Physics-Based Analytical Modeling of Electromigration Reliability for Multi-Branch Interconnect Trees

Jiangtao Peng, Hai-Bao Chen, Taeyoung Kim, Hengyang Zhao, Sheldon X.-D. Tan

***Abstract*—In high performance circuit design, thermal effect on electromigration (EM) reliability has become a recent major research for being a limiting factor. Due to the strong thermal-dependence of leakage power, circuit performance, IC package cost and reliability, considering time-varying temperature effect has been an emerging concern on EM in conducting metal lines with complex interconnect structures. Dynamic temperature-aware reliability modeling has been proposed as a tool to explore the trade-off between the accuracy of lifetime prediction and the system performance. However, average temperature in the EM failure process is commonly used in most existing EM models and analytic techniques considering time-varying temperature are mainly focused on a single metal wire. In this paper, we propose a novel analytic method to calculate the stress evolution considering time-varying temperature effects during the void nucleation phase for some multi-branch interconnect trees, including the straight-line there-terminal wires, the T-shaped four-terminal wires and the cross-shaped five-terminal wires. The proposed closed-form expression can be used to calculate the hydrostatic stress evolution with time-varying temperature. Experiment results show that the obtained analytic solutions match well with the numerical results calculated using COMSOL and thus the proposed models can be used in traditional EM reliability analysis tools.**

***Index Terms*—electromigration, multi-branch interconnect trees, analytical model, time-varying temperature, stress evolution.**

I. Introduction

In this work, for the first time, we propose an analytic method to calculate the stress evolution considering time-varying temperature effects during the void nucleation phase for some multi-branch interconnect trees, including the straight-line there-terminal wires, the T-shaped four-terminal wires and the cross-shaped five-terminal wires. The proposed closed-form expression can be used to calculate the hydrostatic stress evolution with time-varying temperature.

The remainder of the paper is organized as follows. In Section **??**, we induce the analytical expression for the stress evolution in multi-branch interconnect tree considering constant temperature.Then we extend this model in time-varying temperature environment in Section **??**. Experimental results for the straight-line there-terminal wires, the T-shaped four-terminal wires and the cross-shaped five-terminal wires are presented in Section IV. Concluding remarks are drawn in Section V.

II. Physics based interconnect reliability modelling

Electromigration (EM) became of engineering interest since it was first observed as one of the primary failure mechanisms in aluminum IC conductors. Many researchers have studied the evolution of stress due to electromigration. Kirchheim proposed a physically based model in which generation of stress in grain boundaries during electrimigration is caused by the annihilation and generation of vacancies. Korhonen [11] proposed another physically based analytical model for mechanical stress evolution during electromigration in a confined metal line described by a one-dimensional equation, as follows:

$$\frac{\partial \sigma(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{D_a B\Omega}{kT}\left(\frac{\partial \sigma(x,t)}{\partial x} + \frac{Z^* e\rho}{\Omega} j\right)\right] \tag{1}$$

where σ is the hydrostatic stress, t is time, D_a is the atomic diffusivity, B is the effective bulk modulus, Ω is atomic volume,k is the Boltzman's constant, T is absolute temperature, Z^* is the effective charge number, e is electron charge, ρ is resistivity, and j is current density. Our works were mainly based on Equation (1). The advantage of this model is that the evolution of hydrostatic stress in the confined line can be calculated in a closed form, which has been widely used in EM analysis. For the sake of simplicity, we rewrote Equation (1) as follows:

$$\frac{\partial \sigma(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_t\left(\frac{\partial \sigma(x,t)}{\partial x} + G\right)\right] \tag{2}$$

where $D_t = \frac{D_a B\Omega}{kT}$ is the stress diffusivity affected by temperature T and $G = \frac{Z^* e\rho}{\Omega} j$ is EM driving force. Moreover, $D_a = D_0 e^{-\frac{E_a}{kT}}$ is the effective atomic diffusion coefficient. D_0 and E_a stand for the pre-exponential factor and the activation energy, respectively.

