Multiple Regression II

```
##
## Call:
## lm(formula = Weight ~ Age + Poverty, data = nhanes)
##
## Residuals:
     Min 10 Median 30 Max
##
## -47.54 -19.90 0.82 16.96 65.53
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 58.8926 7.0353 8.371 8.69e-13 ***
## Age
      0.3537 0.1310 2.699 0.00835 **
## Poverty -0.3501 1.5890 -0.220 0.82612
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.93 on 87 degrees of freedom
  (10 observations deleted due to missingness)
## Multiple R-squared: 0.07949, Adjusted R-squared: 0.05833
## F-statistic: 3.756 on 2 and 87 DF, p-value: 0.02724
```

- Interpret all coefficients
- Interpret all significance tests of coefficients
- Is it a good model?

Last time

- Introduction to multiple regression
- Interpreting coefficient estimates
- Estimating model fit
- Significance tests (omnibus and coefficients)

Today

- Tolerance
- Hierarchical regression/model comparison
- Categorical predictors

The Data

Stress

Support

group*

0.22

0.19

-2.01

0.17

0.30

0.05

```
stress.data = read.csv(here("data/stress.csv"))
library(psych)
describe(stress.data)
```

```
##
                               sd median trimmed
                                                    mad
                                                         min
           vars
                  n
                      mean
                                                                 max
                                                                      rang
              1 118 488.65 295.95 462.50
                                          485.76 372.13 2.00 986.00 984.0
## id
              2 118
                    7.61
                                    7.75
                                            7.67
                                                 2.26 0.70
                                                              14.64
                                                                      13.9
## Anxiety
                             2.49
## Stress
                                                   1.65 0.62
                                                                       9.7
              3 118
                      5.18
                             1.88
                                    5.27
                                            5.17
                                                               10.32
## Support
              4 118
                    8.73
                             3.28
                                    8.52
                                            8.66
                                                 3.16 0.02
                                                               17.34
                                                                      17.3
## group*
              5 118
                                                   0.00 1.00
                                                               2.00
                      1.53
                             0.50
                                    2.00
                                            1.53
                                                                       1.0
##
           kurtosis
                       se
## id
              -1.29 27.24
## Anxiety
               0.28
                    0.23
```

The Model

```
mr.model <- lm(Stress ~ Support + Anxiety, data = stress.data
summary(mr.model)
```

```
##
## Call:
## lm(formula = Stress ~ Support + Anxiety, data = stress.data)
##
## Residuals:
##
      Min
         10 Median 30
                                   Max
## -4.1958 -0.8994 -0.1370 0.9990 3.6995
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -0.31587 0.85596 -0.369 0.712792
## Support
         ## Anxiety 0.25609 0.06740 3.799 0.000234 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.519 on 115 degrees of freedom
## Multiple R-squared: 0.3556, Adjusted R-squared: 0.3444
                                                            6 / 41
## F-s+z+is+ic : 31 73 on 2 and 115 DF n-vz=1 up : 1 062e-11
```

Standard error of regression coefficient

In the case of univariate regression:

$$se_b = rac{s_Y}{s_X} \sqrt{rac{1-r_{xy}^2}{n-2}}$$

In the case of multiple regression:

$$se_b = rac{s_Y}{s_X} \sqrt{rac{1 - R_{Y\hat{Y}}^2}{n - p - 1}} \sqrt{rac{1}{1 - R_{i.jkl...p}^2}}$$

- As N increases...
- As variance explained increases...

Tolerance

$$se_b = rac{s_Y}{s_X} \sqrt{rac{1 - R_{Y\hat{Y}}^2}{n - p - 1}} \sqrt{rac{1}{1 - R_{i.jkl...p}^2}}$$

- ullet what cannot be explained in X_i by other predictors
- large tolerance (little overlap) means standard error will be small.
- what does this mean for including a lot of variables in your model?

Which variables to include

- Your goal should be to match the population model (theoretically)
- Including many variables will increase degrees of freedom and standard errors; in other words, putting too many variables in your model may make it more difficult to find a statistically significant result
- But that's only the case if you add variables unrelated to Y or X; there are some cases in which adding the wrong variables can lead to spurious results.

Methods for entering variables

Simultaneous: Enter all of your IV's in a single model.

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3$$

 The benefits to using this method is that it reduces researcher degrees of freedom, is a more conservative test of any one coefficient, and often the most defensible action (unless you have specific theory guiding a hierarchical approach).

