Hypothesis Testing II

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Introduction

- We are continuing to make statistical inferences about target populations.
- Now we are going to draw conclusions about population parameters of two populations.
- Example: Is Brighton's passing accuracy better than Chelsea's?
    - Again, we need statistical methods to draw conclusions about the differences in passing accuracy.
- Note: *The formulas are more complicated than the one-sample tests.*

Two-Sample Tests

Test For	Null Hypothesis (H_0)	Test Statistic	Distribution	Use When
Difference of two means ($\mu_1 - \mu_2$)	$\mu_1 - \mu_2 = 0$	$\dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	Z	Both normal distributions, or $n_1, n_2 \geq 30$; σ_1, σ_2 known
Difference of two means ($\mu_1 - \mu_2$)	$\mu_1 - \mu_2 = 0$	$\dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$	t distribution with df = the smaller of $n_1 - 1$ and $n_2 - 1$	$n_1, n_2 < 30$; and/or σ_1, σ_2 unknown
Mean difference μ_d (paired data)	$\mu_d = 0$	$\dfrac{(\bar{d} - \mu_d)}{s_d / \sqrt{n}}$	t_{n-1}	$n < 30$ pairs of data and/or σ_d unknown
Difference of two proportions ($p_1 - p_2$)	$p_1 - p_2 = 0$	$\dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$	Z	$n\hat{p}, n(1-\hat{p}) \geq 10$ for each group

Recall Two-Sample Hypothesis Formulations

- **Two-Sample Tests**:

H_0: *Population parameter (group 1) - Population parameter (group 2)* ≥ 0
H_1: *Population parameter (group 1) - Population parameter (group 2)* < 0

H_0: *Population parameter (group 1) - Population parameter (group 2)* ≤ 0
H_1: *Population parameter (group 1) - Population parameter (group 2)* > 0

H_0: *Population parameter (group 1) - Population parameter (group 2)* $= 0$
H_1: *Population parameter (group 1) - Population parameter (group 2)* $\neq 0$

t-tests for Independent Samples I

- We are now going to compare the population means of two independent populations.
- Again, we can specify the three kinds of tests.

p-values in R

- We can use the `t.test()` function in R.
- In the case of a two-sample t-test we need two vectors as arguments `x` and `y`.
- Produces the test statistic (t), confidence intervals, p-value, and sample means.
- Usage:
  - Lower one-tailed: `t.test(x, y, mu = 0, alternative = "less", conf.level = 0.95, paired = FALSE, var.equal = FALSE)`
  - Upper one-tailed: `t.test(x, y, mu = 0, alternative = "greater", conf.level = 0.95, paired = FALSE, var.equal = FALSE)`
  - Two-tailed: `t.test(x, y, mu = 0, alternative = "two.sided", conf.level = 0.95, paired = FALSE, var.equal = FALSE)`

t-tests for Independent Samples II

- We can set the `mu = difference` value to change the hypotheses about the differences in means.
- `t.test()` function performs a Welch's t-test unless `var.equal = TRUE`.
- **This formulation does not work if the samples *are not* independent.**

Example 1

- Import the *iris* dataset into your environment and conduct the following hypothesis tests ($\alpha = 0.05$):
  - The mean petal length of virginica irises is smaller than the mean petal length of setosa irises.
  - The mean petal width of virginica irises is larger than the mean petal width of versicolor irises.
  - The mean petal length of virginica irises is different than the mean petal length of versicolor irises.
- *Hint: Start by identifying H_0 and H_1*

t-tests for Paired Samples I

- We are now going to compare the population means of two paired populations.
- This occurs when the observations are naturally paired
  - Pre- and post-treatment individuals (before and after study).
  - Comparing injured and non-injured limbs.
  - Repeated measures.
- Hypothesis tests are more accurate than assuming observations are independent.

p-values in R

- We can use the `t.test()` function in R.
- In the case of a two-sample t-test we need two vectors as arguments `x` and `y` (must be the same length and order).
- Produces the test statistic (t), confidence intervals, p-value, and sample means.
- Usage (change `paired = TRUE`):
  - Lower one-tailed: `t.test(x, y, mu = 0, alternative = "less", conf.level = 0.95, paired = TRUE, var.equal = FALSE)`
  - Upper one-tailed: `t.test(x, y, mu = 0, alternative = "greater", conf.level = 0.95, paired = TRUE, var.equal = FALSE)`
  - Two-tailed: `t.test(x, y, mu = 0, alternative = "two.sided", conf.level = 0.95, paired = TRUE, var.equal = FALSE)`

Example 2

- Import the Paired.csv file into your environment and test the following hypotheses ($\alpha = 0.1$):
    - The average post-treatment values are smaller than the pre-treatment measures.
    - The average post-treatment values are different than the pre-treatment measures.
    - The average post-treatment values are larger than the pre-treatment measures.
- *Hint: Start by identifying H_0 and H_1*

Z-tests for Differences in Proportions

- We may also conduct hypothesis tests for differences in proportions.
- Recall, that the sampling distribution of proportions is assumed to be normal.
- Again, we can specify the three kinds of tests.

