Week 2: Making Best Decisions in Settings with Low Uncertainty

- A resource allocation example: Zooter Industries **Session 1**
- Converting a verbal problem description into an algebraic model: variables decisions, objective, constraints
- From an algebraic model to a spreadsheet implementation: optimizing with Excel Solver **Session 2**
- Matching demand and supply across space: Keystone Dry Goods Logistics **Session 3**

start from low uncertainty.

Zooter Industries: Products, Profits, Demand

- *Zooter Industries* (ZI) manufactures high-end kick-scooters for the North American market
- ZI's main product models are Razor and Navajo, with profit contributions of $150 and $160 per unit
- At present, ZI's scooters are so popular that the company can sell all the units it makes

Zooter Industries: Manufacturing Process

- The production process for each model includes three main steps:
  - frame manufacturing
  - wheels and deck assembly
  - quality assurance and packaging

- Each unit of the two scooter models requires the following processing times in these production steps:

Model	**Frame Manufacturing (hours)**	**Wheels and Deck Assembly (hours)**	**Quality Assurance and Packaging (hours)**
Razor	4.0	1.5	1.0
Navajo	5.0	2.0	0.8

Zooter Industries: Supply Side

- ZI's capacity available at each production step is shown below for the coming week

Production Step	Available Time in the Coming Week (hours)
Frame Manufacturing	5610
Wheels and Deck Assembly	2200
Quality Assurance and Packaging	1200

- How many units of each model should ZI produce in the coming week in order to maximize its weekly profit?

Assuming Away Uncertainty: Pros and Cons

- The Zooter example treats profit contributions, manufacturing requirements, supply availabilities as non-random quantities
- If ZI decides to make a certain number of units of each scooter model in the coming week, it will know for sure
  - How much profit it will make
  - Whether it will have sufficient supply of each resource
- The "no uncertainty" assumption simplifies the search for the best production plan
- In practice, it allows us to tackle analytics models with large numbers of products and resources

Assuming Away Uncertainty: Pros and Cons

- May be a reasonable assumption when a decision maker has substantial control over his/her business environment
  - Short-term planning
  - Longer-term planning when existing contracts ensure stability of prices, costs, and demand and supply parameters
- May result in problematic recommendations in settings with significant data uncertainty
- When uncertainty is significant and must be included in the analysis, the task of finding the best decision may become far more complex
- In Weeks 3 and 4 we will look at how to evaluate choices and make best decisions in such settings

Evaluating a Production Plan: Decision Variables

- Before approaching a task of finding the best production plan, or **optimizing** production, we must know how to evaluate any given production plan

- In optimization lingo, the term **"decision variables"** describes the quantities that a decision maker can change to achieve a desired performance.

- In the ZI example, there are two decision variables:
  - *R*, the number of Razor scooters to produce in the coming week
  - *N*, the number of Navajo scooters to produce in the coming week

- A particular choice of values for decision variables is called a **"solution"**. For example, *R*=500 and *N*=500 is a solution

Evaluating a Production Plan: Objective Function

- If ZI decides to produce *R*=500 Razor and *N*=500 Navajo scooters in the coming week, how much profit will ZI make in this case?
  - Profit (in $) = $150*500*+$160*500 = $75000 + $80000 = $155000

- The **“objective”** is a performance metric we want to maximize or minimize. In this example, profit is an objective to be maximized

- For *R*=500 and *N*=500, the profit value is $155000. How much profit will ZI make for an arbitrary pair of values *R* and *N*?
  - Profit (in $) = 150*$R$ + 160*N

- 150*R + 160*N is an **“objective function”**, i.e., an objective expressed as a function of decision variables

- $155000 is an “**objective function value**” (OFV) for solution *R*=500, *N*=500

plug in

Evaluating a Production Plan: Constraints

- If ZI decides to produce *R*=500 Razor and *N*=500 Navajo scooters in the coming week, how much of each resource will it require?
- Required number of **frame manufacturing hours**:
  4*500+5*500 = 4500 – does not exceed 5610 hours available
- In general, for any potential production plan, the required number of frame manufacturing hours may not exceed the number of hours available
- In the optimization lingo, we use the term "**constraint**" to describe this requirement

Evaluating a Production Plan: Constraints

- Does the R=500 and N=500 production plan have enough of other resources to be implemented?

