

# Multiple Regression & Interactions

# With 1 predictor

## Intercept

- **Continuous:** The mean value of Y (outcome) when X (predictor) is 0
- **Categorical:** The mean value of Y (outcome) when X (predictor) is 0, except one of your groups is coded as 0. So it's the mean value of Y for the group coded as 0.

## Regression Coefficient

- **Continuous:** A 1-unit change in X predicts a  $b_1$  change in Y
- **Categorical:** A 1-unit change in X predicts a  $b_1$  change in Y, except a 1-unit change is going from the category coded as 0 to the category coded as 1

## Omnibus test

- How well does the model fit the data?
- $F$ -test,  $R^2$  etc.

# Multiple Regression

- Now we are going to add in more variables
  - Maybe you have a covariate you want to **control** for
  - Maybe you want to look at the effects of X and Z on Y...independently

# Multiple Regression with Continuous Predictors

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + \cdots + b_kX_k$$

- Intercept is the value of  $Y$  when all predictors = 0
- Regression coefficients are the predicted change in  $Y$  for a 1 unit change in  $X$ , *holding all other predictors constant*
- Residual in simple regression can be thought of as a measure of  $Y$  that is left over after accounting for your DV

```
library(here)
stress.data = read.csv(here::here("R", "stress.csv"))
library(psych)
describe(stress.data$Stress)
```

```
##      vars    n mean    sd median trimmed  mad   min    max range skew kurtosis
## X1      1 118 5.18 1.88   5.27    5.17 1.65 0.62 10.32  9.71 0.08    0.22 0.
```

# Example

```
mr.model <- lm(Stress ~ Support + Anxiety, data = stress.data)
summary(mr.model)
```

```
##
## Call:
## lm(formula = Stress ~ Support + Anxiety, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.1958 -0.8994 -0.1370  0.9990  3.6995
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.31587    0.85596  -0.369  0.712792
## Support      0.40618    0.05115   7.941 1.49e-12 ***
## Anxiety      0.25609    0.06740   3.799 0.000234 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.519 on 115 degrees of freedom
## Multiple R-squared:  0.3556,    Adjusted R-squared:  0.3444
## F-statistic: 31.73 on 2 and 115 DF,  p-value: 1.062e-11
```

# Visualizing multiple regression

```
library(visreg)
```

```
visreg2d(mr.model, "Support", "Anxiety", plot.type = "persp")
```

```
mr.model <- lm(Stress ~ Support + Anxiety, data = stress.data)
summary(mr.model)
```

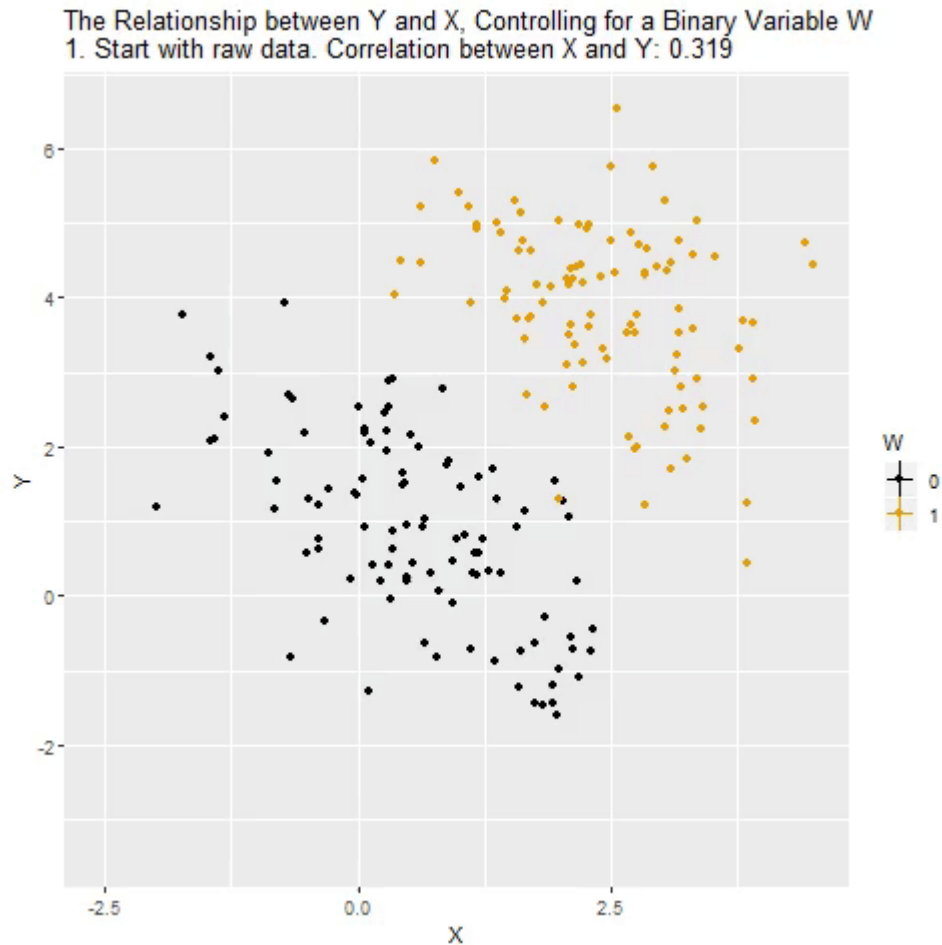
```
##
## Call:
## lm(formula = Stress ~ Support + Anxiety, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.1958 -0.8994 -0.1370  0.9990  3.6995
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## Coefficients:
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```
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## (Intercept) -0.31587    0.85596  -0.369  0.712792
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##
## Residual standard error: 1.519 on 115 degrees of freedom
## Multiple R-squared:  0.3556,    Adjusted R-squared:  0.3444
## F-statistic: 31.73 on 2 and 115 DF,  p-value: 1.062e-11
```

# "Controlling for"



# Estimating model fit

```
mr.model <- lm(Stress ~ Support + Anxiety, data = stress.data)
summary(mr.model)
```

```
##
## Call:
## lm(formula = Stress ~ Support + Anxiety, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.1958 -0.8994 -0.1370  0.9990  3.6995
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.31587    0.85596  -0.369  0.712792
## Support      0.40618    0.05115   7.941 1.49e-12 ***
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##
## Residual standard error: 1.519 on 115 degrees of freedom
## Multiple R-squared:  0.3556,    Adjusted R-squared:  0.3444
## F-statistic: 31.73 on 2 and 115 DF,  p-value: 1.062e-11
```

# R-squared, $R^2$

- Same interpretation as before
- Adding predictors into your model will increase  $R^2$  – regardless of whether or not the predictor is significantly correlated with Y.
- Adjusted/Shrunken  $R^2$  takes into account the number of predictors in your model

# Categorical predictors

One of the benefits of using regression is that it can handle both continuous and categorical predictors and allows for using both in the same model.

Categorical predictors with more than two levels are broken up into several smaller variables. In doing so, we take variables that don't have any inherent numerical value to them (i.e., nominal and ordinal variables) and ascribe meaningful numbers that allow for us to calculate meaningful statistics.

You can choose just about any numbers to represent your categorical variable. However, there are several commonly used methods that result in very useful statistics.

# Dummy coding

In dummy coding, one group is selected to be a reference group. From your single nominal variable with  $K$  levels,  $K - 1$  dummy code variables are created; for each new dummy code variable, one of the non-reference groups is assigned 1; all other groups are assigned 0.

| Occupation | D1 | D2 |
|------------|----|----|
| Engineer   | 0  | 0  |
| Teacher    | 1  | 0  |
| Doctor     | 0  | 1  |

The dummy codes are entered as IV's in the regression equation.

| Person  | Occupation | D1 | D2 |
|---------|------------|----|----|
| Billy   | Engineer   | 0  | 0  |
| Susan   | Teacher    | 1  | 0  |
| Michael | Teacher    | 1  | 0  |
| Molly   | Engineer   | 0  | 0  |
| Katie   | Doctor     | 0  | 1  |

## Example

Solomon's paradox describes the tendency for people to reason more wisely about other people's problems compared to their own. One potential explanation for this paradox is that people tend to view other people's problems from a more psychologically distant perspective, whereas they view their own problems from a psychologically immersed perspective. To test this possibility, researchers asked romantically-involved participants to think about a situation in which their partner cheated on them (self condition) or a friend's partner cheated on their friend (other condition). Participants were also instructed to take a first-person perspective (immersed condition) by using pronouns such as I and me, or a third-person perspective (distanced condition) by using pronouns such as he and her.

```
solomon <- read.csv(here::here("R", "solomon.csv"))
```