The research for multi-branch interconnect trees is still a large and challenging problem. Due to the strong thermal-dependence of leakage power, circuit performance, IC package cost and reliability, thermal effects has become a recent major research for being a limiting factor in high performance circuit design. Among the thermal effects, varying temperature is an important part to EM analysis. We have made the EM experiments considering the changing temperature and the average temperature. In Fig.1, we create three types of changing temperature. As we can see, function $T_0(t)$ is the average temperature of square wave temperature $T_1(t)$ and sine wave

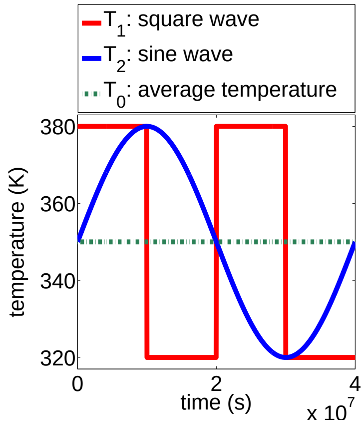
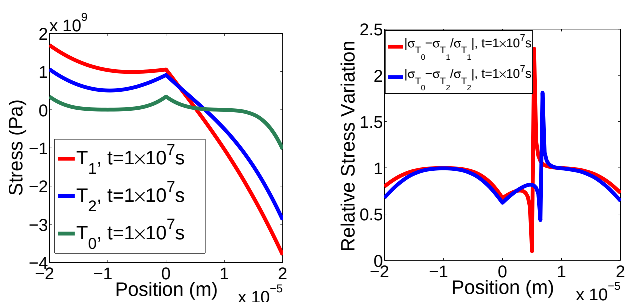


Fig. 1. The different temperature function in later experiments.

temperature $T_2(t)$. The stress evolution results for straight line interconnect tree simulated in COMSOL are showed in Fig.2, indicating that the relative stress variation is mostly higher than 0.5. Explicitly, we can't use average temperature to replace the changing temperature.



(a) (b)

Fig. 2. The real results considering different temperature environment in COMSOL. (a) the stress evolution; (b) the relative stress variation.

In this work, for the first time, we propose an analytic method to calculate the stress evolution considering time-varying temperature effects during the void nucleation phase for some multi-branch interconnect trees, including the straight-line there-terminal wires, the T-shaped four-terminal wires and the cross-shaped five-terminal wires. The proposed closed-form expression can be used to calculate the hydrostatic stress evolution with time-varying temperature.

The remainder of the paper is organized as follows. In Section **??**, we induce the analytical expression for the stress evolution in multi-branch interconnect tree considering constant temperature.Then we extend this model in time-varying temperature environment in Section **??**. Experimental results for the straight-line there-terminal wires, the T-shaped four-terminal wires and the cross-shaped five-terminal wires are presented in Section IV. Concluding remarks are drawn in Section V.

III. Accurate Analytical Modeling for Hydrostatic Stress Evolution

In this paper, an accurate analytical modeling for transient hydrostatic stress analysis for a general realistic interconnect structure as shown in Fig.3, which represents a long metal wire with a number of the voltage and current ports. Since many complex interconnect structures such as power grids could be decomposed to hierarchically in smaller interconnect components, the accurate analytic closed-form expressions describing hydrostatic stress evolution for multi-branch interconnect trees will lay the groundwork for the EM reliability analysis of complex interconnect networks. Based on the Laplace transformation technique, an accurate closed-form expression during the void nucleation phase for the long metal wire shown in Fig.3 will be proposed.

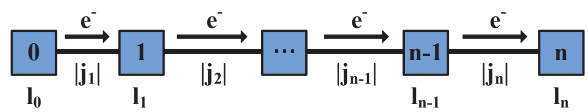


Fig. 3. An example of general realistic interconnect structure.