- Required number of **wheels and deck assembly hours**:
  1.5*500+2.0*500 = 1750 – does not exceed 2200 hours available

- Required number of **quality assurance and packaging hours**:
  1.0*500+0.8*500 = 900 – does not exceed 1200 hours available

- A production plan that, like R=500 and N=500, satisfies all constraints is called **feasible**

Evaluating a Production Plan: Constraints

- What if ZI decides to produce *R*=500 Razor and *N*=750 Navajo scooters?

- Required number of **frame manufacturing hours**:
  4*500+5*750 = 5750 – exceeds 5610 hours available

- Required number of **wheels and deck assembly hours**:
  1.5*500+2.0*750 = 2250 – exceeds 2200 hours available

- Required number of **quality assurance and packaging hours**:
  1.0*500+0.8*750 = 1100 – does not exceed 1200 hours available

- A production plan that, like *R*=500 and *N*=750, violates at least one constraint is called **infeasible**

Evaluating a Production Plan: Constraints

- For a production plan that makes R Razor and N Navajo scooters, how does one express a constraint on the number of available frame manufacturing hours?

- In words, we have "number of required frame manufacturing hours may not exceed the number of available hours"

- Using variables R and N, we can write this statement as

$$4*R + 5*N \leq 5610$$

- In the same way, the constraints on the number of available wheels and deck assembly hours and the number of available quality assurance and packaging hours can be written as

$$1.5*R + 2.0*N \leq 2200$$

$$1.0*R + 0.8*N \leq 1200$$

Other Constraints?

- Numbers R and N must be integer

$$R, N = \text{integer}$$

- Numbers R and N cannot be negative

$$R, N \geq 0$$

Searching for the Best Production Plan: A Complete Model

- Putting the decision variables, objective function and constraints together, we can express our model as

algebraic formulation

Maximize $150*R + 160*N$

subject to

$4*R + 5*N \leq 5610$ (frame manufacturing hours)

$1.5*R + 2.0*N \leq 2200$ (wheel and deck manufacturing hours)

$1.0*R + 0.8*N \leq 1200$ (QA and packaging hours)

R, N = integer

$R, N \geq 0$

- We will use Solver to "optimize" this model

A Comment on Objective and Constraints

- An optimization model can have any number of decision variables and constraints but it must have one objective to be maximized or minimized

- In practice, there could be a number of quantities, **key performance indicators**, that a manager may want to keep track of: profit, cost, customer service levels, utilization of resources, etc.

- If one of the key performance indicators, such as profit, is selected as the objective, the rest of the key performance indicators can be used in constraints

- For example, the model can “maximize profit while making sure that the resource utilization does not exceed 95%”

Model Types: “Easier” …

- Zooter model involves only
  - constant parameters, like 5610
  - products of decision variables and constant parameters, like 150*R and 0.8*N
  - adding and/or subtracting the resulting expressions, like 1.5*R + 2.0*N

- Such models are called **“linear”** and easier to optimize in practice

… And “Harder”

- Sometimes it is necessary to use models that involve **“nonlinear”** expressions of decisions variables, for example, $R*N$, R^2, $N/(R+N)$ or $\sqrt{N}$

- Nonlinear models are much harder to optimize, especially as the number of variables and constraints grows

- In the Zooter model, the numbers of scooters produced must be round, or **integer**

- In general, imposing such a requirement can significantly complicate optimization even in linear models, especially in models with large numbers of variables and constraints

Additional References

- More on optimization, linear, non-linear models as well as models with integer variables:
  - “Business Analytics” by James R. Evans
  - “Spreadsheet Modeling and Decision Analysis: A Practical Introduction to Business Analytics” by C. Ragsdale