Incomplete Block Designs II

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Topics

Introduction

- Sometimes a BIB design requires more blocks or EUs than are available or needed (power of test).

- When this happens the following BIB design requirements may be relaxed:
    1. Each treatment level will be equally replicated.
    2. Each treatment level appears within a block with every other treatment level the same number of times.

- The frequency of occurrences of pairs of factor levels in blocks will not be the same.
    - **Some pairwise comparisons of treatment levels will have smaller standard errors than others.**

Road Map

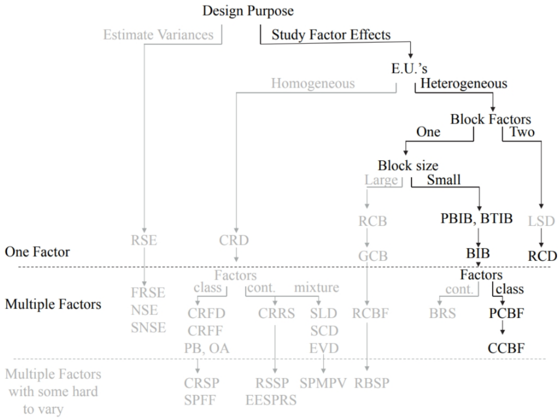

Figure: Source: (1)

Balanced Treatment Incomplete Block (BTIB) Designs

- In **balanced treatment incomplete block** (BTIB) designs one treatment level is designated as the control.
  - More interest in comparing the other treatment levels.
- Each treatment must appear the same number of times (λ_0) with the control.
- Each treatment must appear the same number of times (λ_1) in a block with every other test treatment.
- *The design that is more efficient in comparing each treatment with the control but less efficient in comparisons among the other treatment levels.*

Example 1 Preliminaries I

- Design to study the diastolic blood pressure readings of three different pharmacies.
- Blood pressure is most consistent in one person within a short period of time.
  - blocked into homogeneous groups by subject and time.
- It is impossible to move subjects immediately to the different pharmacies.
  - A portable blood pressure model was taken to each pharmacy to act as a control level for the factor.
- *The model and analysis will be the same as the BIB designs, but the efficiency of the comparisons will be different.*

Example 1

- Use the *BPmonitor* dataset from the daewr package to conduct the following analysis:
  1. Take some time to get to know the data.
     - What do you notice about the frequency of the different factor levels?
  2. Construct an appropriate ANOVA table for this data.
  3. Make pairwise comparisons to see if there are any significant differences in the blood pressure readings at the different pharmacies.
     - What do you notice about the standard error?

Partially Balanced Incomplete Block (PBIC) Designs

- In **partially balanced incomplete block** (PBIC) designs each pair of treatments are either *first associates* or *second associates*.

- **First associates** occur together in a block λ_1 times.
- **Second associates** occur together in a block λ_2 times.
  - $\lambda_1 > \lambda_2$

- The standard error for first associates is smaller than that of second associates.

Generalized Cyclic Incomplete Block Designs

- **Generalized cyclic incomplete block** designs are a class of PBIB designs that are easy to create and have good statistical properties.
- Algorithm to create a generalized cyclic incomplete block design with block size k and $b = t$ blocks:
  1. Start with a subset of k treatment factor levels as the initial block.
  2. Add 1 to each treatment level in the initial block to form the next block (rolls over if last level)
  3. Continue adding blocks in this manner until you have t blocks.

Generalized Cyclic Incomplete Block Designs in R

- Assuming the inequalities are satisfied we can use R to create generalized cyclic incomplete block designs:
  - `set.seed(2030)`
  - `library(agricolae)`
  - `treat <- c(1, ..., t)`
  - `des <- design.cyclic(treat, k = k, r = r)`
  - `des$book`

- Inequalities:
  1. $b \geq t$
  2. $tr = bk$
  3. $\lambda(t-1) = r(k-1)$

Example 2

- Use the `design.cyclic()` function to create a generalized cyclic incomplete block design with $t = 6$, $k = 3$, and $r = 3$.
- Identify λ_1 and λ_2

Generalized Cyclic Incomplete Block Designs Comments

- The `design.cyclic()` function automatically randomizes the plan.
- Analysis is performed in the same way as BIB and BTIB designs.
- The model assumptions of normality and homogeneity of experimental error variance are expected to hold.

Recall: Latin-square Block Designs I

- **Latin-square design** (LSD) is blocked both horizontally and vertically.
- Each treatment level is assigned only once to each row and each column.
- **The number of row blocks *must* equal the number of column blocks.**

Recall: Latin-square Block Designs II

- Latin-square designs may be used when there are two independent blocking factors.
- Illustrative Example:
    - Experiment to determine the effect of tread design on the wear life of automobile tires.
        - EU: A wheel on the car.
        - Treatment: The tread design of the mounted tire.
        - Block 1: Type of automobile (weights may impact wear)
        - Block 2: Wheel position on car (front differs from back)
- The number of row blocks **must** equal the number of column blocks.

Row Column Designs

- It may be impractical to have the number of row blocks equal to the number of column blocks.
- A **row column design** (RCD) uses a *complete* block design in the column blocks and an *incomplete* block design in the row blocks.
- *The modelling is the same as what is used for Latin-square designs.*

Road Map

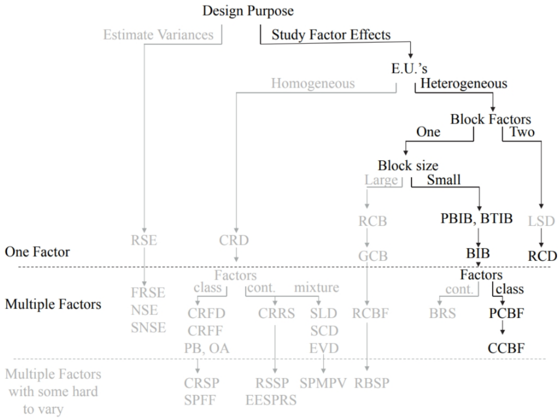

Figure: Source: (1)

Row Column Designs in R

- We can use R to create a row column design:
  - `set.seed(2030)`
  - `library(agricolae)`
  - `treat <- c(1, ..., t)`
  - `RCD <- design.cyclic(treat, k = k, r = r, rowcol = TRUE)`

Example 3

- Researchers are interested in studying the effect of shelf facing on the sales of toothpaste in drugstores.
- Assume there are 8 stores (blocking factor) and four weeks of sales (blocking factor).
  - Test 8 different shelf configurations (treatment factor).
- Use R to construct a row column design for this study.

Analysis of RCD in R

- We can analyse the data from a RCD in R:
  - `model <- aov(response ~ Block.row + Block.column + factor, data = data)`
  - `summary(model)`
- We can also examine the means and test the differences in factor levels:
  - lsmeans(model, pairwise $\sim$ factor, adjust $=$ ("tukey"))

Example 4

- Import the *RCD.csv* data into R.
- Take some time to get to know the data.
- Construct an ANOVA table to examine the effects of the different treatment levels.
- Make pairwise comparisons to see if there are any significant differences in the effects of the formulations.

References & Resources

1. Lawson, J. (2014). *Design and Analysis of Experiments with R (Vol. 115)*. CRC press.

- `design.cyclic()`
- `lsmeans()`
- daewr