<u>Analysis of a proton's motion in a magnetic and electric field using the Euler and the Euler-Cromer method.</u>

1. Introduction

The aim of this report is to demonstrate the findings of a java code used to produce plots of the position of a proton undergoing motion in a magnetic and electric field. This was done using two different methods, which will be analysed separately in this report.

2.1 Plotting the motion of a proton in a magnetic field using two different methods.

Originally using the Euler method provided in the code the plot *fig1* is produced showing the path of a proton moving in a magnetic field for 100 orbits. Next using a different method – the Euler-Cromer method – provided by the paper the path of the proton can then be plotted again and compared for differences. This method is different as it advances on the velocity first then uses that to advance on the x position based on the previous x position, this could produce different results if the protons are not placed at a start point at the origin, for example, something that will be discussed later. The plot of the proton position can be seen in *fig2.* With these both being particles in a magnetic field, I would expect this to produce a largely circular orbit with radius roughly equal to that which was calculated – around 0.1. As is evident in both plots this is the case – they appear elliptical due to the axis – however they are both largely circular orbit.

By finding the difference in position between the two methods – by subtracting one away from the other *fig3* – and over plotting the two graphs to show the overlap *fig4*, the similarities can be seen. The difference between them shows that most of the points lie on the axis meaning there is very little difference between the two and in general they provide good estimates within a small error. This can be seen further from the over plotted graph of the two.

2.2 Plotting the motion of a proton in a magnetic field using two different methods.

Next by setting a point charge at the origin the behaviour of a proton in an electric field – due to that point charge – can be plotted. This I expected to once again to be a circular orbit – as Coulombs law is analogous to gravity and produces a circular orbit. This, however, was not what I found. I found that if the start position of the particle was placed at the centre you would have a particle oscillating in one direction independent of the method used – see *fig5* – this seems logical however as if the particle only has velocity component in one direction the charge would only serve to accelerate in one direction and produce an oscillation. To combat this, I placed the particle at a position away from the centre. This however – for both methods - gave rise to a spiral pattern as seen in *fig6* and *fig7.* One thing this did show, was that there was much greater variation in position arising from the Euler-Cromer method – likely due to the high acceleration and low velocity at maximum points meaning the x values experience more of a spread.

3. Final observations

From these observations I can't find too much of a difference between the two methods, the Euler method seems to perform worse when the start position is further away from the zero point and provide a slight underestimate. The Euler-Cromer method, however, seems to have more of a spread with the electric field, possibly more accurate than that of the Euler method as the small changes in the values of the velocity could impact on the x position to produce a more accurate value.