Methods for entering variables

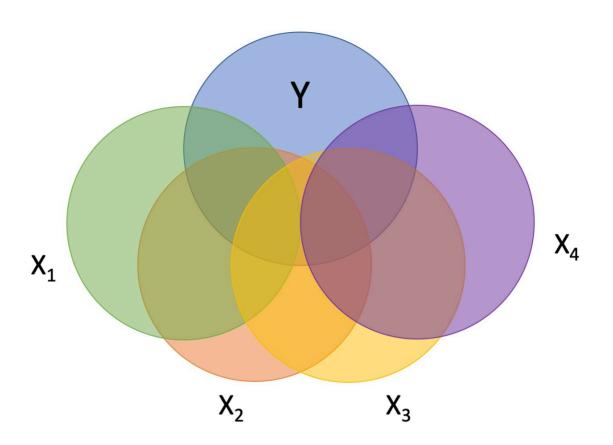
$$Y = b_0 + e$$
 $Y = b_0 + b_1 X_1 + e$
 $Y = b_0 + b_1 X_1 + b_2 X_2 + e$
 $Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + e$

This is known as **hierarchical regression**. Hierarchical regression is a subset of **model comparison** techniques.

Hierarchical regression / Model Comparison

If we're comparing nested models by incrementally adding or subtracting variables, this is known as **hierarchical regression**.

- Multiple models are calculated
- Each predictor (or set of predictors) is assessed in terms of what it adds (in terms of variance explained) at the time it is entered
- Order is dependent on an a priori hypothesis



R-square change

distributed as an F

$$F(p.\,new,N-1-p.\,all) = \ rac{R_{m.2}^2 - R_{m.1}^2}{1 - R_{m.2}^2} (rac{N-1-p.\,all}{p.\,new})$$

can also be written in terms of SSresiduals

```
m.1 <- lm(Stress ~ Support, data = stress.data)
m.2 <- lm(Stress ~ Support + Anxiety, data = stress.data)
anova(m.1, m.2)

## Analysis of Variance Table
##
## Model 1: Stress ~ Support
## Model 2: Stress ~ Support + Anxiety
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 116 298.72
## 2 115 265.41 1 33.314 14.435 0.0002336 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
## Analysis of Variance Table

##
## Response: Stress

## Df Sum Sq Mean Sq F value Pr(>F)

## Support 1 113.151 113.151 49.028 1.807e-10 ***

## Anxiety 1 33.314 33.314 14.435 0.0002336 ***

## Residuals 115 265.407 2.308

## ---

## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Call:
## lm(formula = Stress ~ Support + Anxiety, data = stress.data)
##
## Residuals:
      Min
         10 Median 30 Max
##
## -4.1958 -0.8994 -0.1370 0.9990 3.6995
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -0.31587 0.85596 -0.369 0.712792
         ## Support
## Anxiety 0.25609 0.06740 3.799 0.000234 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.519 on 115 degrees of freedom
## Multiple R-squared: 0.3556, Adjusted R-squared: 0.3444
## F-statistic: 31.73 on 2 and 115 DF, p-value: 1.062e-11
```

```
##
## Call:
## lm(formula = Stress ~ Support, data = stress.data)
##
## Residuals:
      Min 10 Median 30 Max
##
## -3.8215 -1.2145 -0.1796 1.0806 3.4326
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 2.56046 0.42189 6.069 1.66e-08 ***
## Support 0.30006 0.04527 6.629 1.12e-09 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.605 on 116 degrees of freedom
## Multiple R-squared: 0.2747, Adjusted R-squared: 0.2685
## F-statistic: 43.94 on 1 and 116 DF, p-value: 1.12e-09
```

```
m.0 <- lm(Stress ~ 1, data = stress.data)
m.1 <- lm(Stress ~ Support, data = stress.data)
anova(m.0, m.1)

## Analysis of Variance Table
##
## Model 1: Stress ~ 1
## Model 2: Stress ~ Support
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 117 411.87
## 2 116 298.72 1 113.15 43.939 1.12e-09 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Partitioning the variance

 It doesn't make sense to ask how much variance a variable explains (unless you qualify the association)

$$R_{Y.1234...p}^2 = r_{Y1}^2 + r_{Y(2.1)}^2 + r_{Y(3.21)}^2 + r_{Y(4.321)}^2 + \dots$$

In other words: order matters!

Categorical predictors

One of the benefits of using regression (instead of partial correlations) is that it can handle both continuous and categorical predictors and allows for using both in the same model.

Categorical predictors with more than two levels are broken up into several smaller variables. In doing so, we take variables that don't have any inherent numerical value to them (i.e., nominal and ordinal variables) and ascribe meaningful numbers that allow for us to calculate meaningful statistics.

Dummy coding

One group is selected to be a **reference** group. K-1 dummy coded variables are created; for each new dummy code variable, one of the non-reference groups is assigned 1; all other groups are assigned 0.

Occupation	D1	D2
Engineer	0	0
Teacher	1	0
Doctor	0	1

	Person	erson Occupation		D2
Billy		Engineer	0	0
	Susan	Teacher	1	0
	Michael	Teacher	1	0
	Molly	Engineer	0	0
	Katie	Doctor	0	23/4

The dummy codes are entered as IV's

Solomon's Paradox

Describes the tendency for people to reason more wisely about other people's problems compared to their own. Maybe people tend to view other people's problems from a more psychologically distant perspective, whereas they view their own problems from a psychologically immersed perspective. To test this possibility, researchers asked romantically-involved participants to think about a situation in which their partner cheated on them (self condition) or a friend's partner cheated on their friend (other condition). Participants were also instructed to take a first-person perspective (immersed condition) by using pronouns such as I and me, or a third-person perspective (distanced condition) by using pronouns such as he and her.

Grossmann, I., & Kross, E. (2014). Exploring Solomon's paradox: Self-distancing eliminates self-other asymmetry in wise reasoning about close relationships in younger and older adults. *Psychological Science*, *25*, 1571-1580.

