Fixing quantum errors with quantum tricks: A brief introduction to Quantum Error Correction

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Towards Scalable Quantum Devices

"Quantum phenomena do not occur in a Hilbert space, they occur in a laboratory." - Asher Peres

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Desiderata for a universal quantum computer

1. A scalable system with well defined qubits
2. Initialization to a simple fiducial state
3. Long decoherence times
4. The ability to perform a universal set of gates
5. Permit efficient, single-qubit measurements

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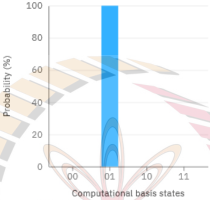
Decoherence: Irretrievable loss of information (coherence) in a quantum state due to interactions with its environment (bath).

Ideal vs Noisy QC



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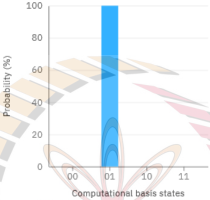

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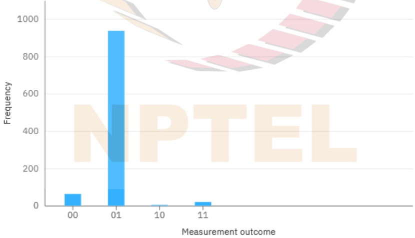

Result of Grover search on the 5-qubit Lima processor

Enter QEC

*"Noisy Intermediate-Scale Quantum (**NISQ**) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably.*

Quantum Error Correction (is) our basis for thinking that quantum computers are scalable to large devices solving hard problems."

– John Preskill, Quantum Computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).

Noise on classical bits

- Bit-flip errors: The Binary Symmetric Channel

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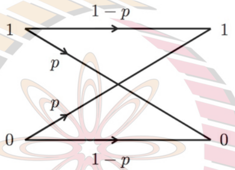

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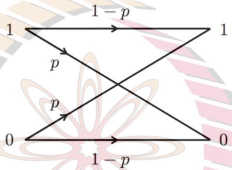

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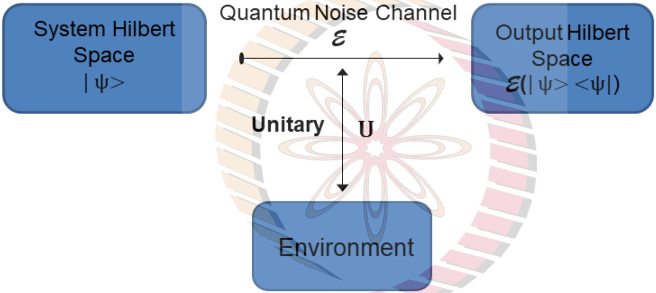
- The 3-bit repetition code : Encode 0 as 000 and 1 as 111. Single-bit errors lead to distinct 3-bit strings:

$$000 \rightarrow 001/010/100;\ 111 \rightarrow 110/101/011$$

- Error **detection**: The decoder decides whether the noisy string corresponds to 000 or 111 by parity checks (majority voting!).
- The 3-bit code corrects errors upto $O(p^2)$.

Noise in quantum systems



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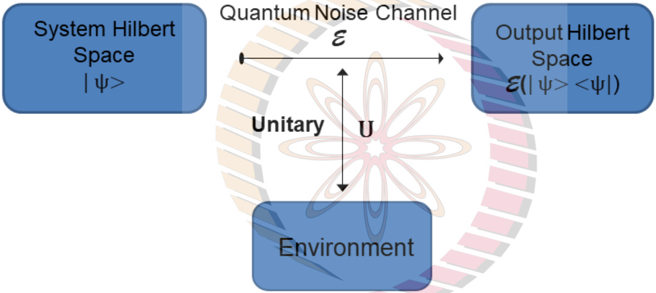
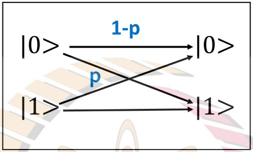


- Noisy evolution is *not unitary* $\Rightarrow$ Irreversible!
- Example: quantum bit-flip noise

Quantum bit-flip noise



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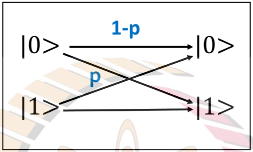

- State of the qubit is affected by an X error with probability p:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

It is left unchanged, with probability $1 - p$.

