Comparing Means

Recap

- Descriptive Stats
- Normal Distributions
- Sampling Distributions

This time

- t-tests through oneway ANOVA
- BUT, we're going to take a different approach...

Model Comparisons

Scenario 1

Gerrymandering

- Depending on the estimate you pick, about 53% of voters in Wisconsin were Democrats in 2016.
- So our best estimate of the percentage of voters that are Democrat in any *district* might be 53%
- Now that 2016 feels like a million years ago, you find that in actuality it was 52% of voters in Wisconsin were Democrats in 2016.
- *Question: Was our population estimate of 53% significantly different from our sample estimate of 52%?*

one-sample t-test

Model Comparisons

In the normal **one-sample t-test**

- $\bar{x} = \mu$
- $H_0 : \bar{x} - \mu = 0$
- $H_A : \bar{x} - \mu \neq 0$

Model Comparisons

Let's break it down into **full** and **restricted** models:

- **Restricted Model:** reflects what we are testing *against*.
- **Full Model:** allows us to fully include all information we might have.
- Size of the effect is calculated as the following:

$$\frac{(E_r - E_f)/(df_r - df_f)}{E_f/df_f}$$

where:

- E_r is the error from the restricted model
- E_f is the error from the full model
- df_r is the degrees of freedom from the restricted model
- df_f is the degrees of freedom from the full model

The Data

```r
dems <- data.frame(Dem = c(30, 69, 99, 77, 29, 37, 38, 37))
dems
```

```
##   Dem
## 1  30
## 2  69
## 3  99
## 4  77
## 5  29
## 6  37
## 7  38
## 8  37
```

The Restricted Model

Step 1: Get the deviation scores. In the restricted model, we are subtracting from our population estimate of 53%

```
dems$deviationScores <- dems$Dem - 53
dems
```

```
##   Dem deviationScores
## 1  30             -23
## 2  69              16
## 3  99              46
## 4  77              24
## 5  29             -24
## 6  37             -16
## 7  38             -15
## 8  37             -16
```

The Restricted Model

Step 2: Square the deviation scores

```
dems$dev2 <- dems$deviationScores ^2
dems
```

```
##   Dem deviationScores dev2
## 1  30             -23  529
## 2  69              16  256
## 3  99              46 2116
## 4  77              24  576
## 5  29             -24  576
## 6  37             -16  256
## 7  38             -15  225
## 8  37             -16  256
```

The Restricted Model

Step 3: Get the sum of the square deviation scores. This is our **ERROR** term. It is the squared errors.

```
Er <- sum(dems$dev2)
dems
```

```
##   Dem deviationScores dev2
## 1  30             -23  529
## 2  69              16  256
## 3  99              46 2116
## 4  77              24  576
## 5  29             -24  576
## 6  37             -16  256
## 7  38             -15  225
## 8  37             -16  256
```

```
Er
```

```
## [1] 4790
```

The Restricted Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this restricted model, there is nothing that we are "guessing" or estimating. So there is nothing to subtract. $df = n$, **df = 8**

```
dfr <- 8
```

The Full Model

Step 1: Get the deviation scores. In the full model, we are subtracting from our population estimate of 52%

```
demsFull = dems %>%
  select(Dem)

demsFull$deviationScores <- demsFull$Dem - 52
demsFull
```

```
##   Dem deviationScores
## 1  30             -22
## 2  69              17
## 3  99              47
## 4  77              25
## 5  29             -23
## 6  37             -15
## 7  38             -14
## 8  37             -15
```

The Full Model

Step 2: Square the deviation scores

```
demsFull$dev2 <- demsFull$deviationScores^2
demsFull
```

```
##   Dem deviationScores dev2
## 1  30             -22  484
## 2  69              17  289
## 3  99              47 2209
## 4  77              25  625
## 5  29             -23  529
## 6  37             -15  225
## 7  38             -14  196
## 8  37             -15  225
```

The Full Model

Step 3: Get the sum of the square deviation scores. This is our **ERROR** term. It is the squared errors.

```
Ef <- sum(demsFull$dev2)
demsFull
```

```
##   Dem deviationScores dev2
## 1  30             -22  484
## 2  69              17  289
## 3  99              47 2209
## 4  77              25  625
## 5  29             -23  529
## 6  37             -15  225
## 7  38             -14  196
## 8  37             -15  225
```

```
Ef
```

```
## [1] 4782
```