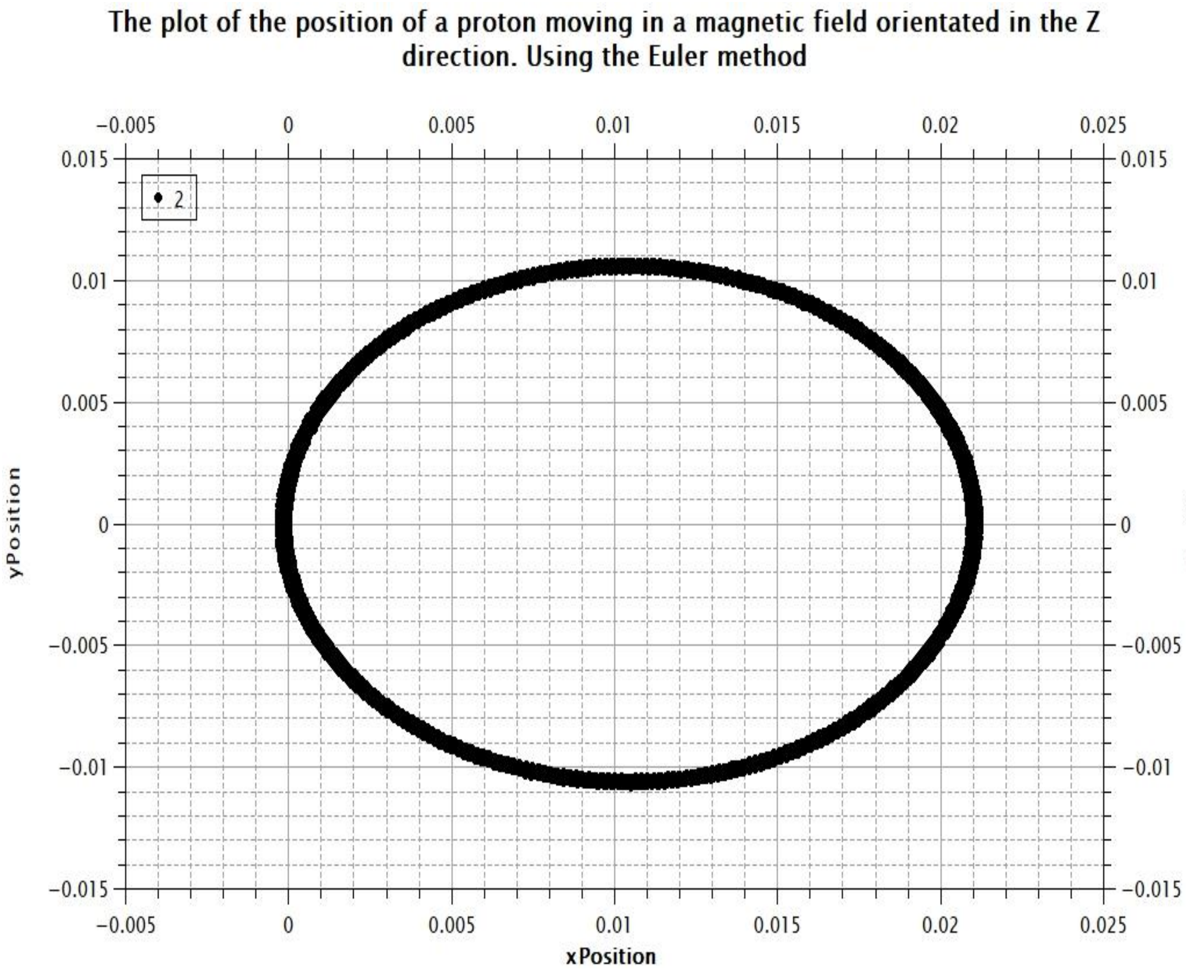

Fig1: A plot to show the position of a proton calculated via the Euler method undergoing motion in a magnetic field of strength 1e-7 T in the z and velocity 0.1 m/s in the y direction

