CLASSES FOR FAST MAXIMUM ENTROPY TRAINING

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ABSTRACT

Maximumentropymodelsareconsideredbymanytobe of the most promising avenues of language modeling research. Unfortunately, long training times make maximum ent ropy research difficult. We present a novel speedup tec hnique: we changetheformofthemodeltouseclasses.Ours peedupworks by creating two maximum entropy models, the first o f which predictstheclassofeachword, and the second of whichpredicts the word itself. This factoring of the model leads to fewernonzero indicator functions, and faster normalization, achieving speedups of up to a factor of 35 over one of the be st previous techniques. Italsoresults in typically slightly lowerperplexities. The same trick can be used to speed training of oth er machine learningtechniques, e.g. neuralnetworks, applied toanyproblem withalargenumberofoutputs, such as languagemo deling.

1. INTRODUCTION

Maximum entropy models [1] are perhaps one of the m ost promising techniques for language model research. These techniques allow diverse sources of information to becombined. Foreach source of information, a set of constraint sonthemodel can be determined, and then, using an algorithm suc h as Generalized Iterative Scaling (GIS), a model can be found that satisfies all of the constraints, while being as sm oothaspossible. However, training maximum entropy models can be ext remely time consuming, taking weeks, months, or more. We showthat by using word classing, the training time can be si gnificantly reduced, by up to a factor of 35. In particular, w e change the formofthemodel. Instead of predicting words dir ectly, we first predict the class that the next word belongs to, an dthen predict the word itself, conditioned on its class. Thete chniqueusedis actually more general: it can be applied to any pro blem where there are a large number of outputs to predict, wit h language modeling being just one example. Furthermore, the technique $applies\,not\,only\,to\,maximum entropy\,models, but to$ almostany machine learning technique for predicting probabili ties that is slowed by a large number of outputs, including many uses of decisiontreesandneuralnetworks.

In this paper, we first give a very brief introduc tion to maximum entropy techniques for language modeling. We then go on to describe our speedup. After this, we desc ribeprevious researchinspeedingupmaximumentropytraining,a ndcompare ittoourtechnique. Next, we give experimentalre sults, showing both the increased speed of training, and a slight reduction in perplexity of the resulting models. Finally, we co nclude with a short discussion of how these results can be applie d to other machinelearningtechniquesandtootherproblems.

We begin with a very quick introduction to language models in general, and maximum entropy-based language mode ls in particular. Language models assign probabilities to word sequences $P(w_1...w_n)$. Typically, this is done using the trigram approximation:

$$P(w_1...w_n) = \prod_{i=1}^n P(w_i \mid w_1...w_{i-1}) \approx \prod_{i=1}^n P(w_i \mid w_{i-2}w_{i-1})$$

Maximum entropy models for language modeling [2] do not necessarily use the n-gram approximation and can in condition on arbitrary length contexts. The genera conditionalmaximumentropymodelisas follows:

$$P'(w \mid w_1...w_{i-1}) = \frac{\exp\left(\sum_{j} \lambda_j f_j(w, w_1...w_{i-1})\right)}{Z_{\lambda}(w_1...w_{i-1})}$$

The λ_j are real-valued constants learned in such a way as optimize the perplexity of training data. $Z_{\lambda}(w_1...w_{i-1})$ is a normalizing constant so that the sum of all probabi lities is 1,

simply set equal to
$$\sum_{w} \exp\left(\sum_{j} \lambda_{j} f_{j}(w, w_{1}...w_{i-1})\right)$$
. The f_{j}

represent a large set of indicator functions that a lways have the value1or0.Forinstance,wecoulduse f_i ("Tuesday", w $_1$... w_{i-1}) = 1 if w_{i-1} = "on" and w_{i-2} = "meet" (and, implicitly, otherwise 0). If λ_i were given a positive value, then the probability o "Tuesday" in the context of "meet on" would be rais making many indicator functions of this type, we co uld capture alloftheinformationcapturedbyatrigram. Simi larly, we could make a bigram indicator function with $f_i("Tuesday", w_1...w_{i-1})$ = 1 if $w_{i-1} = "on"$. Or we could make a unigram indicator function with the simple $f_i("Tuesday", w_1...w_{i-1})=1$ for all $w_1...w_{i-1}$. In principle, any set of indicator functions that dependson w,w 1...wi-1canbeused,includingn-grams,caching, skippingn-grams,wordtriggers,etc.

