Miscellaneous

Last time

- 2x2 ANOVA with interactions
- NHST and effect sizes

Today

A series of shpiels

- Finish up between-subjects ANOVA
- Within-subjects designs
- Non-parametric tests

OG data

	Normal Equal Sign Problems				Tricky Equal Sign Problems				
8-9 year olds	3	4	3	4	0	0	0	1	μ_8
9-10 year olds	4	4	4	3	2	3	2	1	μ_9
	μ_{Normal}				μ_{Tricky}				

What we did...

- Found the means for each cell in our 2x2 ANOVA
- Took away the effect of each of the 2 factors (Age and Problem)
- Added back the grand mean

Another way of doing the interaction

$H_{0.3}$: *The differences between age groups are the same regardless of problem type (tricky or normal)*

8-9 Year Olds with Normal = Problems	9-10 Year Olds with Normal = Problems	8-9 Year Olds with Tricky = Problems	9-10 Year Olds with Tricky = Problems
3.50	3.75	0.25	2.00

Null hypothesis for interactions

$H_{0.3}$: *The differences between age groups are the same regardless of problem type (tricky or normal)*

$$H_{0.3} : (\mu_{N8} - \mu_{N9}) = (\mu_{T8} - \mu_{T9})$$

$$H_{0.3} : (\mu_{N8} - \mu_{N9}) - (\mu_{T8} - \mu_{T9}) = 0$$

$$H_{0.3} : \mu_{N8} - \mu_{N9} - \mu_{T8} + \mu_{T9} = 0$$

8-9 Year Olds with Normal = Problems	9-10 Year Olds with Normal = Problems	8-9 Year Olds with Tricky = Problems	9-10 Year Olds with Tricky = Problems
3.50	3.75	0.25	2.00
1	-1	-1	1

First rule of contrasts is that they must sum to 0

Coontrasts are your friends!

$$F = \frac{\psi^2}{MS_{\text{Within}}\Sigma(c_j^2/n_j)}$$

$$\psi = (1)(3.50) + (-1)(3.75) + (-1)(0.25) + (1)(2.00) = 1.50$$

$$F = \frac{1.50^2}{.375\Sigma(c_j^2/n_j)}$$

$$F = \frac{1.50^2}{.375(\frac{1^2}{4} + \frac{-1^2}{4} + \frac{-1^2}{4} + \frac{1^2}{4})}$$

$$F = \frac{2.25}{.375} = 6$$

Did we get it right?

	SS	df	MS	F
Age	4.00	1	4.00	10.667
Problem	25.00	1	25.00	66.667
Age x Problem	2.25	1	2.25	6.00
Error / Residuals	4.50	12	0.375	

Testing Ourselves

New 2x2 design comparing treatments to reduce depression

- Therapy = Cognitive Behavioral vs. Meditation Training
- Medication = With meds vs. Without meds

	SS	df	MS	F
Therapy				
Medication				
Therapy x Medication				
Error / Residuals				

Fill in the blanks

	SS	df	MS	F
Therapy	300			20
Medication			250	
Therapy x Medication				
Error / Residuals		20		
Total	1000			

Fill in the blanks -- Answer

	SS	df	MS	F
Therapy	300	1	300	20
Medication	250	1	250	16.667
Therapy x Medication	150	1	150	10
Error / Residuals	300	20	15	
Total	1000			

Fill in the blanks

	SS	df	MS	F
Therapy			50	
Medication	100			
Therapy x Medication	50			
Error / Residuals			10	
Total	320			

Fill in the blanks -- Answer

	SS	df	MS	F
Therapy	50	1	50	5
Medication	100	1	100	10
Therapy x Medication	50	1	50	5
Error / Residuals	120	12	10	
Total	320			

Assumptions of ANOVA

1. Experimental errors are normally distributed (more later)

2. Equal variances between treatments. Aka homogeneity of variances.

3. Independence of samples (more next)

What happens when we break some of these assumptions?

Within-subjects

What is it?

- Other names: within-groups, repeated measures
- Each subject contributes a score to each level of an independent variable (breaking independence assumption)
- We've done this before...paired t-test
    - Pre vs. Post

Why does this matter?

- Some of the variability in the scores within a level of a factor is predictable if you know which participant contributed the score.

- If you could remove the variability that goes with the differences between the participants, you could reduce the variability within a level of the factor.

    - Same participants, or paired in some way (matched)

Repeated Measures ANOVA

- You can calculate, and then discard, the variability among the means that comes from differences between the subjects
  - What does this mean if you want to know about individual differences?
- The remaining variability in the dataset is then partitioned into 2 components:
  1. variance due to differences between treatments/conditions/levels
  2. variance due to error (like measurement error, random noise etc.)