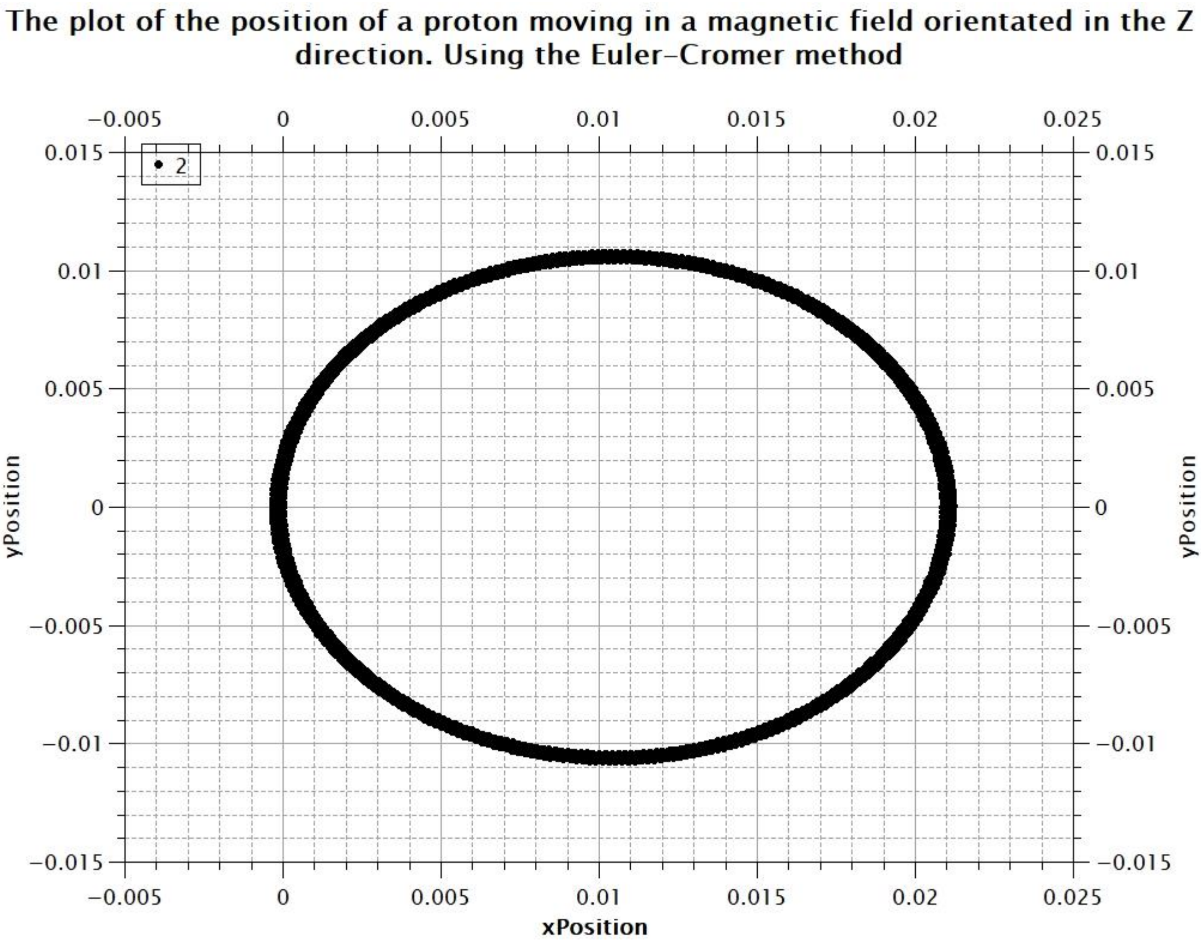

Fig2: *A plot to show the position of a proton calculated via the Euler-Cromer method undergoing motion in a magnetic field of strength 1e-7 T in the z and velocity 0.1 m/s in the y direction*