The optimal $\,\lambda$ -values must be learned. The algorithm – Generalized Iterative Scaling [1] – for optimizing (basedonsomesetoftrainingdata)canbeverysl ow. Itrequires many iterations, and at each iteration, it involves words in the training data. We give here a very ro the algorithm, with only the inner loop of this code, and the most time-cons uming

The inner loop of this code, and the most time-cons uming part is lines 4 to 12. Notice that the inner loop contains several loops over all words in the vocabulary (lines 4,7 and 8). Notice

```
For each iteration
2
       observed[1..# of indicators] \leftarrow \{0\}
      For i = 1 to |training data|
3
         For each word w in Vocabulary
4
5
            \texttt{unnormalized[w]} \leftarrow \exp(\sum \lambda_j f_j(w, w_1 ... w_{i-1}))
6
         z \leftarrow \sum unnormalized[w]
7
         For each word w in Vocabulary
8
            For each j such that f_i(w, w_1...w_{i-1}) \neq 0
9
              observed[j]+=f; Xunnormalized[w]/z
10
11
           Next i
12
         Next w
13
      Next i
14
      For each indicator fi
         re-estimate \lambda_j using observed[j]
15
16
      Next j
17
    Next iteration
```

that the sum in line 5 is typically bounded by the number of different types of indicator functions. In particu lar, a given system will typically have only a few types of indi cators-e.g. unigram, bigram, and trigram - and typically, for e ach of these types, and for a given word w, and history, there c anbeonlyone non-zero indicator function. This means that the s um in line 5 and the loop in line 9 are bounded by the number of types of indicator function. Overall, then, the inner loop oflines4to12 istypicallyboundedbythenumberoftypesofindi catorfunction, times the vocabulary size. This means that decreas ing the vocabulary size leads to a decrease in the runtime of the inner loop. Certain types of indicator functions (e.g. t riggers) and optimizations to line 5 (summing only over non-zero f i) change the exact analysis of run-time, but not the intuiti on that innerlooprun-timeisroughlyproportionaltovocabulary size.

2. CLASS-BASED SPEEDUP

We now describe our speedup. We assign each word in the vocabulary to a unique class. For instance, catan ddog might be in the class of ANIMAL, while Tuesday and Wednesday might be in the class of WEEKDAY. Next, we observe that

 $P(w \mid w_1...w_{i-1}) = P(class(w) \mid w_1...w_{i-1}) \times P(w \mid w_1...w_{i-1}, class(w))$ This equality holds because each word is in a single eclass, and is easily proven. Conceptually, it says that we can d ecomposethe prediction of a word given its history into predict ionofitsclass given the history, and the probability of the word given the history and the class. For "true" probabilities, t his equality is exact. If the probabilities are not true, but inst ead are, for instance, the results of estimating a model, or are smoothed, or are the results of computing a maximum entropy mode 1. the equalitywillnotbeexact,butwillbeaverygood approximation. Indeed, the approximation is so good that typically classing is usedtolowertheperplexityofmodels.

This decomposition is the basis for our technique. Rather than create a single maximum entropy model, we create the two different models, the first of which predicts the class of a word given its context $P(class(w)/w_1...w_{i-1})$, and the second of which

predicts a word given its class and its context, $P(w/w_1...w_{i-1}, class(w_i))$. The process of training each of these two mode ls is completely separate. If we have 100 classes in our the inner loop of the training code for predicting bounded by a factor of 100, rather than a factor of size. Thus, this model can be computed relatively quickly.