The Full Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this full model, we are guessing/estimating our sample mean of 52. $df = n - 1$, **df = 8 - 1 = 7**

```
dff <- 7
```

The Effect

$$\frac{(E_r - E_f)/(df_r - df_f)}{E_f/df_f}$$

```
effect <- ((Er - Ef) / (dfr - dff)) / (Ef/dff)
effect
```

```
## [1] 0.01171058
```

This is our F-statistic. Remember that $t^2 = F$. So to get our t-statistic, let's take the square root of our `effect`.

```
tstat <- sqrt(effect)
round(x = tstat, digits = 3)
```

```
## [1] 0.108
```

Model Comparison Approach

Is 0.108 more extreme than our critical value?

- For an $\alpha = .05$ in a two-tailed test with $df - 7$, the critical t value is 2.3650.

Conclusion: No, it's not more extreme than the critical value. The error terms between the null and restricted models are not meaningfully different -- therefore the means are not statistically significantly different

Model Comparison Approach

We just did a one-sample t-test! Let's verify our results:

```
t.test(x = dems$Dem, mu = 53)
```

```
##
##      One Sample t-test
##
## data:  dems$Dem
## t = -0.10822, df = 7, p-value = 0.9169
## alternative hypothesis: true mean is not equal to 53
## 95 percent confidence interval:
##  30.14892 73.85108
## sample estimates:
## mean of x
##        52
```

Look at the errors!

In general, we try to *minimize* error!

Restricted

```r
as.data.frame(dems$dev2)
```

```
##   dems$dev2
## 1       529
## 2       256
## 3      2116
## 4       576
## 5       576
## 6       256
## 7       225
## 8       256
```

Full

```r
as.data.frame(demsFull$dev2)
```

```
##   demsFull$dev2
## 1           484
## 2           289
## 3          2209
## 4           625
## 5           529
## 6           225
## 7           196
## 8           225
```

Scenario 2

- What if now we want to compare the difference in means (of % Democrats) between the 2010 election?
- *Question: are the means of % Democrats significantly different between 2010 and 2016?*

The Data

```
dems <- data.frame(Dem = c(30, 69, 99, 77, 29, 37, 38, 37,
                           30, 62, 50, 69, 27, 29, 44, 45),
                   Year = c(rep("2016", times = 8),
                            rep("2010", times = 8)))
dems$Year <- factor(dems$Year)

dems
```

```
##    Dem Year
## 1   30 2016
## 2   69 2016
## 3   99 2016
## 4   77 2016
## 5   29 2016
## 6   37 2016
## 7   38 2016
## 8   37 2016
## 9   30 2010
## 10  62 2010
## 11  50 2010
## 12  69 2010
## 13  27 2010
## 14  29 2010
## 15  44 2010
## 16  45 2010
```

The Hypotheses

- $H_0 : \bar{x}_{2010} - \bar{x}_{2016} = 0$
- $H_A : \bar{x}_{2010} - \bar{x}_{2016} \neq 0$
- Restricted Model: the best way of minimizing errors is to use the overall grand mean
- Full Model: the best way of minimizing errors is to use the group-specific mean.

The Means

Let's get the grand mean to use in our Restricted model and the means of each group (% Democrat in 2010 vs. % Democrat in 2016):

```
grandMean <- mean(dems$Dem)

groupMeans <- dems %>%
  group_by(Year) %>%
  summarize(means = mean(Dem))

grandMean
```

```
## [1] 48.25
```

```
groupMeans
```

```
## # A tibble: 2 x 2
##   Year  means
##   <fct> <dbl>
## 1 2010   44.5
## 2 2016   52
```

The Restricted Model

```
dems$Mean <- rep(grandMean, times = nrow(dems))
dems
```

```
##     Dem Year  Mean
## 1    30 2016 48.25
## 2    69 2016 48.25
## 3    99 2016 48.25
## 4    77 2016 48.25
## 5    29 2016 48.25
## 6    37 2016 48.25
## 7    38 2016 48.25
## 8    37 2016 48.25
## 9    30 2010 48.25
## 10   62 2010 48.25
## 11   50 2010 48.25
## 12   69 2010 48.25
## 13   27 2010 48.25
## 14   29 2010 48.25
## 15   44 2010 48.25
## 16   45 2010 48.25
```

