

Multilevel Modeling

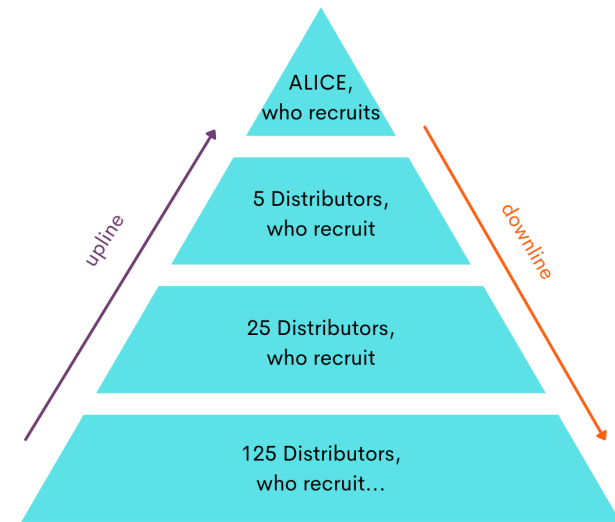
An Introduction

Roadmap

- Why MLM?
- Some technical features
- What are things to consider when you run an MLM?
- Running an MLM in R
- Resources

Many words for the same thing

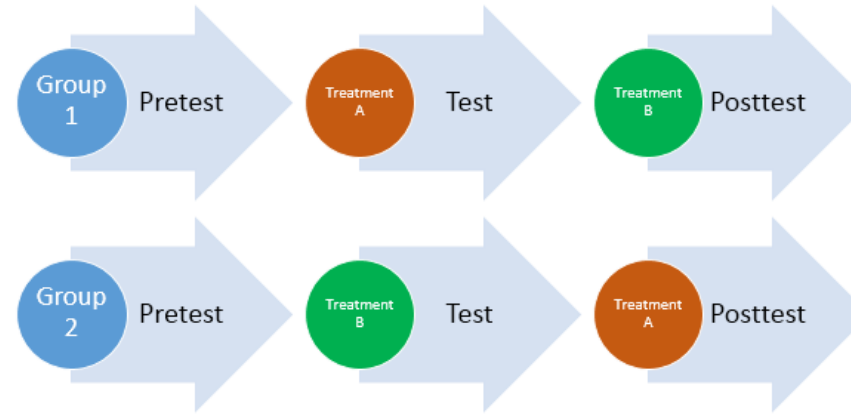
- Multilevel Modeling (not Multilevel Marketing)
- Hierarchical Linear Modeling
- Mixed Effects Modeling
- ...



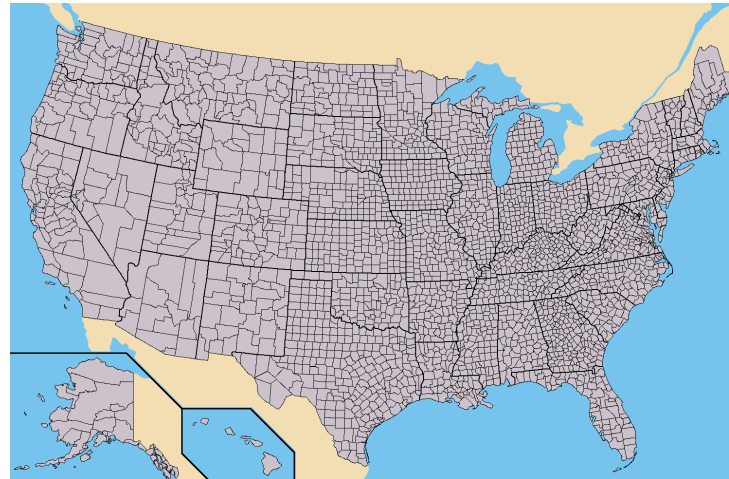
MLM Distribution Structure

Why MLM?

