

Week 3: Risk and Evaluation of Alternatives

- ◆ Making Decisions in Low-Uncertainty vs. High-Uncertainty Settings
- ◆ Example: Evaluating a Wireless Data Plan
- ◆ Reward and Risk
- ◆ Connecting Random Inputs and Random Outputs
- ◆ Simulating Uncertain Outcomes in Excel
- ◆ Interpreting Simulation Results: “Short” vs. “Long” Simulations
- ◆ Using Histograms to Visualize Simulation Results

Example: Evaluating a Wireless Data Plan

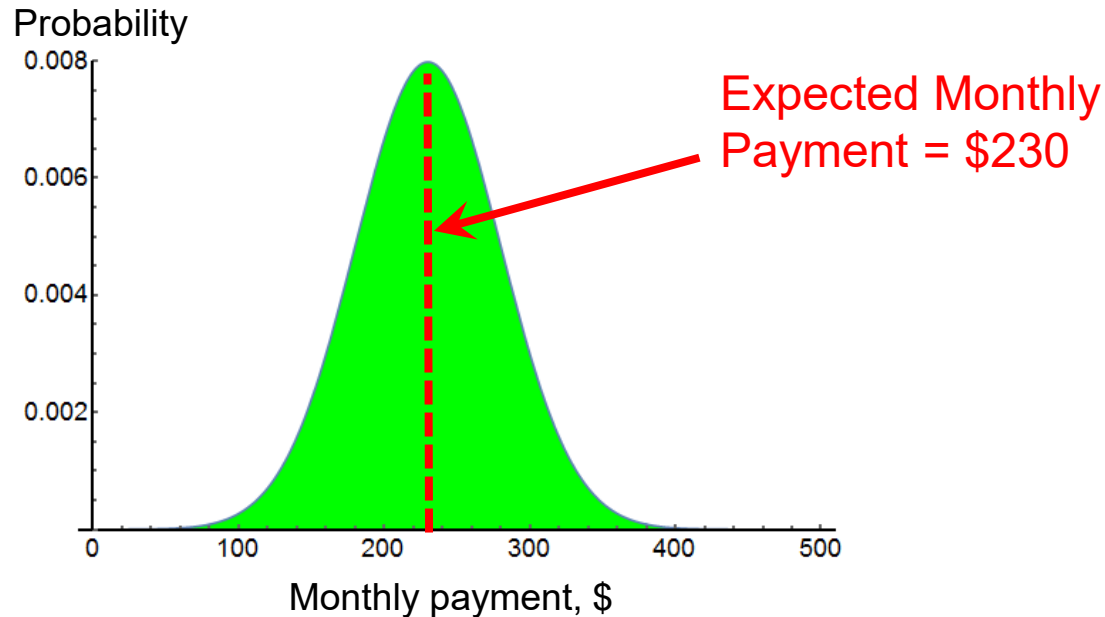
- ◆ A business analytics consultant based in Philadelphia is considering changing her wireless data plan to accommodate her family's growing use of video streaming services
- ◆ Under her current data plan called "Family Share" she pays \$10 for each GB of data her family uses
- ◆ After doing research on data plans offered by her wireless carrier, the consultant has decided to select the plan her carrier calls "Superior Share"
- ◆ Under the Superior Share plan, the consultant will pay a flat fee of \$160 for up to 20GB of data per month. This data allowance may be shared among the members of her family

Example: Evaluating a Wireless Data Plan

- ◆ If her family's actual monthly data usage exceeds 20GB, she will then have to pay for any data usage above this threshold at the rate of \$15 per GB
 - For example, if her family's monthly data usage is 22GB, her monthly payment will be $\$160 + (22 - 20) * \$15 = \$190$

- ◆ If her family's actual monthly data usage does not exceed 20GB, she will still have to pay the full \$160 amount, and the amount of unused data under 20GB will not “roll over” to the next month
 - For example, if her family's monthly data usage is 17GB, her monthly payment will be \$160

We Have a Complete Description of the Random Future Monthly Payments Under the Old Plan



- ◆ Consultant estimates that her monthly data usage is distributed as a normal random variable with the mean 23 GB and the standard deviation 5 GB
- ◆ So, the expected value of monthly payments under the old plan is **\$230**
- ◆ The standard deviation of monthly payments under the old plan is **\$50**