The Restricted Model

Step 1: Deviation Scores

```
dems$deviationScores <- dems$Dem - dems$Mean
dems
```

```
##    Dem Year  Mean deviationScores
## 1   30 2016 48.25          -18.25
## 2   69 2016 48.25           20.75
## 3   99 2016 48.25           50.75
## 4   77 2016 48.25           28.75
## 5   29 2016 48.25          -19.25
## 6   37 2016 48.25          -11.25
## 7   38 2016 48.25          -10.25
## 8   37 2016 48.25          -11.25
## 9   30 2010 48.25          -18.25
## 10  62 2010 48.25           13.75
## 11  50 2010 48.25            1.75
## 12  69 2010 48.25           20.75
## 13  27 2010 48.25          -21.25
## 14  29 2010 48.25          -19.25
## 15  44 2010 48.25           -4.25
## 16  45 2010 48.25           -3.25
```

The Restricted Model

Step 2: Square Deviation Scores

```
dems$dev2 <- dems$deviationScores ^2
dems
```

```
##     Dem Year  Mean deviationScores      dev2
## 1    30 2016 48.25          -18.25  333.0625
## 2    69 2016 48.25           20.75  430.5625
## 3    99 2016 48.25           50.75 2575.5625
## 4    77 2016 48.25           28.75  826.5625
## 5    29 2016 48.25          -19.25  370.5625
## 6    37 2016 48.25          -11.25  126.5625
## 7    38 2016 48.25          -10.25  105.0625
## 8    37 2016 48.25          -11.25  126.5625
## 9    30 2010 48.25          -18.25  333.0625
## 10   62 2010 48.25           13.75  189.0625
## 11   50 2010 48.25            1.75    3.0625
## 12   69 2010 48.25           20.75  430.5625
## 13   27 2010 48.25          -21.25  451.5625
## 14   29 2010 48.25          -19.25  370.5625
## 15   44 2010 48.25           -4.25   18.0625
## 16   45 2010 48.25           -3.25   10.5625
```

The Restricted Model

Step 3: Sum of Squares -- our **ERROR** term

```
Er <- sum(dems$dev2)
dems
```

```
##    Dem Year  Mean deviationScores      dev2
## 1   30 2016 48.25          -18.25  333.0625
## 2   69 2016 48.25           20.75  430.5625
## 3   99 2016 48.25           50.75 2575.5625
## 4   77 2016 48.25           28.75  826.5625
## 5   29 2016 48.25          -19.25  370.5625
## 6   37 2016 48.25          -11.25  126.5625
## 7   38 2016 48.25          -10.25  105.0625
## 8   37 2016 48.25          -11.25  126.5625
## 9   30 2010 48.25          -18.25  333.0625
## 10  62 2010 48.25           13.75  189.0625
## 11  50 2010 48.25            1.75    3.0625
## 12  69 2010 48.25           20.75  430.5625
## 13  27 2010 48.25          -21.25  451.5625
## 14  29 2010 48.25          -19.25  370.5625
## 15  44 2010 48.25           -4.25   18.0625
## 16  45 2010 48.25           -3.25   10.5625
```

```
Er
```

```
## [1] 6701
```

The Restricted Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this restricted model, we are guessing/estimating our grand mean of 48.25. $df = n - 1$, **df = 16 - 1 = 15**

```
dfr <- 15
```

The Full Model

```r
dems <- data.frame(Dem = c(30, 69, 99, 77, 29, 37, 38, 37,
                          30, 62, 50, 69, 27, 29, 44, 45),
                   Year = c(rep("2016", times = 8),
                            rep("2010", times = 8)))
dems$Year <- factor(dems$Year)

dems$Mean <- c(rep(groupMeans$means[2], times = 8),
               rep(groupMeans$means[1], times = 8))
dems
```

```
##    Dem Year Mean
## 1   30 2016 52.0
## 2   69 2016 52.0
## 3   99 2016 52.0
## 4   77 2016 52.0
## 5   29 2016 52.0
## 6   37 2016 52.0
## 7   38 2016 52.0
## 8   37 2016 52.0
## 9   30 2010 44.5
## 10  62 2010 44.5
## 11  50 2010 44.5
```