MLM helps when assumptions of independence are violated



Independence



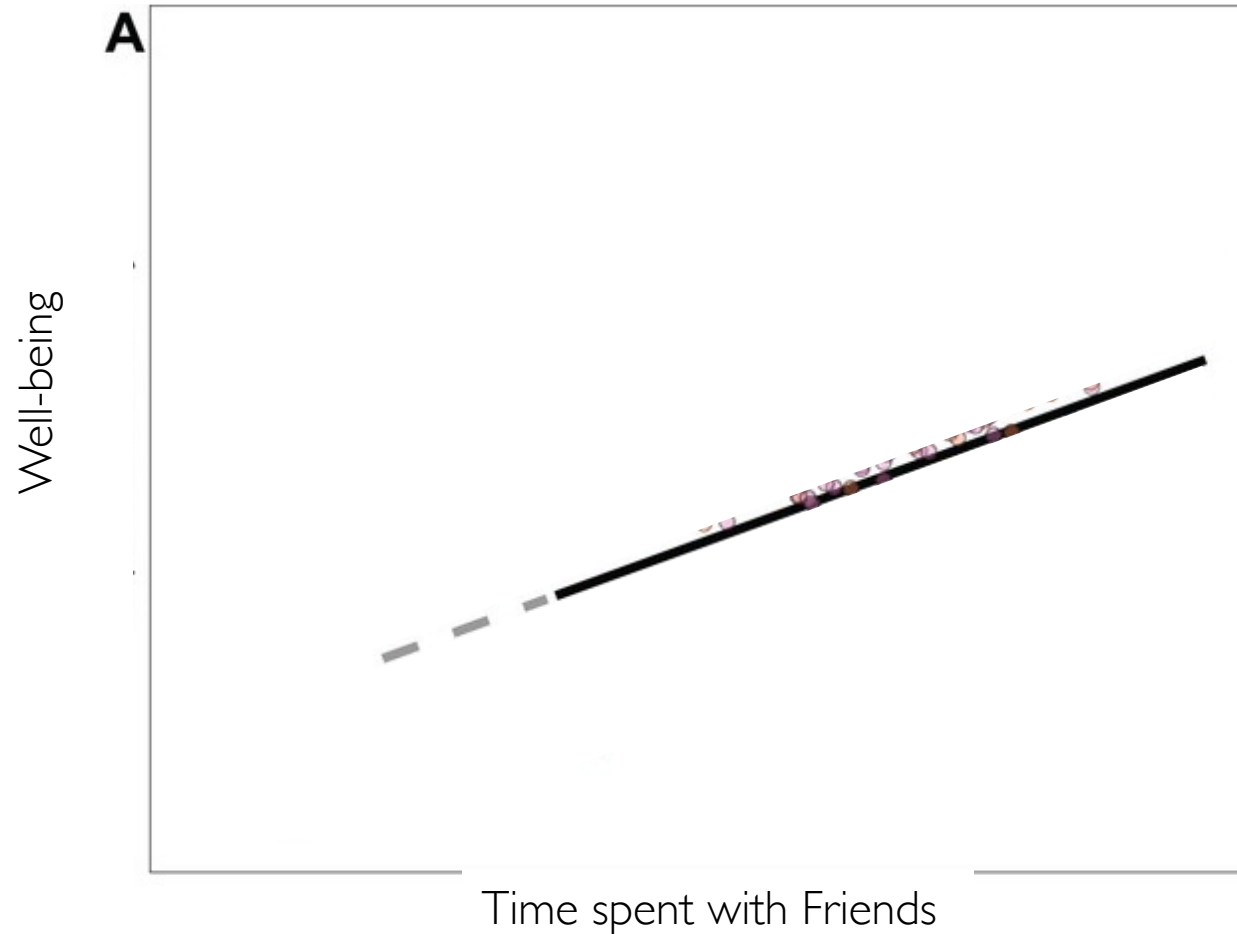
Individual differences matter

(Even if you are not a personality psychologist)

- Example:
 - Does spending more time with friends predict higher well-being?
 - 6 people in our study (terribly under-powered)
 - We ask them once every day for a week a) how much time they spent with their friends and b) how high they rate their well-being that day

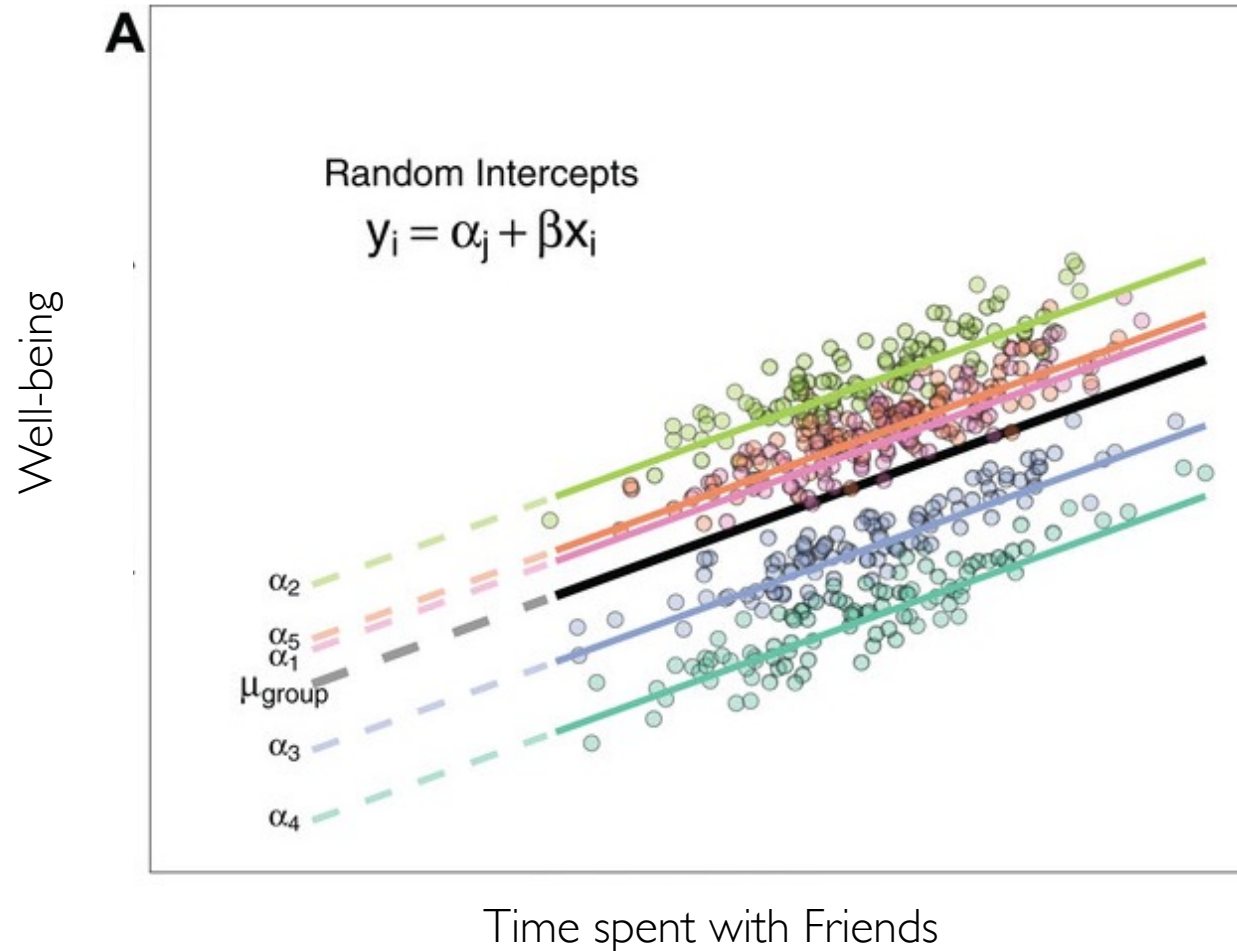
Individual differences matter

(Even if you are not a personality psychologist)



