Two Between-Subjects Factors

Recap

- We've taken a model comparison approach to comparing group means with ANOVA
  - Restricted
  - Full
  - *F* test
- Explored different approaches to comparing means
  - Multiple comparisons
  - Contrast coefficients

This time

- Adding another independent variable:

Between Subjects Factors

Nomenclature

Consider the following design

Kindergarten and 1st Graders' knowledge of double letters in words

Allowed doubles

- *baff*
- *holl*
- *dess*

Unallowed doubles

- *bbaf*
- *hhol*
- *ddes*

Two-way between-subjects, completely crossed (balanced)

```
##      doublet
## grade allowed unallowed
##     1       4         4
##     K       4         4
```

Null Hypothesis for "Main Effects"

$H_{0.1}$: Main effect of grade: The two grade groups perform similarly, regardless of doublet type.

$H_{0.2}$: Main effect of doublet legality: Children perform similarly with allowed and unallowed doublets, regardless of what grade they are in.

$H_{0.1}$: $\bar{x}_K = \bar{x}_{1st}$

$H_{0.2}$: $\bar{x}_{allowed} = \bar{x}_{unallowed}$

Null Hypothesis for "Main Effects"

$H_{0.1}$: $\bar{x}_K = \bar{x}_{1st}$

$H_{0.2}$: $\bar{x}_{allowed} = \bar{x}_{unallowed}$

```r
(grandmean <- mean(data$correct))
```

```
## [1] 6.0625
```

```r
(groupmeans <- aggregate(correct ~ grade + doublet, data, mean))
```

```
##   grade   doublet correct
## 1     1   allowed    8.50
## 2     K   allowed    2.75
## 3     1 unallowed    9.50
## 4     K unallowed    3.50
```

Model Comparison Approach for Main Effects

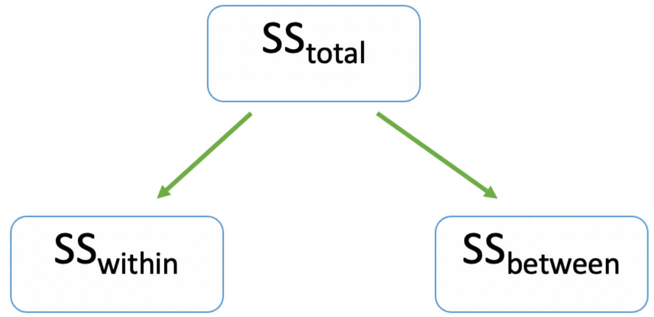
- **Two Restricted Models:** reflects what we are testing *against*.
  - grand mean is our best guess for the main effect of grade
  - grand mean is our best guess for the main effect of doublet position

- **Two Full Models:** allows us to fully include all information we might have.
  - best way of minimizing errors is to use grade-specific mean
  - best way of minimizing errors is to use doublet-specific mean

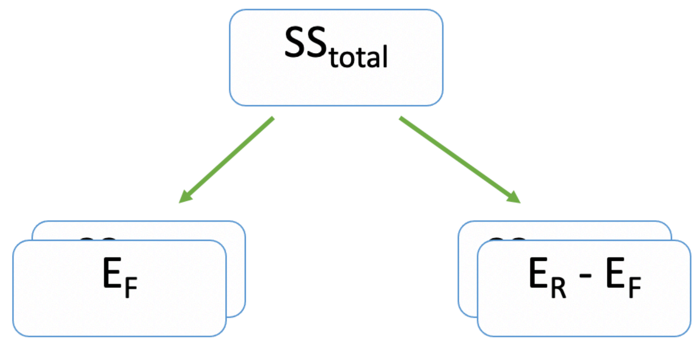
F Table

	SS	Df	MS	F
Grade	SS_{grade} (same as $E_R - E_F$ if there was one factor!)	df_g	SS_{grade} / df_{grade}	MS_{grade} / MS_W
Doublet	$SS_{doublet}$ (same as $E_R - E_F$ if there was one factor!)	df_d	$SS_{doublet} / df_{doublet}$	$MS_{doublet} / MS_W$
Error (residual)	SS_W	df_W	SS_W / df_W	