The Full Model

Step 1: Deviation Scores

```
dems$deviationScores <- dems$Dem - dems$Mean
dems
```

```
##     Dem Year Mean deviationScores
## 1    30 2016 52.0           -22.0
## 2    69 2016 52.0            17.0
## 3    99 2016 52.0            47.0
## 4    77 2016 52.0            25.0
## 5    29 2016 52.0           -23.0
## 6    37 2016 52.0           -15.0
## 7    38 2016 52.0           -14.0
## 8    37 2016 52.0           -15.0
## 9    30 2010 44.5           -14.5
## 10   62 2010 44.5            17.5
## 11   50 2010 44.5             5.5
## 12   69 2010 44.5            24.5
## 13   27 2010 44.5           -17.5
## 14   29 2010 44.5           -15.5
## 15   44 2010 44.5            -0.5
## 16   45 2010 44.5             0.5
```

The Full Model

Step 2: Square Deviation Scores

```
dems$dev2 <- dems$deviationScores ^2
dems
```

```
##    Dem Year Mean deviationScores    dev2
## 1   30 2016 52.0           -22.0  484.00
## 2   69 2016 52.0            17.0  289.00
## 3   99 2016 52.0            47.0 2209.00
## 4   77 2016 52.0            25.0  625.00
## 5   29 2016 52.0           -23.0  529.00
## 6   37 2016 52.0           -15.0  225.00
## 7   38 2016 52.0           -14.0  196.00
## 8   37 2016 52.0           -15.0  225.00
## 9   30 2010 44.5           -14.5  210.25
## 10  62 2010 44.5            17.5  306.25
## 11  50 2010 44.5             5.5   30.25
## 12  69 2010 44.5            24.5  600.25
## 13  27 2010 44.5           -17.5  306.25
## 14  29 2010 44.5           -15.5  240.25
## 15  44 2010 44.5            -0.5    0.25
## 16  45 2010 44.5             0.5    0.25
```

The Full Model

Step 3: Sum of Squares -- our **ERROR** term

```
Ef <- sum(dems$dev2)
dems
```

```
##    Dem Year Mean deviationScores    dev2
## 1   30 2016 52.0           -22.0  484.00
## 2   69 2016 52.0            17.0  289.00
## 3   99 2016 52.0            47.0 2209.00
## 4   77 2016 52.0            25.0  625.00
## 5   29 2016 52.0           -23.0  529.00
## 6   37 2016 52.0           -15.0  225.00
## 7   38 2016 52.0           -14.0  196.00
## 8   37 2016 52.0           -15.0  225.00
## 9   30 2010 44.5           -14.5  210.25
## 10  62 2010 44.5            17.5  306.25
## 11  50 2010 44.5             5.5   30.25
## 12  69 2010 44.5            24.5  600.25
## 13  27 2010 44.5           -17.5  306.25
## 14  29 2010 44.5           -15.5  240.25
## 15  44 2010 44.5            -0.5    0.25
## 16  45 2010 44.5             0.5    0.25
```

```
Ef
```

```
## [1] 6476
```

The Full Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this full model, we are guessing/estimating our 2 means (mean for 2010 and mean for 2016). $df = n - 2$, **df = 16 - 2 = 14**

```
dff <- 14
```

The Effect

$$\frac{(E_r - E_f)/(df_r - df_f)}{E_f/df_f}$$

```
effect <- ((Er - Ef) / (dfr - dff)) / (Ef/dff)
effect
```

```
## [1] 0.4864114
```

This is our F-statistic. Remember that $t^2 = F$. So to get our t-statistic, let's take the square root of our `effect`.

```
tstat <- sqrt(effect)
round(x = tstat, digits = 3)
```

```
## [1] 0.697
```

Model Comparison Approach

Is 0.697 more extreme than our critical value?

- For an $\alpha = .05$ in a two-tailed test with $df - 7$, the critical t value is 2.131.