The stress evolution process for each segment in Fig.3 is described by the Korhonen equation. Stress evolution equations for each segment are coupled to each other by the boundary conditions representing the continuity of stress and fluxes at the junctions in the long metal wire. The general system of equation in this case can be described as follows:

$$
\begin{aligned}
\frac{\partial \sigma_1}{\partial t} &= \frac{\partial}{\partial x}[\kappa_1(\frac{\partial \sigma_1}{\partial x} + G_1)],\ l_0 \le x \le l_1\\
\frac{\partial \sigma_2}{\partial t} &= \frac{\partial}{\partial x}[\kappa_2(\frac{\partial \sigma_2}{\partial x} + G_2)],\ l_1 \le x \le l_2,\\
&\cdots\cdots\\
\frac{\partial \sigma_{n-1}}{\partial t} &= \frac{\partial}{\partial x}[\kappa_{n-1}(\frac{\partial \sigma_{n-1}}{\partial x} + G_{n-1})],\ l_{n-2} \le x \le l_{n-1},\\
\frac{\partial \sigma_n}{\partial t} &= \frac{\partial}{\partial x}[\kappa_n(\frac{\partial \sigma_n}{\partial x} + G_n)],\ l_{n-1} \le x \le l_n.
\end{aligned}
\tag{3}
$$

Boundary conditions for these equations are given as the follows:

$$
\begin{aligned}
&\kappa_1(\frac{\partial \sigma_1}{\partial x} + G_1) = 0,\ x = l_0,\\
&\sigma_1 = \sigma_2,\ x = l_1,\\
&\kappa_1(\frac{\partial \sigma_1}{\partial x} + G_1) = \kappa_2(\frac{\partial \sigma_2}{\partial x} + G_2),\ x = l_1,\\
&\sigma_2 = \sigma_3,\ x = l_2,\\
&\kappa_2(\frac{\partial \sigma_2}{\partial x} + G_2) = \kappa_3(\frac{\partial \sigma_3}{\partial x} + G_3),\ x = l_2,\\
&\cdots\cdots\\
&\kappa_n(\frac{\partial \sigma_n}{\partial x} + G_n) = 0,\ x = l_n.
\end{aligned}
\tag{4}
$$

Initial conditions for the void nucleation phase are given by $\sigma_i = 0$ at $t = 0$ at each segment. The Laplace transformation technique can be used to obtain an accurate closed-form expression for the void stress evolution equations (3). For the sake of simplicity, we need to assume that $\kappa_1 = \kappa_2 = \cdots = \kappa_k$. After transforming (3)-(4) by the Laplace transformation technique, we get a system of ordinary differential equations (ODEs). In order to derive the solution of each ODE, we first need to introduce the following notations:

$$
\begin{aligned}
&\xi^i_{n,1} = l_n + 2m(l_n - l_0) - x,\\
&\xi^i_{n,2} = (2l_n - l_0) + 2m(l_n - l_0) - x,\\
&\xi^i_{n,2j+1} = (2l_n - l_j) + 2m(l_n - l_0) - x,\\
&\xi^i_{n,2j+2} = \begin{cases} (2l_n - 2l_0 + l_j) + 2m(l_n - l_0) - x, \ 0 < j < i,\\ l_j + 2m(l_n - l_0) - x, \ i \le j \le n-1, \end{cases}\\
&\eta^i_{n,1} = (l_n - 2l_0) + 2m(l_n - l_0) + x,\\
&\eta^i_{n,2} = -l_0 + 2m(l_n - l_0) + x,\\
&\eta^i_{n,2j+1} = (l_j - 2l_0) + 2m(l_n - l_0) + x,\\
&\eta^i_{n,2j+2} = \begin{cases} -l_j + 2m(l_n - l_0) + x, \ 0 < j < i,\\ (2l_n - l_j - 2l_0) + 2m(l_n - l_0) + x, \ i \le j \le n-1. \end{cases}
\end{aligned} \tag{5}
$$

