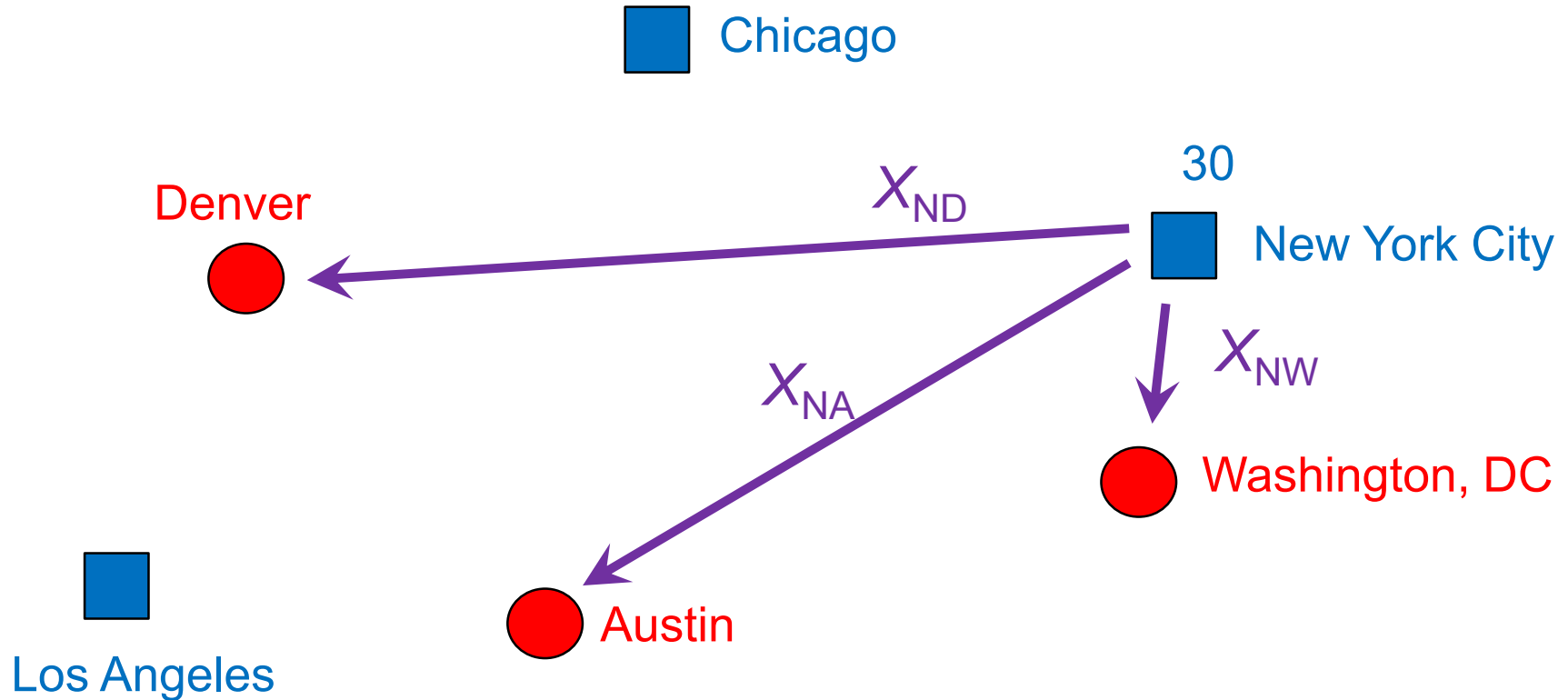
$$\text{Number of tons shipped from Los Angeles} = X_{LD} + X_{LA} + X_{LW} = 15$$

KDGL: Supply Constraints



Number of tons shipped from Chicago = $X_{CD} + X_{CA} + X_{CW} = 20$

KDGL: Supply Constraints



$$\text{Number of tons shipped from New York City} = X_{ND} + X_{NA} + X_{NW} = 30$$