Conclusion: No, it's not more extreme than the critical value. The error terms between the null and restricted models are not meaningfully different -- therefore the means are not statistically significantly different

Model Comparison Approach

We just did an independent-samples t-test! Let's verify our results:

```
t.test(dems$Dem ~ dems$Year)
```

```
##
##      Welch Two Sample t-test
##
## data:  dems$Dem by dems$Year
## t = -0.69743, df = 11.406, p-value = 0.4995
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -31.06632  16.06632
## sample estimates:
## mean in group 2010 mean in group 2016
##               44.5               52.0
```

Scenario 3

- We have a dataset that looks at the lengths and widths of petals & sepals of the iris flower. It includes 3 different species of irises.
- *Question: are the sepal lengths different amongst the 3 species of irises?*

The Data

```r
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

```r
iris <- iris %>%
  select(Sepal.Length, Species)
```

The Hypotheses

- $H_0 : \bar{x}_{setosa} = \bar{x}_{versicolor} = \bar{x}_{virginica}$
- $H_A : \bar{x}_{setosa} \neq \bar{x}_{versicolor} \neq \bar{x}_{virginica}$
- Restricted Model: the best way of minimizing errors is to use the overall grand mean
- Full Model: the best way of minimizing errors is to use the group-specific mean.

The Means

Let's get the grand mean to use in our Restricted model and the means of each group:

```
grandMean <- mean(iris$Sepal.Length)

groupMeans <- iris %>%
  group_by(Species) %>%
  summarize(means = mean(Sepal.Length))

grandMean
```

```
## [1] 5.843333
```

```
groupMeans
```

```
## # A tibble: 3 x 2
##   Species    means
##   <fct>      <dbl>
## 1 setosa      5.01
## 2 versicolor  5.94
## 3 virginica   6.59
```

The Restricted Model

```
restricted <- iris
restricted$Mean <- rep(grandMean, times = nrow(restricted))
head(restricted)
```

```
##   Sepal.Length Species     Mean
## 1          5.1  setosa 5.843333
## 2          4.9  setosa 5.843333
## 3          4.7  setosa 5.843333
## 4          4.6  setosa 5.843333
## 5          5.0  setosa 5.843333
## 6          5.4  setosa 5.843333
```

The Restricted Model

Step 1: Deviation Scores

```
restricted$deviationScores <- restricted$Sepal.Length - restricted$Mean
head(restricted)
```

```
##   Sepal.Length Species     Mean deviationScores
## 1          5.1  setosa 5.843333      -0.7433333
## 2          4.9  setosa 5.843333      -0.9433333
## 3          4.7  setosa 5.843333      -1.1433333
## 4          4.6  setosa 5.843333      -1.2433333
## 5          5.0  setosa 5.843333      -0.8433333
## 6          5.4  setosa 5.843333      -0.4433333
```

The Restricted Model

Step 2: Square Deviation Scores

```r
restricted$dev2 <- restricted$deviationScores ^2
head(restricted)
```

```
##   Sepal.Length Species     Mean deviationScores      dev2
## 1          5.1  setosa 5.843333      -0.7433333 0.5525444
## 2          4.9  setosa 5.843333      -0.9433333 0.8898778
## 3          4.7  setosa 5.843333      -1.1433333 1.3072111
## 4          4.6  setosa 5.843333      -1.2433333 1.5458778
## 5          5.0  setosa 5.843333      -0.8433333 0.7112111
## 6          5.4  setosa 5.843333      -0.4433333 0.1965444
```

The Restricted Model

Step 3: Sum of Squares -- our **ERROR** term

```
Er <- sum(restricted$dev2)
head(restricted)
```

```
##   Sepal.Length Species     Mean deviationScores      dev2
## 1          5.1  setosa 5.843333      -0.7433333 0.5525444
## 2          4.9  setosa 5.843333      -0.9433333 0.8898778
## 3          4.7  setosa 5.843333      -1.1433333 1.3072111
## 4          4.6  setosa 5.843333      -1.2433333 1.5458778
## 5          5.0  setosa 5.843333      -0.8433333 0.7112111
## 6          5.4  setosa 5.843333      -0.4433333 0.1965444
```