Grossmann, I., & Kross, E. (2014). Exploring Solomon's paradox: Self-distancing eliminates self-other asymmetry in wise reasoning about close relationships in younger and older adults. *Psychological Science*, 25, 1571-1580.

```
psych::describe(solomon[,c("ID", "CONDITION", "WISDOM")], fast = T)
```

```
##          vars    n  mean    sd   min   max  range   se
## ID              1 120 64.46 40.98  1.00 168.00 167.00 3.74
## CONDITION       2 120  2.46  1.12  1.00  4.00   3.00 0.10
## WISDOM          3 115  0.01  0.99 -2.52  1.79   4.31 0.09
```

```
library(knitr)
library(kableExtra)
library(tidyverse)
head(solomon) %>%
  select(ID, CONDITION,
         WISDOM) %>%
  kable() %>% kable_styling()
```

| ID | CONDITION | WISDOM     |
|----|-----------|------------|
| 1  | 3         | -0.2758939 |
| 6  | 4         | 0.4294921  |
| 8  | 4         | -0.0278587 |
| 9  | 4         | 0.5327150  |
| 10 | 2         | 0.6229979  |
| 12 | 2         | -1.9957813 |



```

solomon = solomon %>%
  mutate(dummy_2 = ifelse(CONDITION == 2, 1, 0),
         dummy_3 = ifelse(CONDITION == 3, 1, 0),
         dummy_4 = ifelse(CONDITION == 4, 1, 0))
solomon %>%
  select(ID, CONDITION, WISDOM,
         matches("dummy")) %>%
  kable() %>% kable_styling()

```

| ID | CONDITION | WISDOM     | dummy_2 | dummy_3 | dummy_4 |
|----|-----------|------------|---------|---------|---------|
| 1  | 3         | -0.2758939 | 0       | 1       | 0       |
| 6  | 4         | 0.4294921  | 0       | 0       | 1       |
| 8  | 4         | -0.0278587 | 0       | 0       | 1       |
| 9  | 4         | 0.5327150  | 0       | 0       | 1       |
| 10 | 2         | 0.6229979  | 1       | 0       | 0       |
| 12 | 2         | -1.9957813 | 1       | 0       | 0       |
| 14 | 3         | -1.1514699 | 0       | 1       | 0       |
| 18 | 2         | -0.6912011 | 1       | 0       | 0       |
| 21 | 2         | 0.0053117  | 1       | 0       | 0       |

```
mod.1 = lm(WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solomon)
summary(mod.1)
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6809 -0.4209  0.0473  0.6694  2.3499
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.5593     0.1686  -3.317 0.001232 **
## dummy_2       0.6814     0.2497   2.729 0.007390 **
## dummy_3       0.7541     0.2348   3.211 0.001729 **
## dummy_4       0.8938     0.2524   3.541 0.000583 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
## (5 observations deleted due to missingness)
## Multiple R-squared:  0.1262,    Adjusted R-squared:  0.1026
## F-statistic: 5.343 on 3 and 111 DF,  p-value: 0.001783
```

# Interpreting coefficients

When working with dummy codes, the intercept can be interpreted as the mean of the reference group.

$$\hat{Y} = b_0 + b_1 D_2 + b_2 D_3 + b_3 D_2$$

$$\hat{Y} = b_0 + b_1(0) + b_2(0) + b_3(0)$$

$$\hat{Y} = b_0$$

$$\hat{Y} = \bar{Y}_{\text{Reference}}$$

What do each of the slope coefficients mean?

From this equation, we can get the mean of every single group.

```
newdata = data.frame(dummy_2 = c(0,1,0,0),  
                      dummy_3 = c(0,0,1,0),  
                      dummy_4 = c(0,0,0,1))  
predict(mod.1, newdata = newdata, se.fit = T)
```

```
## $fit  
##           1           2           3           4  
## -0.5593042  0.1220847  0.1948435  0.3344884  
##  
## $se.fit  
##           1           2           3           4  
## 0.1686358  0.1841382  0.1634457  0.1877848  
##  
## $df  
## [1] 111  
##  
## $residual.scale  
## [1] 0.9389242
```

From this equation, we can get the mean of every single group.

```
solomon %>%  
  mutate_at("CONDITION", ~as.factor(.)) %>%  
  group_by(CONDITION) %>%  
  drop_na() %>%  
  summarize(meanWisdom = mean(WISDOM))
```

```
## # A tibble: 4 × 2  
##   CONDITION meanWisdom  
##   <fct>      <dbl>  
## 1 1      -0.559  
## 2 2       0.122  
## 3 3       0.195  
## 4 4       0.334
```

And the test of the coefficient represents the significance test of each group to the reference. This is an independent-samples  $t$ -test.

The test of the intercept is the one-sample  $t$ -test comparing the intercept to 0.

```
summary(mod.1)$coef
```

| ## |             | Estimate   | Std. Error | t value   | Pr(> t )     |
|----|-------------|------------|------------|-----------|--------------|
| ## | (Intercept) | -0.5593042 | 0.1686358  | -3.316641 | 0.0012319438 |
| ## | dummy_2     | 0.6813889  | 0.2496896  | 2.728944  | 0.0073896074 |
| ## | dummy_3     | 0.7541477  | 0.2348458  | 3.211247  | 0.0017291997 |
| ## | dummy_4     | 0.8937927  | 0.2523909  | 3.541303  | 0.0005832526 |

What if you wanted to compare groups 2 and 3?

```

solomon = solomon %>%
  mutate(dummy_1 = ifelse(CONDITION == 1, 1, 0),
         dummy_3 = ifelse(CONDITION == 3, 1, 0),
         dummy_4 = ifelse(CONDITION == 4, 1, 0))
mod.2 = lm(WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solomon)
summary(mod.2)

```

```

##
## Call:
## lm(formula = WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6809 -0.4209  0.0473  0.6694  2.3499
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.12208    0.18414   0.663  0.50870
## dummy_1      -0.68139    0.24969  -2.729  0.00739 **
## dummy_3       0.07276    0.24621   0.296  0.76816
## dummy_4       0.21240    0.26300   0.808  0.42104
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
## (5 observations deleted due to missingness)
## Multiple R-squared:  0.1262,    Adjusted R-squared:  0.1026

```

```
solomon = solomon %>%
  mutate_at("CONDITION", ~as.factor(.))

mod.3 = lm(WISDOM ~ CONDITION, data = solomon)
summary(mod.3)
```

```
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
```

|  | Min     | 1Q      | Median | 3Q     | Max    |
|--|---------|---------|--------|--------|--------|
|  | -2.6809 | -0.4209 | 0.0473 | 0.6694 | 2.3499 |

```
##
## Coefficients:
```

|             | Estimate | Std. Error | t value | Pr(> t ) |     |
|-------------|----------|------------|---------|----------|-----|
| (Intercept) | -0.5593  | 0.1686     | -3.317  | 0.001232 | **  |
| CONDITION2  | 0.6814   | 0.2497     | 2.729   | 0.007390 | **  |
| CONDITION3  | 0.7541   | 0.2348     | 3.211   | 0.001729 | **  |
| CONDITION4  | 0.8938   | 0.2524     | 3.541   | 0.000583 | *** |

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
## (5 observations deleted due to missingness)
## Multiple R-squared:  0.1262,    Adjusted R-squared:  0.1026
## F-statistic: 5.343 on 3 and 111 DF,  p-value: 0.001783
```



# Omnibus test

```
summary(mod.1)
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6809 -0.4209  0.0473  0.6694  2.3499
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.5593     0.1686  -3.317  0.001232 **
## dummy_2       0.6814     0.2497   2.729  0.007390 **
## dummy_3       0.7541     0.2348   3.211  0.001729 **
## dummy_4       0.8938     0.2524   3.541  0.000583 ***
## ---
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```

# Omnibus test

```
summary(mod.2)
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6809 -0.4209  0.0473  0.6694  2.3499
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.12208    0.18414   0.663  0.50870
## dummy_1       -0.68139    0.24969  -2.729  0.00739 **
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## ---
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```

# Omnibus test

```
summary(mod.3)
```

```
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6809 -0.4209  0.0473  0.6694  2.3499
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.5593     0.1686  -3.317 0.001232 **
## CONDITION2     0.6814     0.2497   2.729 0.007390 **
## CONDITION3     0.7541     0.2348   3.211 0.001729 **
## CONDITION4     0.8938     0.2524   3.541 0.000583 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
## (5 observations deleted due to missingness)
## Multiple R-squared:  0.1262,    Adjusted R-squared:  0.1026
## F-statistic: 5.343 on 3 and 111 DF,  p-value: 0.001783
```

# Omnibus test

```
anova(mod.3)
```

```
## Analysis of Variance Table
```

```
##
```

```
## Response: WISDOM
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
```

```
## CONDITION    3 14.131   4.7105   5.3432 0.001783 **
```

```
## Residuals 111 97.855   0.8816
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

# What are interactions?

When we have two variables, A and B, in a regression model, we are testing whether these variables have **additive effects** on our outcome, Y. That is, the effect of A on Y is constant over all values of B.