KDGL: Supply Constraints

$$X_{LD} + X_{LA} + X_{LW} = 15 \quad (\text{Los Angeles supply})$$

$$X_{CD} + X_{CA} + X_{CW} = 20 \quad (\text{Chicago supply})$$

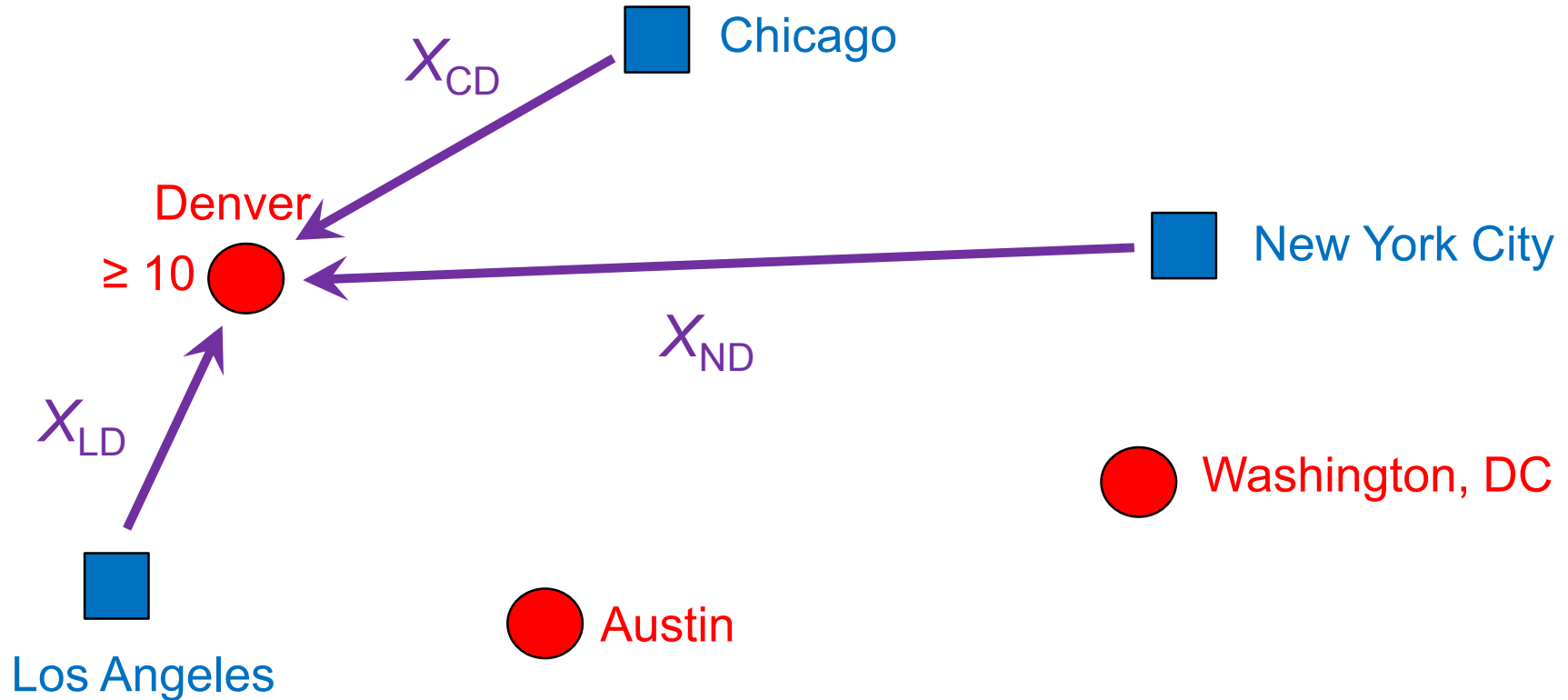
$$X_{ND} + X_{NA} + X_{NW} = 30 \quad (\text{New York supply})$$