Individual differences matter

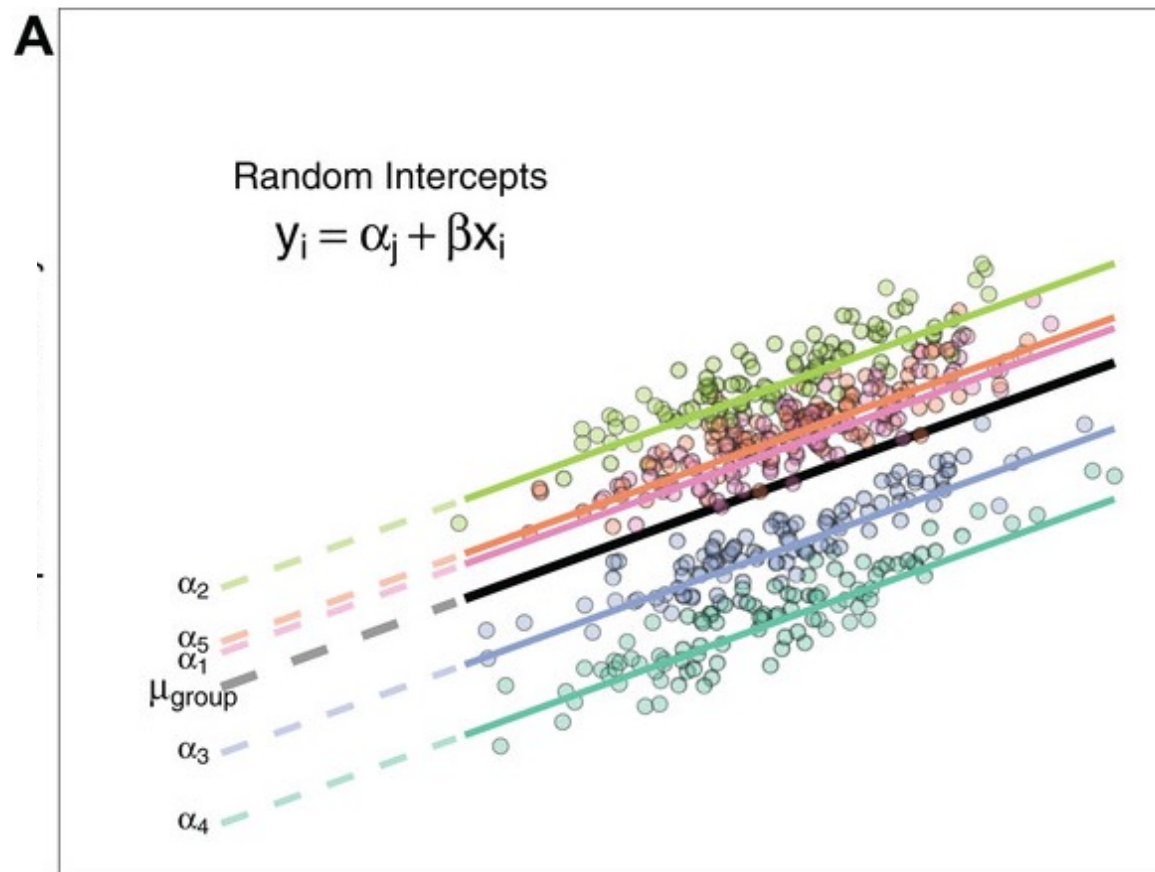
(Even if you are not a personality psychologist)