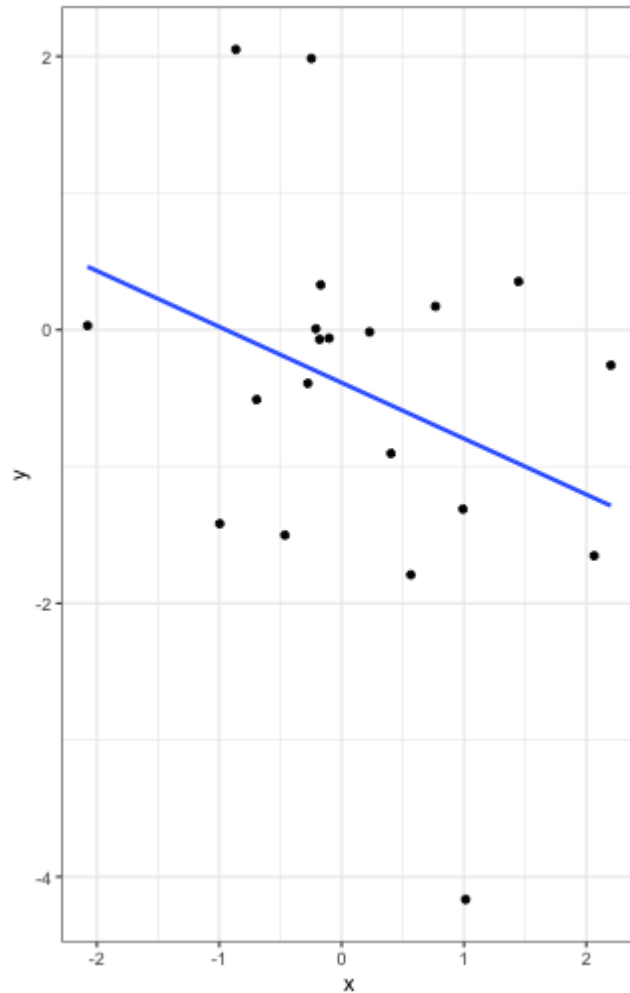
- Example: Drinking coffee and hours of sleep have additive effects on alertness; no matter how many hours I slept the previous night, drinking one cup of coffee will make me .5 SD more awake than not drinking coffee.

# What are interactions?

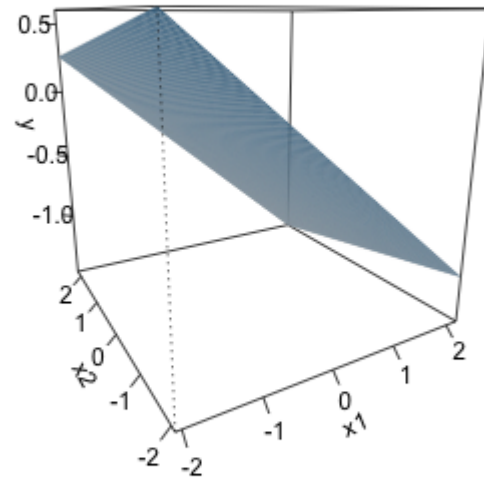
However, we may hypothesize that two variables have **joint effects**, or interact with each other. In this case, the effect of A on Y changes as a function of B.

- Example: Chronic stress has a negative impact on health but only for individuals who receive little or no social support; for individuals with high social support, chronic stress has no impact on health.
- This is also referred to as **moderation**.

# Univariate regression



# Multivariate regression



# Multivariate regression with an interaction



# Example

Let's use data about stress. We have an outcome (Stress) that we are interested in predicting from trait Anxiety and levels of Social Support. We can ignore the **group** status for the time being.

```
library(here)
stress.data = read.csv(here("R/stress.csv"))
library(psych)
describe(stress.data)
```

```
##          vars    n   mean      sd median trimmed   mad  min   max   range  skew
## id           1 118 488.65 295.95 462.50  485.76 372.13 2.00 986.00 984.00  0.
## Anxiety      2 118   7.61   2.49   7.75   7.67   2.26 0.70  14.64  13.94 -0.
## Stress       3 118   5.18   1.88   5.27   5.17   1.65 0.62  10.32   9.71  0.
## Support      4 118   8.73   3.28   8.52   8.66   3.16 0.02  17.34  17.32  0.
## group*      5 118   1.53   0.50   2.00   1.53   0.00 1.00   2.00   1.00 -0.
##          kurtosis    se
## id          -1.29 27.24
## Anxiety      0.28  0.23
## Stress       0.22  0.17
## Support      0.19  0.30
## group*     -2.01  0.05
```

# In R

```
i.model1 = lm(Stress ~ Anxiety + Support + Anxiety:Support,  
              data = stress.data)  
i.model2 = lm(Stress ~ Anxiety*Support, data = stress.data)
```

Both methods of specifying the interaction above will work in R. Using the `*` tells R to create both the main effects and the interaction effect. Note, however that the following code *gives you the wrong results*:

```
imodel_bad = lm(Stress ~ Anxiety:Support,  
                data = stress.data)  
# This does not create main effects.  
# It is VERY WRONG  
# Don't do this
```

```
i.model1 = lm(Stress ~ Anxiety*Support, data = stress.data)
summary(i.model1)
```

```
##
## Call:
## lm(formula = Stress ~ Anxiety * Support, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.8163 -1.0783  0.0373  0.9200  3.6109
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.73966     1.12101   -2.444  0.01606 *
## Anxiety         0.61561     0.13010    4.732 6.44e-06 ***
## Support        0.66697     0.09547    6.986 2.02e-10 ***
## Anxiety:Support -0.04174     0.01309   -3.188  0.00185 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.462 on 114 degrees of freedom
## Multiple R-squared:  0.4084,    Adjusted R-squared:  0.3928
## F-statistic: 26.23 on 3 and 114 DF,  p-value: 5.645e-13
```

# All our tidying functions work!

```
library(broom)
library(knitr)

kable(tidy(i.model))
```

| term            | estimate   | std.error | statistic | p.value   |
|-----------------|------------|-----------|-----------|-----------|
| (Intercept)     | -2.7396625 | 1.1210052 | -2.443934 | 0.0160605 |
| Anxiety         | 0.6156122  | 0.1301016 | 4.731780  | 0.0000064 |
| Support         | 0.6669669  | 0.0954746 | 6.985802  | 0.0000000 |
| Anxiety:Support | -0.0417408 | 0.0130933 | -3.187954 | 0.0018497 |

# All our tidying functions work!

```
kable(head(augment(i.model1)))
```

| Stress  | Anxiety  | Support | .fitted  | .resid     | .hat      | .sigma   | .cooksd   | .st  |
|---------|----------|---------|----------|------------|-----------|----------|-----------|------|
| 3.19813 | 10.18520 | 6.1602  | 5.020185 | -1.8220554 | 0.0205374 | 1.458248 | 0.0083121 | -1.2 |
| 7.00840 | 5.58873  | 8.9069  | 4.563653 | 2.4447470  | 0.0173247 | 1.450055 | 0.0125411 | 1.6  |
| 6.17400 | 6.58500  | 10.5433 | 5.448214 | 0.7257861  | 0.0131721 | 1.466888 | 0.0008333 | 0.4  |
| 8.69884 | 8.95430  | 11.4605 | 6.133020 | 2.5658202  | 0.0379024 | 1.447732 | 0.0315283 | 1.7  |
| 5.26707 | 7.59910  | 5.5516  | 3.880245 | 1.3868246  | 0.0200085 | 1.462572 | 0.0046863 | 0.9  |
| 5.12485 | 8.15600  | 7.5117  | 4.734061 | 0.3907895  | 0.0100296 | 1.468032 | 0.0001828 | 0.2  |

# All our tidying functions work!

```
kable(glance(i.model1))
```

| r.squared | adj.r.squared | sigma    | statistic | p.value | df | logLik    | AIC     | BIC      |
|-----------|---------------|----------|-----------|---------|----|-----------|---------|----------|
| 0.4083528 | 0.3927831     | 1.462042 | 26.22746  | 0       | 3  | -210.2205 | 430.441 | 444.2944 |

# Conceptual interpretation

$$\hat{Y} = b_0 + b_1X + b_2Z + b_3XZ$$

You can interpret the interaction term in the same way you normally interpret a slope coefficient -- this is the effect of the interaction controlling for other variables in the model.

You can also interpret the intercept the same way as before (the expected value of Y when all predictors are 0).

But here,  $b_1$  is the effect of X on Y when Z is equal to 0.

# Conceptual interpretation

$$\hat{Y} = b_0 + b_1X + b_2Z + b_3XZ$$

**Lower-order terms** change depending on the values of the higher-order terms. The value of  $b_1$  and  $b_2$  will change depending on the value of  $b_3$ .

- These values represent "conditional effects" (because the value is conditional on the level of the other variable). In many cases, the value and significance test with these terms is either meaningless (if Z is never equal to 0) or unhelpful, as these values and significance change across the data.

**Higher-order terms** are those terms that represent interactions.  $b_3$  is a higher-order term.

- This value represents how much the slope of X changes for every 1-unit increase in Z AND how much the slope of Z changes for everyone 1-unit increase in X.