KDGL: Demand Constraints

- ◆ There are minimum quantities that must be shipped to each of the distribution centers.
- ◆ KDGL may ship more than these quantities to ensure that all 65 tons of powdered drink are removed from the warehouses, but it cannot ship less

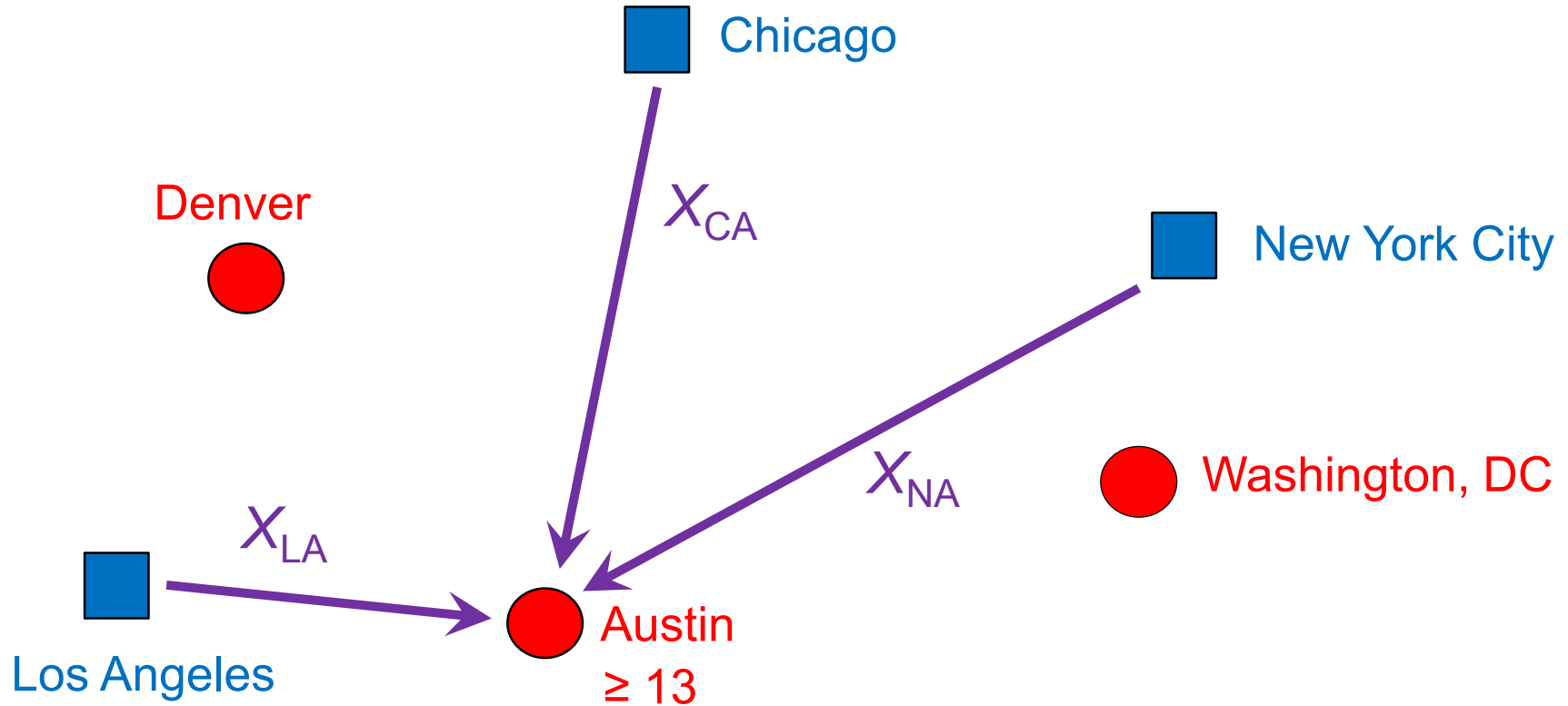
Distribution Center	Minimum To Be Shipped In (tons)
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KDGL: Demand Constraints