Next,considertheunigram,bigramandtrigramindi catorsfor a class-based model. An example unigram indicator would be $f_j($ "Tuesday", $w_1...w_{i-1},class(w_i)$)=1 if $class(w_i)$ =WEEKDAY. Forthebigramindicator,wewouldhave $f_j($ "Tuesday", $w_1...w_{i-1}, class(w_i)$)=1 if $class(w_i)$ =WEEKDAY and w_{i-1} ="on"; and a trigramindicatorwouldbe $f_j($ "Tuesday", $w_1...w_{i-1},class(w_i)$)=1 if $class(w_i)$ =WEEKDAYand w_{i-1} ="on" and w_{i-2} ="on" and on "on".

Notice an important fact: for a word w not in the same class as w_i , $P(w/w_1...w_{i-1}, class (w_i))=0$. This means that we can modify the loops of lines 4, 7 and 8 to loop only o ver those words w such that w is in the same class as w_i . Now, if each class has 100 words, then the run time of the inner loop is bounded by a factor of 100. (If we were to explicit lyperform the computations, the unigram λ 's for words w not in $class(w_i)$ would be set to ∞ , and the unnormalized probabilities would be 0, leading to no contribution in lines 7 and 10).

Consider a hypothetical example, with a 10,000 word vocabulary,100 classes and 100 wordsperclass. T heinnerloop of the standard training algorithm would require time proportional to 10,000. Alternatively, we can use speedup. Both the innerloop for learning the classed smodel, and the inner loop for running the word-given-class model are bounded by a factor of 100, leading to an overall hypothetical improvement of 10,000/(100+100)=50.

We can extend this result to 3 or more levels, by p redicting first a super-class, e.g. NOUN, and then a class, e.g. WEEKDAY, and finally the word, Tuesday. Such a dec omposition will further reduce the maximum number of indicator functions, but, since there is some overhead to eac h level, we havenotfoundimprovements by extending beyond 31 evels.

3. PREVIOUS RESEARCH

Maximumentropy has been well studied. [1] gives the classic Generalized Iterative Scaling algorithm, although i n a form suitable for joint probabilities, as opposed to the probabilities given here, and is somewhat dense; [2] is a classic introduction to the use of maximum entropy models or language modeling, but despite the fact that [2] uses condit ional probabilities, most of the discussion is of joint probabilities.

[2] has previously used a simple form of classes with maximumentropy-based language models. However, the eywere used only as conditioning variables; i.e., indicator functions like $f_j(w/w_{i-1}) = 1$ if w = y and class $(w_{i-1}) = x$ were used. They were not used for p redicting outputs, and thus did not lead to speed ups.

Word classes have, of course, been used extensively in language modeling, including [2][3][4][5]. However researchhasfocusedmostlyonimproving perplexity orreducing language model size, and neverto our knowledge for speed. Note that we have previously used a model f

similartotheoneusedhereforreducinglanguage modelsize,by uptoafactorof3,atthesameperplexity[5].

Therehavebeenthreenoteworthypreviousattempts to speed up maximum entropy models: unigram caching, Improve diterativeScaling(IIS)[6], and cluster expansion [7][8].

Unigram caching makes use of the following observat ion: mostbigramandtrigramindicators are not used in practice(e.g. ifthestring"YorkFrancisco"neveroccurredinth etrainingdata, then there will not be any bigram indicators for th at case). On theotherhand, all possible unigramindicators typ ically areused. Thismeansthattypically, the vast majority of ind icatorfunctions thatarenon-zeroforagivencontextareunigrami ndicators; also notice that these unigram indicators are independen t of context, meaning computation can be easily shared. In unigr amcaching, the effect of the unigram indicators is pre-compute d and the computationsoftheinnerlooparerearrangedsoth attheydepend onlyonthosenon-unigramindicatorsthattakeano n-zero value. In practice, the number of non-zero indicators stil I tends to be proportionaltothevocabularysize(sincethenumb erofnon-zero bigrams, trigrams, and similar indicator-functions for a given historyisboundedbythevocabularysize).

