3. $P\{Y_{n+1}=i \mid Y_1=x_1, Y_2=x_2, \cdots, Y_n=x_n\}$

$= P\{Y_n + X_{n+1}=i \mid Y_1=x_1, \cdots\ Y_n=x_n\}$

$$= \begin{cases} \frac{1}{n+1} & i = x_n+1, x_n+2, \cdots, x_n+n+1. \\ 0 & \text{其它}. \end{cases}$$

显出转移条件概率仅与 n 与 x_n 有关,即与当时时刻和当前状态有关 故为马氏链.

一步转移矩阵.

$$P^{(1)}(n) = \begin{pmatrix} 0 & & & & & & \\ & 0 & \cdot & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & & \\ & & & 0. & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \\ & 0 & & & \ddots & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\ & & & & & & \ddots & & \end{pmatrix}$$

5. $P=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $P^2=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $P^3=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. - - - 极限 $\lim\limits_{n\to\infty} P_{ij}^{(n)}$ 不存在,非遍历.

$V=(v_1, v_2)$.

$$V = VP \Rightarrow \begin{cases} v_1 = v_2 \\ v_2 = v_1 \\ v_1+v_2=1 \end{cases} \quad \text{即 } P\ v_1=v_2=\frac{1}{2}.$$

8. (1) $T=\{0,1,2,\cdots\}$ $E=\{1,2,3,4\}$.

$$P = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

(2) $$P^{(2)}=P^2=\begin{bmatrix} \frac{1}{3} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{1}{3} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{3} & \frac{2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{3} \end{bmatrix} \qquad P^{(3)}=P^3=\begin{bmatrix} \frac{2}{9} & \frac{7}{27} & \frac{7}{27} & \frac{7}{27} \\ \frac{7}{27} & \frac{2}{9} & \frac{7}{27} & \frac{7}{27} \\ \frac{7}{27} & \frac{7}{27} & \frac{2}{9} & \frac{7}{27} \\ \frac{7}{27} & \frac{7}{27} & \frac{7}{27} & \frac{2}{9} \end{bmatrix}$$

(3). $\sum_{i=1}^{4} P\{X(1)=i, X(3)=i\} = \sum_{i=1}^{4} P\{X(1)=i\}\cdot P\{X(3)=i \mid X(1)=i\}$

$$= \sum_{i=1}^{4} \frac{1}{4} \times \frac{1}{3} = \frac{1}{3}.$$

(4). $\sum_{i=1}^{4} P\{X(0)=i, X(3)=i\} = \sum_{i=1}^{4} P\{X(0)=i\} P\{X(3)=i \mid X(0)=i\}$

$$= \sum_{i=1}^{4} \frac{1}{4} \times P_{ii}^{(3)} = \frac{2}{9}.$$

10. 假设，一件新产品由A公司推销，则称其处于状态1.

…………B……………………2

…………C……………………3.

$X(n)$ 为第 n 年新产品销售状态，

初始分布为 $\pi(0)=(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

转移矩阵

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.3 & 0.1 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$$

第一年 $\pi(1) = \pi(0)P = (\frac{8}{15}, \frac{7}{30}, \frac{7}{30})$

因 P 是正则阵，则该马氏链为遍历的，设极限分布为 $\pi=(\pi_1, \pi_2, \pi_3)$ 则

$\pi = \pi P$ 即 $\begin{cases} \pi_1 = 0.4\pi_1 + 0.6\pi_2 + 0.6\pi_3 \\ \pi_2 = 0.3\pi_1 + 0.3\pi_2 + 0.1\pi_3 \\ \pi_3 = 0.3\pi_1 + 0.1\pi_2 + 0.3\pi_3 \end{cases}$ 且 $\pi_1+\pi_2+\pi_3=1$.

解得 $\pi = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$

17. (1) 状态空间 $E=\{0, 1, 2, \cdots a, a+1, \cdots a+b\}$. 转移矩阵.

$$P = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \cdots & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & 0\ \frac{1}{2}\ 0 & \frac{1}{2} \\ 0 & 0 & \cdots & \cdots\ 0 & 1 \end{bmatrix}$$

(2) 甲输光概率为最终概率

$$f_{a0} = \sum_{n=a}^{+\infty} f_{a0}^{(n)} = \left(\tfrac{1}{2}\right)^{a} + \left(\tfrac{1}{2}\right)^{a+2} + \cdots + \left(\tfrac{1}{2}\right)^{a+2k} + \cdots$$

$$= \left(\tfrac{1}{2}\right)^{a} \left[\tfrac{1}{4} + \cdots \left(\tfrac{1}{4}\right)^{k} + \cdots\right]$$

$$= \left(\tfrac{1}{2}\right)^{a} \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}\left(\tfrac{1}{2}\right)^{a}.$$

19. (1) 因 $P^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{8} \\ \frac{1}{2} & 7/12 & 7/24 \\ 1/6 & 7/18 & 13/36 \end{bmatrix}$ ∴ P 为正则阵. 故马氏链满足遍历性.