What changes in the calculations?

- Restricted Model: ΣD_i^2
  - The null is that there is no difference in the participants' scores between conditions
  - Take the difference scores, square them, sum them up
  - df_r = how many difference scores there are; you do not remove a df because you don't use a mean

What changes in the calculations?

- Full Model: $\Sigma(D_i - \bar{D})^2$
  - The full model/alternative hypothesis is that there is a difference in the participants' scores between conditions
  - Take each individual's difference score and find the deviation for that difference score from the mean difference score of the sample. Square it, sum it up.
  - EX: pre = 8, post = 10, diff = 2
  - EX: if the average of the difference scores was 5, you would then do `2 - 5 = -3`
  - df_f = number of difference scores - 1; you lose 1 df because you calculated the mean of difference scores

The Consequences -- Good

- Same participants = less time and effort
- Statistical POWER!
  - What goes into our F-statistic? $\frac{MS_{btwn}}{MS_{error}}$
  - The smaller MS_{error}, the larger the F-statistic
  - The repeated design allows you to remove the between-subject variability, so MS_{error} gets smaller

The Consequences -- Bad

- Often not feasible
- Order effects
  - Everyone gets Chocolate -> Vanilla -> Strawberry
  - How do you know that eating chocolate ice cream first doesn't change the way vanilla tastes to participants?
  - You need to counterbalance! (some get c -> v -> s, some get v -> s -> c etc...)
- Time elapsed can be...tricky
  - Take Josh Jackson's Applied Longitudinal class
  - Take Mike Strube's Hierarchical Linear Modeling class

Shelly hates within-subjects ANOVAs

- When you have a within-subjects variable, our homogeneity assumption expands out to "sphericity"
- Rather than the groups having the same variances, now it's the DIFFERENCE SCORES WITHIN THE GROUP that have to have the same variances
- **At best you get the same answer as you would with a multilevel model**
- At worst, you've violated assumptions (rather aggressively) and therefore it's not valid to interpret the results
- More important: if you have within-subject variance, just explicitly model it!

Venn Diagram Exercise

How do we break down variances?

Analysis of Covariance (conceptually)

Logic of ANCOVA

Say you want to do an ANOVA, but there's an additional variable (continuous or categorical) whose influence you wish to *control for*. We call that a **covariate**.

Ex:

- Effect of treatment, controlling for initial levels of some disorder
- Controlling for known effects like age, sex, SES, etc...

Logic of ANCOVA

We just need to add another term to our restricted and full models.

Restricted:

$$Y_{ij} = \mu + Covariate + e_{ij}$$

Full:

$$Y_{ij} = \mu + Age_j + Covariate + e_{ij}$$

ANCOVA Thoughts

The Good:

- Can increase power by reducing SS_{within}
- Some use to control for initial levels of a disorder (meh)

The Bad:

- We will talk about regression extensively next semester; IMHO, use formal regression framework instead
- You're losing a degree of freedom in both your full and restricted models. So if you include a covariate that doesn't matter much, and now you've lost a df, then you might inadvertently wind up with less statistical power. Oy!

Non-parametric Tests

Normality Assumption

- For t-test, we said the data needed to be normally distributed
  - can use QQ plots (quantile-quantile plots)
  - can use Shapiro-Wilk test
- For ANOVA, it's slightly different...
  - $H_0 : Y_{ij} = \mu + \epsilon_{ij}$
  - $H_1 : Y_{ij} = \mu_j + \epsilon_{ij}$
  - The assumption is that ϵ_{ij}, or your residuals, need to be normally distributed -- lots more next semester
  - Can use QQ plots and Shaprio-Wilk -- QQ plots more common