# Conceptual interpretation

Higher-order interaction terms represent:

- the change in the slope of X as a function of Z
- the degree of curvature in the regression plane
- the linear effect of the product of independent variables

```
stress.data$AxS = stress.data$Anxiety*stress.data$Support  
head(stress.data[,c("Anxiety", "Support", "AxS")])
```

| ## |   | Anxiety  | Support | AxS       |
|----|---|----------|---------|-----------|
| ## | 1 | 10.18520 | 6.1602  | 62.74287  |
| ## | 2 | 5.58873  | 8.9069  | 49.77826  |
| ## | 3 | 6.58500  | 10.5433 | 69.42763  |
| ## | 4 | 8.95430  | 11.4605 | 102.62076 |
| ## | 5 | 7.59910  | 5.5516  | 42.18716  |
| ## | 6 | 8.15600  | 7.5117  | 61.26543  |

```
summary(lm(Stress ~ Anxiety + Support + AxS, data = stress.data))
```

```
##
## Call:
## lm(formula = Stress ~ Anxiety + Support + AxS, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.8163 -1.0783  0.0373  0.9200  3.6109
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.73966     1.12101  -2.444  0.01606 *
## Anxiety      0.61561     0.13010   4.732 6.44e-06 ***
## Support      0.66697     0.09547   6.986 2.02e-10 ***
## AxS          -0.04174     0.01309  -3.188  0.00185 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.462 on 114 degrees of freedom
## Multiple R-squared:  0.4084,    Adjusted R-squared:  0.3928
## F-statistic: 26.23 on 3 and 114 DF,  p-value: 5.645e-13
```

```
summary(lm(Stress ~ Anxiety*Support, data = stress.data))
```

```
##  
## Call:  
## lm(formula = Stress ~ Anxiety * Support, data = stress.data)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -3.8163 -1.0783  0.0373  0.9200  3.6109   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)   -2.73966    1.12101  -2.444  0.01606 *      
## Anxiety         0.61561    0.13010   4.732 6.44e-06 ***   
## Support        0.66697    0.09547   6.986 2.02e-10 ***   
## Anxiety:Support -0.04174    0.01309  -3.188 0.00185 **     
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 1.462 on 114 degrees of freedom  
## Multiple R-squared:  0.4084,    Adjusted R-squared:  0.3928   
## F-statistic: 26.23 on 3 and 114 DF,  p-value: 5.645e-13
```

**They're the same!!**

# Conditional effects and simple slopes

The regression line estimated in this model is quite difficult to interpret on its own. A good strategy is to decompose the regression equation into **simple slopes**, which are determined by calculating the conditional effects at a specific level of the moderating variable.

- Simple slope: the equation for Y on X at different levels of Z; but also refers to only the coefficient for X in this equation
- Conditional effect: the slope coefficients in the full regression model which can change. These are the lower-order terms associated with a variable. E.g., X has a conditional effect on Y.

Which variable is the "predictor" (X) and which is the "moderator" (Z)?

# Getting Simple Slopes

The conditional nature of these effects is easiest to see by "plugging in" different values for one of your variables. Return to the regression equation estimated in our stress data:

$$\hat{Stress} = -2.74 + 0.62(Anx) + 0.67(Sup) + -0.04(Anx \times Sup)$$

## Set Support to 5

$$\begin{aligned}\hat{Stress} &= -2.74 + 0.62(Anx) + 0.67(5) + -0.04(Anx \times 5) \\ &= -2.74 + 0.62(Anx) + 3.35 + -0.2(Anx) \\ &= 0.61 + 0.42(Anx)\end{aligned}$$

## Set Support to 10

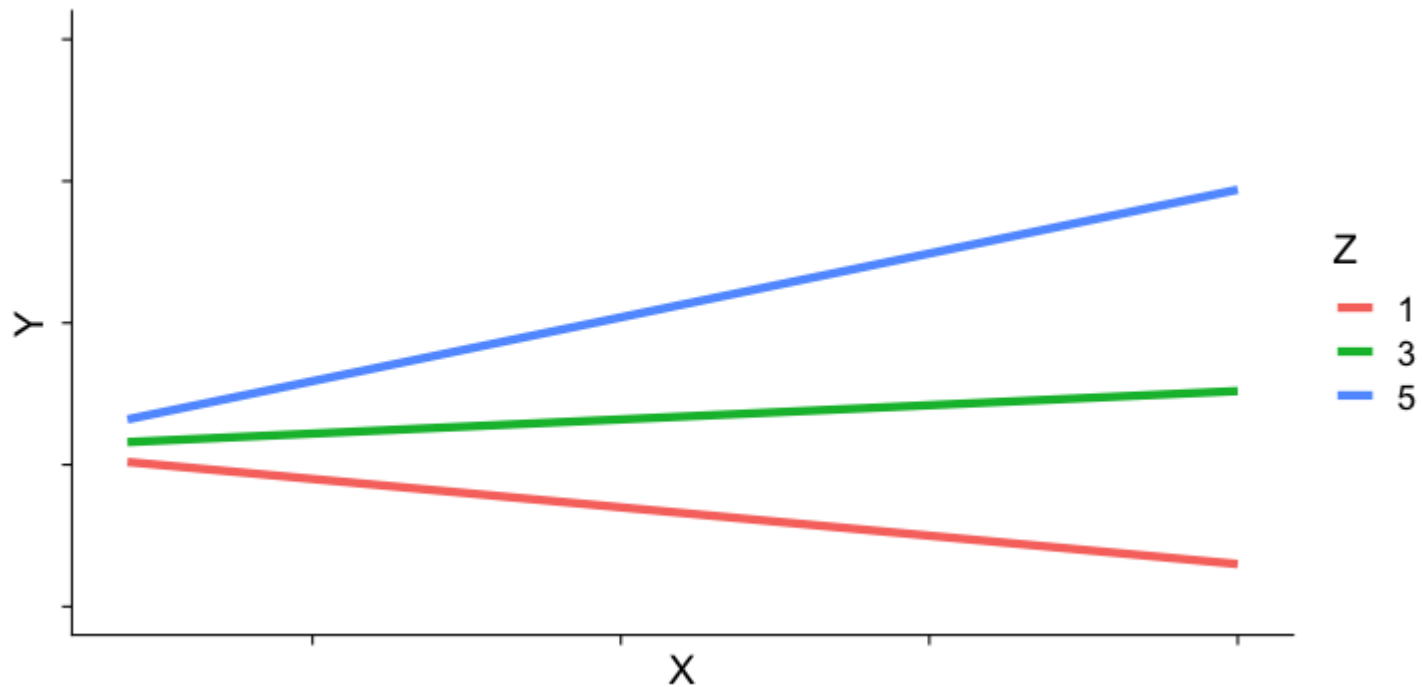
$$\begin{aligned}\hat{Stress} &= -2.74 + 0.62(Anx) + 0.67(10) + -0.04(Anx \times 10) \\ &= -2.74 + 0.62(Anx) + 6.7 + -0.4(Anx) \\ &= 3.96 + 0.22(Anx)\end{aligned}$$

# Interaction shapes

Often we graph the simple slopes as a way to understand the interaction. Interpreting the shape of an interaction can be done using the numbers alone, but it requires a lot of calculation and mental rotation. For those reasons, consider it a requirement that you graph interactions in order to interpret them.