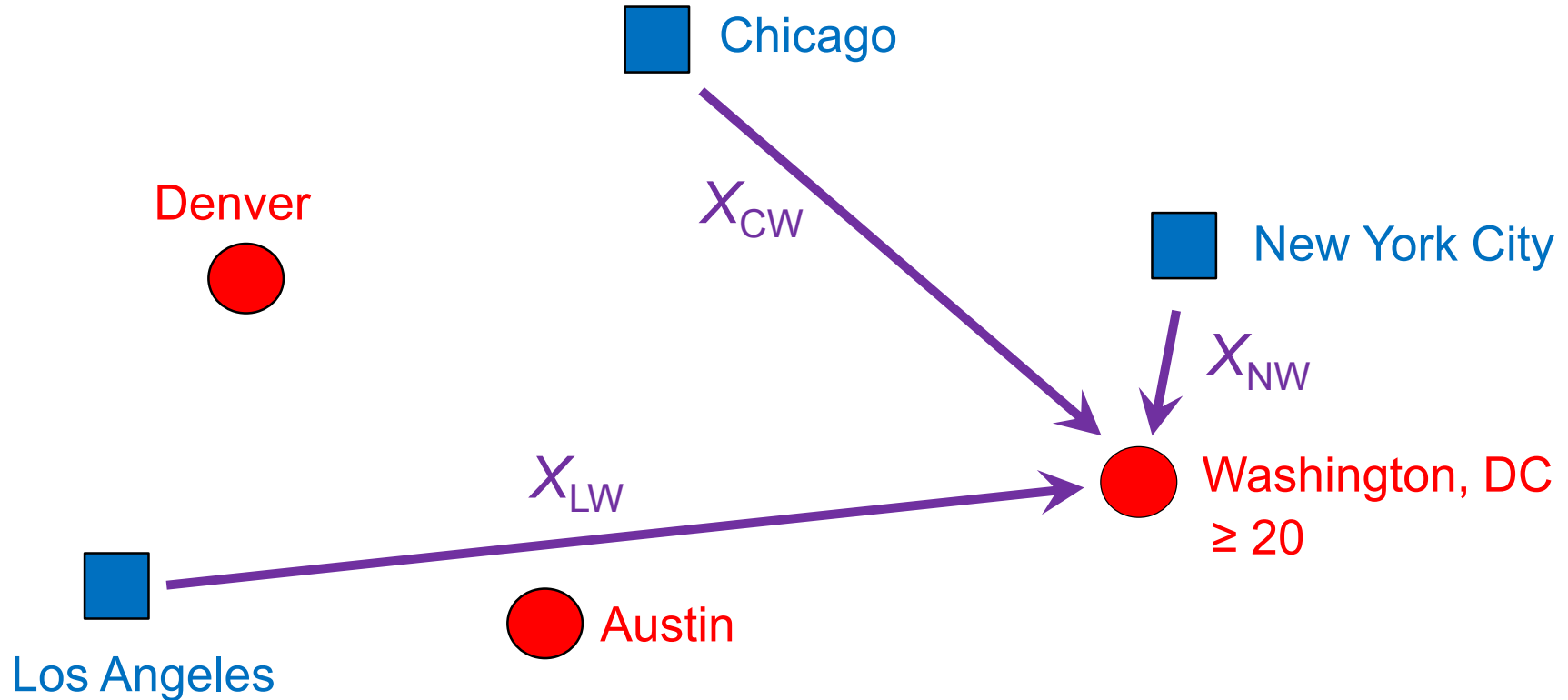
Number of tons shipped to Denver = $X_{LD} + X_{CD} + X_{ND} \geq 10$