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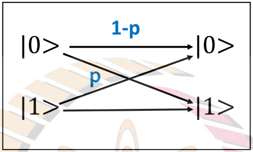

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$$\rho = (1 - p)\,|0\rangle\langle 0| + p|1\rangle\langle 1|\,.$$

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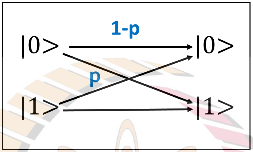

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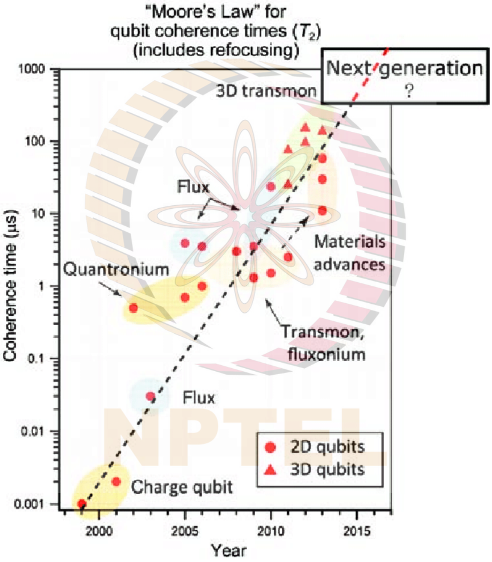
$$\rho = (1 - p)\,|0\rangle\langle 0| + p|1\rangle\langle 1|\,.$$

- The error probability behaves as, $p = \frac{1}{2}\left(1 - \exp -t/T\right)$.
  T is the *coherence* time.

Coherence Times : Superconducting Qubits



Superconducting quantum circuits and circuit-QED, Sep 2017.

Quantum Error Correction

- **The challenges:**
  1. The encoding process has to circumvent the no-cloning theorem.
  2. A continuum of several possible errors can occur on a single qubit: bit-flip errors, phase-flip errors, amplitude-damping errors etc.
  3. Measurement disturbs the state! How do we decode?

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  3. Measurement disturbs the state! How do we decode?
- **Solutions:**
  1. Entanglement to the rescue! We encode into entangled states.
  2. Discretize the errors in terms of a finite, unitary set of errors: Unitary error basis (linearity helps!)
  3. Decode using controlled gate operations!

Quantum Error Correction

"With group and eigenstate, we've learned to fix,
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- The 3-qubit code encodes one qubit into three:

$$|0_L\rangle \rightarrow |0\rangle|0\rangle|0\rangle, \ |1_L\rangle \rightarrow |1\rangle|1\rangle|1\rangle, \text{ so that,}$$
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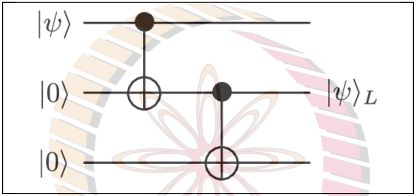
- The states $|0_L\rangle$ and $|1_L\rangle$ are the codewords or logical states.
- The no-cloning theorem forbids an operation of the form,

$$\alpha|0\rangle + \beta|1\rangle \to (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

The encoding circuit



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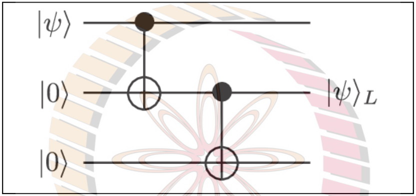

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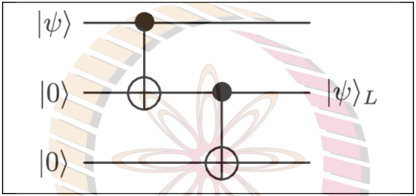

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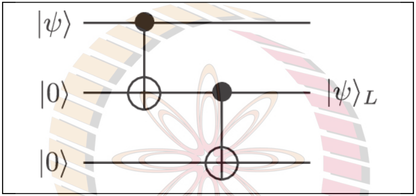

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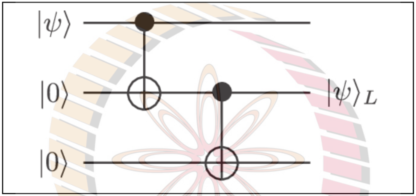

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- The noise now acts on all 3 qubits, but is assumed to act independently and identically on each qubit.
- The leading source of noise is the set of single-qubit errors – XII, IXI and IIX – which occur with probability p.
- Multi-qubit errors occur with probability $O(p^2)$ or $O(p^3)$.