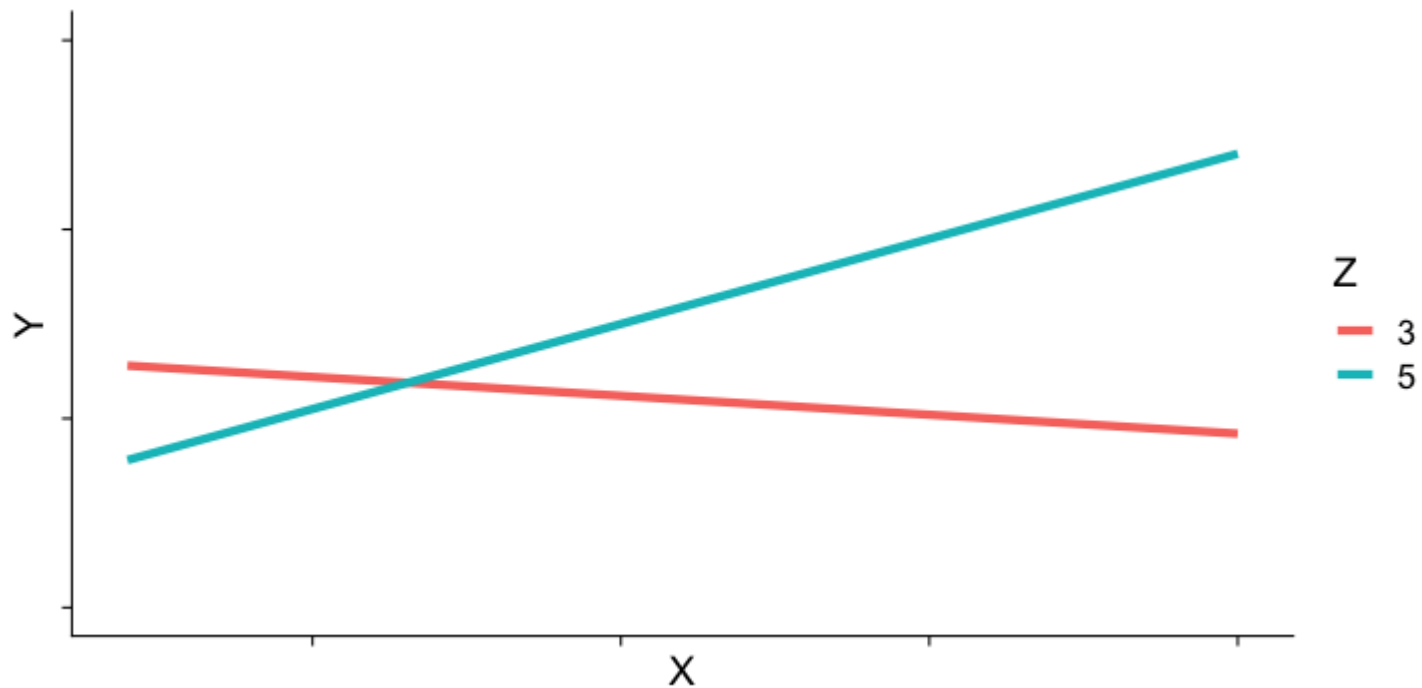
# Interaction shapes

## Ordinal interactions



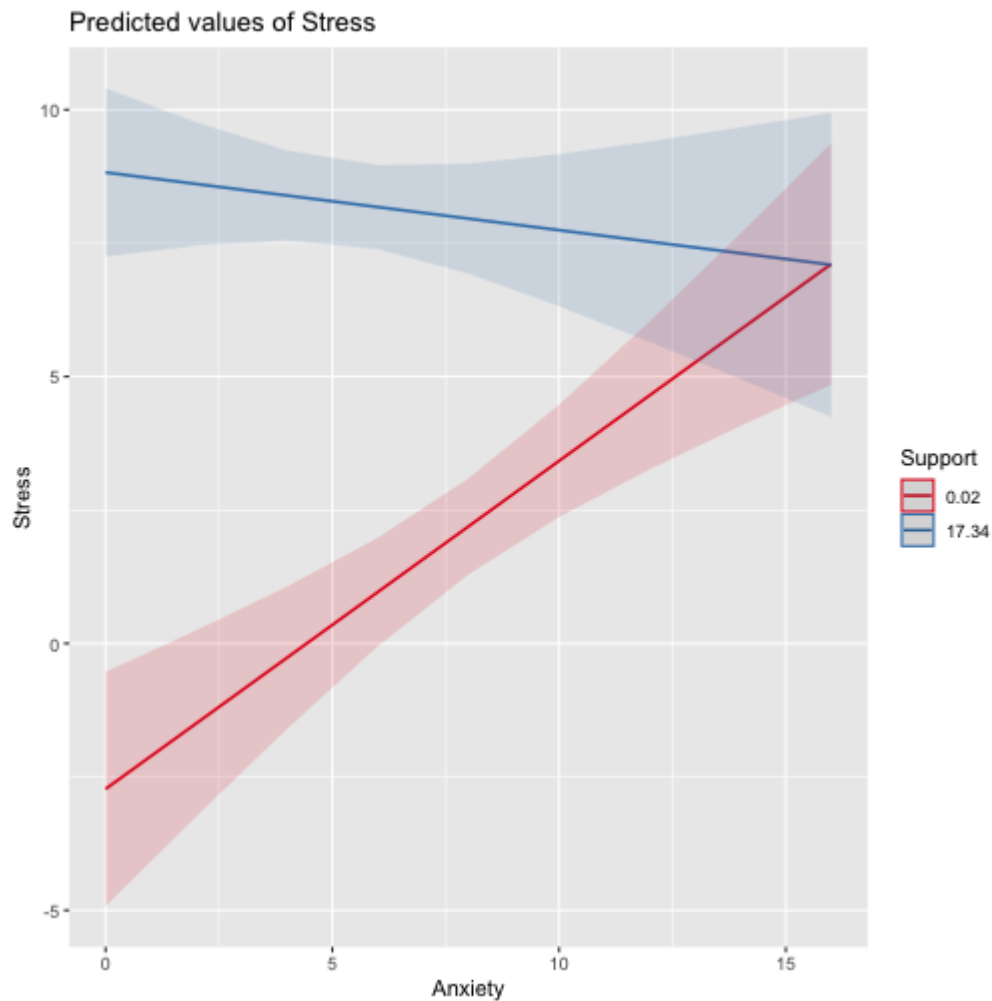
# Interaction shapes

## Cross-over (disordinal) interactions

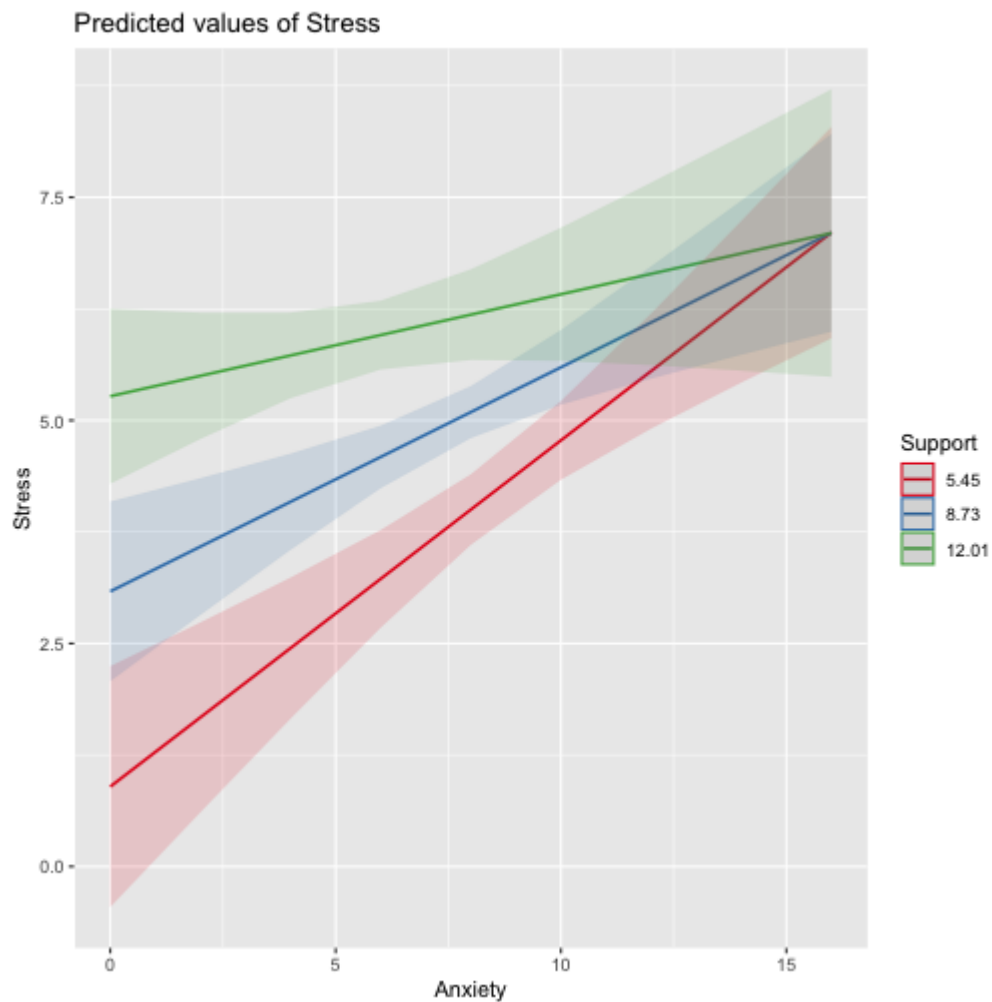




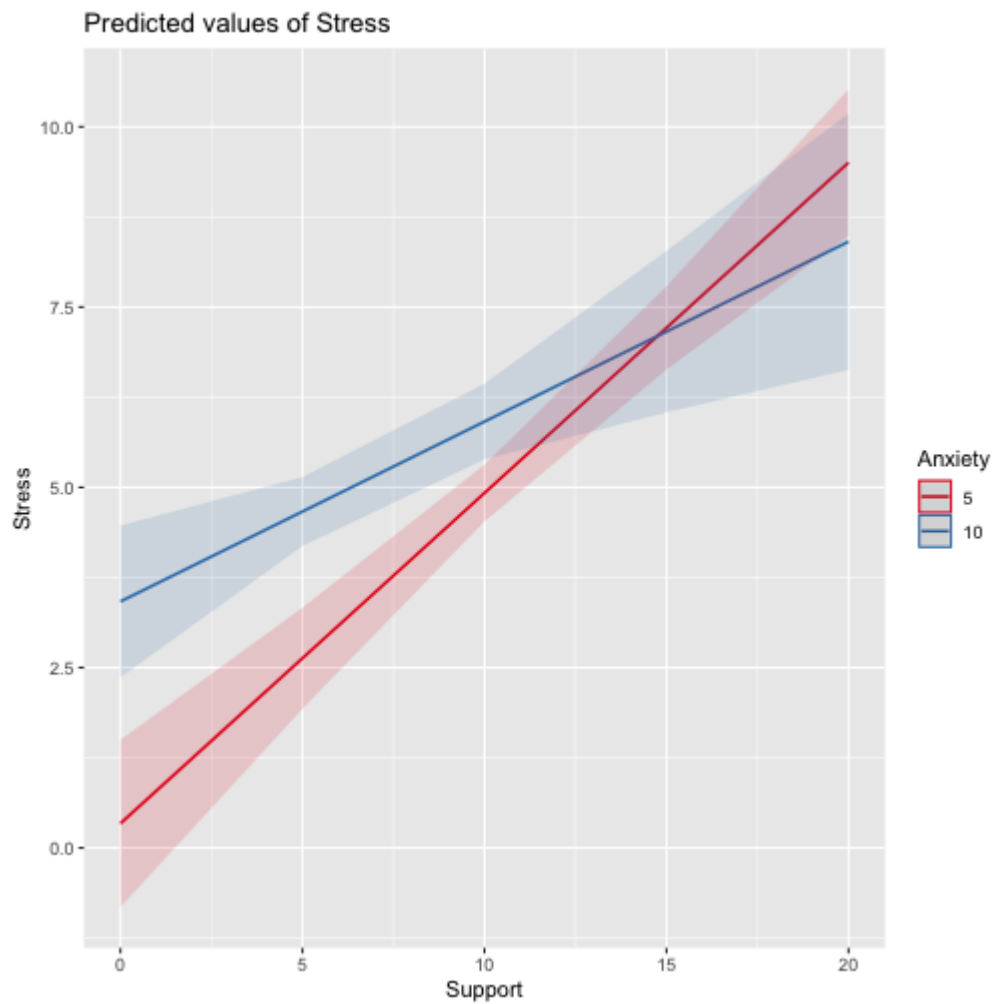
```
library(sjPlot)
plot_model(imodel, type = "int")
```



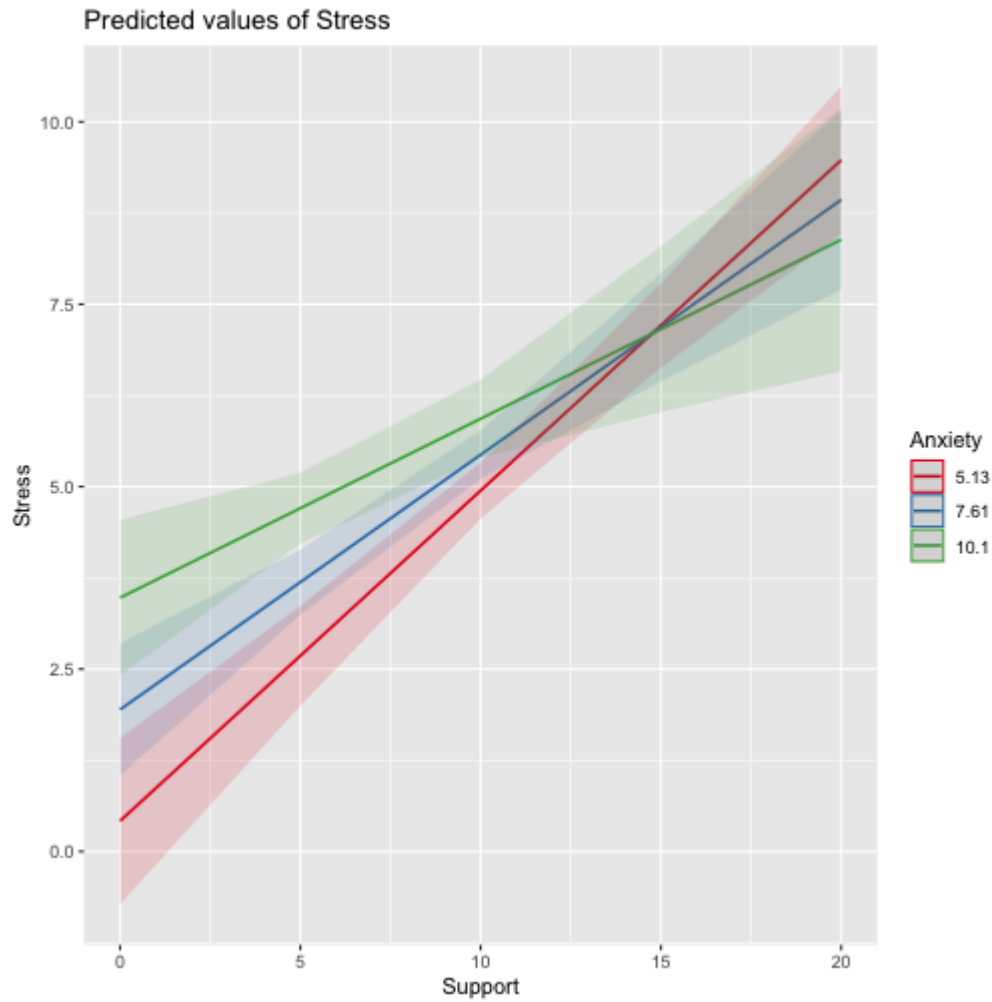
```
plot_model(imodel, type = "int", mdrt.values = "meansd")
```



```
plot_model(imodel, type = "pred", terms = c("Support", "Anxiety[5,10]"))
```



```
plot_model(imodel, type = "pred", terms = c("Support", "Anxiety"), mdrt.
```



# Simple slopes - Significance tests

$$\hat{Stress} = -2.74 + 0.62(Anx) + 0.67(Sup) + -0.04(Anx \times Sup)$$

We want to know whether anxiety is a significant predictor of stress at different levels of support.

```
library(reghelper)
simple_slopes(imodel, levels = list(Support = c(4,6,8,10,12)))
```

| ##   | Anxiety | Support | Test | Estimate | Std. Error | t value | df  | Pr(> t )  | Sig. |
|------|---------|---------|------|----------|------------|---------|-----|-----------|------|
| ## 1 | sstest  | 4       |      | 0.4486   | 0.0886     | 5.0617  | 114 | 1.610e-06 | ***  |
| ## 2 | sstest  | 6       |      | 0.3652   | 0.0733     | 4.9791  | 114 | 2.289e-06 | ***  |
| ## 3 | sstest  | 8       |      | 0.2817   | 0.0654     | 4.3095  | 114 | 3.488e-05 | ***  |