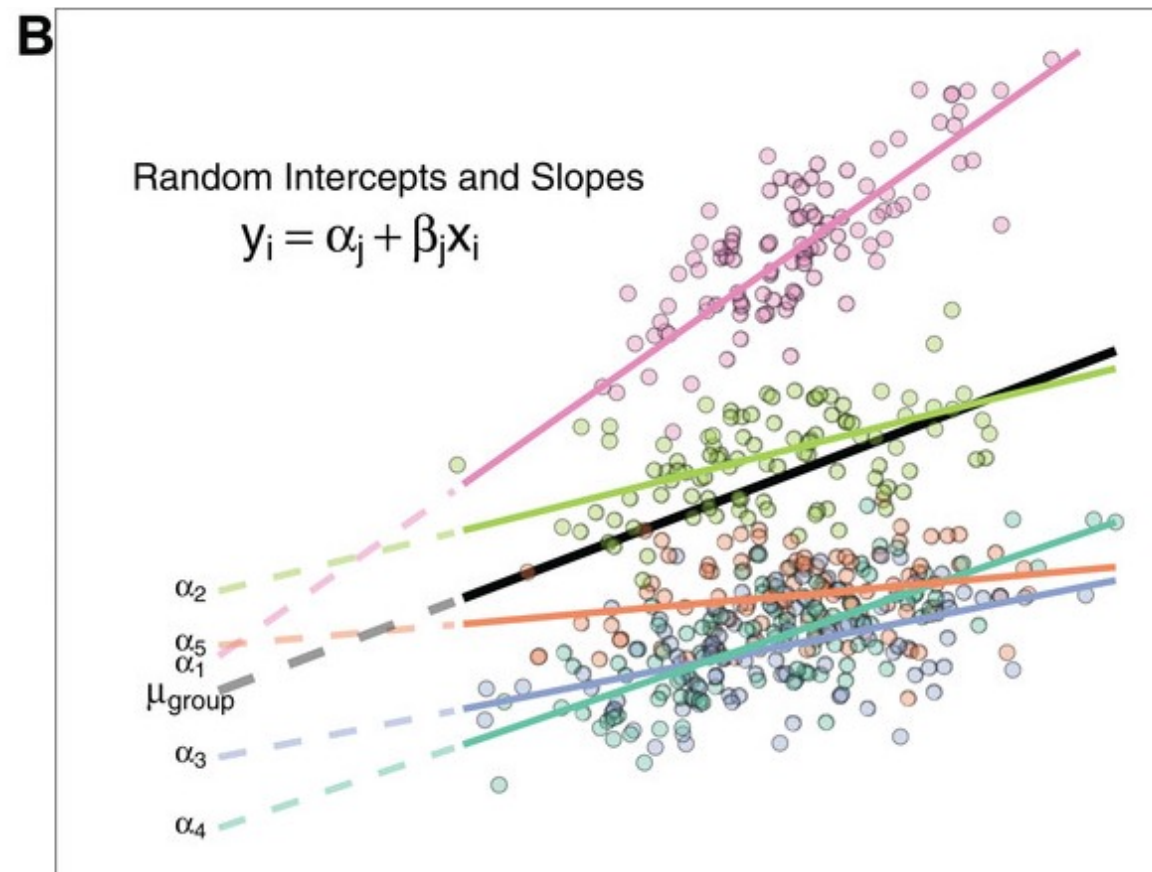
Individual differences matter

(Even if you are not a personality psychologist)

Well-being

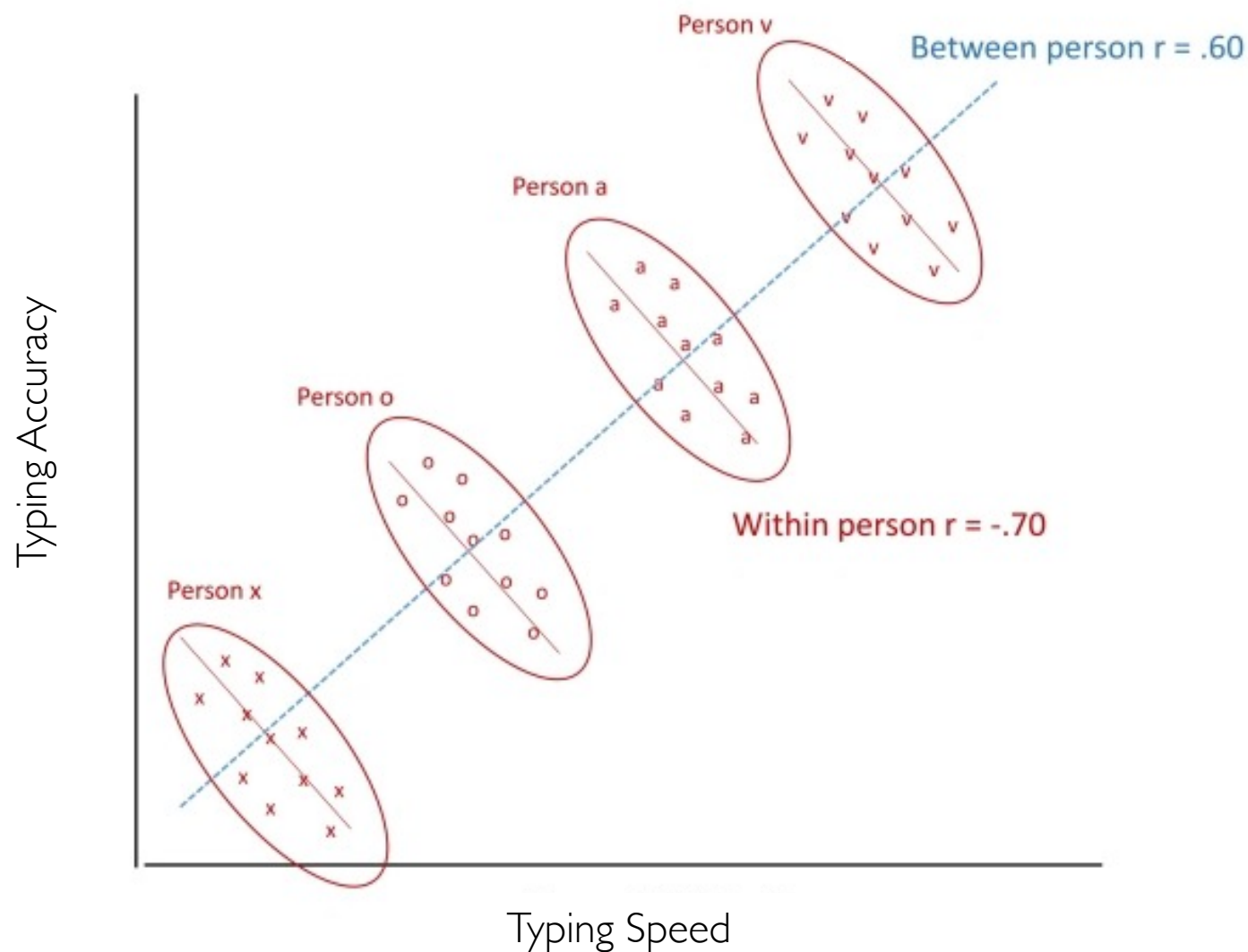


Time spent with Friends



Time spent with Friends

Between- and within-person effects matter



MLM beats RM-ANOVA

- Simultaneous estimation of within and between effects
- RM ANOVA uses listwise deletion
- Participants or items with more missing cases have weaker influences on parameter estimates (i.e., the parameter estimates are precision weighted), and extreme values are “shrunk” toward the mean
- Categorical and Continuous predictors possible in MLM
- Can be adapted to continuous and categorical outcomes (GLMER)
 - dependent variable is continuous and the independent variables are categorical in ANOVA

Technical Aspects

Maximum Likelihood

- No ordinary least squares for MLM
- Instead, we are using Maximum Likelihood Estimation

Pooling

- Grouping variable is treated as coming from a population. All groups are alike (because they are from the same population), but different in their own way.
- Because of this it is helpful to use information from other groups to help make predictions.
- MLM is running a regression for each group. We want to pool as this leads to better predictions as we are not overfitting our data!