The variance tree

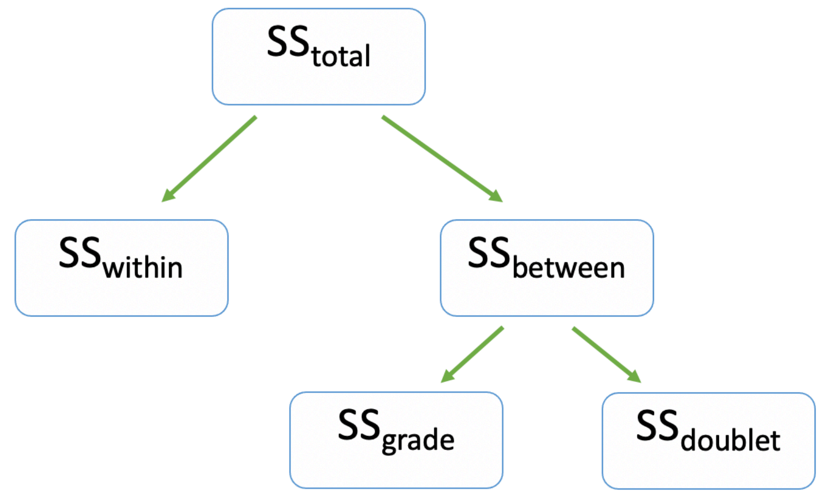


The variance tree



* This is basically our F test!

The variance tree



Let's start with Grade!

$$SS_{grade} = E_R - E_F$$

Restricted: our best guess for any student is the grand mean + some error

Full: our best guess for any student is their grade's mean + some error

Step 1: The Restricted Model for Grade

```
##    grade   doublet correct prediction     dev       dev2
## 1      K   allowed       2     6.0625 -4.0625 16.5039062
## 2      K   allowed       3     6.0625 -3.0625  9.3789062
## 3      K   allowed       4     6.0625 -2.0625  4.2539062
## 4      K   allowed       2     6.0625 -4.0625 16.5039062
## 5      K unallowed       3     6.0625 -3.0625  9.3789062
## 6      K unallowed       4     6.0625 -2.0625  4.2539062
## 7      K unallowed       3     6.0625 -3.0625  9.3789062
## 8      K unallowed       4     6.0625 -2.0625  4.2539062
## 9      1   allowed       8     6.0625  1.9375  3.7539062
## 10     1   allowed       9     6.0625  2.9375  8.6289062
## 11     1   allowed       7     6.0625  0.9375  0.8789062
## 12     1   allowed      10     6.0625  3.9375 15.5039062
## 13     1 unallowed       9     6.0625  2.9375  8.6289062
## 14     1 unallowed      10     6.0625  3.9375 15.5039062
## 15     1 unallowed       9     6.0625  2.9375  8.6289062
## 16     1 unallowed      10     6.0625  3.9375 15.5039062
```

```
## [1] 150.9375
```

Step 2: The Full Model for Grade

Get Ef for grade. Our best guess is the group's mean.

```
(grade_means <- aggregate(correct ~ grade, data, mean))
```

```
##   grade correct
## 1     1   9.000
## 2     K   3.125
```

Step 2: The Full Model for Grade

```
##    grade   doublet correct prediction    dev      dev2
## 1      K   allowed       2      3.125 -1.125 1.265625
## 2      K   allowed       3      3.125 -0.125 0.015625
## 3      K   allowed       4      3.125  0.875 0.765625
## 4      K   allowed       2      3.125 -1.125 1.265625
## 5      K unallowed       3      3.125 -0.125 0.015625
## 6      K unallowed       4      3.125  0.875 0.765625
## 7      K unallowed       3      3.125 -0.125 0.015625
## 8      K unallowed       4      3.125  0.875 0.765625
## 9      1   allowed       8      9.000 -1.000 1.000000
## 10     1   allowed       9      9.000  0.000 0.000000
## 11     1   allowed       7      9.000 -2.000 4.000000
## 12     1   allowed      10      9.000  1.000 1.000000
## 13     1 unallowed       9      9.000  0.000 0.000000
## 14     1 unallowed      10      9.000  1.000 1.000000
## 15     1 unallowed       9      9.000  0.000 0.000000
## 16     1 unallowed      10      9.000  1.000 1.000000
```

```
## [1] 12.875
```

Step 3: Calculate SS.

$$SS_{grade} = E_r - E_f$$

```
(SS_grade <- Er_grade - Ef_grade)
```

```
## [1] 138.0625
```

Step 4: Determine the degrees of freedom.