| ## 4 | sstest  | 10      |      | 0.1982   | 0.0674     | 2.9424  | 114 | 0.003946  | **   |
| ## 5 | sstest  | 12      |      | 0.1147   | 0.0786     | 1.4600  | 114 | 0.147036  |      |

If you don't list levels, then this function will test simple slopes at the mean and 1 SD above and below the mean.

# Simple slopes - Significance tests

What if you want to compare slopes to each other? How would we test this?

The test of the interaction coefficient is equivalent to the test of the difference in slopes at levels of Z separated by 1 unit.

```
coef(summary(imodel))
```

| ## |                 | Estimate    | Std. Error | t value   | Pr(> t )     |
|----|-----------------|-------------|------------|-----------|--------------|
| ## | (Intercept)     | -2.73966246 | 1.12100519 | -2.443934 | 1.606052e-02 |
| ## | Anxiety         | 0.61561220  | 0.13010161 | 4.731780  | 6.435373e-06 |
| ## | Support         | 0.66696689  | 0.09547464 | 6.985802  | 2.017698e-10 |
| ## | Anxiety:Support | -0.04174076 | 0.01309328 | -3.187954 | 1.849736e-03 |

# Centering

The regression equation built using the raw data is not only difficult to interpret, but often the terms displayed are not relevant to the hypotheses we're interested.

- $b_0$  is the expected value when all predictors are 0, but this may never happen in real life
- $b_1$  is the slope of X when Z is equal to 0, but this may not ever happen either.