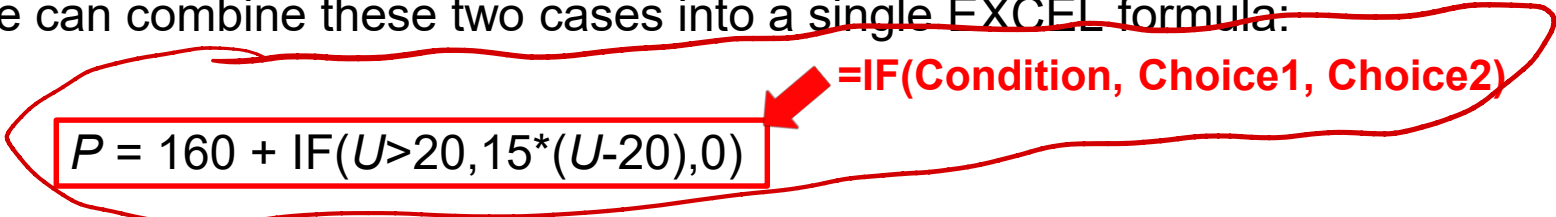
What About the Distribution of Monthly Payments Under the New Data Plan?

- ◆ What is the expected monthly payment under the new data plan?
- ◆ What is the standard deviation of the monthly payments under the new data plan?

An Algebraic Formula: Monthly Payment for Any Value of Data Usage

- ◆ We can calculate the monthly payment value P (in \$) for any value of data usage U (in GB)
- ◆ If U is below or at 20, then the monthly payment P is 160
- ◆ If U is above 20, then the monthly payment is $160 + 15*(U-20)$

- ◆ We can combine these two cases into a single EXCEL formula:


$$P = 160 + \text{IF}(U > 20, 15*(U-20), 0)$$

=IF(Condition, Choice1, Choice2)

- ◆ The IF function looks at the **Condition**: if the **Condition** is true, then the value of IF is equal to **Choice1**; if the **Condition** is false, then the value of IF is equal to **Choice2**

An Algebraic Formula: Monthly Payment for Any Value of Data Usage

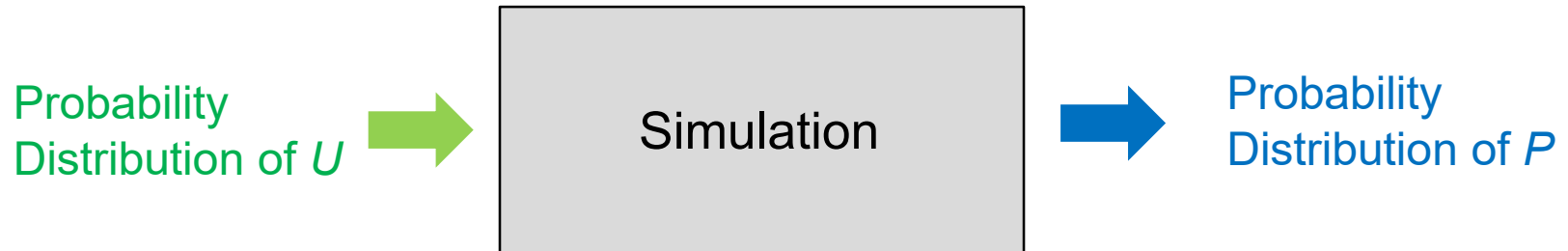
- ◆ $P = 160 + \text{IF}(U > 20, 15 * (U - 20), 0)$
- ◆ U is distributed as a normal random variable with a mean of 23, and a standard deviation of 5
- ◆ What is the distribution of P ?
- ◆ What is the expected value of P ?
- ◆ What is the standard deviation of P ?

Expected Value of Monthly Payment Under the New Plan?

- ◆ $P = 160 + \text{IF}(U > 20, 15 * (U - 20), 0)$
- ◆ Expected value of U is 23
- ◆ So, shouldn't the expected value of P be $160 + 15 * (23 - 20) = \mathbf{205}$?
- ◆ In general, we do not get the correct value for the expected monthly payment that way
- ◆ Example: suppose that U takes only 2 values, 18 with probability 50% and 28 with probability 50% (so that the expected data usage value of U is still 23)
 - If $U=18$, then $P=160$
 - If $U=28$, then $P=160+15*(28-20) = 280$
 - The expected value of P is $0.5*160+0.5*280 = \mathbf{220}$
 - This value is very different from 205, the value one gets after plugging in the expected data usage value into the monthly payment formula