```
Er
```

```
## [1] 102.1683
```

The Restricted Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this restricted model, we are guessing/estimating our grand mean of 5.843. $df = n - 1$, **df = 150 - 1 = 149**

```
dfr <- 149
```

The Full Model

```
full <- iris
full$Mean <- c(rep(groupMeans$means[1], times = 50),
               rep(groupMeans$means[2], times = 50),
               rep(groupMeans$means[3], times = 50))
head(full)
```

```
##   Sepal.Length Species  Mean
## 1          5.1  setosa 5.006
## 2          4.9  setosa 5.006
## 3          4.7  setosa 5.006
## 4          4.6  setosa 5.006
## 5          5.0  setosa 5.006
## 6          5.4  setosa 5.006
```

The Full Model

Step 1: Deviation Scores

```
full$deviationScores <- full$Sepal.Length - full$Mean
head(full)
```

```
##   Sepal.Length Species  Mean deviationScores
## 1          5.1  setosa 5.006           0.094
## 2          4.9  setosa 5.006          -0.106
## 3          4.7  setosa 5.006          -0.306
## 4          4.6  setosa 5.006          -0.406
## 5          5.0  setosa 5.006          -0.006
## 6          5.4  setosa 5.006           0.394
```

The Full Model

Step 2: Square Deviation Scores

```
full$dev2 <- full$deviationScores ^2
head(full)
```

```
##   Sepal.Length Species  Mean deviationScores     dev2
## 1          5.1  setosa 5.006           0.094 0.008836
## 2          4.9  setosa 5.006          -0.106 0.011236
## 3          4.7  setosa 5.006          -0.306 0.093636
## 4          4.6  setosa 5.006          -0.406 0.164836
## 5          5.0  setosa 5.006          -0.006 0.000036
## 6          5.4  setosa 5.006           0.394 0.155236
```

The Full Model

Step 3: Sum of Squares -- our **ERROR** term

```
Ef <- sum(full$dev2)
head(full)
```

```
##   Sepal.Length Species  Mean deviationScores      dev2
## 1          5.1  setosa 5.006           0.094 0.008836
## 2          4.9  setosa 5.006          -0.106 0.011236
## 3          4.7  setosa 5.006          -0.306 0.093636
## 4          4.6  setosa 5.006          -0.406 0.164836
## 5          5.0  setosa 5.006          -0.006 0.000036
## 6          5.4  setosa 5.006           0.394 0.155236
```

```
Ef
```

```
## [1] 38.9562
```

The Full Model

Step 4: Determine the degrees of freedom.

- Degrees of freedom deals with how much information is *free to vary*
- You get a feel for this the more you practice
- In this full model, we are guessing/estimating our 3 means (mean for each species). $df = n - 3$, **df = 150 - 3 = 147**

```
dff <- 147
```

The Effect

$$\frac{(E_r - E_f)/(df_r - df_f)}{E_f/df_f}$$

```
effect <- ((Er - Ef) / (dfr - dff)) / (Ef/dff)
effect
```

```
## [1] 119.2645
```

This is our F-statistic. This is an ANOVA, so we can stick with the F-statistic.

```
round(x = effect, digits = 3)
```

```
## [1] 119.265
```

Model Comparison Approach

Is 119.265 more extreme than our critical value?

- A significant F-statistic is anything above 1. Yes, our value is larger than 1.

Conclusion: The error terms between the null and restricted models are meaningfully different -- therefore the means are statistically significantly different

Model Comparison Approach

We just did a oneway ANOVA! Let's verify our results:

```
summary(aov(Sepal.Length ~ Species, data = iris))
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## Species       2  63.21  31.606   119.3 <2e-16 ***
## Residuals   147  38.96   0.265
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Extras

- Could you do this with a paired samples t-test? **YES**
- Could you do this with a 2x2 ANOVA (or any other form)? **YES**
- So. Why is it called ANOVA?

Utility

We have programs like R. In the workforce, no one will expect you to calculate this stuff by hand. So why go through the effort of showing you this?

A model is what **YOU** define. It's how you think the world works. The restricted model is really just an emobodiement of the null hypothesis! The full model is the embodiment of the alternative hypothesis!

Minimizing error terms is how we evaluate multitudes of models!

Plus, model comparison frameworks come up more formally in some advanced types of statistics.