**Centering** your variables by subtracting the mean from all values can improve the interpretation of your results.

- Remember, a linear transformation does not change associations (correlations) between variables. In this case, it only changes the interpretation for some coefficients

# Centering

```
stress.data = stress.data %>%  
  mutate(Anxiety.c = Anxiety - mean(Anxiety),  
         Support.c = Support - mean(Support))  
head(stress.data[,c("Anxiety", "Anxiety.c", "Support", "Support.c")])
```

| ##   |  | Anxiety  | Anxiety.c   | Support | Support.c  |
|------|--|----------|-------------|---------|------------|
| ## 1 |  | 10.18520 | 2.57086873  | 6.1602  | -2.5697997 |
| ## 2 |  | 5.58873  | -2.02560127 | 8.9069  | 0.1769003  |
| ## 3 |  | 6.58500  | -1.02933127 | 10.5433 | 1.8133003  |
| ## 4 |  | 8.95430  | 1.33996873  | 11.4605 | 2.7305003  |
| ## 5 |  | 7.59910  | -0.01523127 | 5.5516  | -3.1783997 |
| ## 6 |  | 8.15600  | 0.54166873  | 7.5117  | -1.2182997 |

**DO NOT CENTER YOUR DEPENDENT VARIABLE (Y; STRESS)**



```
summary(lm(Stress ~ Anxiety.c*Support.c, data = stress.data))
```

```
##
## Call:
## lm(formula = Stress ~ Anxiety.c * Support.c, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.8163 -1.0783  0.0373  0.9200  3.6109
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      4.99580    0.14647   34.108 < 2e-16 ***
## Anxiety.c         0.25122    0.06489    3.872 0.000181 ***
## Support.c         0.34914    0.05238    6.666 9.82e-10 ***
## Anxiety.c:Support.c -0.04174    0.01309   -3.188 0.001850 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.462 on 114 degrees of freedom
## Multiple R-squared:  0.4084,    Adjusted R-squared:  0.3928
## F-statistic: 26.23 on 3 and 114 DF,  p-value: 5.645e-13
```

```
summary(imodel)
```

```
##
## Call:
## lm(formula = Stress ~ Anxiety * Support, data = stress.data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.8163 -1.0783  0.0373  0.9200  3.6109
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.73966    1.12101   -2.444  0.01606 *
## Anxiety         0.61561    0.13010    4.732 6.44e-06 ***
## Support        0.66697    0.09547    6.986 2.02e-10 ***
## Anxiety:Support -0.04174    0.01309   -3.188  0.00185 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.462 on 114 degrees of freedom
## Multiple R-squared:  0.4084,    Adjusted R-squared:  0.3928
## F-statistic: 26.23 on 3 and 114 DF,  p-value: 5.645e-13
```

What changed? What stayed the same?

# Standardized regression equation

So far, we've only discussed the unstandardized regression equation. If you're interested in getting the standardized regression equation, you can follow the same procedure of standardizing your variables first and then entering them into your linear model.

An important note: You must take the product of the Z-scores, not the Z-score of the products to get the correct regression model.

## This is OK

$$Y \sim z(X) + z(Z) + z(X)*z(Z)$$

$$Y \sim z(X)*z(Z)$$

## This is not OK

$$Y \sim z(X) + z(Z) + z(X*Z)$$

# Extensions of Interactions

Interactions are all over the place and we can extend these concepts out:

- Mixing continuous & categorical variables. *"does the slope of x & y change between group 1 and group 2?"*
- Polynomials are also interactions

# Mixing categorical and continuous

Consider the case where  $D$  is a variable representing two groups. In a univariate regression, how do we interpret the coefficient for  $D$ ?

$$\hat{Y} = b_0 + b_1 D$$

$b_0$  is the mean of the reference group, and  $D$  represents the difference in means between the two groups.

# Interpreting slopes

Extending this to the multivariate case, where  $X$  is continuous and  $D$  is a dummy code representing two groups.

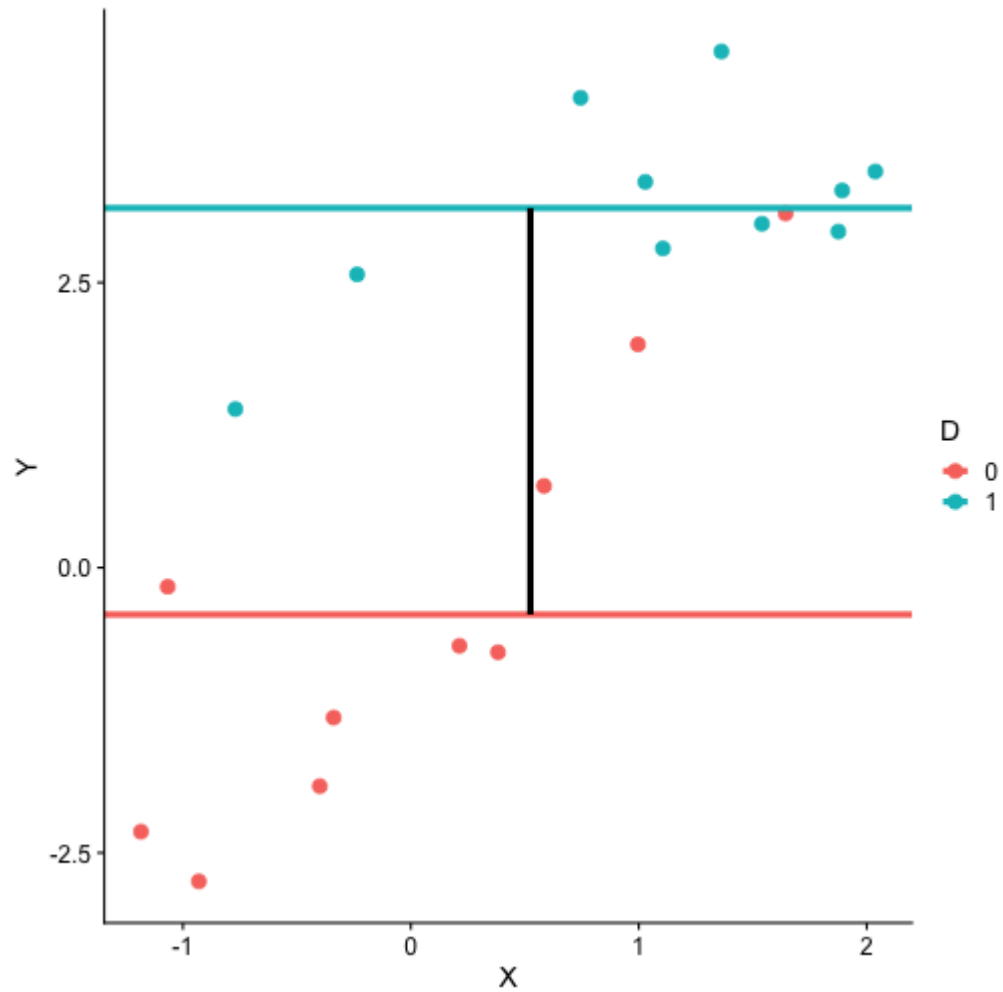
$$\hat{Y} = b_0 + b_1 D + b_2 X$$

How do we interpret  $b_1$ ?

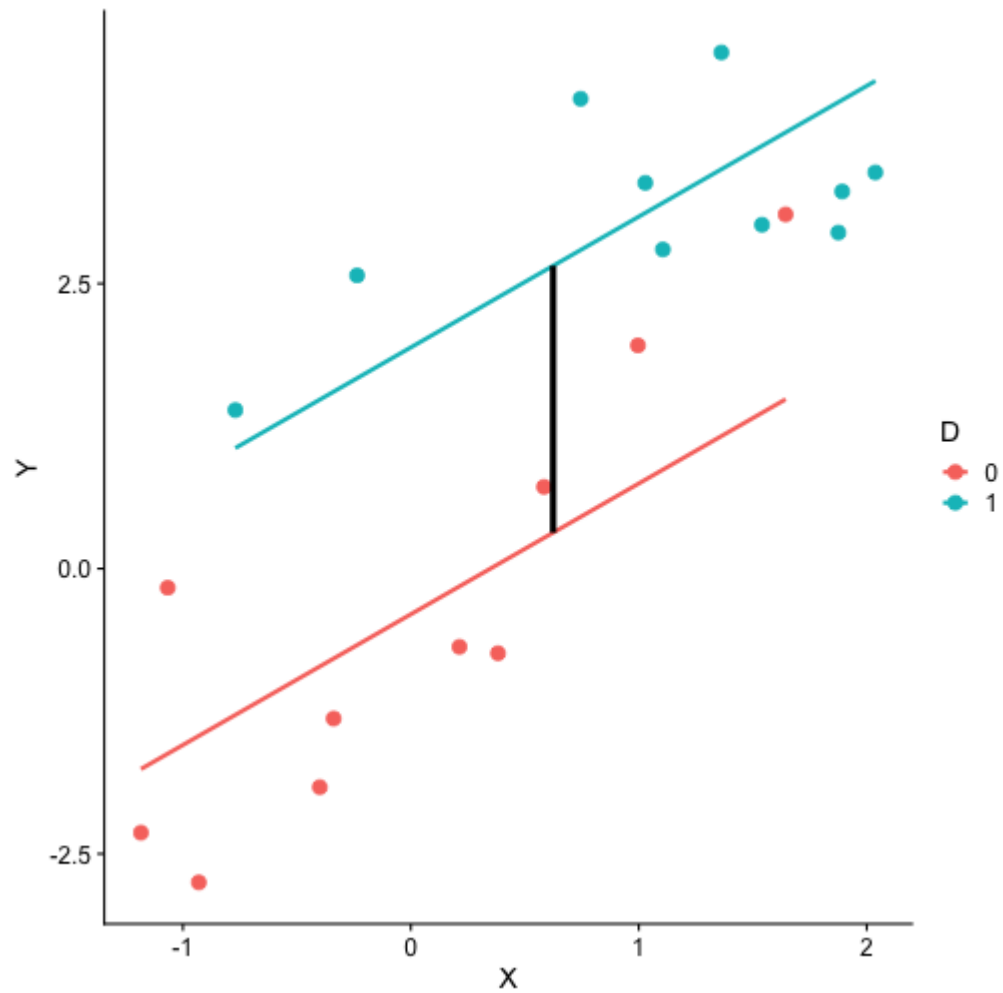
$b_1$  is the difference in means between the two groups *if the two groups have the same average level of  $X$*  or holding  $X$  constant.