We have implemented unigram caching and it leads t o considerable speedups over the naïve implementation timesspeedupisaspeedupoverunigramcaching.O urtechnique can be used with or without unigram caching, but be some extra overhead involved in unigram caching, an our technique drastically reduces the number of uni usually besttouseourtechnique without unigram aching.

In Improved Iterative Scaling [6], a different upda technique is used. It introduces additional overhea d that slows down the time for each iteration of the iterative s caling algorithm, but allows larger steps to be taken at e ach time, leading to fewer iterations, and overall faster per formance. It also introduces additional memory overhead and codi ng complexity. The main benefits from improved iterat ive scaling come from certain models in which the total numberofindicator functions that can be true for a certain time is hi ghly variable. The learning speed of Generalized Iterative Scaling is inversely $\max_{i} \sum_{j} f_{j}(w, w_{1}...w_{i-1})$. proportional to the value of

GIS uses the maximum of this value to slow learning , while IIS slowslearningonacase-by-casebasis.Insomemo dels.thissum can be very different for different w, i. In particular, models using caching and triggering techniques can lead to these different numbers of active indicators. In other m odels, such as n-gram-stylemodels, there is a fixed maximum fort henumberof non-zero indicators. In these models, IIS would le adtolittleor noreductioninthenumberofiterationsofiterati vescaling, and, because of the additional overhead for each iterati on, might actuallyleadtoaslowdown.

The last technique we consider is the most powerful one, clusterexpansion,introducedin[7]andexpandedi n[8].Cluster expansion can be regarded as a natural extension of unigram caching to n-grams. Consider a simple trigram mode 1. With some straightforward rearrangement of the equations , for two trigrams with a common bigram, most of the computat ioncanbe shared. In [8], this technique is extended to hand le cases in which there is limited interaction between hierarch ical constraints, and still achieves good speedups (a fa ctor of 15.) However, as [7] concludes, cluster expansion "is li mited in its

usefulness... When the number of interacting constrai nts is large...the cluster expansion is of little use in compexact maximum entropy solution." We believe that the same conclusionappliesto [8]. In particular, a simple model combining bigram 1-back, 2-back, ..., 5-back constraints would probably shown o, or only small, gains from the techniq ues of [7] or [8], while for our technique, the gains would be about the same.

In theory, our speedup can be used in conjunction w ith IIS, unigramcaching, or cluster expansion. However, in conjunction with unigram caching, in experiments, it typically leads to only small speed improvements, or sometimes actual slowd owns (because unigram caching introduces overhead in oth er parts of the algorithm). Similarly, we suspect that with cl usterexpansion speedupmightbelimited. Wehavenottestedoura lgorithmwith IIS, but in principle, there is no reason they coul d not be combined, and we guess the combination would work w ell.

4. RESULTS

Weranourexperimentsusing four different learnin simple GIS, GIS with unigram caching, GIS with a two o level clustering, and GIS with athreelevel clustering. Weranon four different sizes of training data. The model used is a "skipping, classing" model with the following types of indicator functions, where W, Y, and Zarevariables filled inforspecific instances of the indicator functions.

Weusedallandonlyindicatorfunctionswherether ewereat least three matching cases in the training data. W e found our word classes by using a top-down splitting algorith m that attemptedtominimizeentropyloss, as described in [9].Weused differentnumbers of classes for different purposes .Forthetwolevel splitting, we used approximately 60-250 class es. For the three-level splitting, we used approximately 8-30 c lasses for the first level, and 100-2000 classes for the second le vel. In all cases, we optimized the number of classes by runnin g one iterationoftraining with varying numbers of classes, andpickingthe fastest. The classes used in the indicator functio ns are not typicallythesameastheclassesusedinourfactorin g;fortheindicator classes, we used 64 classes. We linearly inte rpolated each maximum entropy model with a trigram model, to smoo th and avoid zero probabilities. Our technique interpolat ed with a trigram model reduced overall perplexity from 1% to 5% versusa maximumentropymodelwithoutourtechniqueinterpo latedwith the same trigram model; we were not able to run the baseline perplexity at 10,000,000 words, because the version withoutour $speedups was too slow. \ We used subsets of Wall Str$ eetJournal