- $df_{grade} = k_{grade} - 1$
- where k = number of levels in the factor

F Table

	SS	Df	MS	F
Grade	SS_{grade} 138.0625 (same as $E_R - E_F$ if there was one factor!)	df_g 1	SS_{grade} / df_{grade}	MS_{grade} / MS_W
Doublet	$SS_{doublet}$ (same as $E_R - E_F$ if there was one factor!)	df_d	$SS_{doublet}$ / $df_{doublet}$	$MS_{doublet}$ / MS_W
Error (residual)	SS_W	df_W	SS_W / df_W	

Rinse and repeat for doublet legality

$SS_{doublet} = E_R - E_F$

Restricted: our best guess for any student is the grand mean + some error

- that is, regardless of whether students saw allowable or unallowable doublets, they have the same predicted number of correct items

Full: our best guess for any student is their doublet type group's mean + some error

Step 1: The Restricted Model for Doublet Type

```
##    grade   doublet correct prediction     dev       dev2
## 1      K   allowed       2     6.0625 -4.0625 16.5039062
## 2      K   allowed       3     6.0625 -3.0625  9.3789062
## 3      K   allowed       4     6.0625 -2.0625  4.2539062
## 4      K   allowed       2     6.0625 -4.0625 16.5039062
## 5      K unallowed       3     6.0625 -3.0625  9.3789062
## 6      K unallowed       4     6.0625 -2.0625  4.2539062
## 7      K unallowed       3     6.0625 -3.0625  9.3789062
## 8      K unallowed       4     6.0625 -2.0625  4.2539062
## 9      1   allowed       8     6.0625  1.9375  3.7539062
## 10     1   allowed       9     6.0625  2.9375  8.6289062
## 11     1   allowed       7     6.0625  0.9375  0.8789062
## 12     1   allowed      10     6.0625  3.9375 15.5039062
## 13     1 unallowed       9     6.0625  2.9375  8.6289062
## 14     1 unallowed      10     6.0625  3.9375 15.5039062
## 15     1 unallowed       9     6.0625  2.9375  8.6289062
## 16     1 unallowed      10     6.0625  3.9375 15.5039062

## [1] 150.9375
```

Step 2: The Full Model for Doublet

```
(dbl_means <- aggregate(correct ~ doublet, data, mean))
```

```
##     doublet correct
## 1   allowed   5.625
## 2 unallowed   6.500
```

Step 2: The Full Model for Doublet

```
##    grade   doublet correct prediction    dev      dev2
## 1      K   allowed       2      5.625 -3.625 13.140625
## 2      K   allowed       3      5.625 -2.625  6.890625
## 3      K   allowed       4      5.625 -1.625  2.640625
## 4      K   allowed       2      5.625 -3.625 13.140625
## 5      1   allowed       8      5.625  2.375  5.640625
## 6      1   allowed       9      5.625  3.375 11.390625
## 7      1   allowed       7      5.625  1.375  1.890625
## 8      1   allowed      10      5.625  4.375 19.140625
## 9      K unallowed       3      6.500 -3.500 12.250000
## 10     K unallowed       4      6.500 -2.500  6.250000
## 11     K unallowed       3      6.500 -3.500 12.250000
## 12     K unallowed       4      6.500 -2.500  6.250000
## 13     1 unallowed       9      6.500  2.500  6.250000
## 14     1 unallowed      10      6.500  3.500 12.250000
## 15     1 unallowed       9      6.500  2.500  6.250000
## 16     1 unallowed      10      6.500  3.500 12.250000
```

```
## [1] 147.875
```

Step 3: Calculate SS

$$SS_{dbl} = E_r - E_f$$

```
(SS_dbl <- Er_dbl - Ef_dbl)
```

```
## [1] 3.0625
```