```
psych::describe(solomon[,c("ID", "CONDITION", "WISDOM")], fas
```

```
sd median
##
                                       min
                                                   range skew kurtos
                     mean
                                              max
           vars
## ID
              1 120 64.46 40.98 61.50 1.00 168.00 167.00
                                                         0.49
                                                                 -0.
## CONDITION
              2 120 2.46 1.12 3.00 1.00
                                           4.00 3.00 0.00
                                                                 -1.
## WISDOM
              3 115 0.01 0.99 0.16 -2.52 1.79 4.31 -0.74
                                                                 -0.
```

ID	CONDITION	WISDOM
1	3	-0.2758939
6	4	0.4294921
8	4	-0.0278587
9	4	0.5327150
10	2	0.6229979
12	2	-1.9957813

ID	CONDITION	WISDOM	dummy_2	dummy_3	dummy_4
1	3	-0.2758939	0	1	0
6	4	0.4294921	0	0	1
8	4	-0.0278587	0	0	1
9	4	0.5327150	0	0	1
10	2	0.6229979	1	0	0
12	2	-1.9957813	1	0	0
14	3	-1.1514699	0	1	0
18	2	-0.6912011	1	0	0
21	2	0.0053117	1	0	0
25	4	0.2863499	0	0	1
26	4	-1.8217968	0	0	1
30	1	-1.2823302	0	0	0

```
mod.1 = lm(WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solom
summary(mod.1)
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
      Min
              10 Median 30
##
                                  Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.5593
                     0.1686 -3.317 0.001232 **
## dummy_2
         ## dummy_3 0.7541 0.2348 3.211 0.001729 **
## dummy_4 0.8938 0.2524 3.541 0.000583 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
    (5 observations deleted due to missingness)
##
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

```
summary(lm(WISDOM ~ CONDITION, data = solomon))
```

```
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
##
      Min
               10 Median
                               30
                                      Max
## -2.64827 -0.55096 0.09494 0.72958 2.20076
##
## Coefficients:
##
             Estimate Std. Error t value Pr(>|t|)
## CONDITION 0.28621 0.07956 3.598 0.000478 ***
## ---
  Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.943 on 113 degrees of freedom
    (5 observations deleted due to missingness)
##
## Multiple R-squared: 0.1028, Adjusted R-squared: 0.09482
## F-statistic: 12.94 on 1 and 113 DF, p-value: 0.000478
```

```
solomon$CONDITION <- factor(solomon$CONDITION)</pre>
summary(lm(WISDOM ~ CONDITION, data = solomon))
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
      Min
               10 Median 30
##
                                     Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.5593 0.1686 -3.317 0.001232 **
## CONDITION2 0.6814 0.2497 2.729 0.007390 **
## CONDITION3 0.7541 0.2348 3.211 0.001729 **
## CONDITION4 0.8938 0.2524 3.541 0.000583 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
##
    (5 observations deleted due to missingness)
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

Interpreting coefficients

When working with dummy codes, the intercept can be interpreted as the mean of the reference group.

$$egin{aligned} \hat{Y} &= b_0 + b_1 D_2 + b_2 D_3 + b_3 D_2 \ \hat{Y} &= b_0 + b_1 (0) + b_2 (0) + b_3 (0) \ \hat{Y} &= b_0 \ \hat{Y} &= ar{Y}_{ ext{Reference}} \end{aligned}$$

What do each of the slope coefficients mean?

From this equation, we can get the mean of every single group.