Important Considerations

Centering

- Centering is very important for the interpretation of variables in MLM
- What does 0 mean?
- What do deviations from 0 mean? (important for within-person effects)

Centering

- Grand-mean centering
 - Similar to what we have been doing in regression (subtracting the overall mean from each value)

Participant	Happiness _{ji}	
Shelly	2	-1
Shelly	3	0
Messi	3	0
Messi	4	1

Grand Mean of Happiness = 3

Centering

- Group-mean centering (or Person-mean centering)
 - New! (subtracting the person (or group) mean from each value)

Participant	Happiness _{ji}	
Shelly	2	-0.5
Shelly	3	0.5
Messi	3	-0.5
Messi	4	0.5

Shelly Mean of Happiness = 2.5

Messi Mean of Happiness = 3.5

Centering

- Group-mean centering (or Person-mean centering)
 - New! (subtracting each value from the person (or group) mean)

Participant	Happiness _{ji}	
Shelly	2	-0.5
Shelly	3	0.5
Messi	3	-0.5
Messi	4	0.5



When Shelly feels -.5 less happy than she normally feels



When Messi feels -.5 less happy than he normally feels

Shelly Mean of Happiness = 2.5

Messi Mean of Happiness = 3.5

Power

Choose a model (more information in panel About the Method):

Model 2: Effect of a level-2 continuous predictor on the mean level ▼

Model 2: Effect of a level-2 continuous predictor on the mean level

Level 1: $Y_{it} = \gamma_{0i} + \epsilon_{it}$

Level 2: $\gamma_{0i} = \beta_{00} + \beta_{01}W_i + \nu_{0i}$

W_i is the level-2 variable which is normally distributed $N(\mu_W^2, \sigma_W^2)$

AR(1) errors ϵ_{it} with autocorrelation ρ_ϵ and variance σ_ϵ^2

Number of participants: introduce an increasing sequence of positive integers (comma-separated).

Number of participants

15,30,45,60,80,100

Number of time points

70

PowerAnalysisIL Shiny App

Lafit, Ginette, Janne Adolf, Egon Dejonckheere, Inez Myin-Germeys, Wolfgang Viechtbauer, & Eva Ceulemans. (in press). Selection of the Number of Participants in Intensive Longitudinal Studies: A User-Friendly Shiny App and Tutorial for Performing Power Analysis in Multilevel Regression Models That Account for Temporal Dependencies. *Advances in Methods and Practices in Psychological Science*.

Fixed intercept: β_{00}

43.01

Effect of the level-2 continuous variable on the intercept: β_{01}

1.50

Standard deviation of level-1 errors: σ_ϵ

12.62

Autocorrelation of level-1 errors: ρ_ϵ

0.46

Standard deviation of random intercept: σ_{ν_0}

12.90

Mean of level-2 variable W:

15.70

Standard deviation of level-2 variable W:

5.00

☒ Center the level-2 variable W

☒ Estimate AR(1) correlated errors ϵ_{it}

Type I error: α

0.05

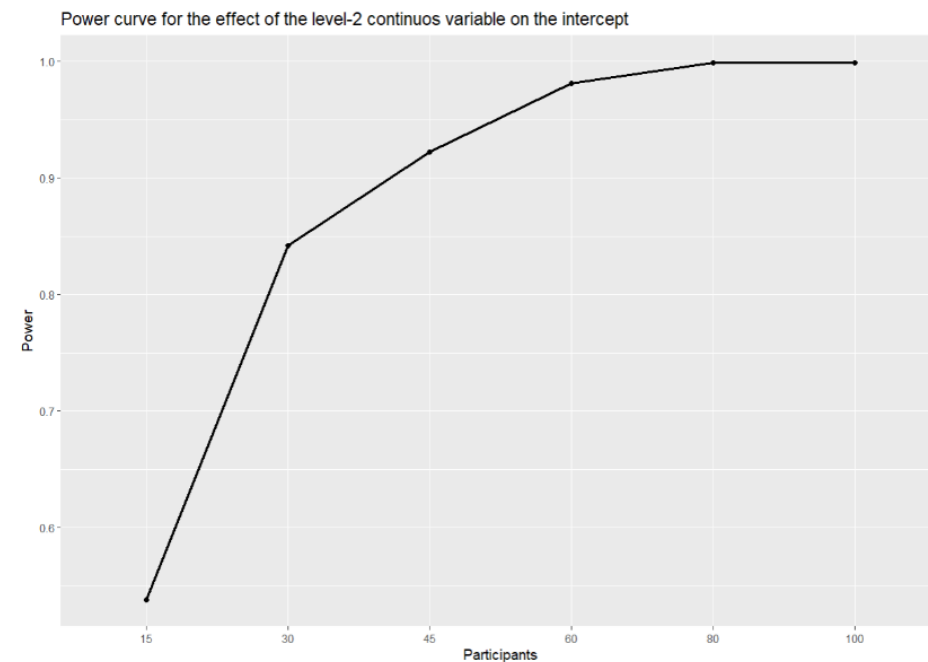
Monte Carlo Replicates

1000

Choose the method to fit linear mixed-effects model

Maximizing the restricted log-likelihood ▼

Estimate Computational Time Compute Power Reset Page



Preregistration

- Be explicit about different parts of your model (researcher degrees of freedom!)
 - Random effects included? Which ones?
 - What happens if the models do not converge?
 - How are you centering your variables?