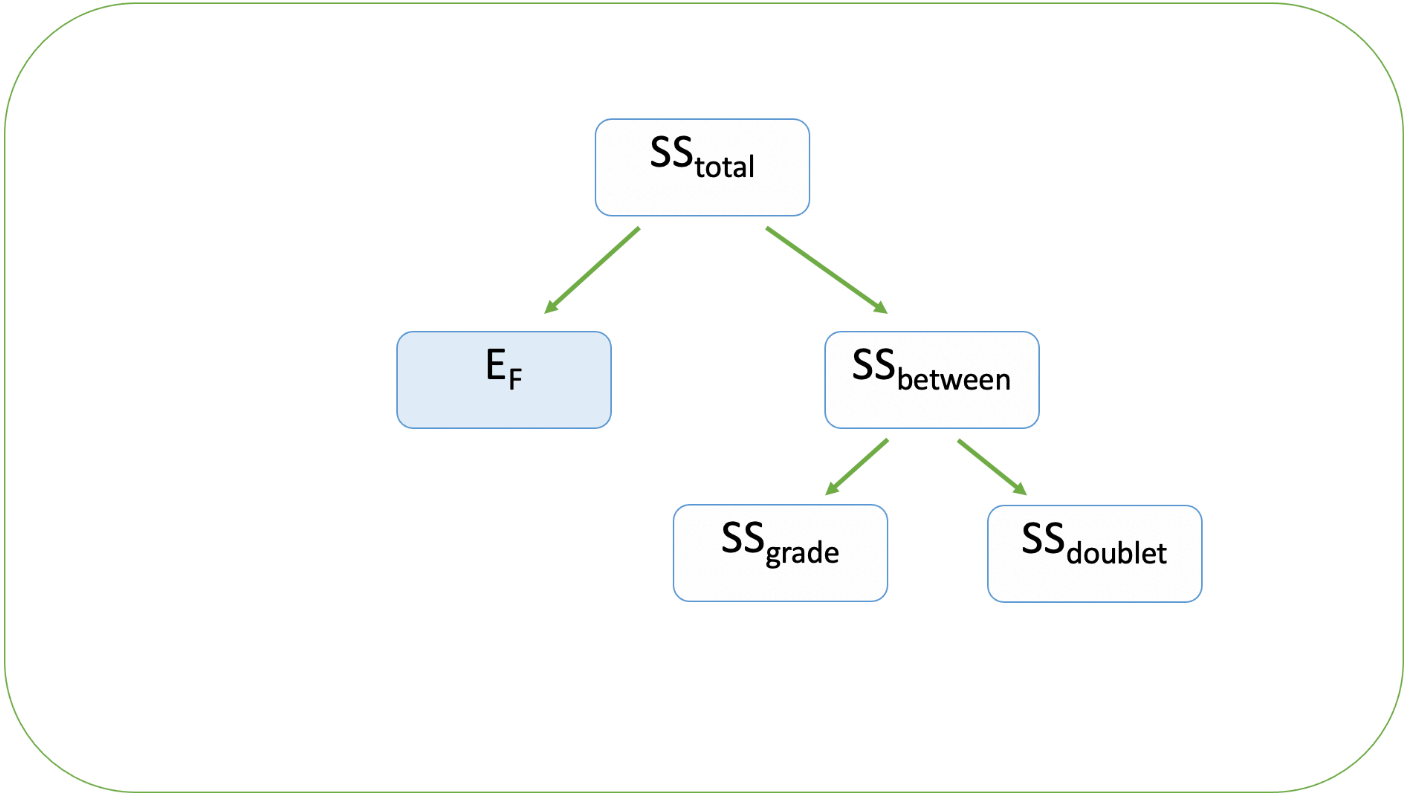
Step 4: Determine the degrees of freedom

- $df_{dbl} = k_{dbl} - 1$
- where k = number of levels in the factor

F Table

	SS	Df	MS	F
Grade	SS_{grade} **138.0625** (same as $E_R - E_F$ if there was one factor!)	df_g **1**	SS_{grade} / df_{grade}	MS_{grade} / MS_W
Doublet	$SS_{doublet}$ **3.0625** (same as $E_R - E_F$ if there was one factor!)	df_d **1**	$SS_{doublet} / df_{doublet}$	$MS_{doublet} / MS_W$
Error (residual)	SS_W	df_W	SS_W / df_W	

What about that Error?



This is our **leftover variation** after taking our grade and doublet variables into account.

Calculating SS error

$$SS_{Total} = SS_{grade} + SS_{doublet} + SS_{error}$$

Conceptually:

$$SS_{error} = SS_{Total} - SS_{grade} - SS_{doublet}$$

Calculating SS error (Full Model perspective)

Our full model allows for a grade effect and a doublet effect.

So from the full model perspective, the best guess for any student is...