Similar to the analytical method introduced in [], we need to construct the following basic function:

$$
g(x,t) = 2\sqrt{\frac{\kappa t}{\pi}} e^{-\frac{x^2}{4\kappa t}} - x \times \mathtt{erfc}\{\frac{x}{2\sqrt{\kappa t}}\}. \tag{6}
$$

where the complementary error function $\mathtt{erfc}\{x\}$ is defined as $\mathtt{erfc}\{x\} = \frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-t^2}dt$. Thus, a general form of analytical solutions of stress evolution equations for all segments can be written as follows

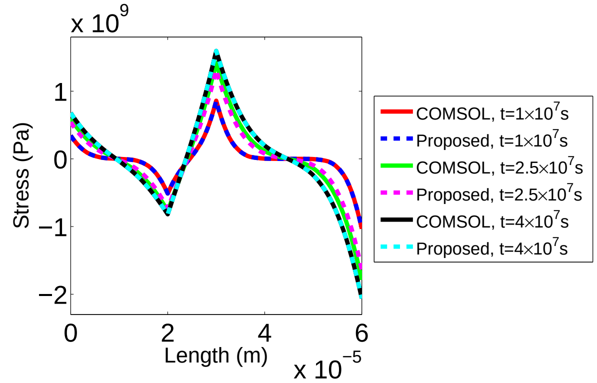
$$
\begin{aligned}
&\sigma_{n,i}(x,t) = -\sum_{m=0}^{+\infty}\{G_n g(\xi^i_{n,1},t) - G_1 g(\xi^i_{n,2},t)\\
&-\sum_{j=1}^{n-1}\frac{G_{j+1}-G_j}{2} g(\xi^i_{n,2j+1},t) - \sum_{j=1}^{n-1}\frac{G_{j+1}-G_j}{2} g(\xi^i_{n,2j+2},t)\}\\
&-\sum_{m=0}^{+\infty}\{G_n g(\eta^i_{n,1},t) - G_1 g(\eta^i_{n,2},t) - \sum_{j=1}^{n-1}\frac{G_{j+1}-G_j}{2} g(\eta^i_{n,2j+1},t)\\
&-\sum_{j=1}^{n-1}\frac{G_{j+1}-G_j}{2} g(\eta^i_{n,2j+2},t)\}.
\end{aligned} \tag{7}
$$

It should be noted that the present analysis method in analytical characterization is similar to what has been reported in our early work of analytical modeling of electromigration for multi-branch interconnect tree. However, the purpose of this paper is not to develop any new analysis method, but to illustrate the closed-form expression for the solution of stress evolution equations for more complex interconnect trees embedded in the frequently employed circuits.

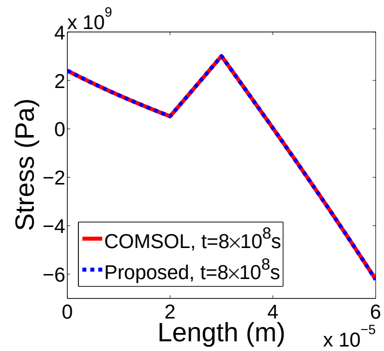
IV. Experimental results and discussions

In order to show the accuracy of the proposed model of EM reliability for multi-branch interconnect three with line structure shown in Fig. 3, the analytical solution (7) of stress evolution equations is calculated with the MATLAB enviroment, and we also compare the results from (7) with those obtained by the finite element tool COMSOL [**?**]. In our experiments, we test three cases of interconnect tree structure by changing n in (7) which is the number of wire segments in the interconnect tree. In the simulations to be described below, the following parameter values will be used: $Z^* = 10$, $\rho = 3 \times 10^{-8}\Omega/m$, $\Omega = 8.78 \times 10^{-30}m^3$, $B = 5.5 \times 10^{10}Pa$, $D_0 = 5.5 \times 10^{-5}m^2/s$, $E_a = 1.1eV$, $e = 1.6 \times 10^{-19}C$, $k = 1.38 \times 10^{-23}J/K$, and $T = 350K$.