Action of the single bit-flip errors

- Under the action of the single qubit errors, the codewords get modified as,

$$
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|0\rangle|0\rangle|0\rangle &\rightarrow |1\rangle|0\rangle|0\rangle \text{ (or) } |0\rangle|1\rangle|0\rangle \text{ (or) } |0\rangle|0\rangle|1\rangle \\
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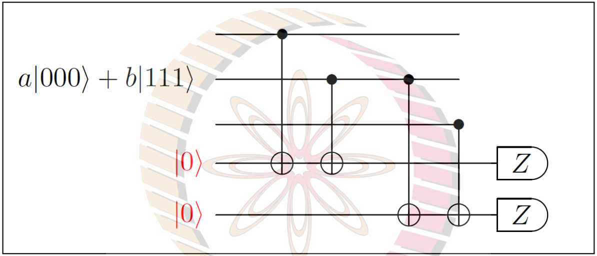
- Bit flip on different qubits lead to distinct states, which are distinguishable $\Rightarrow$ Errors can be detected and corrected.
- Error detection requires a set of measurements in the Z- basis ($\{|0\rangle, |1\rangle\}$ basis). The measurements are not done on the encoded qubits, but on an additional pair of qubits called ancilla qubits.

Error detection and Recovery



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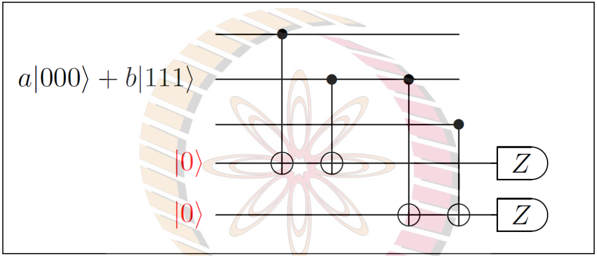

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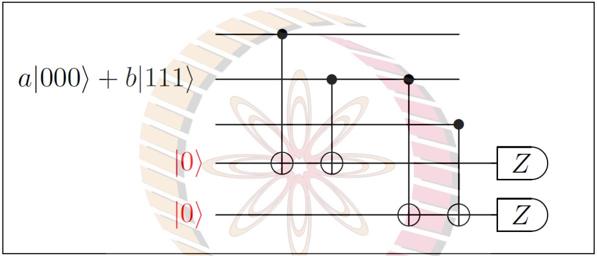

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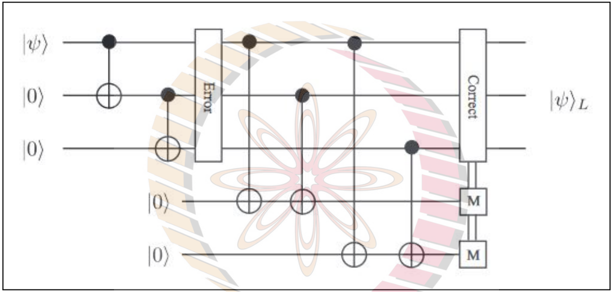
- Based on the value of the syndrome bits, an X gate is applied to one of the 3 encoded qubits.

Syndrome table for the 3-qubit code

$\mathcal{S}$ / $\mathcal{E}$	ZZI	IZZ	Comments	Recovery operator
III	+1	+1	No error	Identity
XII	- 1	+1	bit flip on the first qubit	apply Pauli X on the first qubit
IXI	- 1	- 1	bit flip on the second qubit	apply Pauli X on the second qubit
IIX	+1	- 1	bit flip on the third qubit	apply Pauli X on the third qubit

The 3-qubit QEC circuit



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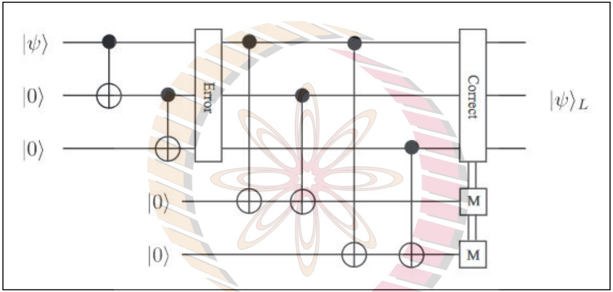

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- QEC improves the fidelity and coherence time of the qubit, provided the error probability $p < \frac{1}{2}$.