Calculating SS error

```
##    grade   doublet correct prediction deviationScores   dev2
## 1      K   allowed       2       2.75           -0.75 0.5625
## 2      K   allowed       3       2.75            0.25 0.0625
## 3      K   allowed       4       2.75            1.25 1.5625
## 4      K   allowed       2       2.75           -0.75 0.5625
## 5      K unallowed       3       3.50           -0.50 0.2500
## 6      K unallowed       4       3.50            0.50 0.2500
## 7      K unallowed       3       3.50           -0.50 0.2500
## 8      K unallowed       4       3.50            0.50 0.2500
## 9      1   allowed       8       8.50           -0.50 0.2500
## 10     1   allowed       9       8.50            0.50 0.2500
## 11     1   allowed       7       8.50           -1.50 2.2500
## 12     1   allowed      10       8.50            1.50 2.2500
## 13     1 unallowed       9       9.50           -0.50 0.2500
## 14     1 unallowed      10       9.50            0.50 0.2500
## 15     1 unallowed       9       9.50           -0.50 0.2500
## 16     1 unallowed      10       9.50            0.50 0.2500
```

```
(SSe <- sum(full_error$dev2))
```

```
## [1] 9.75
```

Degrees of Freedom in two factor between subjects

df_{Between}

- For *each* factor:
  - $df_{factor} = \text{Levels of factor} - 1$
  - e.g., grade has **2 levels** (K, first)
  - $df_{grade} = 2 - 1 = 1$

Degrees of Freedom in two factor between subjects

$df_{within/error/residual}$

16 observations, 1 additional mean for grade factor, 1 additional mean for doublet factor, and 1 grand mean

$df_w = N - (\text{Levels of } F_1 - 1) - (\text{Levels of } F_2 - 1) - 1$

simplifies to
$N - \text{Levels of } F_1 - \text{Levels of } F_2 + 1$

$$df_w = 16 - 1 - 1 - 1 = 13$$

$$df_w = 16 - (2 - 1) - (2 - 1) - 1 = 13$$

$$df_w = 16 - 2 - 2 + 1 = 13$$

Putting the pieces together

	SS	Df	MS	F
Grade	SS_{grade} (same as $E_R - E_F$ if there was one factor!)	$(G - 1)$	SS_{grade} / df_{grade}	MS_{grade} / MS_W
Doublet	$SS_{doublet}$ (same as $E_R - E_F$ if there was one factor!)	$(D - 1)$	$SS_{doublet} / df_{doublet}$	$MS_{doublet} / MS_W$
Error (residual)	SS_W	$N - 1 - (G - 1) - (D - 1)$ = $N - G - D + 1$	SS_W / df_W	

F Table

	SS	Df	MS	F
Grade	SS_{grade} **138.0625** (same as $E_R - E_F$ if there was one factor!)	df_g **1**	SS_{grade} / df_{grade}	MS_{grade} / MS_W
Doublet	$SS_{doublet}$ **3.0625** (same as $E_R - E_F$ if there was one factor!)	df_d **1**	$SS_{doublet}$ / $df_{doublet}$	$MS_{doublet}$ / MS_W
Error (residual)	SS_W **9.75**	df_W **13**	SS_W / df_W	

F Table

	SS	Df	MS	F
Grade	SS_{grade} **138.0625** (same as $E_R - E_F$ if there was one factor!)	df_g **1**	SS_{grade} / df_{grade} = 138.0625	MS_{grade} / MS_W = 184.08
Doublet	$SS_{doublet}$ **3.0625** (same as $E_R - E_F$ if there was one factor!)	df_d **1**	$SS_{doublet} / df_{doublet}$ = 3.0625	$MS_{doublet} / MS_W$ = 4.08
Error (residual)	SS_W **9.75**	df_W **13**	SS_W / df_W = .75	

Without stars

1. We know that when F > 1, effect is significant (generally)

2. Use a critical value to determine whether to reject the null.

```
(F_crit = qf(.05, df1 = 1, df2 = 13, lower.tail = F))
```

```
## [1] 4.667193
```

$F_{grade} = 184.08$ is more extreme than our $F_{crit} = 4.67$ so we reject our $H_{0.1}$

$F_{dbl} = 4.08$ is *not* more extreme than our $F_{crit} = 4.67$ so we fail to reject our $H_{0.2}$

Is it always this painful?

No, because we have R!

```
summary(aov(correct ~ grade + doublet, data))
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)    
## grade        1 138.06  138.06 182.911 4.92e-09 ***
## doublet      1   3.06    3.06   4.057   0.0652 .  
## Residuals   13   9.81    0.75                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Next time

- What if we expect performance to depend on factors?
- We would add an *interaction* to our ANOVA: grade x doublet
- We will translate all of this into formal equations (eek!)