KDGL: Demand Constraints



Number of tons shipped to Austin = $X_{LA} + X_{CA} + X_{NA} \geq 13$

KDGL: Demand Constraints



Number of tons shipped to Washington, DC = $X_{LW} + X_{CW} + X_{NW} \geq 20$

KDGL: Demand Constraints

$$X_{LD} + X_{CD} + X_{ND} \geq 10 \quad (\text{Minimum Denver demand})$$

$$X_{LA} + X_{CA} + X_{NA} \geq 13 \quad (\text{Minimum Austin demand})$$

$$X_{LW} + X_{CW} + X_{NW} \geq 20 \quad (\text{Minimum Washington, DC demand})$$

KDGL: Model Formulation

Minimize the Total Shipping Cost

$$105*X_{LD} + 135*X_{LA} + 153*X_{LW} + 110*X_{CD} + 140*X_{CA} + 137*X_{CW} \\ + 130*X_{ND} + 132*X_{NA} + 115*X_{NW}$$

$$X_{LD} + X_{LA} + X_{LW} = 15 \quad (\text{Los Angeles supply})$$

$$X_{CD} + X_{CA} + X_{CW} = 20 \quad (\text{Chicago supply})$$

$$X_{ND} + X_{NA} + X_{NW} = 30 \quad (\text{New York supply})$$

$$X_{LD} + X_{CD} + X_{ND} \geq 10 \quad (\text{Minimum Denver demand})$$

$$X_{LA} + X_{CA} + X_{NA} \geq 13 \quad (\text{Minimum Austin demand})$$

$$X_{LW} + X_{CW} + X_{NW} \geq 20 \quad (\text{Minimum Washington, DC demand})$$

$$X_{LD}, \dots, X_{NW} \geq 0 \quad (\text{Non-negativity})$$

Spreadsheet Solution

	A	B	C	D	E	F	G
1	Keystone.xlsx	Keystone Dry Goods Logistics Problem					
2	Operations Analytics MOOC						
3						Total Cost (in \$)	7,485
4	Shipping Cost (\$ per ton)					=SUMPRODUCT(B6:D8,B12:D14)	
5	From/To	Denver	Austin	Washington			
6	Los Angeles	105.00	135.00	153.00			
7	Chicago	110.00	140.00	137.00			
8	New York City	130.00	132.00	115.00			
9						=SUM(B12:D12)	
10	Shipping quantities (tons)					Total Shipped	Supply
11	From/To	Denver	Austin	Washington			
12	Los Angeles	15	0	0	15	=	15
13	Chicago	17	3	0	20	=	20
14	New York City	0	10	20	30	=	30
15	Total	32	13	20			
16	=SUM(B12:B14)	=>	=>	=>			
17	Minimum Demand	10	13	20			

decision variables, objective function, constraints. ✓

- ◆ Solver states that the best decision is to send the entire Los Angeles supply to Denver, split the supply in Chicago between Denver and Austin, and split New York City's supply between Austin and Washington
- ◆ The shipping cost associated with this decision is \$7485

Making Optimal Decisions in Practice: Commercial Optimization Packages

- ◆ Solver is a useful tool for learning optimization, but for problems of practical size, with thousands of variables and constraints, commercial alternatives may offer an advantage
- ◆ Whether using Solver, or commercial packages, optimization requires specifying decision variables, an objective function and constraints
- ◆ A recent comparison of optimization software by the *OR/MS Today*: <http://www.orms-today.org/surveys/LP/lp12.html> (while the table is called “linear programming survey”, the software packages mentioned can handle non-linear models)

Optimization in Practice

- ◆ Examples of the use of optimization methods: articles published in *Interfaces*



Optimizing Chevron's Refineries

Ted Kutz, Mark Davis, Robert Creek, Nick Kenaston, Craig Stenstrom, and Margery Connor
Interfaces 2014, 44:1, 39-54.



Zara Uses Operations Research to Reengineer Its Global Distribution Process

Felipe Caro, Jérémie Gallien, Miguel Díaz, Javier García, José Manuel Corredoira, Marcos Montes, José Antonio Ramos, and Juan Correa
Interfaces 2010, 40:1, 71-84.

- ◆ In practice, optimization often needs to be done in uncertain environments