p-values in R

- We can use the `prop.test()` function in R.
- In the case of a two-sample Z-test for proportions we need the number of *successes* for each group and the number of *trials*.
  - We include vectors of *successes* and *trials* into the function.
- Produces the test statistic, confidence intervals, p-value, and sample proportions.
- Usage:
  - Lower one-tailed: `prop.test(x=c(`x_1`,`x_2`) n=c(`n_1`,`n_2`), alternative = "less", conf.level = 0.95, correct = FALSE)`
  - Upper one-tailed: `prop.test(x=c(`x_1`,`x_2`) n=c(`n_1`,`n_2`), alternative = "greater", conf.level = 0.95, correct = FALSE)`
  - Two-tailed: `prop.test(x=c(`x_1`,`x_2`) n=c(`n_1`,`n_2`), alternative = "two.sided", conf.level = 0.95, correct = FALSE)`

Example 3

- Import the *Cars93* dataset from the *MASS* package into your environment and test the following hypotheses ($\alpha = 0.05$):
    - The proportion of vehicles that **do not** have manual transmissions available (*Man.trans.avail*) is less than those that do have manual transmissions available.
    - There is a larger proportion of vehicles that are rear wheel drive than vehicles that are four-wheel drive (4WD) (*DriveTrain*).
    - The proportion of Vans on the road is different than the proportion of Compact vehicles (*Type*).
- *Hint: Start by identifying H_0 and H_1*

F-tests for Differences in Variances

- **Assuming that our data come from a normal distribution**, we can test hypotheses about the equality of variances.
- The F-test has the following test statistic:

$$F = \frac{s_1^2}{s_2^2} \tag{1}$$

- It also has two values of degrees of freedom: $df_1 = n_1 - 1$ & $df_2 = n_2 - 1$
- If the variances differ significantly, we would expect F to be much larger than 1; the closer F is to 1, the more likely it is that the variances are the same.
- Again, we can specify the three kinds of tests.

p-values in R

- We can use the `var.test()` function in R.
- In the case of a two-sample F-test for variances we need two vectors as arguments `x` and `y`.
- Produces the test statistic F, confidence intervals, p-value, and sample variance ratio.
- Usage:
  - Lower one-tailed: `var.test(x, y, ratio = 1, alternative = "less", conf.level = 0.95)`
  - Upper one-tailed: `var.test(x, y, ratio = 1, alternative = "greater", conf.level = 0.95)`
  - Two-tailed: `var.test(x, y, ratio = 1, alternative = "two.sided", conf.level = 0.95)`
- The `ratio = 1` argument specifies which ratio value you are testing.

Example 4

- Generate the samples `x`, `y`, and `z` in the example code. Use those variables to conduct the following hypothesis tests ($\alpha = 0.05$):
  - The variance of `x` is greater than the variance of `y`.
  - The variance of `y` is different than the variance of `z`.
  - The variance of `z` is smaller than the variance of `x`.
- *Hint: Start by identifying H_0 and H_1*

Hypothesis Tests & Confidence Intervals

- Confidence intervals and the hypothesis tests we have looked at are related.
  - They rely on essentially the same information.
  - We even used the same R functions for both.
- When testing the difference between two population parameters, if the confidence interval for the difference contains 0, then we would not reject the null hypothesis.
- They can be used interchangeably under the correct conditions.

Which Test to Use?

- Understanding the assumptions needed for each test allows you to decide which test to use and how to use it.
- Information to help with deciding:
  - Are there one- or two-samples being tested?
  - What is the sampling distribution of the population parameter we are interested in?
  - Lower, upper, or two-tailed test?
  - What is the confidence level we are interested in?

Example 5

- Determine which hypothesis test to use for the following situations, write down the null and alternative hypotheses, population parameter(s) of interest, and perform the test:

1. I believe with 95% confidence that the average price of a car in 1993 in the U.S.A. was $18000. Am I correct? (Use the *Cars93* dataset from MASS)

2. I believe with 90% confidence that the average price of Midsize cars was lower than or equal to the average price of Small cars in 1993 in the U.S.A.. Am I correct?

3. I believe with 99% certainty that the proportion of cars equipped for 5 passengers in 1993 in the U.S.A. was 0.5. Am I correct?

Example 6

- Import the *Trials.csv* dataset into R. Determine which hypothesis test to use for the following situations, write down the null and alternative hypotheses, population parameter(s) of interest, and perform the test:

1. Is there a difference between the variance of the results of Trial 1 and the results of Trial 2? ($\alpha = 0.05$)
2. Does Trial 2 effect the averages of Test Group B and Test Group C differently? ($\alpha = 0.1$)
3. Are the baseline results on average lower than the results of Trial 1? ($\alpha = 0.01$)
4. (Assuming the data come from a much larger experiment) Is there a significant difference in the proportions of individuals in Test Group A and Test Group B? ($\alpha = 0.05$)

Exercise 1

- Use the *Trials.csv* to test the following:

  1. Is there a difference between the variance of the Baseline and the results of Trial 2? ($\alpha = 0.05$)
  2. Do participants from Test Group A have on average a larger distance than Participants from Test Group C? ($\alpha = 0.05$)
  3. Are the average Baseline results larger than the results of Trial 1? ($\alpha = 0.05$)
  4. Are the Baseline variances different between Test Group A and Test Group B? ($\alpha = 0.05$)

Exercise 2

- Use your new skills to come up with and test three hypotheses from the *Trials.csv* data.

References & Resources

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