Reporting Standards

- Subfield dependent
 - Focus on fixed effects (like for a regression)
 - Highlight random effects
 - Random effect correlations
 - Report model fit - model comparisons

Running MLMs in R

Our data

```
fPID esm_happy interaction_pleasure
<dbl> <dbl> <dbl>
302 2 2
302 1 5
302 5 5
302 5 5
302 5 5
302 5 5
302 5 5
302 1 5
302 5 5
302 5 5
302 3 5
.. with 11,801 more rows
```

60 times for each person

Centering - Getting Person Means

```
df<-data%>%group_by(fPID)%>%summarise(  
  interaction_pleasure_pm=mean(interaction_pleasure,na.rm=T))  
df$interaction_pleasure_pm_c<-df$interaction_pleasure_pm - mean(df$interaction_pleasure_pm,na.rm=T)
```

fPID	interaction_pleasure_pm	interaction_pleasure_pm_c
<dbl>	<dbl>	<dbl>
302	4.6	1.29
303	3.55	0.238
305	2.95	-0.359
306	3.55	0.238
308	3.18	-0.137
309	3.4	0.0866
311	3.32	0.00239
312	3.67	0.353
314	2.96	-0.357
315	2.98	-0.338

Centering - Getting Person-Mean-Centered Variables

fPID	esm_happy	interaction_pleasure	interaction_pleasure_pm	interaction_pleasure_pm_c
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
302	2	2	4.6	1.29
302	1	5	4.6	1.29
302	5	5	4.6	1.29
302	5	5	4.6	1.29
302	5	5	4.6	1.29
302	5	5	4.6	1.29
302	1	5	4.6	1.29
302	5	5	4.6	1.29
302	5	5	4.6	1.29
302	3	5	4.6	1.29

```
data$interaction_pleasure_pmc<-data$interaction_pleasure-data$interaction_pleasure_pm
```

fPID	esm_happy	interaction_pleasure	interaction_pleasure_pm	interaction_pleasure_pm_c	interaction_pleasure_pmc
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
302	2	2	4.6	1.29	-2.6
302	1	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	1	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	5	5	4.6	1.29	0.400
302	3	5	4.6	1.29	0.400

Running the MLM with a Random Intercept

```
model.int<-lmer(esm_happy~1+interaction_pleasure_pm_c+interaction_pleasure_pmc+(1|fPID),data=data)
```

Running the MLM with a Random Intercept

Fixed effects:

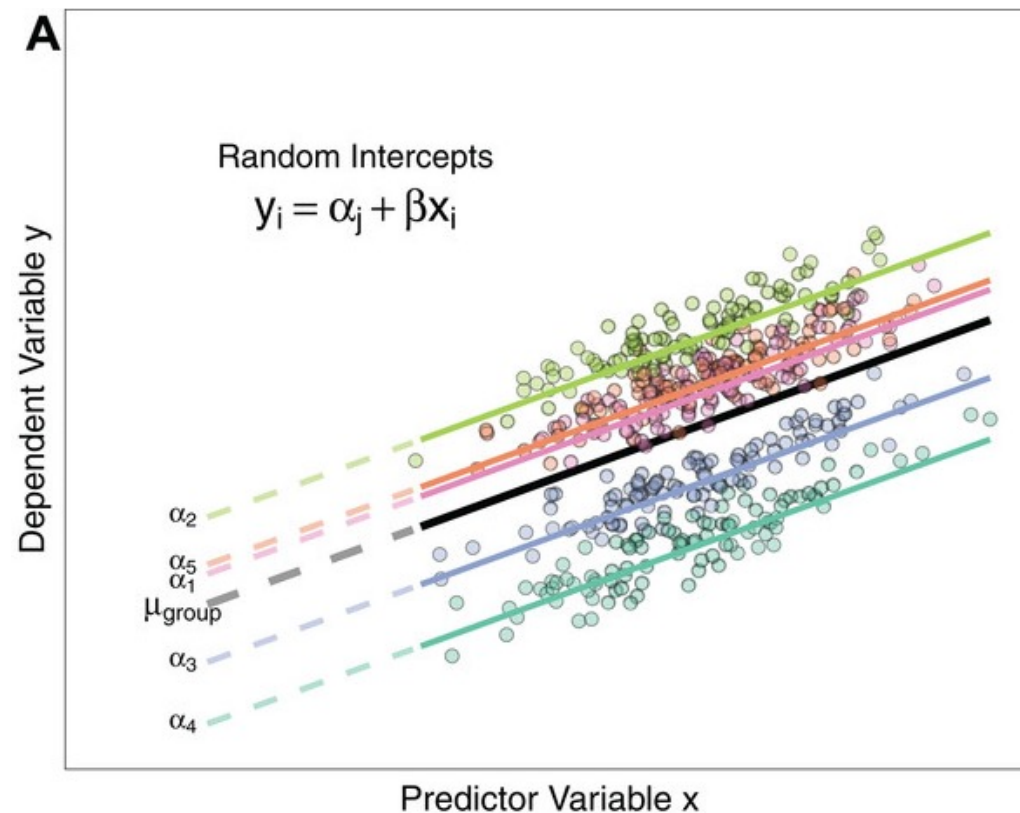
	Estimate	Std. Error	df	t value	Pr(> t)	
(Intercept)	4.013923	0.058997	278.828284	68.04	<0.0000000000000002	***
interaction_pleasure_pm_c	1.133289	0.079146	285.944382	14.32	<0.0000000000000002	***
interaction_pleasure_pmc	0.368370	0.009438	11513.711258	39.03	<0.0000000000000002	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Running the MLM with a Random Intercept

```
Random effects:
Groups   Name      Variance Std. Dev.
fPID     (Intercept) 0.9621  0.9809
Residual                1.2532  1.1195
Number of obs: 11811, groups: fPID, 290
```

An individual whose Intercept is one SD above the Mean would be almost 1 unit higher in happiness



Running the MLM with Random Intercept and Slope

```
model.int<-lmer(esm_happy~1+interaction_pleasure_pm_c+interaction_pleasure_pmc+(1|fPID),data=data)
```

```
model.intslope<-lmer(esm_happy~1+interaction_pleasure_pm_c+interaction_pleasure_pmc+(1+interaction_pleasure_pmc|fPID),data=data)
```

