Probabilistic Solution to the Jumping Monkey

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1 The Jumping Monkey Problem

The *Jumping Monkey* problem is stated as follows; given a forest with n trees, a sneaky monkey (Bob), and a heartless hunter (Chuck), we are concerned with the life-expectancy of Bob. Specifically, we suppose a known adjacency structure among the trees that compose the forest, and grant Chuck with a powerful – but tree-friendly – weapon capable of killing Bob if aimed at the right tree, while preserving the tree itself. Chuck never sees Bob, except when he dies. To give Bob a better chance of survival, we further impose that if Chuck shoots the wrong tree, Bob will invariably jump randomly to another neighbor tree. Given this context, we ask the two following question: given an unlimited amount of bullets, what is the best strategy Chuck can employ to kill Bob?

2 Markov Process Modeling

The jumping process followed by Bob can be modeled as a Markov process, in which the murder attempts of Chuck are represented as a temporary deletion of outgoing flow from the tree being shot. Let us denote $\{T_i\}_{i=1}^n$ the set of nodes/trees, and $\Lambda(t) = [\lambda_{i,j}(t)]_{1\leq i,j\leq n}$ the flow structure on the forest at time t. We have:

$$\lambda_{i,j}(t) = \begin{cases} \dfrac{\delta_{i,j}}{d_i} & \text{if } S(t) \neq i \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where t denotes the discrete time, $S(t)$ is the tree shot by Chuck at time t, $\delta_{i,j} = 1$ if trees $i \neq j$ are neighbors (0 otherwise), and d_i is degree of node i. Given this transition process, we use Chapman-Kolmogorov equation to describe the evolution of the probability $\pi_i(t)$ for Bob to be in tree i at time $t+1$:

$$\pi_i(t+1) = \sum_{j\neq i} \pi_j(t)\lambda_{j,i}(t) \tag{2}$$

Initializing this system with $S(0) = 0$ – and $S(k) \in [\![1, n]\!] \ \forall k > 0$ – the best strategy for Chuck simply consists in shooting the tree where Bob most likely hides. The temporary deletion of outgoing transitions from the tree being shot at time t will most efficiently affect the *propagation* process by penalizing the most significant "redistribution" of probabilities from time t to $t + 1$.

3 The Thinking Monkey

Up to now, we denied Bob any significant cognitive faculty. But as shown on figure 1, Bob can think, and we wonder how the previous strategy can be adapted if Bob is free to decide at each time step whether he jumps to another tree or continues to hide. For simplicity, we assume that there is a static probability p that he jumps. In this case, the previous transitions can simply be updated to:

$$\lambda_{i,j}(t) = \begin{cases} \dfrac{p \ \delta_{i,j}}{d_i} & \text{if } S(t) \neq i \text{ and } i \neq j \\ (1-p) & \text{if } S(t) \neq i \text{ and } i = j \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

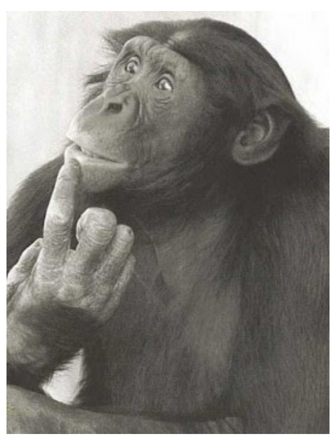

Fig. 1. Bob thinks