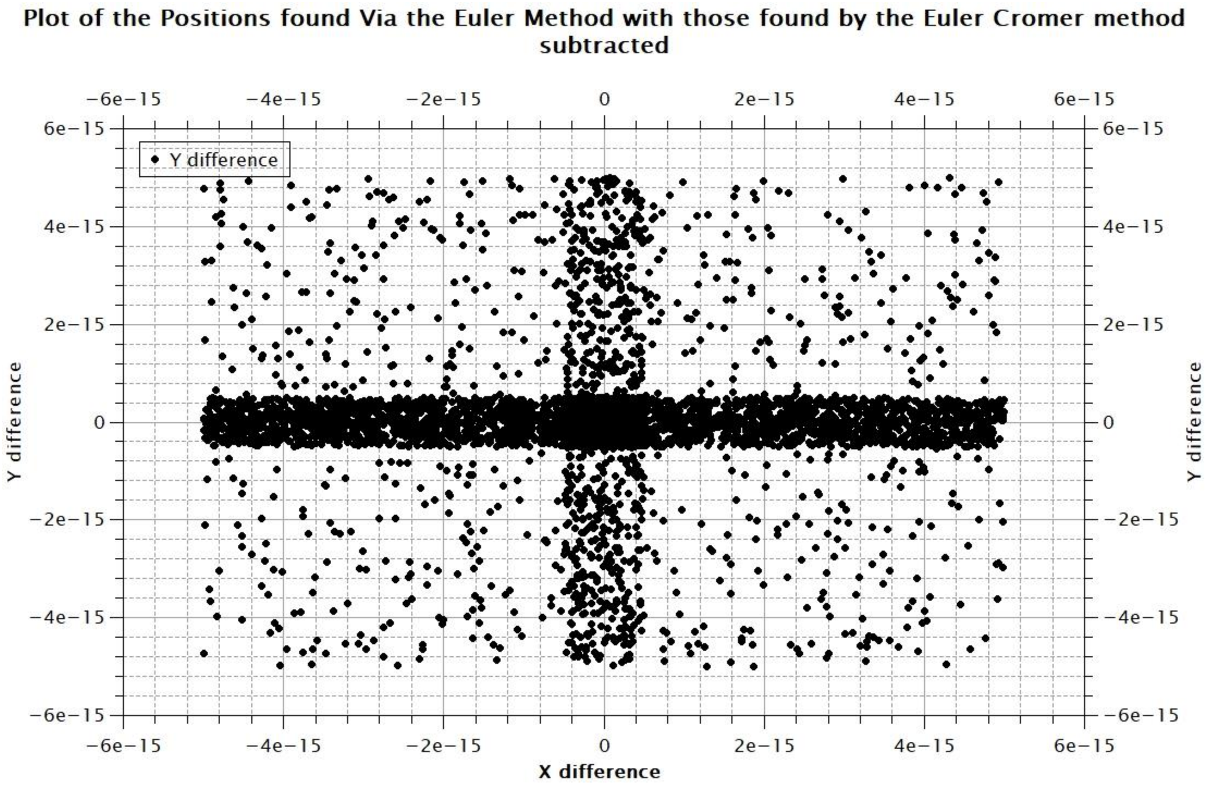

Fig1: A plot to show the difference in position of a proton calculated via the Euler method and Euler-Cromer undergoing motion in a magnetic field of strength 1e-7 T in the z and velocity 0.1 m/s in the y direction