```
newdata = data.frame(dummy_2 = c(0,1,0,0),
                      dummy 3 = c(0,0,1,0),
                      dummy_4 = c(0,0,0,1)
predict(mod.1, newdata = newdata, se.fit = T)
## $fit
##
## -0.5593042 0.1220847 0.1948435 0.3344884
##
## $se.fit
##
## 0.1686358 0.1841382 0.1634457 0.1877848
##
## $df
## [1] 111
##
## $residual.scale
## [1] 0.9389242
```

From this equation, we can get the mean of every single group.

```
solomon %>%
  mutate_at("CONDITION", ~as.factor(.)) %>%
  group_by(CONDITION) %>%
  drop_na() %>%
  summarize(meanWisdom = mean(WISDOM))
```

And the test of the coefficient represents the significance test of each group to the reference. This is an independent-samples *t*-test.

The test of the intercept is the one-sample *t*-test comparing the intercept to 0.

```
summary(mod.1)$coef
```

```
## (Intercept) -0.5593042 0.1686358 -3.316641 0.0012319438

## dummy_2 0.6813889 0.2496896 2.728944 0.0073896074

## dummy_3 0.7541477 0.2348458 3.211247 0.0017291997

## dummy_4 0.8937927 0.2523909 3.541303 0.0005832526
```

What if you wanted to compare groups 2 and 3?

```
solomon = solomon %>%
  mutate(dummy_1 = ifelse(CONDITION == 1, 1, 0),
         dummy_3 = ifelse(CONDITION == 3, 1, 0),
         dummy 4 = ifelse(CONDITION == 4, 1, 0))
mod.2 = lm(WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solom
summary(mod.2)
##
## Call:
## lm(formula = WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##
      Min
               10 Median 30
                                    Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.12208 0.18414 0.663 0.50870
## dummy_1 -0.68139 0.24969 -2.729 0.00739 **
## dummy_3 0.07276 0.24621 0.296 0.76816
## dummy_4 0.21240 0.26300 0.808 0.42104
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
                                                              34 / 41
```

(5 observations deleted due to missingness)

##

We have to consider the correlations between the IVs, as highly correlated variables make it more difficult to detect significance of a particular X. One useful way to conceptualize the relationship between any two variables is "Does knowing someone's score on X_1 affect my guess for their score on X_2 ?"

Are dummy codes associated with a categorical predictor correlated or uncorrelated?

dummy_1 -0.3239068 -0.3872466 -0.3318469 1.0000000

What do you think of this model?

```
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
##
      Min
          10 Median 30
                                    Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -0.5593 0.1686 -3.317 0.001232 **
## CONDITION2 0.6814 0.2497 2.729 0.007390 **
## CONDITION3 0.7541 0.2348 3.211 0.001729 **
## CONDITION4 0.8938 0.2524 3.541 0.000583 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
    (5 observations deleted due to missingness)
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_2 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##
      Min
          10 Median 30 Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.5593 0.1686 -3.317 0.001232 **
## dummy_2 0.6814 0.2497 2.729 0.007390 **
## dummy_3 0.7541 0.2348 3.211 0.001729 **
## dummy_4 0.8938 0.2524 3.541 0.000583 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
    (5 observations deleted due to missingness)
##
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

```
##
## Call:
## lm(formula = WISDOM ~ dummy_1 + dummy_3 + dummy_4, data = solomon)
##
## Residuals:
##
      Min
          10 Median 30 Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.12208 0.18414 0.663 0.50870
## dummy_1 -0.68139 0.24969 -2.729 0.00739 **
## dummy_3 0.07276 0.24621 0.296 0.76816
## dummy_4 0.21240 0.26300 0.808 0.42104
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
    (5 observations deleted due to missingness)
##
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

```
##
## Call:
## lm(formula = WISDOM ~ CONDITION, data = solomon)
##
## Residuals:
##
      Min
          10 Median 30
                                    Max
## -2.6809 -0.4209 0.0473 0.6694 2.3499
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.5593 0.1686 -3.317 0.001232 **
## CONDITION2 0.6814 0.2497 2.729 0.007390 **
## CONDITION3 0.7541 0.2348 3.211 0.001729 **
## CONDITION4 0.8938 0.2524 3.541 0.000583 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9389 on 111 degrees of freedom
    (5 observations deleted due to missingness)
##
## Multiple R-squared: 0.1262, Adjusted R-squared: 0.1026
## F-statistic: 5.343 on 3 and 111 DF, p-value: 0.001783
```

Next time...

Exam 1!