Running the MLM with Random Intercept and Slope

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)	
(Intercept)	4.01416	0.05901	278.79300	68.02	<0.00000000000000002	***
interaction_pleasure_pm_c	1.15206	0.07803	287.95575	14.77	<0.00000000000000002	***
interaction_pleasure_pmc	0.36082	0.01560	277.95534	23.12	<0.00000000000000002	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Running the MLM with Random Intercept and Slope

Random effects:

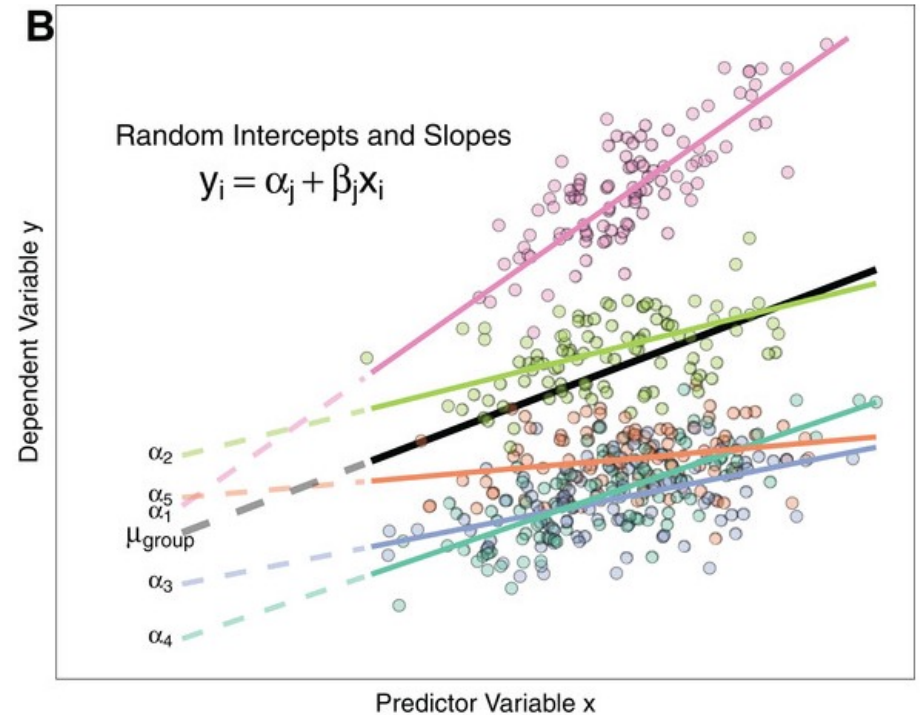
Groups	Name	Variance	Std.Dev.	Corr
fPID	(Intercept)	0.96441	0.9820	
	interaction_pleasure_pmc	0.03901	0.1975	-0.25
Residual		1.20304	1.0968	

Number of obs: 11811, groups: fPID, 290

An individual whose Intercept is one SD above the Mean would be almost 1 unit higher in happiness

AND

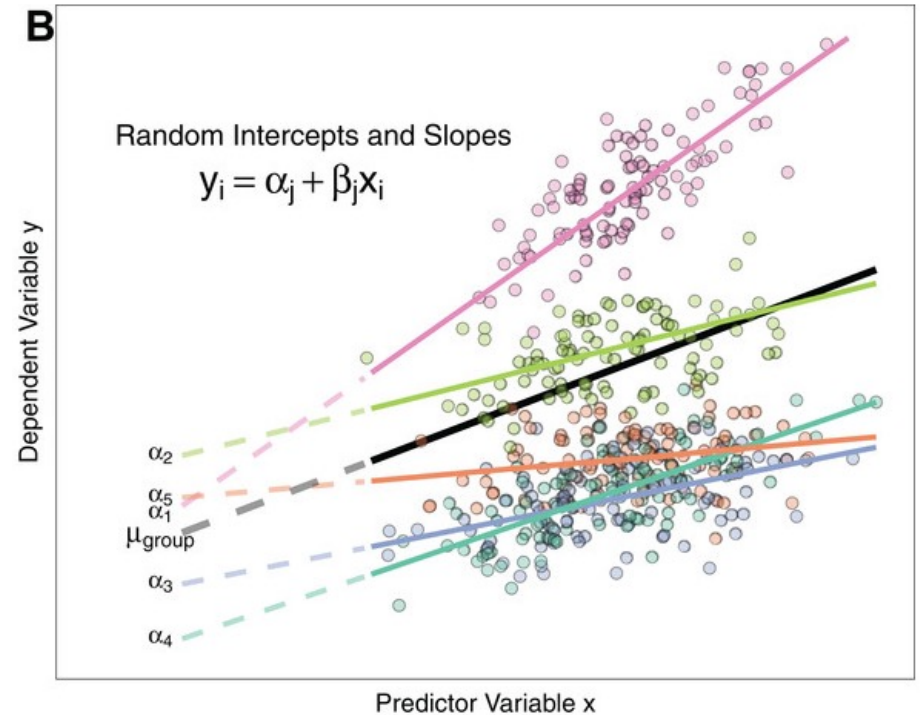
An individual whose Slope is one SD above the Mean would have a slope of $\sim .56$



