Week 2 Review: Making Best Decisions in Settings with Low Uncertainty

- Resource Allocation Example: Natural Disaster Housing Relief
- Network Optimization Example: Colombi Parquet

- ◆ A UN agency that manages relief efforts is organizing medium-term housing for survivors in a country that recently suffered from a natural disaster. The agency is considering building four types of housing units: A, B, C, and D
- ◆ Each of four types of housing uses the same basic set of building materials (bricks, corrugated iron sheets, and wooden poles), as well as labor, in the following amounts:

Unit Type	Bricks	Iron Sheets	Wooden Poles	Labor Hours
Α	400	20	35	300
В	600	10	32	200
С	400	25	26	200
D	300	18	30	400

 Once built, housing units will provide shelter for 7, 6, 5 and 7 people for each unit of A, B, C, and D, respectively

The availability of building materials and labor is limited by the following amounts:

Resource	Available
Bricks	1500000
Iron Sheets	100000
Wooden Poles	125000
Labor Hours	1500000

◆ The agency would like to decide how many units of each type of housing to build to maximize the number of people receiving shelter without exceeding the resource availabilities

Natural Disaster Housing Relief: Decision Variables

How many units of each type to build?

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N_A = number of type A units to build,
N_B = number of type B units to build,
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 $N_{\rm C}$ = number of type C units to build,

 $N_{\rm D}$ = number of type D units to build.

Natural Disaster Housing Relief: Objective Function

- Maximize the number of people receiving shelter:
 - Each type A unit provides shelter for 7 people. N_A type A units provide shelter for $7*N_A$ people
 - Each type B unit provides shelter for 6 people. $N_{\rm B}$ type B units provide shelter for $6*N_{\rm B}$ people
 - Each type C unit provides shelter for 5 people. N_C type C units provide shelter for 5*N_C people
 - Each type D unit provides shelter for 7 people. N_D type D units provide shelter for $7*N_D$ people
- ◆ The total number of people receiving shelter: 7*N_A + 6*N_B + 5*N_C + 7*N_D

Resource Availabilities

Number of required bricks cannot exceed brick availability

 N_A type A units require 400* N_A bricks

 $N_{\rm B}$ type B units require 600* $N_{\rm B}$ bricks

 $N_{\rm C}$ type C units require 400* $N_{\rm C}$ bricks

 $N_{\rm D}$ type D units require 300* $N_{\rm D}$ bricks

$$400*N_A + 600*N_B + 400*N_C + 300*N_D \le 1500000$$

Number of required iron sheets cannot exceed iron sheet availability

 N_A type A units require $20*N_A$ iron sheets

 $N_{\rm B}$ type B units require $10^*N_{\rm B}$ iron sheets

 $N_{\rm C}$ type C units require 25* $N_{\rm C}$ iron sheets

 $N_{\rm D}$ type D units require $18*N_{\rm D}$ iron sheets

$$20*N_A + 10*N_B + 25*N_C + 18*N_D \le 100000$$

Resource Availabilities

Number of required wooden poles cannot exceed wooden pole availability.

 N_A type A units require $35*N_A$ wooden poles.

 $N_{\rm B}$ type B units require $32*N_{\rm B}$ wooden poles.

 $N_{\rm C}$ type C units require 26* $N_{\rm C}$ wooden poles.

 $N_{\rm D}$ type D units require 30* $N_{\rm D}$ wooden poles.

$$35*N_A + 32*N_B + 26*N_C + 30*N_D \le 125000$$

Number of required labor hours cannot exceed labor hours availability

 N_A type A units require 300* N_A labor hours

 $N_{\rm B}$ type B units require 200* $N_{\rm B}$ labor hours

 $N_{\rm C}$ type C units require 200* $N_{\rm C}$ labor hours

 $N_{\rm D}$ type D units require 400* $N_{\rm D}$ labor hours

$$300*N_A + 200*N_B + 200*N_C + 400*N_D \le 1500000$$

Resource Availabilities

$$400*N_A + 600*N_B + 400*N_C + 300*N_D \le 1500000$$
 (bricks)
 $20*N_A + 10*N_B + 25*N_C + 18*N_D \le 100000$ (iron sheets)
 $35*N_A + 32*N_B + 26*N_C + 30*N_D \le 125000$ (wooden poles)
 $300*N_A + 200*N_B + 200*N_C + 400*N_D \le 1500000$ (labor hours)

◆ All decision variables must be integer

$$N_A$$
, N_B , N_C , N_D = integer

◆ All decision variables must be non-negative

$$N_{\rm A}, N_{\rm B}, N_{\rm C}, N_{\rm D} \ge 0$$

Natural Disaster Housing Relief: Complete Model

Maximize
$$7*N_A + 6*N_B + 5*N_C + 7*N_D$$

Subject to

$$400*N_A + 600*N_B + 400*N_C + 300*N_D \le 1500000$$

$$20*N_A + 10*N_B + 25*N_C + 18*N_D \le 100000$$

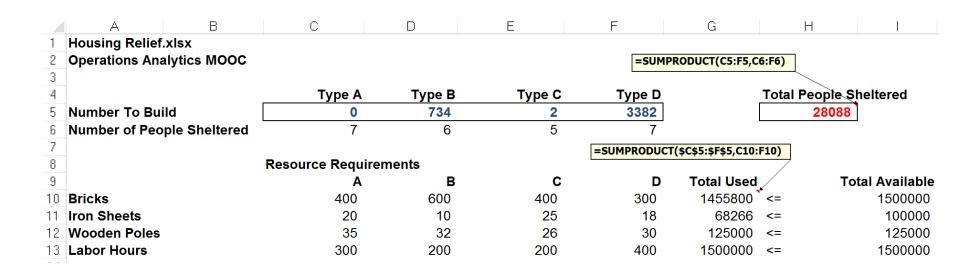
$$35*N_A + 32*N_B + 26*N_C + 30*N_D \le 125000$$