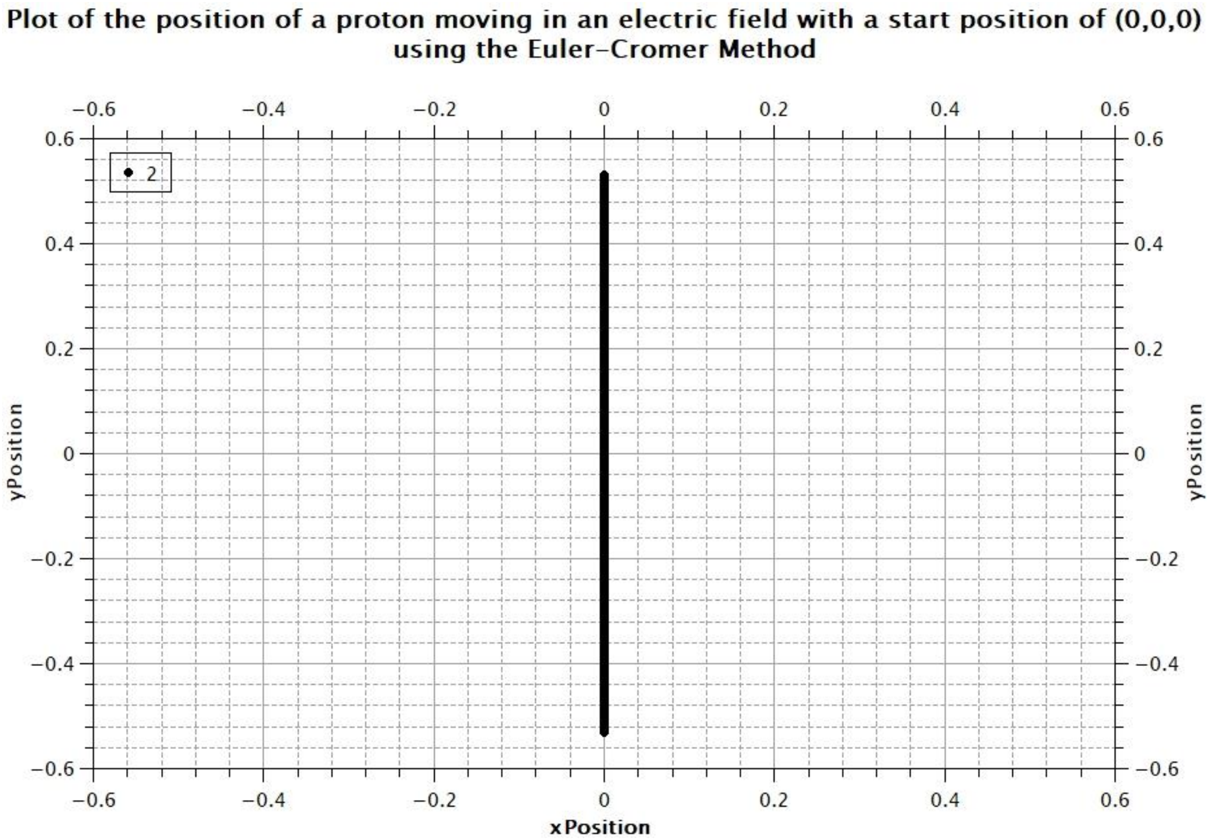

Fig4: A plot to show the position of a proton calculated via the Euler and Euler-Cromer method undergoing motion in an electric field with velocity 0.1 m/s in the y direction starting at the origin.