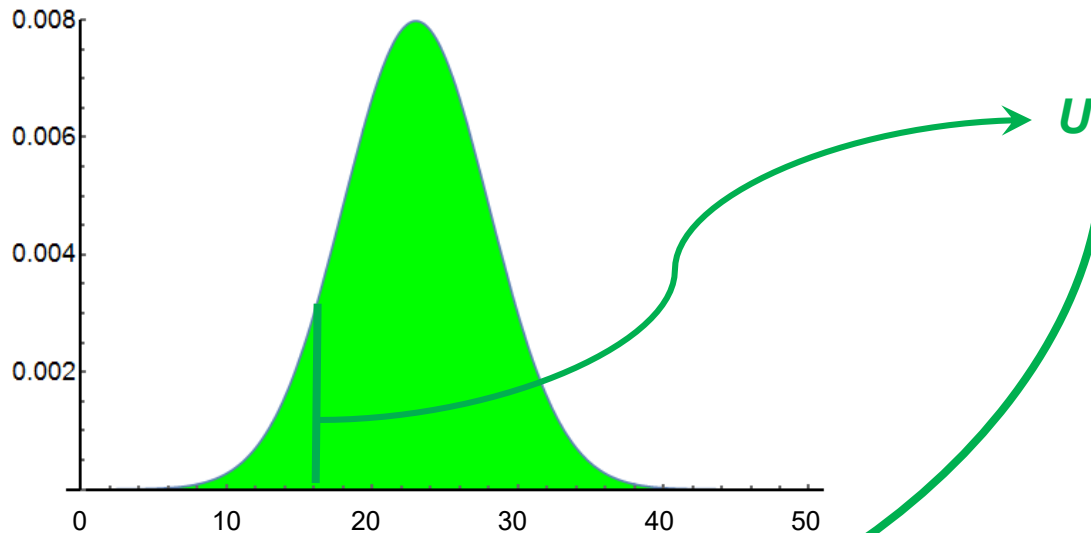
Simulation as an Analytics Tool

- ◆ Simulation is a tool that uses a probability distribution of the “input” random variable (such as data usage U) to create a distribution of the “output” random variable (such as monthly payment P)



Simulation as an Analytics Tool

- ◆ In each step of a simulation, a **random instance of the “input” variable is generated...**



- ◆ ... and the **resulting value of the “output”** is calculated:
 $P = 160 + \text{IF}(U > 20, 15 \cdot (U - 20), 0)$

Simulation as an Analytics Tool

- ◆ These simulation steps (called “**simulation runs**”) can be repeated as many times as necessary to generate the “**sample distribution**” of “output” values
- ◆ Once this “sample distribution” of output is generated, it can be analyzed to determine estimates for the expected value, standard deviation, etc. – and any other reward and risk measures we choose
- ◆ Excel can be used for both running the simulation and for the follow-up analysis

Running Simulation in Excel: Analysis ToolPak

- ◆ Likely to be a part of standard Excel installation on Windows
- ◆ On Mac (see <https://support.microsoft.com/en-us/kb/2431349>)
 - Included on Excel 2016 for Mac
 - Not included on earlier versions of Excel, but you can use a similar free software called StatPlus:mac LE, available here:
<http://www.analystsoft.com/en/products/statplasmacle/>
- ◆ On Google Sheets: an equivalent add-on called XLMiner Analysis ToolPak is available
measure of reward: expected monthly payment under the new plan.
standard deviation of monthly payment as measure of risk.

Simulated Data Usage Values and Corresponding Monthly Payment Values: Excel Implementation

	A	B	C	D	E	F
1	DataPlan10.xlsx	Wireless Data Plan	Simulation Run	Data Usage, U (GB)	Payment, P (\$)	
2	Operations Analytics MOOC		1	11.9319952	160	
3			2	24.0282690	220.4240354	
4	Data Allowance (GB)	20	3	=B\$5+IF(D2>B\$4,B\$6*(D2-B\$4),0)		
5	Fixed Payment (\$)	160	4	21.7321587	185.9823805	
6	Rate Above Allowance (\$/GB)	15	5	34.2335329	373.5029929	
7			6	16.5820597	160	
8	Expected Data Usage (GB)	23	7	30.7079676	320.619514	
9	St. Dev. of Data Usage (GB)	5	8	36.9010808	413.5162123	
10			9	20.3471859	165.2077878	
11			10	28.3229996	284.8449946	
12			=AVERAGE(D2:D11)		=AVERAGE(E2:E11)	
13			Sample Mean	25.0470054	252.9339988	→ measure of reward.
14			Sample St. Dev.	7.787935101	92.19007977	
15			=STDEV(D2:D11)		=STDEV(E2:E11)	

measure of risk

◆ See DataPlan10.xlsx