$$300*N_{\rm A} + 200*N_{\rm B} + 200*N_{\rm C} + 400*N_{\rm D} \le 1500000$$

$$N_A$$
, N_B , N_C , N_D = integer

$$N_A$$
, N_B , N_C , $N_D \ge 0$

Natural Disaster Housing Relief: Excel Implementation



- According to the Solver, the optimal solution is to build 734 units of type
 B, 2 units of type C, and 3382 units of type D
- The total number of sheltered people corresponding to this decision is 28088

Colombi Parquet: Context

- Colombi Parquet (CP) is a German company that specializes in manufacturing high-quality hardwood floors
- CP operates two plants: one located in Saxony-Anhalt (plant A) and another one located in Baden-Württemberg (plant B)
- The capacities (in thousands of square feet) of these two plants for the next quarter are shown below

Plant	Capacity
Α	2500
В	3000

Colombi Parquet: Context

- CP's hardwood floors are distributed within three marketing regions in Northern and Central Europe
- ◆ The minimum demand requirements (in thousands of square feet) for each marketing region for the next quarter are shown below:

Region 1	Region 2	Region 3	
2000	930	2200	

Colombi Parquet: Context

◆ The shipping costs (in € per thousand square feet) from the plants to the marketing regions are shown below:

From	to Region 1	to Region 2	to Region 3
Plant A	15	21	17
Plant B	23.5	25.5	22

◆ The company wants to determine the quantities of hardwood floors it should ship from each plant to each marketing region in order to minimize the total transportation cost for the next quarter while satisfying the demand requirements at each marketing region and the capacity limitations at each plant

Colombi Parquet: Decision Variables

How many square feet of hardwood floors should it ship from each plant to each marketing region?

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X_{A1} = 1000's of square feet of hardwood floors to ship from plant A to region 1, X_{B1} = 1000's of square feet of hardwood floors to ship from plant B to region 1, X_{A2} = 1000's of square feet of hardwood floors to ship from plant A to region 2, X_{B2} = 1000's of square feet of hardwood floors to ship from plant B to region 2, X_{A3} = 1000's of square feet of hardwood floors to ship from plant A to region 3, X_{B3} = 1000's of square feet of hardwood floors to ship from plant B to region 3
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Colombi Parquet: Objective Function

- Minimize the total transportation cost
 - Total Transportation Cost:

$$15^*X_{A1} + 23.5^*X_{B1} + 21^*X_{A2} + 25.5^*X_{B2} + 17^*X_{A3} + 22^*X_{B3}$$

Colombi Parquet: Constraints

Supply Constraints:

- The amount shipped from plant A cannot exceed plant A's capacity $X_{A1} + X_{A2} + X_{A3} \le 2500$
- The amount shipped from plant B cannot exceed plant B's capacity $X_{B1} + X_{B2} + X_{B3} \le 3000$

Colombi Parquet: Constraints

Demand Constraints:

- The amount shipped to region 1 cannot be less than 2000 $X_{A1} + X_{B1} \ge 2000$
- The amount shipped to region 2 cannot be less than 930 X_{A2} + X_{B2} ≥ 930
- The amount shipped to region 3 cannot be less than 2200 X_{A3} + X_{B3} ≥ 2200

Colombi Parquet: Constraints

Shipping amounts cannot be negative:

$$X_{A1}, X_{B1}, X_{A2}, X_{B2}, X_{A3}, X_{B3} \ge 0$$

Colombi Parquet: Complete Model

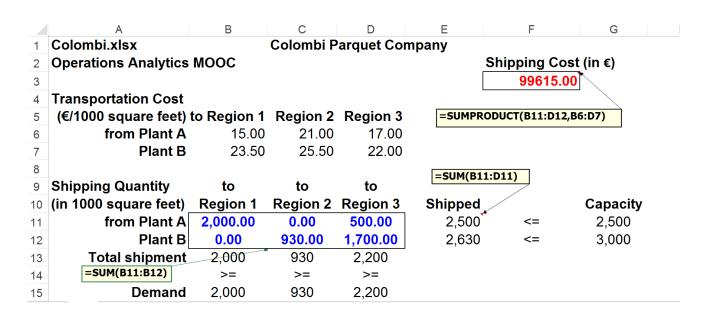
Minimize
$$15^*X_{A1} + 23.5^*X_{B1} + 21^*X_{A2} + 25.5^*X_{B2} + 17^*X_{A3} + 22^*X_{B3}$$

Subject to:

$$X_{A1} + X_{A2} + X_{A3} \le 2500$$

 $X_{B1} + X_{B2} + X_{B3} \le 3000$
 $X_{A1} + X_{B1} \ge 2000$
 $X_{A2} + X_{B2} \ge 930$
 $X_{A3} + X_{B3} \ge 2200$
 $X_{A1}, X_{B1}, X_{A2}, X_{B2}, X_{A3}, X_{B3} \ge 0$

Colombi Parquet: Excel Implementation



- ◆ According to the Solver, the optimal solution is to
 - produce 2500 thousand square feet in Plant A and split this production between regions 1 and 3
 - produce 2630 thousand square feet in Plant B and split this production between regions 2 and 3
- ◆ The optimal cost is 99615€