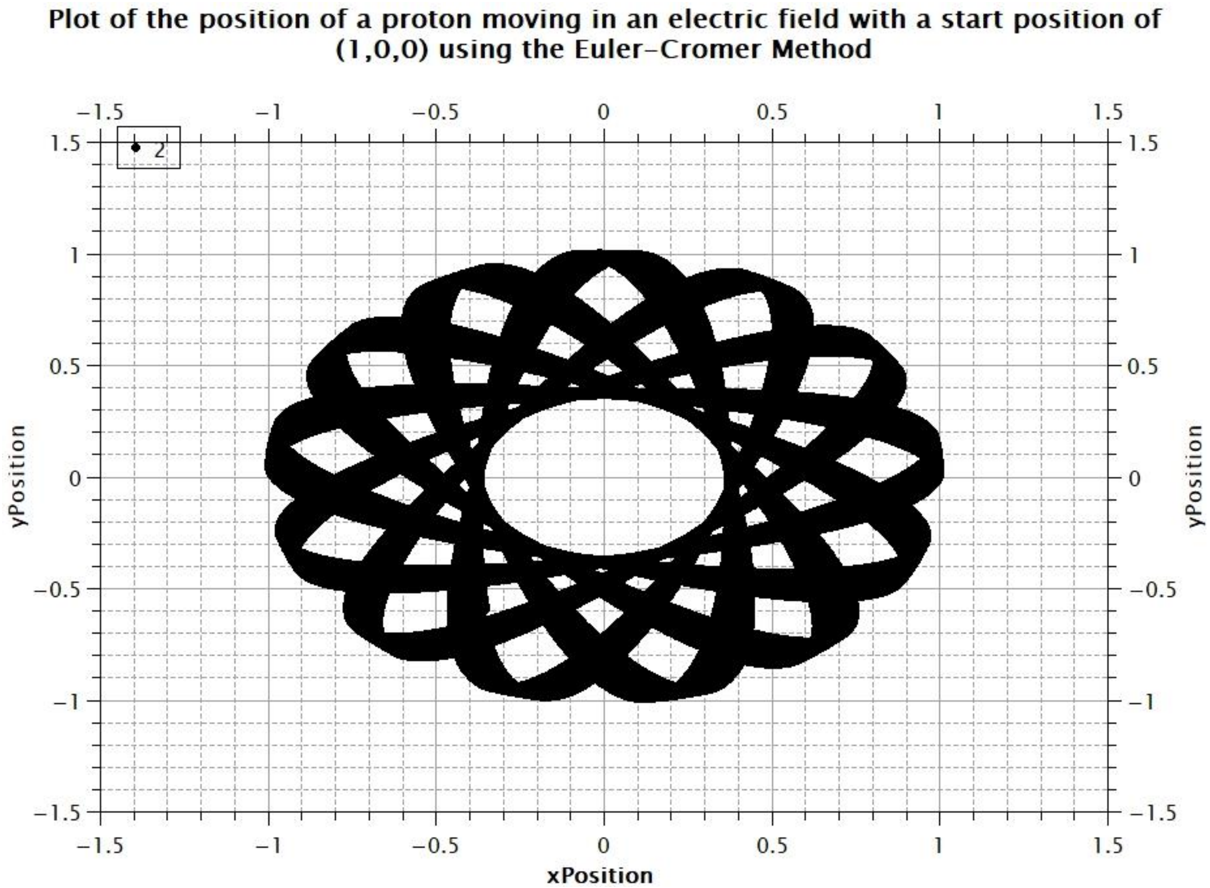

Fig6: A plot to show the position of a proton calculated via the Euler-Cromer method undergoing motion in an electric field with velocity 0.1 m/s in the y direction starting at the origin.

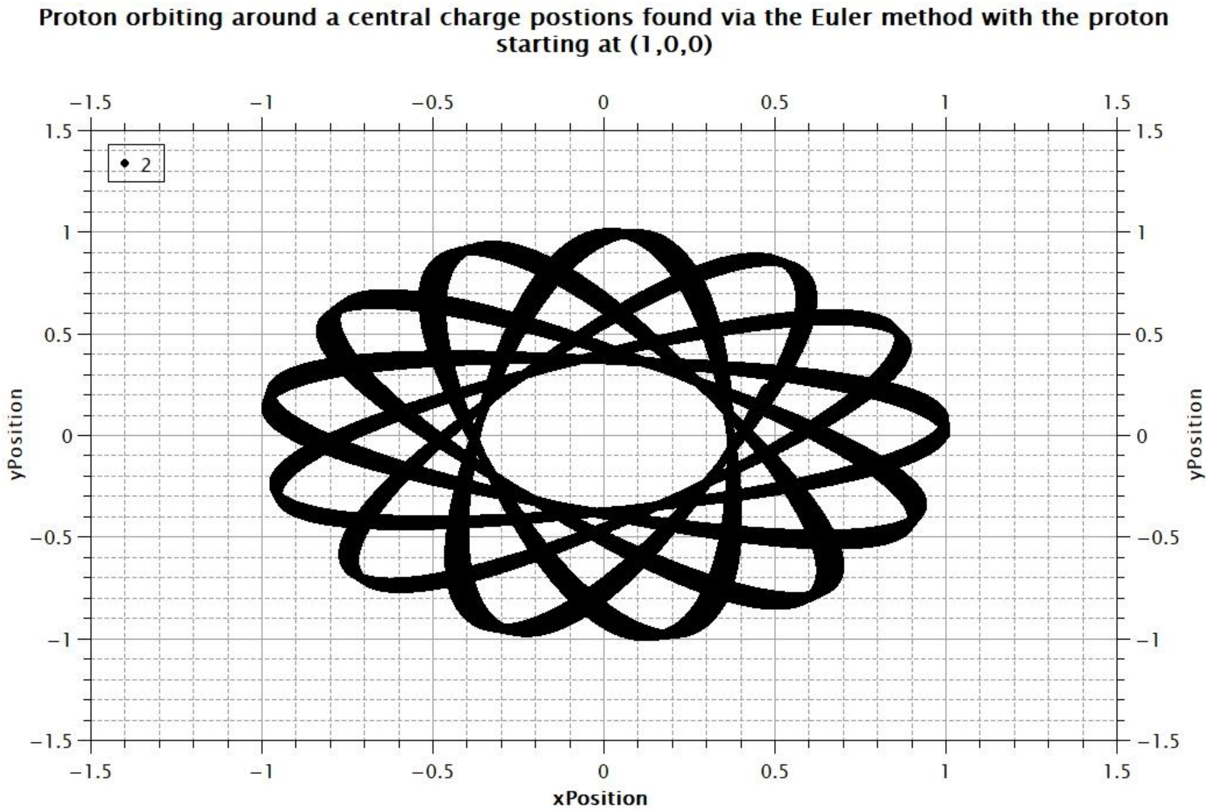

Fig7: A plot to show the position of a proton calculated via the method undergoing motion in an electric field with velocity 0.1 m/s in the y direction starting at the origin.