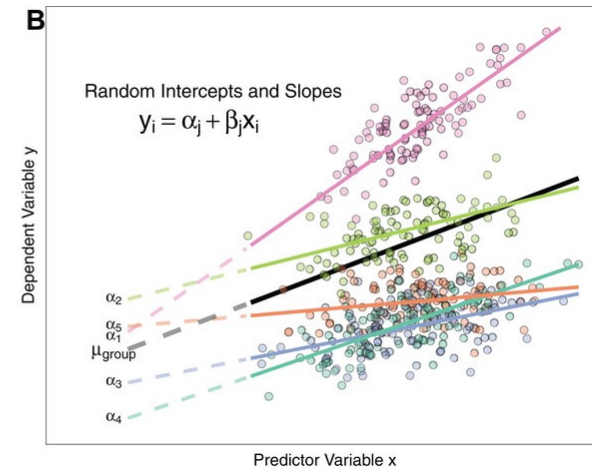
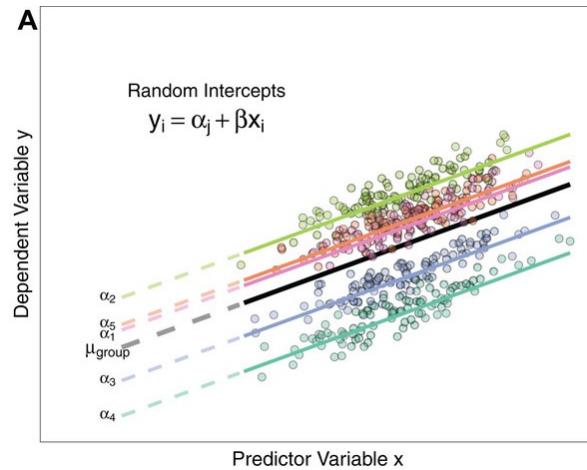
Running the MLM with Random Intercept and Slope

```
Random effects:
Groups      Name                Variance Std.Dev. Corr
fPID        (Intercept)            0.96441  0.9820
            interaction_pleasure_pmc 0.03901  0.1975 -0.25
Residual                                1.20304  1.0968
Number of obs: 11811, groups: fPID, 290
```

People who have higher mean levels of happiness when they experience average pleasure during a social interaction show a lower association between interaction pleasure and happiness



What do the individual estimates look like for these models?



	(Intercept)	interaction_pleasure_pm_c	interaction_pleasure_pmc
302	2.456996	1.133289	0.3683698
303	3.280112	1.133289	0.3683698
305	3.398064	1.133289	0.3683698
306	3.419967	1.133289	0.3683698
308	4.992923	1.133289	0.3683698
309	4.038525	1.133289	0.3683698

	(Intercept)	interaction_pleasure_pm_c	interaction_pleasure_pmc
302	2.429388	1.152062	0.4824074059
303	3.271820	1.152062	0.4472887873
305	3.396268	1.152062	0.4853060932
306	3.411305	1.152062	0.5035916966
308	4.990474	1.152062	0.5041272775
309	4.037405	1.152062	0.3390767027

Should we include a random slope?

- Let's compare the models we just ran and see which one provides more information!

```
> anova(model.int,model.intslope)
refitting model(s) with ML (instead of REML)
Data: data
Models:
model.int: esm_happy ~ 1 + interaction_pleasure_pm_c + interaction_pleasure_pmc + (1 | fPID)
model.intslope: esm_happy ~ 1 + interaction_pleasure_pm_c + interaction_pleasure_pmc + (1 + interaction_pleasure_pmc | fPID)
      npar   AIC    BIC logLik deviance Chisq Df          Pr(>Chisq)
model.int      5 37166 37202 -18578    37156
model.intslope  7 36943 36994 -18464    36929 226.8  2 < 0.00000000000000022 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Yes! Including the random slope results in a better model (significant Chi-Square test, lower BIC)

Convergence Issues

- Estimation issues can occur when we are trying the model to do a lot with little information (e.g., too many random slopes, too little variance)
- Adjusting the optimizer can help

Other ways of running MLMs in R

- Glmer
 - For non-continuous outcomes (e.g., binary outcomes)
- Nlme
 - Precursor to lmer, can do some things lmer cannot do like modifying covariance structure
- Brms (Bayesian MLM)
 - Can predict variance parameter and run multivariate models more easily

Example: MLM in Experimental Settings

Unaggregated data set				Aggregated data set		
PID	modality	stim	RT	PID	modality	RT
301	Audio-only	gown	1024	301	Audio-only	1027
301	Audio-only	might	838	301	Audiovisual	1002
301	Audio-only	fern	1060	302	Audio-only	1047
301	Audio-only	vane	882	302	Audiovisual	1043
301	Audio-only	pup	971	303	Audio-only	883
301	Audio-only	rise	1064	303	Audiovisual	938

Note: PID = participant identification number; stim = stimulus; RT = response time.

Example: MLM in Experimental Settings

```
rt_full.mod <- lmer(RT ~ 1 + modality + (1 + modality|PID) +  
(1 + modality|stim), data = rt_data)
```

Response time predicted by modality (audiovisual, audio only). Random intercepts and slopes added for both participants and stimulus

```
> summary(rt_full.mod)
```

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	1044.14	23.36	52.14	44.704
modality	83.18	12.58	52.10	6.615

Example: MLM in Experimental Settings

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
stim	(Intercept)	303.9	17.43	
	modality	216.6	14.72	0.16
PID	(Intercept)	28552.7	168.98	
	modality	7709.8	87.81	-0.17
Residual		65258.8	255.46	

- SD for by- item random intercepts (in boldface in the output above) indicates that response times for particular items varied around the average intercept of 1,044 ms by about 17 ms.
- Similarly, the standard deviation for by-participant random slopes (in boldface in the output above) indicates that participants' estimated slopes varied around the average slope of 83 ms by about 88 ms.

Resources

- Violet Brown's amazing paper for an application to experimental settings
 - Brown, V.A. (2021). An introduction to linear mixed-effects modeling in R. *Advances in Methods and Practices in Psychological Science*, 4(1)
- Take classes
 - Hierarchical Linear Modeling (Mike Strube)
 - Applied Longitudinal Data Analysis (Josh Jackson)
 - APA Advanced Training Institute Longitudinal Modeling
 - APA Methods lectures
 - CenterStat (classes for free sometimes)

Resources

- Books
 - Longitudinal Analysis: Modeling within person fluctuation and change - Lesa Hoffman
 - Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling - Tom Snijders and Roel Bosker