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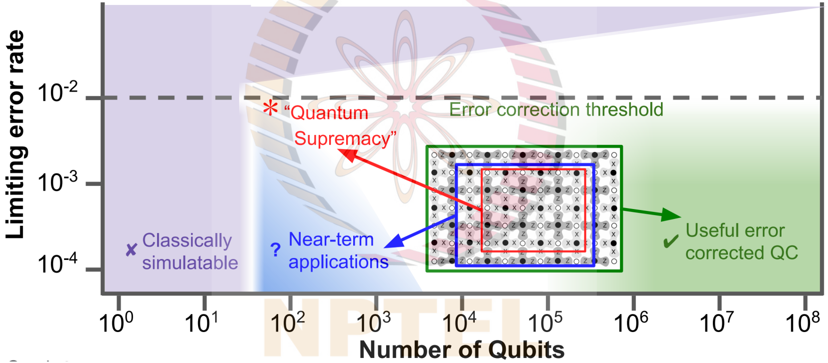
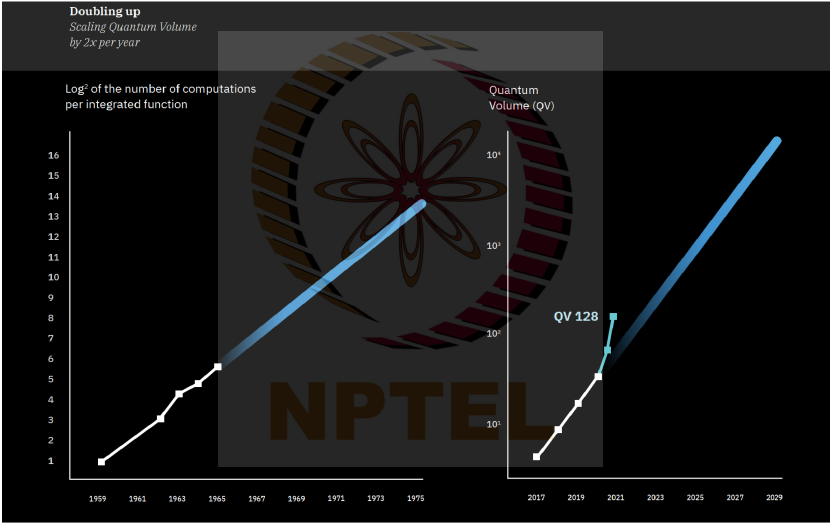
- Further resource requirements:
  - Dealing with faulty gates and circuits : **Fault-tolerance**
  - Bringing down the logical error rate : **concatenation**
    Eg. A concatenated 7-qubit code $\Rightarrow$ 49 physical qubits for one logical qubit!

The road ahead ...



The Google Roadmap (2018)

Future Outlook



The IBM Roadmap (2021)

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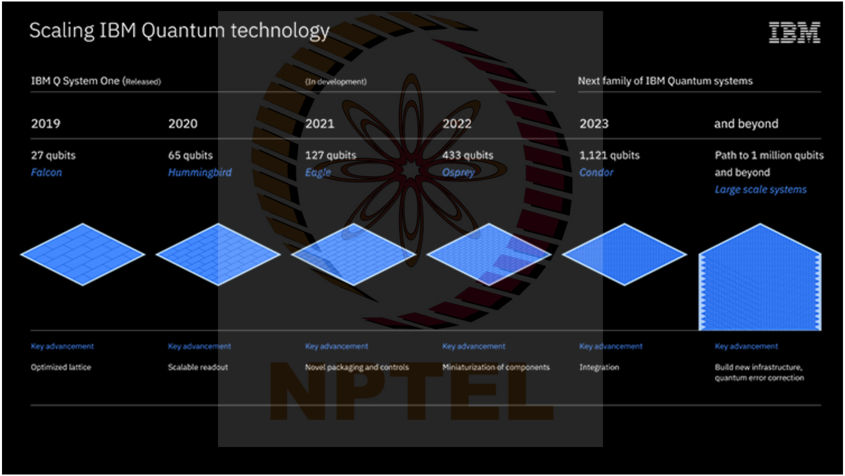

The IBM Roadmap (2021)

References

- Quantum Computing and Quantum Information, Nielsen and Chuang (Chapter 10)
- Lecture notes by Prof. John Preskill (Chapter 7) http://theory.caltech.edu/ preskill/ph229/notes/chap7.pdf