A. *Four-terminal interconnect tree* ($n = 3$)

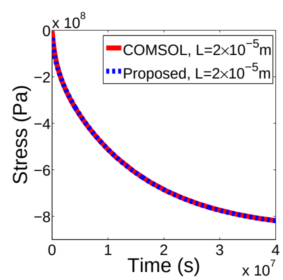


(a)

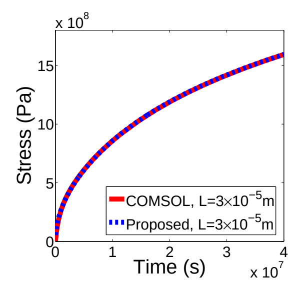


(b)

Fig. 4. The experiments results of straight line at changing temperatures, $j_1 = 2 \times 10^{10}A/m^2$, $j_2 = 6 \times 10^{10}A/m^2$ (a) the comparison of stress evolution at square wave temperature at a fixed time; (b) the comparison of stress evolution at sine wave temperature at a fixed time; (c) the comparison of stress evolution at square wave temperature at a fixed position; (d) the comparison of stress evolution at sine wave temperature at a fixed position



(a)



(b)

Fig. 5. The experiments results of straight line at changing temperatures, $j_1 = 2 \times 10^{10}A/m^2$, $j_2 = 6 \times 10^{10}A/m^2$ (a) the comparison of stress evolution at square wave temperature at a fixed time; (b) the comparison of stress evolution at sine wave temperature at a fixed time; (c) the comparison of stress evolution at square wave temperature at a fixed position; (d) the comparison of stress evolution at sine wave temperature at a fixed position

We first analyze the four-terminal interconnect tree with three wire segments with the current flow directions as shown in Fig. []. A constant current density of $j_1 =??$ is applied in the left segment, $j_2 =$ is used to stress the middle wire segment, and $j_3 =$ represent the current density of the right wire segment. The lengths for each wire segment in this interconnect tree are set to be $?m$, $?m$, and $?m$, respectively.

We first analyze the 3-terminal interconnect tree with two segments with the current flow directions as showed in Fig.**??**. Fig.**??** shows obtained evolution of the stress distribution. From Fig.**??** and Fig.**??**, analytical solution obtained with the proposed method fits well to the results of the numerical simulation at every time instance. For a fixed position shown in Fig.**??** and Fig.**??**, the stress changes in agreement with the varying temperatures. In the case of square ware temperature, when temperature rises up to $380K$, the stress is going through dramatic changes. On the contrary, when temperature is $320K$, the stress is smoothly continuous. In the case of sine wave temperature, when temperature falls down to below $250K$, the stress has a process of slow change. And average temperature is useless for the interconnect tree analysis. Thermal effects depended on the varying temperature are remarkable.

We first apply the COMSOL to compute an accurate numerical solution of (1). We set the width of 3D interconnect tree as $10^{-8}m$, considering the thin and narrow interconnect line. Then we use the proposed approach to estimate the stress in MATLAB. Comparisons are shown in Fig. **??**, Fig. **??**, and Fig. **??**.

B. Five-terminal interconnect tree ($n = 4$)

C. Six-terminal interconnect tree ($n = 5$)

V. Conclusion

In this paper, we have proposed a new analysis methods for multi-terminal interconnect tree considering changing temperature. The new time-varying model show an excellent agreement with the detailed numerical analysis obtained from COMSOL.