QQ Plots

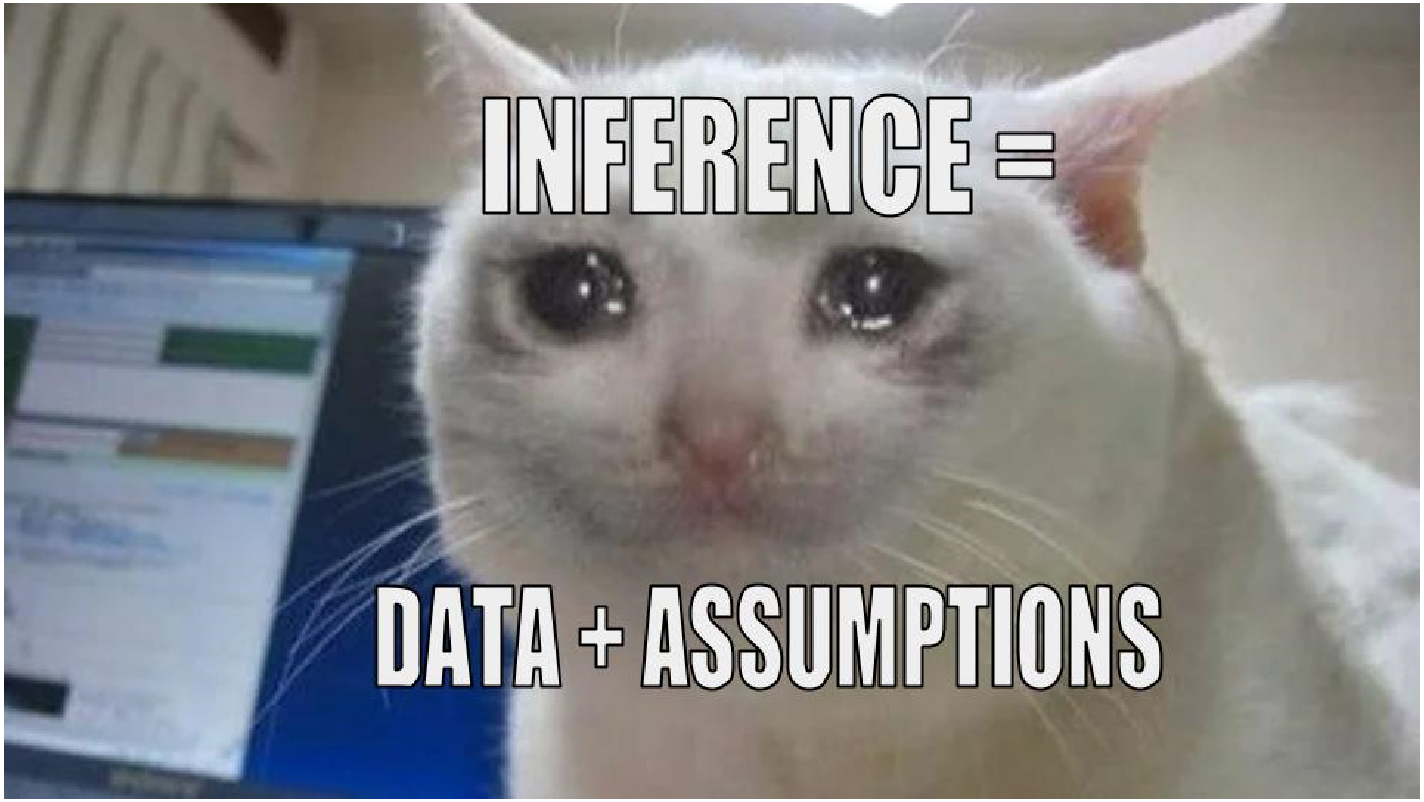
Try this (you might be limited to # of times you can click):

https://xiongge.shinyapps.io/QQplots/

Check out this Stack Exchange on how to interpret QQ plots.

Who cares?

If we violate our assumptions, *any of them*, we cannot make any valid inferences!



Non-parametric tests

Enter non-parametric tests. Often these are based on using the rank order of data (and the median instead of the mean).

Helpful when:

- Dependent variable is nominal
- Independent or dependent variable is ordinal
- Sample size is small
- Underlying population is skewed (reaction times, household income)

Non-parametric tests

Limitations:

- CI and effect size calculations aren't always possible (or if they are, they're a pain)
- Less powerful typically
- Increased risk of a Type II error; maybe that's OK?
- Nominal/ordinal scales provide less detail than continuous data

Parametric Test	Situation	Non-Parametric Version
Single sample *z*	sample mean vs population mean with known σ	¯\\(ツ)/¯
Single sample *t*	sample mean vs population mean with unknown σ	*Wilcoxon Signed-Rank Test*
Paired samples *t*	Compare 2 means with within-groups design	*Wilcoxon Signed-Rank Test*
Independent samples *t*	Compare 2 means with between-groups design	*Mann-Whitney U Test*
Oneway ANOVA (btwn groups)	Compare 3+ levels of IV	*Kruskal-Wallis H Test*
Oneway ANOVA (repeated measures)	Compare 3+ levels of IV	*Friedman Test*
Twoway ANOVA	2+ IVs (main effects/interactions)	*Kruskal-Wallis H Test*
Correlation	Relationship between 2 continuous vars	*Spearman Rank-Order Correlation*

Next time

Validity!