This, by the way, is ANCOVA.

# Visualizing

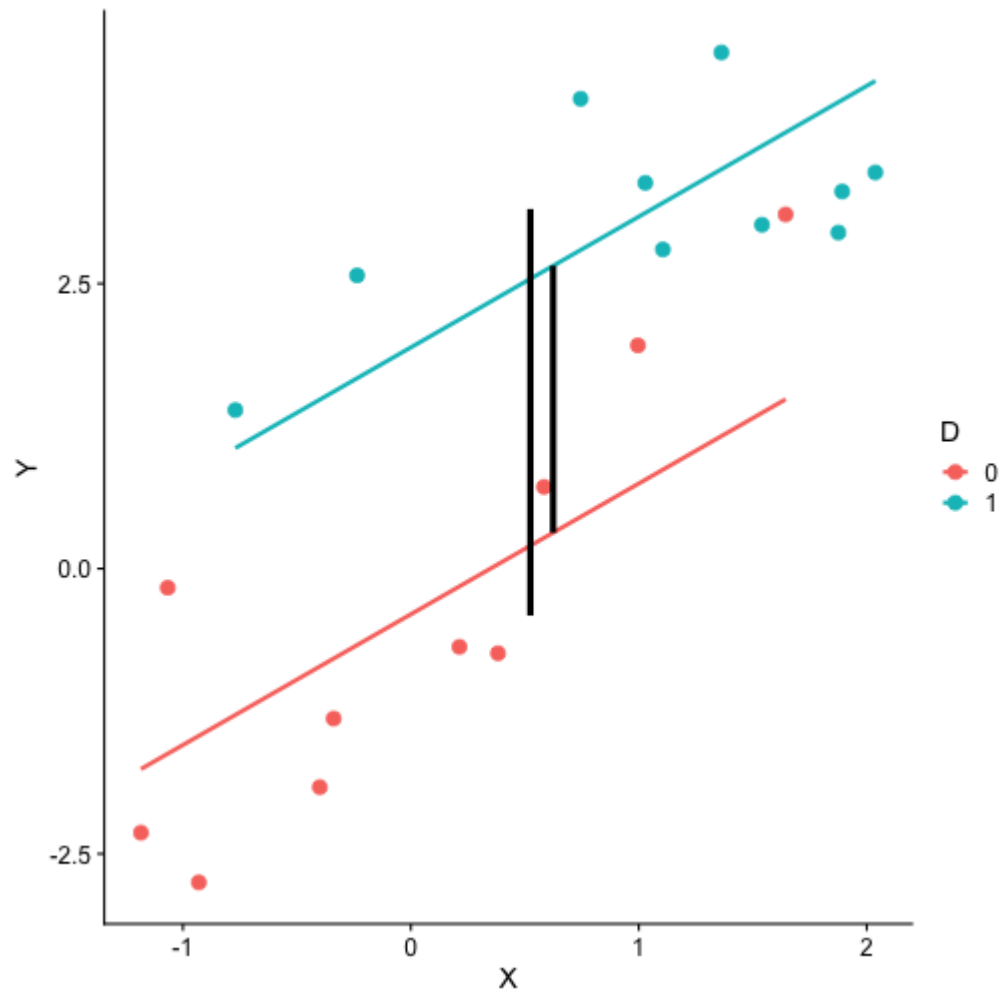


# Visualizing





# Visualizing



# Interactions

Now extend this example to include joint effects, not just additive effects:

$$\hat{Y} = b_0 + b_1D + b_2X + b_3DX$$

How do we interpret  $b_1$ ?

$b_1$  is the difference in means between the two groups *when X is 0*.

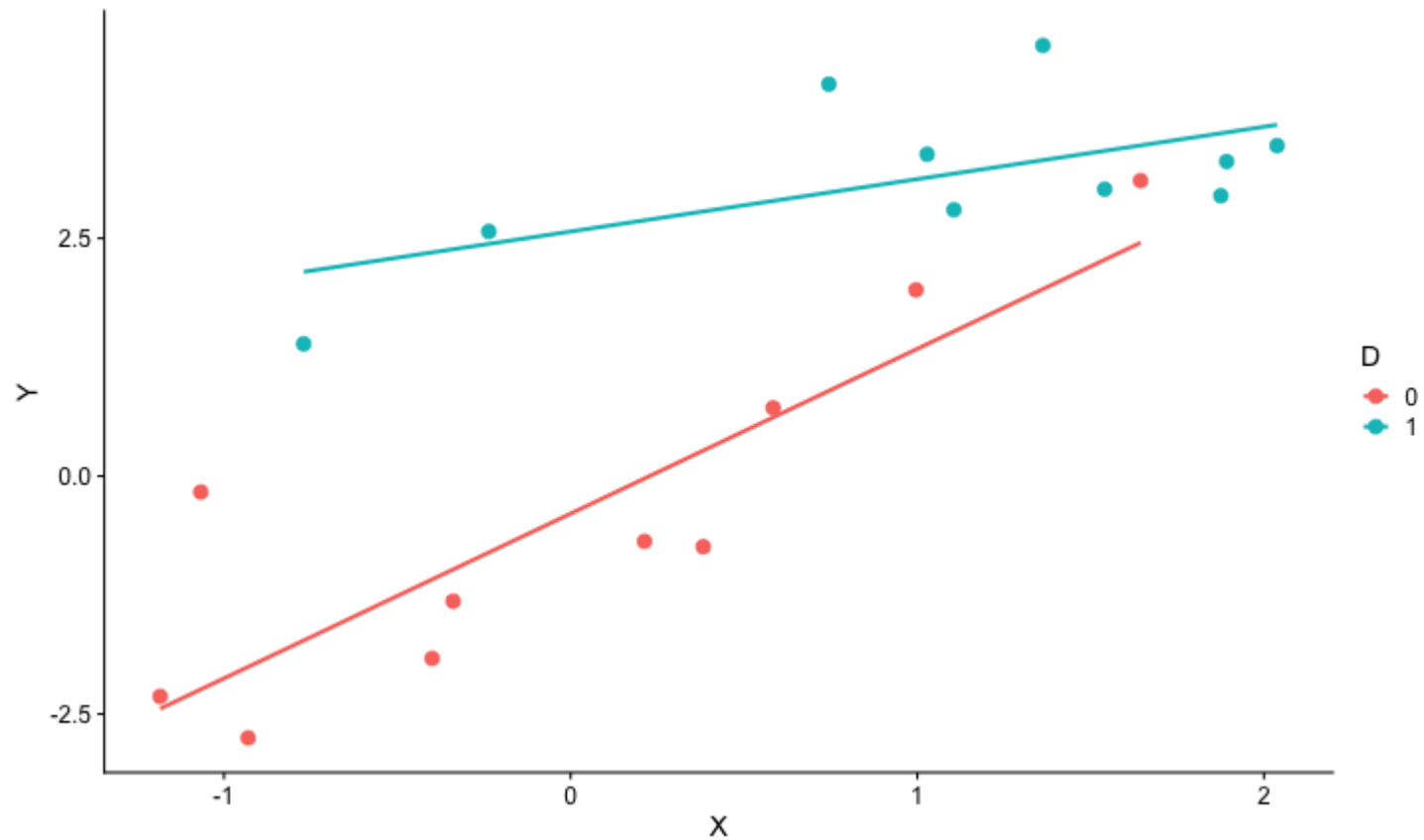
What is the interpretation of  $b_2$ ?

$b_2$  is the slope of X among the reference group.

What is the interpretation of  $b_3$ ?

$b_3$  is the difference in slopes between the reference group and the other group.

# Visualizing



# Polynomial Regression

Polynomial regression (nonlinear) is most often a form of hierarchical regression that systematically tests a series of higher order functions for a single variable.

**Linear:**  $\hat{Y} = b_0 + b_1X$

**Quadratic:**  $\hat{Y} = b_0 + b_1X + b_2X^2$

**Cubic:**  $\hat{Y} = b_0 + b_1X + b_2X^2 + b_3X^3$

**You need 16x the sample size to detect an interaction as you need for a main effect of the same size**