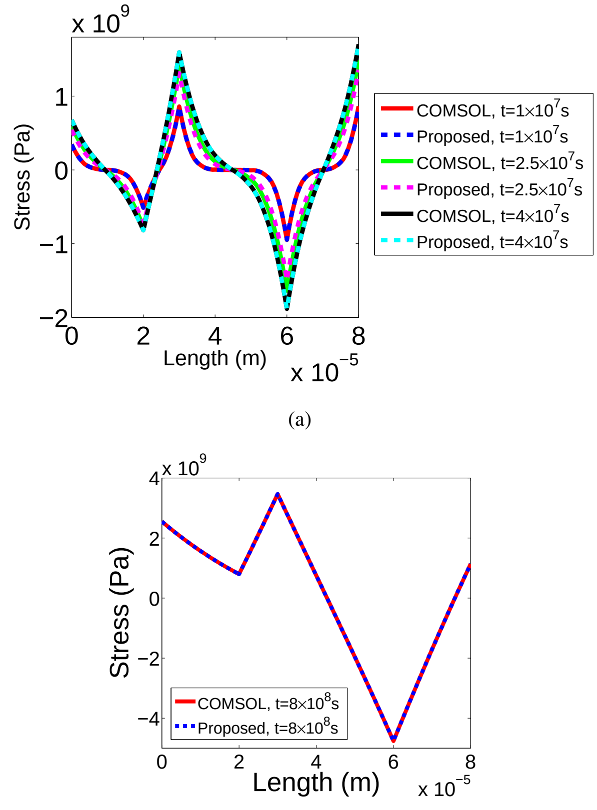


Fig. 6. The experiments results of straight line at changing temperatures, $j_1 = 2 \times 10^{10} A/m^2$, $j_2 = 6 \times 10^{10} A/m^2$ (a) the comparison of stress evolution at square wave temperature at a fixed time; (b) the comparison of stress evolution at sine wave temperature at a fixed time; (c) the comparison of stress evolution at square wave temperature at a fixed position; (d) the comparison of stress evolution at sine wave temperature at a fixed position

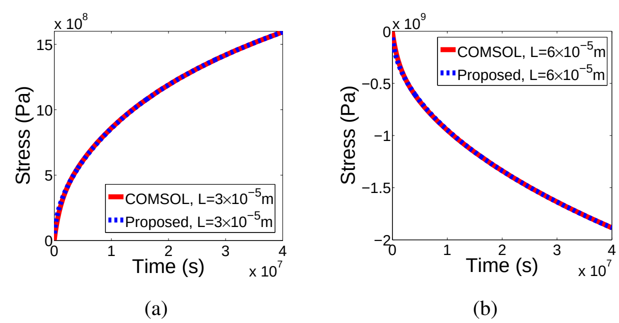


Fig. 7. The experiments results of straight line at changing temperatures, $j_1 = 2 \times 10^{10} A/m^2$, $j_2 = 6 \times 10^{10} A/m^2$ (a) the comparison of stress evolution at square wave temperature at a fixed time; (b) the comparison of stress evolution at sine wave temperature at a fixed time; (c) the comparison of stress evolution at square wave temperature at a fixed position; (d) the comparison of stress evolution at sine wave temperature at a fixed position

References

[1] B. Bailey, "Thermally challenged," *Semiconductor Engineering*, pp. 1–8, Dec. 2013.

[2] S. Chatterjee, M. B. Fawaz, and F. N. Najm, "Redundancy-aware electromigration checking for mesh power grids," in *IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, 2013.

[3] V. Mishra and S. S. Sapatnekar, "The impact of electromigration in Copper interconnects on power grid integrity," in *Proc. Design Automation Conf. (DAC)*, June 2013.

[4] I. A. Blech, "Electromigration in thin aluminum films on titanium nitride," *Journal of Applied Physics*, vol. 47, no. 4, pp. 1203–1208, 1976.

[5] J. R. Black, "Electromigration-a brief survey and some recent results," *IEEE Transactions on Electron Devices*, vol. 16, no. 4, pp. 338–347, 1969.