data, building the classes from scratchateach siz e, and using the 60,000 most common words in the training data, or a llwords, if therewerefewerthan 60,000 unique words in the tr ainingdata.

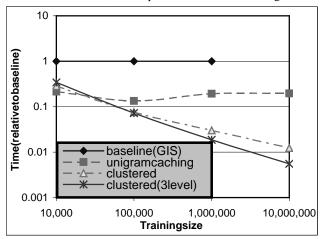


Figure1:SpeedupResults

Figure 1 shows our results, giving relative speeds. Notice that we achieve a speedup of up to a factor of 35 o ver the unigram caching result. We believe this is the lar gest speedup reported. Notice also that at the smallest data si ze, the classing methods actually result in minor slowdowns compared to unigramcaching, but that as the training data size increases, the speedupfromourtechniquealsorapidlyincreases.

5. DISCUSSION

We have discussed our speedup technique in the cont ext of training. However, it can in many cases also be us edfortesting. Inparticular, if in the test situation one needst heprobabilitiesof mostorallwordsinaparticularcontext,ourspee dupwillnotbe helpful.Ontheotherhand,ifoneweretousemax imumentropy models to rescore n-best lists, the speedup would w ork just as well for testing as for training. In the case of, say, rescoring thelatticesdid lattices, the speedup would be helpful, as long as notallowfortoomanywordsineachcontext.

Notice that our speedup technique could be applied to a variety of other problems and to a variety of other learning methods. In particular, there is nothing in partic ular specific to language modeling in our speedup technique, except that we are predictingtheprobabilitiesofalargenumberofo utputs(possible nextwords). Anyotherproblempredicting the prob abilitiesofa large number of outputs could benefit from these me thods. Similarly, there is little that is specific to maxi mum entropy modelsinourtechnique. Forinstance, considertr aininganeural network to learn the probabilities of 10,000 outputs.Eachstepof training would require back-propagating 9,999 zeros and one 1. Alternatively, one could place the outputs into 100 classes. A first network could be trained to learn the class-p robabilities. Each step of training would require back-propagatin g 99 zeros and one 1. Next, we would learn 100 neural network s, for predicting the probability of the outputs given the class, one neural network for each class, predicting a probabi lity for each output in that class. Network i would learn the conditional

probabilities of outputs in class i given that class Each step of training would need only train the net corresponding to the correct class, meaning that ag zeros and one 1 would need to be back-propagated. the number of hidden units of these smaller network only 100 values (for the class, or the outputs give would also be much smaller than the number of hidde networkforpredicting10,000outputsdirectly.

Similarly, there are at least two ways to train dec handle large numbers of outputs (train decision tre outputs at each leaf, or train a binary decision tr possible output and normalize). Again, in both of ourmethodcanbeapplied.

More generally, almost any learning algorithm that at training time when there are a large number of o benefit from our approach. Similarly, any algorith testtime by a large number of outputs, but used in whichonlyafewofthoseoutputsareneeded, would

Ourtechnique is an extremely promising one. Altho only an approximation, rather than an exact techniq thatboththeoreticallyandempiricallyreducesper plexity; itadds very little complexity to coding; it leads to perha ps the largest reported speedups – a factor of 35; these speedups whenneededmost,onlarge,complexproblems;itca independently of the form of the model; and it can both to other learning algorithms and to other prob lemdomains. Wearehopefulthatotherswilluseit, both forma ximumentropy modelingappliedtolanguagemodelingandinmanyo

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