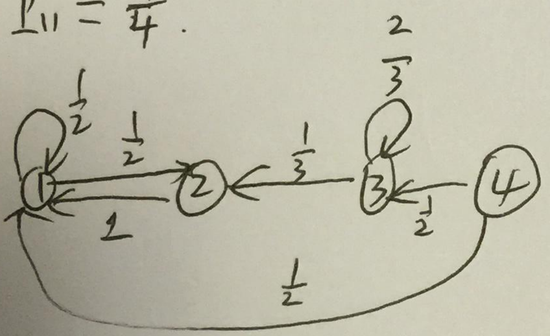
(2). 设 $V=(v_1, v_2, v_3)$ ∵ $V=VP$. 解得. $V=\left(\frac{4}{11}, \frac{4}{11}, \frac{3}{11}\right)$

(3) $P(X_4=3 \mid X(1)=1, X(2)=1) = P\{X(4)=3 \mid X(2)=1) = P(X(2)=3 \mid X(0)=1) = p_{13}^{(2)} = \frac{1}{8}$

$P\{X_2=1 \;\; X_3=2 \mid X_1=1) = P(X_3=2 \mid X_1=1, X_2=1)\, P(X_2=1 \mid X_1=1)$

$= P_{12}^{(1)} P_{11}^{(1)} = \frac{1}{4}.$

23. (1).



(2). $f_{44}^{(1)}=0$, $f_{44}^{(n)}=0$ $n \geq 1$. 故 $f_{44}=0$, 为非常返态.

$f_{33}^{(1)} = \frac{2}{3}$. $f_{33}^{(n)} = 0$ $(n>1)$ 故 $f_{33} = \frac{2}{3}$. 为非常返

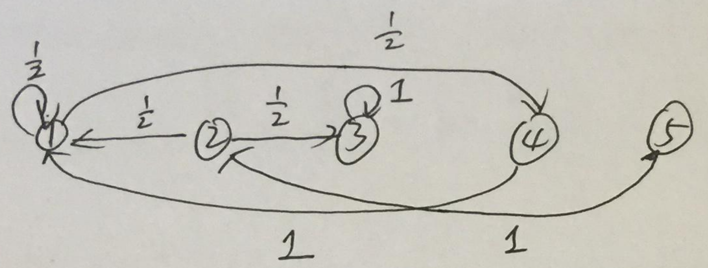
$f_{11}^{(1)} = \frac{1}{2}$, $f_{11}^{(2)} = \frac{1}{2}$ ∴ $f_{11} = 1$. 为常返态.

$\mu_{11} = \sum_{n=1}^{+\infty} n f_{11}^{(n)} = f_{11}^{(1)} + 2 f_{11}^{(2)} = \frac{3}{2} < +\infty$. (正常返)

$d_{11} = 1$ (非周期) 1为遍历态. 又1.2互通. 故 2也为遍历态.

(3) $E = \{1, 2\} \cup \{3, 4\}$.

25. ①



② 因 $f_{33}=f_{33}^{(1)}=1$. 故3为非周期正常返即遍历态.

$f_{22}^{(n)}=0$ $n\geq1$, 故 $f_{22}=0$, 则 2为非常返态.

同理 5亦为非常返态.

$f_{11}^{(1)}=\frac{1}{2}$, $f_{11}^{(2)}=\frac{1}{2}\times2=\frac{1}{2}$, $f_{11}^{(n)}=0$ $n\geq3$.

$f_{11}=1$ $d_{11}=1$. 故 1为遍历态. 又 1. 4 互通故 4为遍历态.

③ 由互通. 状态空间分解为

$E=\{1,4\}\cup\{3\}\cup\{2,5\}$.

$$a-1 \xleftrightarrow{\quad a \quad} a+1$$
$$\tfrac{1}{2} \qquad\qquad \tfrac{1}{2}$$

$$f_{a,0} = P_{a,a+1} f_{a+1,0} + P_{a,a-1} \cdot f_{a-1,0} \qquad (1)$$

记 $f_{a,0} = f_a$，(1)为

$$\begin{cases} f_a = \cancel{\phi}\,\frac{1}{2} f_{a+1} + \frac{1}{2} f_{a-1}. \\ f_0 = f_{0,0} = 1. \qquad f_{a+b,0} = 0. \end{cases}$$

$$\frac{f_a}{2} + \frac{f_a}{2} = \frac{1}{2} f_{a+1} + \frac{1}{2} f_{a-1} \Rightarrow f_{a+1} - f_a = f_a - f_{a-1}$$

$\cancel{f_{n+1}}$

加和
$$\begin{cases} f_n - f_{n-1} = f_{n-1} - f_{n-2} = \cdots = f_1 - f_0 \\ f_{n-1} - f_{n-2} = f_{n-2} - f_{n-3} = \cdots = f_1 - f_0 \\ \vdots \\ f_2 - f_1 = f_1 - f_0 \end{cases}$$

$$f_n - f_1 = (n-1)(f_1 - f_0)$$

令 $n = a+b$. $f_0 = 1$, $f_{a+b} = 0$. 求出 f_1

$$\underline{f_n = n f_1 + (n-1)}.$$