[6] J. R. Lloyd, "New models for interconnect failure in advanced IC technology," in *International Symposium on the Physical and Failure Analysis of Integrated Circuits*, pp. 1–7, 2008.

[7] M. Hauschildt, C. Hennesthal, G. Talut, O. Aubel, M. Gall, K. B. Yeap, and E. Zschech, "Electromigration early failure void nucleation and growth phenomena in Cu and Cu(Mn) interconnects," in *IEEE International Reliability Physics Symposium (IRPS)*, pp. 2C.1.1–2C.1.6, 2013.

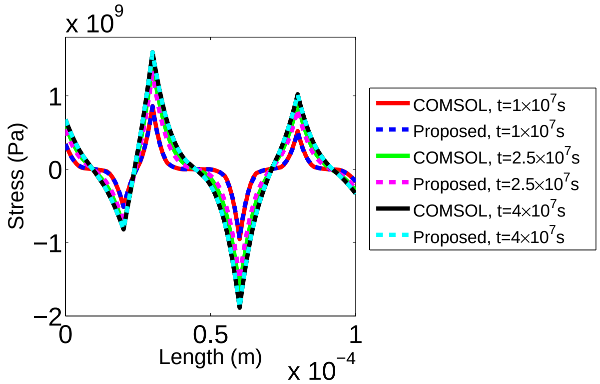
[8] M. Pathak, J. S. Pak, D. Pan, and S. K. Lim, "Electromigration modeling and full-chip reliability analysis for BEOL interconnect in TSV-based 3D ICs," in *IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, pp. 555–562, 2011.

[9] X. Huang, T. Yu, V. Sukharev, and S. X.-D. Tan, "Physics-based electromigration assessment for power grid networks," in *Proc. Design Automation Conf. (DAC)*, June 2014.

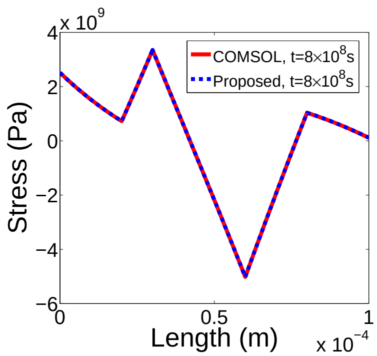
[10] V. Sukharev, "Beyond Black's equation full-chip EM/SM assessment in 3D IC stack," *Microelectronic Engineering*, vol. 120, pp. 99–105, May 2014.

[11] M. A. Korhonen, P. Borgesen, K. N. Tu, and C. Y. Li, "Stress evolution due to electromigration in confined metal lines," *Journal of Applied Physics*, vol. 73, no. 8, pp. 3790–3799, 1993.

[12] V. Sukharev, A. Kteyan, E. Zschech, and W. D. Nix, "Microstructure

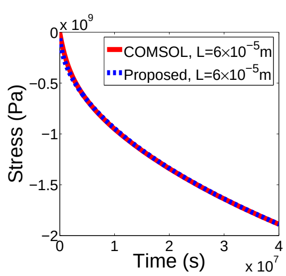


(a)

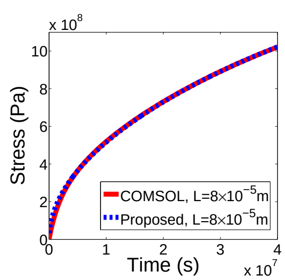


(b)

Fig. 8. the



(a)



(b)

Fig. 9. The experiments results of straight line at changing temperatures, $j_1 = 2 \times 10^{10} A/m^2,\ j_2 = 6 \times 10^{10} A/m^2$ (a) the comparison of stress evolution at square wave temperature at a fixed time; (b) the comparison of stress evolution at sine wave temperature at a fixed time; (c) the comparison of stress evolution at square wave temperature at a fixed position; (d) the comparison of stress evolution at sine wave temperature at a fixed position

effect on EM-induced degradations in dual inlaid Copper interconnects," *IEEE Transactions on Device and Materials Reliability*, vol. 9, no. 1, pp. 87–97, 2009.

[13] "Comsol multiphysics." http://www.comsol.com.

[14] S. P. Hau-Riege and C. V. Thompson, "Experimental characterization and modeling of the reliability of interconnect trees," *Journal of Applied Physics*, vol. 89, pp. 601–609, January 2001.

[15] A. V. Vairagar, S. G. Mhaisalkar, M. A. Meyer, E. Zschech, A. Krishnamoorthy, K. N. Tu, and A. M. Gusak, "Direct evidence of electromigration failure mechanism in dual-damascene Cu interconnect tree structures," *Applied Physics Letters*, vol. 87, pp. 081909–1–081909–4, August 2005.

[16] C. Thompson, S. Hau-Riege, and V. Andleigh, "Modeling and experimental characterization of electromigration in interconnect trees," in *AIP Conference Proceedings*, pp. 62–73, 1999.

[17] C. L. Gan, C. V. Thompson, K. L. Pey, and W. K. Choi, "Experimental characterization and modeling of the reliability of three-terminal dual-damascene Cu interconnect trees," *Journal of Applied Physics*, vol. 94, pp. 1222–1228, July 2003.

[18] S. M. Alam, *Design Tool and Methodologies for Interconnect Reliability Analysis in Integrated Circuits*. PhD thesis, Massachusetts Institute of Technology, September 2004.

[19] S. P. Hau-Riege and C. V. Thompson, "Electromigration saturation in a simple interconnect tree," *Journal of Applied Physics*, vol. 88, pp. 2382–2385, September 2000.

[20] S. Hau-Riege, *New Methodologies for Interconnect Reliability Assessments of Integrated Circuits*. PhD thesis, Massachusetts Institute of Technology, June 2000.

[21] J. J. Clement, S. P. Riege, R. Cvijetic, and C. V. Thompson, "Methodology for electromigration critical threshold design rule evaluation," *IEEE Trans. on Computer-aid Design of Integrated Circuits and Systems*, vol. 18, no. 5, pp. 576–581, 1999.

[22] A. Abbasinasab and M. Marek-Sadowska, "Blech effect in interconnects: applications and design guidelines," in *International Symposium on Physical Design (ISPD)*, pp. 111–118, 2015.

[23] F. Abraham, *Homogeneous Nucleation Theory*. New York: Academic Press, 1974.

[24] Z. Suo, *Reliability of Interconnect Structures*, vol. 8 of *Comprehensive Structural Integrity*. Amsterdam: Elsevier, 2003.

[25] M. Lin and A. Oates, "An electromigration failure distribution model for short-length conductors incorporating passive sinks/reservoirs," *IEEE Transactions on Device and Materials Reliability*, vol. 13, pp. 322–326, March 2013.

[26] V. Sukharev, X. Huang, and S. X.-D. Tan, "Electromigration induced stress evolution under alternate current and pulse current loads," *Journal of Applied Physics*, vol. 118, pp. 034504–1–034504–10, 2015.

[27] S. Alam, G. Lip, C. Thompson, and D. Troxel, "Circuit level reliability analysis of cu interconnects," in *International Symposium on Quality Electronic Design (ISQED)*, pp. 238–243, 2004.

[28] C. Gan, C. Thompson, K. Pey, W. Choi, C. Chang, and Q. Guo, "Experimental characterization of the reliability of multi-terminal dual-damascene copper interconnect trees," in *Materials Research Society Symposium Proceedings (MRSSP)*, pp. 121–126, 2003.

[29] H. Chen, S. X.-D. Tan, V. Sukharev, X. Huang, and T. Kim, "Interconnect reliability modeling and analysis for multi-branch interconnect trees," in *Proc. Design Automation Conf. (DAC)*, June 2015.